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Supply Disruptions, Asymmetric Information and a Dual-Sourcing Option

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Abstract

We study a manufacturer's strategic use of a dual-sourcing option when facing suppliers who possess private information about their likelihood of experiencing a supply disruption. The manufacturer can diversify its supply by ordering from both suppliers, but we find that the cost of doing so is inflated under asymmetric information due to the suppliers' incentives to misrepresent their reliabilities. If the manufacturer instead sole-sources, competition between the suppliers curbs their informational rents. Therefore, asymmetric information pushes the manufacturer away from diversification and towards sole-sourcing. Surprisingly, the additional cost that asymmetric information imposes on diversification may cause the manufacturer to cease diversifying in reaction to uniformly eroding supply base reliability, while it would do just the opposite under symmetric information. Despite these trends away from diversification, the value of the dual-sourcing option should not be underestimated for manufacturers who are unsure of their suppliers' reliabilities — the dual-sourcing option is actually more valuable under asymmetric information than under symmetric information if the manufacturer's cost of replacing a unit lost due to a disruption is moderate. We also analyze the effect of codependence between supply disruptions, and find that a reduction in supplier codependence increases the manufacturer's value of information. Therefore, strategic actions to reduce codependence between supply disruptions should not be seen as a substitute for learning about the suppliers' reliabilities.

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1. Introduction

The average US manufacturer spends roughly half its revenue procuring goods and services (U.S. Census Bureau, 2006). While outsourcing production of a critical component can offer cost advantages for the manufacturer, it can also introduce the risk that a supplier's failure to deliver will halt the manufacturer's production. A supplier's facility might suffer from fire, flood, or an earthquake. A labor strike or financial bankruptcy might shut down supplier operation (see Babich, 2007, for examples). Changes in a supplier's ownership status can also trigger disruptions. For example, after it purchased Dovatron in April 2000, Flextronics announced that it would completely shut down Dovatron's low-volume specialty circuit board facility in Anaheim as part of restructuring to focus on low-mix, high-volume products. As a result of Flextronics's decision, Beckman Coulter Inc., a medical device manufacturer who single-sourced a critical component from Dovatron's Anaheim facility, lost its supplier. The supply disruption cost Beckman Coulter millions of dollars.[1]

To mitigate such risks, manufacturers often employ a *dual-sourcing option*:[2] they widen a critical component's supply base to include more than one supplier. Practitioner surveys (Wu and Choi, 2005) identify two main benefits of a dual-sourcing option. First, a dual-sourcing option enables the manufacturer to reduce risk by *diversifying* it supply, that is, by simultaneously ordering the component from two suppliers. Second, a dual-sourcing option encourages competition among suppliers, resulting in lower procurement costs for the manufacturer.

Previous research assumes that the manufacturer and suppliers have identical knowledge about the possibility that a supplier experiences a disruption. However, suppliers might be privileged with better information about their susceptibility to disruptions. For instance, Dovatron likely enjoyed better information than Beckman Coulter had about Dovatron's acquisition by Flextronics. A supplier might also outsource sub-components to second-tier suppliers without telling the

[1] For more details, see `www.callahan-law.com/verdicts-settlements/fraud-beckman-coulter/index.html`.

[2] In this paper, we refer to the case in which there is only one supplier in the supply base as *single-sourcing*, and the case in which there are two suppliers as *dual-sourcing*. In the case of dual-sourcing, we refer to the manufacturer's action of ordering from only one supplier as *sole-sourcing*, and the action of ordering from both as *diversification*.

manufacturer it is doing so, creating risks of disruptions caused by second-tier suppliers.[3]

Intuitively, asymmetric information about suppliers' reliabilities might change the way the manufacturer mitigates disruption risks. For instance, Beckman Coulter might have sought to diversify its circuit board supply had it known that Dovatron was in talks with Flextronics and that a restructuring might follow. Conversely, employing risk-mitigating strategies might make the manufacturer more or less sensitive to asymmetric information about suppliers' reliabilities. In this paper, we seek to understand how the use of a dual-sourcing option is affected by asymmetric information about suppliers' reliabilities and how the value of information is affected by the dual-sourcing option.

To this end, we utilize a stylized, one-period model with a manufacturer and two suppliers. Each supplier privately knows whether they are a high or a low reliability-type supplier. The manufacturer seeks to contract with one or possibly both suppliers for production. In addition to setting quantity and payment terms, contracts ensure that the suppliers have an incentive to deliver by specifying penalties for non-delivery.[4] The manufacturer maximizes its expected profit, and in so doing must strategically account for each supplier's incentive to misrepresent their reliability. Using a mechanism design approach, we solve this model (detailed in §3), and explore the effect of asymmetric information about suppliers' reliabilities on a manufacturer's application of a dual-sourcing option to mitigating disruption risks. Next, we highlight some of our findings and briefly describe the paper's organization.

§4 presents the benchmark model of symmetric information. In §5 we analyze how asymmetric information about suppliers' reliabilities changes the manufacturer's use of its dual-sourcing option. Asymmetric information pushes the manufacturer towards sole-sourcing (and away from diversification) as a way to leverage supplier competition and limit each supplier's ability to misrepresent its reliability. Moreover, as the reliabilities of both the high and low supplier-types decrease, a manufacturer will diversify more under symmetric information, but may diversify less under asymmetric

[3]This is what happened to Menu Foods in March 2007. Its supplier ChemNutra outsourced production of wheat gluten, an ingredient in the pet food, to a Chinese supplier Xuzhou Anying Biologic Technology Development Co. Ltd.. In an effort to increase its profit margin, Xuzhou Anying introduced melamine into wheat gluten, resulting in deaths of numerous pets. More examples are provided in Yang et al. (2008).

[4]In practice, such penalties are commonly referred to as "liquidated damages", e.g., Corbin (2007).

information. Thus, even if diversification is a useful tool under symmetric information for a manufacturer facing a supply base whose overall reliability has declined (e.g., as the supply base moves offshore), the same need not be true under asymmetric information about suppliers' reliability.

In §6 we study the manufacturer's value of information about suppliers' reliabilities. It is conceivable that better information about suppliers' reliabilities could be obtained by the manufacturer, at least for risk factors the manufacturer can identify and learn about. For instance, the manufacturer might audit the suppliers for bankruptcy risk, the soundness of their fire prevention measures, structural earthquake proofing, or location relative to flood-prone coastlines. We find that information about reliability, which would likely be costly to obtain, is most beneficial for the manufacturer when the item's value is high. In such a case, the manufacturer forgoes leveraging competition and instead uses diversification to help mitigate the overall risk of non-delivery. Surprisingly, we also find that information may become more valuable even as a high-type supplier's reliability becomes closer to that of a low-type, because in such a case supply diversification becomes more important. Thus, having "less to learn" about the suppliers' reliabilities (more similarity between supplier reliability types) should not be seen as a substitute for information.

In §7 we examine the value of the dual-sourcing option for the manufacturer, that is, the incremental benefit the manufacturer enjoys by having a supply base with two suppliers instead of one. Expanding the supply base can be difficult and time-consuming for the manufacturer. For example, it took Beckman Coulter months of searching and testing before Dovatron was discovered and deemed capable of producing the specialty circuit boards that Beckman Coulter needed (see Footnote 1). Furthermore, this process may not reveal full information about the suppliers. Beckman Coulter was shocked to learn in May 2000 of Dovatron's impending closure of the Anaheim plant (see Footnote 1). Thus, the manufacturer might like to know when it is most beneficial to have either an additional supplier (the dual-sourcing option) or better information (symmetric information about suppliers' reliabilities), or both. We find that better reliability information and the dual-sourcing option are substitutes when the manufacturer's value for the item being procured is low. However, information and the dual-sourcing option become complements when the manufacturer's value for the item being procured is high, at which point having two suppliers whose reliabilities are known makes the dual-sourcing option more valuable for the manufacturer.

In §8 we examine the possibility that the suppliers' disruption probabilities are correlated, for instance, the suppliers share vulnerabilities to the same underlying risk factors such as earthquakes or floods. We find that as the two suppliers' disruptions become more correlated, the manufacturer finds diversification less desirable and, hence, relies more on sole-sourcing and leveraging supplier competition. Due to increased competition, the suppliers have smaller incentives to misrepresent their reliabilities, and thus asymmetric information about suppliers' reliabilities is less of a concern for the manufacturer. In §9 we provide concluding remarks. All proofs are in the e-companion.

2. Literature Review

Our paper contributes to the important and fast growing research and applications area of supply disruptions management (see review papers by Kleindorfer and Saad, 2005; Tang, 2006). We study multi-sourcing as the risk-mitigation tool. This tool is commonly used in operations and examples of recent articles that consider it are Babich et al. (2005); Tomlin (2006); Tomlin and Wang (2005); Dada et al. (2007). These papers focus on the risk-reduction benefits of multi-sourcing due to diversification. However, as we highlight in this paper, multi-sourcing has additional strategic benefits, because it encourages competition among suppliers. Babich (2006) and Babich et al. (2007) also found that, similar to our results, the manufacturer must strike the balance between diversification and competition.

Unlike the majority of papers on supply risk (including the ones mentioned in the paragraph above), we model the practical situation where the suppliers are better informed about the likelihoods of supply disruptions than the manufacturer is. Note that our paper is different from those in procurement and economics literatures, where asymmetric information is about suppliers' costs (e.g., Dasgupta and Spulber, 1989; Corbett, 2001; Beil and Wein, 2003; Elmaghraby, 2004;

Kostamis et al., 2006; Wan and Beil, 2008). As discussed in Yang et al. (2008), asymmetric information about disruptions affects not only procurement cost but also the manufacturer's risk profile. Our paper is not unique in studying asymmetric information about disruption risks. Examples of other papers that do the same are Tomlin (2005); Gurnani and Shi (2006); Lim (1997); Baiman et al. (2000). The latter three, unlike our paper, do not use multi-sourcing. Tomlin (2005) studies multi-sourcing, but relies on Bayesian updating over time as a mechanism for the manufacturer to learn the supplier's reliability, whereas we invoke optimal incentive-compatible contracts instead.

The work closest to the present paper is Yang et al. (2008). In that model, the manufacturer sources from one supplier and uses backup production options to manage supply disruption risk. The supplier's reliability (the likelihood of disruption) is its private information. The manufacturer designs a menu of incentive contracts to reveal the supplier's private information. The present paper studies a wholly different risk-management tool: dual-sourcing. Our findings broaden the understanding of how asymmetric information about suppliers' reliabilities interacts with the manufacturer's risk management policies.

Another work related to our paper is by Chaturvedi and Martínez-de-Albéniz (2008). They assume that the supplier's reliability and cost are its private information and study a procurement auction with multi-sourcing. Furthermore, they use contracts with a structure similar to that in our paper, consisting of transfer payment, order quantity, and penalty. Similar to us, they find that under asymmetric information the manufacturer diversifies less. However, there are several important differences between the two papers including the following. We focus explicitly on valuing the manufacturer's dual-sourcing option as a risk-management tool and relating this value to asymmetric information about supply risk. Our penalty in the contract between the manufacturer and a supplier is variable, depending on the size of the shortfall in the delivery from the supplier. We find that as the suppliers become less reliable the manufacturer could actually stop diversifying under asymmetric information about suppliers' reliabilities, which we believe will not happen in their model. In addition, we study the case in which the suppliers' disruptions are correlated.

3. Model

We model a stylized supply chain with a manufacturer and two suppliers. The suppliers' production processes are subject to random disruptions. When a disruption occurs, the production process yields zero output and the supplier delivers nothing. For instance, fire could destroy inventory and halt production, or contamination could force a pharmaceutical firm to scrap vaccines. There are two types of suppliers in the market: high reliability (H) and low reliability (L). The two supplier types differ from each other in their likelihoods of disruptions and their costs of production, with low-reliability suppliers being more prone to production disruptions. Let $t_n \in \{H, L\}$ denote the type of supplier $n = 1, 2$. The commonly known probability of a supplier being of high-type is α^H, and the probability of a supplier being of low-type is α^L, where $\alpha^H + \alpha^L = 1$. We assume that the two suppliers' types are independent of one another.[5] To capture the manufacturer's lack of visibility into the suppliers' reliabilities and costs, we assume that a supplier's reliability type is its private information, unknown to the manufacturer and the other supplier.[6]

In keeping with our assumption that a disruption results in non-delivery, we represent supplier-n's proportional random yield as a Bernoulli random variable, and denote it by $\rho_n^{t_n}$:

$$\rho_n^{t_n} \stackrel{\text{def}}{=} \begin{cases} 1 & \text{with probability } \theta^{t_n} \\ 0 & \text{with probability } 1 - \theta^{t_n}, \end{cases} \tag{1}$$

where $\theta^H = h$ and $\theta^L = l$, $h > l$. For the time being, we assume that the two suppliers' production disruption processes are independent of each other, that is, $\rho_1^{t_1}$ and $\rho_2^{t_2}$ are independent (§8 relaxes this assumption). The probability θ^{t_n} is a measure of supplier-n's reliability. Notice that reliability depends only on the supplier's type.

A production attempt costs a type-t supplier ($t \in \{H, L\}$) c^t per unit regardless of whether it is successful or not. Although we allow c^H and c^L to be different, the high-type is assumed to be more cost-efficient than the low-type, that is, the expected cost of successfully producing one unit is smaller for high-type suppliers: $c^L/l > c^H/h$.[7]

[5] See §9 for discussion of dependent types.

[6] Our results can be extended to the case where the suppliers perfectly know the type of each other.

[7] Note that, for one unit of input going into production, the expected output of a type-t supplier is θ^t. Hence, were repeated production attempts allowed, the expected cost of successfully producing one unit would be c^t/θ^t.

The manufacturer incurs a setup cost for ordering from a supplier, denoted by K, in addition to the cost of purchasing parts.The setup cost can be the cost of transferring technology to the supplier, or administrative costs incurred to manage the procurement process. We assume the parts from the two suppliers are perfect substitutes, for example, the suppliers produce the part to the manufacturer's specifications or the part is standardized.

The manufacturer uses the parts to produce a product and sells it to meet demand. To focus on the effects of supply risk, we assume that the manufacturer faces a demand, D, that is known at the time the manufacturer places its orders. When supplier deliveries do not cover the entire demand D, the manufacturer loses r per unit shortfall. The value r represents the manufacturer's revenue per item, or alternatively its per item recourse cost to secure a backup supply.[8]

To govern its relationship with the two suppliers, the manufacturer uses a pair of contracts, one for each supplier. Each contract consists of a transfer payment, X, an order quantity q, and a non-delivery unit penalty, p. The penalty serves to hold the supplier liable in case of disruption.

Intuitively, the manufacturer would like the transfer payments, order quantities and penalties to depend on the suppliers' true reliabilities. However, the manufacturer does not know the suppliers' true reliabilities, and instead—as is standard in the economics literature—we assume that the manufacturer offers several contract options (a contract "menu") from which the suppliers choose. To find the manufacturer's optimal contract menu, we apply the Revelation Principle (Myerson, 1979) and focus on the class of incentive-compatible direct-revelation menus. Under such a contract menu, both suppliers will truthfully report their reliability types to the manufacturer. For simplicity in the analysis, we assume that the suppliers cannot collude.[9]

The contract menu is a modeling construct that captures the general practice of tailoring contracts to specific suppliers in a procurement process. In particular, the contract menu captures two salient features of a typical contracting process. First, the contract for a supplier (e.g., the size of non-delivery penalty) is tailored according to the reliability risk perceived by the manufacturer.

[8]For instance, after its supplier Dovatron's production was shut down, Beckman Coulter created their own in-house specialty circuit board production line, which cost them 2.1 million dollars to construct (see Footnote 1).

[9]For instance, the suppliers might not even know who they are competing against. Jap (2003) surveys implementations of reverse auctions for procurement and notices that most bidding events are anonymous.

Thus, a high- and a low-type suppliers can end up with different contracts. Second, the contract for one supplier depends on the reliability of the other supplier, both of whom compete for business from the manufacturer: intuitively, a supplier's likelihood of receiving an order decreases as the other supplier becomes more reliable. Therefore, the contract for one supplier must be a function of the reliability types of both suppliers.

Thus, the contract menu in this model consists of four pairs of contracts, each pair corresponding to one of four combinations of supplier types:

$$\left\{[X_n(t_n, t_{\overline{n}}), q_n(t_n, t_{\overline{n}}), p_n(t_n, t_{\overline{n}})], n = 1, 2\right\} \text{ for } t_1, t_2 \in \{H, L\},$$

where $\overline{n}$ indicates the supplier other than supplier n (e.g., if $n = 2$, then $\overline{n}$ indicates supplier 1). To ease the notation, we use the shorthand notation $(X_n, q_n, p_n)(t_n, t_{\overline{n}})$ to denote the contract.

We consider a single-period problem in which the manufacturer has only one contracting opportunity. The timing of events is shown in Figure 1. The problem can be divided into two stages: contracting and execution. At the beginning of the contracting stage, nature selects the types of the two suppliers and reveals each supplier's type to that supplier only. Next, the manufacturer offers a menu of contracts to the suppliers. The suppliers make their participation decisions and then report their types to the manufacturer. Based on their reports, the manufacturer chooses a pair of contracts from the contract menu and assigns them to the respective suppliers. This concludes the contracting stage. In the execution stage, the suppliers receive their transfer payments from the manufacturer, run production, make delivery, and pay a penalty, if necessary.

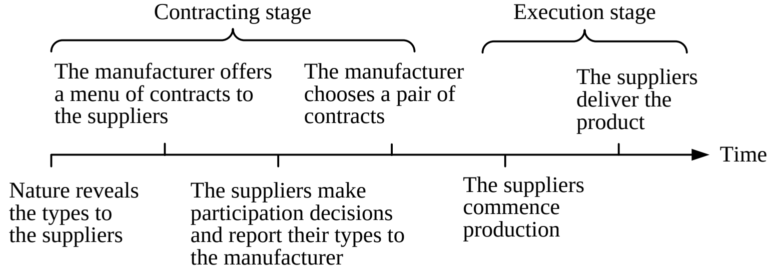

Figure 1: Timing of events.

We solve the problem by working backward from the execution stage. The next subsection presents the analysis of the supplier's execution stage decisions.

3.1 Supplier's Production Decisions

To simplify the notation in this subsection, we suppress subscript n for the suppliers. In the execution stage, given a contract (X, q, p) from the manufacturer, a supplier of type $t \in \{H, L\}$ chooses the size of its production run, z, to maximize its expected profit. Subsequently, if the production output, $\rho^t z$, is less than the order quantity q, the supplier pays a penalty p per unit of shortfall $(q - \rho^t z)^+$. (The $^+$ operation is defined such that $x^+ = x$ if $x > 0$ and $x^+ = 0$ if $x \leq 0$.) The following is supplier n's optimization problem:[10]

$$\pi^t(X, q, p) = \max_{z \geq 0} \Big\{ X - c^t z - pE(q - \rho^t z)^+ \Big\}. \tag{2}$$

When choosing the size of production run, z, the supplier trades off the cost of production, $c^t z$, against the expected non-delivery penalty, $pE(q - \rho^t z)^+$. Notice that the choice of z is independent of the transfer payment X, and, hence, the optimal z depends on q and p only (although X affects the supplier's participation decision). Let $z^t(q, p)$ denote the optimal size of the production run, given contract (X, q, p). The lemma below presents the supplier's optimal production run size and optimal expected profit. As the lemma shows, the supplier will produce the entire order quantity as long as its expected cost of successfully producing one unit, c^t/θ^t, is lower than the penalty, p.

Lemma 1. *For a given contract (X, q, p), the supplier's optimal production size, $z^t(q, p)$, and expected profit, $\pi^t(X, q, p)$, are:*

Case	$z^t(q, p)$	$\pi^t(X, q, p)$
(1) $p < c^t/\theta^t$	0	$X - p\,q$
(2) $p \geq c^t/\theta^t$	q	$X - c^t\,q - (1 - \theta^t)p\,q$

In case (1) of Lemma 1, where the penalty is even lower than the expected cost of successfully producing one unit, the supplier makes no production attempt. As we will see later, this situation never arises under the manufacturer's optimal contract menu.

[10] This is a special case of the supplier's problem (2) in Yang et al. (2008).

Lemma 1 shows that the supplier's expected profit is increasing in its reliability, θ^t. (In this paper, we use increasing and decreasing in the weak sense.) Thus, a high-type supplier earns a larger expected profit than a low-type supplier if both suppliers are offered the same contract. We define a high-type supplier's reliability advantage over a low-type supplier to be the difference between their optimal expected profits under the same contract.

Definition 1. *Under contract* (X, q, p)*, the supplier's reliability advantage for being of high-type as opposed to low-type is* $\Gamma(q,p) \stackrel{\text{def}}{=} \pi^H(X,q,p) - \pi^L(X,q,p)$.

Notice that Γ is not a function of the transfer payment, X, because it cancels out in the calculation. Applying the expression for the supplier's optimal profit in Lemma 1 to the definition yields:

$$\Gamma(q,p) = \begin{cases} 0 & p < c^H/h \\ (h\,p - c^H)q & c^L/l > p \geq c^H/h \\ [(h-l)\,p - (c^H - c^L)]q & p \geq c^L/l\,. \end{cases} \tag{3}$$

Using equation (3), $\Gamma(q,p)$ can be shown to be positive for all $q \geq 0$ and $p \geq 0$, increasing in p, q, and h, and decreasing in l. These properties of $\Gamma(q,p)$ will be instrumental in developing insights about the effects of asymmetric information on the manufacturer's procurement actions.

3.2 Manufacturer's Contract Design Problem

We now explore the manufacturer's decisions in the contracting stage. Recall that we model the manufacturer's decisions as a mechanism design problem, and, by the Revelation Principle, we focus on incentive-compatible direct-revelation contract menus.

Recall that t_n and $t_{\overline{n}}$ are the reliability types of supplier n and supplier $\overline{n}$. At the beginning of the contracting stage, the manufacturer, who does not know the suppliers' types, designs and offers a contract menu to maximize its expected profit. Let $s_n, s_{\overline{n}} \in \{H, L\}$ denote the types reported by supplier n and the other supplier upon observing the contract menu offered by the manufacturer. (Notice that if $s_n \neq t_n$, then the supplier is misrepresenting itself.) Based on the reported types, supplier n receives a contract $(X_n, q_n, p_n)(s_n, s_{\overline{n}})$, runs production of optimal size $z_n^{t_n}[(q_n, p_n)(s_n, s_{\overline{n}})]$ and earns a profit of $\pi_n^{t_n}[(X_n, q_n, p_n)(s_n, s_{\overline{n}})]$ (see Lemma 1). At the time supplier n is reporting its type, it does not know the type of the other supplier and hence supplier-n's expected profit is $\Pi_n^{t_n}(s_n) \stackrel{\text{def}}{=} E_{t_{\overline{n}}}\Big\{\pi_n^{t_n}[(X_n, q_n, p_n)(s_n, t_{\overline{n}})]\Big\}$, where the expectation is taken over supplier-$\overline{n}$'s type.

The manufacturer designs its contract menu to optimize its expected profit while inducing the suppliers to report their true reliability types. The manufacturer's contract design problem is the following optimization problem:

$$\max_{\substack{(X_n, q_n, p_n)(t_n, t_{\overline{n}}) \\ n=1,2;\, t_1, t_2 \in \{H, L\}}} \Bigg\{ \sum_{t_1, t_2 \in \{H,L\}} \alpha^{t_1} \alpha^{t_2} \Bigg[\, rE \min\Big\{D,\, \rho_1^{t_1}\, z_1^{t_1}[(q_1, p_1)(t_1, t_2)] + \rho_2^{t_2}\, z_2^{t_2}[(q_2, p_2)(t_2, t_1)]\Big\}$$
$$- X_1(t_1, t_2) + p_1(t_1, t_2)\, E\Big[q_1(t_1, t_2) - \rho_1^{t_1}\, z_1^{t_1}[(q_1, p_1)(t_1, t_2)]\Big]^+ - K\, \mathbf{1}_{\{q_1(t_1,t_2)>0\}} \tag{4a}$$
$$- X_2(t_2, t_1) + p_2(t_2, t_1)\, E\Big[q_2(t_2, t_1) - \rho_2^{t_2}\, z_2^{t_2}[(q_2, p_2)(t_2, t_1)]\Big]^+ - K\, \mathbf{1}_{\{q_2(t_2,t_1)>0\}} \, \Bigg] \Bigg\},$$

Subject to For $n = 1, 2$

(I.C.) $$\Pi_n^H(H) \geq \Pi_n^H(L), \quad \Pi_n^L(L) \geq \Pi_n^L(H) \tag{4b}$$

(I.R.) $$\Pi_n^H(H) \geq 0, \quad \Pi_n^L(L) \geq 0 \tag{4c}$$

$$q_n(t_n, t_{\overline{n}}) \geq 0,\ p_n(t_n, t_{\overline{n}}) \geq 0,\ \text{for } t_1, t_2 \in \{H, L\}.$$

The manufacturer's objective function (4a) is the manufacturer's expected revenue minus the transfer payments to the two suppliers minus the setup costs of ordering from the suppliers plus the expected penalties received, where the expectation is over the suppliers' true types t_1 and t_2.

Constraints (I.C.) are *incentive compatibility* constraints for high-type and low-type supplier-n, respectively. The left-hand-side of each of these constraints is supplier-n's expected profit when it truthfully reports its type, given that supplier-$\overline{n}$'s type is unknown. The right-hand-side is supplier-n's expected profit when it misrepresents itself. The constraints ensure that supplier n finds it optimal to report its true type. Constraints (I.R.) are the *individual rationality* constraints for high-type and low-type supplier-n. These constraints ensure that supplier-n's expected profit is greater than its reservation profit, which is normalized to zero. This assumption is common in the mechanism design field, and is adopted in both the economics (e.g., Myerson, 1981; Che, 1993) and operations management (e.g., Lim, 1997; Corbett et al., 2004) literatures.

For problem (4), we have assumed that neither supplier perfectly knows the other supplier's reliability type, leading to a Bayesian mechanism. However, there are situations where each of the two suppliers has perfect information about the other supplier's reliability type. The contract menu under such an assumption is called a dominant-strategy mechanism in the information economics literature. For a general mechanism design problems in which the payoff functions of the principal and the agents are quasilinear in the transfer payments (as in our model), the optimal dominant-strategy mechanism is also an optimal Bayesian mechanism (Mookherjee and Reichelstein, 1992). That is, the set of optimal Bayesian mechanisms subsumes the set of optimal dominant-strategy mechanisms. We later show that in our model we can choose a Bayesian-mechanism optimal contract menu such that it is also an optimal dominant-strategy mechanism.

4. Optimal Contracts under Symmetric Information

To provide a benchmark we first solve a variant of problem (4) in which suppliers' reliabilities are common knowledge. We refer to this variant as the symmetric information model and use it to explore the effect of asymmetric information. The *incentive compatibility* constraints (4b) are no longer required. The *individual rationality* constraints (4c) become

$$\pi_n^H[(X_n, q_n, p_n)(H, t_{\overline{n}})] \geq 0 \text{ and } \pi_n^L[(X_n, q_n, p_n)(L, t_{\overline{n}})] \geq 0, \quad \text{for } n \in \{1, 2\},\ t_{\overline{n}} = H, L.$$

Given two suppliers with types $t_1, t_2 \in \{H, L\}$, we refer to the more reliable supplier as the *primary* supplier, and the less reliable one as the *secondary* supplier. If both suppliers are of the same reliability type, we use the convention of designating supplier 1 as the primary supplier. Hereafter, we use subscripts m and $\overline{m}$ to indicate the primary and the secondary suppliers. The following proposition states the manufacturer's optimal procurement actions, illustrated in Figure 2.

Proposition 1. *Under symmetric information, given the reliability types of the primary and secondary suppliers, t_m and $t_{\overline{m}}$, there exist two thresholds, $\tilde{r}^{t_m}$ and $\tilde{r}^{t_m, t_{\overline{m}}}$ (given in Table AT-1), such that the manufacturer does not order from either supplier if $r \leq \tilde{r}^{t_m}$, orders only from the primary* *supplier if $\tilde{r}^{t_m} < r \leq \tilde{r}^{t_m,t_{\overline{m}}}$, and orders from both suppliers if $r > \tilde{r}^{t_m,t_{\overline{m}}}$. The manufacturer's optimal contract menu, $\{(\tilde{X}_n^*, \tilde{q}_n^*, \tilde{p}_n^*)(t_n, t_{\overline{n}}), n = 1, 2\}$, $t_1, t_2 \in \{H, L\}$, is provided in Table AT-2.*

The suppliers earn zero profit, $\pi_n^H[(\tilde{X}_n^*, \tilde{q}_n^*, \tilde{p}_n^*)(H, t_{\overline{n}})] = \pi_n^L[(\tilde{X}_n^*, \tilde{q}_n^*, \tilde{p}_n^*)(L, t_{\overline{n}})] = 0$, for $t_{\overline{n}} \in \{H, L\}$, $n = 1, 2$. The manufacturer's expected profit is

$$\begin{aligned}&\left(\alpha^H\right)^2\left(\left[(h\,r - c^H)D - K\right]^+ + \left\{\left[(1-h)h\,r - c^H\right]D - K\right\}^+\right)\\ &+2\left(\alpha^H\alpha^L\right)\left(\left[(h\,r - c^H)D - K\right]^+ + \left\{\left[(1-h)l\,r - c^L\right]D - K\right\}^+\right)\\ &+\left(\alpha^L\right)^2\left(\left[(l\,r - c^L)D - K\right]^+ + \left\{\left[(1-l)l\,r - c^L\right]D - K\right\}^+\right).\end{aligned} \tag{5}$$

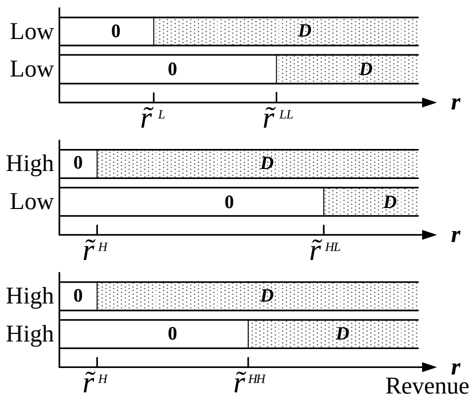

Figure 2: The manufacturer's optimal procurement actions under symmetric information in relation to the revenue, r. The label 0 marks the regions where a supplier receives no order, and the label D marks the regions where a supplier receives an order of size D. The manufacturer orders only from the primary supplier if the revenue, r, is small, and diversifies if r is large.

Given two suppliers of known types, once the unit revenue, r, becomes sufficiently large, the manufacturer will diversify, that is, it will order from both suppliers. Observe from Figure 2 that when the two suppliers are of different types, the threshold for diversification is greater compared to the cases where the suppliers are of the same type, i.e., $\tilde{r}^{HL} > \tilde{r}^{HH}$ and $\tilde{r}^{HL} > \tilde{r}^{LL}$. In other words, having two suppliers with different types makes it less appealing for the manufacturer to diversify. Intuitively, the secondary supplier in the case where the two suppliers have different types is less reliable than the secondary supplier in the case where both suppliers are of high-type. Similarly, the primary supplier in the case where the two suppliers have different types is more reliable than the primary supplier in the case where both suppliers are of low-type. Either way, the additional value from the secondary supplier is smaller when the two suppliers have different types.

Per Proposition 1, under symmetric information the manufacturer extracts all channel profit. Therefore, the channel's profit is also maximized at the manufacturer's optimal contract menu. We let $\pi_C^{t_1,t_2}[(q_1,p_1),(q_2,p_2)]$ be the channel's profit under a pair of contracts $\{(X_1,q_1,p_1),(X_2,q_2,p_2)\}$, when the two suppliers are of types t_1 and t_2. Note that $\pi_C^{t_1,t_2}$ is independent of the transfer payments X_1 and X_2, because they are payments within the channel. The optimal channel profit equals $\pi_C^{t_1,t_2}[(\tilde{q}_1^*,\tilde{p}_1^*)(t_1,t_2),(\tilde{q}_2^*,\tilde{p}_2^*)(t_2,t_1)]$. The channel loses a profit when $\{(q_1,p_1),(q_2,p_2)\}$ are different from the channel-optimal contract terms in Proposition 1. We define the channel loss as:

Definition 2. *The channel loss under a pair of contracts* $\{(X_1,q_1,p_1),(X_2,q_2,p_2)\}$*, when the two suppliers' types are* t_1 *and* t_2*, is*

$$\Delta^{t_1,t_2}[(q_1,p_1),(q_2,p_2)] \stackrel{\text{def}}{=} \pi_C^{t_1,t_2}[(\tilde{q}_1^*,\tilde{p}_1^*)(t_1,t_2),(\tilde{q}_2^*,\tilde{p}_2^*)(t_2,t_1)] - \pi_C^{t_1,t_2}[(q_1,p_1),(q_2,p_2)].$$

5. Optimal Contracts under Asymmetric Information

In this section we explore the manufacturer's contract design problem (4) under asymmetric information. We first explain the tradeoff underlying the manufacturer's contracting decisions in the face of privately informed suppliers. Proposition 2 presents the optimal contract menu. We then compare the optimal contract menus under symmetric and asymmetric information to identify the effect of asymmetric information.

The manufacturer's tradeoff. Incentive problems arise when the suppliers have private information. Recall that the optimal contract menu under symmetric information, denoted by $(\tilde{X}_n^*,\tilde{q}_n^*,\tilde{p}_n^*)(t_n,t_{\overline{n}})$, $n=1,2$, $t_1,t_2 \in \{H,L\}$, is designed so that the manufacturer extracts the entire channel profit and leaves zero profit to the suppliers. Under asymmetric information, if the manufacturer offered the same contract menu, then a high-type supplier would have an incentive to misrepresent itself. For example, if supplier n is a high-type supplier, it would claim to be a low-type supplier and receive the contract $(\tilde{X}_n^*,\tilde{q}_n^*,\tilde{p}_n^*)(L,t_{\overline{n}})$. This contract, which yields zero profit to a low-type supplier, would bring a strictly positive profit to high-type supplier-n equal to its reliability advantage, denoted by $\Gamma_n[(\tilde{q}_n^*,\tilde{p}_n^*)(L,t_{\overline{n}})]$ (see Definition 1). Therefore, under asymmetric information, if the manufacturer offered contract menu $(\tilde{X}_n^*,\tilde{q}_n^*,\tilde{p}_n^*)(t_n,t_{\overline{n}})$, $n=1,2$, $t_1,t_2 \in \{H,L\}$, it would have to make an *incentive payment* to high-type supplier-n in the amount of

$\Gamma_n[(\tilde{q}_n^*, \tilde{p}_n^*)(L, t_{\overline{n}})]$. Alternatively, the manufacturer may offer a different contract menu to reduce this incentive payment, which nonetheless may not optimize the channel profit. The resulting channel loss (see Definition 2) will then be borne by the manufacturer. Hence, in designing the contract menu, the manufacturer must strike a balance between the incentive payment to a high-type supplier and the channel loss.

The optimal contract menu. The next proposition describes the optimal contract that achieves this goal. Using the analytical results in Proposition 2, we illustrate the manufacturer's optimal procurement actions on the right panel of Figure 3. We continue to use the convention that when both suppliers are the same type, supplier 1 is designated as the primary supplier.

Proposition 2. *Under asymmetric information, given the reliability types of the primary and secondary suppliers, t_m and $t_{\overline{m}}$, there exist two thresholds, r^{t_m} and $r^{t_m,t_{\overline{m}}}$ (given in Table AT-3), such that the manufacturer does not order from either supplier if $r \leq r^{t_m}$, orders only from the primary supplier if $r^{t_m} < r \leq r^{t_m,t_{\overline{m}}}$, and orders from both suppliers if $r > r^{t_m,t_{\overline{m}}}$. The manufacturer's optimal contract menu, $\{(X_n^*, q_n^*, p_n^*)(t_n, t_{\overline{n}}),\ n = 1, 2\}$, $t_1, t_2 \in \{H, L\}$, is presented in Table AT-4. Without loss of optimality, we choose the optimal transfer payments $X_n^*(t_n, t_{\overline{n}})$ such that the optimal contract menu is also a dominant-strategy mechanism (i.e., when the suppliers perfectly know the type of each other).*

The high- and low-type supplier-n's profits are $\pi_n^H[(X_n^*, q_n^*, p_n^*)(H, t_{\overline{n}})] = h\left(\frac{c^L}{l} - \frac{c^H}{h}\right) q_n^*(L, t_{\overline{n}})$, and $\pi_n^L[(X_n^*, q_n^*, p_n^*)(L, t_{\overline{n}})] = 0$; the manufacturer's expected profit (before knowing supplier types) is

$$\begin{aligned}
&\left(\alpha^H\right)^2\left(\left[(h\,r - c^H)D - K\right]^+ + \left\{\left[h(1-h)r - c^H\right]D - K\right\}^+\right)\\
&+2\left(\alpha^H\alpha^L\right)\left(\left[(h\,r - c^H)D - K\right]^+ + \left\{\left[l(1-h)r - c^L\right]D - K - \frac{\alpha^H}{\alpha^L}h\left(\frac{c^L}{l} - \frac{c^H}{h}\right)D\right\}^+\right)\\
&+\left(\alpha^L\right)^2\left(\left[(l\,r - c^L)D - K - \frac{\alpha^H}{\alpha^L}h\left(\frac{c^L}{l} - \frac{c^H}{h}\right)D\right]^+ + \left\{\left[l(1-l)r - c^L\right]D - K - \frac{\alpha^H}{\alpha^L}h\left(\frac{c^L}{l} - \frac{c^H}{h}\right)D\right\}^+\right).
\end{aligned} \tag{6}$$

Effect of asymmetric information on ordering decisions. As in the case of symmetric information, under the optimal contract menu the manufacturer places no orders when the unit revenue is sufficiently small, orders only from the primary supplier once the unit revenue becomes larger, and orders from both the primary and secondary suppliers once the unit revenue is sufficiently

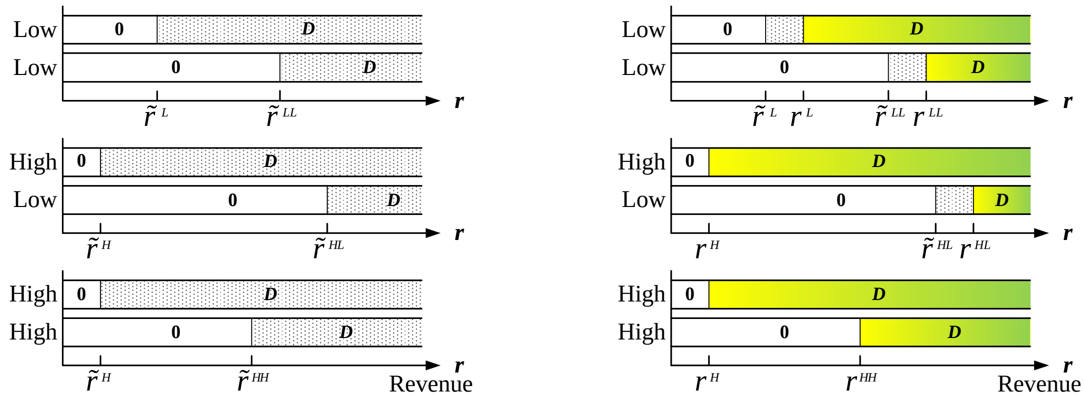

Figure 3: The manufacturer's optimal procurement actions in relation to the revenue, r, under symmetric information (the left panel) and asymmetric information (shaded with solid color on the right panel). The difference between the manufacturer's procurement actions under symmetric and asymmetric information is shown on the right panel. Under asymmetric information, the manufacturer forgoes ordering from the low-type supplier when r falls in the intervals corresponding to the dotted bars.

large. However, as the right panel of Figure 3 shows, the thresholds for ordering from the low-type supplier, be it a primary or secondary supplier, are higher under asymmetric information than under symmetric information. In particular, the manufacturer stops diversifying when $\max\{r^L, \tilde{r}^{LL}\} \leq r < r^{LL}$ and both suppliers are of low-type, or $\tilde{r}^{HL} \leq r < r^{HL}$ and the two suppliers are of different types. Under symmetric information, the manufacturer sole-sources when the anticipated revenue brought by the secondary supplier does not outweigh the additional ordering costs. However, under asymmetric information there is an additional benefit to sole-sourcing, which pushes the manufacturer to diversify less. If the manufacturer will not order from a secondary supplier of low-type, then a high-type supplier knows that pretending to be of low-type might backfire – they could lose the order. Thus, when the manufacturer rolls back diversification, the suppliers find themselves in more intense competition, reducing their incentives to misrepresent themselves.

As a consequence of forgoing ordering from a low-type supplier, the manufacturer receives less supply under asymmetric information. Thus, we have the following result:

Corollary 1. *The total quantity received by the manufacturer from the two suppliers under asymmetric information is stochastically smaller than under symmetric information.*

Sensitivity to supply-base reliability. A worsening of reliability in the supply base may have different effects on the use of diversification under symmetric and asymmetric information. One may intuitively expect that whenever the supply base becomes less reliable, the manufacturer will find it more attractive to diversify. Indeed, with symmetric information under a mild restriction on supplier reliabilities, $h + l > 1$, the manufacturer may start diversifying when the supply base becomes less reliable, but will never stop diversifying. However, under asymmetric information, the opposite may be true: a worsening of reliability may cause the manufacturer to stop diversifying under asymmetric information. These observations are formalized in the following corollary.

Corollary 2. *Suppose the non-disruption probabilities are such that $h+l > 1$. Consider the optimal contract pair offered to one high- and one low-type suppliers. Under asymmetric information, there exist h, l and unit revenue r such that the manufacturer diversifies, but if both h and l decrease by some $\epsilon \in (0, l)$ the manufacturer will stop diversifying. Under symmetric information, the manufacturer would never stop diversifying in response to a reliability decrease.*

The explanation of this result lies in high-type suppliers' incentives to misrepresent themselves. When both supplier types are sufficiently unreliable (as made precise in the proof of the above corollary), a further reduction in their reliabilities leads to an increase in a high-type supplier's reliability advantage, which in turn translates into a larger incentive payment from the manufacturer. Even though a worsening of the supply base reliability increases the chance that a low-type supplier's delivery would be critical for meeting demand, the manufacturer can find it attractive to stop diversifying in order to circumvent a ballooning incentive payment for the high-type supplier.

Informational rents and channel loss. We use Proposition 2 to compute the suppliers' incentive payments and the channel loss under the optimal contract menu. These results will be crucial in analyzing the value of information. Following standard information economics terminology, we refer to high-type supplier-n's expected incentive payment under the optimal contract menu as its informational rent, denoted by $\gamma_n(r)$, where the expectation is over supplier-$\overline{n}$'s type: $\gamma_n(r) \stackrel{\text{def}}{=} E_{t_{\overline{n}}}\Big\{\Gamma[(q_n^*, p_n^*)(L, t_{\overline{n}})]\Big\}$.

<table>
<tr><th rowspan="2">Revenue, r</th><th colspan="2">Informational Rent</th></tr>
<tr><th>$\gamma_1(r)$, for supplier 1</th><th>$\gamma_2(r)$, for supplier 2</th></tr>
<tr><td>$r \leq r^L$</td><td>0</td><td rowspan="2">0</td></tr>
<tr><td>$r^L < r \leq r^{LL}$</td><td rowspan="2">$\alpha^L h \left(\frac{c^L}{l} - \frac{c^H}{h} \right) D$</td></tr>
<tr><td>$r^{LL} < r \leq r^{HL}$</td><td>$\alpha^L h \left(\frac{c^L}{l} - \frac{c^H}{h} \right) D$</td></tr>
<tr><td>$r > r^{HL}$</td><td>$h \left(\frac{c^L}{l} - \frac{c^H}{h} \right) D$</td><td>$h \left(\frac{c^L}{l} - \frac{c^H}{h} \right) D$</td></tr>
</table>

Table 1: Informational rents earned by high-type supplier-n, $\gamma_n(r)$, $n = 1, 2$.

The optimal penalty $p_n^*(L, t_{\overline{n}})$ and optimal order quantity $q_n^*(L, t_{\overline{n}})$ are given by Proposition 2. Table 1 presents the closed-form expressions for $\gamma_n(r)$, $n = 1, 2$, revealing that a high-type supplier's informational rent increases in the revenue, r. In particular, as revenue increases it becomes important for the manufacturer to avoid lost sales, encouraging the manufacturer to order from a low-type supplier and thereby allowing a high-type supplier to exploit its reliability advantage.

Given the optimal contracts for two suppliers of types t_1 and t_2 under asymmetric information, the channel loss is denoted by δ^{t_1,t_2} and is given by $\delta^{t_1,t_2}(r) \stackrel{\text{def}}{=} \Delta^{t_1,t_2}\left[(q_1^*, p_1^*)(t_1, t_2), (q_2^*, p_2^*)(t_2, t_1)\right]$.

Revenue, r	$\boldsymbol{\delta^{HH}(r)}$
$r > 0$	0
Revenue, r	$\boldsymbol{\delta^{HL}(r)}$ **or** $\boldsymbol{\delta^{LH}(r)}$
$\tilde{r}^{HL} < r \leq r^{HL}$	$\left[(1-h)l\,r - c^L\right] D - K$
All other r	0
Revenue, r	$\boldsymbol{\delta^{LL}(r)}$
$\tilde{r}^L < r \leq r^L$	$(l\,r - c^L)D - K$
$\tilde{r}^{LL} < r \leq r^{LL}$	$\left[(1-l)l\,r - c^L\right] D - K$
All other r	0

Table 2: The channel loss under the optimal contract menu, $\delta^{t_1,t_2}(r)$.

Table 2 provides a closed-form expression for the channel loss, $\delta^{t_1,t_2}(r)$, $t_1, t_2 \in \{H, L\}$. Under the optimal contract menu, channel loss is strictly positive whenever asymmetric information causes the manufacturer to stop ordering from a low-type supplier from whom it would order under symmetric information. For example, given two suppliers, one of high-type and the other of low-type, if revenue r is such that $\tilde{r}_{HL} < r \leq r_{HL}$, then introducing asymmetric information causes the manufacturer to stop ordering from the low-type supplier, resulting in a profit loss of

$\left[(1-h)l\,r-c^L\right]D-K$, equal to channel loss $\delta^{HL}(r)$.

6. Value of Information

Section 5 shows that asymmetric information about suppliers' reliabilities changes the manufacturer's procurement actions and causes losses in the manufacturer's and the channel's profits. These losses can be avoided by acquiring information, which could be costly and, thus, needs to be justified by the benefits of having such information. In this section, we study the value of information for the suppliers, channel and manufacturer, where the value of information for an entity is defined to be the difference between its expected profits in symmetric and asymmetric information models.[11]

For a high-type supplier, the value of information is the negative of its informational rent (Table 1). For the channel, the value of information equals channel loss (Table 2) weighted by the probability of drawing suppliers of types $t_1, t_2 \in \{H, L\}$. From the manufacturer's perspective, information about suppliers' reliabilities creates value by eliminating both informational rent and channel loss. Hence, the manufacturer's value of information equals the sum of the informational rents paid to the two suppliers (each weighted by the probability that the supplier is of high-type) and the expected channel loss: $\alpha^H\left[\gamma_1(r)+\gamma_2(r)\right]+\left[2\left(\alpha^H\alpha^L\right)\delta^{HL}(r)+\left(\alpha^L\right)^2\delta^{LL}(r)\right]$.

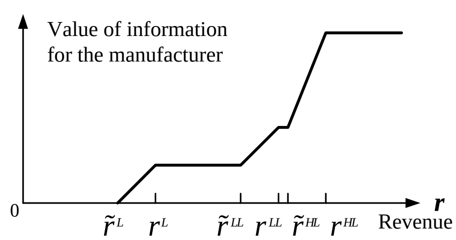

Figure 4: The values of information for the manufacturer increases in the revenue, r.

Figure 4 plots the manufacturer's value of information as a function of r, revealing that as the revenue increases, the manufacturer's value of information increases, alternating between regions where it is strictly increasing and regions where it is flat. For example, when $\tilde{r}^L < r \leq r^L$, given two low-type suppliers, the manufacturer would order from one of them under symmetric information,

[11]Recall from Proposition 2 that in the asymmetric information model, even if a supplier perfectly knows the other supplier's reliability type, it would receive the same contract and thus earn the same profit. Therefore, in this model there is no value for the suppliers only to acquire information about the reliability type of the other supplier.

but would strategically not order at all under asymmetric information. Hence, the manufacturer incurs channel loss, which becomes all the more costly as r continues to increase. This explains the increase in the value of information as r increases from $\tilde{r}^L$ to r^L. Once r is slightly above r^L, the channel loss becomes so onerous that the manufacturer starts ordering from one of the two low-type suppliers even under asymmetric information. While this strategic decision allows the manufacturer to avoid channel loss, it risks incurring informational rent if a high-type supplier is drawn. The information rent does not change with r, so the value of information levels off. This behavior repeats itself as further increases in r create new strategic shifts in the manufacturer's decision to order from a low-type supplier under asymmetric information. Overall, as the revenue, r, increases, the manufacturer becomes more willing to order to avert lost sales, allowing more opportunities for the high-type supplier(s) to exploit their private information and thus enhancing the value of information.

Sensitivity of the manufacturer's value of information to reliability gap $h - l$. Intuitively, one may expect that the manufacturer's value of information increases if the reliability gap between the two supplier-types expands, because high-type suppliers would have stronger incentives to misrepresent their reliabilities. In fact, in the case where there is only one supplier in the supply base, the manufacturer's value of information increases as the reliability gap increases (see Yang et al., 2008, Corollary 6). However, we find that in the dual-sourcing model the value of information may decrease, depending on the size of revenue, r.

Corollary 3. *Suppose that the low-type's reliability, l, is fixed. If $\max\{\tilde{r}^{HL}, c^L/l^2\} < r \leq r^{HL}$, then the manufacturer's value of information is decreasing in h.*

Note that when $l > \frac{1}{2}$, we have $\tilde{r}^{HL} > c^L/l^2$. Therefore, when $l > \frac{1}{2}$, for all $\tilde{r}^{HL} < r \leq r^{HL}$ the value of information is decreasing in h. To see the intuition behind Corollary 3, note that when $\tilde{r}^{HL} < r \leq r^{HL}$, asymmetric information causes the manufacturer to strategically forgo diversification when facing one high- and one low-type supplier. Doing so puts the manufacturer in peril of regretting its decision to not contract with a secondary supplier, but intuitively this risk diminishes as the high-type supplier (the primary supplier) becomes more reliable.

Corollary 3 implies that, even as the reliability gap between the high and low reliability supplier-types shrinks — meaning there is "less to learn" about the suppliers — information is actually more valuable. Thus, having "less to learn" about the suppliers' reliabilities (more similarity between supplier reliability types) should not be seen as a substitute for information.

7. Effect of Dual-Sourcing Option

In this section, we analyze the manufacturer's benefit of acquiring the dual-sourcing option. We find that the dual-sourcing option increases the manufacturer's expected profit while reducing the informational rent of a supplier. We further show that the benefit of the dual-sourcing option for the manufacturer could be enhanced or diminished by improved information about the suppliers. In e-companion D, we examine a benchmark model with a single supplier in the supply base, while retaining all other assumptions of our main model (§3). To differentiate from the sole-sourcing case in the dual-sourcing model, we hereafter refer to this model as the *single-sourcing* model.

7.1 Effect of the Dual-Sourcing Option on the Informational Rent

We now examine how the manufacturer's dual-sourcing option affects a high-type supplier's informational rent. Without loss of generality, we assume that the supplier in question is the one that is favored in case of a tie in the dual-sourcing model (i.e., supplier 1). We compare the informational rent extracted by high-type supplier-1 when it is the only supplier against the rent when it is one of two suppliers that the manufacturer can order from. The result is presented in Table 3.

Revenue, r	**Reduction in informational rent**
$r^L < r \leq r^{HL}$	$\alpha^H h \left(\frac{c^L}{l} - \frac{c^H}{h}\right) D$
$r \leq r^L$ or $r > r^{HL}$	0

Table 3: Reduction in informational rent extracted by high-type supplier-1 if the manufacturer switches from single-sourcing to dual-sourcing.

From Table 3, the dual-sourcing option reduces the high-type supplier's informational rent, if and only if the revenue, r, is such that $r^L < r \leq r^{HL}$. In region $r > r^{HL}$, the revenue is large enough that even when there are two suppliers both of them will receive an order regardless of their reliability types. Hence, when a high-type supplier is one of two suppliers, it has just as much incentive to misrepresent itself as it does when it is the only supplier. This explains why the dual-sourcing option does not change the high-type supplier's profits when $r > r^{HL}$.

In region $r^L < r \leq r^{HL}$, under the single-sourcing model, the manufacturer will order from a supplier regardless of its reliability type. Therefore, a high-type supplier will earn informational rent, because it can always pretend to be of low-type and still receive an order. In contrast, under the dual-sourcing model, a low-type supplier will not receive an order if the other supplier is of high-type. Given the manufacturer's reluctance to order from a low-type supplier, the high-type suppliers now know that pretending to be of low-type comes with the risk of not receiving an order. Thus, by not ordering from a low-type supplier, the manufacturer in effect creates competitive pressure that limits the high-type suppliers' incentive to misrepresent themselves. This competition manifests itself in the reduction of supplier 1's informational rent, shown in Table 3.

Finally, when $r \leq r^L$, the revenue is low enough that a low-type supplier does not receive an order with or without the dual-sourcing option. Hence, in this region, the high-type supplier has no incentive to misrepresent itself and cannot earn informational rent, regardless of whether or not the dual-sourcing option exists.

7.2 Value of the Dual-Sourcing Option for the Manufacturer

We now compute the manufacturer's values of the dual-sourcing option in the symmetric and asymmetric information models, and then compare them to examine the effect of asymmetric information on the benefit of having a dual-sourcing option. We define the value of the dual-sourcing option for the manufacturer as the difference between its expected profits in the dual-sourcing and single-sourcing models, given by (5) and (A.1) for the symmetric information model, and (6) and (A.2) for the asymmetric information model.

Symmetric information. Consider the scenario where in the single-sourcing model the only supplier is of low-type and in the dual-sourcing model the additional supplier is of high-type. In this scenario, the manufacturer enjoys one of two kinds of benefits by moving from the single-sourcing model to the dual-sourcing model. First, if revenue is high and the manufacturer orders from both suppliers in the dual-sourcing model, then the manufacturer benefits from diversification, reducing the probability of lost sales. Second, if revenue is low and the manufacturer chooses to order from only one supplier in the dual-sourcing model, the manufacturer enjoys the benefit of having access to the more efficient, high-type supplier. In the other scenarios, where either the single supplier is of high-type or both suppliers in the dual-sourcing model are of low-type, the dual-sourcing option does not yield benefits of having access to a more reliable supplier. Nonetheless, the dual-sourcing option continues to generate diversification benefits when the revenue is large enough.

Asymmetric information. The value of the dual-sourcing option under asymmetric information is similar to that under symmetric information. One important difference, however, is that the manufacturer yields informational rent(s) to the high-type supplier(s), and the dual-sourcing option may increase or decrease the total informational rent to be paid by the manufacturer. On one hand, the dual-sourcing option has the potential to reduce the informational rent paid to a supplier. As discussed in §7.1, this happens when a manufacturer with the dual-sourcing option chooses not to order from a low-type supplier, thus putting a competitive pressure on high-type suppliers. On the other hand, in cases where the dual-sourcing option enables the manufacturer to order from two high-type suppliers, the manufacturer may have to pay informational rents to both suppliers, reducing the appeal of the dual-sourcing option. Thus, it is not necessarily obvious whether the dual-sourcing option is more or less valuable under asymmetric information compared to under symmetric information. We next explore this comparison.

Information and the value of the dual-sourcing option for the manufacturer. Proposition 3 below formalizes the comparison between the values of the dual-sourcing option under symmetric and asymmetric information, which are plotted in Figure 5.

Proposition 3. *Information and the dual-sourcing option are substitutes when the revenue is small and are complements when the revenue is large. Specifically, there exists a value r_0, $r^{HL} > r_0 > \tilde{r}^{LL}$, such that, for $r > r_0$, the manufacturer's value of the dual-sourcing option is larger under symmetric information than under asymmetric information; the converse is true for $\tilde{r}^L < r < r_0$.*

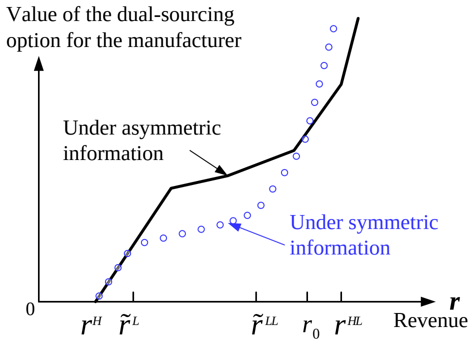

Figure 5: The values of the dual-sourcing option for the manufacturer under symmetric and asymmetric information. The value of the dual-sourcing option under asymmetric information is greater than under symmetric information for $\tilde{r}^L < r < r_0$, but is smaller for $r > r_0$.

Intuitively, one may expect that the manufacturer would always benefit more from the dual-sourcing option under symmetric information because the manufacturer's ability to identify the suppliers' types may help it take better advantage of the option. Proposition 3 shows that information reduces the value of the dual-sourcing option when revenue, r, is small (i.e., $\tilde{r}^L < r < r_0$), but increases the value of the dual-sourcing option when revenue, r, is large (i.e., $r > r_0$).

This observation is a manifestation of the manufacturer's tradeoff between the benefit of supplier competition and the benefit of diversification. When the revenue is small, the competition effect dominates and hence the manufacturer prefers not to order from a secondary supplier. This reduces the informational rent extracted by a high-type supplier. Because this benefit is absent under symmetric information, the dual-sourcing option is more valuable under asymmetric information. When the revenue is large, the benefit of diversification becomes larger than the benefit of competition, and hence the manufacturer finds it useful to diversify. The resulting inflation of informational rents makes the dual-sourcing option less valuable under asymmetric information.

8. Codependent Supplier Production Disruptions

Thus far we assumed that the two suppliers' production disruption processes are independent. In reality, they could be correlated due to common infrastructure, geographic proximity, similar production technologies, overlapping supply bases and other factors. In this section we extend our model and analysis to capture codependence between the two suppliers' disruption processes.

8.1 Model and Optimal Contract Menu

We capture codependence between the two suppliers' disruption processes by allowing the Bernoulli yield random variables of the two suppliers to be statistically dependent. If the two suppliers are of reliability types, $t_1, t_2 \in \{H, L\}$, we may represent codependence between their disruption processes via a joint probability matrix:

$$\Omega^{t_1,t_2} \stackrel{\text{def}}{=} \begin{bmatrix} \omega^{t_1,t_2}(1,1) & \omega^{t_1,t_2}(1,0) \\ \omega^{t_1,t_2}(0,1) & \omega^{t_1,t_2}(0,0) \end{bmatrix}, \tag{7}$$

where $\omega^{t_1,t_2}(x_1, x_2) = P\{\rho_1^{t_1} = x_1, \rho_2^{t_2} = x_2\}$, $x_1, x_2 \in \{0, 1\}$, is the joint probability that the yield rates of the two suppliers' production runs are x_1 and x_2, respectively. Matrix Ω^{t_1,t_2} can be uniquely characterized by one of its four elements and the two suppliers' marginal probabilities of successful production, $P\{\rho_1^{t_1} = 1\} = \theta^{t_1}$ and $P\{\rho_2^{t_2} = 1\} = \theta^{t_2}$, because $\theta^{t_1} = \omega^{t_1,t_2}(1,1) + \omega^{t_1,t_2}(1,0)$ and $\theta^{t_2} = \omega^{t_1,t_2}(0,1) + \omega^{t_1,t_2}(1,1)$.[12] We choose the two suppliers' joint success probability, $\omega^{t_1,t_2}(1,1)$, to be the indicator of the level of codependence between the two suppliers' disruption processes. The larger $\omega^{t_1,t_2}(1,1)$, the greater the codependence.

The manufacturer's contract design problem under asymmetric information is given by problem (4), where the manufacturer's expected sales, $E \min\Big\{D,\ \rho_1^{t_1} z_1^{t_1} + \rho_2^{t_2} z_2^{t_2}\Big\}$ in (4a), is now an expectation over Ω^{t_1,t_2}, the joint probability distribution of $\rho_1^{t_1}$ and $\rho_2^{t_2}$. To obtain an analytical solution, we assume that $\omega^{HL}(1,1)/l \geq \omega^{HH}(1,1)/h$. This assumption is equivalent to the following restriction on the conditional probability of high-type supplier-1 producing successfully, given that supplier 2 produced successfully:

$$P\{\rho_1^H = 1 | \rho_2^L = 1\} > P\{\rho_1^H = 1 | \rho_2^H = 1\}. \tag{8}$$

In words, high-type supplier-1's probability of successful production conditional on supplier 2 producing successfully is decreasing in supplier-2's reliability. For example, consider a situation where the two suppliers are located in the same region and share an unreliable infrastructure and where

[12] In the exhaustive set of joint probability matrices, $\{\Omega^{HH}, \Omega^{HL}, \Omega^{LH}, \Omega^{LL}\}$, matrix Ω^{LH} is the transpose of Ω^{HL}, because we assumed that the two suppliers are symmetric if they are of the same type.

a supplier's reliability increases in its experience of working with this infrastructure. Production success of an inexperienced supplier-2 provides a stronger signal than success of an experienced supplier-2 regarding the chance that the other supplier (supplier 1) will succeed as well.

Proposition 4. *Given that the two suppliers' disruption processes are codependent and condition* (8) *holds, the manufacturer's optimal contract menu and the suppliers' profits under symmetric and asymmetric information are as given in Propositions 1 and 2, respectively, with the thresholds* $\tilde{r}^{t_m,t_{\overline{m}}}$ *and* $r^{t_m,t_{\overline{m}}}$ *redefined per Table AT-5. The manufacturer's expected profits under symmetric and asymmetric information are obtained by replacing* $(1-h)h$, $(1-h)l$ *and* $(1-l)l$ *in* (5) *and* (6) *with* $h - \omega^{HH}(1,1)$, $l - \omega^{HL}(1,1)$ *and* $l - \omega^{LL}(1,1)$, *respectively.*

To obtain the optimal contract menu and the profits of the suppliers and the manufacturer, we can simply rewrite the joint probabilities $(1-h)h$, $(1-h)l$ and $(1-l)l$ in the thresholds of revenue and profits in the previous sections as $h - \omega^{HH}(1,1)$, $l - \omega^{HL}(1,1)$ and $l - \omega^{LL}(1,1)$, respectively.

8.2 Sensitivity Analysis

We analyze how the manufacturer's ordering decisions, the supply chain firms' profits, and the value of information for the manufacturer change as the codependence between the two suppliers' disruption processes increases. To model an increase in codependence, we increase at least one of the three joint success probabilities, $\omega^{HH}(1,1)$, $\omega^{HL}(1,1)$ and $\omega^{LL}(1,1)$, by a small amount ϵ, while keeping the others fixed. We further restrict the increment ϵ to be such that the resulting level of codependence does not violate inequality (8).

The manufacturer's diversification decision. From Proposition 4, the thresholds for the manufacturer to diversify in both symmetric and asymmetric information models, $\tilde{r}^{HH}$, $\tilde{r}^{HL}$ and $\tilde{r}^{LL}$; r^{HH}, r^{HL} and r^{LL}, increase as codependence increases. In other words, greater supplier codependence makes diversification statistically less valuable and pushes the manufacturer towards sole-sourcing, in both symmetric and asymmetric information models.

The supply chain firms' profits under asymmetric information. As the joint success probabilities, $\omega^{HH}(1,1)$, $\omega^{HL}(1,1)$ or $\omega^{LL}(1,1)$, increase, both the manufacturer's expected profit (6) and the high-type suppliers' informational rents (provided in Table 1, with r^{LL} and r^{HL} redefined as in Proposition 4) decrease in the asymmetric information model. To explain this observation, we note that greater codependence makes diversification less valuable for the manufacturer. This reduces the manufacturer's profit in both symmetric and asymmetric information models. Under asymmetric information, however, greater codependence also causes an increase in supplier competition, leading to a reduction in the informational rents extracted by the high-type suppliers. This compensates for the reduction in the benefit of diversification for the manufacturer, making the manufacturer's profit under asymmetric information less sensitive to an increase in codependence than under symmetric information.

Manufacturer's value of information. We replace joint probabilities $(1-h)h$, $(1-h)l$ and $(1-l)l$ in the expression for the manufacturer's value of information with $h-\omega^{HH}(1,1)$, $l-\omega^{HL}(1,1)$ and $l-\omega^{LL}(1,1)$. As codependence increases, the manufacturer's value of information decreases. Thus, as the two suppliers' disruptions become more correlated, the manufacturer becomes less concerned with asymmetric information despite the fact that higher codependence between supplier disruptions makes the supply base less reliable overall. To see why, recall that greater codependence makes diversification less desirable, leading to increased supplier competition. This in turn reduces the suppliers' incentives to misrepresent their reliability, and hence reduces the value of information for the manufacturer. The managerial implication is that strategic actions to reduce codependence between supplier disruptions (e.g., sourcing from different geographic regions) should not be seen as a substitute for obtaining better information about suppliers' reliabilities.

9. Concluding Remarks

Supply disruptions lead to significant losses in shareholders' value (Hendricks and Singhal, 2003). A common operational tool for controlling supply disruption risks is placing orders with several suppliers (diversification), so that if one of the suppliers experiences a problem, others might still deliver parts to the manufacturer. However, working with several suppliers is administratively expensive, and managers must carefully weigh the benefits of diversification against its costs. This tradeoff depends on a number of factors such as whether suppliers are located in the same geographical area (and are exposed to the same causes of disruptions) and how much the manufacturer would lose if a disruption did occur. Furthermore, this paper shows that another important factor is whether suppliers are better informed than the manufacturer about the disruption likelihood.

First, we observe that a dual-sourcing option (the option of the manufacturer to order from one or two suppliers) has several benefits, with the risk-reduction due to diversification being just one of them. The other important benefit comes from competition. Consistent with the extant economics and operations literatures, we find that suppliers use their private information about their reliabilities to extract informational rents from the manufacturer. If the manufacturer orders from both suppliers (i.e., diversifies) then it pays high rents to both suppliers, which leads to very high effective diversification costs. On the other hand, if the manufacturer commits to ordering from only one of the two suppliers, the competition between suppliers keeps these rents down. Thus, we find that asymmetric information about suppliers' reliabilities pushes the manufacturer away from diversification and towards sole-sourcing. As a consequence, the additional cost that asymmetric information imposes on diversification may cause the manufacturer to cease diversifying even as the supply base reliability erodes, which would never happen under symmetric information.

Second, diversification is still used by the manufacturer under asymmetric information about suppliers' reliabilities, but only if the manufacturer's costs in case of a disruption are very high. Because diversification results in high informational rents, manufacturers that choose to diversify have a strong incentive to learn about suppliers' reliabilities, for example, by investing in long-term relationships with suppliers, sending representatives to supplier factories, etc. In contrast, when costs of disruptions are low and consequently the manufacturer orders from only one supplier, competition between suppliers is very effective in curtailing informational rents and the manufacturer would not gain much by knowing everything suppliers know. Thus, in such cases, arm's-length relationships between the manufacturer and the suppliers are more tenable. Moreover, we find that information may become more valuable even as a high-type supplier's reliability becomes closer to that of a low-type, because in such a case supply diversification becomes more important.

Thus, surprisingly, having "less to learn" about the suppliers' reliabilities (more similarity between supplier reliability types) should not be seen as a substitute for information.

Third, having less information about supplier reliability does not necessarily decrease the value of a dual-sourcing option for the manufacturer. If disruptions costs are low, this option is more valuable for a manufacturer who is not as knowledgeable as the suppliers, because it enables such a manufacturer to leverage competition to drive down suppliers' informational rents.

Fourth, introducing codependence between supplier disruptions does not alter any of the above insights, but adds new findings. Although greater supplier codependence reduces the manufacturer's profit, it also heightens supplier competition by making suppliers more similar. Consequently, the decrease in the manufacturer's profit due to greater supplier codependence is less severe under asymmetric information (where competition is more vital) than under symmetric information. In fact, greater codependence between supplier disruptions reduces the suppliers' incentives to misrepresent their reliabilities and reduces the value of information for the manufacturer. Hence, strategic actions to reduce supplier codependence (such as choosing suppliers from different regions) should not be seen as a substitute for learning suppliers' reliabilities.

We believe that if we extended our analysis to include multi-sourcing, the effects of competition, the benefits of selecting the best available supplier(s), and additional costs of diversification due to informational rents would continue to shape the solution of the manufacturer's problem. In fact, our insights suggest that with more suppliers these effects will intensify and the manufacturer will rely more on competition and less on diversification. However, there will be many more combinations of suppliers that are candidates for receiving an order, increasing the complexity of the analysis.

In our model each supplier can be one of two possible types, and the manufacturer's demand is known at the time it places its order. While we suspect that having more than two supplier types or modeling random demand would leave the spirit of our insights unchanged, we believe that the analysis would become substantially more complex. In particular, the monotonicity condition, which enables the incentive compatibility conditions to be verified, might be violated, making the derivation of the optimal contract very cumbersome.

Finally, we assumed that the supplier types are independent. Applying the results from Fudenberg and Tirole (1991, page 292), if the suppliers' types in our model were correlated, the manufacturer could implement the same contract as if the supplier types were public information. Additionally, as in Maskin and Tirole (1990) (also see discussion in Yang et al., 2008), we could allow the demand to be the manufacturer's private information, but the manufacturer would not do any better than if the demand information were public.

In future work, it could be interesting to consider the effects of supplier collusion on the contract design and the value of the dual-sourcing option. We expect that supplier competition will be weakened in the presence of supplier collusion, while at the same time informational rents could be significantly higher if the manufacturer diversifies. Therefore, it is difficult to say, a priori, if collusion will encourage or discourage diversification.

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Electronic Companion for "Supply Risk, Asymmetric Information and a Dual-Sourcing Option"

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A. Tables for Proposition 1

Primary	**Secondary**	$\tilde{r}^{t_m}$	$\tilde{r}^{t_m,t_{\overline{m}}}$
High-type	High-type	$\tilde{r}^H = \frac{c^H}{h} + \frac{K}{h\,D}$	$\tilde{r}^{HH} = \frac{c^H}{h(1-h)} + \frac{K}{h(1-h)\,D}$
High-type	Low-type	$\tilde{r}^H = \frac{c^H}{h} + \frac{K}{h\,D}$	$\tilde{r}^{HL} = \frac{c^L}{l(1-h)} + \frac{K}{l(1-h)\,D}$
Low-type	Low-type	$\tilde{r}^L = \frac{c^L}{l} + \frac{K}{l\,D}$	$\tilde{r}^{LL} = \frac{c^L}{l(1-l)} + \frac{K}{l(1-l)\,D}$

Table AT-1: Expressions for thresholds $\tilde{r}^{t_m}$ and $\tilde{r}^{t_m,t_{\overline{m}}}$ separating revenue into intervals within which the manufacturer does not order from either supplier, orders from only the primary supplier, or orders from both suppliers, under symmetric information.

Revenue	**Supplier**	$\tilde{X}_n^*$	$\tilde{q}_n^*$	$\tilde{p}_n^*$
Two high-type suppliers				
$r \le \tilde{r}^H$	Both	- No contract -		
$\tilde{r}^H < r \le \tilde{r}^{HH}$	Supplier 1	$[c^H + (1-h)\tilde{p}_1^*]D$	D	any $\tilde{p}_1^* \in [\frac{c^H}{h}, r)$
	Supplier 2	- No contract -		
$r > \tilde{r}^{HH}$	Supplier 1	$[c^H + (1-h)\tilde{p}_1^*]D$	D	any $\tilde{p}_1^* \in [\frac{c^H}{h}, r)$
	Supplier 2	$[c^H + (1-h)\tilde{p}_2^*]D$	D	any $\tilde{p}_2^* \in [\frac{c^H}{h}, r)$
High-type supplier-m and low-type supplier-$\overline{m}$				
$r \le \tilde{r}^H$	Both	- No contract -		
$\tilde{r}^H < r \le \tilde{r}^{HL}$	Supplier m	$[c^H + (1-h)\tilde{p}_m^*]D$	D	any $\tilde{p}_m^* \in [\frac{c^H}{h}, r)$
	Supplier $\overline{m}$	- No contract -		
$r > \tilde{r}^{HL}$	Supplier m	$[c^H + (1-h)\tilde{p}_m^*]D$	D	any $\tilde{p}_m^* \in [\frac{c^H}{h}, r)$
	Supplier $\overline{m}$	$[c^L + (1-l)\tilde{p}_{\overline{m}}^*]D$	D	any $\tilde{p}_{\overline{m}}^* \in [\frac{c^L}{l}, r)$
Two low-type suppliers				
$r \le \tilde{r}^L$	Both	- No contract -		
$\tilde{r}^L < r \le \tilde{r}^{LL}$	Supplier 1	$[c^L + (1-l)\tilde{p}_1^*]D$	D	any $\tilde{p}_1^* \in [\frac{c^L}{l}, r)$
	Supplier 2	- No contract -		
$r > \tilde{r}^{LL}$	Supplier 1	$[c^L + (1-l)\tilde{p}_1^*]D$	D	any $\tilde{p}_1^* \in [\frac{c^L}{l}, r)$
	Supplier 2	$[c^L + (1-l)\tilde{p}_2^*]D$	D	any $\tilde{p}_2^* \in [\frac{c^L}{l}, r)$

Table AT-2: The optimal contract menu under symmetric information.

B. Tables for Proposition 2

Primary	**Secondary**	r^{t_m}	$r^{t_m,t_{\overline{m}}}$
High-type	High-type	$r^H = \tilde{r}^H$	$r^{HH} = \tilde{r}^{HH}$
High-type	Low-type	$r^H = \tilde{r}^H$	$r^{HL} = \tilde{r}^{HL} + \frac{\alpha^H}{\alpha^L}\frac{h}{l(1-h)}\left(\frac{c^L}{l} - \frac{c^H}{h}\right)D$
Low-type	Low-type	$r^L = \tilde{r}^L + \frac{\alpha^H}{\alpha^L}\frac{h}{l}\left(\frac{c^L}{l} - \frac{c^H}{h}\right)D$	$r^{LL} = \tilde{r}^{LL} + \frac{\alpha^H}{\alpha^L}\frac{h}{l(1-l)}\left(\frac{c^L}{l} - \frac{c^H}{h}\right)D$

Table AT-3: Expressions for thresholds r^{t_m} and $r^{t_m,t_{\overline{m}}}$ separating revenue into intervals within which the manufacturer does not order from either supplier, orders from only the primary supplier, or orders from both suppliers, under asymmetric information.

Revenue	**Supplier**	X_n^*	q_n^*	p_n^*
		Two high-type suppliers		
$r \leq r^H$	Both	- No contract -		
$r^H < r \leq r^{HH}$	Supplier 1	$[c^H + (1-h)p_1^*]D$	D	any $p_1^* \in [\frac{c^H}{h}, r)$
	Supplier 2	- No contract -		
$r^{HH} < r \leq r^{HL}$	$n = 1, 2$	$[c^H + (1-h)p_n^*]D$	D	any $p_n^* \in [\frac{c^H}{h}, r)$
$r > r^{HL}$	$n = 1, 2$	$[c^H + (1-h)p_n^*]D + h(\frac{c^L}{l} - \frac{c^H}{h})D$	D	any $p_n^* \in [\frac{c^L}{l}, r)$
		High-type supplier-1 and low-type supplier-2		
$r \leq r^H$	Both	- No contract -		
$r^H < r \leq r^L$	Supplier 1	$[c^H + (1-h)p_1^*]D$	D	any $p_1^* \in [\frac{c^H}{h}, r)$
	Supplier 2	- No contract -		
$r^L < r \leq r^{HL}$	Supplier 1	$[c^H + (1-h)p_1^*]D + h(\frac{c^L}{l} - \frac{c^H}{h})D$	D	any $p_1^* \in [\frac{c^L}{l}, r)$
	Supplier 2	- No contract -		
$r > r^{HL}$	Supplier 1	$[c^H + (1-h)p_1^*]D + h(\frac{c^L}{l} - \frac{c^H}{h})D$	D	any $p_1^* \in [\frac{c^L}{l}, r)$
	Supplier 2	$\frac{c^L}{l}D$	D	$p_2^* = \frac{c^L}{l}$
		Low-type supplier-1 and high-type supplier-2		
$r \leq r^H$	Both	- No contract -		
$r^H < r \leq r^{LL}$	Supplier 1	- No contract -		
	Supplier 2	$[c^H + (1-h)p_2^*]D$	D	any $p_2^* \in [\frac{c^H}{h}, r)$
$r^{LL} < r \leq r^{HL}$	Supplier 1	- No contract -		
	Supplier 2	$[c^H + (1-h)p_2^*]D + h(\frac{c^L}{l} - \frac{c^H}{h})D$	D	any $p_2^* \in [\frac{c^L}{l}, r)$
$r > r^{HL}$	Supplier 1	$\frac{c^L}{l}D$	D	$p_1^* = \frac{c^L}{l}$
	Supplier 2	$[c^H + (1-h)p_2^*]D + h(\frac{c^L}{l} - \frac{c^H}{h})D$	D	any $p_2^* \in [\frac{c^L}{l}, r)$
		Two low-type suppliers		
$r \leq r^L$	Both	- No contract -		
$r^L < r \leq r^{LL}$	Supplier 1	$\frac{c^L}{l}D$	D	$p_1^* = \frac{c^L}{l}$
	Supplier 2	- No contract -		
$r > r^{LL}$	$n = 1, 2$	$\frac{c^L}{l}D$	D	$p_n^* = \frac{c^L}{l}$

Table AT-4: The optimal contract menu under asymmetric information.

C. Table for Proposition 4

$\tilde{r}^{t_m, t_{\overline{m}}}$	$r^{t_m, t_{\overline{m}}}$
$\tilde{r}^{HH} = \frac{c^H}{h-\omega^{HH}(1,1)} + \frac{K}{[h-\omega^{HH}(1,1)]D}$	$r^{HH} = \tilde{r}^{HH}$
$\tilde{r}^{HL} = \frac{c^L}{l-\omega^{HL}(1,1)} + \frac{K}{[l-\omega^{HL}(1,1)]D}$	$r^{HL} = \tilde{r}^{HL} + \frac{\alpha^H}{\alpha^L} \frac{h}{l-\omega^{HL}(1,1)} \left(\frac{c^L}{l} - \frac{c^H}{h}\right) D$
$\tilde{r}^{LL} = \frac{c^L}{l-\omega^{LL}(1,1)} + \frac{K}{[l-\omega^{LL}(1,1)]D}$	$r^{LL} = \tilde{r}^{LL} + \frac{\alpha^H}{\alpha^L} \frac{h}{l-\omega^{LL}(1,1)} \left(\frac{c^L}{l} - \frac{c^H}{h}\right) D$

Table AT-5: Expressions for thresholds $\tilde{r}^{t_m, t_{\overline{m}}}$ and $r^{t_m, t_{\overline{m}}}$ when the suppliers' disruptions are codependent.

D. Benchmark: Single-Sourcing Model

Lemma 2 below summarizes the manufacturer's ordering decisions, the manufacturer's profit, and the supplier's profit at the optimal contract menu under asymmetric and asymmetric information. Thresholds $\tilde{r}^H$, $\tilde{r}^L$, r^H and r^L are defined in Propositions 1 and 2.

Lemma 2. *In the symmetric information model with a single supplier, at the optimal contract menu, the manufacturer will order from the high-type supplier if and only if $r > \tilde{r}^H$, and will order from the low-type supplier if and only if $r > \tilde{r}^L$. The expected profits of both the high-type and the low-type suppliers are zero. The expected profit of the manufacturer is*

$$\alpha^H \left[(h\, r - c^H)D - K\right]^+ + \alpha^L \left[(l\, r - c^L)D - K\right]^+ . \tag{A.1}$$

In the asymmetric information model with a single supplier, at the optimal contract menu, the manufacturer will order from the high-type supplier if and only if $r > r^H$, and will order from the low-type supplier if and only if $r > r^L$. The expected profit of the low-type supplier is zero. The expected profit of the high-type supplier equals informational rent, $h\left(c^L/l - c^H/h\right) D$, for $r > r^L$, and equals zero for $r \leq r^L$. The expected profit of the manufacturer is

$$\alpha^H \left[(h\, r - c^H)D - K\right]^+ + \alpha^L \left[(l\, r - c^L)D - K - \frac{\alpha^H}{\alpha^L} h \left(\frac{c^L}{l} - \frac{c^H}{h}\right) D\right]^+ . \tag{A.2}$$

Proof. In the single-supplier model, the manufacturer's optimization problem under asymmetric information is as follows:

$$\max_{\substack{(X,q,p)(t)\\ t\in\{H,L\}}} \quad \Bigg\{ \sum_{t\in\{H,L\}} \alpha^t r E \min\Big\{D,\ \rho^t\, z^t[(q,p)(t)]\Big\}$$

$$- X(t) + p(t)\, E\Big[q(t) - \rho^t\, z^t\, [(q,p)(t)]\Big]^+ - K\, \mathbf{1}_{\{q(t)>0\}} \Bigg\},$$

Subject to

(I.C. H) $\quad \pi^H[(X,q,p)(H)] \geq \pi^H[(X,q,p)(L)],$

(I.C. L) $\quad \pi^L[(X,q,p)(L)] \geq \pi^L[(X,q,p)(H)],$

(I.R. H) $\quad \pi^H[(X,q,p)(H)] \geq 0,$

(I.R. L) $\quad \pi^H[(X,q,p)(L)] \geq 0,$

$$q(t) \geq 0,\ p(t) \geq 0,\ \text{ for } t \in \{H,L\}.$$

Under symmetric information, the manufacturer's optimization problem is the same except that constraints (I.C.H) and (I.C.L) are omitted from the model.

The solution procedures for the above optimization problems under symmetric and asymmetric information in the single-supplier model are similar to the solution procedures for the corresponding problems in the dual-sourcing model, which are presented in the proofs for Propositions 1 and 2. ■

E. Proofs of Statements

Proof of Lemma 1. The supplier's objective function (2) is piecewise linear. We focus on corner-point solutions: $z = 0$ or q. The result follows. ■

Proof of Proposition 1. The manufacturer's contract-design problem under symmetric information is given by the following maximization program:

$$\max_{\substack{(X_n,q_n,p_n)(t_n,t_{\overline{n}})\\ n=1,2;\, t_1,t_2\in\{H,L\}}} \Bigg\{ \sum_{t_1,t_2\in\{H,L\}} \alpha^{t_1}\alpha^{t_2} \Big[rE\min\Big\{D,\ \rho_1^{t_1} z_1^{t_1}[(q_1,p_1)(t_1,t_2)] + \rho_2^{t_2} z_2^{t_2}[(q_2,p_2)(t_2,t_1)]\Big\}$$
$$- X_1(t_1,t_2) + p_1(t_1,t_2)E\Big\{q_1(t_1,t_2) - \rho_1^{t_1} z_1^{t_1}[(q_1,p_1)(t_1,t_2)]\Big\}^+ - K\,\mathbf{1}_{\{q_1(t_1,t_2)>0\}}$$
$$- X_2(t_2,t_1) + p_2(t_2,t_1)E\Big\{q_2(t_2,t_1) - \rho_2^{t_2} z_2^{t_2}[(q_2,p_2)(t_2,t_1)]\Big\}^+ - K\,\mathbf{1}_{\{q_2(t_2,t_1)>0\}} \Big]\Bigg\} \tag{A.3}$$

Subject to for $n = 1,2,\ t_n, t_{\overline{n}} \in \{H,L\}$,

$\pi_n^{t_n}[(X_n,q_n,p_n)(t_n,t_{\overline{n}})] \geq 0$,

$q_n(t_n,t_{\overline{n}}) \geq 0,\ p_n(t_n,t_{\overline{n}}) \geq 0$.

From the supplier's optimal profit (2), the manufacturer's transfer payment to supplier n net of the expected penalty received equals supplier-n's optimal profit plus its cost of production. Under contract $(X_n,q_n,p_n)(t_n,t_{\overline{n}})$, this equality has the following form:

$$X_n(t_n,t_{\overline{n}}) - p_n(t_n,t_{\overline{n}})E\Big\{q_n(t_n,t_{\overline{n}}) - \rho_n^{t_n} z_n^{t_n}[(q_n,p_n)(t_n,t_{\overline{n}})]\Big\}^+$$
$$= \pi_n^{t_n}[(X_n,q_n,p_n)(t_n,t_{\overline{n}})] + c^{t_n} z_n^{t_n}[(q_n,p_n)(t_n,t_{\overline{n}})]. \tag{A.4}$$

We substitute (A.4) into the objective function of (A.3), obtaining

$$\sum_{t_1,t_2\in\{H,L\}} \alpha^{t_1}\alpha^{t_2} \Big[rE\min\Big\{D,\ \rho_1^{t_1} z_1^{t_1}[(q_1,p_1)(t_1,t_2)] + \rho_2^{t_2} z_2^{t_2}[(q_2,p_2)(t_2,t_1)]\Big\}$$
$$- \pi_1^{t_1}[(X_1,q_1,p_1)(t_1,t_2)] - c^{t_1} z_1^{t_1}[(q_1,p_1)(t_1,t_2)] - K\,\mathbf{1}_{\{q_1(t_1,t_2)>0\}}$$
$$- \pi_2^{t_2}[(X_2,q_2,p_2)(t_2,t_1)] - c^{t_2} z_2^{t_2}[(q_2,p_2)(t_2,t_1)] - K\,\mathbf{1}_{\{q_2(t_2,t_1)>0\}} \Big] \tag{A.5}$$

Observe from (A.4) and (A.5) that, to maximize its profit given any $(q_n,p_n)(t_n,t_{\overline{n}})$ for $n = 1,2$, the manufacturer must choose $X_n(t_n,t_{\overline{n}})$ so that the suppliers' expected profits equal their reservation profits, that is, the *individual rationality* constraints for the two suppliers must be binding at the optimal contract menu. Thus, we substitute $\pi_n^{t_n}[(X_n,q_n,p_n)(t_n,t_{\overline{n}})] = 0$ in (A.5), and write the *symmetric information* problem as the sum of four maximization problems, each over (q_1,p_1) and (q_2,p_2) for a combination of the types of the two suppliers:

$$\sum_{t_1,t_2\in\{H,L\}} \Bigg[\alpha^{t_1}\alpha^{t_2} \max_{\substack{(q_1,p_1)(t_1,t_2)\geq 0\\ (q_2,p_2)(t_2,t_1)\geq 0}} \Bigg\{ rE\min\Big[D,\ \rho_1^{t_1} z_1^{t_1}[(q_1,p_1)(t_1,t_2)] + \rho_2^{t_2} z_2^{t_2}[(q_2,p_2)(t_2,t_1)]\Big]$$
$$- \Big[c^{t_1} z_1^{t_1}[(q_1,p_1)(t_1,t_2)] + K\,\mathbf{1}_{\{q_1(t_1,t_2)>0\}}\Big] - \Big[c^{t_2} z_2^{t_2}[(q_2,p_2)(t_2,t_1)] + K\,\mathbf{1}_{\{q_2(t_2,t_1)>0\}}\Big]\Bigg\}\Bigg]. \tag{A.6}$$

From Lemma 1, supplier n will run production of size $z_n^{t_n}[(q_n,p_n)(t_n,t_{\overline{n}})] = q_n(t_n,t_{\overline{n}})$ when $p_n(t_n,t_{\overline{n}}) \geq c^{t_n}/\theta^{t_n}$, or $z_n^{t_n}[(q_n,p_n)(t_n,t_{\overline{n}})] = 0$ when $p_n(t_n,t_{\overline{n}}) < c^{t_n}/\theta^{t_n}$. Notice that the production size $z_n^{t_n}[(q_n,p_n)(t_n,t_{\overline{n}})] = 0$ is attainable even if $p_n(t_n,t_{\overline{n}}) \geq c^{t_n}/\theta^{t_n}$, by setting an order quantity of $q_n(t_n,t_{\overline{n}}) = 0$. Furthermore, solutions $\{(q_n,p_n)(t_n,t_{\overline{n}}) : p_n(t_n,t_{\overline{n}}) < c^{t_n}/\theta^{t_n}\}$ and $\{(q_n,p_n)(t_n,t_{\overline{n}}) : p_n(t_n,t_{\overline{n}}) \geq c^{t_n}/\theta^{t_n}, q_n(t_n,t_{\overline{n}}) = 0\}$ leads to the same objective value of (A.6). Therefore, it is sufficient to focus on the case of $p_n(t_n,t_{\overline{n}}) \geq c^{t_n}/\theta^{t_n}$. (If $q_n(t_n,t_{\overline{n}}) = 0$ at the optimal solution, then $p_n(t_n,t_{\overline{n}})$ is irrelevant.) We roll $z_n^{t_n}[(q_n,p_n)(t_n,t_{\overline{n}})] = q_n(t_n,t_{\overline{n}})$, $n = 1,2$, into each of four maximization problems in (A.6), obtaining for $t_1,t_2 \in \{H,L\}$

$$\max_{\substack{q_1(t_1,t_2)\geq 0\\ q_2(t_2,t_1)\geq 0}} \Big\{ rE\min\Big[D,\ \rho_1^{t_1} q_1(t_1,t_2) + \rho_2^{t_2} q_2(t_2,t_1)\Big] - \Big[c^{t_1} q_1(t_1,t_2) + K\,\mathbf{1}_{\{q_1(t_1,t_2)>0\}}\Big] - \Big[c^{t_2} q_2(t_2,t_1) + K\,\mathbf{1}_{\{q_2(t_2,t_1)>0\}}\Big]\Big\}. \quad \text{(A.7)}$$

Now we solve problem (A.7) above to find the optimal order quantities $q_1(t_1,t_2)$ and $q_2(t_2,t_1)$. Recall that, for two suppliers of types t_1 and t_2, we use label m to indicate the primary supplier, that is, $\theta^{t_m} \geq \theta^{t_{\overline{m}}}$. (We let $m = 1$ if $t_m = t_{\overline{m}}$.) The manufacturer's order quantities for the primary and the secondary suppliers are $q_m(t_m,t_{\overline{m}})$ and $q_{\overline{m}}(t_{\overline{m}},t_m)$. To ease the notation, we hereafter write the order quantities as $(q_m,q_{\overline{m}})$. Because the objective function (A.7) is piecewise linear in the order quantities, without loss of optimality, we focus on the corner-point solutions only: $(q_m,q_{\overline{m}}) \in \big\{(0,0),(D,0),(0,D),(D,D)\big\}$. The objective values of (A.7) at the four corner points are

$$\begin{array}{ll} 0, & (q_m,q_{\overline{m}}) = (0,0),\\ (\theta^{t_m} r - c^{t_m})D - K, & (q_m,q_{\overline{m}}) = (D,0),\\ (\theta^{t_{\overline{m}}} r - c^{t_{\overline{m}}})D - K, & (q_m,q_{\overline{m}}) = (0,D),\\ [(\theta^{t_m} r - c^{t_m})D - K] + \big[\big((1-\theta^{t_m})\theta^{t_{\overline{m}}} r - c^{t_{\overline{m}}}\big)D - K\big], & (q_m,q_{\overline{m}}) = (D,D). \end{array} \quad \text{(A.8)}$$

Observe that $(q_m,q_{\overline{m}}) = (0,D)$ can be dropped from consideration, because it is dominated by $(q_m,q_{\overline{m}}) = (D,0)$. This observation holds trivially, if $t_m = t_{\overline{m}}$. Consider $t_m = H$ and $t_{\overline{m}} = L$. The difference between the objectives at $(q_m,q_{\overline{m}}) = (D,0)$ and $(0,D)$ is $[(h\,r - c^H) - (l\,r - c^L)]D = [h(r - c^H/h) - l(r - c^L/l)]D > [l(r - c^H/h) - l(r - c^L/l)]D$. Under our assumption $c^L/l > c^H/h$ (see page 6), the rightmost term must be positive, and thus $(q_m,q_{\overline{m}}) = (D,0)$ is strictly better.

We now identify the condition under which each of the three candidate optimal solutions, $(q_m, q_{\overline{m}}) = (D, D)$, $(0,0)$ and $(D,0)$, is optimal. It follows from (A.8) that $(q_m, q_{\overline{m}}) = (D, D)$ dominates $(D,0)$, if and only if $\left((1-\theta^{t_{\overline{m}}})\theta^{t_{\overline{m}}} r - c^{t_{\overline{m}}}\right) D - K > 0$, and $(q_m, q_{\overline{m}}) = (D,0)$ dominates $(0,0)$, if and only if $(\theta^{t_m} r - c^{t_m})D - K > 0$. We define $\tilde{r}^{t_m, t_{\overline{m}}}$ and $\tilde{r}^{t_m}$ to be

$$\tilde{r}^{t_m, t_{\overline{m}}} \stackrel{\text{def}}{=} \inf\Big\{ r : \left((1-\theta^{t_m})\theta^{t_{\overline{m}}} r - c^{t_{\overline{m}}}\right) D - K > 0 \Big\} \tag{A.9a}$$

$$\tilde{r}^{t_m} \stackrel{\text{def}}{=} \inf\Big\{ r : (\theta^{t_m} r - c^{t_m})D - K > 0 \Big\} \tag{A.9b}$$

Hence, when $r > \tilde{r}^{t_m, t_{\overline{m}}}$, $(q_m, q_{\overline{m}}) = (D, D)$ dominates both $(D,0)$ and $(0,0)$, and, thus, is optimal. When $r \le \tilde{r}^{t_m}$, $(q_m, q_{\overline{m}}) = (0,0)$ must be optimal. When $\tilde{r}^{t_m} < r \le \tilde{r}^{t_m, t_{\overline{m}}}$, $(q_m, q_{\overline{m}}) = (D,0)$ is optimal.

Finally, given the optimal order quantities $q_n(t_n, t_{\overline{n}})$ we compute the optimal transfer payments and penalties. When $q_n(t_n, t_{\overline{n}}) = D$ is optimal, it must be true that $r > c^{t_n}/\theta^{t_n}$ (see equation (A.8)). In such a case, the optimal penalty $p_n(t_n, t_{\overline{n}})$ can be chosen from interval $[c^{t_n}/\theta^{t_n}, r)$. (All such penalties will cause the supplier to attempt regular production.) When $q_n(t_n, t_{\overline{n}}) = 0$ is optimal, we allow $p_n(t_n, t_{\overline{n}})$ to be any value in $[0, r)$. Given the penalty, the optimal transfer payment $X_n(t_n, t_{\overline{n}})$ is given by equation (A.4). ■

Proof of Proposition 2. First, we roll equation (A.4) into the manufacturer's objective function (4a), obtaining the following maximization problem:

$$\begin{aligned}
\max \Bigg\{ & \sum_{t_1, t_2 \in \{H, L\}} \alpha^{t_1}\alpha^{t_2} \Bigg[rE \min\Big\{ D,\, \rho_1^{t_1} z_1^{t_1}[(q_1,p_1)(t_1,t_2)] + \rho_2^{t_2} z_2^{t_2}[(q_2,p_2)(t_2,t_1)] \Big\} \\
& - \Big\{ c^{t_1} z_1^{t_1}[(q_1,p_1)(t_1,t_2)] + K\,\mathbf{1}_{\{q_1(t_1,t_2)>0\}} \Big\} - \Big\{ c^{t_2} z_2^{t_2}[(q_2,p_2)(t_2,t_1)] + K\,\mathbf{1}_{\{q_2(t_2,t_1)>0\}} \Big\} \Bigg] \qquad \text{(A.10a)} \\
& - \sum_{t_1 = H, L} \Big\{ \alpha^{t_1} \Pi_1^{t_1}(t_1) \Big\} - \sum_{t_2 = H, L} \Big\{ \alpha^{t_2} \Pi_2^{t_2}(t_2) \Big\} \Bigg\}
\end{aligned}$$

Subject to For $n = 1, 2$

$$\Pi_n^H(H) \ge \Pi_n^H(L) \tag{A.10b}$$

$$\Pi_n^L(L) \ge \Pi_n^L(H), \tag{A.10c}$$

$$\Pi_n^H(H) \ge 0, \tag{A.10d}$$

$$\Pi_n^L(L) \ge 0, \tag{A.10e}$$

$$q_n(t_n, t_{\overline{n}}) \ge 0,\ p_n(t_n, t_{\overline{n}}) \ge 0,\ \text{for } t_1, t_2 \in \{H, L\}, \tag{A.10f}$$

where $\Pi_n^{t_n}(s) \stackrel{\text{def}}{=} E_{t_{\overline{n}}}\Big\{\pi_n^{t_n}[(X_n, q_n, p_n)(s, t_{\overline{n}})]\Big\}$ is supplier-n's expected profit when it reports itself as of type-s, $s \in \{H, L\}$, where the expectation is taken over supplier-$\overline{n}$'s type.

Here is our plan to solve problem (A.10). First, we reduce the *incentive compatibility* and *individual rationality* constraints (A.10b–A.10f) to an equivalent set of constraints, among which there are *monotonicity* constraints for the two suppliers. Then, we temporarily relax the *monotonicity* constraints. We show that the optimal solution to the relaxation satisfies the monotonicity constraints and, thus, is optimal for the original problem.

To reduce problem (A.10), we first rearrange the *incentive compatibility* constraints (A.10b) and (A.10c) and the *individual rationality* constraints (A.10d) and (A.10e) for supplier $n = 1, 2$. Applying the definition of a high supplier-type's reliability advantage (Definition 1) under the contract for low-type supplier-n, $(X_n, q_n, p_n)(L, t_{\overline{n}})$, we represent high-type supplier-n's expected profit, when reporting itself as of low-type, as

$$\Pi_n^H(L) = \Pi_n^L(L) + E_{t_{\overline{n}}}\Big\{\Gamma_n[(q_n, p_n)(L, t_{\overline{n}})]\Big\}.$$

Similarly, applying Definition 1 with the contract for high-type supplier-n, $(X_n, q_n, p_n)(H, t_{\overline{n}})$, we represent low-type supplier-n's expected profit, when reporting itself as of high-type, as

$$\Pi_n^L(H) = \Pi_n^H(H) - E_{t_{\overline{n}}}\Big\{\Gamma_n[(q_n, p_n)(H, t_{\overline{n}})]\Big\}.$$

We substitute these two equalities into the right-hand-side of the *incentive compatibility* constraints, (A.10b) and (A.10c), obtaining

$$\Pi_n^H(H) \geq \Pi_n^L(L) + E_{t_{\overline{n}}}\Big\{\Gamma_n[(q_n, p_n)(L, t_{\overline{n}})]\Big\}, \tag{A.11a}$$

$$\Pi_n^L(L) \geq \Pi_n^H(H) - E_{t_{\overline{n}}}\Big\{\Gamma_n[(q_n, p_n)(H, t_{\overline{n}})]\Big\}. \tag{A.11b}$$

Furthermore, inequalities (A.11a), $\Gamma_n[(q_n, p_n)(L, t_{\overline{n}})] \geq 0$ (see the discussion following (3)) and $\Pi_n^L(L) \geq 0$ (constraint (A.10e), *individual rationality* for the low-type) together imply $\Pi_n^H(H) \geq 0$. That is, the *individual rationality* constraint for high-type supplier-n (A.10d) is redundant.

Using the new *incentive compatibility* constraint (A.11) and *individual rationality* constraint (A.10e), we then choose $X_n(t_n, t_{\overline{n}})$ optimally for any given $(q_n, p_n)(t_n, t_{\overline{n}})$. The objective function (A.10a) suggests that the objective is maximized when $X_n(t_n, H)$ and $X_n(t_n, L)$ are chosen such that the expected profit of supplier n of type-t_n, $\Pi_n^{t_n}(t_n)$, is minimized. Thus, at the optimal solution, the *individual rationality* constraint (A.10e) reduces to

$$\Pi_n^L(L) = 0. \tag{A.12}$$

Similarly, at the optimal solution the *incentive compatibility* constraint (A.11a) reduces to

$$\Pi_n^H(H) = E_{t_{\overline{n}}}\Big\{\Gamma_n[(q_n, p_n)(L, t_{\overline{n}})]\Big\}. \tag{A.13}$$

We substitute (A.12) and (A.13) in the *incentive compatibility* constraints (A.11), obtaining

$$(\textit{Monotonicity}) \quad E_{t_{\overline{n}}}\Big\{\Gamma_n[(q_n, p_n)(H, t_{\overline{n}})]\Big\} \geq E_{t_{\overline{n}}}\Big\{\Gamma_n[(q_n, p_n)(L, t_{\overline{n}})]\Big\},\ n = 1, 2 \tag{A.14}$$

which is commonly called the *monotonicity* constraint in the information economics literature.

So far, we have reduced the original *incentive compatibility* and *individual rationality* constraints (A.10b–A.10e) to constraints (A.12–A.14). We roll (A.12) and (A.13) into the objective function (A.10a), obtaining

$$\begin{aligned} \max \Bigg\{ & \sum_{t_1, t_2 \in \{H, L\}} \alpha^{t_1}\alpha^{t_2}\Bigg[rE\min\Big\{D,\ \rho_1^{t_1} z_1^{t_1}[(q_1, p_1)(t_1, t_2)] + \rho_2^{t_2} z_2^{t_2}[(q_2, p_2)(t_2, t_1)]\Big\} \\ & - \Big\{c^{t_1} z_1^{t_1}[(q_1, p_1)(t_1, t_2)] + K\,\mathbf{1}_{\{q_1(t_1,t_2)>0\}}\Big\} - \Big\{c^{t_2} z_2^{t_2}[(q_2, p_2)(t_2, t_1)] + K\,\mathbf{1}_{\{q_2(t_2,t_1)>0\}}\Big\}\Bigg] \\ & - \alpha^H E_{t_2}\Big\{\Gamma_1[(q_1, p_1)(L, t_2)]\Big\} - \alpha^H E_{t_1}\Big\{\Gamma_2[(q_2, p_2)(L, t_1)]\Big\}\Bigg\}. \end{aligned} \tag{A.15}$$

We expand the summation over t_1 and t_2 and the expectations over t_1 and t_2 in the objective function (A.15). The manufacturer's contract design problem (4) is reduced to

$$\begin{aligned} \max_{\substack{(q_n, p_n)(t_n, t_{\overline{n}}) \\ n=1,2;\, t_1, t_2 \in \{H, L\}}} \Bigg\{ & \left(\alpha^H\right)^2 \Bigg[rE\min\Big\{D,\ \rho_1^H z_1^H[(q_1, p_1)(H, H)] + \rho_2^H z_2^H[(q_2, p_2)(H, H)]\Big\} \\ & - \Big\{c^H z_1^H[(q_1, p_1)(H, H)] + K\,\mathbf{1}_{\{q_1(H,H)>0\}}\Big\} \\ & - \Big\{c^H z_2^H[(q_2, p_2)(H, H)] + K\,\mathbf{1}_{\{q_2(H,H)>0\}}\Big\}\Bigg] \end{aligned}$$

$$
\begin{aligned}
&+ \left(\alpha^H \alpha^L\right) \Big[rE \min\Big\{ D,\ \rho_1^H z_1^H[(q_1,p_1)(H,L)] + \rho_2^L z_2^L[(q_2,p_2)(L,H)] \Big\} \\
&\qquad - \Big\{ c^H z_1^H[(q_1,p_1)(H,L)] + K\,\mathbf{1}_{\{q_1(H,L)>0\}} \Big\} \\
&\qquad - \Big\{ c^L z_2^L[(q_2,p_2)(L,H)] + K\,\mathbf{1}_{\{q_2(L,H)>0\}} \Big\} \Big] \\
&+ \left(\alpha^L \alpha^H\right) \Big[rE \min\Big\{ D,\ \rho_1^L z_1^L[(q_1,p_1)(L,H)] + \rho_2^H z_2^H[(q_2,p_2)(H,L)] \Big\} \\
&\qquad - \Big\{ c^L z_1^L[(q_1,p_1)(L,H)] + K\,\mathbf{1}_{\{q_1(L,H)>0\}} \Big\} \qquad\qquad \text{(A.16)} \\
&\qquad - \Big\{ c^H z_2^H[(q_2,p_2)(H,L)] + K\,\mathbf{1}_{\{q_2(H,L)>0\}} \Big\} \Big] \\
&+ \left(\alpha^L\right)^2 \Big[rE \min\Big\{ D,\ \rho_1^L z_1^L[(q_1,p_1)(L,L)] + \rho_2^L z_2^L[(q_2,p_2)(L,L)] \Big\} \\
&\qquad - \Big\{ c^L z_1^L[(q_1,p_1)(L,L)] + K\,\mathbf{1}_{\{q_1(L,L)>0\}} \Big\} \\
&\qquad - \Big\{ c^L z_2^L[(q_2,p_2)(L,L)] + K\,\mathbf{1}_{\{q_2(L,L)>0\}} \Big\} \Big] \\
&- \left(\alpha^H\right)^2 \Gamma_1[(q_1,p_1)(L,H)] - \alpha^H \alpha^L \Gamma_1[(q_1,p_1)(L,L)] \\
&- \left(\alpha^H\right)^2 \Gamma_2[(q_2,p_2)(L,H)] - \alpha^H \alpha^L \Gamma_2[(q_2,p_2)(L,L)] \Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
\text{Subject to} \quad & \text{for } n = 1,2 \text{ and } t_1, t_2 \in \{H, L\}, \\
(\textit{Monotonicity}) \quad & E\Big\{ \Gamma_n[(q_n,p_n)(H,t_{\overline{n}})] \Big\} \geq E\Big\{ \Gamma_n[(q_n,p_n)(L,t_{\overline{n}})] \Big\}, \\
& q_n(t_n,t_{\overline{n}}) \geq 0,\ p_n(t_n,t_{\overline{n}}) \geq 0.
\end{aligned}
$$

This concludes our first major step of reducing problem (A.10).

Now, we carry on with the second major step: to solve the equivalent problem (A.16) to find the optimal $(q_n, p_n)(t_n, t_{\overline{n}})$. We first temporarily drop the *monotonicity* constraint (A.14) and solve problem (A.16) with only nonnegativity constraints. We move $\Gamma_n[(q_n,p_n)(L,t_{\overline{n}})]$ in (A.16) to be with other terms that depend on $(q_n,p_n)(L,t_{\overline{n}})$. This allows us to rearrange problem (A.16) as a weighted sum of four maximization problems:

$$
\begin{aligned}
&\left(\alpha^H\right)^2 \max_{\substack{(q_1,p_1)(H,H)\geq 0 \\ (q_2,p_2)(H,H)\geq 0}} \Big\{ rE \min\Big\{ D,\ \rho_1^H z_1^H[(q_1,p_1)(H,H)] + \rho_2^H z_2^H[(q_2,p_2)(H,H)] \Big\} \qquad \text{(A.17a)} \\
&\quad - \Big\{ c^H z_1^H[(q_1,p_1)(H,H)] + K\,\mathbf{1}_{\{q_1(H,H)>0\}} \Big\} \\
&\quad - \Big\{ c^H z_2^H[(q_2,p_2)(H,H)] + K\,\mathbf{1}_{\{q_2(H,H)>0\}} \Big\} \Big\}
\end{aligned}
$$

$$+\left(\alpha^H\alpha^L\right)\max_{\substack{(q_1,p_1)(H,L)\geq 0\\(q_2,p_2)(L,H)\geq 0}}\Bigg\{rE\min\Big\{D,\ \rho_1^H z_1^H[(q_1,p_1)(H,L)]+\rho_2^L z_2^L[(q_2,p_2)(L,H)]\Big\} \tag{A.17b}$$

$$-\Big\{c^H z_1^H[(q_1,p_1)(H,L)]+K\,\mathbf{1}_{\{q_1(H,L)>0\}}\Big\}$$

$$-\Big\{c^L z_2^L[(q_2,p_2)(L,H)]+K\,\mathbf{1}_{\{q_2(L,H)>0\}}+\frac{\alpha^H}{\alpha^L}\Gamma_2[(q_2,p_2)(L,H)]\Big\}\Bigg\}$$

$$+\left(\alpha^L\alpha^H\right)\max_{\substack{(q_1,p_1)(L,H)\geq 0\\(q_2,p_2)(H,L)\geq 0}}\Bigg\{rE\min\Big\{D,\ \rho_1^L z_1^L[(q_1,p_1)(L,H)]+\rho_2^H z_2^H[(q_2,p_2)(H,L)]\Big\} \tag{A.17c}$$

$$-\Big\{c^L z_1^L[(q_1,p_1)(L,H)]+K\,\mathbf{1}_{\{q_1(L,H)>0\}}+\frac{\alpha^H}{\alpha^L}\Gamma_1[(q_1,p_1)(L,H)]\Big\}$$

$$-\Big\{c^H z_2^H[(q_2,p_2)(H,L)]+K\,\mathbf{1}_{\{q_2(H,L)>0\}}\Big\}\Bigg\}$$

$$+\left(\alpha^L\right)^2\max_{\substack{(q_1,p_1)(L,L)\geq 0\\(q_2,p_2)(L,L)\geq 0}}\Bigg\{rE\min\Big\{D,\ \rho_1^L z_1^L[(q_1,p_1)(L,L)]+\rho_2^L z_2^L[(q_2,p_2)(L,L)]\Big\} \tag{A.17d}$$

$$-\Big\{c^L z_1^L[(q_1,p_1)(L,L)]+K\,\mathbf{1}_{\{q_1(L,L)>0\}}+\frac{\alpha^H}{\alpha^L}\Gamma_1[(q_1,p_1)(L,L)]\Big\}$$

$$-\Big\{c^L z_2^L[(q_2,p_2)(L,L)]+K\,\mathbf{1}_{\{q_2(L,L)>0\}}+\frac{\alpha^H}{\alpha^L}\Gamma_2[(q_2,p_2)(L,L)]\Big\}\Bigg\}$$

We solve each of the four maximization problems in (A.17). First, for each of the four problems, note that as in the proof of Proposition 1 in the symmetric information case, we may assume without loss of generality that $z_n^{t_n}[(q_n,p_n)(t_n,t_{\overline{n}})]=q_n(t_n,t_{\overline{n}})$, where $p_n(t_n,t_{\overline{n}})\geq c^{t_n}/\theta^{t_n}$ to ensure that supplier n attempts production of size q_n. Next, from equation (3), $\Gamma_n[(X_n,q_n,p_n)(L,t_{\overline{n}})]$ is increasing in $p_n(L,t_{\overline{n}})$. Hence, it is optimal to set $p_n(L,t_{\overline{n}})$ to be its minimum c^L/l, which gives $\Gamma_n[(X_n,q_n,p_n)(L,t_{\overline{n}})]=h\,(\frac{c^L}{l}-\frac{c^H}{h})\,q_n(L,t_{\overline{n}})$. Given these refinements, the procedure to find the optimal order quantities is similar to that for problem (A.7) in the symmetric information case. We present the optimal solution to the relaxation (A.17) in the following table, where we make use of the following thresholds:

$$r^{t_m}\stackrel{\text{def}}{=}\inf\left\{r:(\theta^{t_m}r-c^{t_m})D-K-\frac{\alpha^H}{\alpha^L}h\left(\frac{c^L}{l}-\frac{c^H}{h}\right)D\mathbf{1}_{\{t_m=L\}}>0\right\} \tag{A.18a}$$

$$r^{t_m,t_{\overline{m}}}\stackrel{\text{def}}{=}\inf\left\{r:\left[\theta^{t_{\overline{m}}}(1-\theta^{t_m})r-c^{t_{\overline{m}}}\right]D-K-\frac{\alpha^H}{\alpha^L}h\left(\frac{c^L}{l}-\frac{c^H}{h}\right)D\mathbf{1}_{\{t_{\overline{m}}=L\}}>0\right\} \tag{A.18b}$$

Problem (A.17a), $t_1 = t_2 = H$			
When $r \le r^H$:			
$q_1(H,H) = 0$	$q_2(H,H) = 0$	$p_1(H,H) \ge 0$	$p_2(H,H) \ge 0$
When $r^H < r \le r^{HH}$:			
$q_1(H,H) = D$	$q_2(H,H) = 0$	$p_1(H,H) \ge \frac{c^H}{h}$	$p_2(H,H) \ge 0$
When $r > r^{HH}$:			
$q_1(H,H) = D$	$q_2(H,H) = D$	$p_1(H,H) \ge \frac{c^H}{h}$	$p_2(H,H) \ge \frac{c^H}{h}$
Problem (A.17b), $t_1 = H$ and $t_2 = L$			
When $r \le r^H$:			
$q_1(H,L) = 0$	$q_2(L,H) = 0$	$p_1(H,L) \ge 0$	$p_2(L,H) \ge 0$
When $r^H < r \le r^{HL}$:			
$q_1(H,L) = D$	$q_2(L,H) = 0$	$p_1(H,L) \ge \frac{c^H}{h}$	$p_2(L,H) \ge 0$
When $r > r^{HL}$:			
$q_1(H,L) = D$	$q_2(L,H) = D$	$p_1(H,L) \ge \frac{c^H}{h}$	$p_2(L,H) = \frac{c^L}{l}$
Problem (A.17c), $t_1 = L$ and $t_2 = H$			
The solution is identical to the solution for problem (A.17b), except that the indices of the two suppliers are swapped.			
Problem (A.17d), $t_1 = L$ and $t_2 = L$			
When $r \le r^L$:			
$q_1(L,L) = 0$	$q_2(L,L) = 0$	$p_1(L,L) \ge 0$	$p_2(L,L) \ge 0$
When $r^L < r \le r^{LL}$:			
$q_1(L,L) = D$	$q_2(L,L) = 0$	$p_1(L,L) = \frac{c^L}{l}$	$p_2(L,L) \ge 0$
When $r > r^{LL}$:			
$q_1(L,L) = D$	$q_2(L,L) = D$	$p_1(L,L) = \frac{c^L}{l}$	$p_2(L,L) = \frac{c^L}{l}$

In Lemma 3, we show that the optimal solution to the relaxation problem (A.17) in the above table satisfies the *monotonicity* constraint (A.14) for supplier 1 and supplier 2, as long as we restrict $p_n(H, t_{\overline{n}}) \ge \frac{c^L}{l}$ whenever $q_n(L, t_{\overline{n}}) = D$ at the optimal solution. Therefore, with the additional restriction on $p_n(H, t_{\overline{n}})$, the optimal solution to the relaxation in the table above is also optimal for the original problem (A.10).

We now compute the optimal transfer payments using (A.4), (A.12) and (A.13). Without loss of optimality, we let $\pi_n^H[(X_n, q_n, p_n)(H, t_{\overline{n}})] = \Gamma_n[(q_n, p_n)(L, t_{\overline{n}})]$ and $\pi_n^L[(X_n, q_n, p_n)(L, t_{\overline{n}})] = 0$, for $t_{\overline{n}} = H, L$. Hence, it is optimal to let

$$X_n(H, t_{\overline{n}}) = \Gamma_n[(q_n, p_n)(L, t_{\overline{n}})] + c^{t_n} z_n^{t_n}[(q_n, p_n)(H, t_{\overline{n}})] \qquad \text{(A.19a)}$$

$$+ p_n(H, t_{\overline{n}}) E\left[q_n(H, t_{\overline{n}}) - \rho_n^{t_n} z_n^{t_n}[(q_n, p_n)(H, t_{\overline{n}})]\right]^+,$$

$$X_n(L, t_{\overline{n}}) = c^{t_n} z_n^{t_n}[(q_n, p_n)(L, t_{\overline{n}})] \tag{A.19b}$$
$$+ p_n(L, t_{\overline{n}}) E \left[q_n(L, t_{\overline{n}}) - \rho_n^{t_n} z_n^{t_n}[(q_n, p_n)(L, t_{\overline{n}})]\right]^+ .$$

■

Lemma 3. *The optimal solutions to the relaxation* (A.17) *(in the table above) satisfy the monotonicity constraints* (A.14) *for supplier* 1 *and supplier* 2*, if we restrict* $p_n(H, t_{\overline{n}}) \geq \frac{c^L}{l}$ *whenever* $q_n(L, t_{\overline{n}}) = D$*, for* $t_{\overline{n}} \in \{H, L\}$*.*

Proof. It is sufficient to show that, for $n = 1, 2$ and $t_{\overline{n}} = H, L$, with the additional restriction $p_n(H, t_{\overline{n}}) \geq \frac{c^L}{l}$, the optimal solution to (A.17) satisfies

$$\Gamma_n[(q_n, p_n)(H, t_{\overline{n}})] \geq \Gamma_n[(q_n, p_n)(L, t_{\overline{n}})]. \tag{A.20}$$

When $\Gamma_n[(q_n, p_n)(L, t_{\overline{n}})] = 0$ (i.e., $q_n(L, t_{\overline{n}}) = 0$), the inequality (A.20) holds trivially. We now focus on the case when $q_n(L, t_{\overline{n}}) = D$.

From equation (3), $\Gamma_n(q, p)$ is increasing in both q and p. Therefore, it suffices to show that $q_n(H, t_{\overline{n}}) \geq q_n(L, t_{\overline{n}})$ and $p_n(H, t_{\overline{n}}) \geq p_n(L, t_{\overline{n}})$ for all r. First, it can be verified that the optimal solution to (A.17) satisfies $q_n(H, t_{\overline{n}}) \geq q_n(L, t_{\overline{n}})$ for all r. Next, recall that when $q_n(L, t_{\overline{n}}) = D$, $p_n(L, t_{\overline{n}}) = \frac{c^L}{l}$ is optimal. Since the assumption in the lemma gives $p_n(H, t_{\overline{n}}) \geq \frac{c^L}{l}$ whenever $q_n(L, t_{\overline{n}}) = D$, we have $p_n(H, t_{\overline{n}}) \geq p_n(L, t_{\overline{n}})$. Inequality (A.20) follows. ■

Proof of Corollary 2. We want to show that, for h and l such that $h + l > 1$, as both h and l decrease by ϵ, in the symmetric information model $\tilde{r}^{HL}$ always decreases, but in the asymmetric information model, there exist h and l, $h + l > 1$, such that r^{HL} increases.

Let $h = l + \nu$ for some $\nu > 0$. We write $\tilde{r}^{HL}$ as a function of l:

$$\tilde{r}^{HL}(l) = \frac{c^L + \frac{K}{D}}{(1 - l - \nu)l}.$$

Its first-order derivative with respective to l is strictly positive for $2\,l + \nu > 1$, that is, $h + l > 1$. Therefore, for any $h + l > 1$, as both h and l decrease by ϵ, $\tilde{r}^{HL}$ decreases.

Similarly, we write r^{HL} in the asymmetric information model as:

$$r^{HL}(l) = \frac{c^L + \frac{K}{D} + \frac{\alpha^H}{\alpha^L}\left(\frac{l+\nu}{l}c^L - c^H\right)}{(1-l-\nu)l}.$$

Its first order derivative with respect to l equals

$$\left[r^{HL}(l)\right]' = \frac{\left(c^L + \frac{K}{D}\right) l(2l+\nu-1) - \frac{\alpha^H}{\alpha^L}\Big\{c^L\left[2(l+\nu)(1-\nu-l) - l\right] + c^H l(2l+\nu-1)\Big\}}{l^3(1-\nu-l)^2}.$$

For any c^H, c^L, $\frac{K}{D}$ and α^H, there exist l and $h = l+\nu$, for some $\nu > 0$, such that $h + l > 1$, $h > l$, $c^L/l > c^H/h$ and $[r^{HL}(l)]' < 0$. To see this, we let $h = 1 - \xi + \xi^2$ and $l = \xi$, for some $\xi \in (0,1)$, and substitute $l = \xi$ and $\nu = h - l = 1 - 2\xi + \xi^2$ into $[r^{HL}(l)]'$, to obtain

$$[r^{HL}(l)]' \left[l^3(1-\nu-l)^2\right] = -\frac{\alpha^H}{\alpha^L}c^L\xi + \frac{\alpha^H}{\alpha^L}4c^L\xi^2 + \left[c^L + \frac{K}{D} - \frac{\alpha^H}{\alpha^L}(4c^L + c^H)\right]\xi^3 + \frac{\alpha^H}{\alpha^L}2c^L\xi^4 \quad \text{(A.21)}$$

As ξ approaches zero, the right-hand-side of (A.21) approaches zero from below (since the right-hand-side of (A.21) is polynomial in ξ and its leading term is negative). Therefore, one can always pick some $\xi > 0$ that will yield $[r^{HL}(l)]' < 0$. Note that h need not be extremely close to 1 and l need not be extremely small. For instance, suppose $c^H = c^L$, $K = 0$ and $\alpha^H > 2\alpha^L$. At $l = 1/2$ and $\nu = 1/8$ (i.e., $h = 5/8$), $[r^{HL}(l)]' \left[l^3(1-\nu-l)^2\right] = \frac{1}{16}(1 - \frac{\alpha^H}{2\alpha^L}) < 0$. ■

Proof of Proposition 3. We compare the manufacturer's expected profits under symmetric information in the dual- and single-sourcing models, (5) and (A.1). This yields the following expression for the manufacturer's value of the dual-sourcing option under symmetric information (note that in this expression, the definitions of the thresholds $\tilde{r}^H$, $\tilde{r}^L$, $\tilde{r}^{LL}$, $\tilde{r}^{HL}$ and $\tilde{r}^{HH}$ have been used to replace the positive operators $[\ ]^+$ of (5) and (A.1) with indicator functions):

$$\begin{aligned}
&\left(\alpha^L\alpha^H\right)\Big([(h\,r - c^H)D - K]\mathbf{1}_{\{r>\tilde{r}^H\}} - [(l\,r - c^L)D - K]\mathbf{1}_{\{r>\tilde{r}^L\}}\Big)\\
&+ \left(\alpha^H\right)^2\Big\{\left[h(1-h)r - c^H\right]D - K\Big\}\mathbf{1}_{\{r>\tilde{r}^{HH}\}}\\
&+ 2\left(\alpha^H\alpha^L\right)\Big\{\left[l(1-h)r - c^L\right]D - K\Big\}\mathbf{1}_{\{r>\tilde{r}^{HL}\}}\\
&+ \left(\alpha^L\right)^2\Big\{\left[l(1-l)r - c^L\right]D - K\Big\}\mathbf{1}_{\{r>\tilde{r}^{LL}\}}.
\end{aligned} \quad \text{(A.22)}$$

Similarly, we compare the expected profits of the manufacturer under asymmetric information in the dual- and single-sourcing models, (6) and (A.2), obtaining the expression for the manufacturer's value of the dual-sourcing option under asymmetric information:

$$\begin{aligned}
&\left(\alpha^L\alpha^H\right)\left(\left[(h\,r-c^H)D-K\right]\mathbf{1}_{\{r>r^H\}}-\left[(l\,r-c^L)D-K-\frac{\alpha^H}{\alpha^L}h\left(\frac{c^L}{l}-\frac{c^H}{h}\right)D\right]\mathbf{1}_{\{r>r^L\}}\right)\\
&+\left(\alpha^H\right)^2\left\{\left[h(1-h)r-c^H\right]D-K\right\}\mathbf{1}_{\{r>r^{HH}\}}\\
&+2\left(\alpha^H\alpha^L\right)\left\{\left[l(1-h)r-c^L\right]D-K-\frac{\alpha^H}{\alpha^L}h\left(\frac{c^L}{l}-\frac{c^H}{h}\right)D\right\}\mathbf{1}_{\{r>r^{HL}\}}\\
&+\left(\alpha^L\right)^2\left\{\left[l(1-l)r-c^L\right]D-K-\frac{\alpha^H}{\alpha^L}h\left(\frac{c^L}{l}-\frac{c^H}{h}\right)D\right\}\mathbf{1}_{\{r>r^{LL}\}}.
\end{aligned} \tag{A.23}$$

The difference between the manufacturer's value of the dual-sourcing option under asymmetric information and symmetric information, (A.23) minus (A.22), is

$$\begin{aligned}
&(\alpha^L\alpha^H)\left([(l\,r-c^L)D-K]\mathbf{1}_{\{r>\tilde{r}^L\}}-\left[(l\,r-c^L)D-K-\frac{\alpha^H}{\alpha^L}h\left(\frac{c^L}{l}-\frac{c^H}{h}\right)D\right]\mathbf{1}_{\{r>r^L\}}\right)\\
&+2\left(\alpha^H\alpha^L\right)\left(\left\{\left[l(1-h)r-c^L\right]D-K-\frac{\alpha^H}{\alpha^L}h\left(\frac{c^L}{l}-\frac{c^H}{h}\right)D\right\}\mathbf{1}_{\{r>r^{HL}\}}\right.\\
&\qquad\qquad\left.-\left\{\left[l(1-h)r-c^L\right]D-K\right\}\mathbf{1}_{\{r>\tilde{r}^{HL}\}}\right)\\
&+(\alpha^L)^2\left(\left\{\left[l(1-l)r-c^L\right]D-K-\frac{\alpha^H}{\alpha^L}h\left(\frac{c^L}{l}-\frac{c^H}{h}\right)D\right\}\mathbf{1}_{\{r>r^{LL}\}}\right.\\
&\qquad\qquad\left.-\left\{\left[l(1-l)r-c^L\right]D-K\right\}\mathbf{1}_{\{r>\tilde{r}^{LL}\}}\right).
\end{aligned} \tag{A.24}$$

The key is to treat (A.24) as a function of r and show that there exists a unique r_0 in the interval $(\tilde{r}^{LL}, r^{HL})$, at which the curve changes from non-negative to strictly negative. Because the lower bound of the interval, $\tilde{r}^{LL}$, could be either greater or smaller than r^L, we consider two cases: $\tilde{r}^{LL} \geq r^L$ and $\tilde{r}^{LL} < r^L$.

Case $\boldsymbol{\tilde{r}^{LL} \geq r^L}$. We prove the result by tracing the value of (A.24) for $r > \tilde{r}^L$. From the tables describing the thresholds $\tilde{r}^L, \ldots, \tilde{r}^{HL}$ and $r^L, \ldots, r^{HL}$ in Propositions 1 and 2, respectively, and the assumption of the case, we get $\tilde{r}^L < r^L \leq \tilde{r}^{LL} < \{r^{LL}, \tilde{r}^{HL}\} < r^{HL}$. One can check that (A.24) is equal to zero at $r = \tilde{r}^L$, strictly positive and strictly increasing in r for $r \in (\tilde{r}^L, r^L]$, constant with respect to r for $r \in (r^L, \tilde{r}^{LL}]$, decreasing in r for $r \in (\tilde{r}^{LL}, r^{HL}]$ (regardless of the ordering of r^{LL} and $\tilde{r}^{HL}$) and constant in r thereafter. Furthermore, (A.24) is negative at $r = r^{HL}$. Thus, there must exist $r_0 \in (\tilde{r}^{LL}, r^{HL})$ such that (A.24) changes from non-negative to strictly negative at r_0.

Case $\tilde{r}^{LL} < r^{L}$. Again using the definitions of the thresholds and the assumption of the case, we know $\tilde{r}^{L} < \tilde{r}^{LL} < r^{L} < r^{LL} < r^{HL}$. We utilize two sub-cases, depending on $\tilde{r}^{HL}$.

Sub-case $\tilde{r}^{LL} < \tilde{r}^{HL} \leq r^{L}$. One can check that (A.24) is zero at $r = \tilde{r}^{L}$, and strictly positive and strictly increasing in r for $r \in (\tilde{r}^{L}, \tilde{r}^{LL}]$. As for $r \in (\tilde{r}^{LL}, r^{L}]$, there are three possibilities for (A.24): increasing throughout, increasing until $\tilde{r}^{HL}$ and then decreasing thereafter, or decreasing throughout. Additionally, (A.24) is decreasing in r for $r \in (r^{L}, r^{HL}]$ and constant in r thereafter. Therefore, there exists $r^* \in [\tilde{r}^{LL}, r^{L}]$ such that (A.24) is strictly positive at $r = r^*$ and decreasing for $r > r^*$. Furthermore, (A.24) is negative at $r = r^{HL}$. Thus, there must exist $r_0 > r^*$ such that (A.24) changes from non-negative to strictly negative at r_0.

Sub-case $r^{L} < \{r^{LL}, \tilde{r}^{HL}\} < r^{HL}$. Similarly, one can check that (A.24) is zero at $r = \tilde{r}^{L}$, and strictly positive and strictly increasing in r for $r \in (\tilde{r}^{L}, \tilde{r}^{LL}]$. As for $r \in (\tilde{r}^{LL}, r^{L}]$, there are two possibilities for (A.24): increasing throughout, or decreasing throughout. For $r \in (r^{L}, r^{HL}]$, (A.24) is decreasing (regardless of the ordering of r^{LL} and $\tilde{r}^{HL}$), and is constant for $r > r^{HL}$. Therefore, the same logic as in the previous sub-case establishes the existence of r_0. ■

Proof of Proposition 4. We provide a sketch of the proof by focusing on the proofs of Propositions 1 and 2 (with independent supplier disruptions) and identifying the steps that change if the suppliers' disruptions are now codependent.

We first consider the symmetric information model. As in the proof of Proposition 1 (independent supplier disruptions), to find the optimal order quantities under codependent supplier disruptions, we can focus on corner-point solutions, $q_n = 0$ or D for $n = 1, 2$. Recall that, given the two suppliers' types, t_1 and t_2, index $m \in \{1, 2\}$ is defined such that $\theta^{t_m} \geq \theta^{t_{\overline{m}}}$. (When $\theta^{t_m} = \theta^{t_{\overline{m}}}$, we break ties in favor of supplier 1.) At corner points $(q_m, q_{\overline{m}}) = (0, 0)$, $(D, 0)$ and $(0, D)$, respectively, supplier disruption codependence does not change the manufacturer's payoff, because the manufacturer orders from at most one supplier. At each of these three corner points the manufacturer's profit under codependent supplier disruptions is given by the first three rows of (A.8) (in the case of independent disruptions).

At corner point $(q_m, q_{\overline{m}}) = (D, D)$, the manufacturer realizes a sale of D when either one or both suppliers run production successfully. Therefore, in the case of codependent supplier disruptions, the manufacturer's expected payoff at $(q_m, q_{\overline{m}}) = (D, D)$ equals

$$\Big[\omega^{t_m,t_{\overline{m}}}(1,0) + \omega^{t_m,t_{\overline{m}}}(0,1) + \omega^{t_m,t_{\overline{m}}}(1,1)\Big] rD - \big(c^{t_m}D + K\big) - \big(c^{t_{\overline{m}}}D + K\big),$$

where $\omega^{t_m,t_{\overline{m}}}(1,0)$, $\omega^{t_m,t_{\overline{m}}}(0,1)$ and $\omega^{t_m,t_{\overline{m}}}(1,1)$ are the joint probabilities for, respectively, only supplier-m, only supplier-$\overline{m}$, and both suppliers running production successfully. From the suppliers' marginal probabilities $\omega^{t_m,t_{\overline{m}}}(1,0) + \omega^{t_m,t_{\overline{m}}}(1,1) = \theta^{t_m}$ and $\omega^{t_m,t_{\overline{m}}}(0,1) = \theta^{t_{\overline{m}}} - \omega^{t_m,t_{\overline{m}}}(1,1)$, we rewrite the manufacturer's expected payoff above as

$$\Big[(\theta^{t_m} r - c^{t_m})D - K\Big] + \Big[\big([\theta^{t_{\overline{m}}} - \omega^{t_m,t_{\overline{m}}}(1,1)]r - c^{t_{\overline{m}}}\big)\, D - K\Big]. \tag{A.25}$$

Under supplier disruption codependence, the steps for identifying the optimal order quantities among the four corner-points are the same as in Proposition 1, except that the manufacturer's payoff at corner point $(q_m, q_{\overline{m}}) = (D, D)$ (the last row in (A.8)) is replaced by (A.25). Comparing (A.25) to the last row in (A.8), the only difference is that the joint probability $(1-\theta^{t_m})\theta^{t_{\overline{m}}}$ is replaced by $\theta^{t_{\overline{m}}} - \omega^{t_m,t_{\overline{m}}}(1,1)$. Specifically, for $t_1, t_2 \in \{H, L\}$, the joint probabilities $(1-h)h$, $(1-h)l$ and $(1-l)l$ are replaced by $h - \omega^{HH}(1,1)$, $l - \omega^{HL}(1,1)$ and $l - \omega^{LL}(1,1)$, respectively. Hence, the thresholds $\tilde{r}^{HH}$, $\tilde{r}^{HL}$ and $\tilde{r}^{LL}$ are modified by replacing the joint probabilities $(1-h)h$, $(1-h)l$ and $(1-l)l$ with $h - \omega^{HH}(1,1)$, $l - \omega^{HL}(1,1)$ and $l - \omega^{LL}(1,1)$, and the optimal contract is the same as in Proposition 1 albeit using the redefined thresholds.

Under asymmetric information, the proof of Proposition 2 holds under codependent supplier disruptions provided that, in the definition $r^{t_m,t_{\overline{m}}}$ (equation (A.18b)), we replace $(1-\theta^{t_m})\theta^{t_{\overline{m}}}$ by $\theta^{t_{\overline{m}}} - \omega^{t_m,t_{\overline{m}}}(1,1)$. (The logic behind this replacement mirrors that discussed above for the symmetric information case.) Recall that, in Proposition 2, Lemma 3 was used to verify the monotonicity conditions. The same verification is needed here as well, and our assumption $\omega^{HH}/h > \omega^{HL}/l$ ensures that the same analysis as in Lemma 3 carries through under codependence (i.e., when $(1-\theta^{t_m})\theta^{t_{\overline{m}}}$ is replaced by $\theta^{t_{\overline{m}}} - \omega^{t_m,t_{\overline{m}}}(1,1)$ in the definition of $r^{t_m,t_{\overline{m}}}$). In particular, with restriction $\omega^{HH}/h > \omega^{HL}/l$, we can verify the key step that $q_n(H,t_{\overline{n}}) \geq q_n(L,t_{\overline{n}})$ for $n = 1,2$ and $t_{\overline{n}} \in \{H,L\}$. That is, the order size received by a supplier is larger when it is of high-type versus when it is of low-type. ■