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A STUDY OF WATERHAMMER INCLUDING EFFECT OF HYDRAULIC LOSSES

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NOMENCLATURE

Symbol	Units	Description
A	ft^2	Cross sectional area of pipe
a	ft/sec	Velocity of pressure wave
b	ft	Thickness of pipe wall
c_1	——	Factor of pipe restriction
D	ft	Inside diameter of pipe
E	lb/ft^2	Modulus of elasticity of pipe wall material
f	——	Friction factor
g	ft/sec^2	Acceleration of gravity
H	ft	Piezometric head at a given time and place on the pipe; $H = \frac{P}{\gamma} + Z$
H_o	ft	Head at the reservoir
h_f	ft	Friction loss in feet
K	lb/ft^2	Bulk modulus of elasticity of the liquid
K,K'	——	Minor loss coefficient
L	ft	Total length of pipe
L_n	ft	Length of any uniform section of pipe
P	lb/ft^2	Pressure intensity at any point in a pipe
R	——	Reynolds number
t	sec	Time
T_c	sec	Time of gate closure
V	ft/sec	Velocity in pipe for surge conditions
V_o	ft/sec	Velocity in pipe for initial steady conditions

NOMENCLATURE*† (CONT'D)

Symbol	Units	Description
x	ft	Distance measured positive from upstream
Z	ft	Elevation
α	——	Angle of slope of pipe
γ	lb/ft^3	Specific weight of the liquid
ϵ	ft	Roughness height
μ	——	Poisson's ratio for the pipe wall material
μ	$lb\text{-}sec/ft^2$	Viscosity
ν	ft^2/sec	Kinematic viscosity
ρ	$slug/ft^3$	Density of the liquid
τ	——	The ratio of effective gate opening to the full gate opening

* See also Figure 1.

† Somewhat different notations are used in the examples of "MAD" language programs because there are only twenty-six Roman capital letters available in the keypunch. They will not cause any confusion since they appear only in the appendices.

I. INTRODUCTION

Not too many problems in water hammer including friction losses have been solved, partly owing to the comparatively small, insignificant quantity of friction loss in this phenomenon, where rapid changes in flow are most likely to occur, and partly owing to the fact that no entirely satisfactory method has been devised for their inclusion in water hammer analysis.[13,15,16] However, this effect sometimes cannot be disregarded in cases of slow gate closure, a long pipe, or a high friction factor.

The existing methods for estimating friction losses are inadequate, owing to some rough approximations. A few authors have developed graphical methods,[2,15] whereby the effect of losses can be approximated by placing one or more hypothetical obstructions at certain selected locations along the pipe line and by lumping the friction losses of each section at these points. These methods will produce an abrupt drop of pressure at each obstruction, which evidently is not the true case. Although the accuracy can be augmented by increasing the number of obstructions, this increases greatly the complexity of the graphical solution.

Some analytical solutions for this class of study have been published also. All of them have employed a linear approximation for friction effect which is different from the actual situation. They generally involve difficult mathematical operations or tedious series.[17,24]

It is, viewed as necessary and useful to devise a better method which can be used to solve the problems in water hammer including friction losses more directly and more rapidly, with closer agreement to the actual situation. An attempt has been made by the author to attack the problem directly from the basic partial differential equations, without resort to graphical methods, indirect mathematical transformations, or linear approximations. In recent years, the coming into existence of electronic digital computers has made accurate calculations describing many complex physical phenomena practical in cases formerly beyond reach. The numerical solutions by digital computer will play an important role in the analytical part of the present study.

In following chapters, the basic partial differential equations for water hammer including friction losses are reviewed first. The direct solution of these nonlinear equations with aid of the computer is then presented with mathematical analysis, computer approach, flow diagram and some notes in programming. The method of characteristics has been employed in the present study. The next part is devoted to a study of the application of the theoretical work. This is necessary for better description of the computer method, for appraisal of the validity of the present method, for comparison with the earlier methods and for the guidance in obtaining experimental verification.

The experimental equipment was built in the Fluid Engineering Laboratory at the University of Michigan for the investigation of the attenuation of pressure surges due to friction losses. The experiments were performed for flow in a long pipe line and their results were compared with the theoretical solutions. The discussion of these results is given in Chapter VI.

This study ends with a comparison with earlier methods, conclusions, and brief suggestions for further investigations.

The analysis has been started from the assumptions of homogeneous and elastic liquid and pipe, uniform velocity and pressure over any cross section, and sufficient minimum pressure to exceed the vapor pressure.

It has been assumed in this study that the effect of hysteresis is negligibly small in comparison with the wall friction. By applying the principle of energy to the problem, and comparing the original kinetic energy in the moving water column and the work done in compressing the water and stretching the pipe walls, it may be shown that during the action no energy is lost or converted into heat, if the pipe and water are perfectly elastic.[14] When the stresses of pipe material are below the elastic limit, the assumption of perfect elasticity is very nearly true and the energy loss due to hysteresis of the pipe is very small.[6,12] This is at least quite true compared with the wall friction for the case of experiments made in this study. The elastic hysteresis of water, although no reliable data are available, is also considered to be very small, because the pressure changes take place in a very short time and comparatively little heat energy will be lost.

II. THE WATER HAMMER EQUATIONS INCLUDING FRICTION EFFECT

Fundamental water hammer equations are derived for a general case of variable flow. The elasticity of the pipe walls, the compressibility of the water, and the hydraulic losses due to the pipe friction in both directions are taken into account. The method employed is generally the same as that described in Reference 15, except more terms are involved in this case. The following assumptions are made:

(a) The pipe line remains full of water at all times, i.e., the law of continuity holds.

(b) The static pressure in the pipe is sufficiently high to sustain the minimum pressure above the vapor pressure.

(c) The velocity of water in the axial direction of the pipe is uniform over any cross section of the pipe.

(d) The pressure is uniform over any transverse cross section and is the same as that at the center line of the pipe.

(e) The water level at the reservoir remains constant during the period of investigation.

For the derivation of equations, an element of water which is bounded by two parallel cross sections normal to the pipe axis is considered. The general water hammer equations, then, can be obtained from the conditions of dynamic equilibrium and continuity.

A. Condition of Dynamic Equilibrium

The condition of dynamic equilibrium can be set up for an element dx at a distance x along the pipe measured from the reservoir towards the valve, as illustrated by Figure 1. If the cross-sectional area at M is A, that at N is $A + \frac{\partial A}{\partial x} dx$, and the angle of slope of pipe is α, then the forces acting on these two faces can be expressed as $\gamma A(H-Z)$ and $\gamma(A + \frac{\partial A}{\partial x} dx)[H-Z + (\frac{\partial H}{\partial x} + \sin \alpha)dx]$ respectively. Here, γ, H, and Z are the specific weight of water, the piezometric head and the elevation at point x.

The gravitational force acting on the mass of element at its center of gravity is

$$\gamma(A + \frac{1}{2} \frac{\partial A}{\partial x} dx)dx \ .$$

The wall resistance acting against the flow in the element is dh_f feet of water.

The unbalanced force acting on the element of water along the axis of the pipe is

$$\gamma(A + \frac{\partial A}{\partial x} dx)[H-Z + (\frac{\partial H}{\partial x} + \sin \alpha)dx] - \gamma A(H-Z)$$
$$- \gamma(A + \frac{1}{2} \frac{\partial A}{\partial x} dx)dx \sin \alpha + \gamma(A + \frac{1}{2} \frac{\partial A}{\partial x} dx)dh_f \ ,$$

where the positive direction of the force is taken opposite to the direction of the normal flow. Neglecting the terms of higher order and simplifying, this force reduces to

$$\gamma[A \frac{\partial H}{\partial x} dx + \frac{\partial A}{\partial x} (H-Z) dx + A dh_f] \ .$$

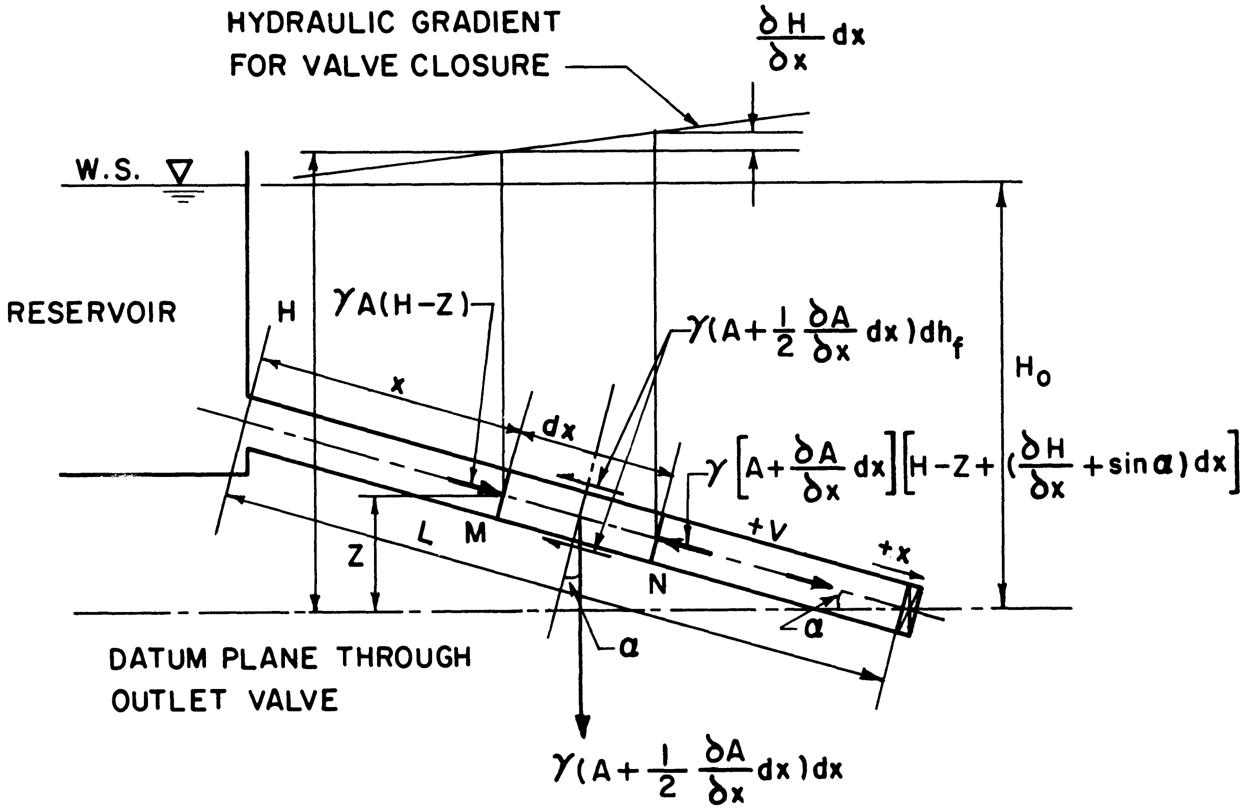


Figure 1. Definition Sketch.

The mass of the element of water to be moved by this unbalanced force is

$$\frac{\gamma(A + \frac{1}{2}\frac{\partial A}{\partial x}\,dx)dx}{g} ,$$

and its deceleration is $-\frac{dV}{dt}$, in which g is the acceleration of gravity, V is the velocity of flow in pipe and t is the time. Again, neglecting the term of higher order, this mass reduces to $\frac{\gamma A dx}{g}$. Then from Newton's second law of motion

$$\gamma[A\frac{\partial H}{\partial x}\,dx + \frac{\partial A}{\partial x}\,(H-Z)\,dx + Adh_f] = \frac{-\gamma A dx}{g}\,\frac{dV}{dt}$$

Since V is a function of both x and t,

$$\frac{dV}{dt} = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x}\frac{dx}{dt} = \frac{\partial V}{\partial t} + V\,\frac{\partial V}{\partial x} .$$

Then

$$\frac{\partial H}{\partial x} + \frac{1}{A}\frac{\partial A}{\partial x}\,(H-Z) + \frac{dh_f}{dx} = -\frac{1}{g}(\frac{\partial V}{\partial t} + V\,\frac{\partial V}{\partial x}) , \qquad (1)$$

which is the equation of motion for the element of water.

When the friction is negligible, this reduces to

$$\frac{\partial H}{\partial x} + \frac{1}{A}\frac{\partial A}{\partial x}\,(H-Z) = -\frac{1}{g}(\frac{\partial V}{\partial t} + V\,\frac{\partial V}{\partial x}) . \qquad (2)$$

If, $\frac{1}{A}\frac{\partial A}{\partial x}\,(H-Z)$ is negligible, as is always the case compared with other terms,(15) Equation (1) reduces to

$$\frac{\partial H}{\partial x} + \frac{dh_f}{dx} = -\frac{1}{g}(\frac{\partial V}{\partial t} + V\,\frac{\partial V}{\partial x}) . \qquad (3)$$

When both $\frac{1}{A}\frac{\partial A}{\partial x}$ and $\frac{dh_f}{dx}$ are negligible, then Equation (1) reduces to the familiar form

$$\frac{\partial H}{\partial x} = -\frac{1}{g}(\frac{\partial V}{\partial t} + V\,\frac{\partial V}{\partial x}) . \qquad (4)$$

Since dh_f is a very small quantity, the assumption

$$dh_f = f \frac{dx}{D} \frac{V^2}{2g}$$

for the element of water dx, is made. Here, f and D are the friction factor and the inside diameter of the pipe respectively. The friction factor is a function of Reynolds number $\mathbf{R}$ and relative roughness ϵ/D, or $f = f(R, \epsilon/D) = f(V,D,\nu,\epsilon/D)$, in which ν is the kinematic viscosity and ϵ is the roughness height of the pipe. The value of f can be obtained from the Moody diagram,[20,21] or from experimental evaluations for a specific pipe of interest.

Substituting $dh_f = f \frac{dx}{D} \frac{V^2}{2g}$ into (1) and (3) respectively, we obtain

$$\frac{\partial H}{\partial x} + \frac{1}{A}\frac{\partial A}{\partial x}(H-Z) + \frac{f}{D}\frac{V^2}{2g} = -\frac{1}{g}\left(\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x}\right) , \tag{5}$$

and

$$\frac{\partial H}{\partial x} + \frac{f}{D}\frac{V^2}{2g} = -\frac{1}{g}\left(\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x}\right) . \tag{6}$$

B. Condition of Continuity

There will not be any change in the equation of continuity whether the wall friction is neglected or considered. Hence, the equations shown in Reference 15 will be used.

$$\frac{\partial H}{\partial t} + V\frac{\partial H}{\partial x} = -\frac{a^2}{g}\frac{\partial V}{\partial x} \tag{7}$$

where the velocity of pressure wave, a, can be expressed as

$$a = \sqrt{\frac{1}{\frac{\gamma}{g}\left(\frac{1}{K} + \frac{Dc_1}{Eb}\right)}} . \tag{8}$$

It is apparent from Equation (8) that the velocity of pressure wave depends on the diameter and the wall thickness of the pipe, the properties of the pipe material and of the fluid, and the ability of the pipe to move in the longitudinal direction. This indicates that a wave reflection occurs at every change in pipe thickness, area, or change in the pipe material, i.e.,

$$a = a[b(x), D(x), E(x)].$$

The factor of pipe restriction, c_1, takes the following values.(15)

(a) $c_1 = \frac{5}{4} - \mu$ for a pipe anchored at the upper end and without expansion joint,

(b) $c_1 = 1 - \mu^2$ for a pipe anchored against longitudinal movement throughout its length,

(c) $c_1 = 1 - \frac{\mu}{2}$ for a pipe with expansion joints.

In these relationships, μ represents the Poisson's ratio for the pipe wall material.

C. Summary

Summarizing the equations shown in the preceding pages, and denoting $V_x = \frac{\partial V}{\partial x}$, $H_t = \frac{\partial H}{\partial t}$, etc., we have the general differential equations for water hammer in a pipe,

$$H_x + \frac{f}{D}\frac{V^2}{2g} = -\frac{1}{g}(V_t + VV_x) \qquad (6)$$

$$H_t + VH_x = -\frac{a^2}{g} V_x \quad . \qquad (7)$$

When the wall friction is negligible,

$$H_x = -\frac{1}{g}(V_t + VV_x) \qquad (4)$$

$$H_t + VH_x = -\frac{a^2}{g} V_x \ . \qquad (7)$$

It is often assumed that in Equations (4) or (6) the term VV_x is small compared with V_t, and in Equation (7) the term VH_x is small compared with H_t.[15,25] Then Equations (6) and (7), and Equations (4) and (7) are again reduced to

$$H_x + \frac{f}{D}\frac{V^2}{2g} = -\frac{1}{g} V_t \qquad (9)$$

$$H_t = -\frac{a^2}{g} V_x \quad , \qquad (10)$$

and

$$H_x = -\frac{1}{g} V_t \qquad (11)$$

$$H_t = -\frac{a^2}{g} V_x \qquad (10)$$

respectively.

In these equations,

$$a = \sqrt{\frac{1}{\frac{\gamma}{g}\left(\frac{1}{K} + \frac{Dc_1}{Eb}\right)}} \quad . \qquad (8)$$

III. A METHOD FOR THE SOLUTION OF WATER HAMMER EQUATIONS BY DIGITAL COMPUTER

The governing partial differential equations, presented in the previous chapter, can be solved by the method of characteristics with the aid of a computer, using specified time intervals and an extrapolation procedure, as described in the following sections.* The abbreviated flow diagram is shown at the end of this chapter.

A. Characteristic Equations for Water Hammer

The partial differential equations for water hammer including friction effect, (6) and (7), are rewritten in a modified form as follows:

$$J_1 = VV_x + V_t + gH_x + \frac{f}{2D} V^2 = 0 \qquad (12)$$

$$J_2 = \frac{a^2}{g} V_x + VH_x + H_t = 0 \qquad (13)$$

where

$$V_x = \frac{\partial V}{\partial x}, \quad H_t = \frac{\partial H}{\partial t}, \quad \ldots\ldots$$

These are two simultaneous quasi-linear partial differential equations of the first order with two independent and two dependent variables.

Combining (12) and (13) linearly,

$$J = J_1 + \lambda J_2$$

$$= (V + \lambda \frac{a^2}{g}) V_x + V_t + (g + \lambda V)H_x + \lambda H_t + \frac{f}{2D} V^2 = 0. \qquad (14)$$

* For the general and thorough mathematical treatment of hyperbolic partial differential equations by the method of characteristics, see Reference 10.

If $V = V(x,t)$ and $H = H(x,t)$ are solutions to (12) and (13), then

$$dV = \frac{\partial V}{\partial x}\,dx + \frac{\partial V}{\partial t}\,dt, \quad dH = \frac{\partial H}{\partial x}\,dx + \frac{\partial H}{\partial t}\,dt\ , \tag{15}$$

or

$$\frac{dV}{dt} = V_x\,\frac{dx}{dt} + V_t, \qquad \frac{dH}{dt} = H_x\,\frac{dx}{dt} + H_t\ . \tag{15'}$$

Now, by examination of Equation (14), with Equations (15') in mind, let

$$(V + \lambda\,\frac{a^2}{g})V_x + V_t = V_x\,\frac{dx}{dt} + V_t = \frac{dV}{dt}\ , \tag{16}$$

$$(\frac{g}{\lambda} + V)H_x + H_t = H_x\,\frac{dx}{dt} + H_t = \frac{dH}{dt}\ , \tag{17}$$

then Equation (14) may be reduced to a form

$$dtJ = dV + \lambda dH + \frac{f}{2D}\,V^2 dt = 0. \tag{18}$$

The conditions for Equation (14) to be in this form are, therefore,

$$\frac{dx}{dt} = V + \lambda\,\frac{a^2}{g}\ , \qquad \frac{dx}{dt} = \frac{g}{\lambda} + V\ , \tag{19}$$

from which

$$V + \lambda\,\frac{a^2}{g} = \frac{g}{\lambda} + V$$

$$\lambda = \pm\,\frac{g}{a}\ , \tag{20}$$

and the characteristic equations for V and H become

$$dtJ = dV + \frac{g}{a}\,dH + \frac{f}{2D}\,V^2 dt = 0 \tag{21}$$

$$dtJ = dV - \frac{g}{a}\,dH + \frac{f}{2D}\,V^2 dt = 0\ . \tag{22}$$

Here, from (19) and (20), the two different characteristic directions at the point (x,t) are given by

$$\zeta_+ = \frac{dt}{dx} = \frac{1}{V+a} , \qquad \zeta_- = \frac{dt}{dx} = \frac{1}{V-a} . \tag{23}$$

Equations (21) and (22) are two separate total differential equations with t as an independent variable and V and H as two dependent variables. If $V = V(x,t)$ and $H = H(x,t)$ satisfy the Equations (12) and (13), then Equations (23) become two separate ordinary differential equations of the first order. These determine two families of characteristic curves, or shortly "characteristics", C_+ and C_- in the (x,t) plane belonging to this solution $V(x,t)$, $H(x,t)$.

Equations (21) to (23) can be rearranged to the following four characteristic equations,

$$\left.\begin{aligned} & dt - \frac{dx}{V+a} = 0 && (24) \\ & dV + \frac{g}{a}\,dH + \frac{f}{2D}\,V^2 dt = 0 && (25) \end{aligned}\right\} \text{ along } C_+$$

$$\left.\begin{aligned} & dt - \frac{dx}{V-a} = 0 && (26) \\ & dV - \frac{g}{a}\,dH + \frac{f}{2D}\,V^2 dt = 0 && (27) \end{aligned}\right\} \text{ along } C_-$$

Equations (24) to (27) are of a particularly simple form and are satisfied, according to the derivation, by every solution of the original system (12) and (13).

B. Finite Difference Approximations

The characteristic equations for water hammer containing the friction terms, (24) to (27), may be solved by employing the first-order or the second-order finite difference approximation. They are

$$\int_{x_o}^{x_1} f(x)dx \approx f(x_o)(x_1-x_o), \tag{28}$$

and

$$\int_{x_o}^{x_1} f(x)dx \approx \frac{1}{2}[f(x_o) + f(x_1)](x_1-x_o) \tag{29}$$

respectively. The former is a linear approximation, whereas the latter uses the trapezoidal rule.

If the extrapolation procedures which is described later are employed with the linear approximation, the degree of accuracy will be increased to that obtainable from the second-order process. Inasmuch as the second-order approximation has the disadvantage of requiring some iterative method, the first-order approximation will be used here.

Referring to Figure 2, C_+ and C_- are two characteristics passing through R and S, and P is their intersection point. Denoting the value of x at R by x_R, etc., for known values of x_R, t_R, V_R, H_R, x_S, t_S, V_S and H_S, the values of x_P, t_P, V_P and H_P can be found by applying the linear approximation (28) to Equations (24) to (27).

$$(t_P - t_R) - \left(\frac{1}{V+a}\right)_R (x_P - x_R) = 0 \tag{30}$$

$$(V_P - V_R) + \frac{g}{a_R} (H_P - H_R) + \left(\frac{f}{2D} V^2\right)_R (t_P - t_R) = 0 \tag{31}$$

$$(t_P - t_S) - \left(\frac{1}{V-a}\right)_S (x_P - x_S) = 0 \tag{32}$$

$$(V_P - V_S) - \frac{g}{a_S} (H_P - H_S) + \left(\frac{f}{2D} V^2\right)_S (t_P - t_S) = 0 \ . \tag{33}$$

These four equations have four unknowns and therefore are solvable.

There are two most typical ways of using the set of Equations (30) to (33) to obtain an approximate numerical solution to the original set of partial differential equations; i.e., (i) Grid of characteristics, (ii) Specified time intervals.

The latter uses specified intervals in the t-direction and relates the values of V and H at the beginning of the interval to those at the end by means of (30) to (33). This method is preferred over the former in case of water hammer problems, because x_P and t_P are known exactly, or they will be assigned exactly, and only two values V_P and H_P are to be determined. Secondly, with method (ii) it is possible to apply extrapolation procedures* to increase the accuracy. Moreover, in the calculation of water hammer problems, method (ii) has an advantage of directly providing the velocity V and the pressure H along the pipe distance x at different times t, in the form most desirable for such studies.** Further, with method (ii) the quantities Δx, Δt are under the control of the individual user. This is a great benefit in treating many different pipe conditions during this study. The process of solving (30) to (33) by method (ii) is described in the following section.

C. Specified Time Intervals

Referring to Figure 3, t_i and t_{i+1} are the beginning and the end of the time interval Δt, and A, C, B are three adjacent points on the line $t = t_i$, Δx apart from each other. Let the point P of

---

* See p.

** See Appendices I to III.

Figure 2 fall on the intersection of $t = t_{i+1}$ and $x = x_C$, and R, S on the line $t = t_i$. Two characteristics C_+ and C_- pass through P, R and P, S as before. Here, the values of V and H at $t = t_i$ are assumed to be known and their values at P are to be found. The steps for the computation are as follows:

(a) From Equations (30) and (32), remembering $x_P = x_C$, $t_R = t_C = t_S$, x_R and x_S can be readily evaluated.

$$x_R = x_C - (V+a)_C\,(t_P - t_C) \tag{34}$$

$$x_S = x_C - (V-a)_C\,(t_P - t_C) \tag{35}$$

Here, it is assumed that $(V+a)_R = (V+a)_C$, $(V-a)_S = (V-a)_C$ because of the sufficiently short distances AC and CB.

(b) Using a linear interpolation,

$$(\zeta_+)_C = \frac{\Delta t}{\Delta x\,\dfrac{V_C-V_R}{V_C-V_A}} = \left(\frac{\Delta t}{\Delta x}\right)\frac{V_C-V_A}{V_C-V_R} \quad ,$$

$$V_C-V_R = \left(\frac{\Delta t}{\Delta x}\right)(\zeta_+)_C^{-1}(V_C-V_A) \quad ,$$

$$V_R-V_C = -\,(V+a)_C\left(\frac{\Delta t}{\Delta x}\right)(V_C-V_A) \quad ,$$

$$V_R = V_C[1 - \Theta(V+a)_C] + V_A\Theta(V+a)_C \ . \tag{36}$$

Similarly

$$V_S = V_C[1 + \Theta(V-a)_C] - V_B\Theta(V-a)_C \ , \tag{37}$$

$$H_R = H_C[1 - \Theta(V+a)_C] + H_A\Theta(V+a)_C \ , \tag{38}$$

$$H_S = H_C[1 + \Theta(V-a)_C] - H_B\Theta(V-a)_C \ , \tag{39}$$

where

$$\Theta = \frac{\Delta t}{\Delta x} \ .$$

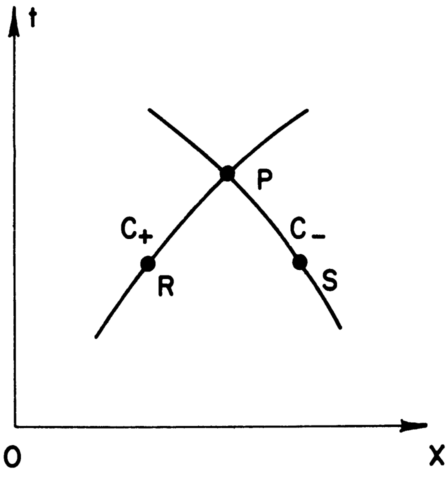


Figure 2. An Intersection of Two Characteristics.

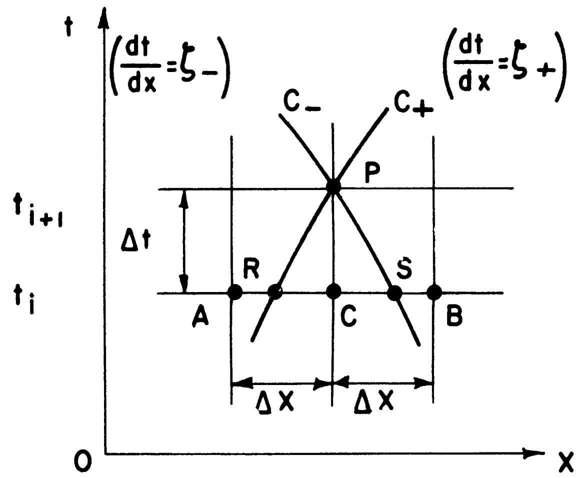


Figure 3. The Specified Time Interval.

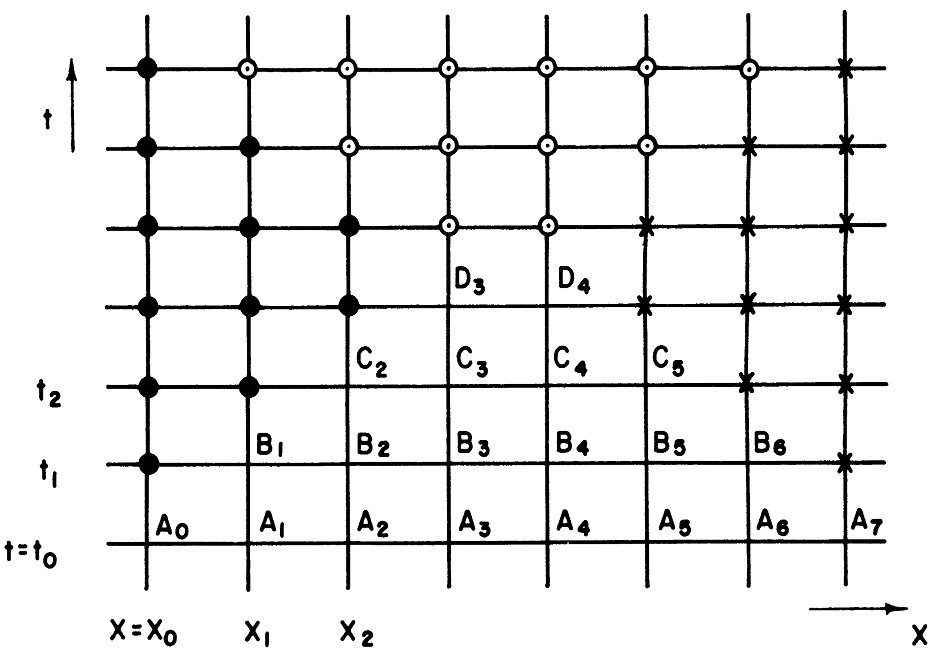


Figure 4. The Procedure of Computation.

As in (a), the assumption of sufficiently small Δt has been made here that C_+ between P and R, and C_- between P and S are straight lines, and that $(\zeta_+)_R = (\zeta_+)_C$, $(\zeta_-)_S = (\zeta_-)_C$ respectively.

(c) Rewriting Equations (31) and (33) as

$$(V_P - V_R) + \frac{g}{a_C}(H_P - H_R) + (\frac{f}{2D} V^2)_C (t_P - t_C) = 0 \ , \qquad (40)$$

$$(V_P - V_S) - \frac{a}{g_C}(H_P - H_S) + (\frac{f}{2D} V^2)_C (t_P - t_C) = 0 \ . \qquad (41)$$

Solving (40) and (41) simultaneously, V_P and H_P are found as

$$V_P = \frac{1}{2}(V_R + V_S) + \frac{g}{2a_C}(H_R - H_S) - (\frac{f}{2D} V^2)_C (t_P - t_C) \ , \qquad (42)$$

$$H_P = \frac{a_C}{2g}(V_R - V_S) + \frac{1}{2}(H_R + H_S) \ . \qquad (43)$$

With these equations, (34) to (43), the computation can be performed as follows: First V and H will be given at A_0, A_1, ..., A_7 in Figure 4, then using Equations (34) - (43), V and H at B_1, ..., B_6 can be evaluated. In a similar manner V and H can be obtained at C_2, ..., C_4, and finally at D_3, D_4. The number of known points at $t = t_0$ will determine how far the computation can proceed. However, if suitable boundary conditions on $x = x_0$ are given, V and H at those points marked by a black circle can be calculated. The same procedures apply to the right end. Those points are marked by a cross. It is then self-evident that once suitable boundary conditions are given at both ends, the computation can be carried out as far as desired.

D. Boundary Conditions

(a) The Right End:

Let $x = x_C$ be the boundary line as shown in Figure 5. Then Equation (34), Equation (36) and Equation (38) can be used for calculating the values x_R, V_R and H_R. Finally, H_P can be obtained by rearranging Equation (40) as

$$H_P = H_R - \frac{a_C}{g}(V_P - V_R) - \left(\frac{af}{2gD} V^2\right)_C (t_P - t_C), \qquad (44)$$

when V_P is obtainable from the given boundary conditions. Alternatively, V_P can be obtained from the following equation

$$V_P = V_R - \frac{g}{a_C}(H_P - H_R) - \left(\frac{f}{2D} V^2\right)_C (t_P - t_C) \ , \qquad (45)$$

when H_P is obtainable.

If the boundary line is at the gate-end, $x = x_C = L$, in which L is the length of the pipe. Then, Equation (34) can be written as

$$x_R = L - (V+a)_C (t_P - t_C) \qquad (46)$$

Other equations remain the same and Equation (44) should be used in this case, because V_P can be obtained, when $t < T_c$ (time of closure), by solving

$$\Delta H = \frac{a \Delta V}{g} \qquad (47)$$

and

$$\frac{V_C - \Delta V}{V_o} = \tau_P \sqrt{\frac{H_C + \Delta H}{H_o'}} \qquad (48)$$

simultaneously for ΔH and ΔV,(21) and by using the relationship

$$V_P = V_C - \Delta V \ . \qquad (49)$$

Here, τ_P is the ratio of the effective gate opening at time P during the gate closure to the effective gate opening at time zero, and H_o' is the head across the gate when $V = V_o$. For $t > T_c$,

$$V_P = 0 \ . \qquad (50)$$

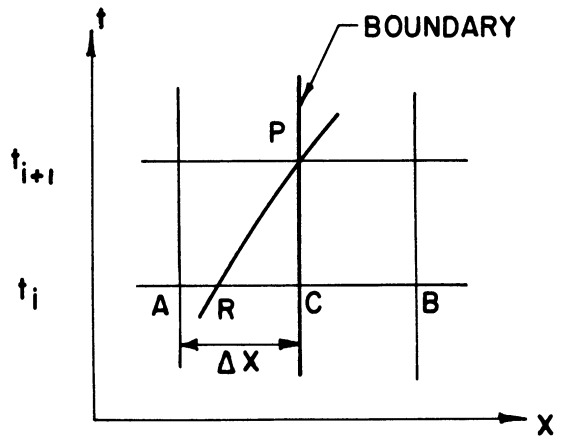


Figure 5. The Right-End Boundary.

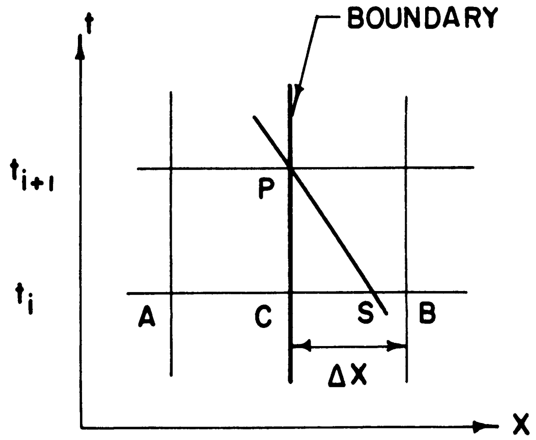


Figure 6. The Left-End Boundary.

(b) The Left End:

Again let $x = x_C$ be the boundary line as shown in Figure 6. Equation (35), Equation (37) and Equation (39) are used to evaluate x_S, V_S and H_S this time. Now V_P can be found by rewriting Equation (41) as

$$V_P = V_S + \frac{g}{a_C}(H_P - H_S) - \left(\frac{f}{2D} V^2\right)_C (t_P - t_C) \qquad (51)$$

if H_P is obtainable from the boundary conditions. Alternatively,

$$H_P = H_S + \frac{a_C}{g}(V_P - V_S) + \left(\frac{af}{2gD} V^2\right)_C (t_P - t_C) \ , \qquad (52)$$

if V_P is obtainable.

E. Extrapolation Procedures[10,11,18]

Since the method of specified time intervals has been used, it is possible to employ extrapolation procedures to increase the accuracy of the computation.

According to the study made by M. Lister and L. Roberts,[11] if a function $f(x,t)$ is to be determined at $t = 2n\Delta t$, in terms of its given value at $t = 0$, this may be achieved by taking n steps of size $2\Delta t$, repeating a linear process at a constant value of x. Let the value of $f(x,t)$ thus obtained be $f_A(x,t)$. Alternatively, we may use $2n$ steps of Δt and the value of $f(x,t)$ thus obtained will be denoted as $f_B(x,t)$. Now, we define

$$\bar{f}(2n\Delta t) = 2f_B(x,t) - f_A(x,t) \ . \qquad (53)$$

If $\bar{f}(2n\Delta t)$ is computed, this value and the true value $f(2n\Delta t)$ agree when terms of the order $(\Delta t)^3$ are neglected. After n steps if $n\Delta t = O(1)$, then the error is of $O(\Delta t)^2$.

The linear procedure given by Equations (34) to (43) is of this form. Computing V and H for two different step sizes $2\Delta t$ and Δt, and combining them by using (53), the error in V and H after n steps is of $O(\Delta t)^2$. This is the accuracy achieved when the second-order process of finite difference approximation is employed [see Equation (29)].

F. The Procedure in Machine Computation and Flow Diagram

The steps in the actual machine computation are stated briefly as follows:

(a) Calculate V and H for a time interval $2\Delta t$, using (34) to (43), (44) to (52).

(b) Calculate V and H for Δt, using the same equations.

(c) Using the results of step (b), calculate V and H for Δt again.

(d) Combining the results of steps (a) and (c) by using (53), calculate extrapolated values of V and H.

(e) Repeat steps (a) - (d) as far as desired. The output is the results obtained at (d).

The abbreviated flow diagram for the solution of water hammer including friction effect in a simple pipe line of constant cross section and constant wall thickness (Figure 7) is shown on the following page. The emphasis is placed on a more logical presentation, omitting some machine programming details.

One of the features in this basic structure of computer programming is that the same block of instructions, which is the core of the analytical solution in this study, can be used repeatedly for the

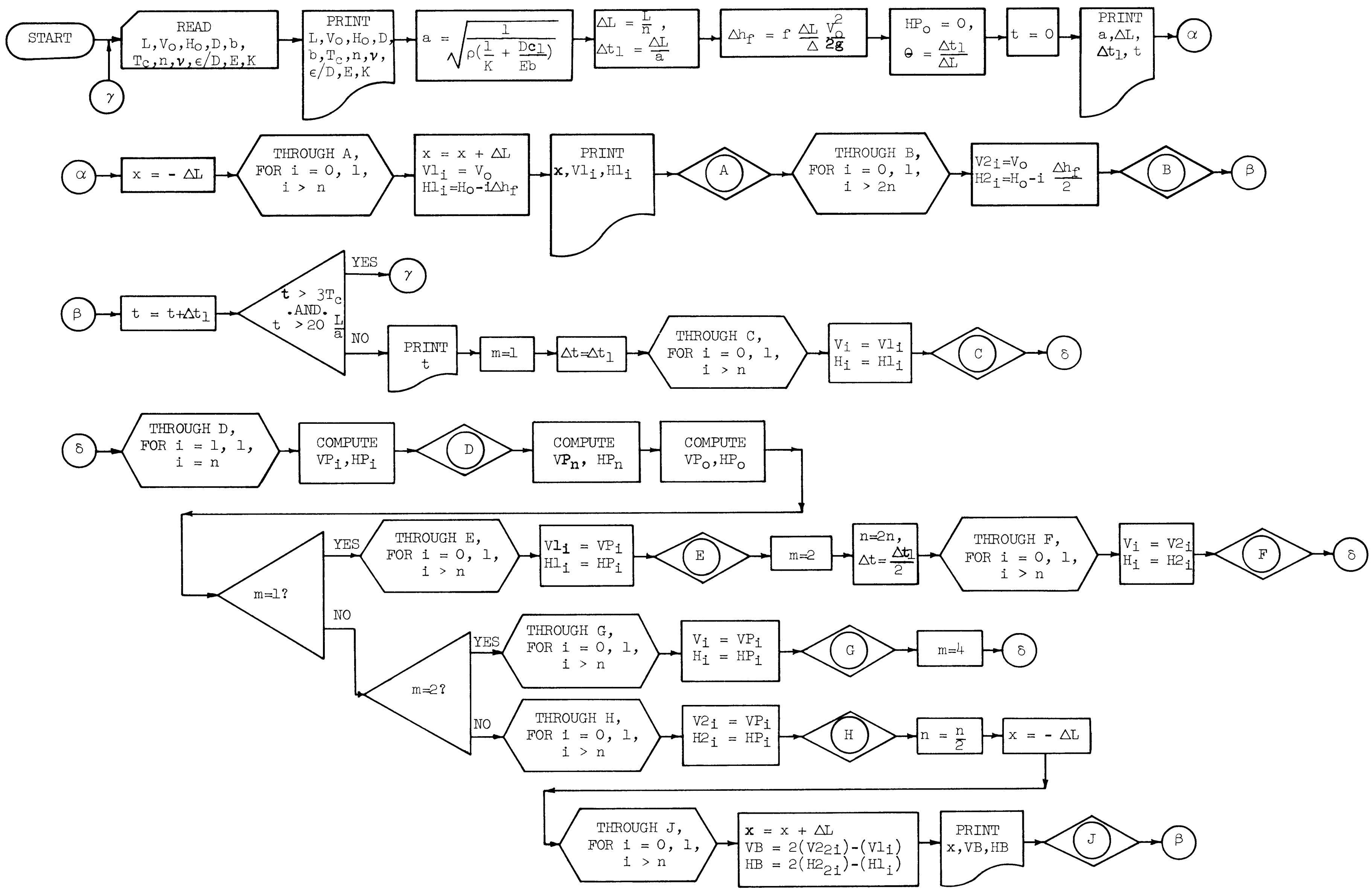


Figure 7. Abbreviated Flow Diagram for the Solution of Water Hammer Including Friction Effect in a Simple Pipe Line of Constant Cross Section and Constant Wall Thickness.

computation of V and H. By suitably storing the previous values of V and H and by using a number m as in Figure 7, the computation can be carried out ingeniously and economically to any desired length.

With this flow diagram as a skelton, MAD language programs(3) can be written by isolating different ways of gate closure and the evaluation of friction factor, $f = f(V,D,\nu,\epsilon/D)$, as subroutines. Some examples of MAD statements can be found in Appendices I to III.

G. Further Remarks

It is well known that the friction always acts against the direction of motion. That means for the reversal of flow the friction term must change its sign accordingly. Unfortunately the effect of friction is expressed in terms of V^2 in the basic differential equations and is incapable of satisfying this requirement. This difficulty can be eliminated by writing V^2 as $V|V|$ which has the same magnitude as before and changes its sign automatically when the flow is reversed. This modification has been provided in the computer programming.

As mentioned in the summary of the previous chapter, usually the nonlinear terms VV_x and VH_x may be neglected in engineering practice. These have been retained in this chapter, inasmuch as the omission of them will not mathematically reduce the difficulty of solution due to one more nonlinear term $\frac{f}{D}\frac{V^2}{2g}$, and by treating the solution more generally it may serve helpfully in some occasions when their effect comes to be appreciable. In the earlier analytical solutions, authors solved the differential equations linearly by omitting these two terms and applying a linear approximation to the friction term.

It is true that in the actual application of the present method to an engineering problem, some computer time can be saved by neglecting these two terms. This can be easily achieved by replacing all terms of $(V+a)$ and $(V-a)$ in the computer program by a and $-a$ respectively.

In case of gradually changing cross-section or wall-thickness $a = a(x)$ must be computed at each point along x. In case of abrupt changes in the pipe line, the pipe can be divided into segments and the same water hammer computation can be performed in each segment with new boundary conditions at each junction. Some further details of computer solution and its application to a few compound systems are presented in the next chapter and Appendices.

IV. APPLICATIONS TO VARIOUS PIPE CONNECTIONS AND DIFFERENT GATE CLOSURES

As briefly mentioned in the preceding chapter, with Equations (36) and (43) and the method of computation as a core, the solution of water hammer phenomena can be extended to a great many different problems, e.g., to various pipe connections and different gate closures. These solutions can be found by adding modifications to the program, by providing appropriate boundary conditions, or by combining sets of computations.

It is helpful and worthwhile here to introduce some applications, because:

(a) The effectiveness and flexibility of this method can be best demonstrated through these applications.

(b) It is useful for comparison with the results obtained by other methods.

(c) Some details in the experimental verification necessitates this kind of study. Since the experiment must be conducted in conformity with the conditions studied in the theoretical analysis, when it is impossible to achieve the theoretical conditions, the theoretical solution should be modified accordingly to meet experimentally feasible conditions.

In the following parts of this chapter, several examples of solutions of water hammer problems are presented. Some of them are essential tools for the comparison of the analytical and experimental results. It is not the purpose of this chapter to attempt to cover the entire field of water hammer, but merely to indicate the usefulness of this tool.

A. Variable Friction Factor

Since the friction factor f in the equation

$$dh_f = f \frac{dx}{D} \frac{V^2}{2g}$$

is a function of the Reynolds number and the relative roughness of the pipe, i.e., a function of D, V, ν, and ϵ/D, it is found more convenient to keep the evaluation of f values outside of the main program. Thus the main program can be kept neat and its structure be kept free from any influence due to change in evaluation of f. Unlike a constant friction factor used hitherto, the variable friction factor f, calculated from given D, V, ν and ϵ/D, should theoretically be able to take care of any flow cases.

For the laminar flow, the value f in a pipe will be,

$$f = \frac{64}{R} = \frac{64}{\frac{DV\rho}{\mu}} = \frac{64\nu}{DV} \quad , \tag{54}$$

in which ρ and μ are the density and the viscosity of the liquid.

For other flow cases with different sizes and materials of pipe, the corresponding equations or values of f can be found from friction factor charts or tables.[20,21]

If for a specific pipe, the friction factor be evaluated in terms of $h_f/(\frac{L}{D}\frac{V^2}{2g})$, then theoretically there should be no need for classifying the type of flow.

In this study, a few subroutines for the evaluation of friction factor have been made. One is a subroutine for the Moody diagram, which employed the same equations used in producing that diagram. With this subroutine, the machine is able to provide any current value of f at any point along the pipe, from the given current data for that point. Needless to say, the usefulness of this subroutine is not limited to this study, but can be extended to any other pipe problems in connection with the computer. (See Appendix IV.)

Others are special subroutines for the particular pipes used in the experiments. They were obtained by careful determination of friction loss through those pipes, and were used in producing the theoretical results in Chapter VI. (See Appendix I.)

B. <u>Rapid and Slow Gate Closures</u>

It was mentioned in the previous chapter that many different types of gate closures can be handled by imposing suitable boundary conditions. In this case, it is also found that the program can be kept orderly and the computation can be carried out more efficiently by providing boundary conditions for gate operation in a subroutine, because:

(a) In case of an instantaneous gate closure, the main program can simply skip the step of calling this subroutine; thus the structure of the main program will remain unaffected. The same applies to the time after the gate closure is completed.

(b) By applying many different ways of gate closure, we can solve a number of water hammer problems produced by those closures. Since all these boundary conditions due to gate closure are supplied by subroutines and can be easily replaced without any change in the main program, great flexibility in computation, results especially in designing gate operations for keeping the pressure below a fixed maximum value.

Some examples of water hammer produced by different gate closure time relations for a simple pipe line are shown in Figure 8 to Figure 10. The examples of instantaneous and slow gate closures for the same pipe are shown in Figure 11 and Figure 12.

C. Pipe Line with Stepwise Changes in Diameter

In the case of a pipe line with stepwise changes in diameter, the pipe line can be divided into segments at each step, and the same water hammer computation can be performed in each segment with suitable boundary conditions at each junction. At junction A, the conditions, (see Figure 13)

$$V_1 D_1^2 = V_2 D_2^2 \tag{55}$$

and

$$H_{A_1} = H_{A_2} \quad , \tag{56}$$

in which V_1, D_1 and V_2, D_2 are velocity and pipe diameter before and after the junction respectively, may be used when the velocity head and the minor loss at the junction are negligible. An example of applying the relationships (55) and (56) can be found in Appendix II.

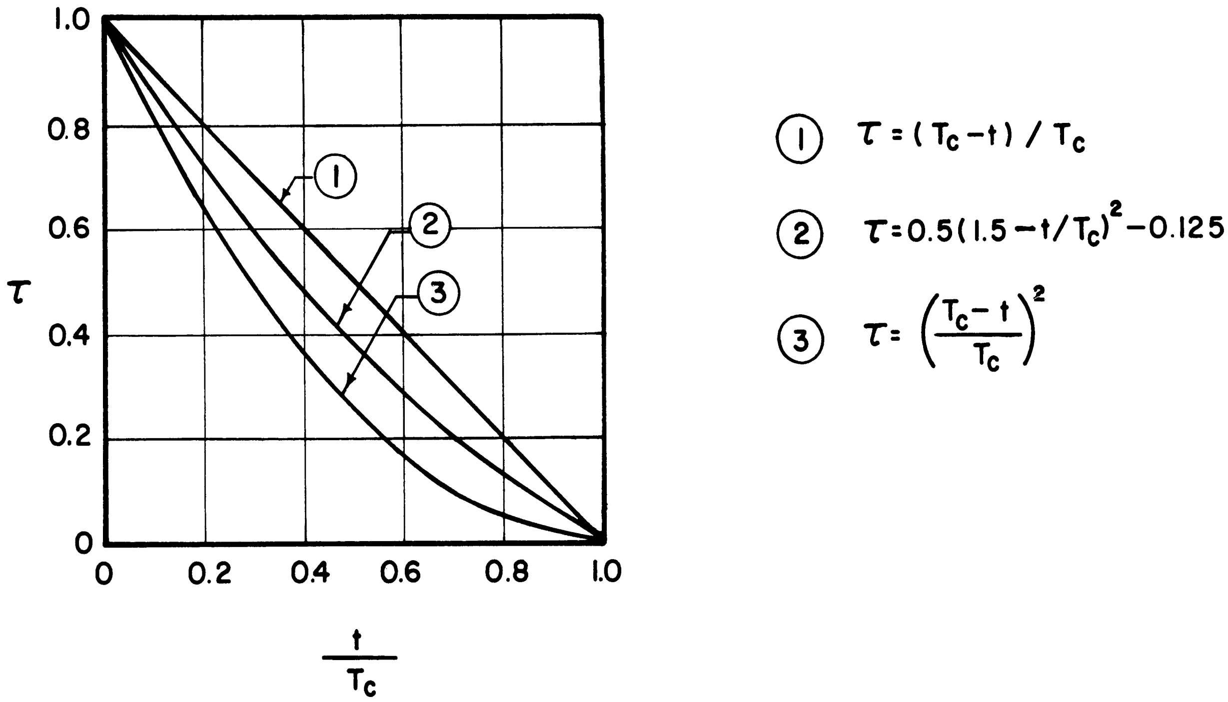


Figure 8. Valve Closure Time Relations.

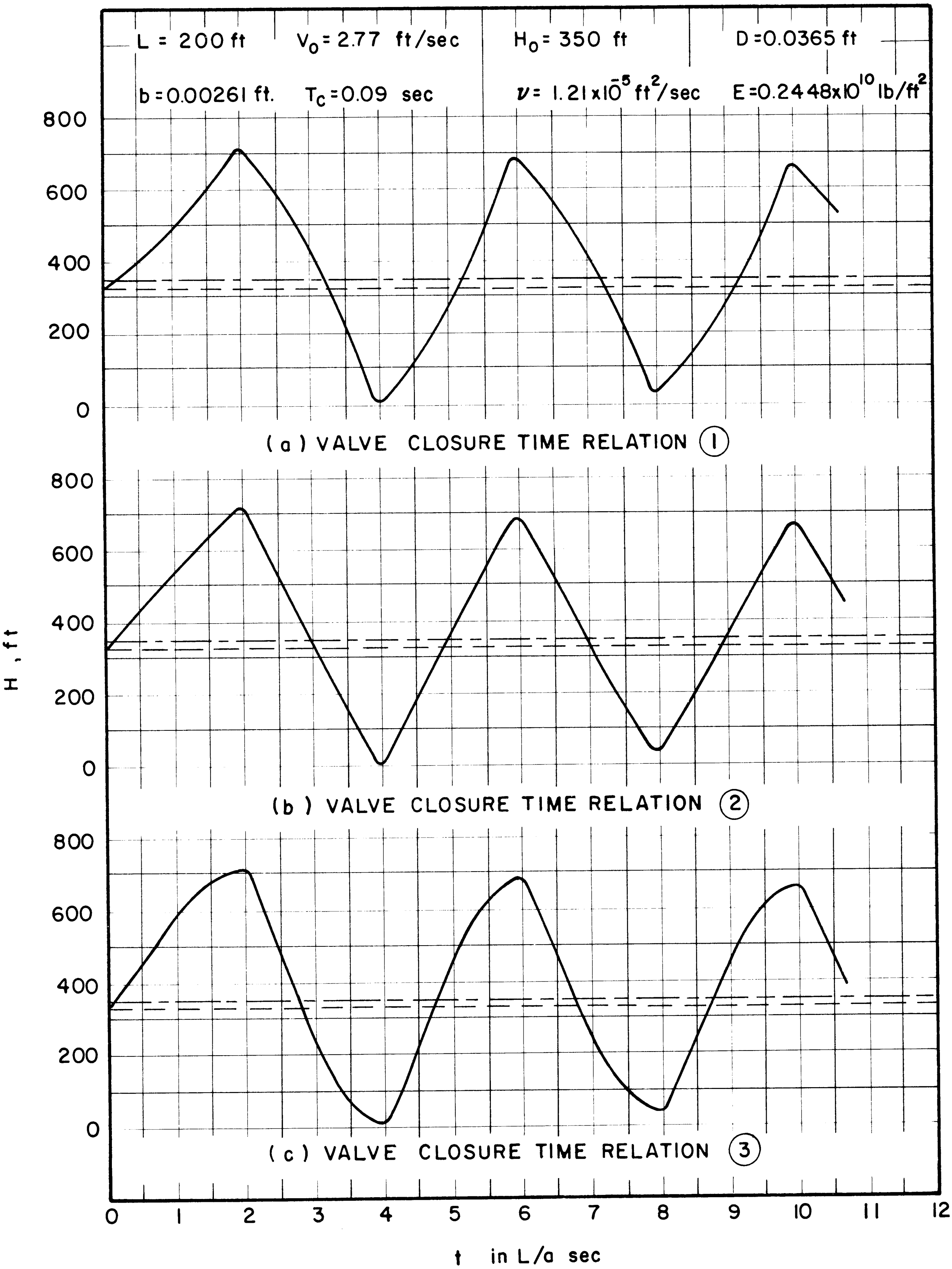


Figure 9. Variation of Water Hammer Pressure with Time, at Valve End, Rapid Closure, $T_c \doteq 2L/a$, for Different Valve Closure Time Relations (See Figure 8).

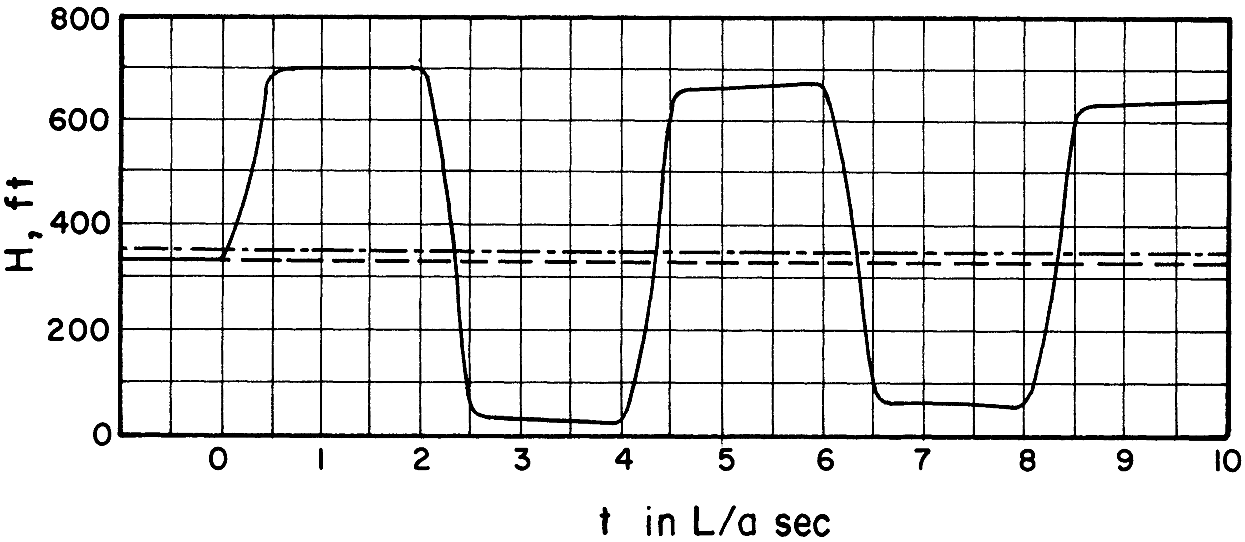


Figure 10. Variation of Water Hammer Pressure with Time, at Valve End, Rapid Closure, $0 < T < 2L/a$, for Valve Closure Time Relation I(1) (See Figure 8).

$L = 200$ ft $V_o = 2.77$ ft/sec $H_o = 350$ ft $D = 0.0365$ ft

$b = 0.00261$ ft $T_c = 0.022$ sec $\nu = 1.21 \times 10^{-5}$ ft^2/sec

$E = 0.2016 \times 10^{10}$ lb/ft^2

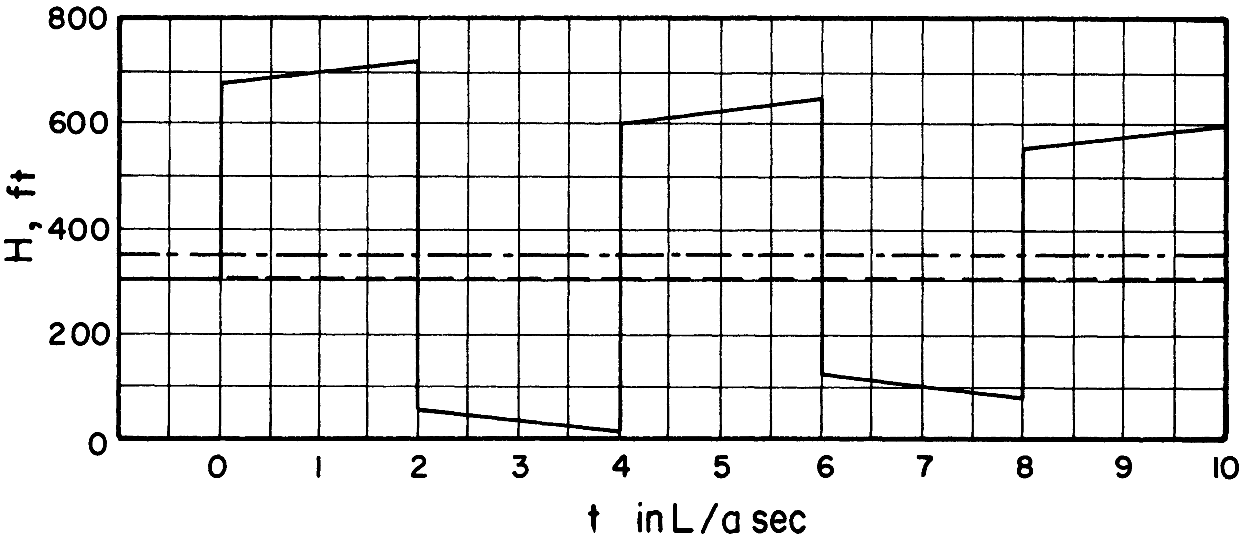


Figure 11. Variation of Water Hammer Pressure with Time, at Valve End, Instantaneous Closure, $T_c = 0$.

$L = 300$ ft $V_o == 2.77$ ft/sec $H_o = 350$ ft $D = 0.026$ ft

$b = 0.00261$ ft $T_c = 0$ $\nu = 1.21 \times 10^{-5}$ ft^2/sec

$E = 0.2448 \times 10^{10}$ lb/ft^2

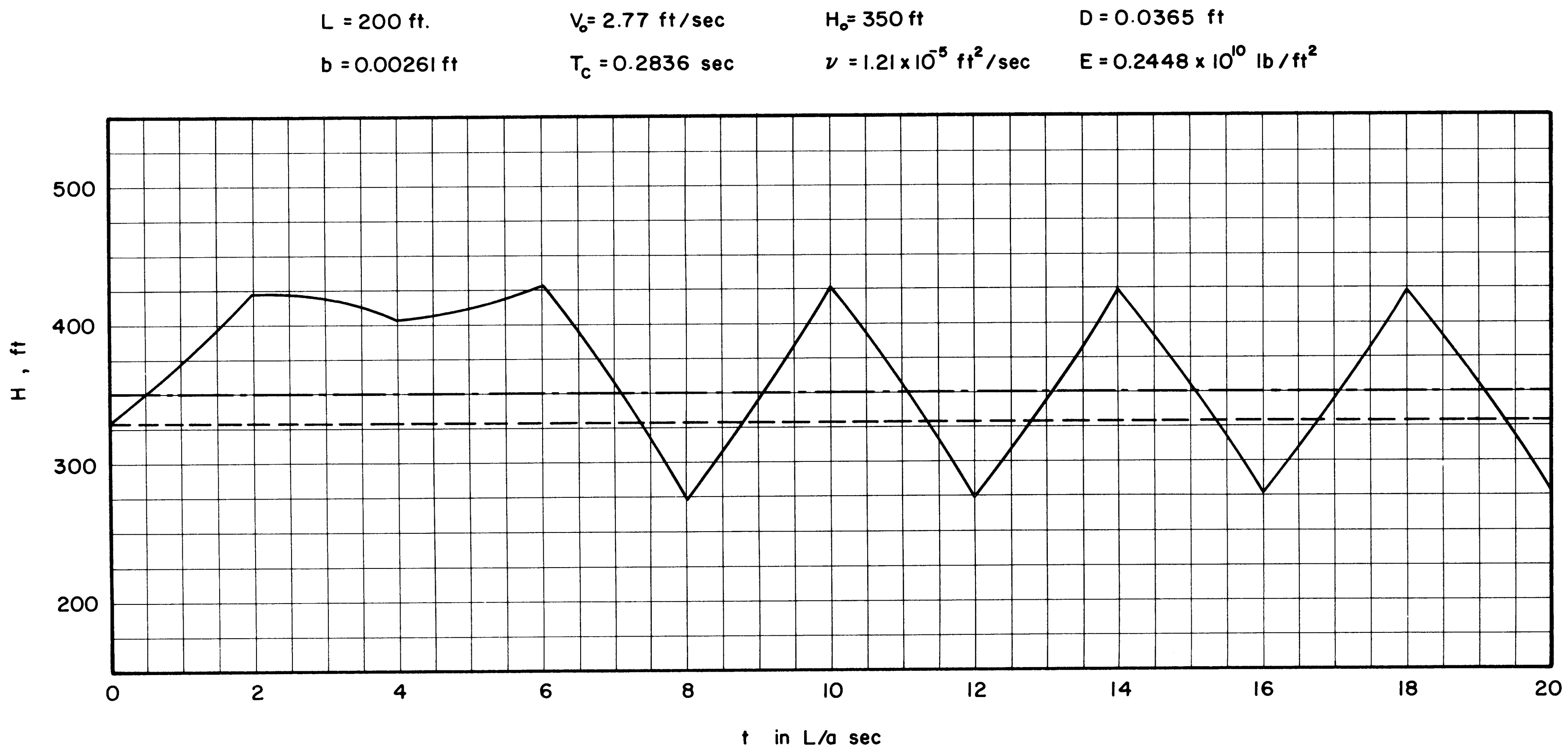


Figure 12. Variation of Water Hammer Pressure with Time, at Valve End, Slow Closure, $T_c > 2L/a$, for Valve Closure Time Relation (1) (See Figure 8)

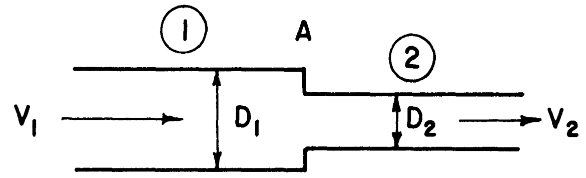


Figure 13. Pipe Line with Stepwise Change in Diameter.

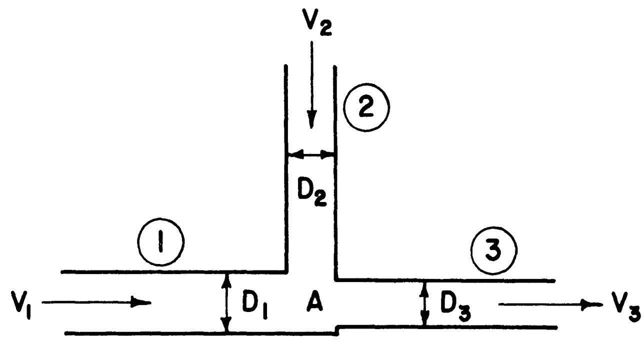


Figure 14. A Junction Where Three Pipes Meet.

D. Compound Pipes

With the same principle mentioned in the previous section, but with more involved boundary conditions, the method of characteristics by the computer can be developed to almost any kind of pipe system. Three examples, which are the conditions employed in the following experiments, are shown here. They are:

(a) a simple pipe line with a dead-end branch connection,

(b) a pipe line with a stepwise change in cross section and a dead-end branch connected near the valve end,

(c) a pipe line with a stepwise change in cross section and a dead-end branch connected at the junction of two segments, as shown in Figure 15.

The conditions to be satisfied at junction A, (see Figure 14) where three pipes meet, are

$$V_1 D_1^2 + V_2 D_2^2 = V_3 D_3^2 \qquad (57)$$

and

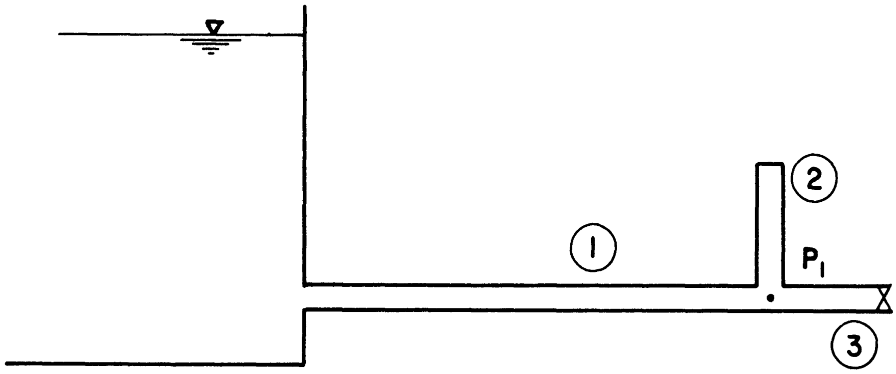
$$H_{A_1} = H_{A_2} = H_{A_3} \qquad (58)$$

if the velocity head and the minor loss are neglected.

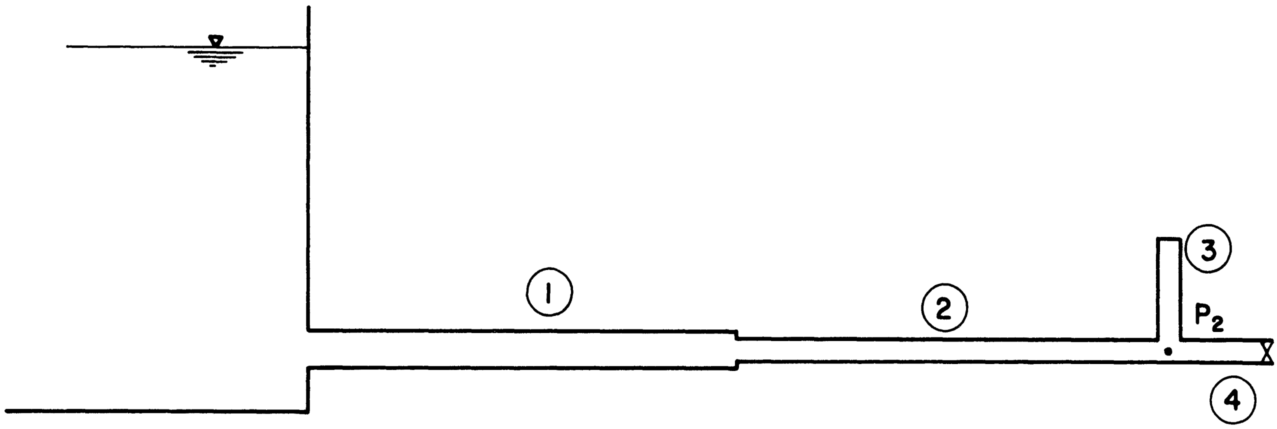
The detailed MAD Language Programs for cases (a), (b) and (c) are shown in the Appendices I to III. Minor losses and velocity heads have been neglected because they are very small compared with the loss due to pipe friction.

E. Nonuniform Pipes

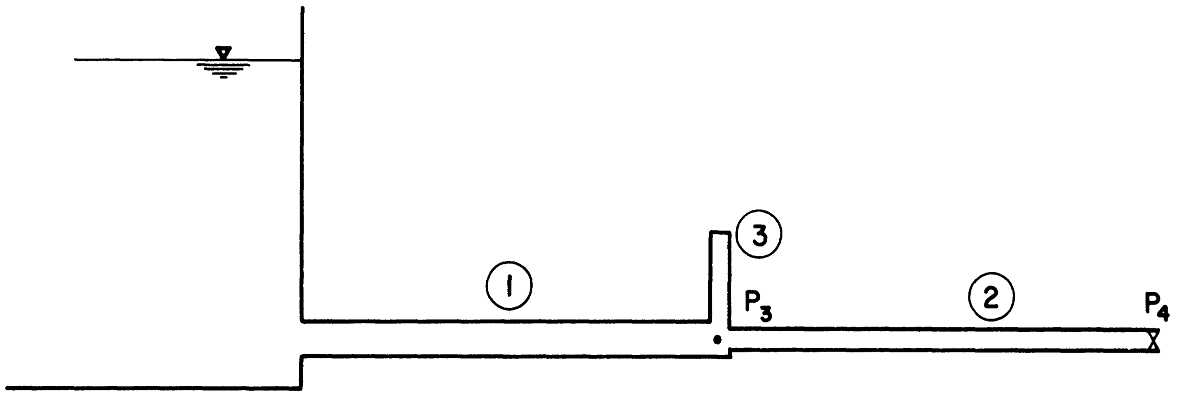
In case of gradually varying cross-section or wall-thickness, the computation introduces no major difficulty so far as the one-dimensional



(a) Case I



(b) Case II



(c) Case III

Figure 15. Three Examples of Compound Complex Pipes.

method is applicable; i.e., the change of factors should not be so large that the assumption of uniform velocity and uniform pressure distribution at any cross section would not be valid. In those computations, instead of using constant a and D, $a = a(x)$, $D = D(x)$ should be used at each point along x.

V. EXPERIMENTAL SET-UP AND PROCEDURES

An experimental check was planned as the next phase in this study. In order to make the friction effect easily observable, a pipe which would produce appreciable head loss between two ends had to be used. Although this can be achieved by increasing the length of pipe and reducing its diameter, there are some difficulties in actual installation in a laboratory. Owing to the limited space in a building, it is usually very difficult to place a sufficiently long straight pipe indoors. Moreover, a long pipe needs many supports, which will produce undesirable disturbances at each point of support. It is also not advisable to bend the pipe, since it gives disturbances to the pressure wave also.

Under these restrictions, two coiled copper tubes, as shown in Plate I, were finally used. One was 300 ft in length, 0.032 in. in thickness and 1/2" O.D., the other one 300 ft.* in length, 0.032 in. in thickness and 3/8" O.D. A core, on which the tubes were to be coiled, was built. It consisted of two wooden plates at each end, about 3'-8" apart, and twelve 1/2" steel bars connecting the plates to frame a core. Each bar was wrapped with a soft rubber tube in order to protect the copper tubes from any damage due to direct contact with the steel bars and to prevent possible disturbance to the transmission of pressure waves at the point of direct contact. The copper tubes were then coiled very carefully, uniformly and loosely around this core. The diameters of the coils were about

* At the end of the experiments, the former was reduced to 299.05' and the latter to 294.30', because by handling, in determination of friction factor, by connection and disconnection to various sources and fittings, the ruined tips had to be cut off each time.

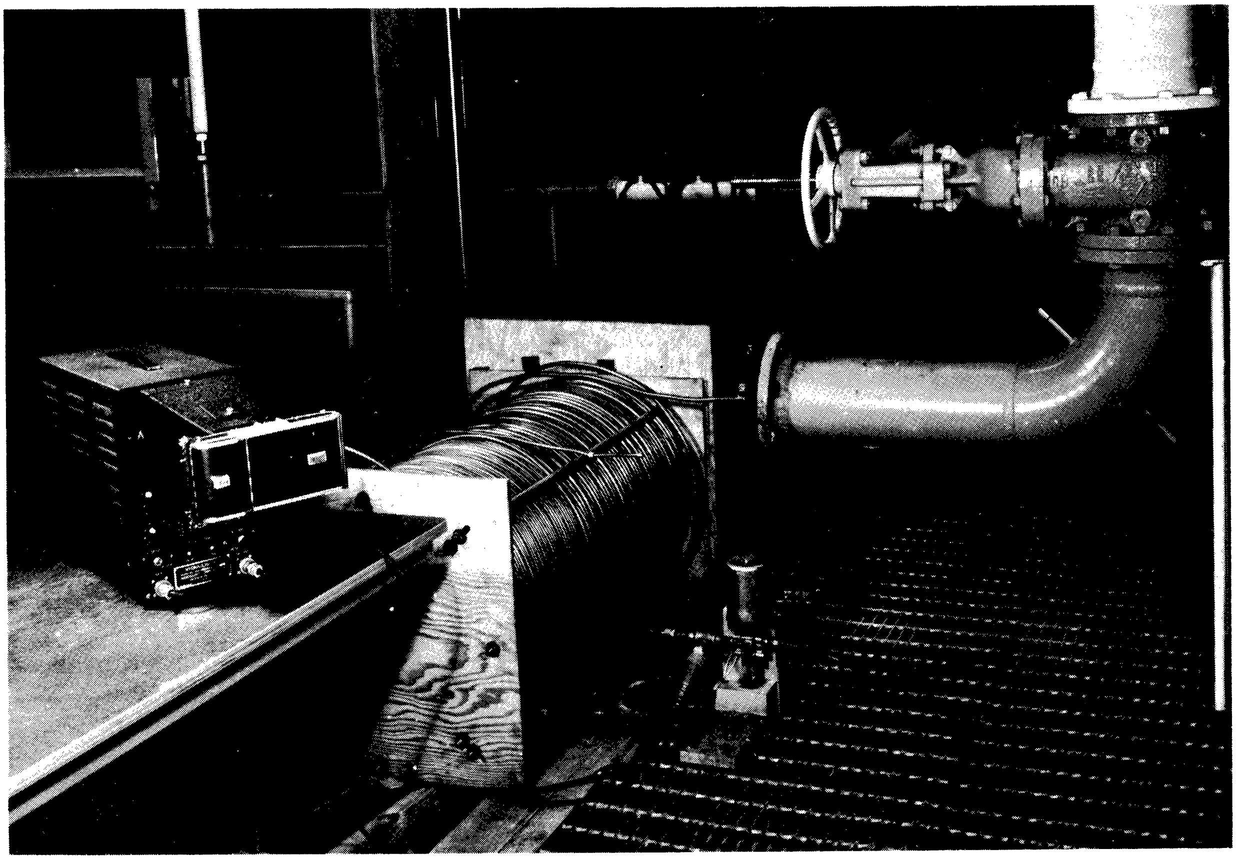


Plate I. The Experimental Set-Up.

two feet. It was considered that although the copper tubes were not straight but coiled, due to the uniform curvature along the entire length of the tubes, there would be no abrupt disturbance along the tubes, and they would act in a manner similar to straight tubes.

The reservoir should have a capacity to provide a constant head during the experiment. A large pipe connected to a very small pipe in which the water hammer phenomena will be investigated, can serve as a reservoir. Two sources were used as intake in the experiments. One was from the 2" I.D. outlet pipe of a pressure pump which could produce about 200 psi. The other one was from a 8" pipe which was connected to a head tank, the elevation of which was about 44' above the outlet of the tube. The former has an advantage of being able to maintain a high pressure inside the tube and thus can provide a higher velocity without causing the minimum pressure to drop below the vapor pressure. However, there is a doubt of it possibly creating some transmission of pressure wave towards upstream from the junction of the tube and the outlet pipe, by which the reflection will be reduced by that amount. The latter, on the contrary, does not have such trouble, because the pipe is sufficiently large in comparison with the tube, but unfortunately the head is quite low in this case.

A considerable time had been consumed for the selection of the valve. If the slow valve closure were also to be investigated, the area of valve opening had to be accurately controllable as a function of time, or at least the time and the way of valve closure should be known. Since there were no such valves in all manuals searched, and the primary interest in the present experiment was to check the analytical solution of differential equations into which the nonlinear friction term was introduced, rather than to observe the difference caused by rapid and slow valve closures, it was decided to limit the test to the instantaneous closure. It was not too difficult to find a solenoid valve which could be closed practically instantaneously as compared to the time of the traveling of pressure wave; however, most of the valves were disc type and there was a possibility of bouncing of the disc during its operation. On that account, a slide-gate type solenoid valve was finally chosen. This type not only has the capability of tight instantaneous closure but also can possibly be used for the case of slow valve closure in the future if a good method is devised to slow down the speed of closure, or at least it can be used for the case of partial valve closure by providing some check. The sluice-gate type solenoid valve used in the test and its connection with the copper tube are shown in Plate II. The gate is closed when no voltage is applied, i.e., it is a normally closed type.

A hydrauliscope, as shown in Plates I and III, was used for the pressure measurements. This is a high speed electronic analyzer which can respond to pressure changes of high frequency and high rate, and is furnished with attachments to permit photographic recording of the curves traced on the screen of the cathode-ray tube.

The pressure pick-up element used in connection with the hydrauliscope is of the resistance type, having a linear response with pressure variation thus permitting the direct static calibration. It is also designed for temperature compensation.

The friction factors of the experimental tubes were determined with great care.

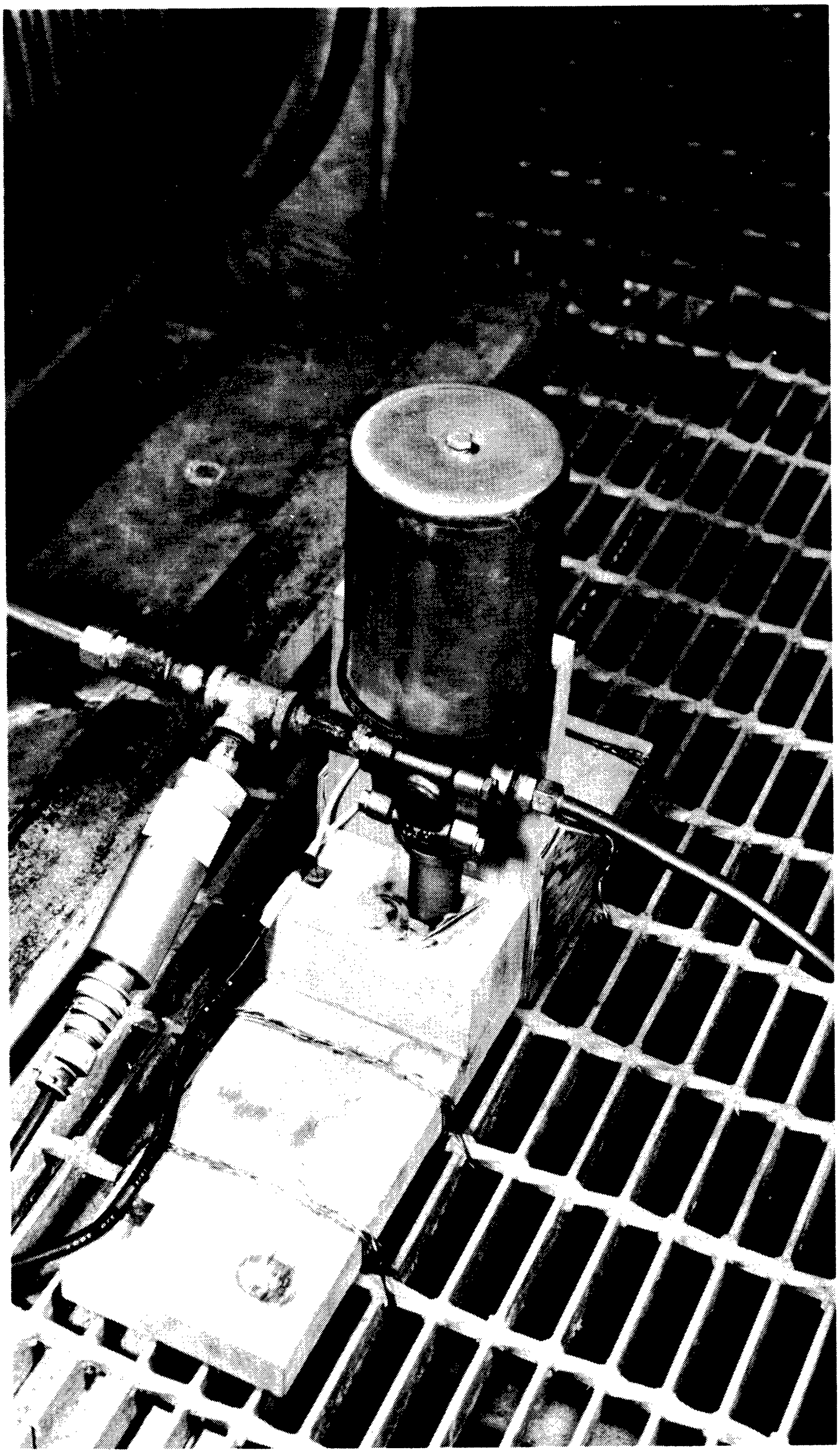


Plate II. The Close-Up of the Solenoid Valve and the Pressure Pick-Up Element.

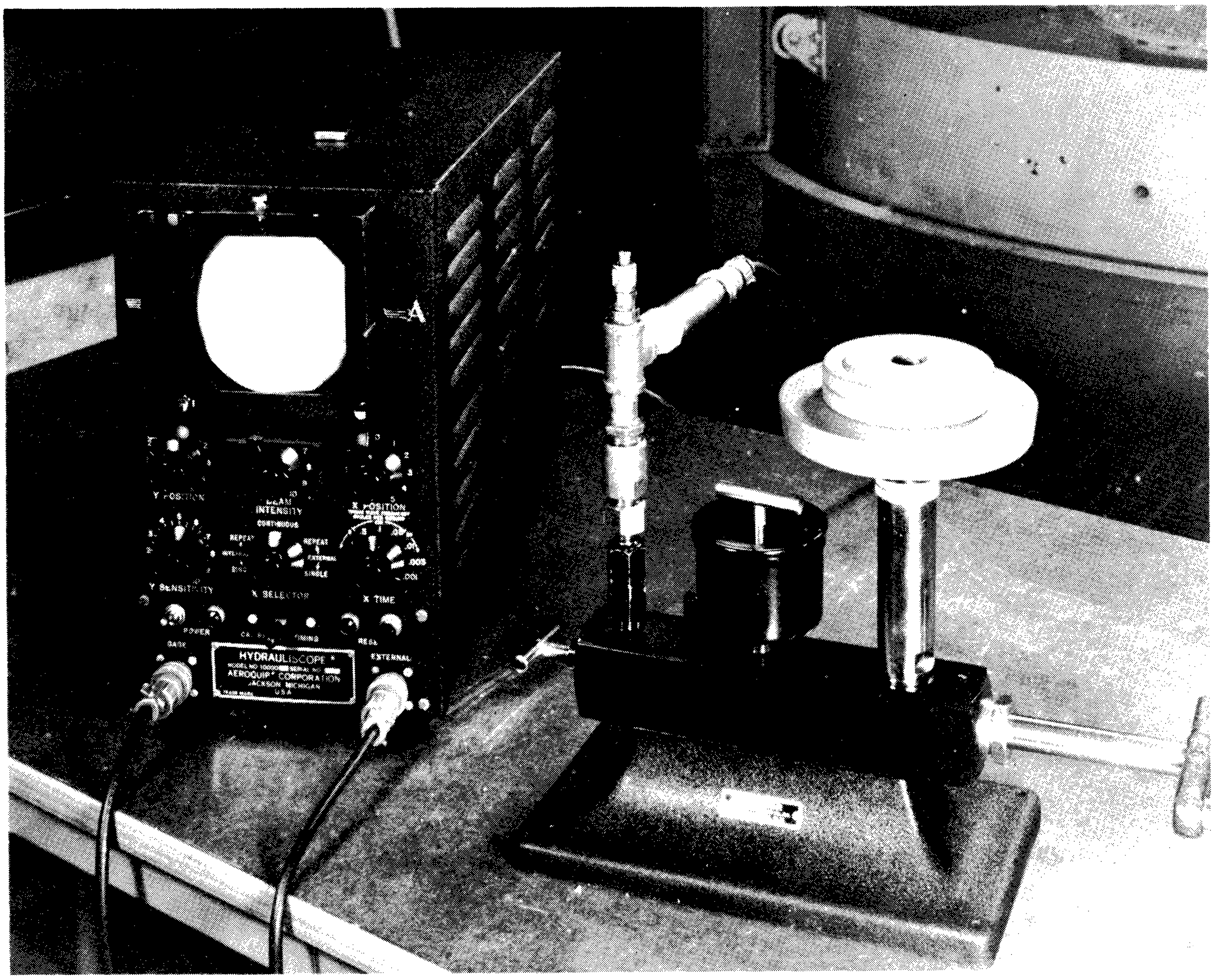


Plate III. The Calibration of Pressure Reading.

Four peizometer rings were made and attached at each end of the two tubes.[8] Four radial holes were carefully drilled at a distance not too close to the end of the tube but within the reach of a small round file so that all burrs could be removed from the inside. The four orifices were placed symmetrically about the vertical bisecting plane with equal spacing, allowing no orifice to be located at the top of the tube, lest air bubbles should enter the hose connected with the manometer. Then the rings were soldered to the outside and at each set of measurements they were connected by rubber hoses to two ends of a differential manometer.

Two kinds of liquid were used for the manometer. Acetylene tetrabromide, specific gravity of which is 2.94, was used for the sensitive measurements in the range of low velocity, and mercury for larger pressure drops.[22] Water was taken from a head tank located under the roof of the Fluid Engineering Laboratory. The water level in the tank was maintained constant during the measurement. The height of weir in the tank above the tube was approximately 43 ft.

Before each series of the measurements, the fastest flow of water, obtainable from the tank, was made to go through the tube to expel any possible bubbles trapped inside. The waste cocks on the top of two limbs of the differential manometer were also widely opened to bleed out the water in order to get free of any trapped air bubbles. The connecting hoses were made short and were continually sloping upward from the rings toward the waste cocks.

The determination of friction factor was then made. The discharge, the temperature of water and the difference in height of the two menisci of the manometer were measured for each different opening of the discharge cock. From these data, the velocity, the kinematic viscosity and the friction loss were evaluated and the curve of friction factor f vs. Reynolds number $\mathbf{R}$ was plotted. Each pair of these two values were obtained from

$$f = \frac{h_f}{(L/D)(V^2/2g)} \tag{59}$$

and

$$\mathbf{R} = \frac{DV}{\nu} \tag{60}$$

The curves, and the MAD program made thereby, are shown in Figure 16, Figure 17, and Appendix I, respectively.

Before proceeding with the experiments of water hammer, the hydrauliscope was checked and adjusted. Then it was calibrated very carefully with the aid of a dead weight gage tester, as shown in Plate III. The Y-sensitivity of the hydrauliscope, which is to adjust the height of the curve or to set the vertical scale at any desired value, say 200 psi for each vertical inch, was calibrated for the two pressure pick-ups used in the experiments. One of them had a 0-500 psi system operating range with 750 psi maximum continuous shock pressure and 1250 psi maximum static pressure permitted. The other one had a 0-200 psi pressure range with 300 psi and 500 psi corresponding permissible values. The pressure pick-up element was installed in the dead weight gage tester. The desired dead weights were applied on the tester and the Y-sensitivity was adjusted to give a desired vertical movement of the beam for the given weight. This value of Y-sensitivity was noted for the experiments.

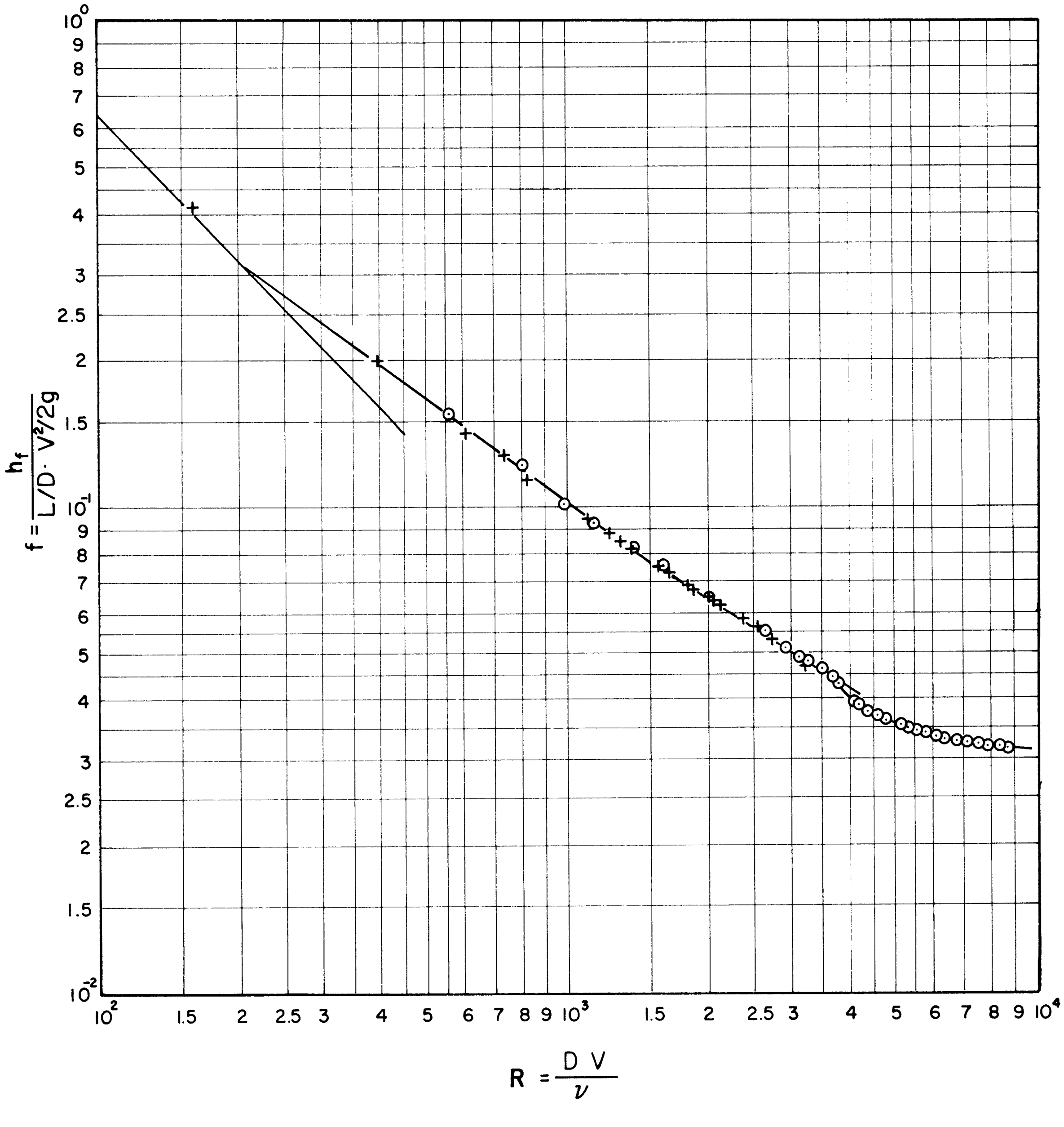


Figure 16. Resistance Diagram for 1/2 in. O.D. Copper Tube.

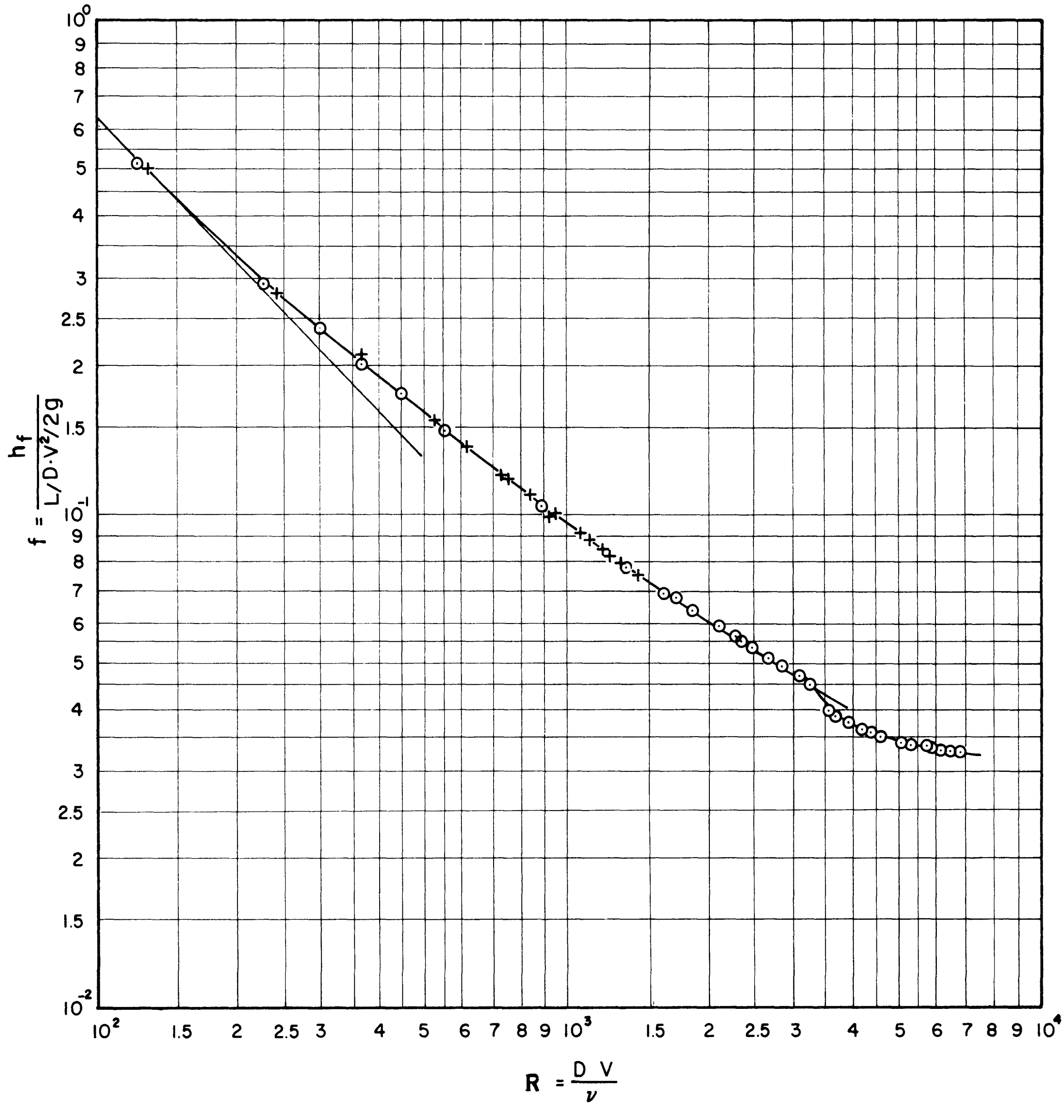


Figure 17. Resistance Diagram for 3/8 in. O.D. Copper Tube.

The experiments were planned for two different flow cases; one for a simple line of a constant cross-section, with the pressure pick-up attached at the valve end, the other one for a line with a stepwise change in cross-section, to which the pressure pick-up would be attached at the valve end and at the junction. However, the practical situation turned out to be the three cases shown in Figure 15 of Chapter IV, owing to the additional projection caused by attaching the pick-up element to the main line. The length of this element is so short in comparison with that of the copper tube that the friction loss caused by its wall shear and by the tee-joint (See Plate II) is quite negligible, yet the effect of storing the additional volume of water in this element can by no means be ignored. Because of the compressibility of water and the elasticity of the pick-up tube, the water stored in this part would be compressed and expanded alternatively, generating a pressure wave moving back and forth with very high frequency as compared to that of the main line. This buffering effect and disturbances caused by emitting successive tiny pressure waves in all directions from the junction soften what otherwise would be a more abrupt pressure versus time curve. This effect has been clearly demonstrated by comparing the photographs taken in the experiments and the two analytical solutions by omitting and including the dead-end branch. (See Plate IV, Figure 11 and Figure 18.)

Each flow case was investigated for both water sources, the pressure pump and the head tank. A micro-switch was connected to the "external" plug-in to facilitate the operation of the hydrauliscope. The camera was installed over the screen of the hydrauliscope, as shown in Plate I, in order to take photographs of pressure versus time curves. The camera consisted only of a metal box with a fixed f 5.6 lens which could focus the screen image continuously on a standard 2-1/4 x 3-1/4 film pack, and with a black plate which could be pulled out for exposure, because there was no shutter or iris.

The steps for the experiments are as follows: to start the flow, the solenoid valve was opened by pressing the switch; then the discharge cock attached at the downstream side of the valve was adjusted to obtain approximately the desired velocity in the tube. The discharge was then measured by using a stop watch and a measuring cylinder or weighing the water. At the same time, the temperature of water was also recorded. The "X-selector" dial of the hydrauliscope was turned to the "external single" (see Plate III) so that only one sweep of the beam would appear on the screen for one push of the microswitch. To start taking a photograph the exposure plate was first pulled out of the camera, and five sweeps of beam were recorded successively to represent five distinct features. (See Plates IV to VI.) The first horizontal sweep is a pressure line at the pressure pick-up for the steady flow. Then the switch of the solenoid valve and the button of the micro-switch were pushed down almost at the same time so that the curve of the water hammer produced by valve closure could be traced out on the screen. After the pressure wave died out due to friction, the third horizontal line was recorded. This represents the static pressure in the reservoir. The zero-pressure line, or the reference line, was recorded fourth, then followed by the last line, the timing wave, which would indicate the time scale used, by providing sine waves of a definite frequency. The screen illumination was increased before the exposure plate was inserted back into the camera, in order to show the grid lines clearly on the film. When the water was taken from the pressure pump, tiny ripples indicating the pressure fluctuation caused by pumping action were observed.

The experimental results thus obtained are shown in the next chapter for the comparison with the theoretical results.

VI. DISCUSSION OF THEORETICAL AND EXPERIMENTAL RESULTS

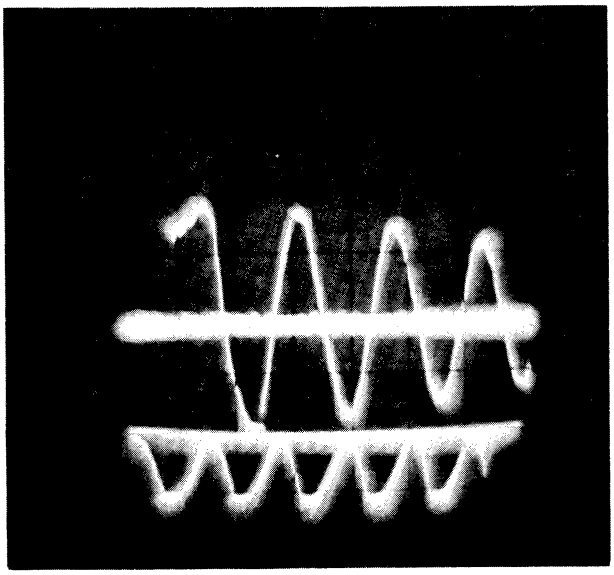
A number of experiments for water hammer under different conditions were performed in the Fluid Engineering Laboratory at the University of Michigan. Owing to the limitation of space and computer time, only several runs which best represent each flow case are listed in Table I. The photographs of pressure versus time curves taken in these runs are also shown in Plates IV, V, and VI. The frequency of timing wave on the hydrauliscope was set at 5 cps for those shown in the photographs. However, a careful calibration of the timing wave, later, indicated that the actual frequency of the hydrauliscope used in the experiments was 5.285 cps. From these photographs and the data given in Table I, the pressure head versus time curves were plotted by solid lines in Figure 18 to Figure 22.

The same data used in the experiments were fed into the electronic computer. The factor of pipe restriction c_1 was taken as $c_1 = 1 - \mu/2$ in these computations. [See Chapter II, B., Case (c).] The velocity and the piezometric head (strictly speaking) at various points along the lines for every assigned time interval were computed and printed out in a tabular form as shown in a few examples given in Appendices I to III. Since in these experiments the datum plane was taken at the level of the pressure pick-up element and the velocity heads are so small, the total head, the piezometric head, and the pressure head at the pick-up are practically identical. From these answers, the pressure head versus time curves at the point of pressure pick-up were plotted by dotted lines in

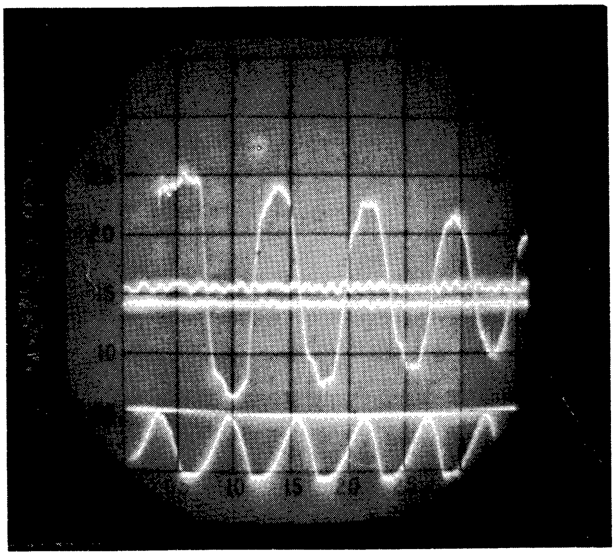
TABLE I

EXPERIMENTAL DATA

No.	Type of Line System	Inside Diameter D ft.	Initial Velocity V_o ft/sec	Head at Reservoir H_o ft.	Temperature °F	Kinematic Viscosity ν ft^2/sec	Reynolds' Number **R**	Bulk Modulus of Elasticity K lb/ft^2	Frequency of Timing Wave f	"Y" Scale Factor psi/in	Source of Water	Theoretical and Experimental Results
1	Fig. 15 (a) Case I	0.03633	2.94	451	106.0	0.692	15330	0.475×10^8	5.285	200	Pressure Pump	Fig. 18 Plate IV
2	Fig. 15 (a) Case I	0.02592	3.08	486	93.1	0.799	9990	0.466	5.285	200	Pressure Pump	Fig. 19 Plate IV
3	Fig. 15 (b) Case II	0.03633 0.02592	1.112 2.183	442	101.8	0.726	5556 7790	0.471	5.285	200	Pressure Pump	Fig. 20 Plate V
4	Fig. 15 (c) Case III	0.03633 0.02592	1.083 2.127	432	103.8	0.712	5530 7740	0.473	5.285	200	Pressure Pump	Fig. 21 Plate V
5	Fig. 15 (a) Case I	0.03633	0.367	45.0	76.9	0.993	1340	0.462	5.285	40	Head Tank	Fig. 22 Plate VI

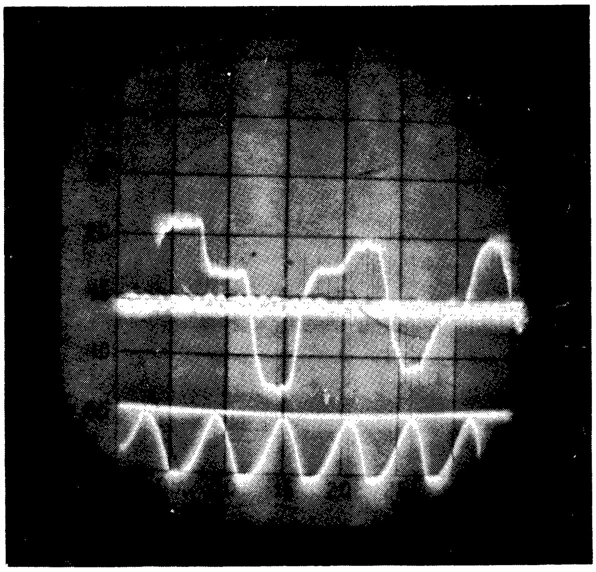


No. 1

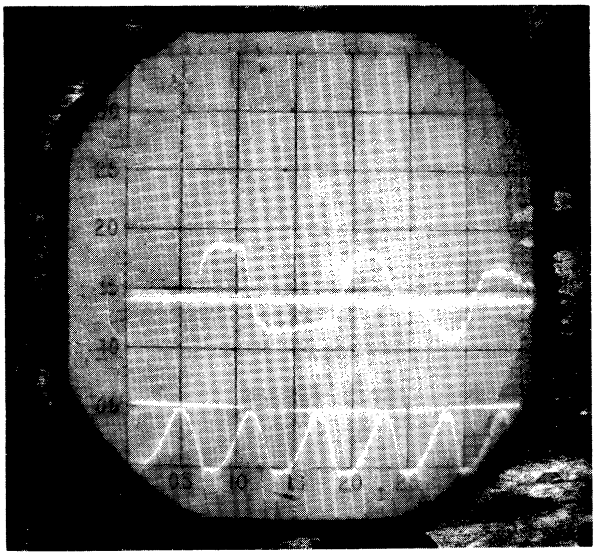


No. 2

Plate IV. The Experimental Results.
(See Table I and Figures 18 and 19)

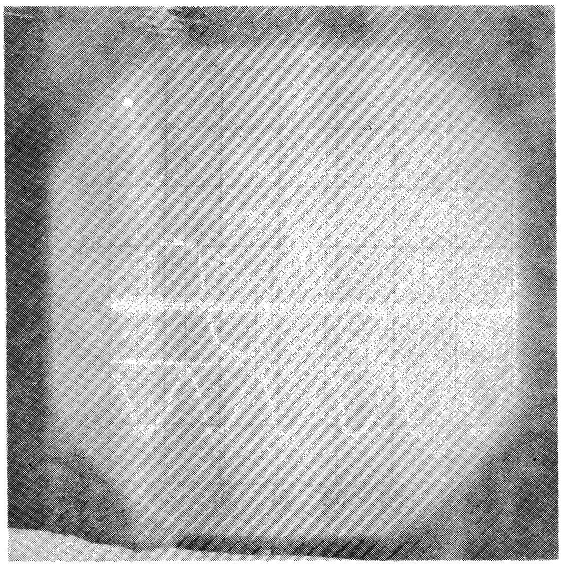


No. 3



No. 4

Plate V. The Experimental Results.
(See Table I and Figures 20 and 21)



No. 5

Plate VI. The Experimental Results.
(See Table I and Figure 22.)

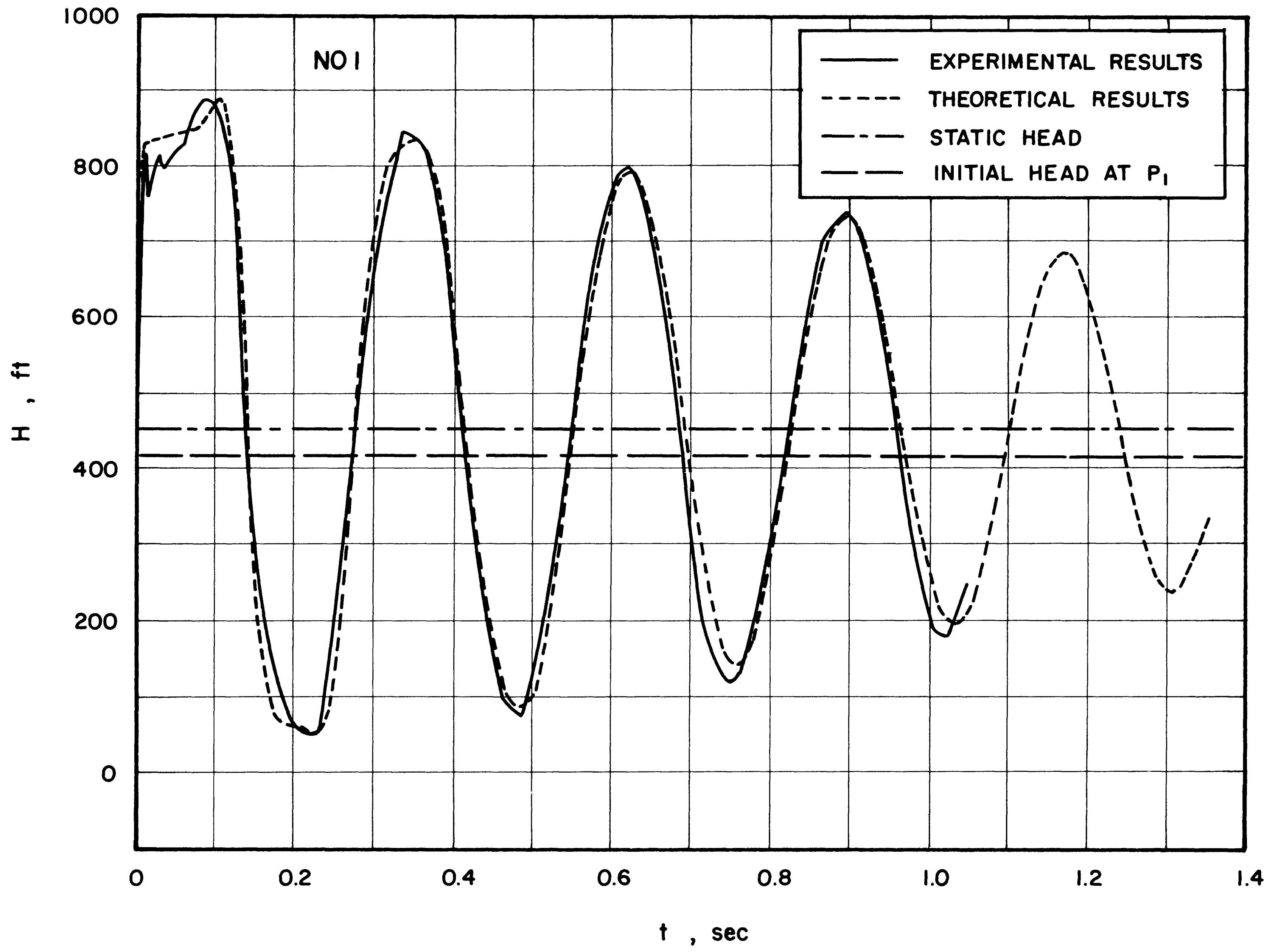


Figure 18. Pressure-Time Curve at P_1; Case I (See Figure 15 a).

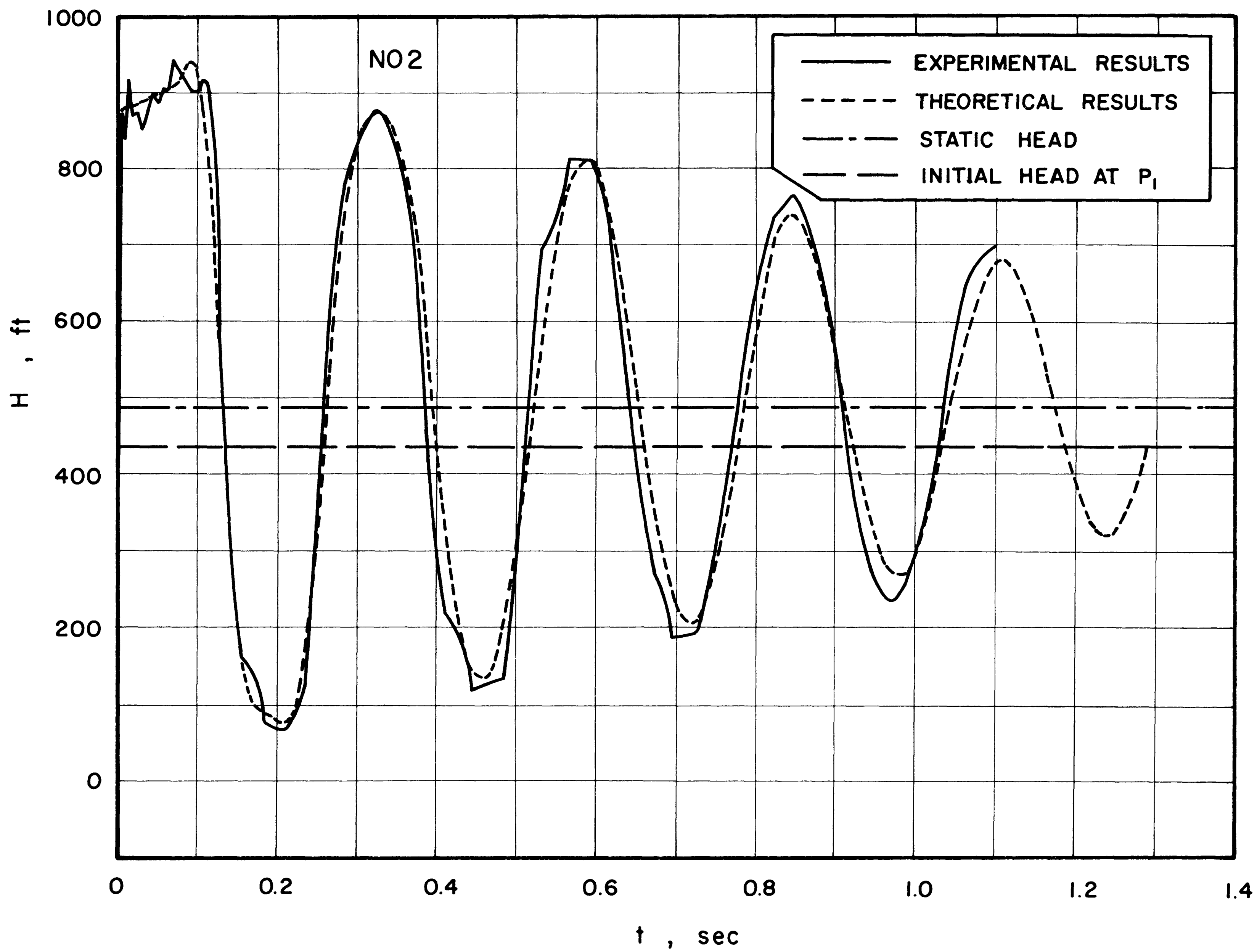


Figure 19. Pressure-Time Curve at P_1; Case I (See Figure 15a).

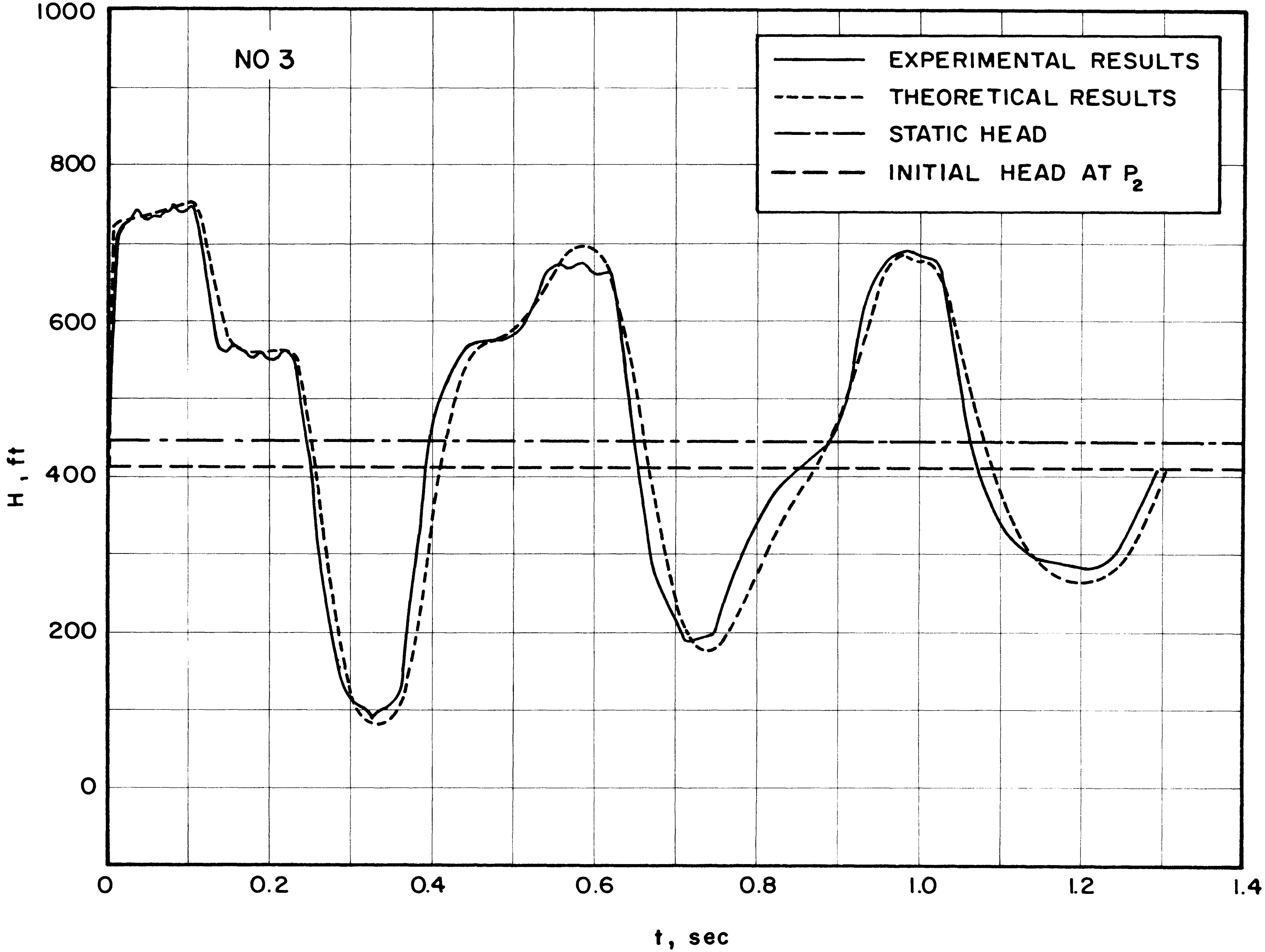


Figure 20. Pressure-Time Curve at P_2; Case II (See Figure 15b).

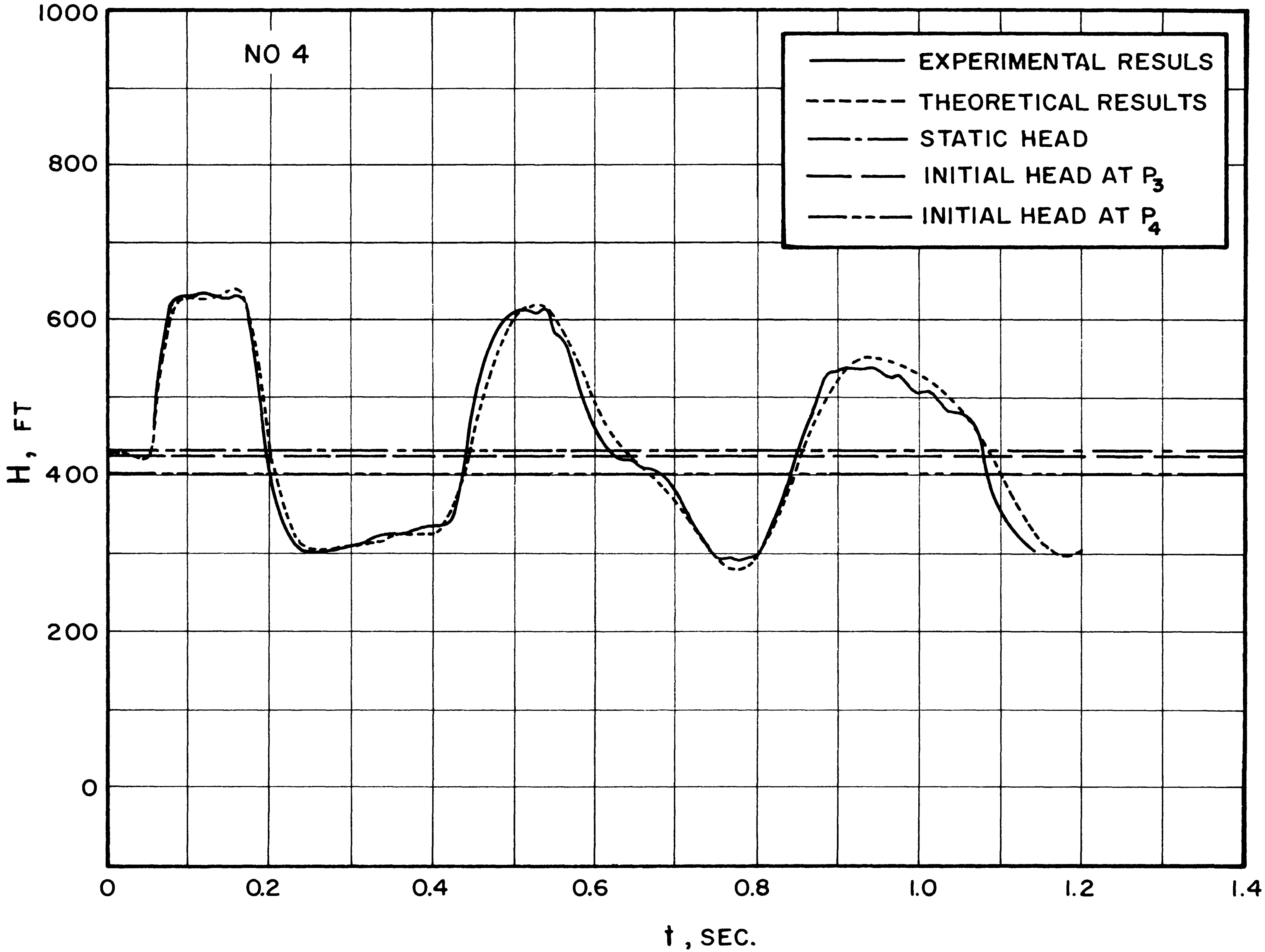


Figure 21. Pressure-Time Curve at P_3; Case III (See Figure 15c).

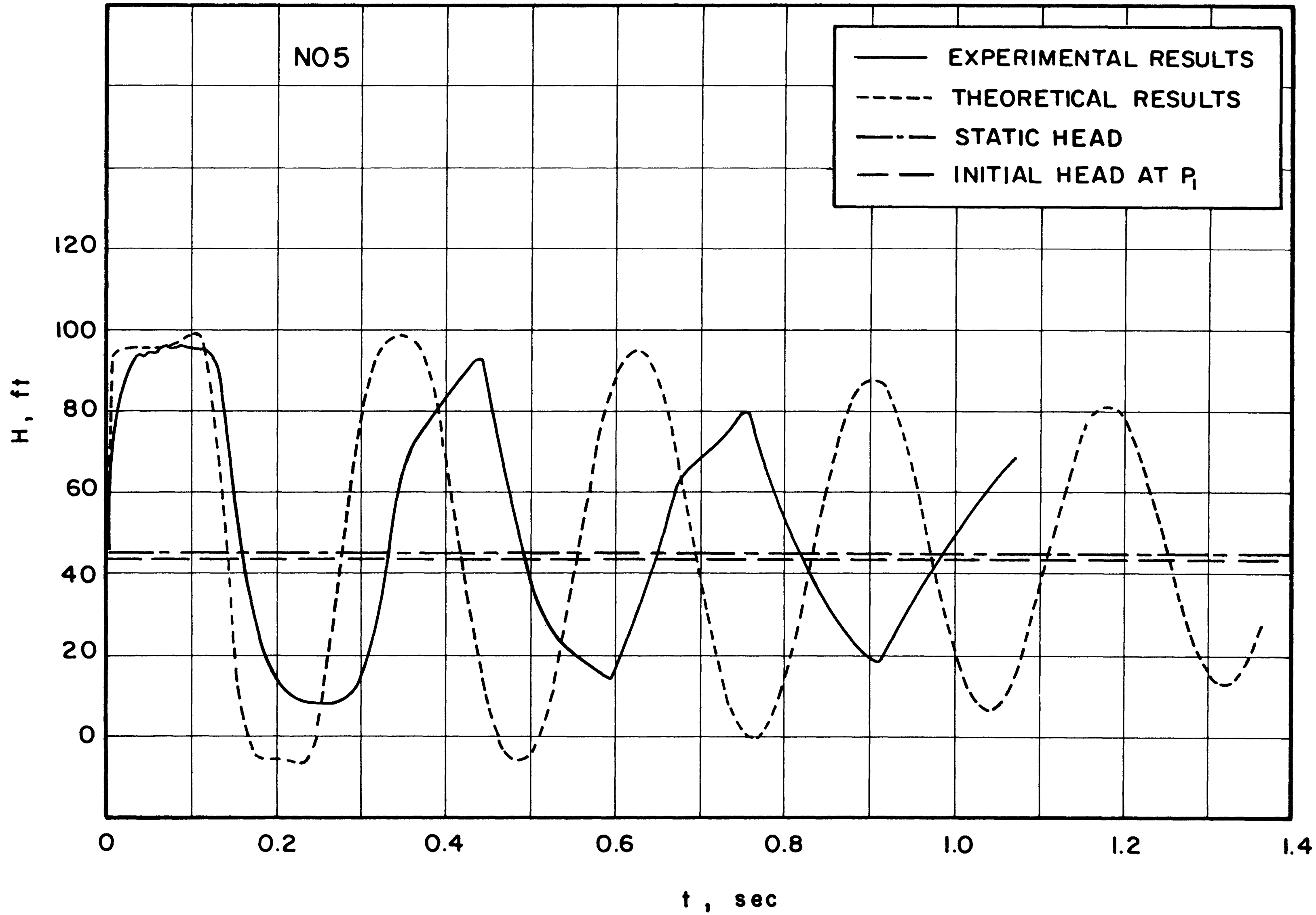


Figure 22. Pressure-Time Curve at P_1; Case I (See Figure 15a).

Figure 18 to Figure 22 for comparison. These are the theoretical solutions corresponding to the above-mentioned experiments.

A few words worth mentioning here are the effect of attaching the pressure pick-up to the main line. The significance of this effect and what type of pipe system has to be employed have been described in the previous two chapters. However, a major difficulty arises in the actual computer application. Since the length of pick-up is so short compared with the length of main line, formidable computer time will be required if the actual length of pick-up be used. This difficulty can be reduced considerably by using a much longer hypothetical dead-end branch so long as its length is sufficiently small relative to the main line, because the significant effect is caused not by the extra wall friction of the short branch, which is essentially negligible, but by its buffering effect and the emission of successive pressure waves therefrom, due to its existence. This effect should cause tiny ripples along the primary curve (this was observed in some plottings which are not shown here), but it did not appear in the theoretical curves shown in this chapter due to the scale used in plotting and the time interval used in printing.

The comparison of the computed and experimental results shows good agreement for the flows with higher velocities. The computer programs, which were compiled according to the analytical procedures described in Chapters II, III, and IV, not only could depict the shape of the curve very closely to the experimental one - even for a very complicated curve -, but also gave numerical values of pressure heads very close to those obtained from the experiments.

The most important value for pipe design is usually the value of the first pressure peak after the valve closure. All of the computed values showed an excellent agreement with the corresponding experimental ones. They also indicate remarkable agreement in rate and way of attenuation of pressure surges. It is not uncommon in a complex or compound pipe system that the different concentrations of reflected pressure waves from different points cause the magnitude of a succeeding peak or trough to exceed that of the previous one. The good agreements between computation and experiment in this respect will warrant the extensive application of this method to complicated pipe systems.

The situation is somewhat different in flows with extremely low velocities. Observing No. 5 in Table I and Figure 22, we find a lack of resemblance in their shapes between the computation and experiment. The speed of pressure wave in the experiment is slower than that obtained by computation. The discrepancy in the case of laminar flow could be conjectured as the violation of the assumption of uniform velocity distribution over any cross section made in Chapter II. For the exact solution of such flow we may have to consider the three-dimensional case, starting from the general Navier-Stokes equations. It may also require some reexamination or refinement in the experimental technique. These are out of the scope of this study and will be left for a future investigation.

VII. COMPARISON WITH EARLIER METHODS

As already mentioned in the introduction, the existing methods for estimating friction losses in water hammer are not only few but inadequate. The direct analytical solution of the basic partial differential equations is the only method capable of distributing the effect of friction losses along the pipe line in agreement with the actual situation. However, the effect of friction losses, which are assumed to vary with V^2, makes the basic differential equations non-linear even when the terms $V\frac{\partial V}{\partial X}$ and $V\frac{\partial H}{\partial X}$ are neglected, as indicated in the equation of dynamic equilibrium (6). The simultaneous solution of this equation with the equation of continuity (7), had been considered as impossible.[13,15,16,24] Thus some approximations, some of which appeared as graphical methods, some as operational mathematics and so forth, had been employed as substitutions because of no exact solution.

Among those existing methods, the graphical methods are probably most widely used. The first approximation of this class is an estimation of loss by a hypothetical obstruction located either at the upstream end[15] or at the valve end[16] of the pipe line. This obstruction has the same total friction loss as the entire pipe line. The gist of the method is to proceed along the same fundamental network of solid lines as in the non-frictional case of the graphical solution but adding the correction of head loss each time at the obstruction. The basic system of equations, from which the graphical method has been developed, remains the same as in the non-friction case, and the entire method is simply a combination of nonfriction case and abrupt pressure deduction at the obstruction. Although this is a simple, good approximation for estimating pressure at a point at certain fixed times, it gives no information about other points along the pipe line.

A better graphical approximation could be achieved by taking a number of hypothetical obstructions at various points along the pipe line lumping the friction losses of each section at these points. As in the first approximation, the same network must be applied, the correction of friction effect must be added at each point, and those lines would be interwoven and developed further as the time goes. In this case, it generally involves some trial-and-error method to locate two points at each side of an obstruction, except those defined by some boundary conditions. This method, as the first one, produces an abrupt drop of pressure at each obstruction which evidently is not be true case.

In the graphical methods, although closer and closer approximations can be obtained by increasing the number of obstructions, this rapidly complicates the solution. This is quite evident if one considers the fact that each obstruction has two points of different heads at each side and each point of each pair emits two lines of different slope, and moreover, the distance between these two points, as well as their locations on the graph, has to be determined by trial-and-error method. These lines develop further and further into a great tangle as time progresses. Successive trial-and-error often results in the accumulation of errors and the values thus obtained may sometimes become doubtful.

Some authors have developed methods of estimating the effect of losses by operational mathematics or other analytical solutions.[17,24,5] They differ in degree of accuracy and complexity, in method of approaching the problem, and in range of application, but they have one thing in common - the linearization of the friction term. Since the effect of friction is expressed in terms of V^2 in the basic differential equations, and since most cases in engineering practice are turbulent flows, the linearization of the friction term cannot exactly represent the actual situation. In other words, they are approximations and hence not wholly satisfactory.

Wood[24] introduced Heaviside's operational calculus in the field of water hammer and presented one example of a simple line with instantaneous gate closure at the lower end, allowing for friction. He assumed friction to be linear and for this purpose he replaced V^2 by $(K_f V_m)V$. Here, V_m is the initial velocity prevailing in the pipe and K_f is a constant for linear approximation. Besides the linear approximation, it gives only "surge pressure" and not the actual pressures and velocities along the pipe.

The pioneer work of Wood was improved by Rich[17], superseding Heaviside's operation by the Laplace-Mellin transformation. It permitted working directly with total pressures and velocities instead of surge pressures and velocities. The solutions have rather involved series in Bessel functions, but the method required replacement of nonlinear terms, as KV^2, by a linear approximation.

The author also tried some Laplace transformations before the present study, and by taking advantage of the digital computer, he used several different values of K_f for corresponding values of V, instead of using a single constant K_f as was done by earlier authors. This is tantamount to dividing the parabolic curve of friction versus velocity into several segments, replacing each segment of curve by a straight line. This naturally should give better agreement with the actual situation.

However, there are still some disadvantages in employing the operational mathematics, even with aid of the computer. The transformation and inverse transformation involve many difficult mathematical manipulations and often result in long series. It has certain limitations because of necessary approximations, and consequently is not quite flexible in application. It is sometimes very difficult or impossible to determine constants in the operational solutions for some given boundary conditions.

The direct analytical solution of the basic partial differential equations, which has been presented in the previous chapters, naturally has greater accuracy over other methods because of its directness. It can handle the V^2 term and consequently is able to distribute the effect of friction losses along the pipe line in accordance with nature, without lumping, approximation or indirect transformation.

Besides its accuracy, this method possesses some other advantages. Since the computation is carried out by the electronic digital computer, it is much faster than old methods. Unlike the case in the present study, much computer time can be saved in actual applications, because there will not be a small pressure pick-up in an actual pipe, and even if the pipe system has a dead-end branch, it will be of the same order of length. Many other details, which were retained in this study for greater accuracy, may be omitted in practical cases. For most purposes, the computer time will not be more than a few minutes to several minutes.

Great flexibility is one of the remarkable features of this method. With the same form of basic program, it can be developed to extensive problems of complicated pipe systems and gate operations. It may also serve in solving some problems formerly beyond scope.

As can be seen in the examples given in the appendices, the answers can be printed out in tabular form so that it becomes extremely easy for anyone to understand the phenomena and the results are ready to plot in any form desired. It supplies all historical and geographical data for both velocity and pressure. These have all been included automatically in the course of computation. For any fixed point along the pipe, we can trace the velocity and pressure following the time variation, or for any particular instant, we can observe how velocity and pressure are distributed along the pipe. Indeed, by this method, we can gather extensive valuable information from answer sheets simultaneously, whereas the earlier methods could give answers for only limited points and limited time intervals.

The program is easily controllable both in time and in location according to accuracy and necessity. After we make up a computer program for one type of pipe system, we can solve as many as we desire for the same type of problems by simply feeding in the data. Those advantages mentioned above are also good for non-friction cases. If we prepare many programs for typical water hammer problems, a great number of solutions can be readily obtained in a form similar to data from mathematical tables.

VIII. CONCLUSIONS

From the foregoing chapters of this study the following conclusions may be drawn:

1. The basic partial differential equations of water hammer including effect of friction, which contain nonlinear terms and have not been solved, can be solved directly by the method of characteristics, with the aid of digital computer.

2. Solutions by the above mentioned method agree with solutions made by earlier methods when the friction term is neglected. Inclusion of this term clearly and sensitively depicts the damping effect.

3. The variety of examples given in this study shows that the method is capable of handling many different gate operations and pipe connections easily and adequately.

4. The experimental studies made for the case of instantaneous valve closure check the validity of the theoretical solutions. The attenuation of pressure due to friction evaluated by this method, agrees with experiment unless the velocity is too low. Hence, the method is able to provide an accurate evaluation of friction effect in most engineering cases.

5. The experiments also verify that the theoretically obtained curves of pressure versus time for different pipe systems and branch connections can correctly and sensitively describe the actual pressure variations.

6. From the analytical work and experimental verification, it is believed that the method presented in this study affords an adequate and rapid means of analyzing water hammer phenomena allowing friction in any pipe system and branch connection for any desired boundary conditions.

7. It has been observed that this method has remarkable advantages of exactness, directness, quickness, flexibility, and wide applicability over earlier methods, whether the friction is included or not.

For a more thorough verification of the analytical work, further experimentation for slow valve closure and for non-uniform pipes are recommended. During the course of experimental investigations, it was observed that the minimum pressure inside the pipe dropped below the vapor pressure a few times. A better analytical study for this case, considering the energy dissipation due to both ordinary wall friction and resurge effect, are also suggested. The theoretical solutions and experimental results showed unsatisfactory agreements in case of extremely low velocity. A further study for this case, both in theoretical analysis and experimental investigation, are highly desired. It may require a three-dimensional analysis or re-examination of assumptions. It may more likely require further modification or improvement in experimental equipment and technique as the first step of the study. Research on water hammer in a very elastic pipe will also be an interesting subject.

It may be worthwhile to prepare at a later date a library of problems in water hammer, in which complete workable computer programs for the rapid solution of many representative problems based upon the principle and method presented in this study will be collected, so that a great number of solutions can be readily found by simply providing the type of problem and necessary data.

There will be certain cases in which the effect of hysteresis plays a significant role in the energy dissipation. Since there are little rigorous mathematical expressions and reliable data available for this effect, the theoretical solution of these problems may become very difficult. Nevertheless, experimental evaluations may be feasible by deducting the computed loss due to wall friction, minor losses, etc., from the total observed loss.

APPENDIX I

A MAD Language Program(3) for the solution of water hammer including effect of friction losses for the pipe system Case I and a part of its computed results. [See Figure 15a.] Subroutines included.*

[See also APPENDIX V.]

* F = 1 for R.L. 64 is used in order to prevent F(=f) → ∞. In this case $V \doteq 0$.

```
 CHINTU LAI                      Q042N 11             005    030
* EXECUTE, DUMP                                                    WH11    7
* COMPILE MAD, PUNCH OBJECT
               INTEGER I,J,K,M,N,TREC,TP
               DIMENSION V(40), H(40), V1(20), H1(20), V2(40), H2(40), VP(40
              2), HP(40)
               PRINT FORMAT TITLE
 HERE          READ FORMAT CARD, L,L2,L3,V0,H0,N,TC,D,D2,B,B2,NU,S1,S2,TP,E,
              2E2,EW
               PRINT FORMAT GIVEN, L,L2,L3,V0,H0,N,TC,D,D2,B,B2,NU,S1,S2,E,
              2E2,EW
               Q=1./EW+0.81*D/(B*E)
               Q2=1./EW+0.81*D2/(B2*E2)
               A=SQRT.(1./(1.93*Q))
               A2=SQRT.(1./(1.93*Q2))
               J=2
               K=1
               DELL=L/N
               HF=FR.(D,V0,NU,S1)*DELL*V0*V0/(D*64.332)
               HF3=HF*L3/DELL
               DELTI=L2/A2
               DELV=0.01
               DELH=0.01
               T=0
               HP(0)=H0
               PRINT FORMAT TEST, Q, A, Q2,A2, DELL, DELTI
               PRINT FORMAT TIME, T
               X=-DELL
               THROUGH ANN, FOR I=0, 1, I.G.N
               X=X+DELL
               V1(I)=V0
               H1(I)=H0-I*HF
 ANN           PRINT FORMAT RESULT,X,V1(I),H1(I)
               PRINT FORMAT BLANK
               THROUGH ARBOR, FOR I=N+J+K,1,I.G.N+2*J
               V1(I)=V0
               H1(I)=H1(N)-(I-N-J-K)*HF3
               PRINT FORMAT RESULT,X,V1(I),H1(I)
 ARBOR         X=X+L3
               HE=H1(N+2*J)
               PRINT FORMAT BLANK
               X=0
               THROUGH MICH, FOR I=N+K,1,I.G.N+J
               V1(I)=0.
               H1(I)=H1(N)
               PRINT FORMAT RESULT,X,V1(I),H1(I)
 MICH          X=L2
               HF12=HF/2.
               HF32=HF3/2.
               THROUGH USA, FOR I=0, 1, I.G.2*N
               V2(I)=V0
 USA           H2(I)= H0-I*HF12
               THROUGH CANADA, FOR I=2*(N+J+K),1,I.G.2*(N+2*J)
               V2(I)=V0
 CANADA        H2(I)=H2(2*N)-(I-2*(N+J+K))*HF32
               THROUGH ENGLAD, FOR I=2*(N+K),1,I.G.2*(N+J)
               V2(I)=0
 ENGLAD        H2(I)=H2(2*N)
               TH=DELTI/DELL
```

```
                TH2=DELTI/L2
                TH3=DELTI/L3
 PHILA          TREC=0
 NEW            T=T+DELTI
                WHENEVER T.G.3*TC.AND.T.G.20.*(L+L3)/A, TRANSFER TO HERE
                TREC=TREC+1
                WHENEVER TREC.NE.TP, TRANSFER TO YORK
                PRINT FORMAT TIME, T
 YORK           M=1
                DELT =DELTI
                TI=T
                THROUGH WASH, FOR I=0,1,I.G.N+2*J
                V(I)=V1(I)
 WASH           H(I)=H1(I)
 BOSTON         THROUGH LONDON, FOR I=1, 1, I.E.N
                VR=V(I)*(1.-TH*(V(I)+A))+V(I-1)*TH*(V(I)+A)
                VS=V(I)*(1.+TH*(V(I)-A))-V(I+1)*TH*(V(I)-A)
                HR=H(I)*(1.-TH*(V(I)+A))+H(I-1)*TH*(V(I)+A)
                HS=H(I)*(1.+TH*(V(I)-A))-H(I+1)*TH*(V(I)-A)
                VP(I)=0.5*(VR+VS)+(16.083/A)*(HR-HS)-0.5*FR.(D,V(I),NU,S1)*.A
               2BS.V(I)*V(I)*DELT/D
 LONDON         HP(I)=(A/64.332)*(VR-VS)+0.5*(HR+HS)
                VS=V(0)*(1.+TH*(V(0)-A))-V(1)*TH*(V(0)-A)
                HS=H0*(1.+TH*(V(0)-A))-H(1)*TH*(V(0)-A)
                VP(0)=VS+(32.166/A)*(H0-HS)-0.5*FR.(D,V(0),NU,S1)*V(0)*.ABS.V
               2(0)*DELT/D
                WHENEVER M.E.1, TRANSFER TO SPAIN
                THROUGH  BELG, FOR I=N+K+1, 1, I.E.N+J
                VR=V(I)*(1.-TH2*(V(I)+A2))+V(I-1)*TH2*(V(I)+A2)
                VS=V(I)*(1.+TH2*(V(I)-A2))-V(I+1)*TH2*(V(I)-A2)
                HR=H(I)*(1.-TH2*(V(I)+A2))+H(I-1)*TH2*(V(I)+A2)
                HS=H(I)*(1.+TH2*(V(I)-A2))-H(I+1)*TH2*(V(I)-A2)
                VP(I)=0.5*(VR+VS)+(16.083/A2)*(HR-HS)-0.5*FR.(D2,V(I),NU,S2)*
               2V(I)*.ABS.V(I)*DELT/D2
 BELG           HP(I)=(A2/64.332)*(VR-VS)+0.5*(HR+HS)
                THROUGH CZECHO, FOR I=N+J+K+1,1,I.E.N+2*J
                VR=V(I)*(1.-TH3*(V(I)+A ))+V(I-1)*TH3*(V(I)+A)
                VS=V(I)*(1.+TH3*(V(I)-A ))-V(I+1)*TH3*(V(I)-A)
                HR=H(I)*(1.-TH3*(V(I)+A ))+H(I-1)*TH3*(V(I)+A)
                HS=H(I)*(1.+TH3*(V(I)-A ))-H(I+1)*TH3*(V(I)-A)
                VP(I)=0.5*(VR+VS)+(16.083/A)*(HR-HS)-0.5*FR.(D,V(I),NU,S1)*.A
               2BS.V(I)*V(I)*DELT/D
 CZECHO         HP(I)=(A /64.332)*(VR-VS)+0.5*(HR+HS)
 SPAIN          VS=V(N+K+1)*TH2*A2
                HS=H(N+K)*(1.-TH2*A2)+H(N+K+1)*TH2*A2
                HP(N+K)=HS-(A2/32.166)*VS
                VP(N+K)=0
                WHENEVER TI.G.TC, TRANSFER TO ITALY
                EXECUTE BC.(TC,TI,A,V0,HE, V(N+2*J), H(N+2*J),DELV,DELH)
                VP(N+2*J)= V(N+2*J)-DELV
                HP(N+2*J)= H(N+2*J)+DELH
                TRANSFER TO OHIO
 ITALY          VP(N+2*J)=0
 OHIO           VR=V(N+2*J)*(1.-TH3*(V(N+2*J)+A))+V(N+2*J-1)*TH3*(V(N+2*J)+A)
                HR=H(N+2*J)*(1.-TH3*(V(N+2*J)+A))+H(N+2*J-1)*TH3*(V(N+2*J)+A)
 VENUS          HP(N+2*J)=HR-(A/32.166)*(VP(N+2*J)-VR)-A*FR.(D,V(N+2*J),NU,S1
               2)*V(N+2*J)*.ABS.V(N+2*J)*DELT/(64.332*D)
                WHENEVER TI.G.TC, TRANSFER TO STAR
```

```
             EXECUTE BC.(TC,TI,A,V0,HE,VP(N+2*J),HP(N+2*J),DELV,DELH)
             VP(N+2*J)=VP(N+2*J)-DELV
             HP(N+2*J)=HP(N+2*J)+DELH
             WHENEVER (.ABS.DELV.GE.0.001) .AND. (.ABS.DELH.GE.0.001), TRA
            2NSFER TO VENUS
STAR         VP(N+J+K)=V(N+J+K)+VP(N+J+K+1)-V(N+J+K+1)
             VR1=V(N)*(1.-TH*(V(N)+A))+V(N-1)*TH*(V(N)+A)
             HR1=H(N)*(1.-TH*(V(N)+A))+H(N-1)*TH*(V(N)+A)
             VR2=V(N+J)*(1.-TH2*(V(N+J)+A2))+V(N+J-1)*TH2*(V(N+J)+A2)
             HR2=H(N+J)*(1.-TH2*(V(N+J)+A2))+H(N+J-1)*TH2*(V(N+J)+A2)
             VS3=V(N+J+K)*(1.+TH3*(V(N+J+K)-A))-V(N+J+K+1)*TH3*(V(N+J+K)-A
            2)
             HS3=H(N+J+K)*(1.+TH3*(V(N+J+K)-A))-H(N+J+K+1)*TH3*(V(N+J+K)-A
            2)
GREECE       HP(N+J+K)=HS3+(A/32.166)*(VP(N+J+K)-VS3)+A*FR.(D,V(N+J+K),NU,
            2S1)*V(N+J+K)*.ABS.V(N+J+K)*DELT/(64.332*D)
             HP(N)=HP(N+J+K)
             HP(N+J)=HP(N+J+K)
             VP(N)=VR1-(32.166/A)*(HP(N)-HR1)-0.5*FR.(D,V(N),NU,S1)*V(N)*.
            2ABS.V(N)*DELT/D
             VP(N+J)=VR2-(32.166/A2)*(HP(N+J)-HR2)-0.5*FR.(D2,V(N+J),NU,S2
            2)*V(N+J)*.ABS.V(N+J)*DELT/D2
             VEL=VP(N)+VP(N+J)*D2*D2/(D*D)
             DIF=VEL-VP(N+J+K)
             WHENEVER .ABS.DIF .L.0.001, TRANSFER TO POLAND
             VP(N+J+K)=VP(N+J+K)+DIF/3.
             TRANSFER TO GREECE
POLAND       WHENEVER M.NE.1, TRANSFER TO BERLIN
             THROUGH PARIS, FOR I=0,1,I.G.N+2*J
             V1(I)=VP(I)
PARIS        H1(I)=HP(I)
             M=2
             N=2*N
             J=2*J
             K=2*K
             DELT=DELT/2.
             TI=T-DELT
             THROUGH ROME, FOR I=0,1,I.G.N+2*J
             V(I)=V2(I)
ROME         H(I)=H2(I)
             TRANSFER TO BOSTON
BERLIN       WHENEVER M.NE.2, TRANSFER TO UNIV
             THROUGH TOKYO, FOR I=0,1,I.G.N+2*J
             V(I)=VP(I)
TOKYO        H(I)=HP(I)
             M=4
             TI=T
             TRANSFER TO BOSTON
UNIV         THROUGH SUN, FOR I=0,1,I.G.N+2*J
             V2(I)=VP(I)
SUN          H2(I)=HP(I)
             N=N/2
             J=J/2
             K=K/2
             WHENEVER TREC.NE.TP, TRANSFER TO NEW
             X=-DELL
             THROUGH MOON, FOR I=0,1,I.G.N
             X=X+DELL
```

```
        VB=2*V2(2*I)-V1(I)
        HB=2*H2(2*I)-H1(I)
MOON    PRINT FORMAT RESULT, X,VB,HB
        PRINT FORMAT BLANK
        THROUGH TURKEY, FOR I=N+J+K,1,I.G.N+2*J
        VB=2*V2(2*I)-V1(I)
        HB=2*H2(2*I)-H1(I)
        PRINT FORMAT RESULT, X,VB,HB
TURKEY  X=X+L3
        X=0
        PRINT FORMAT BLANK
        THROUGH CONGO, FOR I=N+K,1,I.G.N+J
        VB=2*V2(2*I)-V1(I)
        HB=2*H2(2*I)-H1(I)
        PRINT FORMAT RESULT, X,VB,HB
CONGO   X=L2
        TRANSFER TO PHILA
        VECTOR VALUES TITLE=$64H1WATERHAMMER PRODUCED BY GATE CLOSURE
       2 INCLUDING FRICTION EFFECT  /1H0,S10,50HPIPE LINE WITH A SHOR
       3T DEAD-END BRANCH CONNECTION *$
        VECTOR VALUES CARD = $5F10.3,I4/5F8.5,3F10.8,I3/3E12.5*$
        VECTOR VALUES GIVEN = $5H4L = F10.3,S8,5HL2 = F10.3,S8,5HL3 =
       2 F10.3,S8,5HV0 = F10.3,S8,5HH0 = F10.3/5H0N = I4,S8,5HTC = F8
       3.5,S8,4HD = F8.5,S8,5HD2 = F8.5/5H0B = F8.5,S8,5HB2 = F8.5,S8
       4,5HNU = F10.8,S8,5HS1 = F10.8,S8,5HS2 = F10.8/5H0E = E12.5,S4
       5,5HE2 = E12.5,S4,5HEW = E12.5*$
        VECTOR VALUES TEST = $1H0,4HQ = E12.5,S5,4HA = E12.5,S5,5HQ2
       2= E12.5,S5,5HA2 = E12.5/8H0DELL = E12.5,S5,8HDELTI = E12.5*$
        VECTOR VALUES TIME = $8H4TIME = F10.5/1H0,S12,1HX,S15,8HVELOC
       2ITY,S12,8HPRESSURE*$
        VECTOR VALUES RESULT = $1H ,S7,F10.3,S8,E12.4,S8,E12.4*$
        VECTOR VALUES BLANK = $2H0 *$
        END OF PROGRAM
* COMPILE MAD, PUNCH OBJECT                                    WHS8    3
        EXTERNAL FUNCTION (D,V,NU,K)
        ENTRY TO FR.
        R=D*.ABS.V/NU
        WHENEVER D .G. 0.027, TRANSFER TO LARGE
        WHENEVER R.L.64.
          F=1.0
        OR WHENEVER R.L.200.
          F=64./R
        OR WHENEVER R.L.1050.
          F=17.77*R.P.(-0.7581)
        OR WHENEVER R.L.3250.
          F=6.948*R.P.(-0.6232)
        OR WHENEVER R.L.3600.
          F=498.31*R.P.(-1.1516)
        OR WHENEVER R.L.4200.
          F=8.0349*R.P.(-0.6476)
        OR WHENEVER R.L.5000.
          F=0.51216*R.P.(-0.3176)
        OR WHENEVER R.L.6000.
          F=0.13672*R.P.(-0.1625)
        OTHERWISE
          F=0.076236*R.P.(-0.09538)
        END OF CONDITIONAL
        TRANSFER TO BACK
```

```
LARGE    WHENEVER R.L.64.
           F=1.0
         OR WHENEVER R.L.200.
           F=64./R
         OR WHENEVER R.L.1050.
           F=14.49*R.P.(-0.7197)
         OR WHENEVER R.L.3650.
           F=7.525*R.P.(-0.6258)
         OR WHENEVER R.L.4000.
           F=624.98*R.P.(-1.1643)
         OR WHENEVER R.L.4700.
           F=6.3212*R.P.(-0.6104)
         OR WHENEVER R.L.5700.
           F=0.6013*R.P.(-0.3322)
         OR WHENEVER R.L.7000.
           F=0.2424*R.P.(-0.2271)
         OTHERWISE
           F=0.07819*R.P.(-0.09933)
         END OF CONDITIONAL
BACK     FUNCTION RETURN F
         END OF FUNCTION
* COMPILE MAD, PUNCH OBJECT                                    WHS7
         EXTERNAL FUNCTION (TC,TI,A,V0,H0,V,H,DELV,DELH)
         ENTRY TO BC.
         TAU=0.5*(1.5-TI/TC)*(1.5-TI/TC)-0.125
         C=A*V0/(32.2*H0)
         W=V/V0
         Y=TAU*TAU
         X=2.*W+C*Y
         DLV=0.5*(X-SQRT.(X*X-4.*(W*W-Y*H/H0)))
         DLH=C*DLV
         DELV=V0*DLV
         DELH=H0*DLH
         FUNCTION RETURN
         END OF FUNCTION
```

WATERHAMMER PRODUCED BY GATE CLOSURE INCLUDING FRICTION EFFECT

PIPE LINE WITH A SHORT DEAD-END BRANCH CONNECTION

L = 294.300 L2 = 2.000 L3 = 2.000 V0 = 3.080 H0 = 486.000

N = 10 TC = 0.00000 D = 0.02595 D2 = 0.02595

B = 0.00267 B2 = 0.00267 NU = 0.00000799 S1 = 0.00000000 S2 = 0.00000000

E = 0.24480E 10 E2 = 0.24480E 10 EW = 0.46600E 08

Q = 0.24675E-07 A = 0.45824E 04 Q2 = 0.24675E-07 A2 = 0.45824E 04

DELL = 0.29430E 02 DELTI = 0.43645E-03

TIME = 0.00000

X	VELOCITY	PRESSURE
-0.000	0.3080E 01	0.4860E 03
29.430	0.3080E 01	0.4807E 03
58.860	0.3080E 01	0.4754E 03
88.290	0.3080E 01	0.4701E 03
117.720	0.3080E 01	0.4648E 03
147.150	0.3080E 01	0.4595E 03
176.580	0.3080E 01	0.4542E 03
206.010	0.3080E 01	0.4489E 03
235.440	0.3080E 01	0.4436E 03
264.870	0.3080E 01	0.4383E 03
294.300	0.3080E 01	0.4330E 03

294.300	0.3080E 01	0.4330E 03
296.300	0.3080E 01	0.4327E 03

0.000	0.00000000	0.4330E 03
2.000	0.00000000	0.4330E 03

TIME = 0.00655

X	VELOCITY	PRESSURE
-0.000	0.3080E 01	0.4860E 03
29.430	0.3080E 01	0.4807E 03
58.860	0.3080E 01	0.4754E 03
88.290	0.3080E 01	0.4701E 03
117.720	0.3080E 01	0.4648E 03
147.150	0.3083E 01	0.4591E 03
176.580	0.3101E 01	0.4512E 03
206.010	0.3178E 01	0.4349E 03
235.440	0.3130E 01	0.4364E 03
264.870	0.1688E 01	0.6372E 03
294.300	0.4050E-02	0.8742E 03

294.300	-0.1533E 01	0.8742E 03
296.300	0.00000000	0.6567E 03

0.000	0.00000000	0.1092E 04
2.000	-0.1537E 01	0.8742E 03

TIME = 0.01309

X	VELOCITY	PRESSURE
-0.000	0.3080E 01	0.4860E 03
29.430	0.3080E 01	0.4807E 03
58.860	0.3081E 01	0.4753E 03
88.290	0.3086E 01	0.4693E 03
117.720	0.3104E 01	0.4613E 03
147.150	0.3159E 01	0.4482E 03
176.580	0.3210E 01	0.4354E 03
206.010	0.2890E 01	0.4758E 03
235.440	0.1620E 01	0.6525E 03
264.870	0.2141E 00	0.8498E 03
294.300	0.1586E-02	0.8774E 03

294.300	-0.1512E 01	0.8774E 03
296.300	0.00000000	0.1095E 04

0.000	0.00000000	0.6592E 03
2.000	-0.1513E 01	0.8774E 03

APPENDIX II

A MAD Language Program[3] for the solution of water hammer including effect of friction losses for the pipe system Case II and a part of its computed results. [See Figure 15b.] Subroutines omitted.

[See also APPENDIX V.]

```
      CHINTU LAI                    Q042N 12               005    030
     * EXECUTE, DUMP
     * COMPILE MAD, PUNCH OBJECT                                         WH12    5
                  INTEGER G,I,  J,K,M,N,R,TREC,TP
                  DIMENSION V(60), H(60), V1(30), H1(30), V2(60), H2(60), VP(60
                 2), HP(60), L(4), D(4), B(4), S(4), E(4), Q(4), A(4), DELL(4),
                 3VI(2), HF(4), TH(4)
                  PRINT FORMAT TITLE
      START       READ FORMAT CARD,   V0, H0, TC, NU, N, L(1)...L(4), D(1)...D(
                 24), B(1)...B(4), S(1)...S(4), E(1)...E(4), TP, EW
                  PRINT FORMAT GIVEN, V0, H0, TC, NU, N, L(1)...L(4), D(1)...D(
                 24), B(1)...B(4), S(1)...S(4), E(1)...E(4), EW
                  Z=1./EW
                  THROUGH WORLD, FOR G=1, 1, G.G.4
                  Q(G)=Z+0.81*D(G)/(B(G)*E(G))
      WORLD       A(G)=SQRT.(1./(1.93*Q(G)))
                  J=2
                  K=1
                  THROUGH TOUR, FOR G=1, 1, G.G.2
      TOUR        DELL(G)=L(G)/N
                  DELL(3)=L(3)
                  DELL(4)=L(4)
                  VI(1)=V0
                  C1=D(1)*D(1)/(D(2)*D(2))
                  C2=D(3)*D(3)/(D(2)*D(2))
                  VI(2)=V0*C1
                  THROUGH FROM, FOR G=1, 1, G.G.2
      FROM        HF(G)=FR.(D(G),VI(G),NU,S(G))*DELL(G)*VI(G)*VI(G)/(D(G)*64.33
                 22)
                  HF(4)=HF(2)*L(4)/DELL(2)
                  DELTI=L(3)/A(3)
                  DELV=0.01
                  DELH=0.01
                  T=0
                  H1(0)=H0
                  H2(0)=H0
                  HP(0)=H0
                  PRINT FORMAT TEST, Q(1)...Q(4), A(1)...A(4), DELL(1), DELL(2)
                 2, DELTI
                  PRINT FORMAT TIME, T
                  R=0
                  X=0
                  THROUGH ARBOR, FOR G=1, 1, G.G.2
                  X=X-DELL(G)
                  THROUGH ANN, FOR I=R, 1, I.G.N+R
                  X=X+DELL(G)
                  V1(I)=VI(G)
                  H1(I)=H1(R)-(I-R)*HF(G)
      ANN         PRINT FORMAT RESULT, X, V1(I), H1(I)
                  PRINT FORMAT BLANK
                  R=R+N+J
      ARBOR       H1(R)=H1(R-2)
                  THROUGH MICH, FOR I=R+K, 1, I.G.R+J
                  V1(I)=VI(2)
                  H1(I)=H1(R)-(I-R-K)*HF(4)
                  PRINT FORMAT RESULT, X, V1(I), H1(I)
      MICH        X=X+L(4)
                  HE=H1(R+J)
                  PRINT FORMAT BLANK
```

```
          X=0
          THROUGH USA, FOR I=R-K, 1, I.G.R
          V1(I)=0.
          H1(I)=H1(R)
          PRINT FORMAT RESULT, X, V1(I), H1(I)
USA       X=L(3)
          THROUGH OHIO, FOR G=1, 1, G.G.4
OHIO      HF(G)=HF(G)/2.
          R=0
          THROUGH MD, FOR G=1, 1, G.G.2
          THROUGH PA, FOR I=R, 1, I.G.R+2*N
          V2(I)=VI(G)
PA        H2(I)=H2(R)-(I-R)*HF(G)
          R=R+2*(N+J)
MD        H2(R)=H2(R-4)
          THROUGH WASH, FOR I=R+2*K, 1, I.G.R+2*J
          V2(I)=VI(2)
WASH      H2(I)=H2(R)-(I-R-2*K)*HF(4)
          THROUGH NEW, FOR I=R-2*K,1,I.G.R
          V2(I)=0.
NEW       H2(I)=H2(R)
          THROUGH YORK, FOR G=1, 1, G.G.4
YORK      TH(G)=DELTI/DELL(G)
BALTMR    TREC=0
BOSTON    T=T+DELTI
          WHENEVER T.G.3*TC .AND. T.G.16.*(L(1)/A(1)+(L(2)+L(4))/A(2)),
         2TRANSFER TO START
          TREC=TREC+1
          WHENEVER TREC.NE.TP, TRANSFER TO PHILA
          PRINT FORMAT TIME, T
PHILA     M=1
          DELT=DELTI
          TI=T
          THROUGH CANADA, FOR I=0, 1, I.G.2*(N+K+J)
          V(I)=V1(I)
CANADA    H(I)=H1(I)
SCOT      R=0
          THROUGH ENGLAD, FOR G=1, 1, G.G.2
          THROUGH LONDON, FOR I=R+1, 1, I.E.N+R
          VR=V(I)*(1.-TH(G)*(V(I)+A(G)))+V(I-1)*TH(G)*(V(I)+A(G))
          VS=V(I)*(1.+TH(G)*(V(I)-A(G)))-V(I+1)*TH(G)*(V(I)-A(G))
          HR=H(I)*(1.-TH(G)*(V(I)+A(G)))+H(I-1)*TH(G)*(V(I)+A(G))
          HS=H(I)*(1.+TH(G)*(V(I)-A(G)))-H(I+1)*TH(G)*(V(I)-A(G))
          VP(I)=0.5*(VR+VS)+(16.083/A(G))*(HR-HS)-0.5*FR.(D(G),V(I),NU,
         2S(G))*V(I)*.ABS.V(I)*DELT/D(G)
LONDON    HP(I)=(A(G)/64.332)*(VR-VS)+0.5*(HR+HS)
ENGLAD    R=R+N+J
          VS=V(0)*(1.+TH(1)*(V(0)-A(1)))-V(1)*TH(1)*(V(0)-A(1))
          HS=H0*(1.+TH(1)*(V(0)-A(1)))-H(1)*TH(1)*(V(0)-A(1))
          VP(0)=VS+(32.166/A(1))*(H0-HS)-0.5*FR.(D(1),V(0),NU,S(1))*V(0
         2)*.ABS.V(0)*DELT/D(1)
          WHENEVER M.E.1, TRANSFER TO SWEDEN
          THROUGH NORWAY, FOR G=3, 1, G.G.4
          THROUGH OSLO, FOR I=R-K+1, 1, I.E.R
          VR=V(I)*(1.-TH(G)*(V(I)+A(G)))+V(I-1)*TH(G)*(V(I)+A(G))
          VS=V(I)*(1.+TH(G)*(V(I)-A(G)))-V(I+1)*TH(G)*(V(I)-A(G))
          HR=H(I)*(1.-TH(G)*(V(I)+A(G)))+H(I-1)*TH(G)*(V(I)+A(G))
          HS=H(I)*(1.+TH(G)*(V(I)-A(G)))-H(I+1)*TH(G)*(V(I)-A(G))
```

```
                VP(I)=0.5*(VR+VS)+(16.083/A(G))*(HR-HS)-0.5*FR.(D(G),V(I),NU,
               2S(G))*V(I)*.ABS.V(I)*DELT/D(G)
       OSLO     HP(I)=(A(G)/64.332)*(VR-VS)+0.5*(HR+HS)
       NORWAY   R=R+J
       SWEDEN   R=2*N+3*K
                VS=V(R+1)*TH(3)*A(3)
                HS=H(R)*(1.-TH(3)*A(3))+H(R+1)*TH(3)*A(3)
                HP(R)=HS-(A(3)/32.166)*VS
                VP(R)=0
                R=2*(N+K+J)
                WHENEVER TI.G.TC, TRANSFER TO FINLAD
                EXECUTE BC.(TC,TI,A(4),VI(2),HE,V(R), H(R), DELV,DELH)
                VP(R)=V(R)-DELV
                HP(R)=H(R)+DELH
                TRANSFER TO MOSKVA
       FINLAD   VP(R)=0
       MOSKVA   VR=V(R)*(1.-TH(4)*(V(R)+A(4)))+V(R-1)*TH(4)*(V(R)+A(4))
                HR=H(R)*(1.-TH(4)*(V(R)+A(4)))+H(R-1)*TH(4)*(V(R)+A(4))
       RUSSIA   HP(R)=HR-(A(4)/32.166)*(VP(R)-VR)-A(4)*FR.(D(4),V(R),NU,S(4))
               2*V(R)*.ABS.V(R)*DELT/(64.332*D(4))
                WHENEVER TI.G.TC, TRANSFER TO POLAND
                EXECUTE BC.(TC,TI,A(4),VI(2),HE,VP(R),HP(R),DELV,DELH)
                VP(R)=VP(R)-DELV
                HP(R)=HP(R)+DELH
                WHENEVER (.ABS.DELV.GE.0.001).AND.(.ABS.DELH.GE.0.001), TRANS
               2FER TO RUSSIA
       POLAND   R=2*(N+K)
                VP(R+J+K)=V(R+J+K)+VP(R+J+K+1)-V(R+J+K+1)
                VR2=V(R)*(1.-TH(2)*(V(R)+A(2)))+V(R-1)*TH(2)*(V(R)+A(2))
                HR2=H(R)*(1.-TH(2)*(V(R)+A(2)))+H(R-1)*TH(2)*(V(R)+A(2))
                VR3=V(R+J)*(1.-TH(3)*(V(R+J)+A(3)))+V(R+J-1)*TH(3)*(V(R+J)+A(
               23))
                HR3=H(R+J)*(1.-TH(3)*(V(R+J)+A(3)))+H(R+J-1)*TH(3)*(V(R+J)+A(
               23))
                VS4=V(R+J+K)*(1.+TH(4)*(V(R+J+K)-A(4)))-V(R+J+K+1)*TH(4)*(V(R
               2+J+K)-A(4))
                HS4=H(R+J+K)*(1.+TH(4)*(V(R+J+K)-A(4)))-H(R+J+K+1)*TH(4)*(V(R
               2+J+K)-A(4))
       BERLIN   HP(R+J+K)=HS4+(A(4)/32.166)*(VP(R+J+K)-VS4)+A(4)*FR.(D(4),V(R
               2+J+K),NU,S(4))*V(R+J+K)*.ABS.V(R+J+K)*DELT/(64.332*D(4))
                HP(R)=HP(R+J+K)
                HP(R+J)=HP(R+J+K)
                VP(R)=VR2-(32.166/A(2))*(HP(R)-HR2)-0.5*FR.(D(2),V(R),NU,S(2)
               2)*V(R)*.ABS.V(R)*DELT/D(2)
                VP(R+J)=VR3-(32.166/A(3))*(HP(R+J)-HR3)-0.5*FR.(D(3),V(R+J),N
               2U,S(3))*V(R+J)*.ABS.V(R+J)*DELT/D(3)
                VEL=VP(R)+VP(R+J)*C2
                DIF=VEL-VP(R+J+K)
                WHENEVER .ABS.DIF.L.0.001,  TRANSFER TO DEUTCH
                VP(R+J+K)=VP(R+J+K)+DIF/3.
                TRANSFER TO BERLIN
       DEUTCH   VP(N+J)=V(N+J)+VP(N+J+1)-V(N+J+1)
                VR1=V(N)*(1.-TH(1)*(V(N)+A(1)))+V(N-1)*TH(1)*(V(N)+A(1))
                HR1=H(N)*(1.-TH(1)*(V(N)+A(1)))+H(N-1)*TH(1)*(V(N)+A(1))
                VS2=V(N+J)*(1.+TH(2)*(V(N+J)-A(2)))-V(N+J+1)*TH(2)*(V(N+J)-A(
               22))
                HS2=H(N+J)*(1.+TH(2)*(V(N+J)-A(2)))-H(N+J+1)*TH(2)*(V(N+J)-A(
               22))
```

```
NETH       HP(N+J)=HS2+(A(2)/32.166)*(VP(N+J)-VS2)+A(2)*FR.(D(2),V(N+J),
          2NU,S(2))*V(N+J)*.ABS.V(N+J)*DELT/(64.332*D(2))
           HP(N)=HP(N+J)
           VP(N)=VR1-(32.166/A(1))*(HP(N)-HR1)-0.5*FR.(D(1),V(N),NU,S(1)
          2)*V(N)*.ABS.V(N)*DELT/D(1)
           VEL=VP(N)*C1
           DIF=VEL-VP(N+J)
           WHENEVER .ABS.DIF.L.0.001, TRANSFER TO BELG
           VP(N+J)=VP(N+J)+DIF/2.
           TRANSFER TO NETH
BELG       R=R+2*J
           WHENEVER M.NE.1, TRANSFER TO SWISS
           THROUGH PARIS, FOR I=0, 1, I.G.R
           V1(I)=VP(I)
PARIS      H1(I)=HP(I)
           M=2
           N=2*N
           J=2*J
           K=2*K
           DELT=DELT/2.
           TI=T-DELT
           THROUGH FRANCE, FOR I=0, 1, I.G.2*R
           V(I)=V2(I)
FRANCE     H(I)=H2(I)
           TRANSFER TO SCOT
SWISS      WHENEVER M.NE.2, TRANSFER TO ITALY
           THROUGH ROME,  FOR I=0,1,I.G.R
           V(I)=VP(I)
ROME       H(I)=HP(I)
           M=4
           TI=T
           TRANSFER TO SCOT
ITALY      THROUGH SPAIN, FOR I=0, 1, I.G.R
           V2(I)=VP(I)
SPAIN      H2(I)=HP(I)
           N=N/2
           J=J/2
           K=K/2
           WHENEVER TREC.NE.TP, TRANSFER TO BOSTON
           R=0
           X=0
           THROUGH AUST, FOR G=1, 1, G.G.2
           X=X-DELL(G)
           THROUGH WIEN, FOR I=R, 1, I.G.N+R
           X=X+DELL(G)
           VB=2*V2(2*I)-V1(I)
           HB=2*H2(2*I)-H1(I)
WIEN       PRINT FORMAT RESULT, X,VB,HB
           PRINT FORMAT BLANK
AUST       R=R+N+J
           THROUGH GREECE, FOR I=R+K, 1, I.G.R+J
           VB=2*V2(2*I)-V1(I)
           HB=2*H2(2*I)-H1(I)
           PRINT FORMAT RESULT, X,VB,HB
GREECE     X=X+L(4)
           X=0
           PRINT FORMAT BLANK
           THROUGH TURKEY, FOR I=R-K, 1, I.G.R
```

```
                VB=2*V2(2*I)-V1(I)
                HB=2*H2(2*I)-H1(I)
                PRINT FORMAT RESULT, X,VB,HB
       TURKEY   X=X+L(3)
                TRANSFER TO BALTMR
                VECTOR VALUES TITLE = $75H1WATER HAMMER SOLUTIONS FOR COMPOUN
               2D SYSTEMS INCLUDING FRICTION EFFECT (A) *$
                VECTOR VALUES CARD = $2F10.3,2F10.8,I4/4F10.3,4F8.6/4F8.6,4F1
               20.8/4E12.4,I3/E12.4*$
                VECTOR VALUES GIVEN = $6H4V0 = F10.3,S8,5HH0 = F10.3,S8,5HTC
               2= F10.8,S8,5HNU = F10.8,S8,4HN = I4/14H0L(1,2,3,4) = 4(S5,F10
               3.3)/14H0D(1,2,3,4) = 4(S7,F8.6)/14H0B(1,2,3,4) = 4(S7,F8.6)/1
               44H0S(1,2,3,4) = 4(S5,F10.8)/14H0E(1,2,3,4) = 4(S3,E12.4)/6H0E
               5W = E12.4*$
                VECTOR VALUES TEST = $14H0Q(1,2,3,4) = 4(S5,E12.5)/14H0A(1,2,
               23,4) = 4(S5,E12.5)/11H0DELL(1) = E12.5,S5,10HDELL(2) = E12.5,
               3S5,8HDELTI = E12.5*$
                VECTOR VALUES TIME = $8H4TIME = F10.5/1H0,S12,1HX,S15,8HVELOC
               2ITY,S12,8HPRESSURE*$
                VECTOR VALUES RESULT = $1H ,S7,F10.3,S8,E12.4,S8,E12.4*$
                VECTOR VALUES BLANK = $2H0 *$
                END OF PROGRAM
```

WATER HAMMER SOLUTIONS FOR COMPOUND SYSTEMS INCLUDING FRICTION EFFECT (A)

VO = 1.112 HO = 442.000 TC = 0.00000000 NU = 0.00000726 N = 10

L(1,2,3,4) = 299.050 294.300 4.905 4.905

D(1,2,3,4) = 0.036330 0.025920 0.025920 0.025920

B(1,2,3,4) = 0.002667 0.002667 0.002667 0.002667

S(1,2,3,4) = 0.00000000 0.00000000 0.00000000 0.00000000

E(1,2,3,4) = 0.2448E 10 0.2448E 10 0.2448E 10 0.2448E 10

EW = 0.4710E 08

Q(1,2,3,4) = 0.25739E-07 0.24447E-07 0.24447E-07 0.24447E-07

A(1,2,3,4) = 0.44867E 04 0.46037E 04 0.46037E 04 0.46037E 04

DELL(1) = 0.29905E 02 DELL(2) = 0.29430E 02 DELTI = 0.10654E-02

TIME = 0.00000

X	VELOCITY	PRESSURE
-0.000	0.1112E 01	0.4420E 03
29.905	0.1112E 01	0.4415E 03
59.810	0.1112E 01	0.4409E 03
89.715	0.1112E 01	0.4404E 03
119.620	0.1112E 01	0.4398E 03
149.525	0.1112E 01	0.4393E 03
179.430	0.1112E 01	0.4387E 03
209.335	0.1112E 01	0.4382E 03
239.240	0.1112E 01	0.4377E 03
269.145	0.1112E 01	0.4371E 03
299.050	0.1112E 01	0.4366E 03
299.050	0.2185E 01	0.4366E 03
328.480	0.2185E 01	0.4338E 03
357.910	0.2185E 01	0.4311E 03
387.340	0.2185E 01	0.4284E 03
416.770	0.2185E 01	0.4257E 03
446.200	0.2185E 01	0.4229E 03
475.630	0.2185E 01	0.4202E 03
505.060	0.2185E 01	0.4175E 03
534.490	0.2185E 01	0.4147E 03
563.920	0.2185E 01	0.4120E 03
593.350	0.2185E 01	0.4093E 03
593.350	0.2185E 01	0.4093E 03
598.255	0.2185E 01	0.4088E 03
0.000	0.00000000	0.4093E 03
4.905	0.00000000	0.4093E 03

TIME = 0.00639

X	VELOCITY	PRESSURE
-0.000	0.1112E 01	0.4420E 03
29.905	0.1112E 01	0.4415E 03
59.810	0.1112E 01	0.4409E 03
89.715	0.1112E 01	0.4404E 03
119.620	0.1112E 01	0.4398E 03
149.525	0.1112E 01	0.4393E 03
179.430	0.1112E 01	0.4387E 03
209.335	0.1112E 01	0.4382E 03
239.240	0.1112E 01	0.4377E 03
269.145	0.1112E 01	0.4371E 03
299.050	0.1112E 01	0.4366E 03

299.050	0.2185E 01	0.4366E 03
328.480	0.2185E 01	0.4338E 03
357.910	0.2185E 01	0.4311E 03
387.340	0.2185E 01	0.4284E 03
416.770	0.2185E 01	0.4257E 03
446.200	0.2185E 01	0.4229E 03
475.630	0.2186E 01	0.4200E 03
505.060	0.2205E 01	0.4146E 03
534.490	0.2247E 01	0.4058E 03
563.920	0.1568E 01	0.5004E 03
593.350	0.8376E-01	0.7112E 03

593.350	-0.1043E 01	0.7112E 03
598.255	0.00000000	0.8615E 03

0.000	0.00000000	0.5494E 03
4.905	-0.1127E 01	0.7112E 03

TIME = 0.01279

X	VELOCITY	PRESSURE
-0.000	0.1112E 01	0.4420E 03
29.905	0.1112E 01	0.4415E 03
59.810	0.1112E 01	0.4409E 03
89.715	0.1112E 01	0.4404E 03
119.620	0.1112E 01	0.4398E 03
149.525	0.1112E 01	0.4393E 03
179.430	0.1112E 01	0.4387E 03
209.335	0.1112E 01	0.4382E 03
239.240	0.1112E 01	0.4377E 03
269.145	0.1112E 01	0.4371E 03
299.050	0.1112E 01	0.4366E 03

299.050	0.2185E 01	0.4366E 03
328.480	0.2185E 01	0.4338E 03
357.910	0.2185E 01	0.4311E 03
387.340	0.2185E 01	0.4283E 03
416.770	0.2188E 01	0.4251E 03
446.200	0.2206E 01	0.4198E 03
475.630	0.2253E 01	0.4103E 03
505.060	0.2177E 01	0.4185E 03
534.490	0.1449E 01	0.5202E 03
563.920	0.2987E 00	0.6832E 03
593.350	0.5719E-02	0.7238E 03

593.350	0.1080E 01	0.7238E 03
598.255	0.00000000	0.5678E 03

0.000	0.00000000	0.8791E 03
4.905	0.1074E 01	0.7238E 03

APPENDIX III

A MAD Language Program[3] *for the solution of* water hammer including effect of friction losses for the pipe system Case III and a part of its computed results. [See Figure 15c.] Subroutines omitted.

[See also APPENDIX V.]

```
CHINTU LAI                      Q042N 13             005    030
* EXECUTE, DUMP
* COMPILE MAD, PUNCH OBJECT                                        WH13    2
          INTEGER G,I,  J,K,M,N,R,TREC,TP
          DIMENSION V(60), H(60), V1(30), H1(30), V2(60), H2(60), VP(60
         2), HP(60), L(3), D(3), B(3), S(3), E(3), Q(3), A(3), DELL(3),
         2VI(3), HF(3), TH(3)
          PRINT FORMAT TITLE
START     READ FORMAT CARD,   V0, H0, TC, NU, N, L(1)...L(3), D(1)...D(
         23), B(1)...B(3), S(1)...S(3), E(1)...E(3), TP, EW
          PRINT FORMAT GIVEN, V0, H0, TC, NU, N, L(1)...L(3), D(1)...D(
         23), B(1)...B(3), S(1)...S(3), E(1)...E(3), EW
          Z=1./EW
          THROUGH WORLD, FOR G=1, 1, G.G.3
          Q(G)=Z+0.81*D(G)/(B(G)*E(G))
WORLD     A(G)=SQRT.(1./(1.93*Q(G)))
          J=2
          K=1
          THROUGH TOUR, FOR G=1, 1, G.G.2
TOUR      DELL(G)=L(G)/N
          DELL(3)=L(3)
          VI(1)=V0
          C1=D(1)*D(1)/(D(2)*D(2))
          C2=D(3)*D(3)/(D(2)*D(2))
          VI(2)=V0*C1
          THROUGH FROM, FOR G=1, 1, G.G.2
FROM      HF(G)=FR.(D(G),VI(G),NU,S(G))*DELL(G)*VI(G)*VI(G)/(D(G)*64.33
         22)
          DELTI=L(3)/A(3)
          DELV=0.01
          DELH=0.01
          T=0
          H1(0)=H0
          H2(0)=H0
          HP(0)=H0
          PRINT FORMAT TEST, Q(1)...Q(3), A(1)...A(3), DELL(1), DELL(2)
         2, DELTI
          PRINT FORMAT TIME, T
          R=0
          X=0
          THROUGH ARBOR, FOR G=1, 1, G.G.2
          X=X-DELL(G)
          THROUGH ANN, FOR I=R, 1, I.G.N+R
          X=X+DELL(G)
          V1(I)=VI(G)
          H1(I)=H1(R)-(I-R)*HF(G)
ANN       PRINT FORMAT RESULT, X, V1(I), H1(I)
          PRINT FORMAT BLANK
          R=R+N+J
ARBOR     H1(R)=H1(R-2)
          HE=H1(R)
          X=0
          THROUGH MICH, FOR I=R+K, 1, I.G.R+J
          V1(I)=0.
          H1(I)=H1(N)
          PRINT FORMAT RESULT, X, V1(I), H1(I)
MICH      X=L(3)
          THROUGH USA, FOR G=1, 1, G.G.3
USA       HF(G)=HF(G)/2.
```

```
              R=0
              THROUGH MD, FOR G=1, 1, G.G.2
              THROUGH PA, FOR I=R, 1, I.G.R+2*N
              V2(I)=VI(G)
PA            H2(I)=H2(R)-(I-R)*HF(G)
              R=R+2*(N+J)
MD            H2(R)=H2(R-4)
              THROUGH NEW, FOR I=R+2*K, 1, I.G.R+2*J
              V2(I)=0.
NEW           H2(I)=H2(2*N)
              THROUGH YORK, FOR G=1, 1, G.G.3
YORK          TH(G)=DELTI/DELL(G)
BALTMR        TREC=0
BOSTON        T=T+DELTI
              WHENEVER T.G.3*TC .AND. T.G.16.*(L(1)/A(1)+L(2)/A(2)),
             2TRANSFER TO START
              TREC=TREC+1
              WHENEVER TREC.NE.TP, TRANSFER TO PHILA
              PRINT FORMAT TIME, T
PHILA         M=1
              DELT=DELTI
              TI=T
              THROUGH CANADA, FOR I=0, 1, I.G.2*(N+K+J)
              V(I)=V1(I)
CANADA        H(I)=H1(I)
SCOT          R=0
              THROUGH ENGLAD, FOR G=1, 1, G.G.2
              THROUGH LONDON, FOR I=R+1, 1, I.E.N+R
              VR=V(I)*(1.-TH(G)*(V(I)+A(G)))+V(I-1)*TH(G)*(V(I)+A(G))
              VS=V(I)*(1.+TH(G)*(V(I)-A(G)))-V(I+1)*TH(G)*(V(I)-A(G))
              HR=H(I)*(1.-TH(G)*(V(I)+A(G)))+H(I-1)*TH(G)*(V(I)+A(G))
              HS=H(I)*(1.+TH(G)*(V(I)-A(G)))-H(I+1)*TH(G)*(V(I)-A(G))
              VP(I)=0.5*(VR+VS)+(16.083/A(G))*(HR-HS)-0.5*FR.(D(G),V(I),NU,
             2S(G))*V(I)*.ABS.V(I)*DELT/D(G)
LONDON        HP(I)=(A(G)/64.332)*(VR-VS)+0.5*(HR+HS)
ENGLAD        R=R+N+J
              VS=V(0)*(1.+TH(1)*(V(0)-A(1)))-V(1)*TH(1)*(V(0)-A(1))
              HS=H0*(1.+TH(1)*(V(0)-A(1)))-H(1)*TH(1)*(V(0)-A(1))
              VP(0)=VS+(32.166/A(1))*(H0-HS)-0.5*FR.(D(1),V(0),NU,S(1))*V(0
             2)*.ABS.V(0)*DELT/D(1)
              WHENEVER M.E.1, TRANSFER TO SWEDEN
              THROUGH NORWAY, FOR I=R+K+1, 1, I.E.R+J
              VR=V(I)*(1.-TH(3)*(V(I)+A(3)))+V(I-1)*TH(3)*(V(I)+A(3))
              VS=V(I)*(1.+TH(3)*(V(I)-A(3)))-V(I+1)*TH(3)*(V(I)-A(3))
              HR=H(I)*(1.-TH(3)*(V(I)+A(3)))+H(I-1)*TH(3)*(V(I)+A(3))
              HS=H(I)*(1.+TH(3)*(V(I)-A(3)))-H(I+1)*TH(3)*(V(I)-A(3))
              VP(I)=0.5*(VR+VS)+(16.083/A(3))*(HR-HS)-0.5*FR.(D(3),V(I),NU,
             2S(3))*V(I)*.ABS.V(I)*DELT/D(3)
NORWAY        HP(I)=(A(3)/64.332)*(VR-VS)+0.5*(HR+HS)
SWEDEN        R=R+K
              VS=V(R+1)*TH(3)*A(3)
              HS=H(R)*(1.-TH(3)*A(3))+H(R+1)*TH(3)*A(3)
              HP(R)=HS-(A(3)/32.166)*VS
              VP(R)=0
              R=2*(N+K)
              WHENEVER TI.G.TC, TRANSFER TO FINLAD
              EXECUTE BC.(TC,TI,A(2),VI(2),HE,V(R), H(R), DELV,DELH)
              VP(R)=V(R)-DELV
```

```
                HP(R)=H(R)+DELH
                TRANSFER TO MOSKVA
FINLAD          VP(R)=0
MOSKVA          VR=V(R)*(1.-TH(2)*(V(R)+A(2)))+V(R-1)*TH(2)*(V(R)+A(2))
                HR=H(R)*(1.-TH(2)*(V(R)+A(2)))+H(R-1)*TH(2)*(V(R)+A(2))
RUSSIA          HP(R)=HR-(A(2)/32.166)*(VP(R)-VR)-A(2)*FR.(D(2),V(R),NU,S(2))
               2*V(R)*.ABS.V(R)*DELT/(64.332*D(2))
                WHENEVER TI.G.TC, TRANSFER TO POLAND
                EXECUTE BC.(TC,TI,A(2),VI(2),HE,VP(R),HP(R),DELV,DELH)
                VP(R)=VP(R)-DELV
                HP(R)=HP(R)+DELH
                WHENEVER (.ABS.DELV.GE.0.001).AND.(.ABS.DELH.GE.0.001), TRANS
               2FER TO RUSSIA
POLAND          VP(N+J)=V(N+J)+VP(N+J+1)-V(N+J+1)
                VR1=V(N)*(1.-TH(1)*(V(N)+A(1)))+V(N-1)*TH(1)*(V(N)+A(1))
                HR1=H(N)*(1.-TH(1)*(V(N)+A(1)))+H(N-1)*TH(1)*(V(N)+A(1))
                VS2=V(N+J)*(1.+TH(2)*(V(N+J)-A(2)))-V(N+J+1)*TH(2)*(V(N+J)-A(
               22))
                HS2=H(N+J)*(1.+TH(2)*(V(N+J)-A(2)))-H(N+J+1)*TH(2)*(V(N+J)-A(
               22))
                R=R+2*J
                VR3=V(R)*(1.-TH(3)*(V(R)+A(3)))+V(R-1)*TH(3)*(V(R)+A(3))
                HR3=H(R)*(1.-TH(3)*(V(R)+A(3)))+H(R-1)*TH(3)*(V(R)+A(3))
BERLIN          HP(N+J)=HS2+(A(2)/32.166)*(VP(N+J)-VS2)+A(2)*FR.(D(2),V(N+J),
               2NU,S(2))*V(N+J)*.ABS.V(N+J)*DELT/(64.332*D(2))
                HP(N)=HP(N+J)
                HP(R)=HP(N+J)
                VP(N)=VR1-(32.166/A(1))*(HP(N)-HR1)-0.5*FR.(D(1),V(N),NU,S(1)
               2)*V(N)*.ABS.V(N)*DELT/D(1)
                VP(R)=VR3-(32.166/A(3))*(HP(R)-HR3)-0.5*FR.(D(3),V(R),NU,S(3)
               2)*V(R)*.ABS.V(R)*DELT/D(3)
                VEL=C1*VP(N)+C2*VP(R)
                DIF=VEL-VP(N+J)
                WHENEVER .ABS.DIF.L.0.001,  TRANSFER TO BELG
                VP(N+J)=VP(N+J)+DIF/3.
                TRANSFER TO BERLIN
BELG            WHENEVER M.NE.1, TRANSFER TO SWISS
                THROUGH PARIS, FOR I=0, 1, I.G.R
                V1(I)=VP(I)
PARIS           H1(I)=HP(I)
                M=2
                N=2*N
                J=2*J
                K=2*K
                DELT=DELT/2.
                TI=T-DELT
                THROUGH FRANCE, FOR I=0, 1, I.G.2*R
                V(I)=V2(I)
FRANCE          H(I)=H2(I)
                TRANSFER TO SCOT
SWISS           WHENEVER M.NE.2, TRANSFER TO ITALY
                THROUGH ROME,  FOR I=0,1,I.G.R
                V(I)=VP(I)
ROME            H(I)=HP(I)
                M=4
                TI=T
                TRANSFER TO SCOT
ITALY           THROUGH SPAIN, FOR I=0, 1, I.G.R
```

```
          V2(I)=VP(I)
SPAIN     H2(I)=HP(I)
          N=N/2
          J=J/2
          K=K/2
          WHENEVER TREC.NE.TP, TRANSFER TO BOSTON
          R=0
          X=0
          THROUGH AUST, FOR G=1, 1, G.G.2
          X=X-DELL(G)
          THROUGH WIEN, FOR I=R, 1, I.G.N+R
          X=X+DELL(G)
          VB=2*V2(2*I)-V1(I)
          HB=2*H2(2*I)-H1(I)
WIEN      PRINT FORMAT RESULT, X,VB,HB
          PRINT FORMAT BLANK
AUST      R=R+N+J
          X=0.
          THROUGH GREECE, FOR I=R+K, 1, I.G.R+J
          VB=2*V2(2*I)-V1(I)
          HB=2*H2(2*I)-H1(I)
          PRINT FORMAT RESULT, X,VB,HB
GREECE    X=X+L(3)
          TRANSFER TO BALTMR
          VECTOR VALUES TITLE = $75H1WATER HAMMER SOLUTIONS FOR COMPOUN
         2D SYSTEMS INCLUDING FRICTION EFFECT (6) *$
          VECTOR VALUES CARD = $2F10.3,2F10.8,I4/3F10.3,3F8.6/3F8.6,3F1
         20.8/3E12.4,I3,E12.4*$
          VECTOR VALUES GIVEN = $6H4V  = F10.3,S8,5HH0 = F10.3,S8,5HTC
         2= F10.8,S8,5HNU = F10.8,S8,4HN = I4/12H0L(1,2,3) = 3(S6,F10.8
         3)/12H0D(1,2,3) = 3(S8,F8.6)/12H0B(1,2,3) = 3(S8,F8.6)/12H0S(1
         4,2,3) = 3(S6,F10.8)/12H0E(1,2,3) = 3(S4,E12.4)/6H0EX = E12.4
         5*$
          VECTOR VALUES TEST = $12H0Q(1,2,3) = 3(S6,E12.5)/12H0A(1,2,3)
         2 = 3(S6,E12.5)/11H0DELL(1) = E12.5,S5,10HDELL(2) = E12.5,S5,8
         3HDELTI = E12.5*$
          VECTOR VALUES TIME = $8H4TIME = F10.5/1H0,S12,1HX,S15,8HVELOC
         2ITY,S12,8HPRESSURE*$
          VECTOR VALUES RESULT = $1H ,S7,F10.3,S8,E12.4,S8,E12.4*$
          VECTOR VALUES BLANK = $2H0 *$
          END OF PROGRAM
```

WATER HAMMER SOLUTIONS FOR COMPOUND SYSTEMS INCLUDING FRICTION EFFECT (B)

V0 = 1.083   H0 = 432.000   TC = 0.00000000   NU = 0.00000712   N = 10

L(1,2,3) =	299.050	294.300	4.905
D(1,2,3) =	0.036330	0.025920	0.025920
B(1,2,3) =	0.002667	0.002667	0.002667
S(1,2,3) =	0.00000000	0.00000000	0.00000000
E(1,2,3) =	0.2448E 10	0.2448E 10	0.2448E 10
EW =	0.4730E 08		
Q(1,2,3) =	0.25649E-07	0.24357E-07	0.24357E-07
A(1,2,3) =	0.44946E 04	0.46122E 04	0.46122E 04

DELL(1) = 0.29905E 02   DELL(2) = 0.29430E 02   DELTI = 0.10635E-02

TIME = 0.00000

X	VELOCITY	PRESSURE
-0.000	0.1083E 01	0.4320E 03
29.905	0.1083E 01	0.4315E 03
59.810	0.1083E 01	0.4310E 03
89.715	0.1083E 01	0.4305E 03
119.620	0.1083E 01	0.4299E 03
149.525	0.1083E 01	0.4294E 03
179.430	0.1083E 01	0.4289E 03
209.335	0.1083E 01	0.4284E 03
239.240	0.1083E 01	0.4279E 03
269.145	0.1083E 01	0.4274E 03
299.050	0.1083E 01	0.4268E 03

299.050	0.2128E 01	0.4268E 03
328.480	0.2128E 01	0.4243E 03
357.910	0.2128E 01	0.4217E 03
387.340	0.2128E 01	0.4191E 03
416.770	0.2128E 01	0.4165E 03
446.200	0.2128E 01	0.4139E 03
475.630	0.2128E 01	0.4113E 03
505.060	0.2128E 01	0.4087E 03
534.490	0.2128E 01	0.4061E 03
563.920	0.2128E 01	0.4035E 03
593.350	0.2128E 01	0.4009E 03

0.000	0.00000000	0.4268E 03
4.905	0.00000000	0.4268E 03

TIME = 0.00638

X	VELOCITY	PRESSURE
-0.000	0.1083E 01	0.4320E 03
29.905	0.1083E 01	0.4315E 03
59.810	0.1083E 01	0.4310E 03
89.715	0.1083E 01	0.4305E 03
119.620	0.1083E 01	0.4299E 03
149.525	0.1083E 01	0.4294E 03
179.430	0.1083E 01	0.4289E 03
209.335	0.1083E 01	0.4284E 03
239.240	0.1083E 01	0.4279E 03
269.145	0.1083E 01	0.4274E 03
299.050	0.1083E 01	0.4268E 03

299.050	0.2128E 01	0.4268E 03
328.480	0.2128E 01	0.4243E 03
357.910	0.2128E 01	0.4217E 03
387.340	0.2128E 01	0.4191E 03
416.770	0.2128E 01	0.4165E 03
446.200	0.2128E 01	0.4138E 03
475.630	0.2134E 01	0.4103E 03
505.060	0.2183E 01	0.4007E 03
534.490	0.2139E 01	0.4044E 03
563.920	0.9828E 00	0.5681E 03
593.350	0.00000000	0.7076E 03

0.000	0.00000000	0.4268E 03
4.905	-0.3023E-05	0.4268E 03

APPENDIX IV

The Moody Diagram written in MAD Language (3) for the determination of friction factor in pipe flow problems, in the form of a subroutine.*

[See APPENDIX V]

---

* F = 1 for R.L. 64 is used in order to prevent $F(=f) \rightarrow \infty$. In this case $V \doteq 0$.

```
* COMPILE MAD, PUNCH OBJECT
               EXTERNAL FUNCTION (D,V,NU,K)
               ENTRY TO FR.
               R=D*.ABS.V/NU
               WHENEVER R.L.64.
                 F=1.0
               OR WHENEVER R.L.2000.
                 F=64./R
               OR WHENEVER K.E.0.
                 TRANSFER TO SMOOTH
               OTHERWISE
                 TRANSFER TO ROUGH
               END OF CONDITIONAL
               TRANSFER TO BACK
 SMOOTH        WHENEVER R.L.3250., TRANSFER TO CRITIC
               F=0.043
 STURB         KP=0.86858896*ELOG.(R*SQRT.(F))-0.8
               F0=1./(KP*KP)
               WHENEVER .ABS.(F-F0).L.0.0001, TRANSFER TO BACK
               F=F0
               TRANSFER TO STURB
 ROUGH         RS=D/K
               WHENEVER RS.G.400..AND.R.G.3250. .OR. RS.G.100..AND.R.G.3350.
              2 .OR. RS.G.25..AND.R.G.3550. .OR. RS.LE.25..AND.R.G.3900., TR
              3ANSFER TO RTURB
 CRITIC        W=ELOG.(R/2313.)
               X=3.05396*W*W
               F=0.03*EXP.(X)
               TRANSFER TO BACK
 RTURB         W=0.86858896*ELOG.(RS)+1.14
               F=1./(W*W)
               WHENEVER R*SQRT.(F)/RS.G.200., TRANSFER TO BACK
 TRASN         X=1.74-0.86858896*ELOG.(2./RS+18.7/(R*SQRT.(F)))
               F0=1./(X*X)
               WHENEVER .ABS.(F-F0).L.0.0001, TRANSFER TO BACK
               F=F0
               TRANSFER TO TRASN
 BACK          FUNCTION RETURN F
               END OF FUNCTION
```

APPENDIX V

TABLE II

SYMBOLS USED IN THE MAD STATEMENTS THROUGH APPENDICES I ~ IV AND THEIR EQUIVALENT CONVENTIONAL NOTATIONS OR DESCRIPTIONS

Symbols Used In MAD Statements	Equivalent Conventional Notations
A,A1 or A(1),...	$a, a_1, \ldots$
B,B1 or B(1),...	$b, b_1, \ldots$
C1	$(D_1/D_2)^2 = A_1/A_2$
C2	$(D_3/D_2)^2 = A_3/A_2$
D,D1 or D(1),...	$D, D_1, \ldots$
DELH	ΔH
DELL,DELL(1),...	$\Delta L, \Delta L_1, \ldots$
DELT,DELTI	Δt
DELV	ΔV
E,E1 or E(1),...	$E, E_1, \ldots$
EW	K
F,FR.	f
H,HO,...	$H, H_o, \ldots$
HF	h_f
HP	H_P
HR,HS	H_R, H_S
HR1,HS3,...	$(H_R)_1, (H_S)_3, \ldots$
I	i
J	j
K	k
K	ϵ
L,L1 or L(1),...	$L, L_1, \ldots$
M	m
N	n
NU	ν
Q	$1/K + Dc_1/Eb$
S,S1 or S(1),...	$\epsilon, \epsilon_1, \ldots$
TAU	τ
TC	T_c
TH,TH1 or TH(1),..	$\theta, \theta_1, \ldots$
T,TI	t
V,VO,...	$V, V_o, \ldots$
VEL	V
VI(1),VI(2),...	$(V_o)_1, (V_o)_2, \ldots$
VP	V_P
VR,VS	V_R, V_S
VR1,VS2,...	$(V_R)_1, (V_S)_2, \ldots$
X	x

Symbols Used In MAD Statements	Descriptions
DIF	Difference, Δ
G	Integer
HB	Extrapolated value of H
HE	Head across the gate when $V = V_o$
H1	H for n steps of $2\Delta t$
H2	H for 2n steps of Δt
R	Integer
R	Reynolds number
RS	Relative smoothness D/ϵ
TREC	Integer (for time recording or counting)
TP	Integer (for the control of time interval in printing)
VB	Extrapolated value of V
V1	V for n steps of $2\Delta t$
V2	V for 2n steps of Δt

BIBLIOGRAPHY

1. Allievi, L. "Theory of Waterhammer," translated by E. E. Halmos, printed by Richardo Garoni, Rome, Italy, 1925.

2. Angus, R. W. "Water-Hammer Pressures in Compound and Branched Pipes," Proceedings ASCE, 64, January 1938, p. 133-169.

3. Arden, B., Galler, B., and Graham, R. "Michigan Algorithm Decoder," The University of Michigan, Ann Arbor, Michigan, June 1960.

4. Barbarosa, Nicholas L. "Hydraulic Analysis of Surge Tanks by Digital Computer," Journal of the Hydraulics Division, Proc. of ASCE, 85, No. HY4, April 1959.

5. Binnie, A. M. "The Effect of Friction on Surges in Long Pipe-Lines," Quarterly Journal of Mechanics and Applied Mathematics, Vol. IV, Pt. 3, (1951) 330-343.

6. Cazard, R. "Fatigue of Materials," translated by Fenner, A. J., Philosophical Library, Inc., 1953.

7. Durand, W. F. "Hydraulics of Pipe Lines," D. Van Nostrand Company, Inc., New York, 1921.

8. Freeman, John R. "Flow of Water in Pipes and Pipe Fittings," Published by ASME, 1941.

9. Joukowsky, N. "Waterhammer," translated by Miss O. Simin, Proc. Amer. Water Works Assoc., 24, (1904) 341-424.

10. Lister, M. The Numerical Solutions of Hyperbolic Partial Differential Equations by the Method of Characteristics in "Mathematical Method for Digital Computers" edited by Anthony Ralston and Herbert S. Wilf, John Wiley and Sons, Inc., 1960.

11. Lister, M., and Roberts, L. "On the Numerical Solutions of Spherical Waves of Finite Amplitude," Technical Report on the Project of Machine Method of Computation, MIT-Proj. DIC 6915, June 1956.

12. Love, A. E. H. "A Treatise on the Mathematical Theory of Elasticity," Dover Publications, 1944.

13. McNown, J. S. Surges and Water Hammer in "Engineering Hydraulics," edited by H. Rouse, John Wiley and Sons, Inc., New York, 1950.

14. Moody, Lewis F. "Simplified Derivation of Water-Hammer Formula," Symposium on Water Hammer, ASME-ASCE, (1933) 25-28.

15. Parmakian, J. "Waterhammer Analysis," Prentice-Hall, Inc., New York, 1955.

16. Rich, George, R. "Hydraulic Transients," McGraw-Hill Book Co., Inc., 1951.

17. Rich, George, R. "Water-Hammer Analysis by the Laplace-Mellin Transformation," Trans. ASME, 1945.

18. Roberts, L. "On the Numerical Solution of the Equations for Spherical Waves of Finite Amplitude, II," J. Math. and Phys., Vol. XXXVI, No. 4, January, 1958.

19. Rouleau, W. T. "Pressure Surges in Pipelines Carrying Viscous Liquids," Journal of Basic Engineering ASME, December, 1960.

20. Rouse, Hunter. "Elementary Mechanics of Fluids," John Wiley and Sons, Inc., 1953.

21. Streeter, Victor L. "Fluid Mechanics," McGraw-Hill Book Company, Inc., 1958.

22. Streeter, Victor L. "Friction Resistance in Artificially Roughened Pipes," Trans. ASCE, 101 (1936) 681-713.

23. "Symposium on Water Hammer," ASME-ASCE, 1933.

24. Wood, F. M. "The Application of Heavisides Operational Calculus to the Solutions of Problems in Water Hammer," Transactions ASME, 59, Paper Hyd-59-15 (Nov., 1937) 707-713.