PAMM · Proc. Appl. Math. Mech. **7**, 4140005–4140006 (2007) / **DOI** 10.1002/pamm.200700398

Subcritical instability of liquid metal channel flow in the presence of a spanwise magnetic field

Thomas Boeck * [1], **Dmitry Krasnov**[1], **Maurice Rossi**[2], and **Oleg Zikanov**[3]

[1] Department of Mechanical Engineering, Ilmenau University of Technology, P.O. Box 100565, 98684 Ilmenau, Germany.

[2] Institut Jean Le Rond D'Alembert, Université Pierre et Marie Curie, 4 place Jussieu, F-75252 Paris Cedex 05, France.

[3] Department of Mechanical Engineering, University of Michigan - Dearborn, Dearborn MI 48128-1491, USA.

The linear and nonlinear evolution of perturbations is investigated in a magnetohydrodynamic channel flow with electrically insulating walls. The applied magnetic field is parallel to the walls and orthogonal to the stream. Linear optimal perturbations and their maximum amplifications over finite time intervals are computed using a scheme based on the direct and adjoint governing equations. It is shown that dominant optimal perturbations are no more the classical streamwise modes and how the flow is two-dimenzionalized for high enough Hartmann numbers. For fixed Reynolds and Hartmann numbers, direct numerical simulations are applied to investigate how the transition to turbulence is affected by the magnetic field.

1 Introduction

We consider pressure-driven flow of an incompressible, electrically conducting fluid in an infinite plane channel between insulating walls with a magnetic field B in the spanwise direction. Such flows can be found in numerous metallurgical and materials processing applications, e.g. in electromagnetic flow control in continuous steel casting and in growth of large silicon crystals. Another area of applications is the liquid metal cooling blankets of breeder type for fusion reactors. Our numerical study is performed within the quasi-static approximation, whereby the governing equations reduce to the Navier-Stokes system with the additional Lorentz force. The length and velocity scales are the laminar centerline velocity U and the channel half width L. The non-dimensional basic velocity profile is the parabolic Poiseuille profile. Nondimensional parameters are the Reynolds number $Re = UL/\nu$ and the Hartmann number $Ha = BL\sqrt{\sigma/\rho\nu}$, where σ is the electric conductivity and B the magnetic field strength.

A systematic study of the optimal linear perturbations is performed for different Ha at a subcritical Reynolds number $Re = 5000$. We focus on the transition scenario based on the algebraic transient growth of optimal perturbations followed by their three-dimensional breakdown.

2 Transient growth of linear perturbations

We analyse the evolution of decoupled monochromatic Fourier modes with the wavenumbers α and β in the streamwise and spanwise directions. Since the flow is linearly stable, all eigensolutions decay exponentially. However, linear combinations of eigenmodes can experience substantial transient algebraic growth before they eventually decay. The amplification is quantified as $\hat{G}(T) = E(T)/E(0)$, where $E(T)$ is the kinetic energy of perturbations. This ratio is maximized over all possible initial vertical shapes by an optimization procedure [2] to give the maximum amplification $\hat{G}(T, \alpha, \beta)$.

For $Ha = 0$, streamwise vortices with $\alpha = 0$ provide the largest transient amplification. These modes are strongly damped by the magnetic field, and are, therefore, supplanted by oblique modes with $\alpha > 0$ for $Ha \geq 5$. Fig. 1 shows isolines of $\hat{G}$ for $Ha = 50$ at different times T. The contours indicate variation of $\hat{G}$ from low (white regions) to high (black regions) values. The highest amplification occurs for $T \approx 15$ (Fig. 1a), but there may be several co-existing local maxima, e.g. at $T \approx 28$ (Fig. 1b). Maximization with respect to T, α and β provides the maximum amplification $\hat{M}$ and the corresponding wavevector (α, β) as functions of Ha, which are shown in Fig. 2. The transient growth of oblique perturbations is reduced for $Ha > 0$, and the oblique angle of the optimal modes increases monotonically with Ha. For $Ha \geq 100$, the strongest amplification is provided by the spanwise Orr modes unaffected by the magnetic field. We also show the maximum amplification of streamwise rolls with $\alpha = 0$ for comparison. For these modes, the amplification $\hat{M}_{stream}$ eventually reduces to unity, i.e. they experience no transient growth for sufficiently large Ha. Remarkably, we find scaling relations $\beta \sim Ha^{-1}$ and $\hat{M}_{stream} \sim Ha^{-2}$ for these modes [1].

---

* Corresponding author: e-mail: thomas.boeck@tu-ilmenau.de, Phone: +49 3677 69 2427, Fax: +49 3677 69 1281

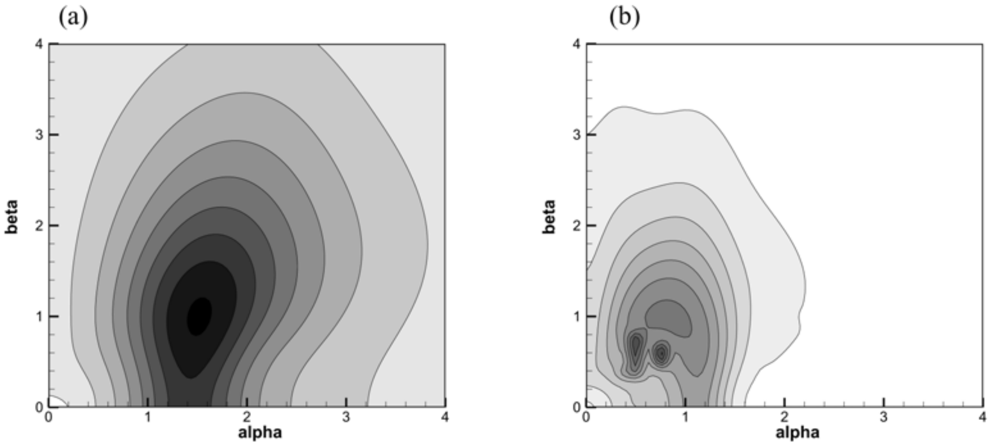

Fig. 1 Isolevels of energy amplification $\hat{G}(\alpha,\beta)$ for $Ha = 50$ at different moments in time T. (a) Global maximum at $T = 15$, (b) three local peaks at $T \approx 28$.

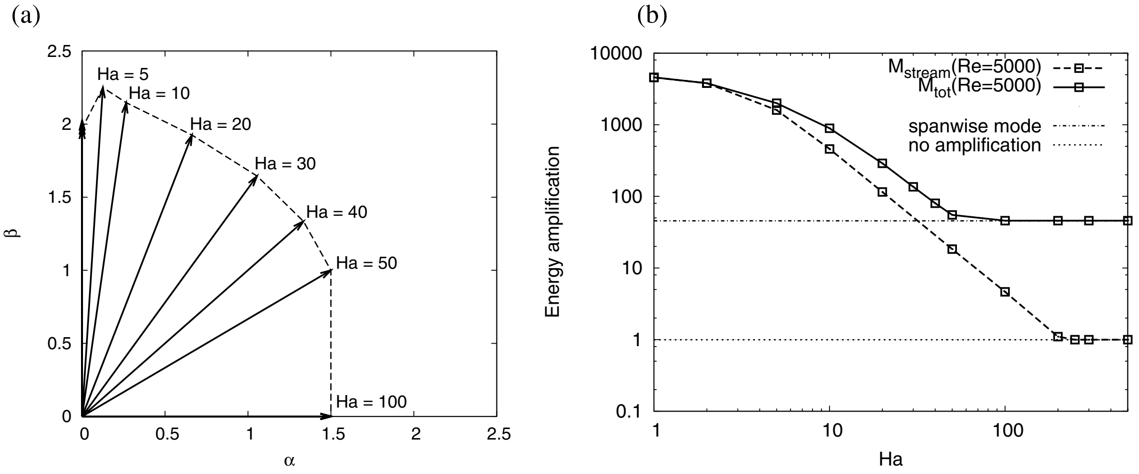

Fig. 2 Optimal wavevector vs. Ha (a) and amplification gains $\hat{M}_{tot}$ and $\hat{M}_{stream}$ for global and streamwise optimal modes (b).

3 Direct simulations of transition

In the transition simulations we study the evolution of the basic flow modulated by the optimal linear mode of a specified amplitude. The initial kinetic energy of the perturbations $E(0)$ varies between 10^{-5} and 10^{-2} relative to that of the basic flow. Weak three-dimensional noise with energy $E_{3D} = 0.01E(0)$ is added at the moment of maximum linear amplification. This approach corresponds to the so-called two-step scenario, which was shown to be pertinent for other parallel shear flows, such as the plane Poiseuille [3], pipe Poiseuille [4], and Hartmann [5] flows. For $Ha = 10$, the maximum energy amplification is provided by an oblique mode. By comparison, the amplification of streamwise vortices is about twice lower. Streamwise vortices do not induce transition to turbulence if $E(0) < 10^{-4}$. For the oblique optimal mode, even $E(0) = 10^{-5}$ is sufficient for the transition when 3D noise is added. Moreover, the transition occurs earlier than in the case of streamwise vortices.

Acknowledgements TB, DK and MR acknowledge financial support from the Deutsche Forschungsgemeinschaft in the framework of the Emmy–Noether Program (grant Bo 1668/2-2). OZ's work is supported by the grant DE FG02 03 ER46062 from the U.S. Department of Energy. Cooperation between the TU-Ilmenau and UM-Dearborn is supported by DAAD and NSF (grant OISE 0338713).

References

[1] D. Krasnov et al: Optimal growth and transition to turbulence in channel flow with spanwise magnetic field. Accepted for publication in J. Fluid Mech.
[2] P. J. Schmid, D. S. Henningson: *Stability and Transition in Shear Flows* (Springer, Berlin Heidelberg New York 2001)
[3] S. C. Reddy et al: On the stability of stream-wise streaks and and transition thresholds in plane channel flow. J. Fluid Mech. **365**, 269 (1998)
[4] O. Zikanov: On the instability of pipe Poiseuille flow. Phys. Fluids **8**, 2923 (1996).
[5] D. S. Krasnov et al: Numerical study of instability and transition to turbulence in the Hartmann flow. J. Fluid Mech. **504**, 183 (2004)