Teaching Algebra in an Inner-city Classroom: Conceptualization, Tasks, and Teaching

by

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A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
(Education)
in The University of Michigan
2008

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For Mom and Dad

ACKNOWLEDGMENTS

I am indebted to members of my committee. Special thanks are extended to Professor Deborah Ball for her wise counsel and patience, to Professor Vilma Mesa, for her constructive criticism and suggestions, to Professor Hyman Bass and Professor Richard Kollar, for pushing my understanding of algebra and, to Professor Lesley Rex, for her support.

I would like to extend my sincere appreciation to my collegues Ronald Tracy and James Sahlaney, for their friendship and support, and to staff and students, for allowing me to grow as a teacher. I would also like to thank my family and friends for their patience and support.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF ABBREVIATIONS

AA	Algebra for All
ACT	American College Testing
CCSM	Cambridge Conference on School Mathematics
JHS	Jefferson High School
MiC	Mathematics in Context
NCRMSE	National Center for Research in Mathematical Science Education
NCTM	National Council of Teachers of Mathematics
NMAP	National Mathematics Advisory Panel
PSSM	Principles and Standards for School Mathematics
PWC	Procedures with Connections tasks
PWOC	Procedures without Connections tasks
SLC	Smaller Learning Communities
TGT	Teacher Generated Tasks

CHAPTER 1

OVERVIEW

For the first six years of my teaching career, I taught algebra the same way I learned it. I chose tasks from the textbook that I enjoyed as a student. Similar to Martin (2000) and Chazan (2000). I also saw bored students sitting in my classroom who really didn't care about the subject matter. Some students talked to each other and some slept through the period. I wasn't alone. Mr. Smith and Ms. Palmer, the two algebra teachers at my school, expressed a lack of connections between algebra and the students, but we didn't have much freedom to change the algebra curriculum. I read academic journals and attended professional development conferences to improve my teaching and understanding of algebra, but grew frustrated. I could see parts of the algebra from Usiskin (1988), Moses (2001), Lampert (2001), and Kieran (1993) that helped me think and teach certain algebra topics, such as the solving of linear equations and exponents, but I could not see the whole picture of high school algebra. What I saw in the algebra textbooks were chapters put together to form algebra with all of the connections that tied algebra together missing.

More than twenty years ago, Kieran (1992) provided an overview of a typical one year course in high school algebra:

> Typical topics [school algebra] include (a) the properties of real and complex numbers; (b) the forming and solving of first- and second-degree equations in one unknown; (c) the simplification of polynomial and rational expressions; (d) the symbolic representation of linear, quadratic, exponential, logarithmic, and trigonometric functions, along with their graphs; and (e) sequences and series. (p. 391)

I recognized these topics as a student and now as a teacher. The historical development of high school algebra has been stagnant with a few minor changes in the topics and approaches to certain topics with the advent of manipulative and computer technology (Dossey, 1997). Why haven't there been any changes in high school algebra?

Burrill (1995) offered the following explanation:

> There is an institutional tradition about algebra. The school community and the public have expectations about the content of algebra, and they do not lightly accept changes that conflict with these expectations. What they learned or did learn is what should be. In addition, tradition has imposed a sequence on the subject that teachers find hard to modify. (p. 55)

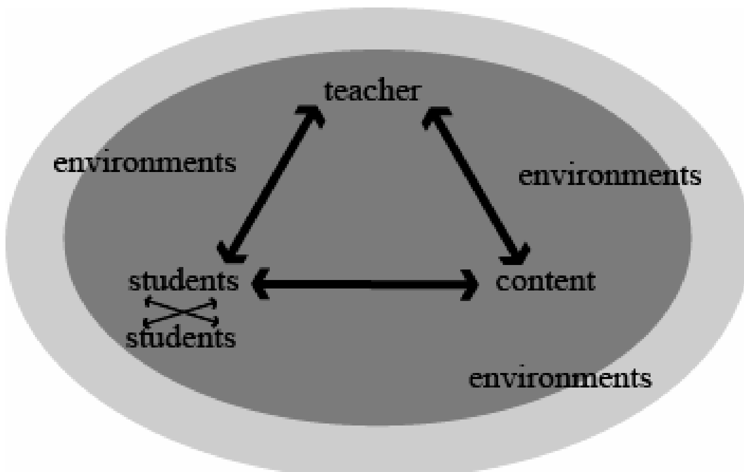
Moses (2001) encountered administrative and historical obstacles when he tried to introduce algebra, a typical 9^{th} grade class, to the 8^{th} grade curriculum. Change would not be that simple. When I first started teaching at Jefferson High School (JHS), I taught algebra using Brown (1992) then Foster (1998) and now Larson et al. (2004), which hereafter is referred to as "Larson." All of these textbooks were similar in their approach to the teaching of algebra and I struggled making the algebra connect to my students. Concentrating only on the first semester of algebra, Larson lists five chapters: (1) Connections to Algebra, (2) Properties of Real Numbers, (3) Solving Linear Equations, (4) Graphing Linear Equations and Functions, and (5) Writing Linear Equations. Little has changed except for the ordering of the algebra topics.

In this chapter, I introduce the teaching framework as a way to frame the discussion for enactments of teaching. I provide a rationale for the research question: What does it take for a teacher to teach algebra in the inner-city high school for understanding? I also give an overview of the literature of surrounding the teaching of high school algebra, conceptualizations of algebra, the content of algebra, and expand the literature review to include researchers working in the field of teaching.

The Instructional Triangle

Understanding the working of the inner-city school is complex. Using Cohen, Raudenbush, and Ball's (2003) model of instruction allows me to untangle these complexities. Figure 1-1 presents a pictorial representation of this model of instruction.

Figure 1- 1. Instruction as interactions (Cohen et al., 2003).



The instructional triangle of teacher, students, and content rests inside the classroom environment. As an inner-city high school teacher, the school environment has a direct effect on my classroom by the implementations and enactments of district policies. This model of instruction functioned well when I was a student in my elementary, Jr. high school, and high school. As a student, I knew what to expect from my peers, my teacher, and my administration. I enjoyed doing the work assigned by my teachers and the school administration made every effort to support our studies by creating an academic school environment.

In the inner-city high school, the instructional triangle remained the same, but the classroom, school, district, and community environment are at times conflicted. For example, my district has a "no cell phone policy" on the basis that these electronic devices distract students from learning. Parents on the other hand, buy cell phones for their children in case of emergencies and for other means of communication. The school administration can enforce this policy when students go through the metal detectors and when the backpacks are checked. Doing this requires resources the school does not have and the school must also deal with angry parents if the cell phones are confiscated.

In my classroom, I have seen cell phones, heard cell phones ringing, and heard students talk on cell phones to their parents. I know that a policy exists for cell phones, but I am unsure what I am supposed to do because the policy is haphazardly enforced. In order to avoid confrontation, I simply asked the students to put the cell phones away. This worked for my students, but for other students who don't know me as their teacher, I was simply ignored. In the end, the district's policy did not get enacted. This is one policy, but the same analysis can be applied to tardiness, truancy, and dress codes. Students and teachers are unsure of polices in the school. This creates an unstable school environment that permeates the classroom.

It took me two years to realize that many of my difficulties came from my inability to separate the school environment from the classroom environment. During those initial years, I tried to teach inside an unstable classroom environment. I had students fighting, chairs being thrown, and the door getting slammed. I lowered my expectations on assessments to build students' self-esteem. This didn't work; their assessment scores remained flat or dipped. I changed my assessment dates for absent students to build rapport. Still, my classes remained unruly. I was still dissatisfied. When I closed my door to teach class, I did it because I was embarrassed to show what was going on in the classroom.

Because of my dissatisfaction in the classroom, I decided to raise my expectation. Toward the end of my second year, I decided to use the regular algebra textbook with my basic algebra students. These basic algebra students came into class loud, never did their homework, so it took me more effort to get the class working. When I told my students that we would be using the regular textbook[1], I found these students attentive to the lesson and had a glimmer of hope as I entered my third year. I was told to go back to the basic algebra textbook two weeks later. Raising my expectation positively affected the classroom environment and I began thinking of other ways to create a positive classroom environment.

Entering my third year, I decided to create my own classroom environment in order to lessen the effects of the school environment. When students arrived, I had already placed a task on the chalkboard. I also gave an assessment every Friday. By doing this, I no longer had to explain the date for assessment and required students to follow my schedule. These basic structures allowed me to focus on teaching. Now, when I closed my door, I did it to block the school environment for entering my classroom.

I gained more control of the classroom, because I had fewer arguments about the assessment dates and fewer discipline issues, but the instructional triangle seemed stuck at the content. Some students participated, but some students seem to be there to do their time. During one conversation with a student, she remarked, “Algebra is stupid. All we do is work with letters.” I was dumbfounded. She was right. No matter what I tried with content, the bidirectional arrows in the instruction triangle felt unidirectional or very weak when the students and I interacted with the content. I decided four years ago to improve my understanding of both algebra and *how to teach* algebra.

Research Question

Opportunely, four years ago, the Smaller Learning Community (SLC) was being implemented by the district. My curriculum leader asked me to be lead teacher for the other two 9th grade mathematics teachers. In our weekly meetings, we saw a disconnect

[1] As a non-native speaker of English, I remembered feeling quite excited when I moved to the regular English class.

between algebra and the students. One teacher suggested building a strong arithmetic foundation. From these discussions, I began formulating a research question: What does it take for a teacher to teach algebra for understanding in an inner-city high school?

The complexity of this research question required that I divided it into four questions, which look at specific elements inside the instructional triangle. The first question deals with the content inside algebra I taught and the district mandated curriculum:

1. How does a numerical pattern approach to algebra compare to district mandated algebra approach using the district's mandated textbook (Larson)?

This question looks at the algebra content, the cognitive demands of the tasks, and the pacing. The second question, deals with the classroom environment:

2. What does teaching a numerical pattern approach look like in the classroom?

This provided a holistic view of the classroom. The third question looks at the instructional triangle:

3. How do instructional practices using this approach work within the classroom?

I analyzed the mathematical tasks, the dialogues, students' work, and their assessment to reveal the tensions and successes that occur when I taught algebra in the classroom. These tensions included the following questions: How should I introduce algebra? How do I teach operations with real numbers? How do teach the solving of linear equation? What obstacles occur in the process of teaching algebra? What are the topics for each chapter?

The fourth question provides the background information required to teach using this approach:

4. What does a week of teaching look like using this approach?

I looked at a weekly teaching cycle and showed how I analyzed assessments, wrote lessons, and enactments.

The research questions above had been forming in my head for the past four years. In many ways, these questions came from trying to survive and thrive as a teacher in the inner-city classroom. I was teaching algebra with little understanding. Kieran (1992) wrote, “However, to cover their lack of understanding, it appears that students resort to memorizing rules and procedures and they eventually come to believe that this activity represents the essence of algebra” (p. 390). This was how I learned algebra and this was how I was teaching algebra. This study describes my role as teacher teaching algebra using a numerical pattern approach as a way to infuse understanding in my classroom and how this reconceptualization looked inside my classroom. The study took place in a 9th grade algebra classroom set in an inner-city high school of a large metropolitan city. This dissertation looks at algebra teaching in the inner-city high school and builds upon the knowledge base for high school algebra, algebra conceptualization, algebra curriculum, and teaching in the inner-city.

Literature Review

Guiding this dissertation, I read researchers working in algebra and algebra teaching, and researches working on the confluence of teaching and difficult teaching environment. I began with an overview of the literature about high school algebra with a focus on the issues surrounding high school algebra, the different conceptualizations of high school algebra, and the algebra content. With so little research on the teaching of algebra, I broadened my literature review to include the teaching of literacy in the high school, mathematics from the middle school, and looked at the scarce research on the teaching of high school algebra. My objective was to look for commonalities across the literature on teaching to create a teaching framework that I could use throughout this dissertation to look at my own high school teaching of algebra.

Algebra Issues

When I began teaching in 1993, I was unaware of the upheaval that was occurring in algebra. Lynn Steen (1993) said the following at the Algebra Initiative Colloquium in 1993, "The major theme of this conference is very simple: Algebra is broken but nonetheless essential" (p. 122). There were four groups working on the following issues: (1) creating an appropriate algebra experience for *all* grades K-12; (2) educating teachers, including K-8 teachers, to provide these algebra experiences; (3) reshaping algebra to serve the evolving needs of the technical work force; and (4) renewing algebra at the college level to serve the future mathematicians, scientists, and engineers (U.S. Department of Education, 1993 p. 1).

Salient to this dissertation is the work of the first working group. The participants in the first group worried that making "algebra for all" (AA) would mean a watering down of the algebra curriculum. To get around this dilemma, the colloquium participants recommended embedding algebra concepts in the K-8 curriculum. Lacampagne (1993) wrote, "Such integration would eliminate algebra for its current gate-keeper function, for algebra would be learned over a period of 8 years" (p. 4). The colloquium participants were unsure of how to translate the algebra curriculum into the K-8 curriculum. Some colloquium participants expressed skepticism with the algebra for all approach and referred the acronym AA as "Algebraists Anonymous" or "Avoiding Algebra" (Steen, 1993). Everyone did agree that algebra should focus on a few big ideas rather than lots of isolated skills.

Two decades later, the issues were "center on the content that should be taught and how it should be taught. Arguments rage over curriculum materials, instructional approaches, and which aspects of the content to emphasize" (RAND Mathematics Study Panel, 2003 p. xiii). Changing high school algebra would be difficult because "we talk about making changes in algebra teaching and learning, we are talking about changing the system of concepts, skills, and knowledge that provided part of the basis for our being here today" (Dossey, 1997 p. 17).

The National Mathematics Advisor Panel (NMAP) (2008) looked into "how students can be best prepared for entry into Algebra" (p.xii). The NMAP centered the discussion around the following five issues: (1) conceptual knowledge and skills; (2) learning processes; (3) instructional practice; (4) teacher and teacher education; and (5) assessment.

The NMAP commissioned a survey to determine the obstacles facing algebra teachers: "The survey revealed that teachers rate their students' background preparation for Algebra 1 as weak. The three areas in which teachers reported their student to have the poorest preparation are rational numbers, word problems, and study habits" (National Mathematics Advisory Panel, 2008 p. 9). These algebra teachers listed these factors: unmotivated students, mixed-ability groupings, lack of family support, and making mathematics assessable and comprehensible as major challenges they face (National Mathematics Advisory Panel, 2008).

I saw this within my own high school, when my district bought two algebra curricula: (1) *Algebra 1* (Larson, Boswell, Kanold, & Stiff, 2004), which is the district mandated textbook; and (2) *Cognitive Tutor: Algebra 1* (Carnegie Learning, 1998), which is an algebra computer software. From this point forward, I will refer to the district mandated algebra textbook as Larson. I was told not to hybridize these curricula. From my perspective the thought of writing two different algebra lessons, utilizing two different pacing charts, and writing two different assessments made no sense. I also didn't believe that either of these curricula met students' needs. Moses (2001) shared my sentiment:

> There are a lot of well-trained curriculum experts and others who know a great deal about math, but, I began to tell myself, what is missing from their work is insight into the minds of the young people they are trying to reach. (p. 102)

Moses saw the disconnection between algebra and the students as more insidious. He correlated the lack of success in algebra to lack of economic access for students, which is a view shared by many (Dossey, 1997; National Mathematics Advisory Panel, 2008; RAND Mathematics Study Panel, 2003; U.S. Department of Education, 1993).

Still, the issues of *what to teach* and *how to teach* remained unanswered. In his article, Dossey presented structures, representation, functions and relations, and modeling as models of understanding for algebra. He stated, "In fact, what we really have to do is think of how to merge them to support a coherent program of study in algebra" (p. 35). The next section, delves further into these models, and reflects my search to get a better understanding of how to create a coherent curriculum.

Algebra Conceptualizations

From my own experience, this lack of coherence was the major obstacle in algebra and that led students into believing that each chapter of the algebra textbook had no connections to the previous or next chapters. Dossey's models of understanding, Chazan's (2000) conceptualizations of algebra, or Bednard et al's (1996) approaches to algebra are ways to organizing algebra into manageable pieces (National Council of Teachers of Mathematics, 1932). Connecting these manageable pieces provides coherence to algebra. In the *Michigan Merit Curriculum: Algebra 1* (Michigan Department of Education, 2006), The Michigan Department of Education members divided algebra into strands, standards, topics, and expectations. For this dissertation, I chose to look at the standards that existed in Larson's chapters.

Through my research, I have found that algebra has six conceptualizations that tied these high school algebra standards: (1) algebra is used to model reality (Moses & Cobb, 2001), (2) algebra is symbolism (Kieran, 1992; Usiskin, 1988, 1993, 1995), (3) algebra is the study of functions (Chazan, 2000; National Council of Teachers of Mathematics, 1926, 1932), (4) algebra is multiple representations (Kaput, 1989), (5) algebra is the study of structures (Aichele & Reys, 1963; Brown et al., 1992; Cambridge Conference on School Mathematics, 1971; Rowen, 1994), and (6) algebra is the search for solutions through the solving of historical problems (Wheeler, 1996).

First, *algebra as a way to model reality* was found in Robert P. Moses' Algebra Project (Moses & Cobb, 2001). Using their surroundings as the seed of inquiry, students would communicate with their peers and teachers through graphical, verbal, and oral presentations. Moses presented algebra as a language developed through informal discussions and drawings of an everyday context that will leads to a formal algebraic language to represent the context. Experiential learning is a circular process of the event, reflection, abstraction, application, leading back to the event (Moses & Cobb, 2001).

Second, *algebra as symbolism* or *generalized arithmetic* appeared in Nunn's *The Teaching of Algebra* (1914). Nunn defines the nature of algebra as analysis, direct use of symbolism, extended use of symbolism, and manipulation of symbols. Nunn (1914) began with an example of the mental process of analysis and generalization. A student was asked to determine the number of squares in a rectangular grid. With some observation, he realized that it was possible and easier to use multiplication instead of addition. With more work, he found a rule exists for finding the area of a rectangle. *Analysis* was the mental process of "playing" with the number and *generalization* was the realization that a rule can be applied to all similar types of problems.

Usiskin (1995) and Kieran (1992) argued algebra should begin at the elementary grades. They rationalized that because algebra is a language, then all human beings can learn it, especially at a younger age (Kieran, 1992; Usiskin, 1993, 1995). In the National Council of Teachers of Mathematics (NCTM) 1988 yearbook, *The Ideas of Algebra, K-12*, Usiskin (1988) wrote "Purposes for algebra are determined by, or are related to, different conceptions of algebra, which correlate with the different relative importance given to the uses of variables" (p. 8). In other words, the variables have a different "feel" depending on their usage(s) and the algebra being done will also be different. Variables are central to the study of algebra in its many forms as generalized arithmetic, study of relationships among different quantities, rules for its manipulations, and the study of structures (Usiskin, 1988).

Third, *algebra as the study of functions* materialized in 1932 with the publication of the *Teaching of Algebra: 7th Yearbook* from the National Council of Teachers of Mathematics (NCTM). NCTM surmised that the jumping point into algebra would begin with students' experiences in their daily lives (Lennes, 1932). Beginning with a function, questions were generated from tables to determine its behavior. These questions allowed for discussions about dependent and independent variables, change, variation, formula, and rate of change with respect to the other variable. These investigations got at the notion of how a function is behaving (Lennes, 1932). The same line of questions could be used to study linear functions with negative numbers, fractional functions, and graphical representation of functions, the square root function, inverse square function, and trigonometric functions to cover all topics in beginning algebra. Using functions as the central theme, multiple representations such tabular, symbolic, and graphical representation can be incorporated (Chazan, 1992; Kaput, 1993).

Fourth, *algebra as multiple representations* developed algebra by moving between the tabular, symbolic, graphical, and verbal representations. Kaput (1991) argued that algebraic development comes from interweaving of different components of algebra such as generalization, opaque formulations, functions and relations, and language. Algebra is not only the collection of patterns, functions, variables, symbolic manipulation, but also the interconnections of these components within a context of a problem that encompasses these components. Multiple representations become easier with the computer's abilities to present problems, draw graphs, and create tables.

Fifth, *algebra as the study of structures* began in the late 1950s as the direction of algebra was slowly moving toward "modern algebra" or "new math" in response to the perceived educational and technological gap with the launching of Sputnik by the Soviet Union (Cambridge Conference on School Mathematics, 1971; National Mathematics Advisory Panel, 2008; Phillips, 1993).

In 1963, a workgroup of mathematicians and mathematics educators at the Cambridge Conference on School Mathematics (CCSM) presented a reform to the contents of algebra. The purpose of algebra was to develop students to become engineers, mathematicians, and scientists capable of competing in a technological world (Katz, 1993).

Set theory, Boolean algebra, and number theory was also be included in this new math (Kline, 1971). CCSM was quite ambitious; their initial curriculum for algebra included the following topics: ring of polynomials over a field, polynomial functions, rational forms and functions, quadratic equations, iterative procedures, difference polynomials, Euclidean algorithm, Diophantine equations, modular arithmetic, and complex numbers as residue classes of polynomials (mod x^2+1). They were met with a lot of criticism by other workgroups for being too abstract and of little use for students not going into the mathematical, technical, and engineering fields (Kline, 1971). Some of these topics (e.g., set theory and Boolean algebra) were implemented in textbooks in the 1970s and 1980s, only to be removed later.

Sixth, *algebra as the search for solutions through the solving of historical problems* was first seen in Cardan's work (1501-1576), where he consolidated the known methods for solving equation in his treatise *Ars Magna* (1545) and within the pages revealed methods for solving cubic and quartic equations (Katz, 1993; Kline, 1985). A curriculum could be constructed with problems that Diophantus (200-284 B.C.), Cardano (1510-1576), and Viète (1540-1603) worked on (Charbonneau & Lefebvre, 1996). Using a problem-solving perspectives, Wheeler (1996) wrote, "If the mathematical developments that took place in history are trustworthy guides to the development of mathematical instruction, then it seems clear that the introduction of algebra should follow the problem solving approach and focus on the solution of equations" (p. 147).

These conceptualizations provided me lenses to analyze and understand the algebra curricula. Using these conceptualizations, I was able to see how the authors of these curricula connected the different algebra topics for a semester, but translating them into a working curriculum would not be easy.

As mentioned above, my high school had two algebra curricula. I would categorize the algebra for the textbook by Larson et al. (2004) as a modified symbolism. The other algebra curriculum *Cognitive Tutor: Algebra 1* (Carnegie Learning, 1998) is an example of the multiple representations, which uses the computer's abilities to present contextual tasks, create tables, make graphs, and plot points to connect the contextual, tabular, graphical, and algebraic representation. I now understood why hybridizing the two curricula would be disastrous. Nevertheless, I did try to use Larson to structure classroom discussion and *Cognitive Tutor: Algebra 1* for independent work. Students were confused because both approaches were too different. I tried using each of them individually, but was not convinced that these were the algebra experiences I wanted for my students.

<u>Patterns</u>. Before the start of the semester, our 9th grade mathematics team met to discuss our approach for the upcoming academic year. We all expressed a desire to make the mathematics meaningful to our students and we believed that finding the appropriate conceptualization would achieve this. We realized that the six high school conceptualizations of algebra described above would not fit the needs of our students. Modeling reality required resources we didn't have. Symbolism and generalized

mathematics were Larson's approach. The study of function, structures, or historical problem was too abstract for my 9th graders. Multiple representations required a computer lab that was not reliable and the other two teachers did not like the algebra software.

I decided to follow the elementary school conceptualization for algebra. Through these discussions, we decided to use numerical patterns as our entry into algebra. For Dossey, algebra "grows from the study of growth itself. One of the first places students see growth is when they look at patterns and patterns of numbers" (p. 20). Schoenfeld (1993) recommended, "the use of variables to describe patterns and give formulas involving geometric, physical, economic, and other relationships" (p. 10) as areas of study for algebra.

With numerical patterns, I could construct tasks using geometric shapes or visual patterns. English and Warren (1999) recommended having a balance with visual patterns and number patterns in a table. They further wrote:

> We should also keep in mind that these patterning activities need not end once the variable concept has been established. They can provide a useful, concrete base in subsequent symbolic work, including the equivalence of algebraic expressions. (p. 145).

From this quote above, English and Warren hint that it may be possible to extend numerical patterns to other algebra topics. Numerical patterns need not be just an entry point into algebra, but a possible model, conceptualization, or approach. A numerical pattern approach has its root in arithmetic which allows for a transition from arithmetic to algebra.

While each of these conceptualizations offered a possible approach to the teaching of algebra, I had to consider my students, the resources available, the administration staff within my school and those within the district. English (1999) wrote in her summary, "In this article we have attempted to show how the patterning approach can be used to introduced elementary algebraic ideas" (p. 145). Dossey (1997) offered a way of tying numerical patterns with geometry. Using these ideas of English and Dossey, an approach to algebra could be built around numerical patterns. This approach offered the following advantages: the comfort of arithmetic to my students; the use of textbook tasks and computer lab software as resources; and a not-too radical approach as to cause problems with my administrative staff. This approach also had a couple of disadvantages: no existing curriculum to work with and numerical pattern which was discreet versus algebra containing continuous functions. I decided to pursue a numerical pattern approach because the advantages outweighed the disadvantages. I had a way to connect algebra, but I had not yet determined the algebra content.

Algebra Content

An obstacle to the teaching of algebra is determining the content for algebra. From the 1993 *Algebra Initiative Colloquium* in Leerburg, Steen (1993) recommended to, "teach a few big ideas well, not many superficially" (p. 123). Accordingly, I focused only on the recommendations made about algebra:

- The representation of phenomena with symbols and use of these symbols sensibly;
- The use of variables to describe patterns and give formulas involving geometric, physical, economic, and other relationships;
- Simple manipulations with these variables to enable other patterns to be seen and variations to be described;
- The solving of simple equations and inequalities and systems by machine; and
- The picturing and examination of relationships among variables using graphs, spreadsheets or other technology. (Schoenfeld, 1993 p. 13)

The list provided an overview of what should be happening in algebra. The use of patterns appeared twice in the list. In *The Nature and Role of Algebra in the K-14 Curriculum* symposium, Williams (1997) suggested, "Every student is capable of (a) learning about the use of symbols, (b) using patterns to look for generalizations, and (c) understanding the use of dependent, systematic relationship to model situations and make predictions" (p. 41). From my perspective, what was missing in these recommendations was how to tie these pieces together.

After many years of teaching the district mandated curriculum, I had decided that the first semester of algebra would be the study of linear equations. The focus for the first semester would include writing of linear equation and solving of linear equation which included the concept of slope and *y*-intercept. I wanted my students to fully understand the linear equation. Thus, the study of the absolute value function and exponents were pushed toward the end of the first semester. I also needed to incorporate the study of operations of rational numbers into the first semester. With the algebra content and algebra conceptualization decided, I next looked at teaching.

Teaching Framework

In considering my teaching approach, I broadened my literature review to include the works of researchers dealing with difficult problems in teaching and learned from them how they approached and taught content. This literature review encompassed literacy from the high school and mathematics from the middle school and high school. I revisited the work of Moses and his Algebra Project.

Lee (2001), an English teacher and researcher, worked with African American students at Fairgate High School in a school community where "middle-class White and Black families have left the city to avoid sending their children to the public school in that area" (p. 99). Fairgate High School is "all-Black high school" (p. 99) with an average ACT score of 13.7. The problem for Fairgate High School English teachers was to find an approach to teach students how to read critically.

During the research period, Fairgate High School implemented a reform program called the Cultural Modeling Project, which provided support for the English department "through curriculum development, technology infusion, professional development, and assessment" (p. 100). The teachers believed that students' language usage already contained the "generative concepts and strategies" (p. 100) that "offers a fertile bridge for scaffolding literacy response, rather than a deficit to be overcome" (p. 101). Working with this premise, the English department at Fairgate met before the start of the academic year to discuss the cultural data set which "included R & B or rap lyrics, rap videos, stretches of signifying dialogues (a genre of talk in African American English Vernacular), as well as film clips and television programs" (Lee, 2007 p. 58) to teach around interpretative problems (e.g., satire, symbolism, irony, and point of view). These cultural data sets are not a static set of texts. They "provide opportunities to model what expert thinking looks like" (p. 35) which "help students to create connections between the known and unknown" (p. 35). New texts could be added if the teachers felt the old text was not adequate and also enlisted students to help choose texts. With each text in the cultural data sets, teachers discussed possible students' reactions, which Lee defined as "idealized model of text analysis" (Lee, 2001 p. 114).

In the classroom, a teacher may introduce a rap lyric to her classroom to teach symbolism. The teacher isn't the expert with the rap lyric so she provides opportunities for students to become the teachers of the class. The teacher takes the role of listener and moderator and through this co-constructs "a culture of inquiry and argumentation based on evidence" (Lee, 2007 p. 67). The teacher's lack of knowledge about rap lyrics shows the students that even expert reader struggle with texts. During these conversations, students talk to each other in small groups but also to the class and this multiparty overlapping talk is a characteristic of Lee's English class. She further elaborated:

> In the midst of what I come to call "the performance floor" of the classroom, I learned very quickly to map what groups of students were

> saying onto my internal map of the domain in order to evaluate what in their statements needed to be elevated to public investigation. (p. 97)

The teacher may not understand the rap lyrics well, but has the reading strategies to make sense of the text and students' responses. This internal map of the domain allows the teacher to position students' responses and provides an appropriate response. By allowing students to take the teacher's role, the students in turn are building their internal map of the domain. Teacher must react to errors, which are mistakes made by the students, and uptakes, which are actions that demonstrate an understanding. The teacher must also react to student's counterscripts, which are moves made by the students unplanned by the teacher.

Within the classroom, teacher's responsibilities include "helping students take responsibility for their close reading and thinking; and engaging the students in close reading of the text" (Lee, 2001 p. 122) and helping co-create a classroom community that builds new norms for reading, values complex problems, models strategies, builds intertextual likes, and use routine artifacts to support critical thinking (p. 115). As students become better readers, the cultural data sets include canonical texts that mirror less of the students' background.

Lensmire (1993) worked with literacy and highlighted the issue of allowing students to take the leadership role. Lensmire wanted to give a voice to his 3rd graders and saw a need to relook at writing. He stated:

> Typically, children compose very little in schools. The writing that is done is tightly controlled by the teacher, who initiates writing tasks; who determines audience, purpose, and format for the writing; and who acts as the sole audience and evaluator. (p. 267)

His solution was to introduce the writing workshop that provided, "opportunities for children to engage in and practice the craft of writing" (p. 267). The writing workshop contained three phases: (1) establishing the norms and routing; (2) writing time for the students; and (3) sharing their work with their peers and the teacher. The power of the writing workshop was that it allowed students to choose what they wanted to write about;

thus the teacher followed the student's lead. Normally, a student would choose a topic and begin writing, the teacher and peers would make comments, and the student rewrote the paper incorporating the comments. Once the paper was finished, the student had the opportunity to present the work to the class.

During the writing workshop, Lensmire encountered a counterscript that could possibly jeopardize the premise of writing workshop and modify negatively the dynamics of the instructional triangle. This occurred when a student, Maya, wrote a paper disparaging another student, Jill, in the classroom. This attack extended to another member of the classroom, Jessie, a student who resided in a trailer park. This was more than one student teasing students but involved "social relations, norms, and the sharing of texts within the writing workshop" (Lensmire, 1994 p. 125). Lensmire wanted to encourage his students to have a voice, but to censure a voice could discourage a student from participation and send a message to the class that their freedom to choose a topic wasn't real and would defeat the premise of giving students a voice in their writing.

Lensmire began by questioning Maya's purpose for writing this text and the meaning in the text and wanted Maya to understand the repercussion of her work. He asked Maya to ask Jill if she agreed to being named in the story. Jill did not agree, but Maya would not back down and "was set on having the character's name be someone in the class—this was her piece, so she had the right to control it" (p. 290).

After many discussions, Maya reluctantly decided to change the name to someone not in the class, and Lensmire had to create rules to prevent this from happening again. Although this solution proved sufficient for this counterscript, he believed a more adequate response would incorporate the student's social and emerging self, peer culture, and "include a vision of the type of classroom community we want our children to write and learn" (p. 294). Incorporating these ideas would strengthen the structure inside the classroom environment and lessen the negative effects of counterscripts. In these studies above, Lee and Lensmire relooked at the content and allowed students to take on the teacher's role. I though that using arithmetic as the jumping point into algebra would also allow students opportunities to take lead in my mathematics classroom.

Gutstein (2003) wanted to "develop students' social and political consciousness, their sense of agency, and their social and cultural identities" (p. 42). He recognized that the district mandated curriculum, *Mathematics in Context* [MiC] (National Center for Research in Mathematical Science Education & Freudenthal Institute, 1998) did not teach for social justice. His solution was to create projects to supplement the curriculum Gutstein created his own projects that used students' background as a way of motivating students to use mathematics to question their reality and, "virtually every project related to and built on my students' lived experiences as urban your from immigrant, Latino, working-class families" (p. 47).

MiC included the following three objectives: (1) multiple perspective, (2) real-life context, and (3) curricular coherence. Within the classroom, students "invented their own solution methods, solved problems in multiple ways, generated multiple solutions when appropriate, reasoned mathematically, communicated their findings both orally and in writing, and developed their mathematical and personal confidence" (p. 67). Students' inputs were used to generate new lines of questions. Gutstein added, "But from the perspective of teaching mathematics and having students develop mathematical power, *MiC* played the leading role" (p. 65). Gutstein created his own content in order to teach for social justice and used the structure built by the *MiC* curriculum to teach his content.

Chazan (2000) saw the dynamics relationship between the teacher and students as an integral part in understanding and motivating lower-tracked students and determining how best to approach his teaching. He recognized that the district mandated curriculum was not connecting to the students. He began his journey by conceptualizing algebra, which took him through the variety of different approaches to algebra and determining the algebra most appropriate for his students. He wrote:

> But such curricular changes were not what I sought when teaching at Holt. I was not convinced that simply changing the material would address the issues of student motivation, conceptual understanding, and classroom discourse that I have described. (p. 60)

He further elaborated:

> Thus, the work I undertook at Holt was a reinterpretation of an existing course. I sought a way of thinking about algebra that would help my students─be they smokers, preppies, stoners, or nerds─see algebraic thinking in the world of their experience. (p. 60)

Chazan solved this problem by building a rapport with his students. This allowed him to tailor a curriculum that was appropriate for his students.

While Gutstein appropriated the structure of another curriculum and Chazan modified his algebra, Moses (2001) decided to create his own curriculum for his students. Moses saw a disconnect between algebra and his students. His solution was to connect the students' environment to algebra. His conception of algebra was to use students' experiences as the seed of inquiry for algebra. He wrote:

> In the Algebra Project this movement from experience to abstraction takes the form of a five-step process that introduces students to the idea that many important concepts of elementary algebra may be accessed through ordinary experiences. (p. 120)

The five steps include: (1) physical events; (2) pictorial representation/modeling; (3) intuitive language/"people talk"; (4) structured language/"feature talk" and; (5) symbolic representation and provides a model of how the interactions within the instructional triangle.

The physical event provides the mathematical content for the students and teacher. The pictorial representation/modeling shows how the students interacted with content. Using their pictorial representations, students discuss and share *features* they took away from the event. The teacher listens and determines how best to introduce the standard terminologies used and presents the symbolic representation for that event. It is hard to determine from his work the mathematical content for algebra and much easier to determine the mechanism for the teaching.

Implementing Moses' Algebra Project proved to be difficult. Martin (2000) began his research as an observer in an inner-city algebra classroom studying Moses' Algebra Project curriculum. He writes:

> Hillside seemed an ideal place to study African-American students' construction of their mathematical knowledge, the mathematical culture and success that the Algebra Project had produced elsewhere, as well as the "algebra for all" philosophy embedded in the curriculum. (p. 3)

He further stated, "After a few weeks at Hillside, however, I saw that things were not going as planned" (p. 3) and saw a lot of bored students who did not participate in the classroom. Martin moved his research out of the classroom to the community and realized that the historical, communal, and societal forces played a role in determining the success and failures of African-American students. Martin believed the lack of success at Hillside could be attributed students questioning "whether what they were doing was, in fact, 'real' mathematics" (p. 180). Students decided not to interact with the content and teaching could not occur in the classroom. I posit a possible reason why the curriculum did not work at Hillside was the teachers did not believe in the curriculum and chose not to modify it for their students.

Discussion

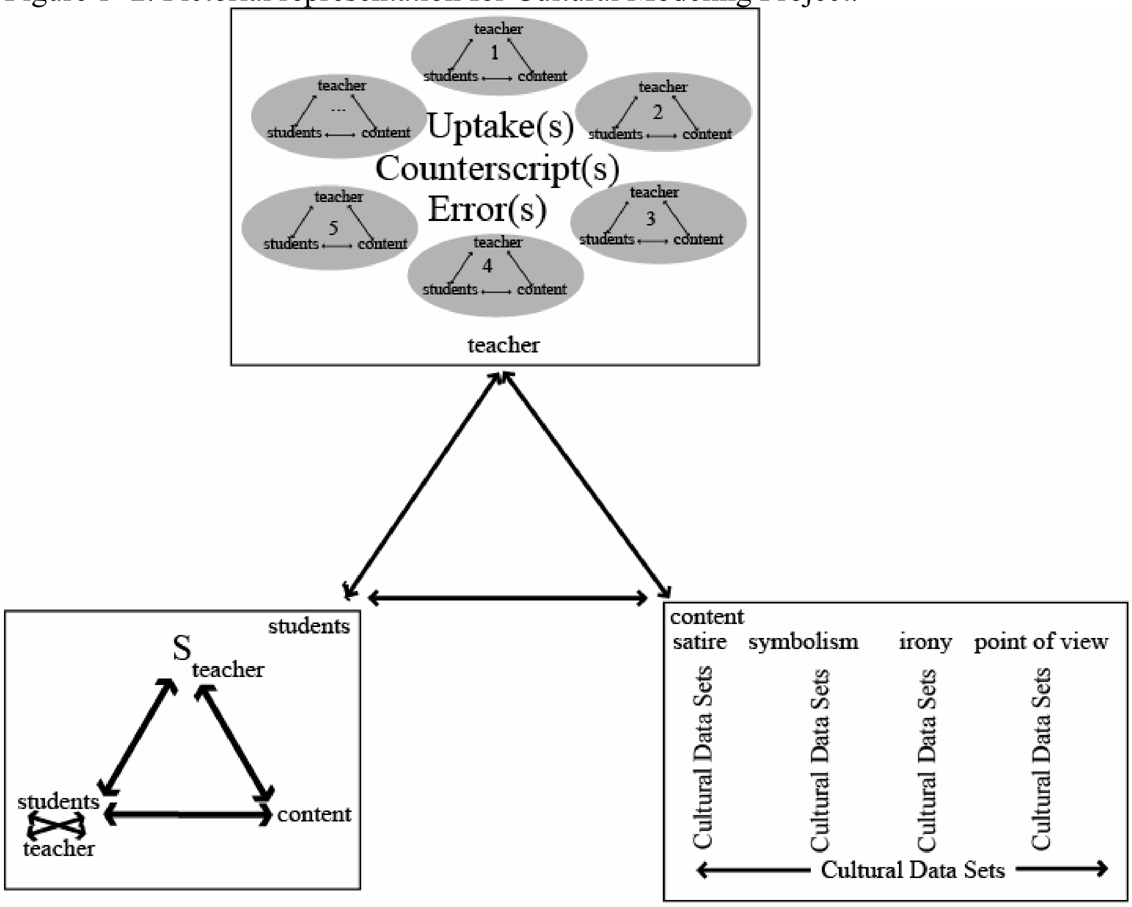
Each of these researchers worked on difficult teaching problems and each researcher provided a solution. For Lee (2001; 2007), teaching her students how to read critically required that she rethink about the content and valued students' background knowledge of African American English Vernacular. Changing the content allowed students opportunities to participate in the literary discussion. For Lensmire (1993; 1994), the problem of giving students a voice proved problematic because the classroom environment had not been established and the community structure negatively impacted his classroom. For Gutstein (2003), using an established curriculum allowed him to pursue his agenda for developing social awareness using mathematics with his students.

For Chazan (2000), connecting algebra to his students required understanding his students and modifying an existing curriculum. For Moses (2001), connecting algebra to his students was solved by using students' environment as the impetus for studying algebra. The Algebra Project process provides a template for the interactions between the teacher, students, and content. On the other hand, Martin(2000) found bored students using the Algebra Project curriculum and determined that students did not believe in the curriculum. Commonalities that exist in these studies were the need to relook at the content and the tailoring of the content to fit the needs of the students.

The obstacles for high school algebra are trying to determine what algebra topics to teach and how to teach these topics in a way that is meaningful to your students. With at least seven conceptualizations for high school algebra, a teacher must determine which of these conceptualizations would work with the students, which required a good rapport between the teacher and students. I decided to use a numerical pattern approached to the teaching of algebra because of my students' needs to review arithmetic and seemed like a good entry point into algebra. With a numerical pattern approach, I decided to make the first semester of algebra as the study of linear equations.

From the literature review, Lee's work resonated strongly with me even though her work focused on literacy. She worked in an inner-city high school, tailored the content for her students, and showed how teacher and students interacted with the content. The work of the Cultural Modeling Project members resembled what I was trying to accomplish with introductory algebra, which was trying to connect the content to the students. Using Cohen, Raudenbush, and Ball's instructional triangle framework in Figure 1-1, I focused on the three vertices of the instructional triangle framework: content, students, and teacher. Using Lee's Cultural Modeling Project framework, I expanded each of the vertices in the instructional triangle framework. Thus, Figure 1-2 shows the incorporation of the Lee's Cultural Modeling Project framework within the instructional triangle framework. This enhanced instructional triangle provided a framework to talk about the vertices and the interactions between the vertices.

Figure 1- 2. Pictorial representation for Cultural Modeling Project.



Before the enactments, the English teachers met to choose, remove, or select texts from the cultural data sets best suited to teach the interpretative problem. By selecting texts specific to the needs of their students, the Cultural Modeling Project members reconceptualized the content. During the enactment of teaching, the teacher presents the text and introduces an open-ended question to the class. As she listens to their responses, she asks students to elaborate, provide evidence, and probe the students with other questions. Doing this, the teacher has provided a model to the students of how an expert reader thinks when she analyzes text. During this discussion, students take the role of the teacher and begin modeling the behavior of the expert reader and build upon their internal domain map. In the model above, I model students' behavior similar to the teacher in the instructional triangle. For the teacher, she has the added responsibility of determining how best to react to the students' inputs. Some inputs, counterscripts, may curtail discussion. Other inputs may show errors while some inputs may show students' understanding of these interpretative problems. These decisions made by the teacher are made in the moment of teaching and guided by the internal domain map and her experience as a teacher. Each response made by the teacher created a new instance of the instructional triangle. I applied this enhanced instructional triangle framework to my own teaching.

This dissertation is divided into six chapters. In this chapter, I introduced the instructional triangle as framework to look at teaching, the research question, and a literature review that included the issues surrounding high school algebra, the different conceptualizations of algebra, the algebra content, and a teaching framework.

The second chapter discusses the method I used to analyze the algebra tasks of the district and the tasks I designed for my classroom. I also provide a framework for looking inside the classroom. The question of bias will also be discussed in the second chapter because I am the teacher and researcher of this study. At the end of the chapter, I provide a discussion section.

The third chapter concerns a comparison of the algebra of the textbook and the teacher's approach to the teaching of algebra. The analysis provides some contrasts between the two curricula and shows that my approach isn't merely a reinterpretation of the textbook, but a different way of looking and ordering algebra topics. A part of this research looks at how the major topics for algebra for the first semester were interpreted through a numerical pattern approach. At the end of the chapter, I provide a discussion section.

The fourth chapter takes the algebraic ideas of the third chapter and places them inside the classroom, which is an implementation of theory into practice and provides two perspectives. In this chapter, I provide two analyses. The first analysis looks at the actions the students and I made. This analysis gives an overall impression of the classroom. The second analysis looks at the teaching of algebra for the first semester. Using assessments, student's journals, my personal journal, and enactments, I weave these data collected to show what teaching looks like in the classroom. At the end of the chapter, I provide a discussion section.

The fifth chapter provides a history of my teaching to show how I adapted as a teacher in a difficult teaching environment. I also provide background information of how assessments are analyzed, how lessons are written, and the enactments of lessons. This teaching cycle of assessment-lesson-enactment shows the work required by me in order to teach algebra in the inner-city classroom. In this chapter, I apply the instructional triangle framework to understand my own teaching. At the end of the chapter, I provide a discussion section.

In the last chapter, I return back to the research literature in this chapter to find teaching obstacles inside the research literature and my dissertation. Since the research literature is not content specific, determining these teaching obstacles provides a template for other researchers and teachers trying to improve their instructional practices. In the last section of this chapter, I provide ideas on how to implement a numerical pattern approach into the classroom. At the end of the chapter, I provide a discussion section.

CHAPTER 2

RESEARCH DESIGN AND METHODOLOGY

This chapter describes the methods used to analyze the following research question: What does it take for a teacher to teach algebra for understanding in the inner-city high school?

I further generated the following questions to guide me:

1. How does a numerical pattern approach to algebra compare to district mandated algebra approach using the textbook by Larson?
2. What does the teaching of algebra using this approach look like in the classroom?
3. How do instructional practices using this approach function within the classroom?
4. What does a week of teaching look like using this approach?

In this chapter, I describe the research design, research context, data collection, analysis, and end with a discussion about bias.

Research Design

My overall objective was to understand what it took for me to teach algebra for understanding in an inner-city classroom. How do members of this classroom interact in the process of learning algebra, and what algebra artifacts (e.g. assessment, journals, students' work, and projects) were produced? I selected particular methodological approaches that allowed me to see the classroom environment in the process of learning algebra. Using ethnographic and discourse analytic frameworks, such as intertexuality and intercontextuality, as ways of conceptualizing and thinking about the processes of learning algebra, I was able to view, think, and reflect about how my algebra classroom was progressing.

Over the course of one semester (80 days), I taught and collected 26 enactments, 11 assessments, wrote journal entries, and collected students' work. The classroom contained 35 students. I applied ethnographic methods during the data collection process to reduce researcher bias. An ethnographic research design allows for, "modification in design is a response to local conditions, to factors previously not known, or to new understandings" (Zaharlick & Green, 1991 p. 209). Audio taping, assessments and students' works, and journals were the data collection tools used for documenting the language used, students' work, and my views of the classroom. The concepts of intertextuality and intercontextuality were utilized to analyze the transcripts.

Research Context

Prior to conducting this research, I taught four classes of Algebra 1 and Pre-Calculus and came to think about an approach to algebra for my students at Jefferson High school (JHS) during the academic year 2003-2004. This research is the culmination of four years of modifying this approach to the teaching of algebra.

A way to describe the research context is to provide an example of my frustration with the tardy policy at JHS. During my decade of teaching at JHS, I have never seen a tardy or an absent policy written on paper. Thus at my school, we have created our own local policies specific to a certain part of the building. In 2003, the administration implemented the Smaller Learning Communities (SLC) initiative. As part of the SLC, we were given more autonomy. The 9th grade teachers decided as a group to close our doors at the sound of the tardy bell and those students caught in the hallways would be moved to a detention area and written up. Our policy lasted for less than a week when we were directed by the principal to allow students into the classroom. We realized that we didn't have any autonomy except for what went on inside our classroom.

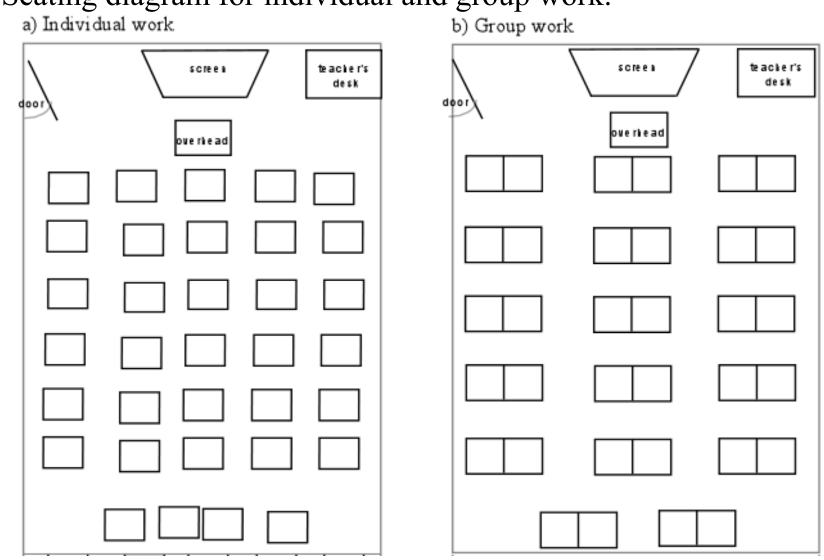
I developed my own routine for tardy students. I allowed late students into the classroom, but they had to wait outside until we had finished the warm-up tasks. This was later modified by the administration into the following: (1) allow the late students into the classroom; (2) they had to sign into the tardy log; and (3) after a certain amount of tardies[1] the student would be sent home. With this policy change, I felt that this was another duty placed upon me and made it harder to create that classroom environment. I presented this example to demonstrate the lack of structure within this school environment.

School

Jefferson High School, a neighborhood high school, is located off a major highway two miles east from the center of a major metropolitan city. JHS is surrounded on three sides by residential homes with the fourth side next to the highway. When I first came to JHS, the student population was just below 1,800. During the academic year 2006-2007, JHS student population dropped below 1,300 (P. Eubanks, Data Presentation, November 8th, 2006). This reduction in student population translated into a reduction of staff and the elimination of "non-essential" classes such as choir and theatre. In the mathematics department, we lost one 9th grade algebra teacher, thereby increasing the average number of 9th graders per class[2]. Even though I had thirty-five students on my roster, the average number of students present ranged from 28 to 32 students.

Although I didn't use a seating chart, after a week, most students remained at their seats of choice. Figure 2-1 presents two desk formations I used in my classroom. For the first week of school, I arranged the desks in row-column format. When we did group work, I pushed two desks together. From week to week, I alternated the desk arrangements depending on our work. There was an overhead in the front of the classroom, which I used often to establish control[3]. During group work, the overhead projector moved to a corner of the classroom to provide more space.

Figure 2- 1. Seating diagram for individual and group work.



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[1] I was never given a specific number of days.

[2] Other mathematics classes had a reduction in the number of students.

[3] Using an overhead allowed me to face the classroom when I taught.

I moved the teacher's desk into the corner and since I never sat at the table, the teacher's desk became a repository for textbooks, tests, and paperwork. There was one worn-out overhead screen in the center front of the room, which I fixed with duct tape to stop from fraying. This was my second year in this room and every attempt at removing the graffiti from the chalkboards and back wall were futile. When I first moved into this room, two windows were bolted shut because in prior years, students snuck out during class and walked on the roof.

Figure 2-2 presents a photograph of the back wall in my classroom. It has a number line and is covered with the students' individual and group work. Because I do not have air conditioning in this classroom, I placed a fan in back of the room and one in the front.

Figure 2- 2. Class photograph in row-column desk arrangement.

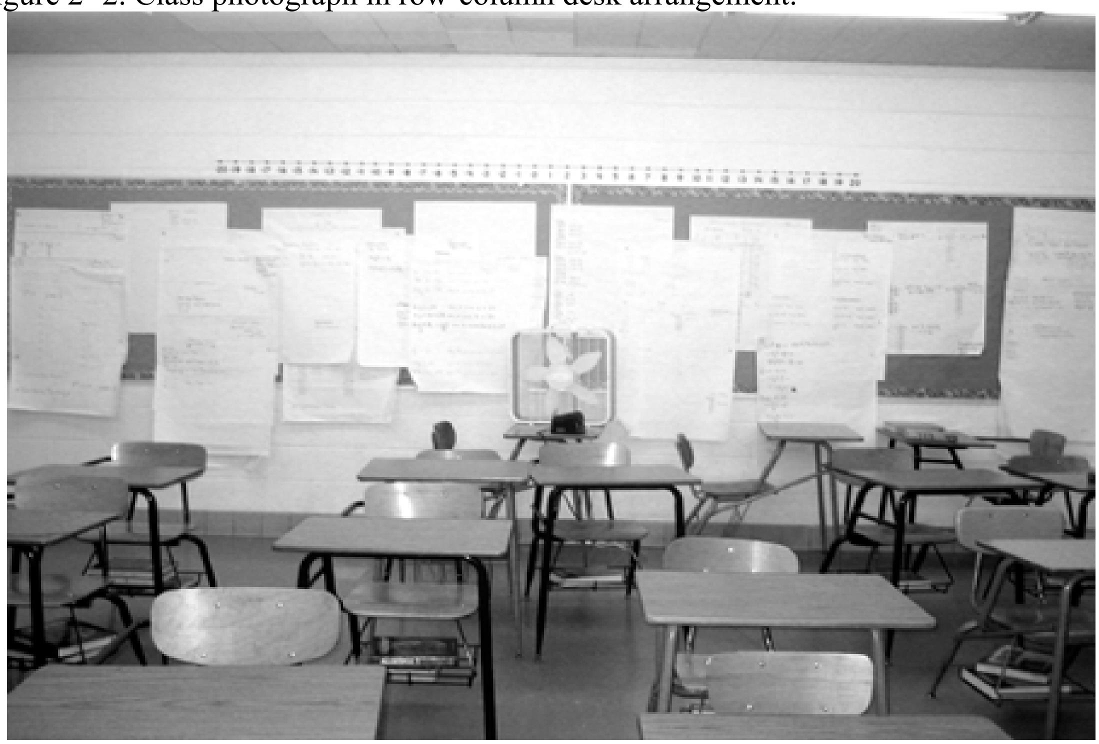


Figure 2-3 presents a side view of the classroom. On the side wall, I have placed another number line. Students used these number lines throughout the semester.

Figure 2- 3. Side view of my classroom.

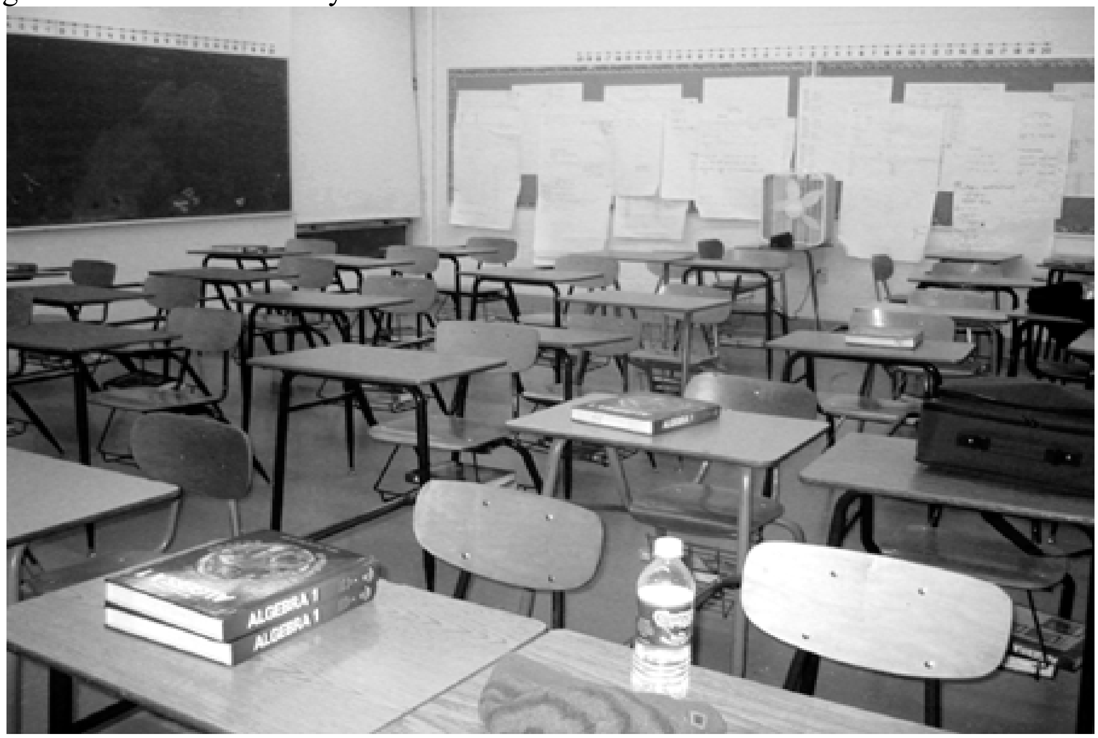


Figure 2-4 is a photograph of my door. The constant pulling by students had finally stripped the threading on the door knob mechanism in November. The engineer didn't have a replacement and had to order another doorknob assembly. In the meantime, a colleague took a plastic bag and made a makeshift doorknob. I received a new doorknob assembly a month later.

Figure 2- 4. Broken door.

This photograph highlights the flexibility and ingenuity needed to work at JHS[4]. Because I had no doorknob, I moved the data projector and computer to another classroom, so they would not be stolen.

Students

The student body in JHS is 99% African-American; 75% of the student body is qualified for free or reduced lunch (Jackson, 1998). Approximately 6% of 2006 graduating seniors in JHS obtained a Michigan endorsement for mathematics (Michigan Department of Education, 2006b). The attendance rate for JHS is 67.2% (Michigan Department of Education, 2006a). The average ACT score for JHS was less than 15 out of 36 (P. Eubanks, Data Presentation, November 8th, 2006), which was below the state average of 19 (Jackson, 1998). During the year of my study, there were a total of 313 ninth graders. On the computerized standardized STAR test for reading, (Renaissance Learning, 2006), 91.1% of the 9th graders read at or below a 6th grade level.

Teacher/Researcher

I came to JHS in the fall of 1993 after a three-year tour with the Peace Corps teaching mathematics in a rural village in Mobaye, Central Africa Republic. During my time in the Peace Corps, I taught algebra, geometry, pre-calculus and calculus. I came away with a deep appreciation of how the French approached the teaching of calculus. I had assumed that everyone learned calculus in the same manner. I took time off in 1998

[4] My colleague and I have fixed the overhead screens and drapes for other teachers in the mathematics department.

to continue my education at a nearby university. I returned back to JHS in 2001 and resigned my teaching position in August 2007. I have been teaching for 13 years.

Teaching Schedule

For the academic year 2006-2007, the year of the study, I taught one Pre-Calculus and four Algebra 1 classes. Because I made changes to the lessons during my lunch hour and further refined them during my 5th hour preparation period, I chose to collect data from 6th hour algebra class. As the last class of the day, all the changes made in the lessons would be incorporated. During the semester, I kept reminding the 6th hour students that they were getting the best lesson of the day. My 6th hour algebra class contained thirty-five students and students were placed into my classroom, if spaces were available, by the counselor. Prior teaching experience had shown me that there is a wide range of mathematics ability within any incoming 9th grade class. As there are three middle schools that feed into JHS, I anticipated a wide range of mathematics experiences due to the different teachers and the mathematics classes taken in the 8th grade.

During 7th hour, students came to do work in the computer lab. During 8th hour, I coached an Academic Games team with a colleague. Table 2-1 lists the periods, times, and names of the classes I taught. I highlighted the class in which I conducted the research.

Table 2- 1. Period, time, and class schedule.

Period	Time	Class
1st	8:15-9:10	Pre-Calculus
2nd	9:15-10:20	Algebra
3rd	10:25-11:20	Algebra
4th	11:25-12:20	Lunch
5th	12:25-1:20	Honors Algebra
6th	1:25-2:20	Algebra
7th	2:25-3:20	Preparation

8th	3:25-4:20	Academic Games

Data Sources

This section describes the type of data collected and the rationale for collecting them. For the first question: How does a numerical pattern approach to algebra compare to district mandated algebra approach using the textbook by Larson? I compared the district mandated curriculum, which uses a textbook authored by Larson, to a numerical pattern approach with a teacher generated task (TGT). By comparing the two curricula, I wanted to show the differences by measuring the algebra standards, cognitive demands, and pacing. I wanted to know if it is possible to use a numerical pattern approach through the first semester of algebra in an inner-city school interweaving the mandated algebra standards in a mathematical sound way. I did this to show why I had relegated Larson as a resource in favor of TGT. In order to compare the two curricula, I collected Larson, curriculum guides, and my enactments to extract the tasks from each curriculum.

For the second question: What does the teaching of algebra using this approach look like in the classroom? I wanted to see what the students and I were doing in the classroom using TGT. I collected the enactments and extracted the conversation between members of this classroom.

For the third question: How do instructional practices using this approach function within the classroom? I wanted to know if it is possible to teach the curriculum in a way that allows inner-city students to question their "known" mathematics, choose appropriate problem solving strategies, and contribute to the building and to the teaching of the numerical pattern approach. I used the enactments to look at the interactions between teacher, students, and content. The assessments and students work illustrate the students' interaction with the content. My personal journal provided an insight into my mental and physical state for that day.

For the fourth question: What is the preparation required by the teacher in order to teach algebra using a numerical pattern approach? I wanted to show what it took to get ready to teach algebra class. I provided a personal journey of how I used the weekly teaching cycle, assessments-lessons-enactments, as a way to provide structure in the class and how I moved through the algebra curriculum. The next section provides further detail of the data sources.

Curriculum Guide and Textbook

The district development team of the Office of Mathematics Education of the district wrote an accompanying curriculum guide (Norde et al., 2006) for our textbook Larson. In this curriculum guide, the development team determined the number of days per lesson and for each one they provided the opening tasks, the tasks for the lesson, the closure task, and the pacing. Using the curriculum guide along with Larson, I extracted the tasks to illustrate what a semester of algebra looks like with Larson.

Enactments

Throughout the semester, I audio taped my teaching. I used a lapel microphone attached to a digital recorder to allow me the freedom to move around the classroom, which was my normal behavior. The data within these enactments contained the tasks used in the lessons and interactions between the members.

Lesson Plans

For the first semester, I wrote weekly lesson plans during the weekend. I collected these lesson plans and placed them in a binder. These lesson plans showed the preparations needed to teach algebra.

Personal Journal

In the course of teaching algebra, the chaos of the school and the obstacles in the teaching of algebra occurred inside the class. I used my lunch hour or 7th hour to write down my thoughts about the school and algebra. The journal was useful in providing a personal account of the classroom, my thoughts on algebra, and my mental state.

Student Work

Throughout the semester, I asked students to do projects, journals, and tasks. Five students allowed me to keep them at the end of the year. These artifacts showed the interactions between the students and content.

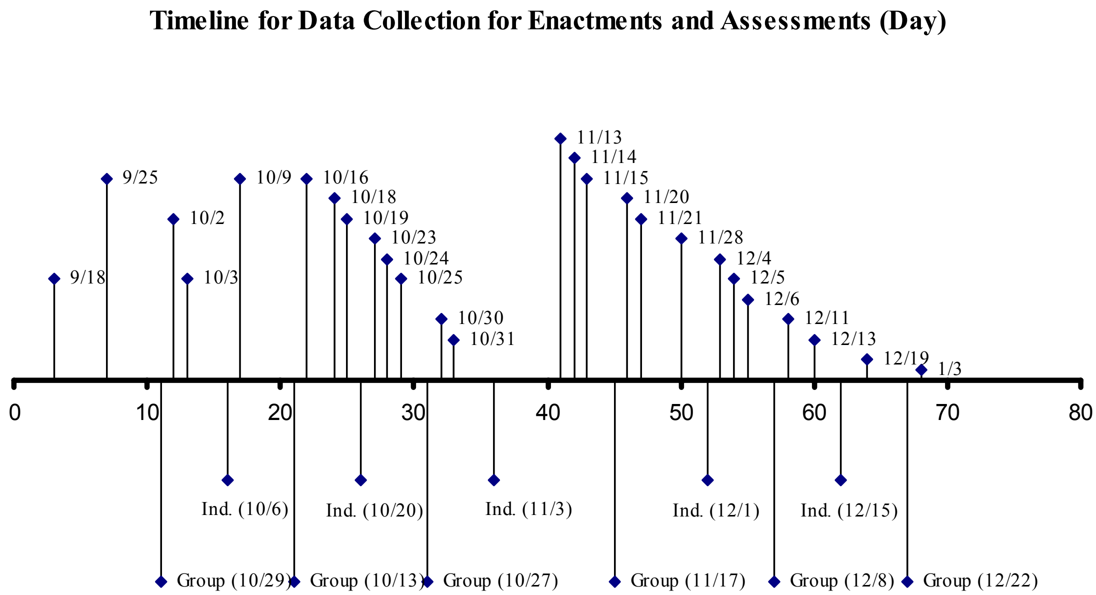
Weekly Assessment

I made photocopies of all of the assessments. These assessments provided a perspective on how students solved tasks. Assessments provided another perspective of the interactions between the students and content.

Data Collection

I collected the student work at the end of the semester and I wrote in my journal. Figure 2-5 presents the dates for the data collection for audio taping of the enactments in the top half of the figure. The bottom half of the Figure 2-5 are the dates for the assessments.

Figure 2- 5. Data collection timeline for enactments and assessments.



Ind. = Individual assessment and Group = Group assessment.

I gave the final exam for algebra on the 70th day; the end of the semester was the 80th day[5]. I collected 26 enactments and had 23 enactments transcribed. I collected 11 assessments. I made sure to collect data for the first day of algebra, the start of algebra topics, and the end of the semester. I collected at least one enactment per week for the first semester in order to capture the range of activities for TGT algebra.

[5] The administration chose the dates for the final exams.

Analysis

Each questions required a different set of analyses. In this section I go into detail how I analyzed the data collected to answer the questions.

Question 1: How does a numerical pattern approach to algebra compare to district mandated algebra approach using the textbook by Larson?

In order to answer this question, I compared the tasks from Larson and TGT to compare the algebra standards, cognitive demands, depth and breadth, and pacing. In the next section, I provide more detail of how I analyzed the two curricula. I do this to show that Larson was not an appropriate algebra curriculum for my students, but I found Larson's tasks useful to build procedural fluency.

Algebra Standards. I consulted the *Mathematics Grade 9 Curriculum Guide* (Norde et al., 2006), the district pacing guide, written by the district development team to choose the tasks for Larson. I used my enactments to extract the tasks for TGT. I used the *Michigan Merit Curriculum: Algebra 1* (Michigan Department of Education, 2006d) and *Mathematics Grade Level Content Expectation* (Michigan Department of Education, 2006c) to code the algebra standards in order to determine how many standards Larson and TGT targeted. Table 2-2 shows the coding schemes for Michigan standards. I coded 983 Larson tasks and 333 TGT tasks.

Table 2- 2. Selected task and Michigan standards.

Task	Michigan Standards
Evaluate the expression. a) $\frac{1}{3}+\frac{1}{3}$ b) $\frac{3}{4}\bullet\frac{1}{3}$ (R. Larson, L. Boswell, T. Kanold, & L. Stiff, 2004a p. 3)	Number and operations N.FL.07.08 Add, subtract, multiply, and divide positive and negative rational numbers fluently
Solve	Expressions and equations A1.2.8 Solve an equation involving several variables

$K = \frac{5}{9}(F - 32) + 273$ for *F*. (Larson et al., 2004a p. 275)	(with numerical or letter coefficients) for designated variable. Justify steps in the solutions.
Find the average speed of a car that traveled 200 miles in 4 hours.	Expressions and equations A1.2.1 Write and solve equations and inequalities with one or two variables to represent mathematical or applied situation.
Evaluate the expression x^4 when $x = 3$.	Expressions and equations A1.1.1 Give a verbal description of an expression that is presented in symbolic form, write an algebraic expression from a verbal description, and evaluate expressions given values of the variables.
The temperature at 6:00 A.M. was 62º F and rose 3º F every hour until 9:00 A.M. Represent the temperature *T* as a function of the number of hours *h*. Write an equation. Make an input-output table. Make a line graph (Larson et al., 2004a p. 49)	Expressions and equations A1.2.1 Write and solve equations and inequalities with one or two variables to represent mathematical or applied situation. Functions A2.4.1 Write the symbolic forms of linear functions (standard [i.e., $Ax + By = C$, where $B \neq 0$], point-slope, and slope-intercept) given appropriate information and convert between forms. Reasoning about systems L1.2.4 Organize and summarize a data set in a table, plot, chart, or spreadsheet; find patterns in a display of data; understand and critique data displays in the media. Algebra A.PA.07.06 Calculate the slope from the graph of a linear function as the ratio of "rise/run" for a pair of points on the graph, and express the answer as a fraction and a decimal; understand that the linear functions have slope that is a constant rate of change.

This table shows that some tasks contained multiple standards.

<u>Cognitive Demands</u>. I used Stein, Smith, Henningsen, and Silver's (2000) *Mathematical Task Analysis Guide* to determine the cognitive demand of the tasks. Table 2-3 lists the complete description of Stein's Task Analysis Guide (Stein et al., 2000).

Table 2- 3.Task Analysis Guide (Stein et al., 2000 p. 16)

Lower-Level Demands	Higher-Level Demands
Memorization Tasks	Procedures with Connection Tasks (PWC)
Involve reproducing previously learned facts, rules, formulae, or definition OR committing facts, rules, formulae, or definition to memory. Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. Are not ambiguous – such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated. Have no connection to the concepts or meaning that underlies the facts, rules, formulae, or definitions being learned or reproduced.	Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding or mathematical concepts and ideas. Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. Usually are represented ways (e.g., visual diagram, manipulative, symbols, and problem situations). Making connections among multiple representations helps to develop meaning. Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.
Procedures Without Connections Tasks (PWOC)	Doing Mathematical Tasks
Are algorithmic. Use of the procedure in either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it.	Require complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example). Require students to explore and understand the nature of mathematical concepts, processes, or relationships.

Have no connection to the concepts or meaning that underlies the procedure being used. Are focused on producing correct answers rather than developing mathematical understanding. Require no explanations, or explanations that focus solely on describing the procedure that was used.	Demand self-monitoring or self-regulation of one's own cognitive processes. Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task. Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. Require considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

I wanted to see Larson's and TGT's mathematical expectation for the students and wanted to see how Larson and TGT introduced each chapter. Table 2-4 shows the coding scheme for cognitive demands. I have included expected student responses. According to this scheme, a lower-level demand task does not require much time or cognitive effort to solve it. A higher-level demand task takes more time and requires more cognitive effort to solve.

Table 2- 4. Coding schemes for cognitive demands.

Lower-Level Demands	Higher-Level Demands
Memorization Evaluate the expression $\frac{1}{3}+\frac{1}{3}$ Expected student response: $\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$ (Larson et al., 2004a p. 3)	Procedures with connections (PWC) A plumber charges a basic service fee plus a labor charge for each hour of service. A 2-hour job costs \$120 and 4-hour job costs \$180. Find the plumber's basic service fee. (Larson et al., 2004a p. 31) Expected student response: Verbal model: $\text{Cost} = \text{rate} \bullet \text{time} + \text{basic fee}$ Guess and Check

	Rate = \$60 and basic fee = \$0 $120 = 2 \bullet 60 + 0$ $180 \neq 4 \bullet 60 + 0$ Rate = \$50 and basic fee = \$20 $120 = 2 \bullet 50 + 20$ $180 \neq 4 \bullet 50 + 20$ Keep decreasing the rate while increasing the basic fee until both algebraic statements are true.
Procedure without connections (PWOC) Find the average speed of a car that traveled 200 miles in 4 hours. Expected student response: $\text{Average speed} = \frac{\text{distance}}{\text{time}}$ $\text{Average speed} = \frac{200\text{ miles}}{4\text{ hours}} =$ (R. Larson, L. Boswell, T. D. Kanold, & L. Stiff, 2004b p. 2)	Doing mathematics A rectangular pool is to be surrounded by a ceramic-tile border. The border will be one tile wide all around. Explain in words, with numbers or tables, visually, and with symbols the number of tiles that will be needed for pools of various lengths and widths. (National Council of Teachers of Mathematics, 2000 p. 282) Possible student responses: I drew several pictures and saw this pattern. You need *L+2* tiles across the top and the same number across the bottom. And you also need *W* tiles on the left and *W* tiles on the right. So altogether, the number of tiles needed is $T = 2(L+2) + 2w$ (National Council of Teachers of Mathematics, 2000 p. 283) I pictured it in my head. First, place one tile at each of the corner of the pool. Then you just need *L* tiles across the top and the bottom, and *W* tiles along each of the sides. So al together, the number of tiles needed is $4 + 2l + 2w$. (National Council of Teachers of Mathematics, 2000 p. 283)

I determined the cognitive demand of a task by first reading the task directions. If the students were asked to follow to a procedure, then I coded the task as PWOC task. If the directions lead students to an underlying algebraic structure, then I coded the task as PWC. Table 2-5 gives examples of the coding used. Except for doing mathematics tasks, all of these tasks were taken from my work or from Larson. I picked a variety of different tasks and provided comments to show how I coded each task.

Table 2- 5. Categorization, tasks, and comments.

Categorization	Tasks	Comments
Memorization	Evaluate a) 2-4 b) 19-12	• Requires little time to solve • Learned facts • Requires limited cognitive effort
PWOC	Solve $K = \frac{5}{9}(F - 32) + 273$ for *F*. (Larson et al., 2004a p. 275)	• Limited cognitive effort • Focus on producing correct answer. • No connections.
PWOC	Evaluate the expression x^4 when $x = 3$.	• Limited cognitive effort • Focus on producing correct answer. • No connections.
PWC	The temperature at 6:00 A.M. was 62º F and rose 3º F every hour until 9:00 A.M. Represent the temperature *T* as a function of the number of hours *h*. Write an equation. Make an input-output table. Make a line graph (Larson et al., 2004a p. 49)	• Focus on procedures to help understand concepts • Multiple approaches • Task instructions guide the students
PWC	**Fundraising** The science club is selling magazine subscriptions at $12 each. How many subscriptions does the club have to sell to raise $276? Use the problem solving plan to answer the question. (Larson et al., 2004a p. 13)	• Focus on procedures to help understand concepts • Multiple approaches

Doing Mathematics	A rectangular pool is to be surrounded by a ceramic-tile border. The border will be one tile wide all around. Explain in words, with numbers or tables, visually, and with symbols the number of tiles that will be needed for pools of various lengths and widths. (National Council of Teachers of Mathematics, 2000 p. 282)	• Multiple approaches • Requires a lot of cognitive effort • Task instruction doesn't help solve the task

Breadth and Depth. Looking at the cognitive demands and the algebraic content allowed me to produce a coarse definition for breadth and depth for algebra. Determining the breadth and depth of the two curricula provided another tool to analyze the two curricula. As a teacher, I used the first semester of algebra as the study of linear functions. I used the second semester as the study of quadratics of algebra. Thus for breadth, I chose from the National Mathematics Advisory Panel's (2008) list of major algebra topics and selected those topics that were relevant to linear functions, which are listed below in Table 2-6.

Table 2- 6. List of algebra topics for first semester of algebra.

My Algebra Topics for Algebra 1
Symbols and Expressions • Arithmetic series **Linear Equations** • Real numbers as points on the number line • Linear equations and their graphs • Solving problems with linear equations • Linear inequalities and their graphs • Graphing and solving systems of simultaneous linear equations **Functions** • Linear functions • Simple nonlinear functions (e.g., absolute value; step functions)

Algebra of Polynomials
• Fundamental theorem of algebra

In order to quantify depth for a standard, I looked at the standards contained in each task and grouped those standards by cognitive demands. For example, this task below:

> The temperature at 6:00 A.M. was 62º F and rose 3º F every hour until 9:00 A.M. Represent the temperature *T* as a function of the number of hours *h*.
> a) Write an equation.
> b) Make an input-output table.
> c) Make a line graph (Larson et al., 2004a p. 49)

This is a PWC task and contains four Michigan standards (refer to Table 2-2). I did this for all of Larson's and TGT's tasks. In order to determine depth for a standard, my rough gauge was to compare the number of PWC and PWOC tasks for that standard. If the number of PWC tasks was greater or equal to the number of PWOC tasks for a particular standard, then the standard had depth.

<u>Pacing</u>. To determine the pacing for Larson, I consulted the curriculum pacing guide to determine the start for each chapter. For TGT, I went back to the enactments and found the start date that corresponded to Larson. I also wanted to show how Larson and TGT introduced each chapter and how each chapter connected together. I did this by using the curriculum guide to select the introductory task for Larson and did the same for TGT.

<u>Intercoder Reliability</u>. To establish intercoder reliability for the cognitive demands and algebra standard, I gave a colleague Stein, Smith, Henningsen, and Silver's

Task Analysis Guide (2000) and MDE standards (Michigan Department of Education, 2006d) (Michigan Department of Education, 2006c) and we coded 10 tasks in the column on page 57 of Larson. We discussed how we coded each task for cognitive demand and content. I gave him a total of 30 tasks with equal number of Larson and my tasks in three sets of 10 tasks.

After my colleague finished, we compared our results for the cognitive demands. In order to match, we needed to have the same coding. For the cognitive demands, we agreed 80% for all 30 tasks. For algebra content, my colleague and I compared the MDE standards and discussed how we arrived at our decision. In order to match, all of MDE standards had to be same. For the first ten tasks, we matched 70% for the first set, 90% for the second set, and 80% for the last set.

Question 2: What does the teaching of algebra using this approach look like in the classroom?

To determine the classroom environment, I looked at the teacher's and students' conversations. I used a constant comparative analysis (Patton, 1990) to look at the conversations in the 23 enactments[6]. Using this approach, I read the transcripts of the enactments, coded the actions, re-coded the actions, developed themes, developed categories, and formed conclusions (Hewitt-Taylor, 2001) (Patton, 1990) about my and my students' actions. The next section goes into further detail of how I came up with these codes.

[6] I have 26 enactments, but used only the 23 transcribed enactments. I did listen to all 26 enactments of teaching.

Initial Categories. Before I began coding, I generated categories that I would like to look at when I coded the transcripts. Table 2-7 lists the initial categories that might occur inside the classroom.

Table 2- 7. List of original categories.

Categories	
Student actions • Bathroom passes • Not having supplies • Fights • Arguments with teacher/students • Tardiness • Talking • Help	School • Announcements • New students • Observations • Fire alarms • Assemblies • Guest speaker • Removing students
Teacher • Explaining • Transitioning • Disciplining • Arguing • Enforcing policy • Broken equipment • Lack of resources	

This table lists possible actions that occurred inside the classroom. Using these categories, I coded the September 25th transcription and generated another set of codes that looked at the mathematics and the non-mathematical events that occurred in the classroom. Table 2-8 lists the categories and the codes along with a brief description.

Table 2- 8. Second set of categories and codes.

Categories	
Mathematics • Algebraic talking: Any discussion between the students and me and between students, about algebra. This could include how to solve a task. I might break this coding into smaller categories. • Arithmetic talking: similar to algebraic talking, this is reserved	Classroom: • Bathroom: Any comments made about bathroom and bathroom passes. • Fights: Any comments between teacher and students and between students during a fight. • Lack of attention: Any comments not associated with mathematics.

for discussion about arithmetic (e.g. order of operations). • Student explanation: I wanted a way to capture when students were at the overhead explaining their steps.	• Lack of understanding: Any comments asking for clarifications, further explanations, or simply, “I don’t get it.” • School policies: Any comments during the class in which I have no control (i.e., new students, tardiness, dress code, and announcements). • Lack of supplies: Any comments about paper, pencil, and pens. • Maintaining control: Any comments made by the teacher to help keep the control the classroom (e.g., sending students out, calling on students, transition, feedback on student work, sending student up to the board). • On task: Any comments made by the students that isn’t about the mathematics but attitudes toward mathematics (e.g., I hate this class, I get it). • Rule: Any discussion about my expectations for class (e.g., how they should keep notes, asking to explain, telling students to behave)

Using this table, I narrowed my focus to mathematics and those events not associated with the mathematics. I divided the mathematical episodes into algebra and arithmetic, depending on the subject matter. A mathematical discussion begins with a task and a discussion about the solving of the task and ends with a transition another task or activity. In class, I also gave students a task and walked around helping individual students. This would not be classified as a mathematical discussion. Teacher’s and students’ actions can occur in mathematical discussions and in between mathematical discussions. After coding the October 23rd, I kept the same codes for the mathematics, but made changes to the classroom codes:

- Off task (Lack of Attention): Initially I thought this would include only when students were talking amongst themselves and not about mathematics. But after Oct. 23rd, I was also off-task when asking students about Homecoming. I might need to group bathroom, computer lab, and fights into this coding, because they occurred but at a very small percentage of the class. Initially, this code was lack of attention, but I found it too restrictive.
- Teacher moves (maintaining control): Any moves by the teacher to move the class along. This includes telling students to be quiet, moving to another task, and transitioning to another activity.
- Teacher explanation: Teacher talks with little input from the students. May need to move it to Algebraic discussion.

I coded four transcripts at one month intervals. Table 2-9 lists the codes and the percentages for each code for the top four categories.

Table 2- 9. Codes, dates, and percentages for top four codes.

Codes	September 25th (Day 7)	October 23rd (Day 27)	November 20th (Day 46)	December 19th (Day 65)
Algebraic discussion	58.2%	61%	58.9%	73.0%
Moves made by teacher	19.7%	8.9 %	18.2%	16.4%
Off-task	1.7%	19.3%	9.8%	4.0%
Lack of understanding	3.0%	1.8%	0.8%	1.1%

This table shows that the code used were able to capture the mathematics along with other actions in the enactments. I consulted with an experienced Nvivo user and she told me the percentages the computer generated were the area of text coverage for a transcript. I wanted a more precise tool to look at the actions made by members of this classroom. Doing this allowed me to code the following, "So 2 plus 3, John turn around, is 5," as two actions. I made the following changes below before my next set of coding:

- Algebraic discussion: At the start of an algebraic discussion, the teacher provides the task and ends either with questions by students or task instructions to another task. By using these endpoints, second level coding will unpack what algebraic discussion looks like.
- On-task: This code is specific to students and how the students interact with the tasks.

- Off-task: This code concerns when the discussion in no longer about the mathematics.
- Teacher actions: These codes are specific to the teacher and are actions made by the teacher within the classroom.
- Procedures and Rules: Discussions about the grades for assessments, homework, and handing in of work.

Final Set of Codes. I separated my actions and those made by the students. I grouped the teacher's actions into mathematical content and management. For the students, I grouped the students' actions into on-task, when they worked on the mathematics, and off-task, when students were not working on the mathematics. Table 2-10 lists the final set of content categories for teacher, a brief discussion, and examples.

Table 2- 10. Teacher's categories for content.

Teacher	
Content: These teacher actions are about the mathematics.	
Categories	Description and example
Clarify/ Explain (C/E)	Teacher restates or clarifies the objective of the task. T: All right. I said that there are how many marbles that box b has than box a? T: It doesn't matter if it's odd or even as long as addition you have to do… T: 6 times evenly, right? Drop the 3. 2 goes into 3?
Direct / Ask (D/A)	Teacher directs the students to focus on a specific part of the task. This direction can be done through asking also. T: Box b. Ok. So it's 3 to? T: 6 go into 9 how many times? T: Wait, wait, 18. And how many teens, three teens. I can also multiply by?
Gauge (G)	Teacher checks to see if students understand the material. T: Did you see how she got that, Ashley? T: Yeah, what did you get? 7/6, yeah that's fine. You can leave it as 7/6. T: What are you stuck on, Breanna? At least write, what are you supposed to write, 1/6?
Verify/ Praise (V/P)	Teacher praises students when they are stuck on a part of the task. T: Yeah you're doing it right because that's the way to do it.

	Then divide by? The other number, yeah. T: 296. (pause) Thank you, you're done, that's it? T: That's correct.

Table 2-11 presents the final set of categories for management. I have included the categories, descriptions, and examples.

Table 2- 11. Teacher's categories for management.

Management: These teacher actions bring the focus back toward the mathematics.	
Categories	Description and examples
Delegate (De)	The teacher delegates different tasks to the students. T: Pass it up to Christian, do you want to grade today, Christian? T: Can you get set up, number seven. And I want either Shamika or LaTreace to go up on the next one. Jayvon, number seven. Get it, set it up. Number seven.
Discipline (Di)	The teacher tells the students to get back to the work. T: 4, 6, 8. All right, come on Jayvon and Mickey. T: Shhh, Toshel. T: Ok. Nolan quiet down please.
Procedures and Rules (P-R)	These are processes such as giving students the homework and handing back the assessments. T: Homework. F: I wish it was computer work. T: Page 329, 21-23. Page 199, 21 and 22. How many people don't have books to take home? I see one, two, three, four. When you're in the computer lab, take that home, that's what I'm telling you, take it home. All right, and you also have a journal tonight. And you know these are worth twenty points, how do you find the algebraic for blank, 3, blank, blank (pause). So maybe we'll do On Sets today, Sydney?
Teacher-student (T-S)	Teacher builds rapport with the students through conversations not about the mathematics. T: When are you guys going to perform, Thursday? F: Friday. T: You're going to be gone tomorrow or is it Wednesday? F: Wednesday. T: Ok, ok, whatever you guys worked out, I don't remember.

	Ashley, how did you do at the tournament? F: I did good.
Transition (Tr)	The teacher signals the start of another task. This tries to get the students ready for the task. T: All right, last one, you ready, April? T: Let's give you another one. T: All right, grab your book, this one here.

I divided the students' actions into on-task and off-task. Table 2-12 lists the last of set of categories to students' on-tasks actions. I have included the categories, descriptions, and examples.

Table 2- 12. Student's categories for on-task.

Students	
On-task: Student speech acts that is about the mathematics.	
Categories	Description and example
Comment (C)	Student commenting on the task. F: So you divided by 4 and each of the boxes has 12. And… Several: 12. F: No it's just subtract 3/6.
Help (H)	Student expressing a lack of understanding or asking for assistance. F: I don't get it. F: You lost me. F: What do I do?
Satisfaction (S)	Student expressing their understanding of the task. F: I figured that out. F: Mr. Pan, I did it. F: Oh I did that all by myself.
Student moves (SM)	Student's acts that help the teaching of algebra. F: Can I help him out? M: How about number seven?
Time (T)	Student asking for more time for the task. F: Wait a minute. F: I'm almost ready. F: Oh so wait a minute Mr. Pan, I didn't know we had to add them.

Table 2-13 lists final set of categories for students' off-task moves. I have included the categories, descriptions, and examples.

Table 2- 13. Student's categories for off-task.

Students	
Off-task: Student speech acts that isn't about the mathematics.	
Categories	Description and examples
Fight (F)	Students fighting. T: Hey, hey, hey, right here, right here. Milton, right over here, right over here. Milton, right over here. You're going to walk; you're going to get in worse trouble. Right here. I have to take care of the other side. Come on don't, don't it's not worth, it's not worth it. It's not worth it. It's not worth it. You got me? It's not worth it. It's not worth it. Sit down. Sit down.)
Places (P)	Student asking to go to the bathroom or computer lab. F: Mr. Pan, can I go to the bathroom? F: I've got to pee real, real bad. M: We're going to the computer room today?
Remarks (R)	Student commenting on a task that isn't about the mathematics. This includes student-student and student-teacher comments. F: I've got a headache. F: Don't be calling me out in the room, I hate that. F: Don't say ooh like you don't want to come over here.
Supplies (Su)	Student expressing a need for supplies (e.g., pens, pencil, and paper). F: Oh man, I don't have any paper. F: We need the book. F: I need some whiteout.

Teacher's and Students' Actions. For the first analysis, I coded 13,799 actions and looked at what teacher and students did inside the classroom. I wanted to be able to describe the classroom environment.

Mathematical Discussions. For the second analysis, I divided the discussion into mathematical discussion when we worked on tasks and in-between mathematical discussion when I gave students time to work on tasks. I looked at my actions and students' actions when we worked on tasks and when I gave them time to work on tasks. This second analysis provided more descriptors for the classroom environment.

"Bad Class." During data collection, I had one bad class where I had no control of the classroom. I presented this enactment to show that the classroom environment can become chaotic without any provocation and to show what it was like for me to teach at JHS when I arrived in 1993.

Intercoder Reliability. Once the codes for students' and teacher's actions were established, I asked a colleague to use these codes to code a small transcript in Figure 2-6. When we compared students' and teacher's codes for the excerpt, our responses matched 70%. We both found it easier to code the students' actions, because their utterances were usually a single action. Teacher's utterances often contained more than one action and we argued the difference between direct/ask and clarify/explain. We decided that direct/ask was used to push students toward the task or procedure (line 111). Clarify/explain was used to provide more information to the students, which is highlighted in line 113. I coded the 23 transcripts and all audible utterances by these categories (line 167). Figure 2-6 presents task, interactions, and the codes used for an algebraic discussion.

Figure 2- 6. Task, transcription, and codes for algebraic discussion.

Task			
Given two linear sequences -10, -8, -6, -4, -2, … and 4, 7, 10, 13, 16,… a) When and where will the pattern meet?			
Line number	Speaker	Statements	Codes
111	T	Just let me finish my lesson. All right, so -10, -8, -6…	Di, (D/A)
112	F	Minus?	C
113	T	Or minus, negative, my fault, negative. You're right. And the other sequence is 4, 7, oh 10, 13 and 16. I want you to find when and where they're going to meet, when and	(C/E) (D/A)

		where the sequences will meet. So go ahead and you can do it the Jefferson way, which is to write all the numbers out.	
114	F	So we go from 10 up to…?	C
115	T	So you look at your number line. Are you going to the right or to the left?	(D/A)
116	F	To the right.	C
117	T	So the numbers getting bigger or smaller?	(D/A)
118	F	Getting smaller.	C
119	F	No it's getting bigger.	C
120	T	So it should be positive, right. Yeah do the algebraic. Yeah.	(V/P), (D/A)
121	F	So that would be 0, and that would be…	C
122	T	You're doing which way? You're going to do the algebraic way? Ok, give me the algebraic way for this one?	(C/E), (D/A)
123	M	$n = 2t - 12$.	C
124	F	-12	C
125	T	Right? And the second one, Breanna?	(D/A), G
126	F	$3t$…	C
127	T	Breanna, ok. Combining her name with another. Breanna, what'd you get, girl?	(D/A), G
128	F	(pause)	
129	F	(whisper) Amber, $3t - 12$.	C.
130	T	Breanna, you need some help, girl. All right, so what do we next, Antonia. I like saying the extra name. I love saying all those. What do we do? Kiara, or Jayvon?	G, (D/A)
131	F	$3t - 12 = 3t +$ …	C
132	T	Very good.	(V/P)
133	F	What?	H
134	T	Now. We have variables on both sides.	(V/P)
135	F	Hold on, I did it a different way, Mr. Pan.	C
136	T	You have variables on both sides. Always try to move the smaller set of variables.	(C/E)
137	F	Wait, I got it a different way.	C
138	T	You set it the other way, it doesn't matter. All right, why do we always subtract or get rid of the smaller amount of variables?	(C/E)
139	F	Because they're smaller.	C
140	F	I don't know.	H
141	M	To make it easy.	C
142	T	Easier because they, the variables will be positive. Ok. So every step I do that's the main reason. I don't want to work with negative numbers, I want to work with?	(C/E)
143	F	Positive.	C
144	T	Positive numbers. So I have $2t$ and $3t$, which one am I going to get rid of?	(C/E), (D/A)
145	Several	$2t$.	C
146	T	How do I remove a $2t$?	(D/A)
147	Several	Subtract.	C
148	T	On this side and? This side. And then what happens? Cross this?	(D/A)
149	F	Out, so it will be canceled.	C
150	T	And I get $1t + 1 = ?$	(D/A)
151	F	12	C
152	T	-12. And how do I do the last step?	(V/P),

			(D/A)
153	F	$t - 1$?	C
154	T	-1, -1, and t = ?	(D/A)
155	F	-13	C
156	T	So are we going to find it going the positive direction, do we actually, or do we have to go to the negative direction?	(C/E), (D/A)
157	F	Mr. Pan, I'm like…	H
158	T	What do you get confused on?	G
159	F	It's so long. (another)	C
160	F	Because I do, when you…	C
161	F	You take that step you bring the 12 down.	C
162	T	Yeah because you got rid of this, right?	(C/E)
163	F	Um hum.	C
164	T	Just bring everything down.	(C/E)
165	F	You got that 12.	C
166	T	That's equal.	(D/A)
167	(many talking at once)		
168	F	I got the answer.	S
169	F	First the 1, then you brought, you take 1 from both sides and you got -13. You can see.	C
170	F	I can't see nothing.	H
171	F	It's easy.	S
172	T	All right, grab the little red book, the one that, this one here.	T

(C/E)= Clarify/Explain, C= comments, Di = Discipline, (D/A)= Direct/ Ask, H= Help, S= Satisfaction, and (V/P)= Verify/Praise

Question 3: How do instructional practices using this approach function within the classroom?

I used my framework for Lee's Cultural Modeling Project of figure 1-2 to guide my analysis of teaching. In this framework, the students' inputs are composed of uptakes/errors and counterscripts. As the teacher, I take in these students' inputs and determine how this maps into my internal domain map for algebra. Students in turn are building their own internal domain map of algebra.

The transcripts of my classes are not simply individual snapshots of my algebra classroom; instead, they are connected to other transcripts by the language used and the procedures learned (Santa Barbara Classroom Discourse Group, 1992a, 1992b). An example of intertextuality and intercontextuality occurred on the 41st day of teaching. I had lost control of the class and students were getting quite angry with me for not explaining how to do the task. Through this noise, a student spoke up[7]:

185 Kiara : As much as we have been going through this, you all telling me you can't do this? Anyway for the next one… Because it's jumping by four and if you all can't tell, there is something wrong with that

The first part of her utterance is an example of intercontextuality, when she reminded the class that they had done this before. The last part of her utterance is an example of intertextuality, where the term jump connects this task to past and future tasks.

This intertextuality, implies that "Any text (oral or written) is infected with the meanings (at least, as potential) of all other texts in which its word have comported" (Gee, 1999 p. 54). Likewise, my transcripts are inter-contextual, they refer "not only to previous texts, but to the social situation in and through which a text was constructed. That is, prior contexts may be interactionally invoked in the local text being constructed" (Floriani, 1994 p. 257). This is also an example of a counterscript, where her response was unexpected by me.

I looked at the transcripts for all the 23 enactments, listened to the audio tapes, re-read my journal entries, looked at students' works, and wrote a description that connected the data collected for that enactment. Figure 2-7 presents a description for 13th day of teaching that illustrates our work with translating verbal expressions with inequalities. As

[7] Pseudonyms are used in all transcripts and reflect the gender and ethnicity of the students.

the researcher, I chose these excerpts or tasks to highlight students' difficulties, approach, prior knowledge, progression, success, and my own perspective of the classroom.

Figure 2- 7. Description for the 13th day of teaching.

Day 13: Last day of Chapter 1
I was still replaying yesterday's lesson and I wrote the following in my journal:

> So how does > get related to greater than? How do I teach the statements: 9 less than *x* and 9 is less than *x*? I felt I was unprepared to teach this piece of mathematics. Students were talking about Pac-Man, open-mouth and close-mouth, which was something new to me.
>
> Mentally I am tired. I wish I could get more sleep. I haven't been able to get the Carnegie to work yet. The 2nd and 3rd hour classes are slowly getting out of hand. Journal Entry: October 3rd, 2006

This past Saturday, Ron, the English 9th grade teacher, and I took our Academic Games[8] students to the first tournament. Instead of resting on the weekend, our Saturday was a work day that began at 6 am and ended around 2 pm. My lesson plans usually suffered for the week along with my ability to maintain control of the classrooms. As a way of building our Academic Games team, we volunteered to teach this as our sixth class where students could receive credits toward graduation[9]. Why do we do this? By working with this group of about 15 students, it gives us a better perspective of our students' lives and balances the difficulties in the classroom.

Why didn't I teach the other topics such as the function notation and the absolute value as part of chapter 1? I would incorporate these algebra topics in later chapters, but as of the 13th day of instruction I had no idea of when. The topic translating verbal expressions would have been done in the first chapter.

Checking for homework

I assigned homework Monday thru Wednesday every week. When students arrive in the classroom, the algebra task is already on the overhead. They take out their homework and during these five minutes, I quickly checked their homework. I would do a more thorough check of the homework during the Friday assessment.

Table 4-1 is the interactions that occur during the first five minutes of class.

Table 4-1. Going over the homework.

Line number	Speaker	Statements
2	Margaret	Yeah, I got my homework.
3	Teacher	Homework, Marcel? Homework?
4	Marcel	I wasn't here yesterday.
5	Teacher	(inaudible) Toshel. Get started on the work today. Jayvon. Ashley?
6	Ashley	Hold on, I've got (inaudible).
7	Teacher	Three minutes to go. You guys have to got to stop coming in late, please. I'm starting to notice this pattern here. Shh. Kwamika, Christian, work please. Guys, you don't have to write the question down, but show me how to find the answer. Ok?
8	Kiara	Mr. Pan, I'm looking for it.
9	Teacher	Shamika?
10	Shamika	I'm looking for it.
11	Teacher	Got anything, Shamika?
12	Shamika	I left mine at (inaudible), I don't got it.
13	Tiara	I'm looking.
14	Teacher	Alonzo?
15	Alonzo	I've got to find it.
16	Teacher	Margaret? This looks good actually. I think that's the right way to do it. Three minutes to go.

As I walked around the classroom, Margaret has her homework (line 2). Others are still trying to get settle (line 5). These students traveled together during the day; therefore students really have no excuse for being late and I am more angered by students' tardiness than lack of homework (line 7).

I see the tardy issue as the more pressing problem. If I don't tackle this problem head on within my classroom, then it will continue to grow. The homework issue will take time. As students become more confident in their mathematics, I would expect to see more homework. I don't get too stressed for the lack of homework.

The task on the overhead is in Table 4-2. I included the Michigan standard and cognitive demand along with the task.

Table 4-2. Task, Michigan Standard, and cognitive demand.

Task	Michigan standard and cognitive demand
Given the following table for the state fair, find the following:	

Tickets	Price
Seniors ($\text{age} \geq 65$)	\$6.50
Adults	\$9.00
Teens (age < 18)	\$5.50

1) Find the price for 2 seniors, 2 adults, and 3 teens
2) Find the price 1 adult and 5 teens
3) If you have \$50, how many teens go to the state fair?
4) What is the algebraic representation for the cost for the number of teens?

L1.2.4 Organize and summarize a data set in a table, plot, chart, or spreadsheet; find patterns in a display of data; **understand** and critique **data** displays **in the media**. (Michigan Department of Education, 2006d p. 8)

This is a PWOC task with limited cognitive effort.

This task was similar to yesterday's task (refer to Figure 4-36) except that I included decimals. As I was walking around the classroom, Ashley and I exchanged the following conversation in Table 4-3.

Table 4- 1. Ashley and I discussing the task.

Line number	Speaker	Statements
54	Ashley	Mr. Pan, how are you supposed to do number three?
55	Teacher	If you had \$50, how many teens can you take? (inaudible response) Well think, you have teens cost how much? Figure out how many you can take. (talking in background)

I may have told her too much information (line 55). I continued monitoring the class for another three minutes.
Solving equation by trial and error

The last time we encountered a task like this (refer to Figure 4-12), I showed them how to solve by plugging in different values. Students had no problem with the first two parts of the task even though I included the decimal into the task. In Figure 4-1, was the interaction that occurred when we solve part 3) of the task. I included the work of one student.

Figure 4- 1. Solving the task.

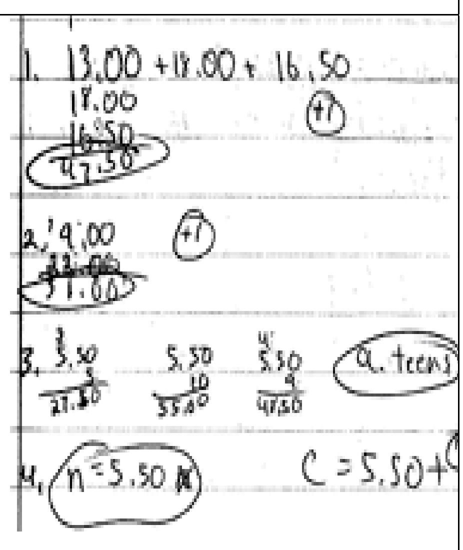
Task

Given the following table for the state fair, find the following:

Ticket	Price
Senior ($\text{age} \geq 65$)	\$6.50
Adult	\$9.00
Teen (age < 18)	\$5.50

1) Find the price for 2 seniors, 2 adults, and 3 teens
2) Find the price 1 adult and 5 teens
3) If you have \$50, how many teens go to the state fair?
4) What is the algebraic representation for the cost for the number of teens?

Ashley's work



Line number	Speaker	Statements
86	Teacher	How many teens can you take if you have \$50?
87	Several	Nine
88	Teacher	Nine. How did we get that answer?
89	Ashley	Multiply 5.50 times 9.
90	Kiara	Nine times 5.50

For this situation above, several students came up with the response that was closest to \$50. For Ashley, the approach for part 3) of the task was to try different values for the number of teens. Ashley tried five, ten, and then ten students, but circled nine as the best response. If this task had been a numerical pattern, I would have expected students to respond no solution for part 3) and I wasn't quite sure students could different between a numerical pattern and a contextual situation. For the algebraic representation, Ashley had written $n = 5.50x$, while I had written $c = 5.50t$. Even thought both answer looked different, she kept her answer (the circled one) by giving herself two points.

Interruptions

Once again, we were transitioning from the overhead to the textbook. I had thought that my direction about the page number was fairly clear yesterday. Table 4-4 was what occurred as I tried to move toward the correction of the homework.

Table 4- 2. Getting started in the book.

Line number	Speaker	Statements
119	Teacher	All right. Grab your book, turn to page 36. We have 30, 32, and 34? Am I right?
120	Christian	Yeah. First we had 14, 16, 18.
121	Teacher	No those were the easy ones, right? I don't have to go over those. The harder ones were…(intercom: Can I get Natasha to come to the office for a moment?) Okay. We'll do 32. Let's go to the 32 first.
122	Ashley	What page?
123	Teacher	Page 36, number 32.
124	Christian	These ones I don't have a problem with, I don't know why.
		(pause-talking in background)
125	Female	Where's my book?
126	Kiara	What page?

I've learned to write the page and task number on the overhead (line 119). A textbook was always beneath the student's desk. I repeated the page number but was interrupted by the intercom (line 121). Interruptions by the intercom were common and often occurred during class time. I repeated the message again (line 123), but I still had students who asked for the page number or still looking for a textbook (lines 125 and 126). I could get frustrated with this situation, but this was part of the growing process for this classroom.

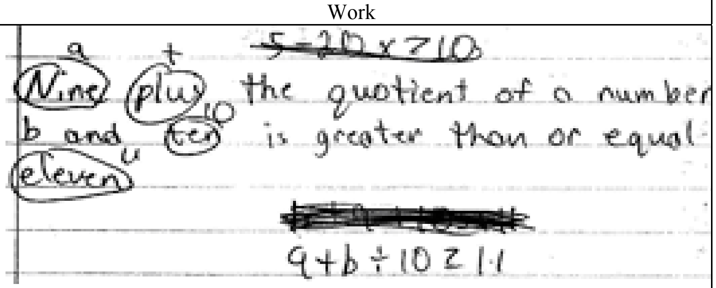
After a few minutes of getting the students on task, we proceeded to translate the verbal expression. In Table 4-5 was one of the verbal expressions we translated that day. This was our third translating task.

Table 4- 3. Task, Michigan standard, and cognitive demand.

Task and cognitive demand	Michigan standard
33. Nine plus the quotient of a number *b* and ten is greater than or equal to eleven. (Larson et al., 2004a p. 36) This is a PWOC task with limited cognitive effort.	A1.1.1 Give a verbal description of an expression that is presented in symbolic form, **write an algebraic expression from a verbal description**, and evaluate expressions given values of the variables. (Michigan Department of Education, 2006d p. 10)

In between the correction of these tasks, I walked around the classroom helping students with the translation. Students asked what quotient was and several students responded with division. After two minutes, I moved toward the overhead and went over the task for the second time in Figure 4-2.

Figure 4- 2. Translating a verbal expression.

Task and cognitive demand	Work
33. Nine plus the quotient of a number *b* and ten is greater than or equal to eleven. This is a PWOC task with limited cognitive effort.	

Line number	Speaker	Statements
259	Teacher	So ok, it goes 9 + *b* divided by.
260	Several	Divided by 10 open mouth, greater than or equal to 11.
261	Teacher	What's this open mouth thing? Is that how your teacher taught you? You said open mouth first or closed mouth first?
262	Britney	Open mouth.
263	Teacher	What if it's the other way, what do you call it if it's this way?
264	Several	Closed mouth.
265	Clorissa	Open mouth, close mouth.
266	Teacher	That's close mouth and that's open mouth?
267	Several	Yeah.
268	Teacher	Ok. All right. I didn't know, so I've never been taught that.
269	Mickey	That's open mouth that side.
270	Teacher	All right, last one. Are we doing pretty good?
271	Christian	No.

272	Britney	No.
273	Teacher	No?
274	Christian	I don't get this.
275	Natasha	I get it.
276	Alonzo	I get it.

Students still used the terminology of open-mouth and close-mouth (line 260). I was still quite confused with the notation (line 263). I had assumed that students understood the material and realized that they still difficulties (lines 271 and 272). We translated six more tasks, before moving to the 2nd chapter. Reflections on the first chapter

I ended chapter 1 questioning myself whether I wasted two days translating verbal expressions. What was accomplished by translating verbal expressions? How does it connect to other algebra topics? Why would you translate a verbal expression? I justify teaching this topic because it is part of the district curriculum, Michigan standards, and I learned it as an algebra student. This internal struggle of my approach to algebra and the algebra of Larson et al. (2004a) manifested itself in the last two days of the lessons. By beginning class with PWOC tasks (refer to Figure 4-35 and Table 4-13), the discourse shifted from algebra to the arithmetic and allowed my class opportunities to practice and fix their misconceptions about arithmetic.

What are the similarities and difference between the district approach with the textbook and my approach for the first chapter? Larson et al. (2004a) and I introduced the variables, variable expressions, exponents, solving equation, algebraic representation, and tables. But our teaching approach to these topics is quite different.

Larson et al. (2004a) teach these topics as individual sections of the textbook and apply these topics in later sections of the chapter. Larson et al. (2004a) also introduce the following topics that I did not include for the first chapter: absolute value, function notation, and graphs. Since incoming students have no experiences with these topics, a majority of these tasks are PWOC tasks. From the students' perspective, they see the pieces of algebra but not the whole picture.

This description above highlights the students' works, students' responses, and my responses to students' uptakes, errors, and counterscripts.

Instructional Practices. With 23 instructional practices or enactments of teaching, I wrote 23 descriptions and began looking for themes for my teaching. I decided to use Larson's five chapters to anchor instructional practices around Larson's objectives. Mirroring Larson provided a time line to look at the progression using TGT. In Larson's Chapter 1, I decided to look at the introduction to algebra. In Larson's Chapter 2, I looked at the subtraction with integers and the division by fraction, which has always been problematic for my students and for me teaching. In Larson's Chapter 3, I looked at the conceptual and procedural approach to solving linear equations. In Larson's Chapter 4, I looked at how I introduced slope and transitioned to the standard terminologies of slope and *y*-intercept. In Larson's Chapter 5, I looked at how to write algebraic representation for a variety of different algebra tasks.

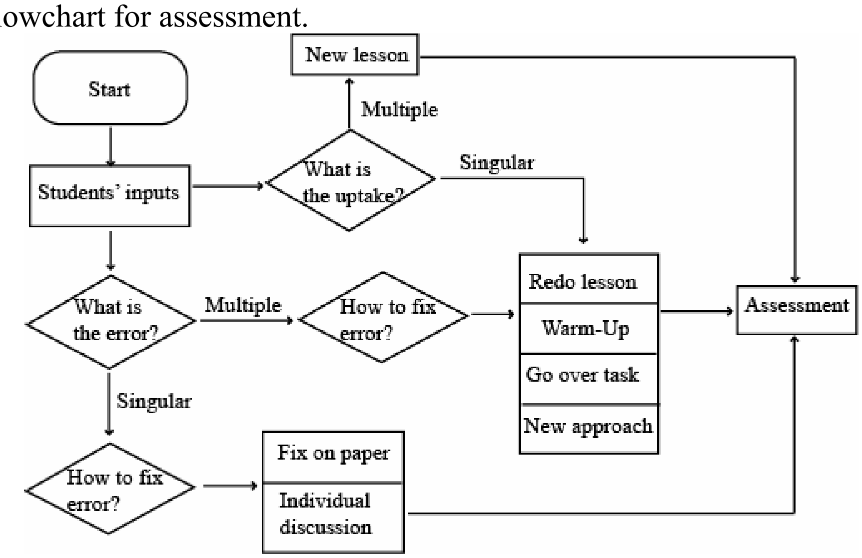
Once I selected the objectives, I went back and selected the appropriate journal entries, student work, and excerpts for each chapter to look at students' uptakes, errors, counterscripts, and domain maps for each of the chapters and my responses to these students' inputs.

Question 4: What does a week of teaching look like using this approach?

In order to answer this question, I selected a week's work of data that included group assessments, lessons written, enactments of teaching, and individual assessments, to show what it took for a teacher to prepare for class. I selected a week because this was how I approach teaching at JHS.

<u>Teaching Cycle</u>. A weekly teaching cycle begins with an assessment, lesson writing, enactments of teaching, and next assessment. Every Friday I gave students an assessment. I graded and analyzed those assessments using the framework in Figure 2-8.

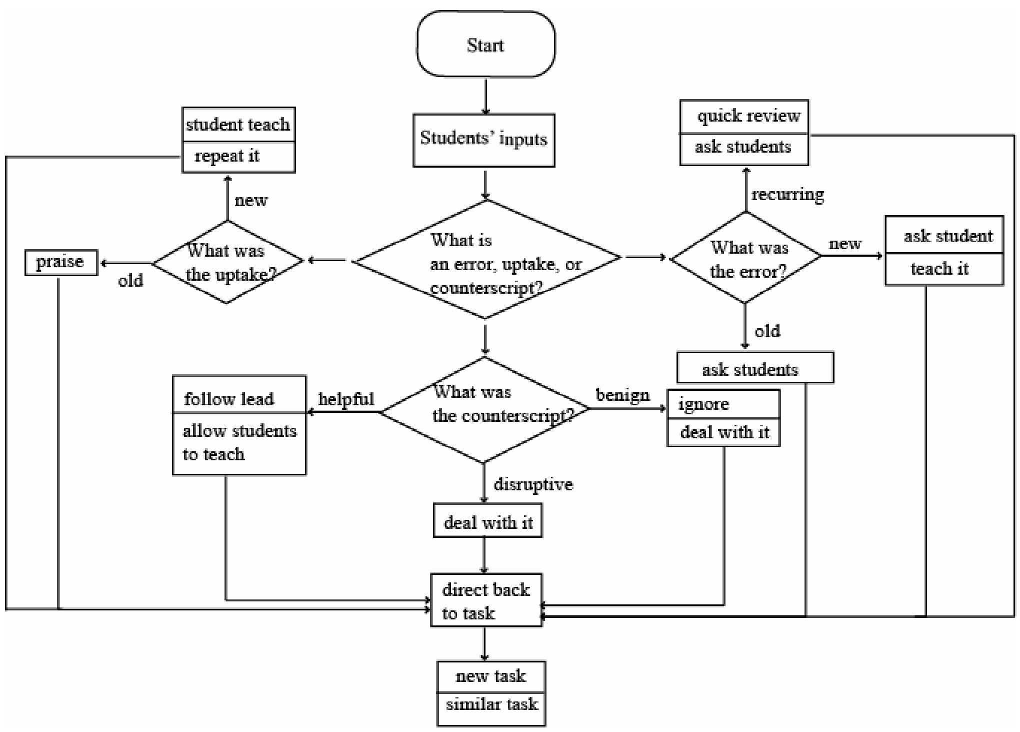
Figure 2- 8. Flowchart for assessment.



If the errors were serious, then I would re-teach a lesson using a different approach. Otherwise, I would write a lesson that incorporated the new type of tasks and included review tasks to fix the errors.

During the enactment of teaching, I followed my lesson plans, but if I felt that students were having difficulties with a task, then I would teach the task and give my students another opportunity to do another task. But if I felt that students understood the task by their participations, then I might increase the cognitive demands by changing the numbers or creating a different task not in my lesson plan. Figure 2-9 is a flowchart in the midst of teaching.

Figure 2- 9. Flowchart for teaching in the classroom.



The following Friday, I would analyze the assessment again following the same flowchart in Figure 2-8.

Once I selected a teaching cycle, I went back and analyze the assessment, followed by the lesson plan, enactment of that particular lesson, and looked at the next assessment.

Data Collection Matrix

Table 2-14 is a data matrix on how I analyzed the data collected to help answer the questions.

Table 2- 14. Data matrix

Research sub-questions	Analysis #1	Analysis #2	Analysis #3	Analysis #4
1. How does a numerical pattern approach to algebra compare to district mandated algebra approach using the textbook by Larson?	Algebra standards	Cognitive demands	Breadth and depth	Pacing
2. What does the teaching of algebra using this approach look like from above classroom?	Teacher and students actions	During mathematical discussions	In-between mathematical discussion	
3. How do instruction practices using this approach function within the classroom?	Description for introducing algebra and writing linear equation	Description for subtraction and division	Description for solving linear equations	Description for slope and writing linear equations
4. What does a week of teaching look like using this approach?	Assessment	Lesson plan	Enactment of a lesson plan	Assessment

Bias

I have identified some basic threats to the validity of this research and address them in this section.

Researcher Bias

I worried that my role as teacher and researcher skewed this research by choosing only the best students' work and highlighting only the best behaviors. As their teacher, I wanted to protect them for the scrutiny of outsiders. I could have written the third sub-question as: What does teaching this approach looks like inside the classroom? By wording the question in this manner, I could highlight the positive aspect of this approach and show only the best work. Instead of doing that, I looked for tensions and obstacles in my teaching and tried to show that I was successful at times but also unsuccessful many times. I also presented the “bad class” to show that teaching in the inner-city was and is extremely difficult, but not impossible.

Descriptive Validity

Johnson (1997) wrote, “descriptive validity refers to the factual accuracy of the account as reported by the qualitative researcher” (p. 11). In this dissertation, I determined the algebra standards and cognitive demands for Larson and my tasks. I also coded my students and my actions for 23 transcripts. I asked a colleague to code a set of tasks and an excerpt of a transcript. I also asked a colleague from JHS to read and make comments on the chapter for my teaching.

Interpretive Validity

Johnson wrote, “interpretive validity is obtained to the degree that the participants’ viewpoints, thoughts, intentions, and experiences are accurate understood and reported by the qualitative researchers” (p.11). As the teacher in this classroom, I needed to monitor and control my bias. Another way to obtain interpretative validity is to use actual data such as an excerpt of a transcript, students’ works, and assessments. This allows the reader to experience the event.

Theoretical Validity

Johnson wrote, “theoretical validity is obtained to the degree that a theory or theoretical explanation developed from a research study fits the data and is, therefore, credible and defensible” (p. 11). In order to ensure theoretical validity, I used multiple data points to collect different types of data for data triangulation.

CHAPTER 3

COMPARING THE TWO ALGEBRAS

Peers and friends often asked, “So what is your research about?” I told them that I was working on an approach to the teaching of algebra for the inner-city. As I thought about this, I wondered how different could this approach be? I still taught in a classroom with thirty-five seats, two chalkboards, two number lines, an overhead projector, and a set of textbooks. For six years, I had difficulties teaching traditional algebra using textbooks similar to Larson. As a novice teacher, I blamed myself for not being able to teach the algebra curriculum, but my struggle with the teaching of algebra continued even as I became an experienced teacher. I began questioning the algebra I was teaching. For example, students’ first exposure to traditional algebra is in the following task: Evaluate $8y$ with $y = 2$. I questioned whether the error made by combining the 8 and 2 together to make 82 instead of 16 was that important. More importantly, I wanted students to first understand when to substitute and then evaluate correctly. Because of my continuing frustration, for the past four years, I have been developing my own algebra using a numerical approach with my students, which I now refer to as teacher generated tasks (TGT).

A numerical pattern approach used students’ prior experiences with arithmetic, which would allow students to transition from arithmetic to algebra. Another advantage of the numerical pattern approach was that I could quickly create these tasks to help my students. I could change the numbers to increase or decrease the cognitive demands to fit the needs of my students. When I began teaching algebra with a numerical pattern approach, I tried to use standard terminologies such as slope and y-intercept and I had to create new terminologies to fit the numerical pattern approach. Since I didn't have an established curriculum, I needed to construct new tasks weekly to implement the mandated district standards. During that first year, I had to use Larson when I couldn't find an equivalent numerical pattern tasks.

In this chapter, I compare the differences between the two curricula. I want to show what was wrong with district mandated approach and how TGT became a more appropriate approach to the teaching of algebra for my inner-city high school students. I do this to answer the following sub-question: How does a numerical pattern approach to algebra compare to district mandated algebra approach using the district's mandated textbook (Larson)? The term "appropriate" refers to tailoring algebra to my students and is guided by the following premises: students are naturally curious about mathematics; students want to be challenged in their work; and my work as a high school algebra teacher is continuous and dynamic. I generated the following questions to help guide me in the analysis:

- Which algebra standards are covered in each curriculum?
- What are the cognitive demands for each curriculum?
- Which algebra curriculum has breadth?
- Which algebra standards have depth?

The second half of the analysis looks at the pacing for the two curricula and I used the following questions:

- How are the chapters divided?
- How are algebra standards introduced?
- How are algebra standards connected in a chapter and between chapters?

The last section of this paper is a discussion of the numerical pattern approach to the teaching of algebra.

The Two Curricula

In this section, I look at the algebra standards, cognitive demands, and the breadth and depth for Larson.

Algebra Standards

Table 3-1 lists the Michigan Standards that are targeted in Larson and TGT, which are marked with the boxes.

Table 3- 1. Description of Michigan Standards for Larson and TGT.

Standards	Descriptions	Larson	TGT
A1.1.1	Give a verbal description of an expression that is presented in symbolic form, write an algebraic expression from a verbal description, and evaluate expressions given values of the variables.	■	■
A1.1.2	Know the definitions and properties of exponents and roots and apply them in algebraic expressions	■	■
A1.2.1	Write and solve equations and inequalities with one or two variables to represent mathematical or applied situations.	■	■
A1.2.3	Solve linear and quadratic equations and inequalities, including systems of up to three linear equations with three unknowns. Justify steps in the solutions, and apply the quadratic formula appropriately.	■	■
A1.2.4	Solve absolute value equations and inequalities and justify	■	
A1.2.8	Solve an equation involving several variables (with numerical or letter coefficients) for designated variable. Justify steps in the solutions.	■	■
A2.1.1	Recognize whether a relationship (given in contextual, symbolic, tabular, or graphical form) is a function and identity its domain and range.	■	
A2.1.3	Represent functions in symbols, graphs, tables, diagrams, or words and translate among representations.	■	■
A2.1.4	Recognize that functions may be defined by different expressions over different intervals of their domains. Such functions are piecewise-defined (e.g., absolute value and greatest integer functions).	■	
A2.1.5	Recognize that functions may be defined recursively. Compute values of and graph simple recursively defined functions.		■
A2.1.7	Identify and interpret the key features of a function from its graph or its formula (e), (e.g., slope, intercept(s), asymptote(s), maximum and minimum value(s), symmetry, and average rate of change over an interval).	■	■
A2.1.2	Read, interpret, and use function notation and evaluate a function at a value in its domain.	■	
A2.2.1	Combine functions by addition, subtraction, multiplication, and division.		■

A2.3.1	Identify a function as a member of a family of functions based on its symbolic or graphical representation. Recognize that different families of functions have different asymptotic behavior at infinity and describe these behaviors.	■	
A2.3.2	Describe the tabular pattern associated with functions having constant rate of change (linear) or variable rates of change.	■	■
A2.4.1	Write the symbolic forms of linear functions (standard [i.e., Ax+By=C, where B not equal to 0], point-slope, and slope-intercept) given appropriate information and convert between forms.	■	■
A2.4.2	Graph lines (including those of the form x=h and y=k) given appropriate information.	■	■
A2.4.4	Find an equation of the line parallel or perpendicular to a given line through a given point. Understand and use the facts that nonvertical parallel lines have equal slopes and that nonvertical perpendicular lines have slopes that multiply to give -1.	■	
L1.1.3	Explain how the properties of associativity, commutativity, and distributivity, as well as identity and inverse elements, are used in arithmetic and algebraic calculations.	■	■
L1.2.4	Organize and summarize a data set in a table, plot, chart, or spreadsheet; find patterns in a display of data; understand and critique data displays in the media.	■	■
L2.1.2	Calculate fluently with numerical expressions involving exponents.	■	
L4.1.1	Distinguish between inductive and deductive reasoning, identifying and providing examples of each.	■	
S2.1.1	Construct a scatterplot for a bivariate data set with appropriate labels and scales.	■	
S2.1.2	Given a scatterplot, identify patterns, clusters, and outliers. Recognize no correlation, weak correlation, and strong correlation.	■	
S2.2.1	For bivariate data that appear to form a linear pattern, find the least squares regression line by estimating visually and by calculating the equation of the regression line. Interpret the slope of the equation for a regression line.	■	
A.FO.06.04	Distinguish between an algebraic expression and an equation.	■	
A.FO.06.12	Understand that adding or subtracting the same number to both sides of an equation creates a new equation that has the same solution.	■	
A.FO.07.12	Add, subtract, and multiply simple algebraic expressions of the first degree, and justify using properties of real numbers.	■	
A.PA.07.06	Calculate the slope from the graph of a linear function as the ratio of "rise/run" for a pair of points on the graph, and express the answer as a fraction and a decimal; understand that the linear functions have slope that is a constant rate of change.	■	■
APA.07.07	Represent linear functions in the form y = x+b, y = mx, and y=mx+b, and graph, interpreting slope and y-intercept.	■	
A.RP.06.02	Plot ordered pairs of integers and use ordered pairs of integers to identify points in all four quadrants of the coordinate plane.	■	
D.PR.06.01	Compute probabilities or events from simple experiments with equally likely outcomes, e.g., tossing dice, flipping coins, spinning spinners, by listing all possibilities and finding the fraction that meets given conditions.	■	

G.TR.07.03	Understand that in similar polygons, corresponding angles are congruent and the ratio of corresponding sides are equal; understanding sides are equal; understand the concepts of similar figures and scale factor.	■	
N.FL.05.20	Solve applied problems involving fractions and decimals; include rounding of answer and checking reasonableness.	■	
N.FL.06.12	Calculate part of a number given the percentage and the number.	■	
N.FL.07.08	Add, subtract, multiply, and divide positive and negative rational numbers fluently.	■	■
N.ME.03.06	Count orally by 6's, 7's, 8's, and 9's starting with 0, making the connection repeated addition and multiplication.	■	
N.ME.06.05	Order rational numbers and place them on number line.	■	
N.ME.06.06	Represent rational numbers as fractions or terminating decimals when possible and translate between these representations.	■	■
N.ME.06.20	Know that the absolute value of a number is the value of the number ignoring the sign; or the distance of the number from 0.	■	
N.ME.08.08	Solve problems involving percent increases and decreases.		■
N.MR.05.22	Express fractions and decimals as percentage and vice versa.	■	
N.MR.06.13	Solve contextual problems involving percentage such as sales taxes and tips.	■	

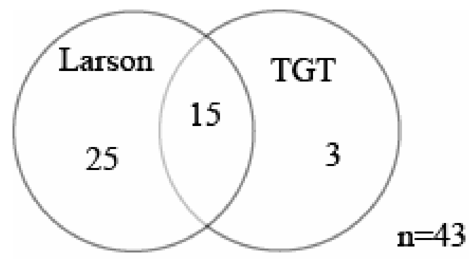
Table 3-2 presents the frequency of the algebra and K-8 standards for each curriculum as elicited by the tasks in each (983 for Larson, and 333 for TGT).

Table 3- 2. Frequency of standards targeted by the Larson and the TGT tasks.

	Larson (n = 983)	TGT algebra (n = 333)
Number of algebra standards	23	14
Number of K-8 standards	17	4
Total	40	18

Larson addressed 23 algebra standards and 17 K-8 standards, whereas TGT addressed 14 algebra standards and 4 K-8 standards. The numbers of standards used differ by 22 standards. Figure 3-1 is a Venn diagram of the overlapping standards for Larson and TGT.

Figure 3- 1. Distribution of standards for the two algebras.

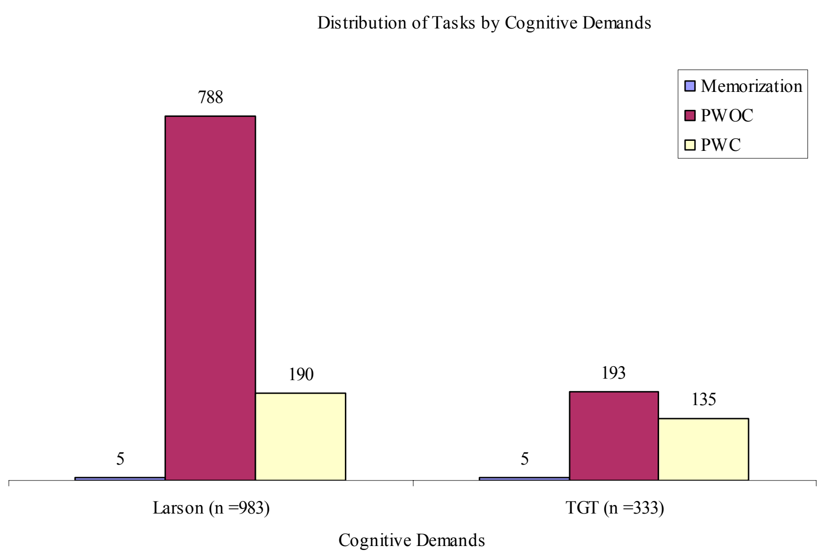


Larson and the TGT algebra overlap on 15 standards (12 algebra standards and 3 K-8 standards). These 15 standards provide a clue to what standards both curricula deemed important for the first semester of algebra. Before I take a closer analysis of these 15 shared standards and 22 different standards, I introduce another tool to help separate the two curricula, the cognitive demands.

Cognitive Demands

Figure 3-2 shows the cognitive demand breakdown for Larson and TGT.

Figure 3- 2. Cognitive breakdown by numbers.



From Table 3-2 shows that for Larson, the ratio of lower-level cognitive demand tasks to higher level cognitive demand tasks is 4 to 1. With a high percentage of lower-level demand tasks, Larson targeted total of 40 MI standards. The high number of standards with a high number of lower-level demand tasks leads to an algebra that has few connections between the standards and is characterized as being too procedural.

On the other hand, the ratio for TGT is closer to 3 to 2 and targeted a total of 18 standards, due to a strike shorten semester, which cut out 10 instructional days. TGT algebra has a mixture of conceptual and procedural tasks.

Breadth and Depth

The breadth for first semester algebra included all standards associated with linear equations. The depth of a particular standard is defined when the number of PWOC standard is less than the number of PWC standard. Most tasks contain multiple parts and each part of a task addresses a particular standard. In other words, if a task contained two parts and addressed two standards or two of same standard, then the task would be counted twice one for each standard addressed. By doing this, the number of tasks changed from 983 to 1329 for Larson and 333 to 474 for TGT. Table 3-3 lists the percentages of the 15 shared standards and highlights which of these standards have depth. Table 3-3 also shows the new number of tasks (1329 for Larson, and 474 for TGT).

Table 3- 3. Percentages for 15 shared Michigan standards and ratio of PWOC to PWC.

	Larson (n=1329) (78 %)	Ratio of tasks	TGT (n=474) (95%)	Ratio of tasks
Standards[1]	% of tasks	PWOC :PWC	% of tasks	PWOC: PWC
Operations with real numbers (N.FL.07.08)	13%	2.4 : 1	15%	2 : 1
Write and solve (A1.2.1)	11%	1 : 1.1	15%	1 : 2.1
Solve linear equation (A1.2.3)	10%	6.1 : 1	12%	28.5 : 1
Evaluate variable	9%	9.1 : 1	13%	3.3 : 1

[1] A complete description of these standards is in Table 3-1.

expression (A1.1.1)				
Organize data into table (L1.2.4)	8%	2.9 : 1	10%	1 : 8.2
Graph lines (A2.4.2)	6%	10.4 : 1	1%	0 : 6
Identify key features of linear equation (A2.1.7)	5%	3.1 : 1	12%	1 : 4.6
Write linear equation (A2.4.1)	5%	1.6 : 1	5%	0 : 6
Solve literal equations (A1.2.8)	4%	47 : 0	1%	3: 0
Calculate slope (A.PA.07.06)	3%	1.9 : 1	4%	1 : 1.6
Evaluate variable expression with exponents (A1.1.2)	3%	37 : 0	2%	11: 0
Explain properties (L1.1.3)	2%	1.5 : 1	1%	4 : 0
Fractions and decimals (N.ME.06.06)	0%	5 : 0	1%	6 : 0
Describe tabular pattern (A2.3.2)	0%	3 : 1	2%	0 : 8
Graphs and tables (A2.1.3)	0%	0 : 1	3%	1 : 5

These 15 MDE standards represent 78% of all of the tasks. For TGT, it represents 95% of the all of the tasks. In the first column, I listed the standards in descending order of frequency of tasks that were coded for each standard for Larson with the corresponding percentages. For example, Larson devoted 13% of its tasks to the operations with real numbers. In the second column, I listed the ratio of PWOC tasks to PWC tasks for that particular standard. I have also listed the percentages for TGT and the ratio of PWOC tasks to PWC tasks for TGT in the last column. Using my rough definition for depth, Larson had only two standards with depth out of 15 and TGT had eight standards out of 15 with depth.

The Other Larson Standards. Larson contains 25 standards not found in TGT algebra. Table 3-4 lists the percentages and ratio of PWOC to PWC for each standard and highlighted those standards that had depth.

Table 3- 4. Percentages and ratio of PWOC to PWC for Larson for 25 standards.

	Larson (n= 1329) (22%)	Ratio of tasks
Standards	% of tasks	PWOC : PWC
Algebraic expression (APA.07.07)	4%	2.4 : 1
Add, subtract, and multiply algebraic expression(A.FO.07.12)	3%	43 : 0
Solve problem involving multiplication (N.MR.05.20)	2%	25: 0
Order number (N.ME.06.05)	2%	22 : 0
Calculate expression with exponent (L2.1.2)	2%	5.7 : 1
Scatterplot (S2.1.1)	1%	3.7 : 1
Regression line (S2.2.1)	1%	4 : 1
Recognize correlation (S2.1.2)	1%	4 : 1
Know absolute (N.ME.06.20)	1%	10 : 0
Recognize a relationship (A2.1.1)	1%	8 : 1
Plot points (A.RP.06.02)	1%	9 : 0
Express fractions (N.MR.05.22)	1%	1.25 : 1
Parallel and perpendicular lines (A2.4.4)	1%	8 : 0
Functional notation (A2.1.2)	1%	7 : 0
Piece-wise (A2.1.4)	1%	7 : 0
Use add and subtract to balance equation (A.FO.06.04)	1%	6 : 0
Deductive and Inductive (L4.1.1)	0%	1 : 4
Express probabilities (D.PR.06.01)	0%	1 : 1.5
Solve absolute (A1.2.4)	0%	4 : 0
Calculate percentage (N.FL.06.12)	0%	4 : 0
Use add and subtract to balance (A.FO.06.12)	0%	0 : 2
Similar polygons (G.TR.07.03)	0%	0 : 2
Repeated addition (N.ME.03.06)	0%	2 : 0
Solve percentage problems (N.MR.06.13)	0%	0 : 2
Identify family of function (A2.3.1)	0%	0 : 1

This table shows that these 25 standards made up 22% of the total number of tasks. Of these 25 standards, five of these have depth, which makes a total of 7 out 40 standards that have depth. There were very few tasks that contained these standards with depth.

A closer look at these 25 standards reveals that the authors could have combined some of these standards, but chose instead to teach each of standards separately by narrowing the objective of the task. Many of the K-8 standards tasks could be re-written as algebra tasks. Table 3-5 list three such tasks.

Table 3- 5. List of tasks coded with K-8 Michigan standards.

Larson	Standard
Task 1 Simplify the expression. a. $4x - x$ (p. 101)	A.F0.07.12 Add, subtract, and multiply simple algebraic expressions of the first degree, and justify using properties of real numbers.
Task 2 What operation do you use to solve an addition equation? To solve a subtraction equation? (p. 131)	A.FO.06.04 Understand that adding or subtracting the same number to both sides of an equation creates a new equation that has the same solution.
Task 3 Write each addition sentence as a multiplication sentence. $2+2+2$ (p. 93)	Count orally by 6's, 7's, 8's, and 9's starting with 0, making the connection repeated addition and multiplication.

It was not possible to code the first task using an algebra standard. This task could have rewritten as, solve $4x - x = 5$, which would be coded as solve linear equation (A1.1.1), but not as add or subtract variable expressions. For the second task, the process of solving of linear equations would allow the students to see the inverse operations with additions and subtractions. The third task could be rewritten as a numerical pattern task. By writing tasks with such a narrow focus, algebra is divided into many standards and creating algebra that has few connections between the standards.

The Other TGT Standards. TGT contains three standards not found in Larson. Table 3-6 presents the two algebra standards and 1 K-8 standard, which represents 5% of the total number of tasks.

Table 3- 6. Percentages and ratio of PWOC to PWC for three TGT standards.

	TGT (n= 474) (5 %)	Ratio of tasks
Standards	% of tasks	PWOC : PWC
Recognize that functions may be defined recursively. Compute values of and graph simple recursively defined functions (A2.1.5)	3%	0 : 13

Combine functions by addition, subtraction, multiplication, and division (A2.2.1)	1%	0 : 4
Solve problems involving percent increases and decreases (N.MR.08.08)	1%	6 : 0

This table shows that two standards have depth, which makes a total of 10 out of 18 standards. The two standards with depth employ the recursive function (A2.1.5) and the addition of functions (A2.2.1).

Discussion

This picture that emerges is that Larson's algebra is characterized by lower-level demand tasks, few connections between the standards, and many tasks that do not address the standards with sufficient depth. Larson could have rewritten some of the tasks to reduce the number of standards; thereby, increasing the connections between standards. Larson's algebra is more procedural and underdeveloped conceptual understanding.

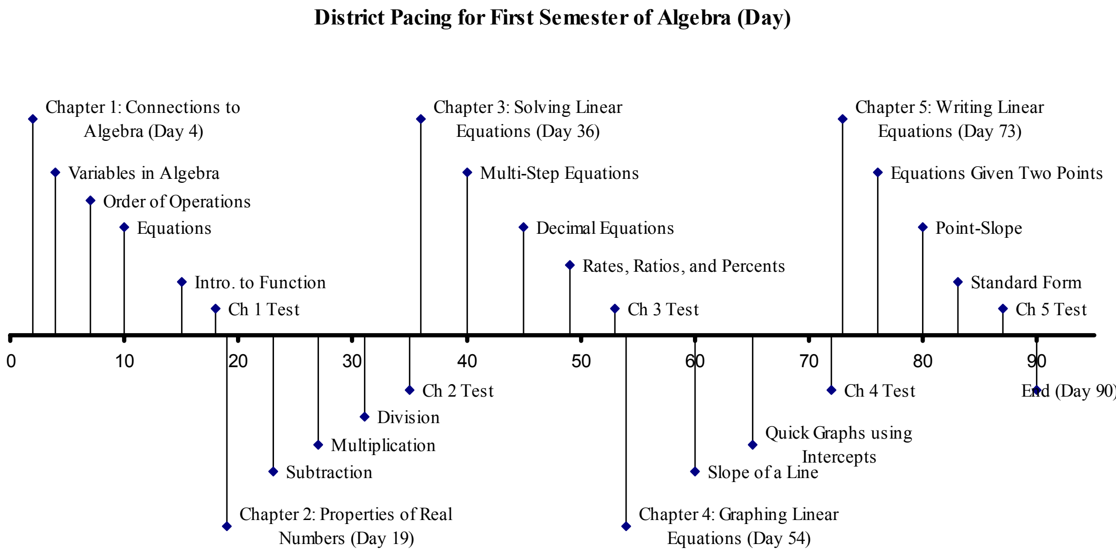
On the other hand, the numerical pattern algebra as given through the teacher generated tasks is characterized by higher-level demand tasks, containing more connections between standards, and addressing them in more depth. TGT does not have breadth, because it does not contain absolute values, scatter plots, parallel lines, and perpendicular lines, but does have depth. With equal amounts of procedural and conceptual tasks, TGT develops conceptual understanding along with procedural fluency.

The Pacing of Algebras

The district pacing chart, the *Mathematics Grade 9 Curriculum Guide* (Norde et al., 2006), lays out which topics to teach and for how long. For the TGT algebra, the teacher determined when to move forward, what topics to teach, and when to teach it. Although there is flexibility with the pacing, the teacher must begin and arrive at the same point as Larson in order to fulfill the district expectation for the first semester of algebra.

Figure 3-3 presents a layout for the first 90 days of algebra as prescribed by the district. The first three days allowed the teacher to get their rooms ready and to register students. Due to the spacing, Figure 3-3 highlights some of the algebra topics.

Figure 3- 3. District Pacing for the first semester of algebra.

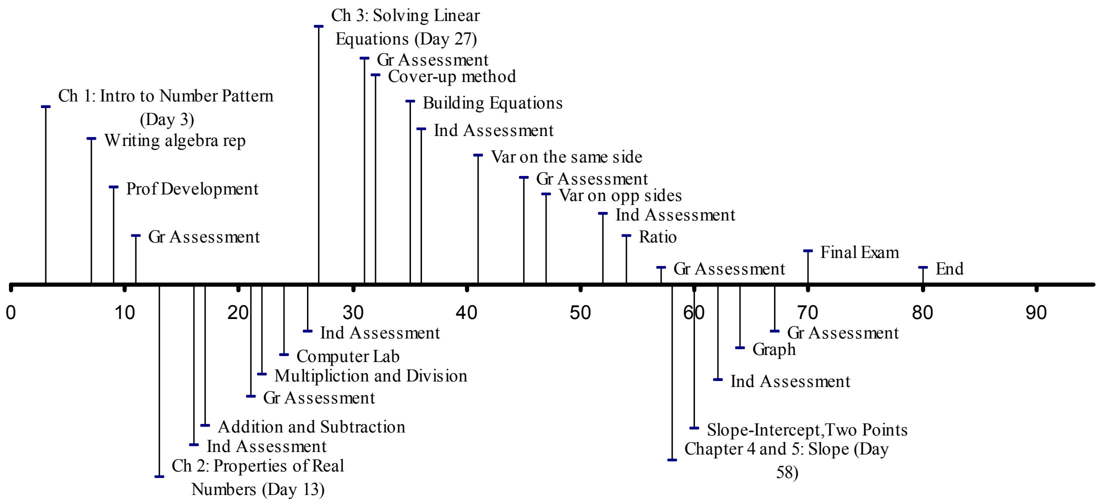


Each chapter takes approximately 20 days to teach; five chapters take approximately 90 days (a semester of instruction). The first chapter begins with the introduction of variables, proceeds to equations, and ends with functions. In the second chapter, the operations are taught individually, moving from addition, to subtraction, to multiplication and ending with division. The third chapter begins with single-step equations and moves to multi-step equations. The fourth chapter starts with slope and moves to the graphing of lines. The fifth chapter starts with writing of linear equations in slope-intercept and moves to point-slope and standard forms. Each of these chapters begins with a standard, from this standard, other standards are introduced.

In Figure 3-4 is pacing for TGT. The first day of algebra begins on the third day.

Figure 3- 4. TGT pacing for Algebra.

TGT Pacing for Algebra (Day)



G r= group assessment and Ind = individual assessment

For the TGT algebra, the chapters are not divided equally. The first chapter begins with the writing of linear equations in slope-intercept form. It involves 10 instructional days, the shortest. The second chapter covers operations with real numbers, and starts with a combination of addition and subtraction as well as multiplication and division. The third chapter, which involves the longest number of days, 31, begins with solving linear equations, moves to equations with the same side, and ends with the solving of equations with variables on opposite sides. The fourth chapter introduces slope and graph. The fifth chapter reviews writing linear equations, actually combining Larson's Chapter 4, slope and functions, with Chapter 5, writing linear equations.

It should be noted that the number of days for the first semester is normally 90 days, but due to a strike at the beginning of the school year, the number of days for the first semester was reduced to 80. The date for the final exam was set by the administrative staff and for this semester it fell on the 70th day of the semester.

Progression

Both algebra curricula arrived at writing linear equations at the end of the first semester, but the curricula achieved this differently. Using the variable along with the procedure for substitution, Larson constructed a pathway for algebra that is consistent with a conceptualization of algebra as symbolism.

The TGT conceptualization, on the other hand, is more aligned with generalized arithmetic with a focus on numerical patterns. The conceptualizations are not so different, but Larson's use of PWOC tasks pushes algebra toward a loosely connected set of topics. TGT's use of PWC tasks connects the chapters of algebra together; therefore, algebra is a set of connected topics.

The next section presents how both curricula begin each of the five chapters: (1) Connections to Algebra; (2) Properties of Real Numbers; (3) Solving Linear Equations; (4) Graphing Linear Equations and Functions; and (5) Writing Linear Equations.

Chapter 1: Connections to Algebra. The start of instruction situates the algebra for the students. From this starting point, Larson and the TGT algebra navigate through the algebra curriculum to arrive at the writing of linear equations. The first algebra tasks given by Larson and TGT are in Figure 3-5.

Figure 3- 5. Introductory tasks from Larson and the TGT.

Larson task	TGT
Evaluate the expression. 1. $8y$ when $y = 2$	Given the following pattern 3, 6, 9, 12, 15… a) Find the 8th , 10th , and 15th term b) What term is equal to 51? c) What is the rule for this pattern?

Larson's is a PWOC task. By beginning with a PWOC task, Larson sets up a procedural (or instrumental, Skemp, 1978) understanding of algebra. Starting with a lower-level cognitive demand task makes it much harder to increase the cognitive demand in a later task (Stein, Smith, Henningsen, & Silver, 2000), which sets up the cognitive expectation for the students. Students begin to see algebra as a set of procedures to be taught (Thorpe, 1989). The procedure for evaluating a variable expression limits the number of algebraic pathways that can be taken. Larson applies this procedure in order of operations and exponential tasks. By the end of the first chapter, students will be exposed to a variety of algebra tasks (e.g. equations, graphs, and functions) that they will encounter for the first semester of algebra.

The TGT algebra task is a PWC task. The task sets up a conceptual understanding (or relational understanding, Skemp, 1978) of algebra. Starting with a higher-level demand task, student's cognitive expectation is higher. Learning the procedure to write the algebraic representation (part c) in the first task can be used throughout the whole semester and opens up other algebraic topics for the other chapters. Instead of using arithmetic to solve part a), students can substitute values into the algebraic representation. For part b), this task sets up the equation to be solved for future chapters. This interplay between arithmetic and algebra occurs throughout the semester. Students would be given opportunities to transition from arithmetic to algebra and see how inefficient arithmetic becomes in the process of solving these TGT tasks. The students would determine when they would algebra.

<u>Chapter 2: Properties of Real Numbers</u>. The second chapter is a review of the arithmetic operations. In order to embed algebra into the chapter, the approach of Larson and TGT are also quite different. Figure 3-6 lists representative tasks for the second chapter.

Figure 3- 6. Algebra tasks for the second chapter.

Larson tasks	TGT
Task 1 Evaluate each function for these values of *x*: -1, 0, 1, and 2. Organize your results in a table. $y = x + 5$ (p. 79)	Task 1 Given 3rd term = 1/2 and jump is ¼ Find the first five terms and algebraic Task 2 Find the first six terms if $n_3 = 1$ and

Task 2 Evaluate the expression when x = -7. a. $2(-x)(-x)$ (p. 94)	$n_{t+1} = \frac{1}{3} n_t$

In Larson's first task, the input and output values are placed into a table and students apply the rules for addition of signed integers to determine the output. In Larson's second task, students substitute the value for x and apply the rules for multiplication of signed integers. The procedure for evaluating a variable expression is used in these two tasks and connects the procedure learned in the first chapter. It is possible to change Larson's tasks to incorporate subtraction and division.

With the TGT model, students have already learned from the first chapter that the term "jump" corresponds to the magnitude of the slope and "0^{th} term" as the y-intercept. In task 1 above, the students use addition to get the 4^{th} and 5^{th} term and subtraction to get the 0^{th}, 1^{st}, and 2^{nd} term. By doing this, the task ties the inverse operations of addition and subtraction. Task 1 is a variation of the task of the first chapter (refer to teacher task Figure 3-5) that foregrounds addition and subtraction and backgrounds the algebraic representation.

For the second task, the TGT model introduces the recursive function so that the inverse operations of multiplication and division can be incorporated into the task. This introduction of the recursive function isn't an extension to the topic but is embedded in the topic. These inverse operations will be further developed in the next chapter, which covers the solving of linear equations.

<u>Chapter 3: Solving Linear Equations</u>. The third chapter is a focused treatment of the solving of linear equations. Larson divided the linear equations into the following order: (1) addition, (2) subtraction, (3) multiplication, and (4) division. Larson moved from one-step equations to multi-step equations, variables on both sides, and variables on opposite sides.

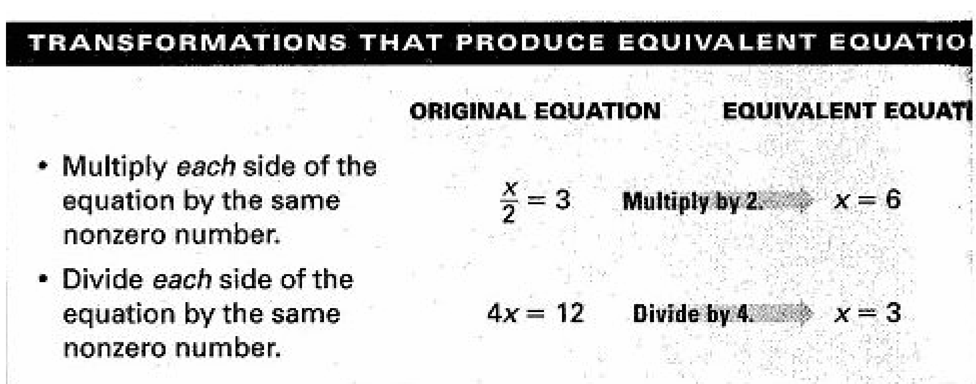
The TGT model does not separate the solving of linear equations by operations or by the number of steps, but depends on the type of equations encountered. Since it is inefficient to always use numerical patterns, TGT uses Larson's tasks as supplement. Figure 3-7 lists two selected tasks used to introduce the third chapter.

Figure 3- 7. Selected tasks for the solving of linear equations.

Larson task	TGT
Use algebra tiles to model and solve the equation. Sketch each step you use. $x+4=6$ (p. 131)	Given the linear pattern 6, 8, 10, 12, 14… a) Find the 12th, 17th, and 10,000th term. b) What term is equal to 24? 58? And 123,456?

Larson also used the algebra tiles for subtraction, variables on the same side, and variables on opposite sides[2]. Additionally, Larson provided the rules for transformations. For solving tasks with multiplication and division, Larson provided the rules for transformations, which are in Figure 3-8.

Figure 3- 8. Transformations rules for multiplication and division (Larson, Boswell, Timothy, & Stiff, 2004 p. 138).



TRANSFORMATIONS THAT PRODUCE EQUIVALENT EQUATIO

	ORIGINAL EQUATION		EQUIVALENT EQUATI
• Multiply *each* side of the equation by the same nonzero number.	$\frac{x}{2}=3$	Multiply by 2.	$x=6$
• Divide *each* side of the equation by the same nonzero number.	$4x=12$	Divide by 4.	$x=3$

The TGT model is an extension of the task of the first task in Figure 3-5. Students continue to practice the evaluation of an equation (part a). By choosing larger numbers, (part b) students get to move from the arithmetic to the algebraic. Doing this provides a rationale for solving linear equations for the students.

[2] All tasks that used algebra tiles were coded as PWC tasks.

Chapter 4: Graphical Linear Equations and Functions. The objectives of Larson's Chapter 4 are for students to be able to graph points, find slopes, and determine when equations are parallel and perpendicular. The chapter ends with the introduction to functional notation. TGT introduces the slope and graphing of lines in this chapter. Central to this chapter is the slope. Figure 3-9 contains two selected tasks used to find the slope.

Figure 3- 9. Selected tasks for finding slopes.

Larson task	TGT
Find the slope of the line passing through the (-2, 1) and (1, -3). (p. 227)	Given the following linear pattern 5, _, _, 11,_… Find the following: a) The missing numbers b) the jump c) the 0^{th} term d) algebraic representation

Larson began the chapter with the concept of slope and also provided the formula for slope. The TGT task is a variation of the task of the first chapter (Figure 3-5) except that numbers in the numerical patterns are removed. Changing the numbers given in the task forces the students to search for a method to find jump. Once the jump is found, students can use arithmetic or algebraic procedures to find the 0^{th} term and algebraic representation. Within this chapter, students transitioned from the term "jump" and "0^{th} term" to slope and *y*-intercept respectively.

Chapter 5: Writing Linear Equations. The objectives for Larson's Chapter 5 included the writing of linear equations in standard form, and point-slope, with an emphasis on the slope-intercept. TGT had already introduced the slope-intercept on the first day. Figure 3-10 presents two tasks used to write the slope intercept form of the linear equation.

Figure 3- 10. Selected tasks for the writing of linear equation in slope-intercept form.

Larson task	TGT
Write an equation of the line that passes through the point (-3, 0) and has a slope of	Write a linear equation with $n_4 = 10$ and slope = $2/3$.

of $\frac{1}{3}$. (p. 280)	

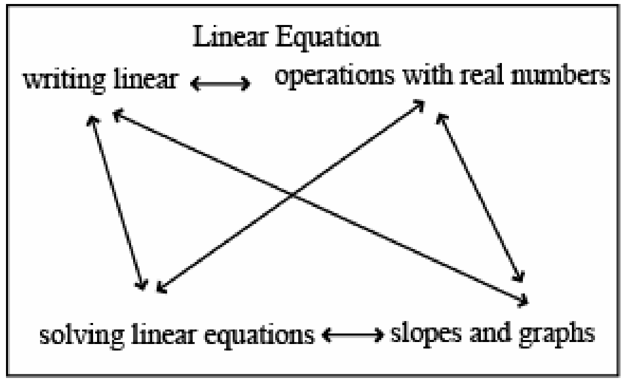
The objective for these tasks is the same, but the approaches are different. Larson applied the slope-intercept equation, which was introduced in this chapter. The TGT is a variation of the task from the second chapter (refer to teacher task 1 Figure 3-6). Slope has replaced "jump" in this task. Students can apply arithmetic or algebraic approaches to write the algebraic representation. Students are continually building their conceptual understanding of linear equation.

Summary

The standard for substituting a value (A1.1.1) is present in all of five Larson's tasks. In Chapter 1, substituting a value into a variable expression leads to a value. In Chapter 2, substituting an input into a function gives an output. In Chapter 3, substituting a value into an equation determines the correctness of a solution. In Chapters 4 and 5, substituting into a formula gives the slope or the slope-intercept respectively.

On the other hand, the TGT tasks share five standards: (1) write and solve (A1.2.1); (2) identify key features of a linear equation (A1.2.7); (3) describe tabular patterns (A2.3.2); (4) write linear equations (A2.4.1); and (5) make tables (L1.2.4). Depending on the objective of the TGT lesson, some standards are foregrounded while other standards are backgrounded. This creates an interweaving of the standards, which is shown by the arrows connecting the standards in the figure below. Figure 3-11 shows the pictorial representation of the standards for TGT.

Figure 3- 11. Pictorial representation of the algebra content for the 1st semester.



Algebra is more than numeric patterns and within the semester, I supplemented the curriculum with geometric shapes and used the algebra software, *Cognitive Tutor: Algebra 1* (Carnegie Learning, 1998) to help build conceptual understanding for contextual tasks.

Discussion

I understood now why my students struggled understanding Larson. Every week I would introduce a new topic with new standards. While I could see where these topics were heading, my students couldn't and gave up trying to understand. I read academic journals to get ideas on how to teach each topic, but I could not maintain the success throughout the semester.

Numerical pattern tasks shifted my understanding and my approach to the teaching of algebra. Instead of thinking how to teach substitution to my students, I shifted my thinking into why students would substitute. As I thought more about numerical pattern, I realized that it was possible to create an algebra curriculum around a numerical pattern approach that has depth and does not water down the high school algebra curriculum. A numerical pattern approach changed the ordering of the algebra curriculum and moved the writing of linear equation to the first week of algebra which changes the approach to the operations of real numbers, the solving of linear equations, and the finding of the slope. It was possible to interweave the mandated standards using a numerical pattern approach to teach algebra.

A reconceptualization of algebra changed my understanding of algebra. With a different approach to algebra, the pacing, procedures, and ordering of algebra topics changed. The pacing for the first five chapters isn't divided evenly between the five chapters. The procedures required different terminologies and the writing of new tasks to incorporate this conceptualization. I also made a decision about the ordering of the algebra topics. I believe that these decisions made me a better algebra teacher, because I was able to compare and contrast my prior and new understanding of algebra. Reconceptualizations of a mathematical subject could be a focus for teacher development that allows new teachers to look holistically at a subject and not focus on the individual topics within a subject.

In this chapter, I attempted to quantify breadth and depth for the algebra that was available to my students. Defining these terms allowed me to quantify the algebra curricula, but these rough definitions of breadth and depth need to be further refined. I am unsure of the breadth of algebra for the first semester. I questioned whether students needed to learn all 40 MDE standards targeted by Larson for the first semester of algebra. The next step is to begin looking at the ordering and how to connect these algebra topics together. I defined depth by looking at the ratio of PWOC to PWC tasks, but I believe that depth is missing a component that incorporates procedural fluency. Defining a process to determine breadth and depth will provide another tool to examine the algebra curriculum.

In this chapter, I presented TGT as a viable approach to the teaching of algebra. In other words, I determined the algebra content and selected a conceptualization that allowed me to construct a curriculum for the first semester of algebra. Using numerical pattern tasks allowed me to introduce each chapter with PWC tasks that interweaves the district mandated standards. By foregrounding and backgrounding district mandated standards I was able to connect the different algebra concepts such as operations with real numbers, solving linear questions, and slope for the first semester of algebra.

Even with this curriculum, was it possible to teach this algebra to a group of 9^{th} graders in an inner-city high school? What were the obstacles encountered in implementing TGT into the classroom? What were the preparations needed by the teacher to teach TGT? How does the theory translate into practice? More importantly, does it work in the classroom? How do teacher and students interact with the content? The next two chapters explore these questions in further detail.

CHAPTER 4

TEACHING

A stereotypical view of the inner-city high school is that of chaos, teachers babysitting students, and bored students learning arithmetic. I must admit that I held this stereotypical view of chaos when I first walked into JHS in 1993. In this chapter, I want to revisit these stereotypical views of the inner-city classroom. Using this instruction as interactions framework, I now return to the second question:

a. What does the teaching of algebra using this approach look like in the classroom?

In order to look at the classroom environment, I held the algebra content constant. In other words, I did not focus on the specific algebra tasks but look at students' actions around the task. How did students interact with task? Did they ask questions? Or were they off-task? This allowed me to look at my interactions with my students while we did the algebra. By doing this, I could see what types of actions were made by the teacher and students. If the teacher and students interacted with the content, the types of actions between the teacher and students would be about the content. The classroom environment level allowed me to answer the following questions:

- What types of interactions occur in the classroom?
- What types of actions are made by the teacher?
- What types of actions are made by the students?

For the third question:

b. How do instructional practices using this approach function within the classroom?

At the instructional triangle level, the algebra content takes the form of tasks. This perspective allows an insider's view of how students and I talk about specific tasks and shifts to describing how students approach, solve, and react to algebra tasks. From the instructional triangle level, the following questions arise:

- How do students and I interact to algebra tasks?
- What teacher decisions are made in the classroom to my students' responses?
- What are the algebra obstacles and how do I address them?

Both perspectives will be used to tackle the overarching question of this study:

> What does it take for a teacher to teache algebra for understanding in an inner-city high school?

These perspectives provide a more complete picture of teaching. I begin first with the classroom environment and in a later section I will present how the instructional triangle functions in the classroom.

Classroom Environment

For this analysis, I begin with the classroom perspective of teaching and focus on the actions made by the students and me. Doing this allows me to get a glimpse of an average classroom. At the end of this analysis, I present a classroom enactment in which I had great difficulties teaching to reveal the tenuous nature of teaching in the inner-city high school, but also illustrate what I encountered as a novice teaching at JHS.

I begin with a global look at the teacher and students' action. After this analysis, I looked more closely at what the teacher and students did in the classroom and finally looked at the teacher and students' action during mathematical discussions and in-between mathematical discussions.

Actions

I begin by showing the actions made by the teacher and students. Table 4-1 contains the frequency and percentages for type of actions made.

Table 4- 1. Frequency and percentages by type of action.

Categories	Number of Actions (*n* = 13,799)	% of actions
Teacher	7,598	55 %
Students On-task	5,347	39 %
Students Off-task	854	6 %

Table 4-1 shows the teacher did not dominate the discussion and students' action were on the mathematics. These figures suggest three possibilities: (1) I taught with no participation from the students; (2) I lectured with students asking questions interspersed in between; or (3) students and I had conversations about the task. In the next section I looked into the teacher and students' actions.

Teacher. Table 4-2 lists the percentages for content and management for teacher's actions.

Table 4- 2. Classification of content and management for teacher's actions with percentages (n=7,598).

Content		Management	
Direct / Ask	50%	Discipline	5%
Clarify / Explain	23%	Transition	2%
Verify / Praise	11%	Delegate	2%
Gauge	6%	Teacher-student	1%
		Procedures and Rules	1%
Total	89%	Total	11%

Table 4-2 shows I made more actions with content (89%) than for management (11%). About 50% of the teacher's actions directed students to focus on a procedure or calculation or asked for the next step in the task. About 5% of teacher's actions were for disciplining students. This asserts that I spent my time discussion mathematics with my students.

Students. Table 4-3 lists the percentages for on-task and off-task in descending order for the students' actions.

Table 4- 3. Percentage of students' actions for on-task and off-task (n = 6201)

On-task		Off-task	
Comments	66%	Remarks	12%
Help	13%	Lack of Supplies	1%
Student moves	4%	Places (Bathroom, Lab)	1%
Satisfaction	3%	Fight	0%
Time	1%		
Total	86%	Total	14%

Students were on-task (86%) and a majority of their actions were about the mathematics (66%) or seeking assistance (13%). When students were off-task, a majority of their actions were comments off-topic or talking with their friends (12%). This asserts that students were on-task and made comments about the mathematics and when students were off-task, students disrupted the class by making comments.

Mathematical Discussions

Table 4-4 provides another view of the classroom that includes the arithmetic and algebraic discussions and the teacher and student's actions when not discussing mathematics.

Table 4- 4. Percentages of actions that includes algebraic and arithmetic discussions (n =13,799).

Categories	Percentages
Algebraic discussion	62%
Arithmetic discussion	2%
Teacher's action	18%
Students' on-task	13%
Students' off-task	5%

The classroom spent 62% on algebraic discussion and only 2% on arithmetic discussion. This classroom was dominated by algebra. When I gave students time to work on a task, students were on-task 13% and off-task 5% during these periods between discussions.

A Typical Class Period. In this section, I separate the mathematical discussions (64%) into the teacher's and students' actions to reveal what the members did during these mathematical discussions and what members did between mathematical discussions (36%). Table 4-5 is the percentage for teacher's and students' actions during mathematical discussion.

Table 4- 5. Percentages for teacher and students during mathematical discussions.

Teacher's action (n = 5,179)		Students' actions (n=3,719)	
Content		On-task	
Direct/Ask	20%	Comment	23%
Clarify/Explain	9%	Help	2%
Verify/ Praise	5%	Student moves	0%
Gauge	1%	Satisfaction	1%
		Time	0%
Total	35%	Total	26%
Management		Off-task	
Discipline	1%	Remarks	1%
Delegate	0%	Lack of Supplies	0%
Transition	1%	Places (Bathroom, Lab)	0%
Teacher-student	0%	Fight	0%
Procedures and rules	0%		
Total	2%	Total	1%

This table shows that during mathematical discussions (64%), students and I worked on the tasks with me directing and asking (20%) and students making comments about the task (23%). During mathematical discussions, students made few off-task remarks (1%) and I had few discipline issues (1%). Table 4-6 presents the actions that are not embedded in mathematical discussion.

Table 4- 6. Percentages for teacher and students in between mathematical discussions.

Teacher's action (n=2,419)	Students' actions (n=2,482)

Content		On-task	
Direct/Ask	8%	Comment	7%
Clarify/Explain	3%	Help	4%
Verify/ Praise	1%	Student moves	1%
Gauge	2%	Satisfaction	1%
		Time	0%
Total	13%	Total	13%
Management		Off-task	
Discipline	2%	Remarks	4%
Delegate	1%	Lack of Supplies	1%
Transition	0%	Places (Bathroom, Lab)	0%
Teacher-student	1%	Fight	0%
Procedures and rules	1%		
Total	5%	Total	5%

It appears that when I gave time to students to work on tasks, students and I continued to work on-task while I walked around the classroom. During these episodes between the discussions, students made more off-task remarks (4%) and I took more discipline actions (2%). Table 4-5 and 4-6 represent for me a period of teaching. Although there are variations in these 23 enactments, the narrative below provides a way of understanding what the percentages look like in a classroom.

As students are entering the classroom, I have already placed a mathematical task on the overhead. I ask them to get started on a task (direct/ask), to take out their homework (procedures and rules), and for some, I ask them to be quiet (discipline). As I walk around the classroom, I make comments about their homework (verify/praise and gauge) and talk to students (teacher-student).

During this period, some students ask about the task (comments and help) and there are some students who continue to talk with friends (remarks). As I pass by their desk, some ask for bathroom passes (places), assistance (help), or paper and a writing utensil (lack of supplies) while some ask if they can do the task on the overhead (student moves). When the timer beeps, some students ask for more time (time). Depending on the noise level in the classroom I either give them more time or ask students to collect the work (delegate) and ask students to grade the work (delegate).

The noise level dies down as I go over the task (fewer discipline issues and remarks). I ask them how to start the task (direct/ask) and students provide a suggestion or a procedure (comment). The students and I continue exchanges for all parts of the task. Depending on the students' reactions to the task (help and/or satisfaction), I either give them another similar task or transition to the objective of the lesson (transition).

There are variations of this cycle of task, time, discussion, transition, and new tasks. But a majority of the classes followed this cycle. Once the lesson is finished, I give them homework. When the class was in computer lab, we worked with the computer software, *Carnegie Tutor: Algebra 1* (Carnegie Learning, 1998) for last 15 minutes of class and I often did not give time for homework in class. Before I end this section, I feel it necessary to provide an example when class behaved poorly. I do this to illustrate that a classroom can erupt into disorder even for an experienced teacher. I also want to show that when I began teaching in 1993, most of classroom enactments were similar or worse than this.

A Bad Class

For this "bad class," I provide the classroom environment and move inside the classroom to look at what occurred between the students and teacher. I begin with the classroom environment and move to the instructional practices that occurred that day.

Classroom Environment of a Bad Class

In this section, I present the classroom environment of this one enactment. This "bad class" occurred only once in the semester and I compared this class to the other 23 enactments. Table 4-7 presents the classification by actions for this class and for the average class.

Table 4- 7. Classification of actions between November 13th class and an average class.

Categories	% of actions for November 13th (n = 360)	% of actions (n= 13,799)
Algebraic discussion	29%	62%
Arithmetic discussion	0%	2%
Teacher's actions	33%	18%
Student on-task	22%	13%
Student off-task	17%	5%

This table shows that during the November 13th class I had to do more actions and achieved less algebraic discussion (29%). Students made more off-tasks actions, but I didn't abandon my lesson plan and we did manage to do some mathematics. In order to understand what I was doing in this classroom, I compared the November 13th enactment to an average class during mathematical discussions, which is in Table 4-8.

Table 4- 8. Percentages of my actions during November 13th and the average enactment during mathematical discussions.

Teacher		
Content		
	November 13th (n = 360)	Average class (n = 13,799)
Direct/Ask	9%	20%
Clarify/Explain	4%	9%
Verify/ Praise	4%	5%
Gauge	0%	1%
Total	17%	35%
Management		
Discipline	1%	2%
Delegate	0%	1%
Transition	0%	0%
Teacher-student	0%	1%
Procedures and rules	0%	1%

Total	1%	5%

This table shows that I did fewer actions during the November 13th enactments, because the mathematical discussions were much less than the 23 other enactments. Table 4-9 provides the percentage for students' action in between mathematical discussions for this one enactment on November 13th and the other 23 enactments.

Table 4- 9. Percentages of students' actions during November 13th and the average enactment during mathematical discussions.

Students		
On-task		
	November 13th (n = 360)	Average class (n = 13,799)
Comment	10%	23%
Help	1%	2%
Student moves	0%	0%
Satisfaction	0%	1%
Time	0%	0%
Total	11%	26%
Off-task		
Remarks	0%	1%
Lack of Supplies	0%	0%
Places (Bathroom, Lab)	0%	0%
Fight	0%	0%
Total	0%	1%

This table shows that with few episodes of mathematical discussions, students made fewer comments (10%) about the algebra tasks. In algebraic discussion (29%), I made 19% of the action and students made 11%.

Since this was a bad class, most of my actions and students' actions occurred between mathematical discussions. Table 4-10 presents the percentages of my actions in between mathematical discussions.

Table 4- 10. Percentages of teacher's actions in between mathematical discussions.

Teacher		
Content		
	November 13th (n = 360)	Average class (n = 13,799)
Direct/Ask	11%	8%
Clarify/Explain	4%	3%
Verify/ Praise	3%	1%

Gauge	3%	2%
Total	21%	13%
Management		
Discipline	7%	2%
Delegate	2%	1%
Transition	1%	0%
Teacher-student	1%	1%
Procedures and rules	1%	1%
Total	12%	5%

During these in-between discussions for the bad class, I took more actions by directing students' action toward the mathematics tasks (11%) but I also had to do more discipline (7%) actions than an average classroom. Overall, I took more actions for content and management than an average classroom. Table 4-11 lists the percentages for the bad class and the average class for students' action in between mathematical discussion.

Table 4- 11. Percentages of students' actions in between mathematical discussions.

Students		
On-task		
	November 13th (n = 360)	Average class (n = 13,799)
Comment	14%	7%
Help	4%	4%
Student moves	2%	1%
Satisfaction	1%	1%
Time	1%	0%
Total	22%	13%
Off-task		
Remarks	16%	4%
Lack of Supplies	0%	1%
Places (Bathroom, Lab)	1%	0%
Fight	0%	0%
Total	17%	5%

For this bad class, I left the students to do the task, because I was unable to do a discussion with them (22%). Students also used these episodes to make more off-task remarks (16%) that disrupted the class.

Overall, these tables (Tables 4-8 through 4-11) hint that I was very close to giving up on this lesson. All of the content and management measures are lower than an average classroom. I was very frustrated at students' behavior. Some students did offer suggestions on the task and asked for help, but much lower than in an average classroom. In an average classroom, there are transitions between tasks or activities and students may talk more during these time period. For this class, the transitions were difficult to determine, because the start of one action was interrupted by another action and the mathematics discussions became fragmented.

Instructional Practices of a Bad Class

For this section, I want to provide an analysis of the instructional practice of this bad class that occurred November 13th to provide context to these percentages of the last section.

> It's been a frustration trying to determine how to handle the stress in this building. I may need to do more exercises or I may need to do less in this building. My head is so tired right now. Journal entry: November 13th, 2006

I wrote this journal entry before the start of my 6th hour class. This was the 41st day of teaching and I was beginning to feel the grind of waking up at 5:30 AM and leaving the building at 5:00 PM. The week before, JHS had standardized testing for seniors and for our 9th graders; they had been coming in at noon for thirty minute classes and this day would be the last day. This 6th class was one of my favorite classes; therefore, even with low energy I felt that I could teach a half-hour class.

For the past few weeks, I had modified my row-column format of desks by pushing the two desks together so partners could help each other. This could also lead to more off-topic conversations. While proctoring the standardized testing, I had a week to think about what I wanted to teach. Two things came to mind: (1) the teaching of solving linear equations with variables on the same side and on opposite sides; and (2) the re-introduction of the iterative notations with the two other operations.

Variables on the Same Side. In the normal progression for the solving of linear equations, students encountered equations with variables on the same side. In prior years for a task $2x+3x+1=26$, I asked students to combine like terms before solving. My approach to algebra was leaning more towards "Why would students solve a task like this" rather than to teaching the actual procedures for solving. I was also the Pre-Calculus teacher; therefore, I realized that the tasks used for the combinations of functions could be modified for my 9th graders. By doing this, I introduced a Pre-Calculus topic for my 9th graders. I went back to the linear pattern and created this task:

> Given the following linear sequences 5, 7, 9, 11, 13,… and 4, 8, 12, 16, 20,…
> a) What is the sum of the sequences for the 5th term? 8th term? And 1000th term?
> b) When will the sum of the sequences equal to 15? 33? And 69?
> c) What is the algebraic representation for the sum of both sequences?

This task was an extension of our work with numerical sequences and I anticipated students to have questions about the sum.

Bathroom Please. After our warm-up tasks, students were not quite settled and there was still a lot of noise in the classroom. The excerpt shows my attempt at trying to get the class back on task.

62 Clorissa: Mr. Pan, can I go to the bathroom?

63 Teacher: Yeah, I believe you asked first, right?

64 Natasha: Do I get to go first?

65 Teacher: I think you asked first, I think she asked first. Antonia, sorry, not Antonia, Natasha. So, instead of giving you…

66 Toshel: Can you all quiet down? God damn [yelling]

67 Natasha: This is not a Kodak moment

68 Teacher: You don't to need cuss dear, just ask nicely.

I was tired and had difficulties determining which student should go to the bathroom (lines 63 and 65). My response to these counterscripts was to try to appease the students and I was trying to balance the students who wanted to leave and those who were waiting for the lesson. During this period between tasks, students became off-task (lines 66 and 67) and I tried to write the task on the overhead and I hoped the task would quiet the students because my discipline tactic wasn't that forceful (line 68).

Another Interruption. This excerpt below that shows how the students continued to talk and I did not have the energy to think of the right tactic to settle them down.

108 Teacher: ….Alright, go and do the eighth term…
Go ahead and do the eighth one….

109 Brittney: Shut up….

110 Teacher: Everybody set yet for this one

111 Christian: Wait, you be moving too fast

112 Toshel: You don't give us no time

113 Tiara: You better wait…

114 Alonzo: He said that for all you people

115 Christian: I am glad today is the last day for this, all these people that acting crack, they're gonna make me flip

We had already discussed how to find the 5^{th} term. In order to find the sum of 8^{th} terms, students can extend the both patterns which was a procedure already taught; therefore, I assumed that students could do this task (line 108). I was wrong. The students didn't ask for more time but they criticized me (lines 111, 112, and 113). I was now a bad teacher, because I didn't give them enough time. Christian attributed all of the noise because of the testing period (line 115). This small disruption in the schedule was wreaking havoc on me and my students. I decided to push forward and finished the lesson to show how simple this lesson was. I believed that by moving toward the content, I could deal with these counterscripts.

Finding the Sum for the 1000th Term. I continued with the task and hoped that the class would settle down. In the excerpt below, I tried to move from arithmetic into algebra by having students find the 1000^{th} term.

150	Teacher:	Alright, the 1000^{th} term
151	Christian:	If you don't put these cameras up?
152	Teacher:	Put them away please, let's go, the 1000^{th} term
153	Alonzo:	How are we gonna do that?
154	Teacher:	Same thing
155	Christian:	Both of them

In order to solve this task, students could find the algebraic representation for both patterns and apply the algebraic representation to find the 1000^{th} term for each pattern. Students were still being disruptive and I felt the need to discipline the class, but also direct the students back to the task (lines 151 and 152). Alonzo was usually a constructive student, but I felt his comment was not conducive to the class and I felt that Alonzo's comment was critical of me and my teaching (line 153). I decided not to tell them to find the algebraic and implied that they have already done this (line 154) and from Christian's response I felt she was heading in the right direction. The next excerpt presents our continued search for the sum for the 1000^{th} term.

163	Alonzo:	Ain't nobody gonna write a 1000^{th} term
164	Teacher:	So what should we do?
165	Christian:	Divide [yelling]
166	Teacher:	[Laughter from class] Christian, Christian, I'm gonna have to put you out, cause you are you are a little too loud for me right now You came running in here and I don't need that right now.
167	Female #2:	I don't know how to do this

168	Chartonya:	Mr. Pan, find your algebraic

I would expect Alonzo's statement from the first week of algebra but not for the 41st day of algebra class and his statement in my opinion was critical of teaching (line 163). Christian who I thought was on right track (line 155) ended up disrupting the classroom by her comment (165). Another student chimed with another negative comment (line 167). I sensed that I was slowly losing control of the class. Every comment made was up to this point negative. Through all of the noise, Chartonya told the class to find the algebraic (line 168). I really didn't know what I would do if this hadn't occurred. I probably would have told the class to find the algebraic representation. I was in no mood to wait for them and had no solution to gain control of the classroom.

Mathematics Amid the Chaos. My patience for this class was very thin. I wanted to get done with this lesson and I was going to drag my students cussing and yelling. With more noise in the classroom, I pushed forward by walking around the classroom. The excerpt below is our continued work in solving for sum for the 1000th term.

178	Teacher:	Britney, come on stay out there
179	Student (outsider):	You didn't hear me at the door…
180	Teacher:	Alright, cause they are bothering the class, I can't stand that. Five, seven, nine, eleven, thirteen, algebraic is?
181	Kiara:	n equal to two t plus three for the first one
182	Teacher:	And I put the one for the first one
183	Christian:	Wait a minute, wait a minute, before you go on, how did you get the algebraic?
184	Teacher:	I am not gonna tell you.

I could not lock my door and left one of my students outside because she was late (line 178). Another student, not a member of my class, walked into my room (line 179) and I immediately told her to get out (line 180). For this particular student, she left but there have been situations when students don't leave the classroom. I was also at fault, because I left one of my students out in the hallway where she could now walk around and misbehave. This illustrated the lack of a tardy policy for the whole school.

Frustrated, I tried to get back to the work (line 180) and Kiara provided the algebraic representation for the first sequence (line 181). When Christian stated she couldn't find the algebraic (line 183), I had no more patience for this and would not explain it to her (line 184). It wasn't just Christian, but the cumulative effect of students' bad behaviors that pushed me over the edge.

A Student Speaks Up. Amid this noise, I continued with the lesson and some of students like Kiara seemed to understand my predicament as a teacher. Seeing that I was incapable of leading the class, Kiara disciplined the students for me in the excerpt below.

185	Kiara:	As much as we have been going through this, you all telling me you can't do this ……Anyway for the next one …Because it's jumping by four and if you all can't tell, there is something wrong with that
186	Breanna:	[Laughter] Everyone don't know it Kiara
187	Kiara:	Well, they need to try learning this cause we been going through this since…
188	Teacher:	September
189	Toshel:	Why do guys have to get mad for you all? [yelling]
190	Teacher:	Two t
191	Christian:	I don't know
192	Kiara:	You all get mad at him, don't get mad at him

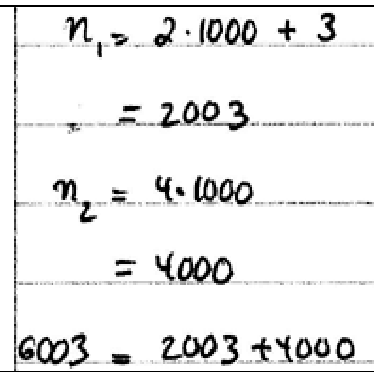
Kiara was able to explain and criticize the students at the same time (line 185). Breanna tried to explain why she couldn't do the task (line 186), but her explanation was rebuffed by Kiara and me (lines 187 and 188). Toshel's yelling made it an even more unbearable situation (line 189). I tried to interject and bring control to the situation (line

190) and Kiara tried to get the students to focus their frustration away from the teacher (line 192).

Kiara's actions were voluntary actions made that help me gain control of the classroom. These actions were rare, because these actions are associated for me. Having Kiara disciplined the students (line 185) seemed more genuine than having me doing it.

The 1000th Term. After all of the yelling, we were almost at the end of the lesson. The class was filled with a lot of anger and frustration. Figure 4-1 shows our work as we arrived at the sum for the 1000th term.

Figure 4- 1. Finding the 1000th term.

Task	Work
Given the following linear sequences 5, 7, 9, 11, 13,… and 4, 8, 12, 16, 20,. b) What is the sum of the sequences for the 5th term? 8th term? And 1000th term?	

Kiara was not finished with her comments to the class and in the excerpt below she continues our work in fining the algebraic representation.

196 Kiara: Don't get mad you all, it's all your fault

197 Christian: I get mad

198 Teacher: So now…..So the first one is ….So..the 1000th term [more noise] so this is two times one thousand plus three

199 Christian: Two thousand and three

200 Brittney: Hey you all, where did the three come from?

201 Toshel: That's what I got

202 Kiara: The other one is four thousand

203 Sarah: I got to write this down, Mr. Peter

204 Kiara: Four thousand plus two thousand and three

205 Jayvon: 6,003

206 Ashley: That’s what I got.

Kiara continued to scold the students (line 196). While other students were quite critical of me, her statement placed the blame back on them. I decided not to interfere and continued with the work (line 198). As a teacher, I don’t often express my emotions outwardly, but I was thinking, how did I get myself into this situation? Kiara took the role of the teacher and was able to teach and reprimand the students. Christian gave me the 1000th for the first number sequence (line 199). I was baffled but pleasantly surprised by Christian’s response (line 199) along with Toshel (line 201). Kiara gave the 1000th term for the second number sequence (line 202) and Kiara showed the class how to find the sum for the 1000th term (line 204). There were a few students who were working on the task.

Removal of a Student. Christian continued to talk and would not quiet down. I decided to remove Christian from the classroom. This was the first of two students I had to remove from class that day.

212 Teacher: Christian, I want you to go, come on. You’re not in for it today. You’re just, you’re too hyper today.

213 Christian: I don’t care.

214 Teacher: Give me your ID. There you go, thank you. (door shuts)

I could no longer take Christian’s antics and I asked her to leave (line 212). I reminded Christian about keeping her behavior (line 166) but she kept pushing. Students often look at the last instance that pushed me over the edge and don’t understand why they got into trouble. During my conversation with her after class, she reiterated that she did nothing wrong in class. Her actions included running into the class and yelling numerous times when I was about to gain control of the classroom. Removing this student was especially difficult because she was an integral member who participated and volunteered in the class. I escorted the student out of the classroom and when I returned I decided to give them a lecture on their behavior in the excerpt below.

250	Teacher:	You guys are 9th graders. When one person gets riled up, you guys all get riled up and you gotta learn not to rile it all the way up.
251	Natasha:	What is that? When are what?

I tried to explain my expectation of them as 9th graders (line 250), but Natasha wanted me to go back to the mathematics. My interpretation of this event was that my students do know how to behave in a classroom, but had chosen to take a different route. I have noticed that after an incident like this, the next day students are more focused on the work. Students would rather do the mathematics than listen to me lecture.

Reflections. Although this class was only 30 minutes long, this was a disastrous class. I had students talking and cussing. I had to remove one of my favorite students from class[1]. I had no energy to maintain control. Days like this occurred more frequently when I first taught in the fall of 1993, because I didn't have the appropriate responses which were compounded by students not connecting with the content I was presenting. Yet, I felt so unprepared this day. Through this noise, Kiara spoke to the class about their behaviors. Another student, Chartonya, worked on the task while other students kept talking. I had at least two students who wanted to learn.

There are many possible factors that affected my classroom: shortened schedule, new lesson, desk arrangement, and/or my lack of energy. Another possible reason for their behavior was that in their other classes, students only talked and took pictures. Many of my classes were like this when I began teaching, because the students were not connecting to the content, but these students though don't have this excuse and Kiara told

[1] Christian was suspended for one day.

them. I could have stopped teaching and allowed the students sit and talk for this shortened class period. I just felt that as a class, we needed to be able to work around the interruption of the school's environment.

Discussion

The research question that guided this analysis was: What does the teaching of algebra using this approach look like in the classroom? Using TGT with a numerical pattern approach to the teaching of algebra, the students and I spent a majority of our actions on the discussion of the algebra tasks. I didn't lecture to my students, but help guide them in the solving of tasks. With students making comments and with me directing them to different parts of the task. My students had a weak understanding of arithmetic and with a numerical pattern approach; I was able to review arithmetic along with the algebra. Thus, this class was not dominated by arithmetic.

I also presented an episode to show how a class can turn chaotic even for an experienced teacher. I did this to show what I experienced daily as a novice teacher, but that chaos in the inner-city classroom can only be buffered until the school environment changed. I did not provoke my students' anger and frustration, but became their target of attack. Kiara was able to deflect some of their frustration, but what would have occurred if she wasn't there? There are not many good options for the teacher in a chaotic classroom. Kiara was one of my better students and would not allow the students to destroy the classroom environment.

These analyses offer another view of the inner-city classroom. It wasn't the chaotic classroom where I taught small groups of students while the others students were off-tasks. It wasn't a classroom where I broke up fights and it wasn't a classroom where I reviewed arithmetic. All of these events did occurred during my first year at JHS, but this chaotic classroom need not be the norm for an inner-city classroom.

Teacher-Students-Content

Teaching is a physically and mentally demanding job. My feet are sore after five hours of standing and directing students. My mind is mentally fatigued from balancing the subject matter and the demands of my students. After a decade of teaching at the same high school, I had developed a routine. I taught Monday through Thursday and gave my students an assessment every Friday. During the weekend, I focused on writing lesson and grading the assessments.

Separating the teaching of the week from the preparation of the weekend allowed me to focus my lesson writing after grading the assessments; thereby, I could decide when to move to another section or try a different approach to a difficult topic. Previously, I provided a description of what transpired in the classroom. For this section, I will further expose what teaching teacher generated tasks, TGT, algebra looks like in my classroom, and how I moved from more chaotic class periods like my "bad class" above to more consistent, effective instructional practice.

After a decade of teaching, many might assume that I have a complete understanding of my pedagogy and content. I find, however, that I am still learning about the topics and how to teach them. By using TGT algebra, I encountered obstacles such as defining and understanding the 0^{th} term, the subscript, or the iterative notation and I struggled along with my students with meaning and understanding. I was re-learning algebra along my students even as I changed the way I taught it.

In this section, I move away from the classroom environment and move toward my teaching. I have identified five different teaching challenges within Larson five chapters: (1) Connections to Algebra; (2) Properties of Real Numbers; (3) Solving Linear Equations; (4) Graphing Linear Equations and Functions; and (5) Writing Linear Equations. I organized it in this manner to maintain a chronological order with Larson, but also to illustrate how I tried to connect the algebra concepts between chapters.

Within each of these chapters, I will try to mix in lessons, discussions, my work, student's works, and journals to provide a view of what instructional practices with TGT looks like.

For each of the five chapters, I present the difficulties I encountered when teaching these topics in the chapter and how the teacher and students interacted with content and unpack how I responded to students' actions. I also show the students work to illustrate how students approached and solved these tasks.

Chapter 1: Connections to Algebra (Day 4)

The first few days of school are filled with confusion at my high school. Students who came for summer orientation pick up their schedules, fill in their forms, and attend classes. For the other students, counselors need to obtain students' records from their middle schools and make schedules for them. Incoming students fill out cards with their contact information so that teachers can use them to keep record of attendance. Instead of working on algebra, students and I spend time filling out forms. Due to the high number of changes in the roster, teachers are not provided with the record keeping books until the fourth or fifth week of school.

The district development team also understands this fact and starts the teaching of algebra on the fourth day of school (refer to Figure 3-5). For the first day of algebra, the third day of the semester, I don't do icebreakers. I don't give a speech about my rules. I don't tell my students of my expectations; instead, I teach them algebra. I do this because my school isn't an "academic" high school, where students are tested or selected before the start of the academic year. My high school is a "social[2]" high school, where the school climate is less about academic success and students come to meet with other

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[2] My peer, Mr. Tracy, coined these terms about our school.

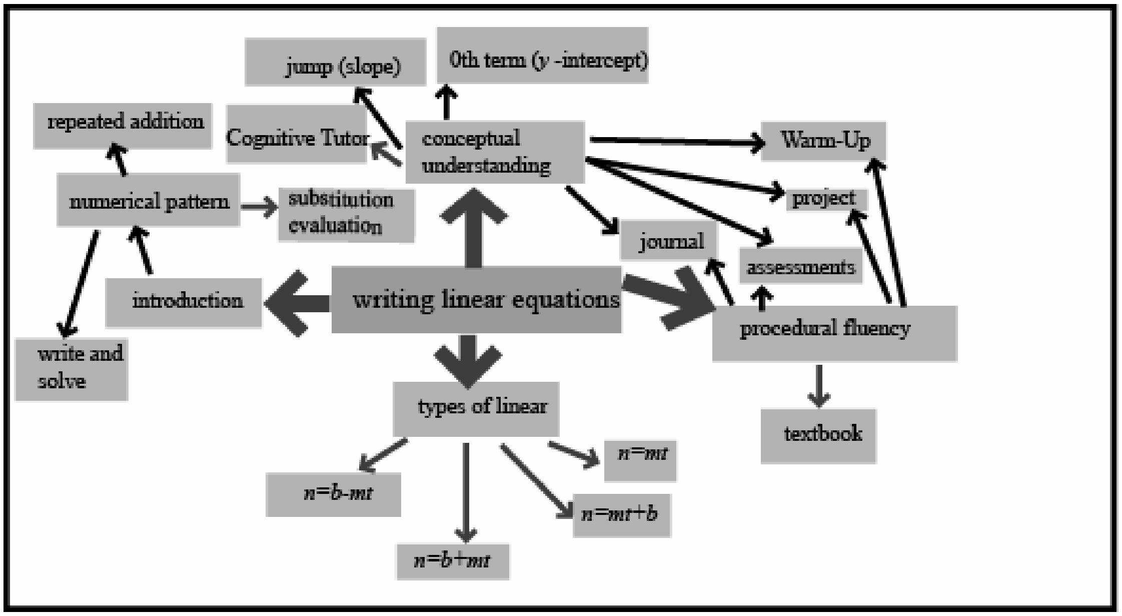
students. I have to provide to my students a reason for showing up other than the "social" atmosphere of this high school. Brooks (1985) writes:

> Within this context, students form their first impressions about the teacher and may even make impressions about the teacher and may even make judgments about the teacher's competence. Veteran teachers believe that teacher behaviors during this session set the tone for subsequent sessions; a tone that may extend for the duration of the school year. (p. 63)

Moreover, my years of teaching have shown that teaching days disappear due to school functions, standardized testing, and school policies for ending a semester. I would not contribute to this by wasting instructional time. I wanted to establish my classroom environment by making the instruction the discipline.

My objective for the first week was to give my students a taste of what algebra *is*. I began with numerical patterns to expose students to how to evaluate, solve, make table, and write algebraic representations albeit at a very basic level. Instead of thinking of these as goals to be mastered (e.g., evaluate, solve, make, and write), I thought of these goals as questions to be asked. Why would you evaluate a variable expression? Why would solve an equation? Why would you make a table? Why would you write an algebraic representation? By thinking in this manner, I was able to construct tasks that provided a rational doing these goals. Figure 4-2 is a pictorial representation of my current domain map for the writing linear equation using TGT.

Figure 4- 2. Pictorial representation for the domain map for writing linear equation.



This figure shows my domain map and serves as a guide for my approach to the writing of linear equations. The domain map contains four elements: (1) introduction, (2) conceptual understanding, (3) procedural fluency, and (4) types of linear equations. The introductory task is a numerical pattern task that offers opportunities for students to use their arithmetic skills before delving into algebra. These introductory tasks are used to build conceptual understanding of the "0^{th} term" and "jump", which represent *y*-intercept and slope respectively. Understanding the "0^{th} term" and "jump" allow students to write linear equations. Depending how well the students understand the task, I might give them more tasks similar to the introductory or Warm-Up tasks or modify it to a different type of linear equations. I also use *Cognitive Tutor* to build conceptual understanding of how to write linear equations and the textbook tasks to build procedural fluency which in this case is the substituting and evaluating of variable expressions. Projects, journals, and assessments are use to help build conceptual understanding and procedural fluency for the different types of linear equations.

The next section presents tasks I used for the first day of algebra and ends with students work to illustrate the type of work done in this classroom.

<u>The First Task (Day 4)</u>. I was ready for my 6th hour by adding a few more tasks to the lesson. I made the following journal entry prior to the class:

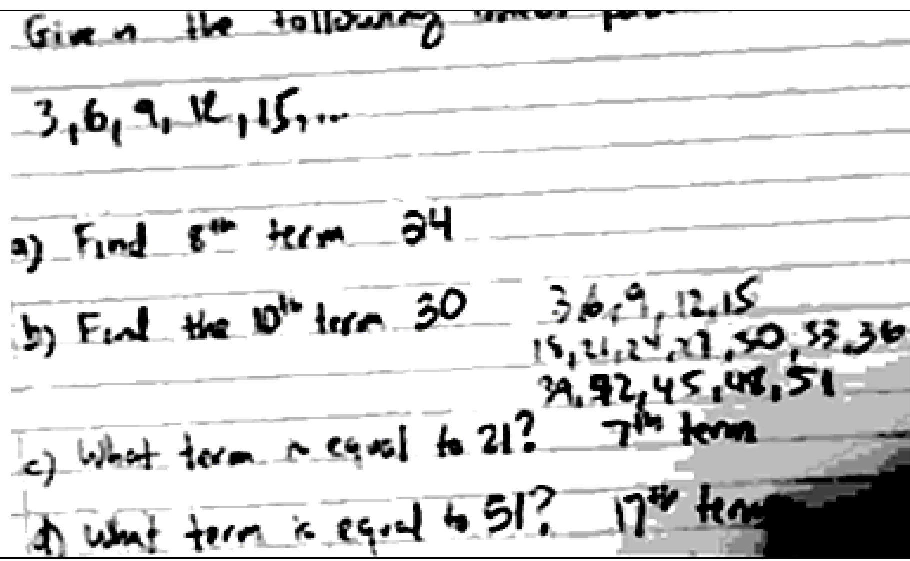
> The 3rd hour class went better than 2nd hour. I felt that I wasn't trying to fill in material to keep the class going. 3rd hour worked well and I felt that they understood the variable expression. I must do a better job of explaining addition and multiplication. Journal Entry: September 18th, 2006

I felt that I corrected the mistakes made in the earlier classes. I wrote the task in Figure 4-3 on the overhead and provided my students five minutes to complete the task. These five minutes allowed my 15 students to settle down and allowed me time to do attendance and help my students. This task in Figure 4-3 has five parts that allows students to achieve some type of success (Lampert, 1990). Figure 4-3 presents the task and my work.

Figure 4- 3. Task and work for the first task.

Warm-Up: Given the following pattern 3, 6, 9, 12, 15…

a) Find the 8th term.
b) Find the 10th term.
c) Find the 15th term.
d) What is the term when the number is 21?
e) What is the rule for this pattern? *(algebraic representation)*



In the excerpt below, I showed how we solved for the 8th term in the sequence and how I handled a situation about the dress code. These were my first mathematical discussion with my students.

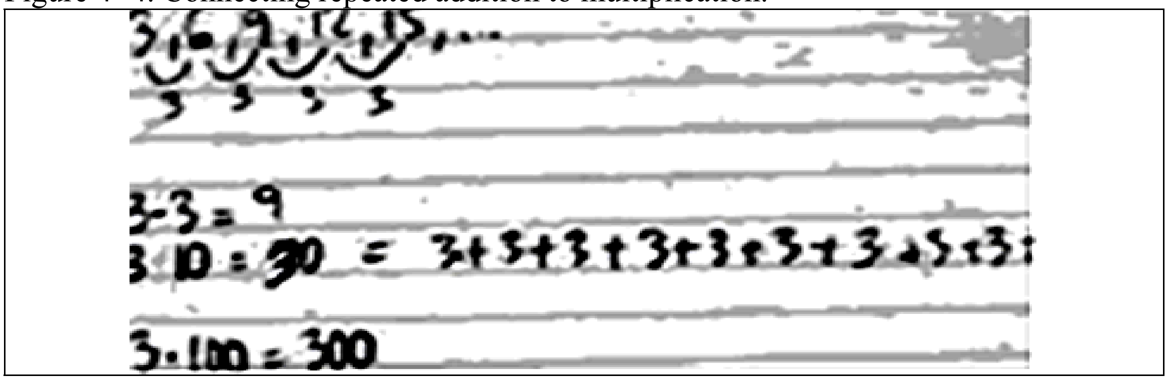
1	Teacher:	Dot, dot, dot, means [pause] it goes on forever. So a, find the 8th term and what did everybody get?
2	Several:	Twenty-four
3	Teacher:	Mickey, put the hood down man or whatever you got there. So it's going 3, 6, 9, 12, 15, 18, 21, 24. So this is actually 24. 1 point. Put a plus one. Second one b. Find the 10th term, so 3, 6, 9, 12, 15, 18, 21, 24, 27, 30 and the answer is?
4	Several:	Thirty

The first four parts of this task could be done without the aid of the algebra. I rewrote the task on a sheet of paper and projected it onto the screen (see work in Figure 4-3. Students had no problem with parts a) and b) (lines 2 and 4). I demonstrated by extending the numerical sequence without the use of algebra.

During the explanation, I ran into my first discipline issue. Depending how I handled this situation was an opportunity to build the classroom environment. My philosophy with students was that they needed to self-monitor their behavior. If a student became out of line, then I would do a friendly reminder. This discipline method worked as shown by the low number of discipline actions (see Table 4-2). I wanted to send the message early that the mathematics is the central to this class, and wearing a "hoodie" was implicitly not allowed in the classroom. Disciplining students in this manner also sent a message that I had high expectation for their behavior.

The writing of linear equations passes through the connection that repeated addition is multiplication. Making my students see this connection would not be easy. In order to do this, I would create a cognitive dissonance by asking students to determine the number for a very large term. I wanted my students to think of why they would need algebra. This is a continuation of the task above and for that first day. Figure 4-4 shows the work.

Figure 4- 4. Connecting repeated addition to multiplication.



This excerpt represents how I tried to get students to think about a different way to think about arithmetic.

26 Teacher:		The third term. If I said the 10th term, what would you do?
27 Mickey:		Three times ten
28 Teacher:		Three times ten or if you said add by three. You would go 3+3+3+…
29 Christian:		Ten times
30 Teacher:		3+3+3+3+3+3+3 ten times. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 (counting the number of threes) which is still equal to
31 Christian:		Thirty
32 Teacher:		Thirty, so this is why you can say adding by three or multiply by three. So what is I said the 100th term like Mickey asked me. What would the answer be?
33 Jayvon:		No one knows.
34 Mickey:		I know

I repeated Mickey's response (line 27) to show that he was corrected and tried to extend it to addition (line 28). With the help of Christian, I connected adding three ten times was the same as multiplying three times ten. Once this was done, I wanted students to think of how to apply this connection between repeated addition and multiplication with much larger number, because one example would not be enough to make this transition. This would serve as my rationale for writing linear algebraic representation and learning algebra. I created a dilemma for my students by asking them to find the 100th term (line 32). Jayvon was sure that this task was impossible to solve (line 33). This was the response I was looking for because Jayvon had presented to the class a challenge. Do I use arithmetic or is there some other tool out there? Even though Mickey was able to see the short cut to finding the 100th term (line 34), I wanted the other students to see this, too.

After repeated addition and multiplication, it was time to move toward the algebraic representation. Historically, this leap from arithmetic to generalized arithmetic was no easy task (Bashmakova & Smirnova, 2000) and from this excerpt below, it remained difficult.

40	Teacher:	So the algebra is? Give me the rule for this pattern. What am I actually doing? The shortcut way
41	Several:	Multiplying
42	Teacher:	Multiplying by what number?
43	Several:	Three
44	Teacher:	And what am I multiplying that number three by?
45	Mickey:	The variable
46	Teacher:	What is that specific variable though? It is called?

I felt that students could see that 100 times three was the same as adding 3s one hundred times. I struggled trying to make this leap to variables (line 40). I decided to have them focus on multiplication as the correct operation and then moved toward the coefficient. Several students responded correctly (line 43) and I moved toward the linear equation by now focusing on the variable. Mickey provided the correct response, but my response was inadequate and I should have asked what the variable stood for (line 46). Students were able to determine the operation and the coefficient but I could not get them to think about using the variables for input and output. This leap from arithmetic to algebra needed more work on my part.

We could write the algebraic representation, but determining what to call the components of an algebraic representation wasn't a matter of using the established terminologies such as slope and *y*-intercept. For Moses (2001) the use of "intuitive language" or local terms may provide a better understanding of the concept and the use of technical language may actually confuse the students. For my first day of algebra, I introduced the terminology "jump" as representing the slope.

I need to digress to talk about the *y*-intercept and the choosing of the variables. Four years ago, it was my year to re-teach Algebra One. During our work with finding the *y*-intercept, which was the term that I used, Chantel[3] looked up and asked me if finding the *y*-intercept was the same as finding the 0^{th} term. I was quite dumbfounded. This question from a student, who had failed the first semester, gave me confidence and the first clue that hints that students maybe connecting with these numerical pattern tasks.

Why didn't I think of that? She created her own terminology for the *y*-intercept, the 0^{th} term. We now used "jump" for the slope and "0^{th} term" for the *y*-intercept. I could see that students gravitating to this new found understanding. Chantel was able to connect information she learned last semester and applied it to this new situation.

Another decision made years ago was to use *t* for the term and *n* for the number[4] instead of *x* and *y*, because *t* and *n* gave more meaning to the numerical task. Internally, I worried that using *t*, *n*, "jump", and "0^{th} term" would confused my students when all the textbooks used *x*, *y*, slope, and *y*-intercept. Using these standard terms made the transition easier, but made understanding of algebra in the moment much more difficult. Explaining why *x* represent the term made no sense to my students.

<u>Writing the Algebraic Representation (Day 5)</u>. The objective for the second day of algebra was to learn how to write the algebraic representation with "jump" and "0^{th} term," which were now common elements in the TGT model. I asked my students to write a journal entry explaining the process with the following prompt: Given the linear pattern: 100, 105, 110, 115, 120,… How would you write the algebraic representation? In Figure 4-5 is one' student response.

Figure 4- 5. Journal entry on how to write algebraic representation.

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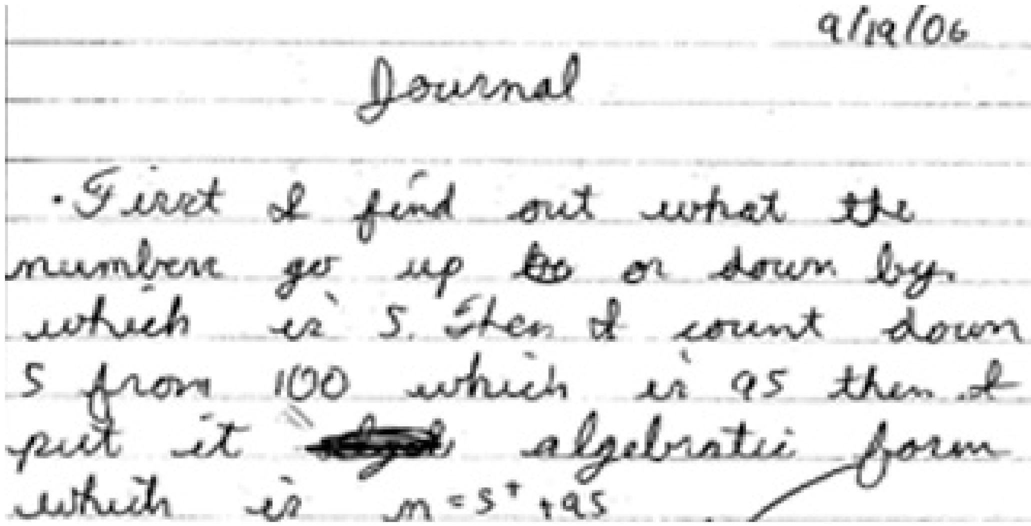
[3] For this academic year, a student made a suggestion on how to write the iterative notation. Another student improved my subscript notation when we worked with systems of equations. I took both of their suggestions.

[4] I changed this after talking to Ms Palmer, another algebra teacher.


9/19/06
Journal
· First I find out what the numbers go up to or down by. which is 5. Then I count down 5 from 100 which is 95 then I put it algebratic form which is n=5t+95

This journal entry shows that this student had no problem explaining how she would determine the algebraic representation from a given sequence of numbers. This was in stark contrast when I first taught the finding of “0th term” (y-intercept) four years ago. Finding the *y*-intercept was a stumbling block for my students, but after Chantel’s suggestion that the “0th term” was the *y*-intercept, finding the *y*-intercept took only one class period. This breakthrough with the “0th term” opened up new algebraic terrain for me and my students. I no longer have to worry about students’ confusion with my method for finding the *y*-intercept and students could devote their energy to finding the “0th term” with arithmetic.

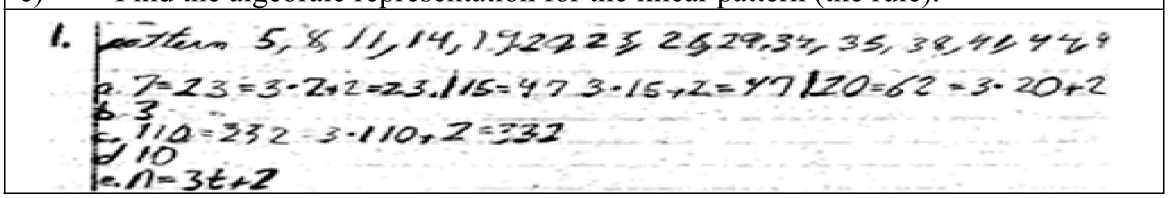
<u>Assessment (Day 11)</u>. The study of numerical patterns was the focus of our discussion for the first week of algebra, but I also introduced students to contextual problems along with geometric tasks and continued using the terms, “0th term” and “jump.” By doing this, a contextual task and geometric task were no different than a numerical pattern task. These terms were more flexible than the standard terms of *y*-intercept and slope, because these standard terms are more often associated with the

graph of linear equation. I have selected students works on an assessment to show what the 11th day of algebra looked like for this classroom, which was major feat for two weeks of teaching.

Figure 4-6 presents a numerical pattern task. Students had to show their work in order to receive full credit. Part b) of the task was a remnant of how I used to teach the finding of the y-intercept by having students determines the multiplication table and calculating the difference between the two numerical patterns. Since students didn't have the tools to solve an equation yet, I made the number small for part d).

Figure 4- 6. Numerical pattern task and work for a group of students.

1.	Given the following pattern 5, 8, 11, 14, 17,…
a)	What is the 7th term, 15th term, and 20th term?
b)	What multiplication table is this pattern derived from?
c)	Find the 110th term.
d)	When will the pattern equal to 32?
e)	Find the algebraic representation for the linear pattern (the rule).

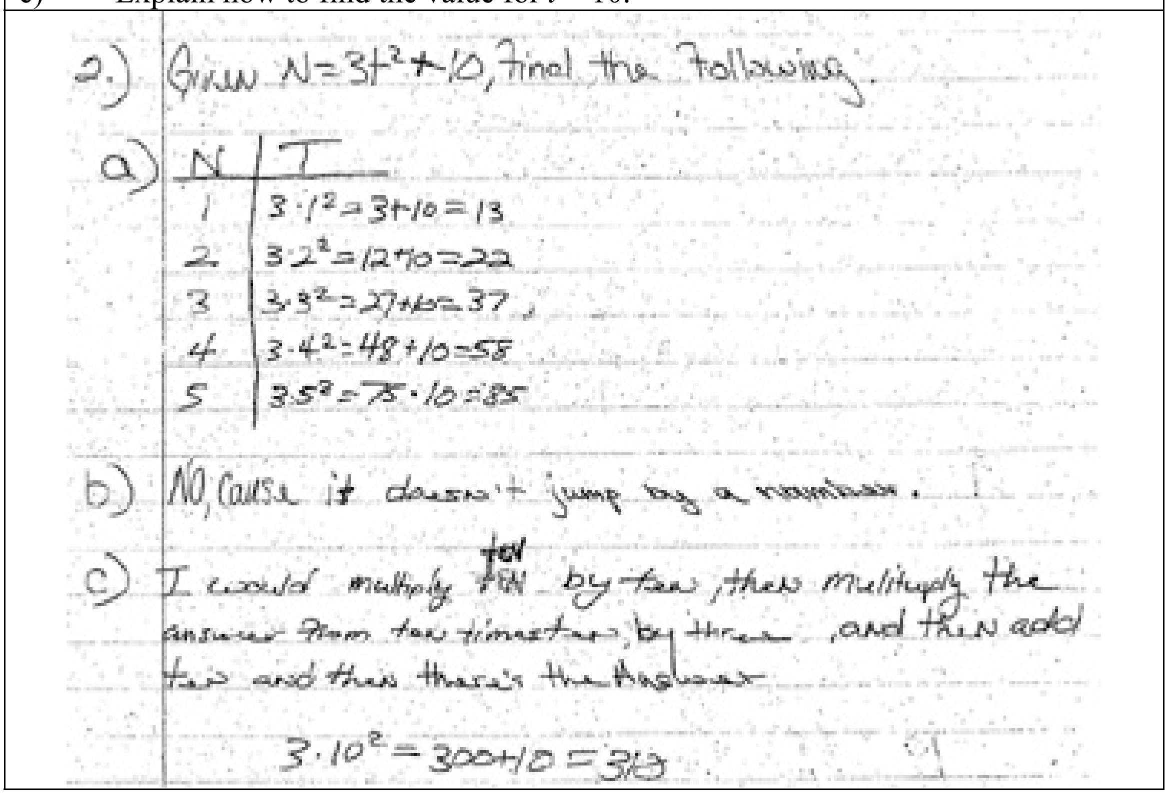


For this group of students, arithmetic was something they were comfortable with and algebra was the "new mathematics." They decided to be safe and extended the pattern and evaluated the algebraic representation for part a), but applied the algebraic representation for part c). Students' uneasiness with algebra would be evident in other assessments.

The task in Figure 4-7 was written because I was fearful that my students would not be getting the same algebra as Larson. Because Larson did work with order of operations and exponents, I decided to write a task that would apply the order of operations with a function. I did this to foreground the order of operation and background function. Since this was a digression from linear patterns, I wanted students to recognize that the pattern formed was not a linear pattern, hence, part b). I could tell students still disliked writing by the amount of groans I heard in the classroom; therefore, I wrote part c) to have them get used to writing in my class, but to explain how they applied the order of operations.

Figure 4- 7. Exponent and order of operation task.

2. Given $n = 3t^2 + 10$, find the following:
a) Make a table for the output *n* for the first five positive integer values for *t*
b) Explain if this pattern is linear.
c) Explain how to find the value for *t* = 10.


2.) Given N=3t²+10, find the following.
a)

N	T
1	3·1² = 3+10 = 13
2	3·2² = 12+10 = 22
3	3·3² = 27+10 = 37
4	3·4² = 48+10 = 58
5	3·5² = 75·10 = 85

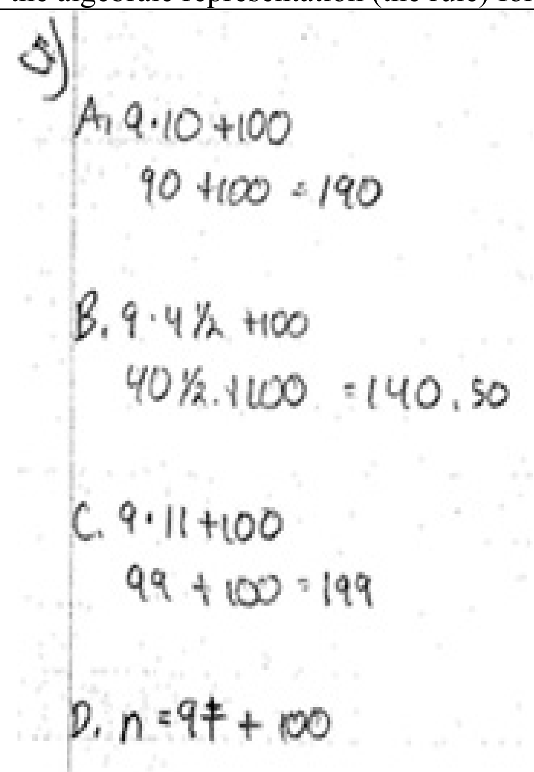
b) NO, cause it doesn't jump by a number.
c) I would multiply ten by ten, then multiply the answer from ten times ten, by three, and then add ten and then there's the answer
3·10² = 300+10 = 310

I understood how they arrived at the answer, but mathematically this was incorrect (refer to part a). This error in usage of the equal sign was minor issue for me and we would correct this in future classes. My main concern was the ability to apply the order of operations correctly. In other words, would they apply exponents before multiplication?

My students were uncomfortable with contextual tasks. Thus, I wrote the following task in Figure 4-8 to give them some experience with contextual tasks. I included the *y*-intercept in this task along with the slope. With a contextual task, the numbers used and solution can be fractional (parts b and c).

Figure 4- 8. Contextual task

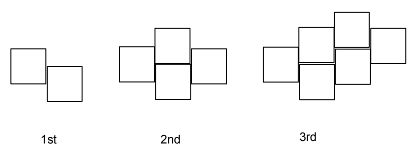
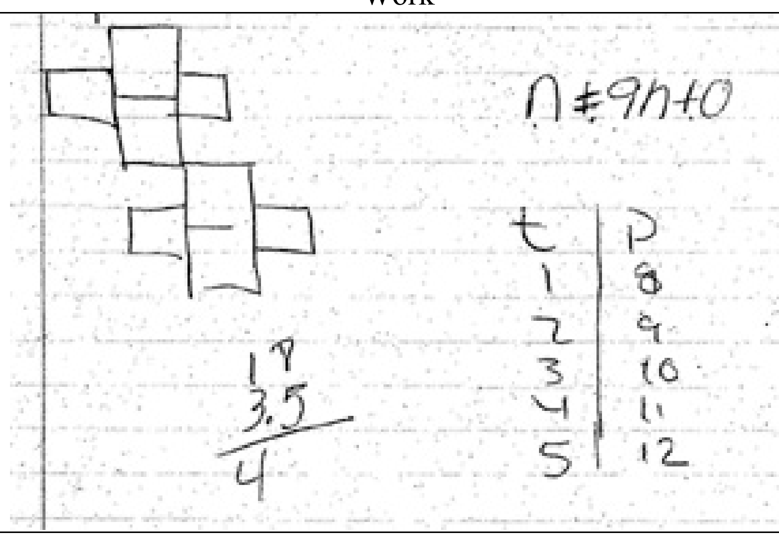
5. Jenny has \$100 in her piggy bank and now works at a job that pays \$9 per hour. Find the amount of money she has if she saved all of her money by:
   a) Working 10 hours
   b) Working 4 ½ hours
   c) How long would it take her to save \$200?
   d) What is the algebraic representation (the rule) for this context?



Some groups ignored the 0th term and some groups couldn’t multiply with a mixed fraction. This was to be expected for the first week of algebra and it allowed me to gauge the arithmetic skills of my students. This group didn’t have difficulties with this task and was quite adept with the mixed fractions (part b), which was not the case for everyone. As mentioned previously, I did not introduce the word "rate" to this task but rather continued to use the terms "jump" and "0th term." I wanted students to see that a contextual situation was analogous to a numerical pattern task[5].

Figure 4-9 is a geometric task along with work of a student. Numerical patterns exist in geometry and for the first two weeks of class, I introduced students to geometric patterns.

Figure 4- 9. Geometric pattern task.

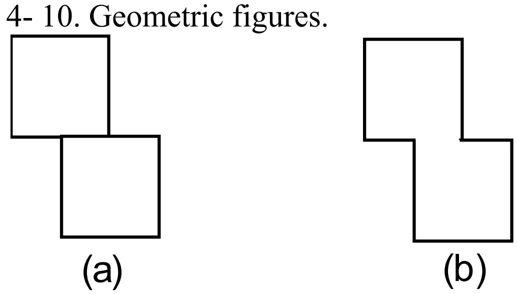
Task
a) Given the following three drawings in the sequence:    a) Draw the 5^{th} figure in the sequence.   b) Find the perimeter for the 5^{th} shape in the pattern.   c) Find the perimeter for the 100^{th} shape in the pattern.   d) What is the algebraic representation for the pattern and perimeter?   e) How did you find the algebraic representation?
Work


[5] I realize that numerical pattern is discreet and this contextual task is continuous, but this point was not important for my students.

I decided to use perimeter as the focus for this pattern, because I believed that area would be too easy. I was wrong. Students did not do well on this task and could not determine the perimeter. I believe they got confused with the perimeter as I looked at the table. Students could not draw or determine the 5th term of the geometric figure.

With the overlapping lines in Figure 4-10a, it is possible to see how the students saw this as two squares; thereby, the perimeter is equal to eight, but this logic doesn't work for the second pattern. Consequently, the next time I teach this lesson, I may use Figure 4-10b in order to avoid confusion. In prior years, I brought in toothpicks when we constructed these geometric figures and I felt that students had a much easier time drawing these figures.

Figure 4- 10. Geometric figures.



Arithmetic and Algebra. I made a decision four years ago to begin algebra with the algebraic representation. Choosing numerical pattern tasks allowed me connect repeated addition with multiplication, but writing the algebraic representation remained elusive. I moved students toward using algebra on that first day by having them decide which approach was more efficient, arithmetic or algebra. We used local terminologies of "jump" and "0th term" to describe the pattern and I worried that students would not be able to transfer to the standard terminologies of slope and *y*-intercept. I also worried that the introduction of the variables *n* and *t* would hinder my students' understanding of *y* and *x*, when we move to graphing and slope-intercept. Teaching students how to write the algebraic representation allowed me to introduce basic notion of writing and solving linear equations.

The assessment showed that some students still struggled with the contextual and geometric tasks. With the time constraint, I had another week to work with geometric figures. For the time-being, I would continue writing contextual tasks into my assessments and when the computer lab was fixed, I would use *Cognitive Tutor* as a resource for further contextual tasks.

Chapter 2: Properties with Real Numbers (Day 19)

The review of operations of real numbers, the second chapter in Larson, is a difficult topic to teach. The Larson approach to this topic includes integer chips, the number line, and rules. All of these approaches contain no algebra; therefore, I often view this as an arithmetic topic rather than an algebraic topic.

I came to this view after many years of unsuccessful attempts and I attributed this to the following obstacles that existed between the arithmetic and my students: (1) students saw this as a middle school topic; (2) students didn't want to show their lack of understanding to others; (3) some students have already mastered the topic and didn't want to re-do the lesson; and (4) students learned this arithmetic concepts incorrectly but didn't know it. Compounding this problem was my lack of pedagogical knowledge on how to teach arithmetic.

To get around these obstacles, I began to think of how to make the arithmetic more algebraic. If this was possible, those students who had mastered the topic would learn some algebra, while the other students would be given another opportunity to master the topic. In order to provide more addition and subtraction tasks, finding the 0^{th} term was the key. Rewriting the task to start with the 3^{rd} term instead of the 1^{st} term allowed students more opportunities to practice addition and subtraction.

Multiplication and division proved to be more difficult, because the pattern generated by repeated multiplication or division was not linear. Students could work with multiplication and division, but finding an easy way to write the algebraic representation proved to be difficult. This breakthrough was not easy, because this wasn't a new idea, but going back to an idea that I had abandoned.

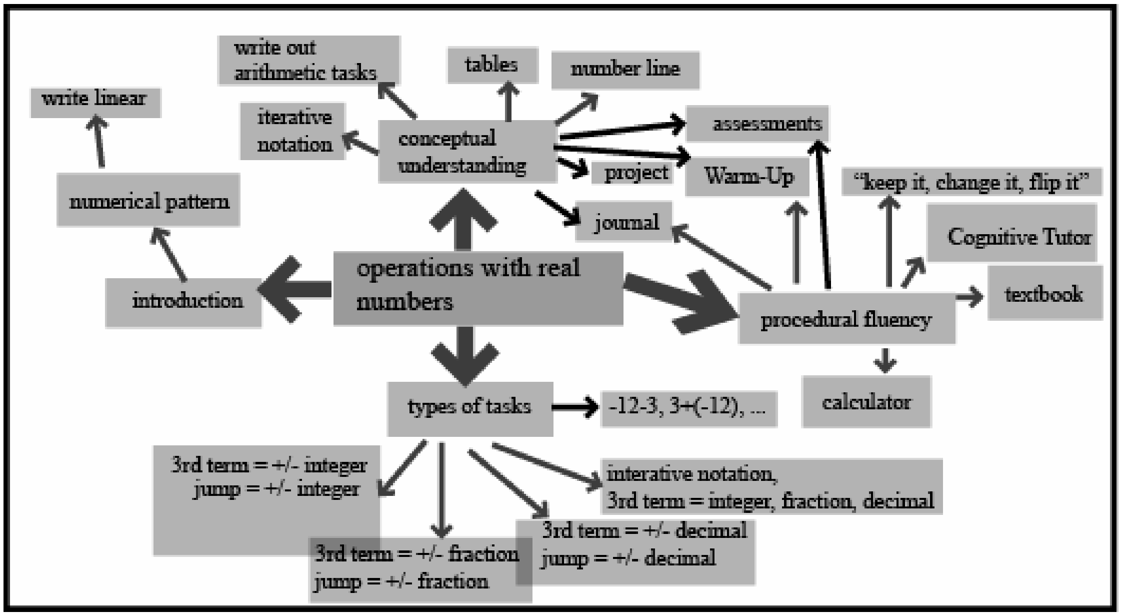
Three years ago, I taught the slope-intercept form along with the iterative notation using NOW and NEXT. When I gave them a linear pattern, my students preferred using NOW and NEXT and I found it difficult to get them to write the slope-intercept form.

I was working with a different approach to the teaching of algebra. A breakthrough for me could be: finding a terminology that makes the problem easier to solve; finding a connections between numerical patterns and the operations with real numbers; or finding the appropriate notation to a task. A breakthrough to connecting multiplication and division was the iterative notation. Here was my journal entry:

> A 2nd hour student remarked that I should change my notation from $U_{n+1} = 2U_n$ to $U_{t+1} = 2U_t$. I will modify this to $n_{t+1} = 2n_t$ for my other classes. Journal entry: October 17th, 2006

She saw that I was using the *n* instead of the variable *t.* As I made the breakthrough, I encountered another obstacle, notation. I have Tiara to thank for helping me get around this obstacle. Once again, a student was able to direct me, the teacher. Internally, I felt that the iterative notation now belonged with TGT algebra. Figure 4-11 shows the current domain map for operations with real numbers.

Figure 4- 11. Pictorial representation for internal map for operations with real numbers



This domain map serves as a guide to the teaching of the operations with real numbers. The domain map contains four elements: (1) the introduction, (2) conceptual understanding, (3) procedural fluency, and (4) the different types of tasks. The introductory tasks build a conceptual understanding by connecting addition, subtraction, table of values, and the number line together. The iterative notation connects multiplication, division, and the table of values together to build conceptual understanding. Calculator, *Cognitive Tutor*, and the textbook build procedural fluency. Projects, Warm-Up tasks, and assessments are used to build conceptual understanding and procedural fluency for the different type of tasks within this chapter.

The next section looks at how these ideas about integrating algebra with the operations of real numbers translated into the classroom. Prior experience had shown that students have difficulties with the following: (1) subtraction of integers; and (2) the division by a fraction.

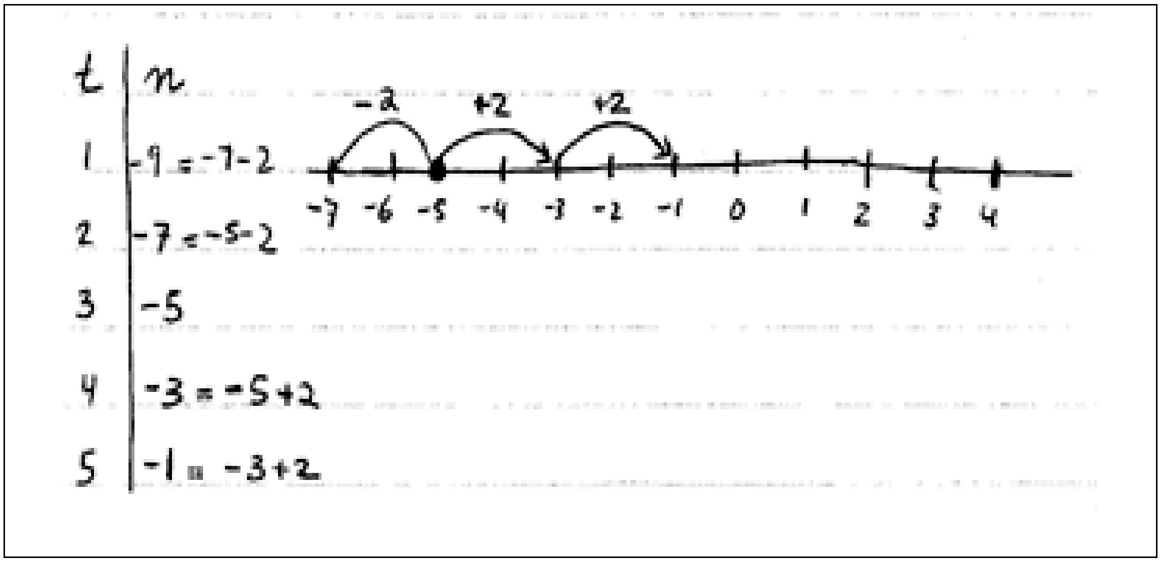
When Subtracting is Adding (Day 17). I believed my students' difficulties with addition and subtraction of integers were the following: (1) confusion with operations; (2) arithmetic tasks had no meaning; and (3) math facts were weak. The confusion came from such tasks as $-2-3$ because the operation is subtraction but to get the answer you must use addition. Students needed to untangle the sign and the actual operation. To help my students untangle their confusion with addition and subtraction, I created a task that contained both addition and subtraction, which was the focus of the lesson. I also wanted students to write and solve task such as $-5-2$. In order to incorporate these criteria, I used the task in Figure 4-12.

This warm-up task was given to students October 9th. In Figure 4-12, I have included the task along with my work and the discussion for finding the second and first term of the sequence. As mentioned before, I began the task with the 3rd term so that students would be able to practice addition in one direction and subtraction in the other direction. I also asked for the algebraic representation to make this task algebraic (part b). The algebraic representation was needed to solve for part c). This discussion took place after we had found the fourth and fifth term and were now discussing how to find the second term. I have included the task, the work I wrote on the overhead. This excerpt the difficulties students have with addition and subtraction of integers.

Figure 4- 12. Task and teacher's work about the 2nd term.

Given the 3rd term is -5 and the jump is 2, find the following:

a) Find the first five terms
b) Find algebraic representation for the linear pattern
c) Find the 100th term



This excerpt below shows that students' prior knowledge conflicted with actual answer.

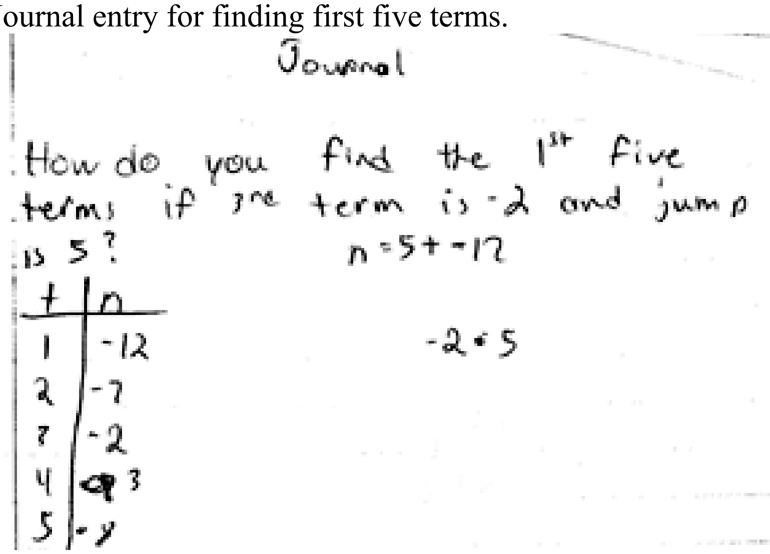
24	Teacher:	1. Ok. Now to go the other direction, you're going to add or subtract?
25	Christian:	Subtract.
26	Teacher:	Yes, Terry?
27	Terry:	I have to get my stuff out of (inaudible).
28	Britney:	Mr. Pan.
29	Teacher:	Yes, dear?
30		(several talking)
31	Teacher:	No, I didn't see any. Did you lose some? All right, so Marsha, when you come in late, all I expect you to do is come in, sit down and quiet down.
32	Christian:	You add.
33	Teacher:	So this direction is going to be -5 – 2.
34	Alonzo:	Which equals 7?
35	Teacher:	-7

The student's interruption may have contributed to disrupting the flow of the class. It was possible that Christian understood the sign of the arithmetic task and the operation needed to get the answer, but it was also possible that she stated both answers without actually understanding the question. Even with the number line drawn, Alonzo still gave me an incorrect response (line 34). I believed Alonzo got confused with the rules for multiplying numbers and adding negative numbers.

I drew a number line and connected it with the arithmetic operation, but Alonzo still stated 7, which even a number that was on the number line. His prior knowledge was so strong that even when faced with overwhelming evidence that this was not the correct answer, he still stated the incorrect answer. My response to Alonzo was to state the correct answer and point to the number line as evidence. This was the reason why I disliked teaching this chapter. I had a much easier time teaching algebraic representation, then trying to change student's incorrect prior knowledge. I also knew that Alonzo would be given many more opportunities to correct his mistake.

At the end of the class period, I asked students to write a journal entry on how to find the first five terms when 3rd term = -2 with a jump of 5. An example of a student work is in Figure 4-13.

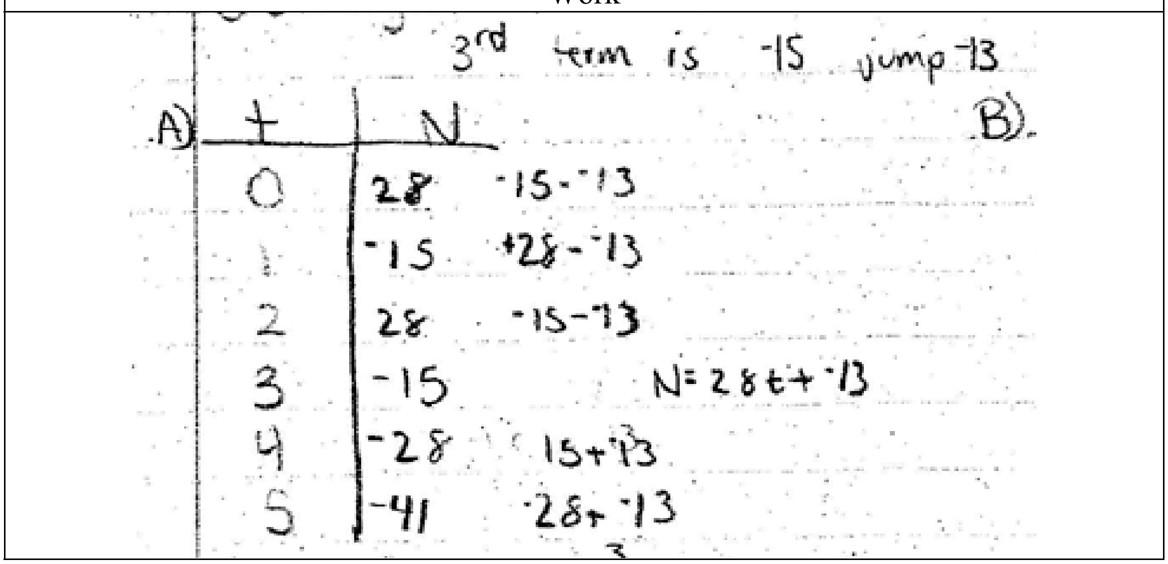
Figure 4- 13. Journal entry for finding first five terms.



We can see in this journal entry that this student did not write out the arithmetic operations or explain how to find the numbers. This student may have used the number line or her own understanding of integers to find the answer. My original assumption was that by writing the arithmetic operation in conjunction with the number line, my students would have a way of self-correcting their mistakes.

During an assessment, I asked students to write the arithmetic operation along with the answer. Figure 4-14 shows the work of a student who struggled with the subtraction and addition of negative integers. This assumption that students were not connecting the arithmetic operation with the solution may need to be revisited; perhaps, students were still confused with the operations. Was the jump too big for this student? Perhaps, I need to rethink my approach? This was only my second year using these types of task and I was still learning.

Figure 4- 14. Student work with addition and subtraction task.

Work


Askey (1999) recommends using contextual situations to give meaning to arithmetic tasks. I tried to provide contextual examples using points and money to make task such as $-8-2$ more meaningful. My attempt at contextual situation example was the following:

618 Teacher: Nolan, Nolan, you're talking too much, I'm going to take 8 points

away, Nolan, you got a pencil in your hand, I'm going to take another 2 points away, how many total points did I take way from you?

619 Christian: 10

I used points instead of money (line 618). Christian responded to my question (line 619). Her response was still problematic because she responded correctly, positive 10, but what happened to the negative sign? The dilemma was that we didn't speak in terms of negative points, but in the context of losing or taking away points.

Not long afterward, I commiserated with Ms. Maple, a teacher from another district, about some of my challenges with teaching operations with integers. She provided me with software for my graphing calculators that provided practice problems. These tasks helped build up my students' math facts. I used the last ten minutes of a class to demonstrate the calculator programs. Figure 4-15 shows two screen shots from the graphing calculator of the type of tasks students encountered. Students typed in the answer. If the answer was incorrect, then the student got two attempts to answer the task correctly. At the end of the program, the graphing calculator gave the students the percentage of correct answers. I told students that they can work with graphing calculator program during their free period and seven out of 35 students took up my offer.

Figure 4- 15. Screen shots of a calculator programs on integer operations.

Calculator tasks	Calculator tasks
QUESTION 1 -7 + -11 ANSWER = ■	QUESTION 2 16 – -13 ANSWER = ■

I used numerical patterns along with arithmetic operations and a number line, contextual situation to the task, and calculator program to foster my students' understanding of adding and subtraction. Students would get another opportunity to work on addition and subtraction with *Cognitive Tutor: Algebra 1*, when the computer lab came online[6].

From assessments, the three approaches worked at some level, but not with the success that I was hoping for. I believed that the numbers used in all three approaches were too small. I believed that I have not created any cognitive dissonance and students reverted back to their prior understanding of how to solve arithmetic tasks.

<u>Multiplication and Division (Day 22)</u>. Another difficult arithmetic topic to teach is the division of a number by a fraction. Citing Ma, Askey (1999) says:

> The teachers who suggested these methods (e.g. changing the problem to decimals, dealing with numerators and denominators separately) also noted that they were not always easier than the standard textbook method of multiplying by the reciprocal. (p. 12)

My dilemma was whether to go back and re-teach division or teach the division of fractions by the rule. My students would determine the pathway. If they had forgotten how to do this, then I would do a review of fractions with box diagrams and moved toward the rule for division of fraction. If they remembered the rule for division of fraction then I would proceed with the lesson. My students' input would determine my action.

Instead of separating division from multiplication like Larson, I wanted to teach multiplication along with division so that students could see the inverse relationships between the two operations. I also wanted students to write and solve their arithmetic tasks. In order to do this, I introduced the iterative notation keeping in mind that I needed to maintain the focus of the lesson, which was still multiplication and division of fractions.

I introduced multiplication and division with integers along with the iterative notation. Figure 4-16 shows the third multiplication/division task. Students were still

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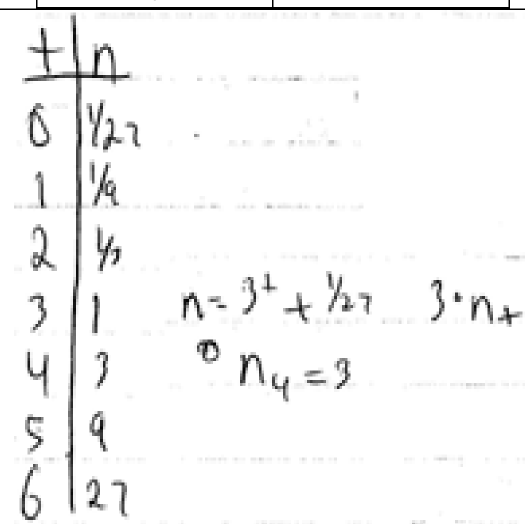
[6] The computer was not fixed until October 17th.

struggling with iterative notation. Different from the first two tasks, this task required division by fraction in order to complete the task and it would be my students' first encounter with the division of fractions in my class. This took place October 16th.

Figure 4- 16.Task and student work.

Given the following table, fill in the missing values and describe the pattern.

T	*n*
0	
1	
2	
3	1
4	3
5	9
6	



This excerpt shows how I used a student's input.

28 Several: 1 divided by 3

29 Teacher: 1 divided by 3 is just? (pause) 1/3. And how do we get the first term?

30 Several: 1/3 divided by 3

31 Teacher: 1/3 divided by 3. Who remembers how to divide?

32 Kiara: Keep it, change it, flip it.

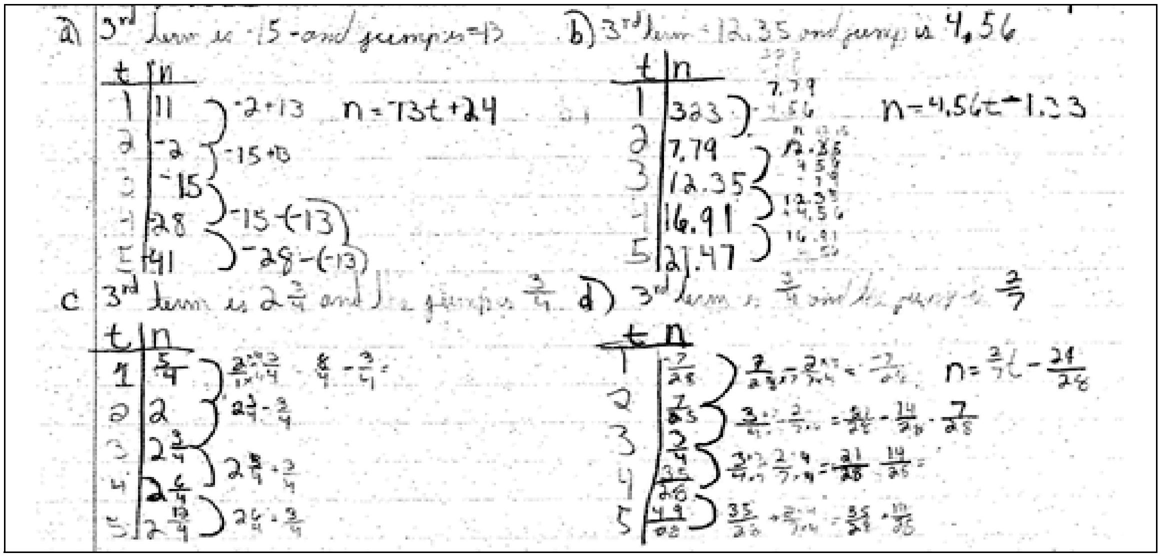
33 Teacher: Keep this as 1, keep it. (giggling)

Building a rapport with students required that I also listened to and used their inputs. If I didn't, my conversation with my students would be one-sided. When I heard Kiara's statement, I decided to follow her lead. I had never heard the multiplication of the reciprocal phrased in this manner (line 32). I decided to follow her lead and used her method whenever we encountered division of fractions. This procedure provided a sense of pride and ownership of the mathematics for the students. I also brought in the graphing calculator to help build basic arithmetic facts for multiplication and division.

Assessments (Day 21 and Day 26). In the second chapter, we also worked with decimals and fractions. Figure 4-17 presents the task and the work of a group assessment that occurred October 10th. I selected this group to highlight students' difficulties with addition and subtraction with integers and simplification of fractions.

Figure 4- 17. Task and the work of a student.

Task
1. Find the first five terms and algebraic representation for the following: a) 3rd term is -15 and jump is -13 b) 3rd term is 12.35 and jump is 4.56 c) 3rd term is $2\frac{3}{4}$ and the jump is $\frac{3}{4}$ d) 3rd term is $\frac{3}{4}$ and the jump is $\frac{2}{7}$
Work



This group of students wrote the arithmetic tasks along with the solutions. For part a), the group correctly determined the number, but the arithmetic operations for the 4^{th} and 5^{th} term are incorrect. Part a) could be done with the number line; therefore, my interpretation was they probably determined the numbers in the table and filled in the arithmetic operations later. For parts b) and c) of the task, this group correctly wrote the arithmetic operations and the numbers in the tables. For part d), this group found the correct numbers to multiply the fractions, but could not multiply 2 times 4 correctly and got 14. I am still unsure why they still struggled with part a) of the task but could do the other parts. For parts b), c), and d), the students needed to write and solve the arithmetic operations. I would continue to write these types of tasks for the next assessment.

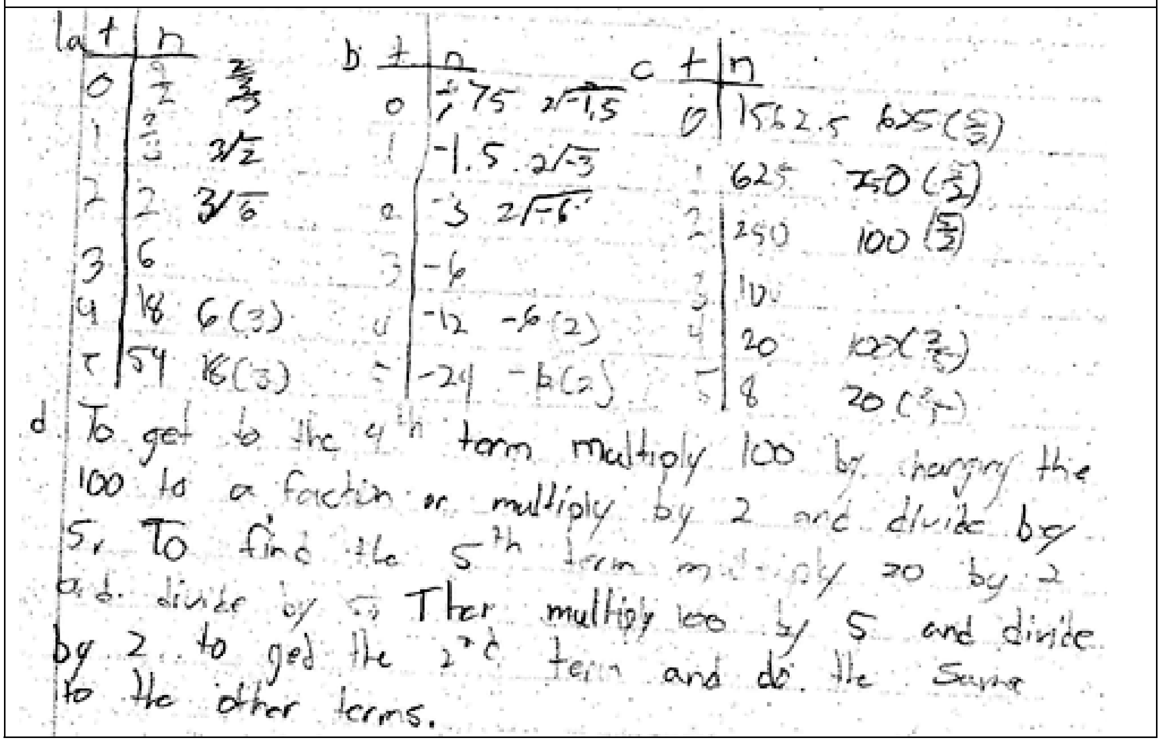
For multiplication and division, I selected the work of one group. Part a) of the task did not involve fractions. Part b) of the task involved negative integers. Part c) was the multiplication and division of fractions. Figure 4-18 contains the task along with the work of one group. This assessment occurred on October 16^{th}.

Figure 4- 18. Task and group work for multiplication and division.

Task
1. Find the 0^{th} term through 5^{th} term. a) $n_{t+1} = 3n_t$ and $n_3 = 6$ b) $n_{t+1} = 2n_t$ and $n_3 = -6$

c)	$n_{t+1} = \frac{2}{5} n_t$ and $n_3 = 100$
d)	Explain in a paragraph how to find the first five terms for $n_{t+1} = \frac{2}{5} n_t$ and $n_3 = 100$.

Work



For part a), this group struggled with the placement of the numerator and denominator with the long division sign, which they did correctly with part b). For part c), this group could apply the procedure for division of fraction, but could not multiply correctly. From this assessment, I questioned my decision of not reviewing fractions or basic multiplication. We continued working with these types of task as we transitioned to the solving of linear equations and I would need to use more fractional equations.

<u>Weak Prior Knowledge</u>. The difficulty with this chapter was trying to correct students' incorrect prior knowledge. I decided to approach this chapter by making it more algebraic. Doing this changed the focus of the lesson to the inverse nature of addition and subtraction and less about writing the algebraic representation. This task also connected the number line and the arithmetic operations, but students' prior knowledge of addition

and subtraction impeded the potential of this task. In order for this task to be more effective, starting with 3rd term with a larger number and jump may push students to rethink their understanding of addition and subtraction.

Using the iterative notation, I was able to connect the multiplication and division together in one task. This was my first year using these types of task and I was proud that a student helped me change the notation. The iterative notation proved at first to be difficult to teach, but once this was accomplished, the focus was the operations and less about the notation.

Another student provided a "teaching device," the "keep it-change it-flip it," for dividing fractions. I have always been quite skeptical of these "teaching devices" because they often don't have any mathematical foundation. I decided to follow my students' lead and strengthened the interactions between teacher, students, and content. Most students were able to apply this "teaching device" for the solving of linear equations with fractions. Overall, I felt more confident in my ability to teach arithmetic operations to algebra students.

Teaching the iterative notation after the slope-intercept form allowed students to see the differences between the two notations. This was the first year I incorporated the iterative notation for multiplication and division. I was worried that my students would associate the iterative notation with only multiplication and division, but students were able to use the notation with other operations, addition and subtraction. For the assessment on November 17th, I wrote the task in Figure 4-19.

Figure 4- 19. Task and student work for iterative notation.

Task
Find the 0th term through 5th term. a) $n_{t+1} = 2n_t$ and $n_3 = 12$ b) $n_{t+1} = n_t + 2$ and $n_3 = 12$ c) $n_{t+1} = \frac{n_t}{2}$ and $n_3 = 12$

d) $n_{t+1} = n_t - 2$ and $n_3 = 12$
Work

a) t	N	b.) t	N	c. t	N	D. t	N
0	1½	0	6	0	96	0	18
1	3	1	8	1	48	1	16
2	6	2	10	2	24	2	14
3	12	3	12	3	12	3	12
4	24	4	14	4	6	4	10
5	48	5	16	5	3	5	8

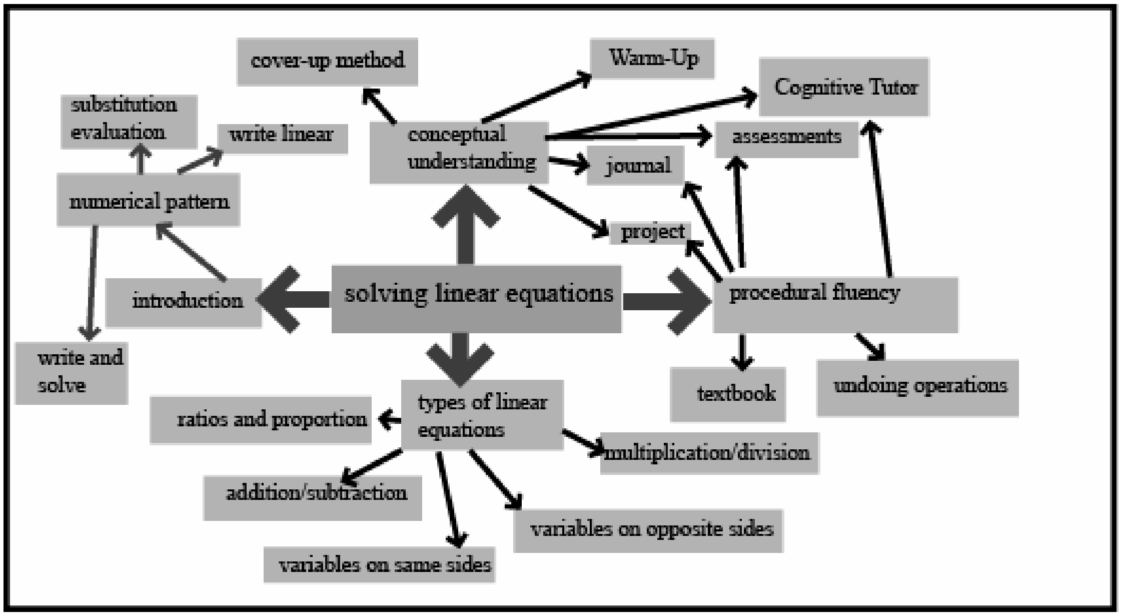
I was pleasantly surprised when all of the groups correctly did this task without any prior work. Students were able to adapt their understanding with the iterative notation. This was another clue that students were connecting to these tasks.

Chapter 3: Solving Linear Equations (Day 36)

> Lesson went well. Students are showing up 7th hour to ask for help. The problem is not finding anytime for me to do other work. More chaos and more fighting in this school. Journal entry: November 15th, 2006

I usually begin this chapter with a conceptual approach to solving linear equations, the cover-up method (Kieran & Chalouh, 1993). This method provides a more conceptual foundation for the solving of linear equations, but using this method required arithmetic flexibility. My students' prior experience with solving linear equation was with undoing operations, which was more procedural and could be applied to more types of equations. I decided to use cover-up to build the conceptual understanding and undoing operations to build procedural fluency. Figure 4-20 shows the domain map for solving linear equations.

Figure 4- 20. Pictorial representation for solving linear equations.



This domain map serves as a guide for the solving linear equation and contains four elements: (1) the introduction, (2) conceptual understanding, (3) procedural fluency, and (4) the different types of linear equations. The introductory task is similar to the introductory tasks of the first chapter. The only difference is to make the number larger to move students toward algebra. I use the cover-up method to build a conceptual understanding on how to solve linear equations. The undoing operations method was used to build procedural fluency for the solving of linear equations. Warm-up tasks, journals, *Cognitive Tutor,* assessments, and projects are used to build conceptual understanding and procedural fluency for the different types of linear equations.

This section illustrates why taking a student's lead might not always be the best and also shows how I introduced the solving of linear equations. This chapter took 31 days to complete, because of the many different types of linear equations.

Cover-up Method (Day 27). Instead of jumping into the solving of equation by undoing, I wanted students to see the meaning behind these operations. Kiaran and Chalouh (1993) recommend emphasizing solving linear equation by thinking about forward operations or the cover-up method instead of undoing operations. For example, in order to solve the equation $x + 4 = 7$, the student covers the variable and generates the following question. What number plus four is equal to seven? The answer is three. With the cover-up method, the student can transform complicated equations into questions.

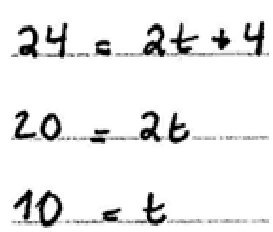
Kiaran and Chalouh (1993)write:

> …how a one-dimensional focus in elementary school mathematics classes on "undoing operations" can sometimes be counterproductive to students' developing an understanding of (a) an equation as a balanced entity and (b) the solving procedure of performing the same operations on both sides of the equation. (p. 182)

I introduced the solving of linear equation by using students' prior experience with numerical patterns. I modified the task in Figure 4-21, by using larger numbers for part b) of the task. I wanted students to see that the algebraic way of solving is more efficient for larger numbers. The excerpt in Figure 4-21 presents my first attempt at using the cover up method for this class. We had already finished part a) and had already found the algebraic representation. We had set up the equation and were beginning to solve for the term when the number was 24. This episode took place October 23rd.

Figure 4- 21. Students' first exposure to the cover-up method.

Given the following pattern 6, 8, 10, 12, 14, ….
b) Find the 12th term, 17th, and 10,000th term.
c) What term is equal to 24? 58? And 123456?



This excerpt below illustrates my first attempt with the cover-up method.

92 Teacher: So let's do it algebraically then, right. So Britney, so we're looking for what value of *t* right here, right Britney and Clorissa, right? So I can cover this up and say what number plus 4 gives me 24. What number is that?

93 Christian: 20

94	Teacher:	Right? 20 + 4 gives me 24, right? 2 times what number gives me 20?
95	Mickey:	10
96	Kiara:	2 times 10.
97	Teacher:	10, right? So the answer should be 10.
98	Clorissa:	I don't understand.
99	Britney:	I still don't…

The cover-up method was a new method to my students and after I finished I could see a lot of confusion in their faces. In prior year, I would continue solving with the cover-up method, but I believed that our work with inverse operations in the last chapter established the foundation for the undoing operations method.

The Undoing Operations Method (Day 27). I met a lot of resistance with the cover-up method and I decided to move to the undoing operations method. I always felt that the undoing operations method was too procedural, but since I had done work with addition and subtraction from the last chapter, I thought the undoing operations method had some conceptual foundation. I covered the variable 2*t* and proceeded with the undoing operations method. This interaction is shown below in Figure 4-22.

Figure 4- 22. Undoing addition with the undoing operations method.

Task	Teacher's work
Given the following pattern 6, 8, 10, 12, 14, …. What term is equal to 24?	$24 = 2t + 4$ $-4 \quad\quad -4$ $20 = 2t$

Our discussion using the undoing operations method is in the excerpt below:

102	Teacher:	So let's do it the other way. You have addition here, how do we

remove addition?

103 Mickey: You subtract.

104 Christian: Subtract.

105 Teacher: Just like we did with the little table, right, you add going on to remove it you have to? Subtract on both sides. Why do we do it on both sides?

106 Mickey: So you can do the easy…

107 Christian: But what if it's an odd number?

108 Teacher: It doesn't matter if it's odd or even as long as addition you have to do…

109 Mickey: Subtraction.

110 Teacher: Subtraction. So what's 4 – 4?

111 Several: 0

112 Teacher: 24 – 4?

113 Several: 20

From Mickey's response, I interpreted this as his recognition of undoing operations (line 106). Christian's response was more troubling (line 107) and I had not encountered this and was quite unsure how to respond. From my interactions with the students, I felt that my students had prior knowledge with undoing operations, but their understanding didn't seem to be robust and the excerpt below showed more difficulties with solving linear equation.

114 Teacher: Now Clorissa, what operation is here? Is this multiplication or division?

115 Clorissa: Multiplication.

116 Teacher: Multiplication, right? So how do you get rid of or remove multiplication? …Divide by?

117 Several: 2

118 Teacher: 2

119 Clorissa: Oh.

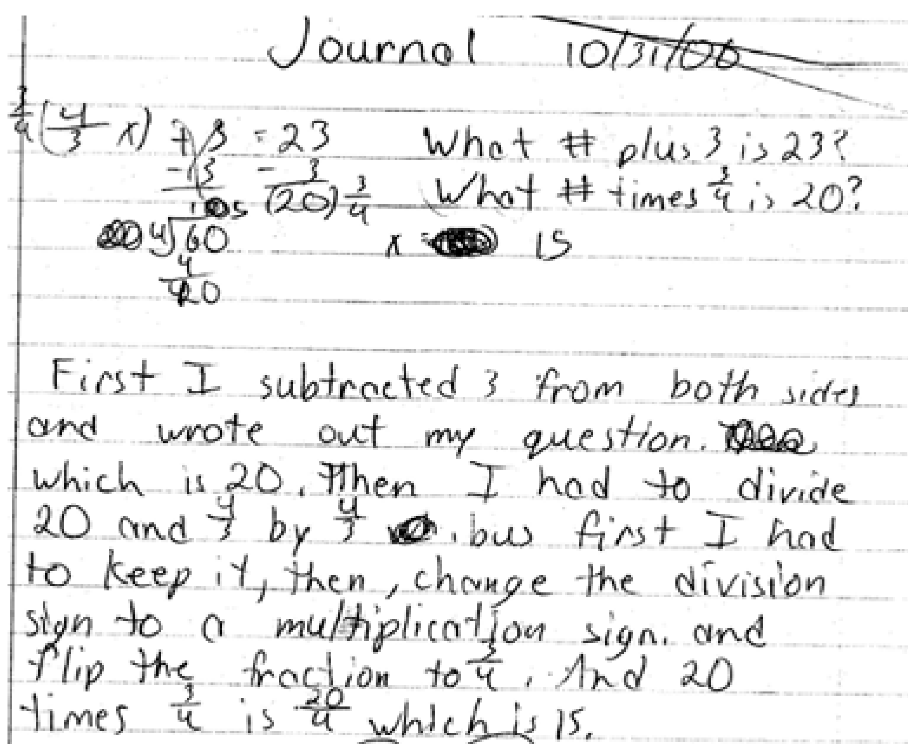
120 Teacher: 20 divided by 2 is?

121 Clorissa: 10

Once again, I led students through the undoing the operations method (lines 116 and 120). Students wanted me to move toward the undoing operations method, but I ended doing all of the work. I believed that students had prior knowledge of the undoing operations, but had forgotten how to apply it.

At the end of the day, I asked students to write using the cover-up method how they would solve $\frac{4}{3}x + 3 = 23$. Figure 4-23 shows the work of a student.

Figure 4- 23. Student journal entry using the cover-up method.



Even though I saw the questions written on the side, I was not sure whether the student was doing this for me or him/herself. The student wrote the first question, which could be done without applying the undoing operations method. If the student applied the cover-up method correctly, then the second question would be what number divided by three is twenty, and the last question would be what number multiplied by four is 60. The second and third questions could be interchanged.

Similar to my struggle with addition and subtraction, students did the cover-up method because I asked them to. In the instruction framework, students chose their prior knowledge of solving over the new approach of solving with the cover-up. Students needed to be convinced that the cover-up method was worth learning, but I didn't give this method enough time. I had seen this before when I tried to get the students to use algebra instead of arithmetic and I had to show them that the arithmetic approach became inefficient with large numbers.

Back to the Cover-up (Day 32). Their understanding of the undoing operations was not as robust and they struggled with the following warm-up task $2(x+3)=30$. Many students removed the 3. I decided to go back and review the cover-up method by asking groups of students to build their equations. My lesson plan is in Table 4-12. I showed students how I wrote the linear equation with one step and then showed how I was able to build more complicated equations.

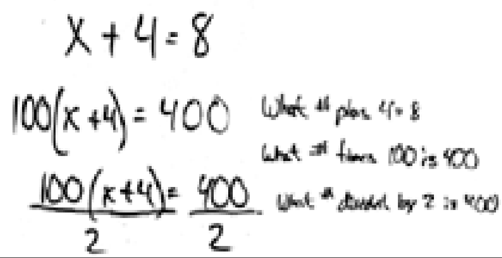
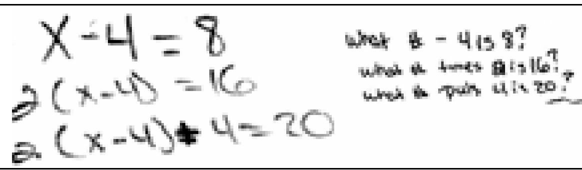
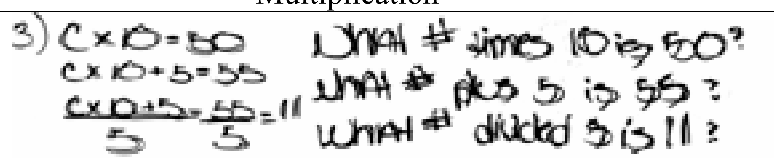
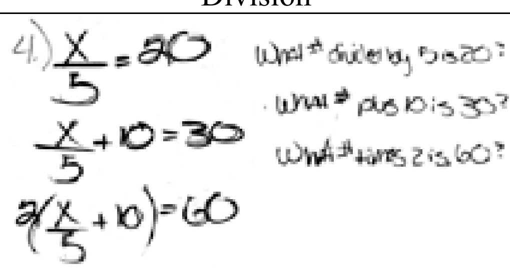
Table 4- 12. Lesson plan for building equations.

Lesson plan
For today's work, I would like you to work in pairs to create and solve problems that have the following property. The solution is the same for all the problems. One-step: $x+4=12$ Notice that I am starting with addition. Two-steps: $2(x+4)=24$ Since I already used addition, I can choose subtraction, multiplication, and division. How did I get 24? Three-steps: $\frac{2(x+4)}{6}=\frac{24}{6}=4$ Since I used addition and multiplication. I ended up using division. Why is the expression equal to 4?

I asked groups of three or four students to create algebraic equations and transferred their work onto presentation paper so that other students could see their work.

Working in groups allowed students to build rapport with other students and allowed students to take charge of their work. I could take the role as the observer. Their work is in Figure 4-24.

Figure 4- 24. Groups of students creating their own task.

<table>
<tr><td>Group work</td></tr>
<tr><td>Addition</td></tr>
<tr><td>
</td></tr>
<tr><td>Subtraction</td></tr>
<tr><td>
</td></tr>
<tr><td>Multiplication</td></tr>
<tr><td>
</td></tr>
<tr><td>Division</td></tr>
<tr><td>
</td></tr>
</table>

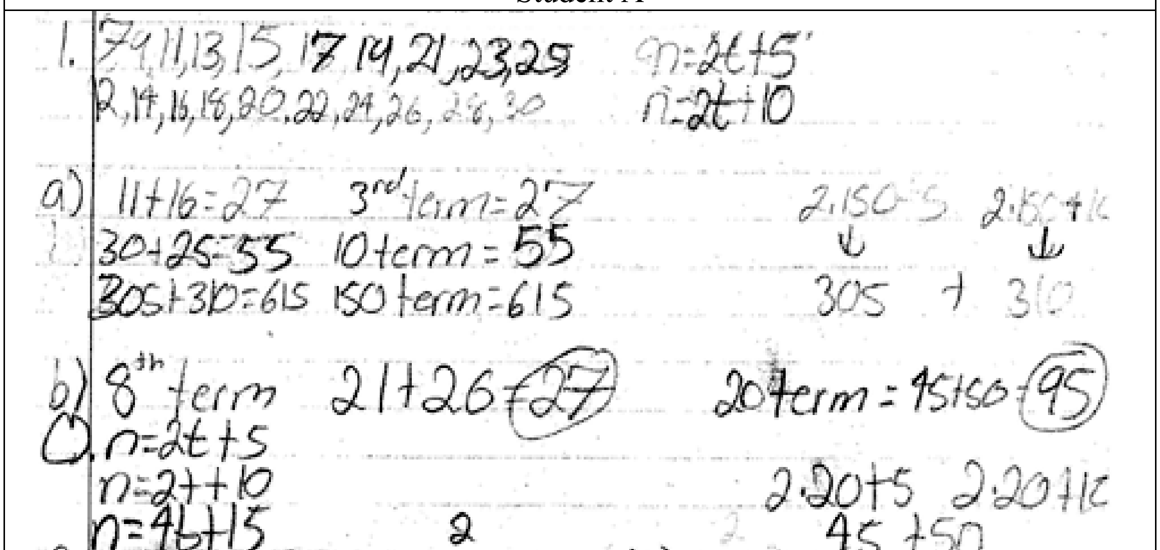
By making them write out the steps for each part of task, students had to make this connection between the equation and the question and focus on a single operation for solving. I made an error in judgment when I abandoned the cover-up method too early. I did not give it enough time. I felt much better as a teacher and I looked forward to the next exam.

Students' Work (Day 36). In order to show the variety of tasks used for the solving of linear equations, I have selected students' work from their assessments. Figure 4-25 presents the task used to demonstrate the solving of linear equations with variables on the same side. Students could use arithmetic or algebraic procedure to solve the task.

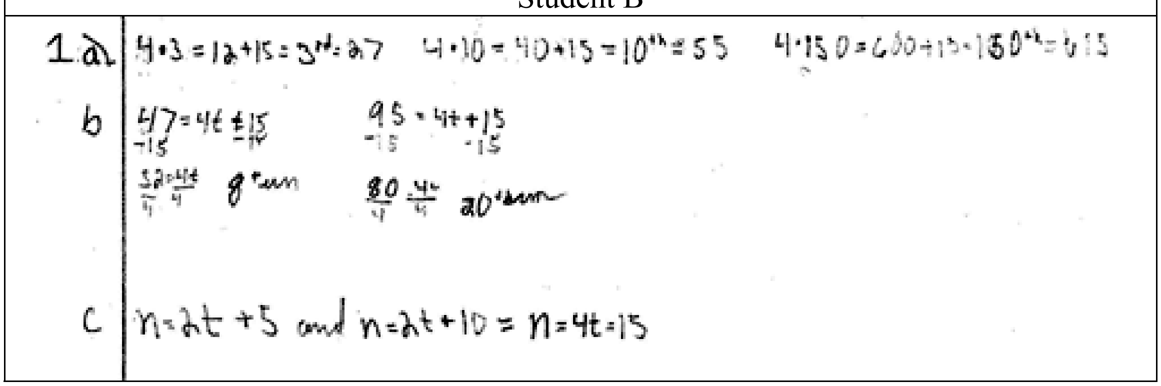
Figure 4- 25. Task and the work of two students.

Task	
1.	Given 7, 9, 11, 13, 15, …. And 12, 14, 16, 18, 20,…
a)	Find the sum of the sequences for the 3rd, 10th, and 150th term.
b)	When will the sum of the sequences be equal to 47? And 95?
c)	What is the algebraic representation for the sum of the sequences?

Student A


1. 7,9,11,13,15,17,19,21,23,25 n=2t+5
12,14,16,18,20,22,24,26,28,30 n=2t+10
a) 11+16=27 3rd term=27 2·150+5 2·150+10
30+25=55 10 term=55 305 + 310
305+310=615 150 term=615
b) 8th term 21+26=47 20 term=45+50=95
c) n=2t+5 2·20+5 2·20+10
n=2t+10 45 +50
n=4t+15

Student B


1a 4·3=12+15=3rd=27 4·10=40+15=10th=55 4·150=600+15=150th=615
b 47=4t+15 95=4t+15
-15 -15 -15 -15
32=4t 8 term 80=4t 20 term
4 4 4 4
c n=2t+5 and n=2t+10 = n=4t+15

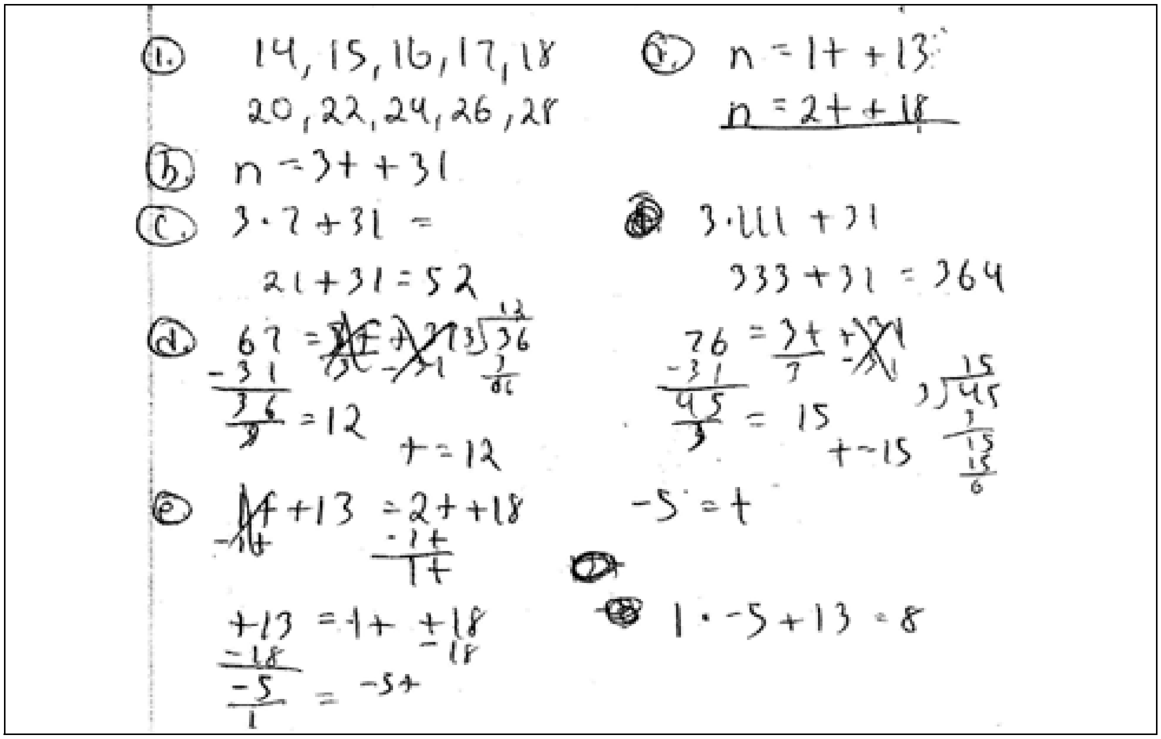
Student A used arithmetic for smaller numbers such as the 3rd and 10th term. For the 150th term, the student used the algebraic representatives separately and then added them together. For part b) Student A chose not to write the equation to solve, but probably used guess and check to solve for 47 and 95. Student B found the algebraic representation for the sum of the sequences and then applied the formula for parts a) and b). When I solved this type of task, I often used the same method as Student B. The cover-up method was not used unless asked. I still hadn't resolved the writing error made by Student B for part a).

These approaches show there were students (like student A), who continued to use arithmetic by extending the pattern and using guess and check. There were students (like Student B), who felt comfortable with the algebra. I realized that it was still their decision to choose arithmetic or algebra. It was up to the students to decide how to interact with the task and helped build their confidence with algebra.

Figure 4-26 presents a task in part e) where they needed to find the point of intersection for the two numerical patterns.

Figure 4- 26. Task and student work.

Task
Given two linear patterns 14, 15, 16, 17, 18,… and 20, 22, 24, 26, 28, … Find the following: a. Find the algebraic representation for the two linear patterns. b. Find the algebraic representation for the sum of the two linear patterns. c. Find the sum of the sequence for the 7th and 111th term. d. Find the term where the sum of the sequences is equal to 67 and 76. e. Find when (at what term) and where (at what number) the sequences meet.
Work

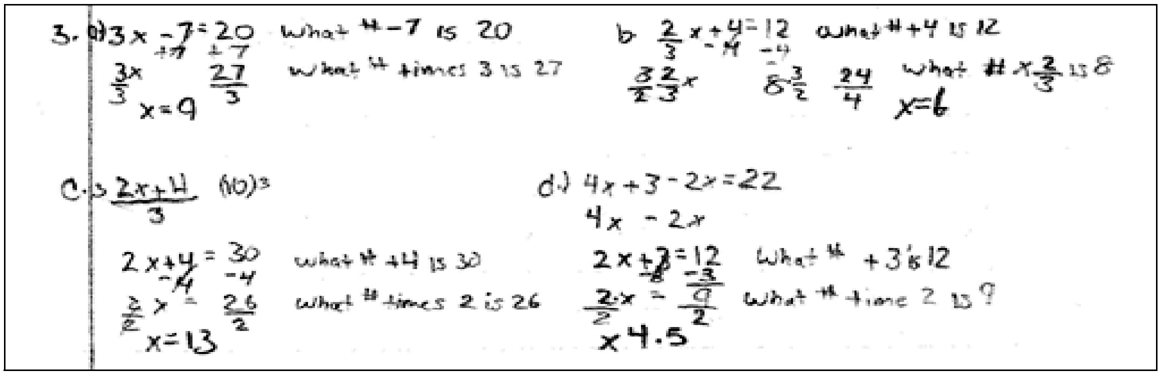


This student was able to manipulate the two numerical patterns and had no problem with any parts of the task. This student used algebra instead of arithmetic. This work also illustrates the type of algebra students were doing in this class.

Figure 4-27 shows the stereotypical tasks for solving linear equations. For this 2nd assessment on solving linear equation, I made sure that students wrote out the questions as part of the cover-up.

Figure 4- 27. Solving linear equation task with student work.

Task			
3.	Solve and write the questions necessary to solve the following problems.		
a)	$3x-7=20$	b)	$\frac{2}{3}x+4=12$
c)	$\frac{2x+4}{3}=10$	d)	$4x+3-2x=27$
Work			

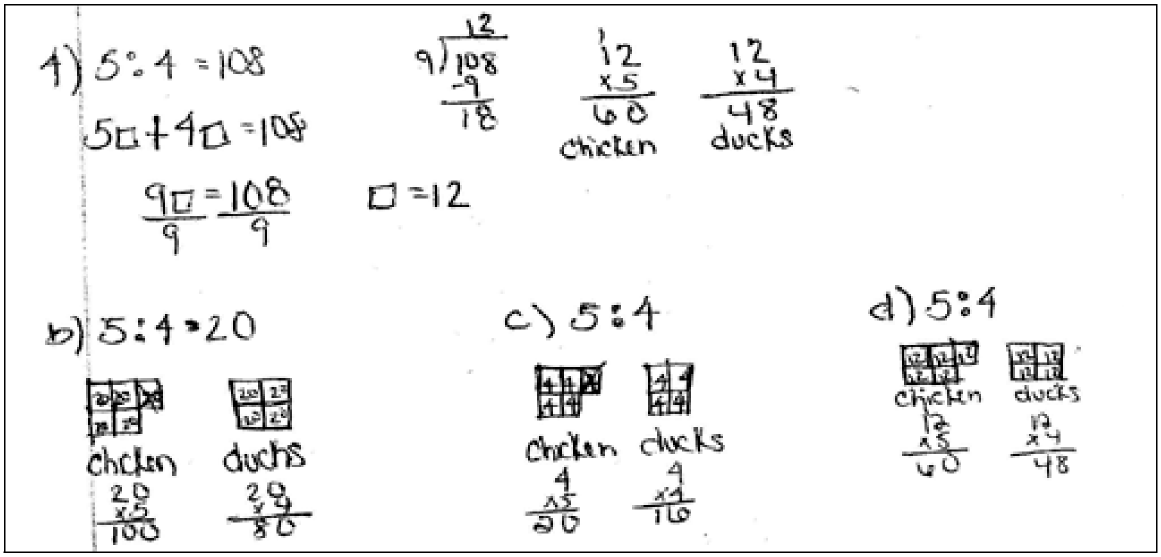


This student was comfortable writing out the questions along with the undoing process. I was much happier with the result of this second assessment for the solving of linear equations. I was beginning to think of a better way to connect the cover-up method with the undoing operations method.

Instead of using numerical pattern for ratio, I decided to use the box method to solve ratio task in order to build some foundation for fractions. Figure 4-28 demonstrates how a student applied the box method.

Figure 4- 28. Ratio task and student work.

Task
4. The ratio of chickens to ducks is 5: 4. Find the following: a. If the total number of chickens and ducks is 108, then what is the number of chickens and ducks? b. If the chickens outnumber the ducks by 20, then what is the number of chickens and ducks? c. If 4 chickens move to a different farm, then the ratio of chickens and ducks are equal. What is the number of chickens and ducks before the chickens leave? d. If the number of ducks is increased by 12, then the ratio of chickens to ducks are equal. What is the number of ducks and chickens before the ducks came?
Work



This student had no difficulties with this task and I wondered if I could have pushed this task even further.

Conceptual and Procedural. The solving of linear equation is a procedural chapter by nature, especially in Larson, and I wanted to provide some conceptual foundation. My mistake was jumping too quickly to the procedural method and had to backtrack to the cover-up method. I made that mistake because I decided to follow my students' wishes instead of following my own intuition. Instead of thinking of the cover-up and undoing as two different opposing methods, I now feel that these methods can work together in the solving for linear equations. For example, when a student wrote a question as part of the cover-up method it would not be difficult to transition to the inverse operations.

I am still unsure of the ordering of these two approaches. Students' prior knowledge with the content appeared when we did operations with real numbers and the solving of linear equation. A task such as $24 = 2t + 4$ triggered their prior knowledge, which did not match with the cover-up method. I created a choice between the two approaches and students chose the undoing operations method. Perhaps, I should have chosen a task where the cover-up method was more efficient than undoing operations such as $\frac{t+2}{4} = 10$, but this task didn't connect to the numerical pattern approach.

I was able to apply the numerical pattern tasks for the sum of the two functions and also for the point of intersection. In prior years, I had time to do group work constructing numerical patterns where there was no point of intersection and also when all points intersect.

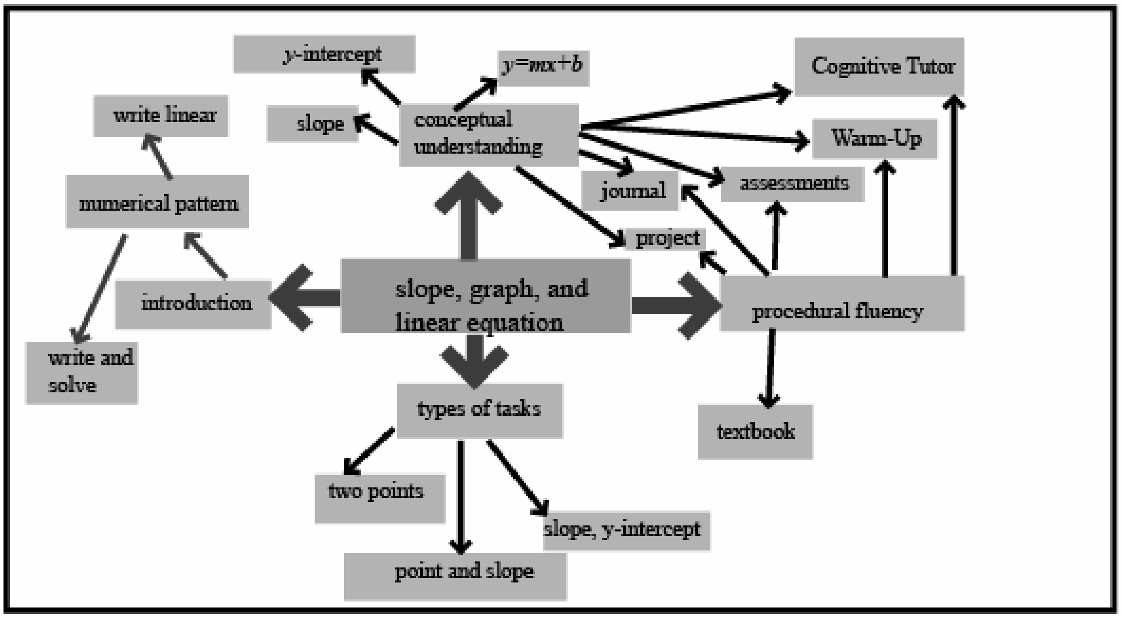
Chapter 4: Graphing Linear Equations and Functions (Day 54)

> Getting closer to Thanksgiving. Never in my years of teaching am I so anxious to get done. I'm mentally and physically drained day after day.
> Journal entry: November 20th, 2006

I would like to believe that I teach on an island and I can teach algebra with impunity. Unfortunately, my students are judged not by me but by standardized tests written by people who may or may not see algebra as I do. I worried that my students' use of "jump" and "0th term" and their algebraic approaches would hinder their success in future mathematics classes. During these moments, I would throw in a few more tasks from Larson into the lesson plan to maintain the connections between my approach and that of Larson.

These moments occurred more frequently as I approached the end the first semester. Students were still using the variables *t* and *n* and I didn't know when I would switch over to *x* and *y*. Within Larson, the approach to slope was to pick the right formula and substitute appropriately. While I believed that formulas were important, being able to solve a task conceptually had a stronger hold on my students. Figure 4-29 shows the domain map for slope, graphs, and linear equations.

Figure 4- 29. Pictorial representation for slope, graphs, and linear equations.



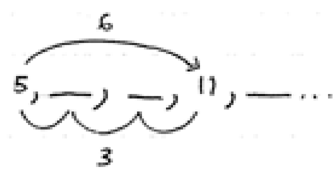
This domain map contains four elements: (1) introduction, (2) conceptual understanding, (3) procedural fluency, and (4) different type of tasks. To build a conceptual understanding of slope, I modify a numerical pattern. In this chapter, I introduce the standard terminologies of slope and *y*-intercept. I use the textbook to build procedural fluency for slope. I use journals, projects, assessments, Warm-Up tasks, and *Cognitive Tutor* to build conceptual understanding and procedural fluency for the different of tasks in this chapter. In this section, I show how I used numerical patterns to develop slope and apply this to write algebraic representation. I also show how I transitioned to the term slope and *y*-intercept.

<u>Slope (Day 58)</u>. In the previous section, I illustrated how my students' prior knowledge impeded their understanding with the cover-up method. This would also be true for slope. My approach to the teaching of slope would not begin with the formula for slope. I would tap into my students' understanding of numerical pattern to have them discover how to determine the slope. Instead of focusing on the "0^{th} term", we now focused on the "jump." I began with numerical patterns that allowed students to use arithmetic to solve and slowly increase the difficulty so that they would have to find a

tool to find the slope. Prior to the task in Figure 4-30, students found the missing numbers by guessing missing numbers.

Figure 4-30 shows the task in which a student found a way to find the slope without guessing the missing numbers. I have included the task and the work for this task. This lesson took place December 11th.

Figure 4- 30. Task and work.

Task
Given the following linear patter 5, __, __, 11, __,…Find the following: a) missing numbers b) jump c) 0^{th} term d) algebraic representation
Work


The discussion of how to find the slope is in the following excerpt:

333 Ashley: 'Cause there's another way you can get the answer doing the same thing.

334 Teacher: Ashley.

335 Ashley: Take 5 from 11 and divide it by 3.

336 Teacher: Where did you get the 3 from?

337 Ashley: Because there's three empty spaces.

338 Teacher: Ok, you got it. All right, Ashley explain one more time, how'd you do it Sydney?

339 Ashley: (inaudible)

340 Teacher: So she found out the big jump here is how many?

341 Several: 6

342 Teacher: 6. And then?

343 Ashley: Divided it by 3.

Some students used guess and check to find the slope, but at least one student was able to find a method that could be used for all points. Instead of providing students with the slope formula, I decided to continue using Ashley's method. My philosophy for high school algebra was not to provide the formulas, but to have students discover them. My rationale was that students would understand the formulas better if they constructed the formulas and procedures from the tasks.

Instead of introducing the slope formula, I decided that I would introduce the term "slope." The discussion that took place is this excerpt below:

354 Teacher: So this is instead of calling jump, people, I'm switching it to the word? (writing the word slope)

355 Several: (pause) Slope.

356 Teacher: Slope. What we've been calling jump is actually the?

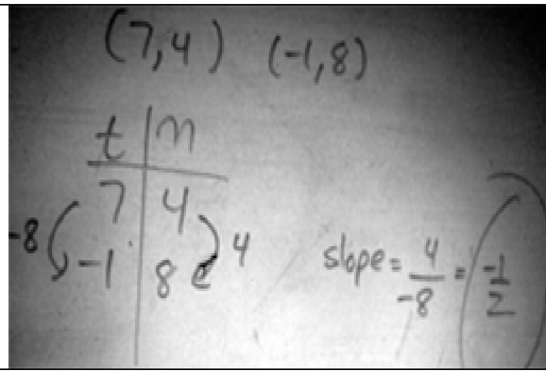
357 Several: Slope.

358 Teacher: Slope.

Transferring to the standard term wasn't that difficult. It is possible that my students had outgrown the term "jump." I wasn't quite sure when I would switch to the *y*-intercept.

<u>Back to Algebraic Representation (Day 60)</u>. We had learned how to find the slope two days before. The day's lesson would go back to the textbook, but instead of just asking for the slope, I would ask students to determine the algebraic representation. I had yet to introduce the formula for slope. I wrote the task on the overhead. The first part of the task was to find the slope. In Figure 4-31, I have included the task and the work.

Figure 4- 31. Task and work for finding the slope

Task
Find the slope (7, 4) , (-1, 8) (Larson, Boswell, Timothy, & Stiff, 2004 p. 230)
Work


This excerpt below was our discussion of how to find the slope.

194 Teacher: …. All right. 4 to 8 is a jump of?

195 Several: 4

196 Teacher: 4. For students who learned this last year I believe you subtracted right, 8 – 4?

197 Christian: Yeah.

198 Teacher: Ok, that's how come we came out to 4. 7 to, 7 to -1 is a jump of, jump of?

199 Christian: 8

200 Teacher: -8. How is it -8?

201 Several: Because it's going down.

202 Ashley: Because it's a 1, -1.

203 Teacher: You're on the seventh floor going down.

204 Christian: So you go down.

205 Teacher: Eight floors. Yep.

The objective for finding the slope was to find the difference between two integers. Christian could not get the sign of the number for the denominator, but several students provided the correct response. I had placed arrows to show the direction we were going, but she still made the error. This also occurred in the 2nd chapter. I felt that they were blurting out the answer before thinking. We had our two numbers to find the slope and this conversation took place is in excerpt below:

235 Teacher: 4 over. 4 over -8, so simplify that, that comes out to?

236 Ashley: -2

237 Christian: ½.

238 Ashley: Oh I get it.

I had both the numerator and denominator. One student got the sign right but divided incorrectly and other could divide and forgot the sign. I felt again quite inadequate to teach arithmetic concepts. Students needed to realize their mistakes and correct them, which was my response. I continued to use contextual situations and used tasks that had negative numbers and were fractional in the class and on assessment.

Even though the objective of the task was to find the slope, my objective was to find the algebraic representation and we needed to find the 0th term. I wanted all of the algebra concepts to connect to each other.

Students could count backward from 7 down to zero or they could count up from -1 to get to zero[7]. Previously, I showed them that we could split the jump of ½ to mean a jump of 1 for the output and 2 for the input. A jump of a ½ could also mean a jump by ½ for the output and 1 for the input. This was one of the longest discussions we had. I let them explain to each other how to find the "0th term." In the excerpt was their discussion on how to find the "0th term."

244 Teacher: …How do we get the zero term?

245 Christian: Oh, you go down.

246 Kiara: You go negative by a half.

---

[7] I taught them to use the slope-intercept form in a later lesson.

247 Christian: No you don't.

248 Ashley : You go up by a half. 8 to 4

249 Kiara: -1/2, you subtracting a ½.

250 Ashley: You go backwards, 8, 4.

251 Mickey: And it jumps down.

252 Christian: No, on the inside you're going up, so it's 8, I mean 4, 8, 12, 16.

253 Mickey: I'm not (inaudible), it's in the middle that you're going.

254 Christian: On the top saying you want a negative on the inside it's supposed to be 4, 8, 12, 16.

255 Teacher: All right, let's…

Even though Ashley and Christian were incorrect with the slope, they both participated in our discussion with finding the "0^{th} term." I had not embarrassed them in front of the class and they felt comfortable enough to voice their opinions. As the teacher, I needed to hear these errors as a way of gauging the class. I would hope the number of errors diminished as we progressed in the semester and when I looked at the assessment.

I redirected their focus to which term allowed us to get to the 0^{th} term the quickest and this was our discussion in excerpt below:

276 Teacher: Right? Just use your jump here, right?

278 Ashley: So it would be…

279 Kiara: 7 ½.

280 Christian: That's bold.

281 Kiara: You're jumping by a ½.

282 Teacher: It could have been either 8 ½ or 7 ½, you have to figure out. So the algebraic is n=?

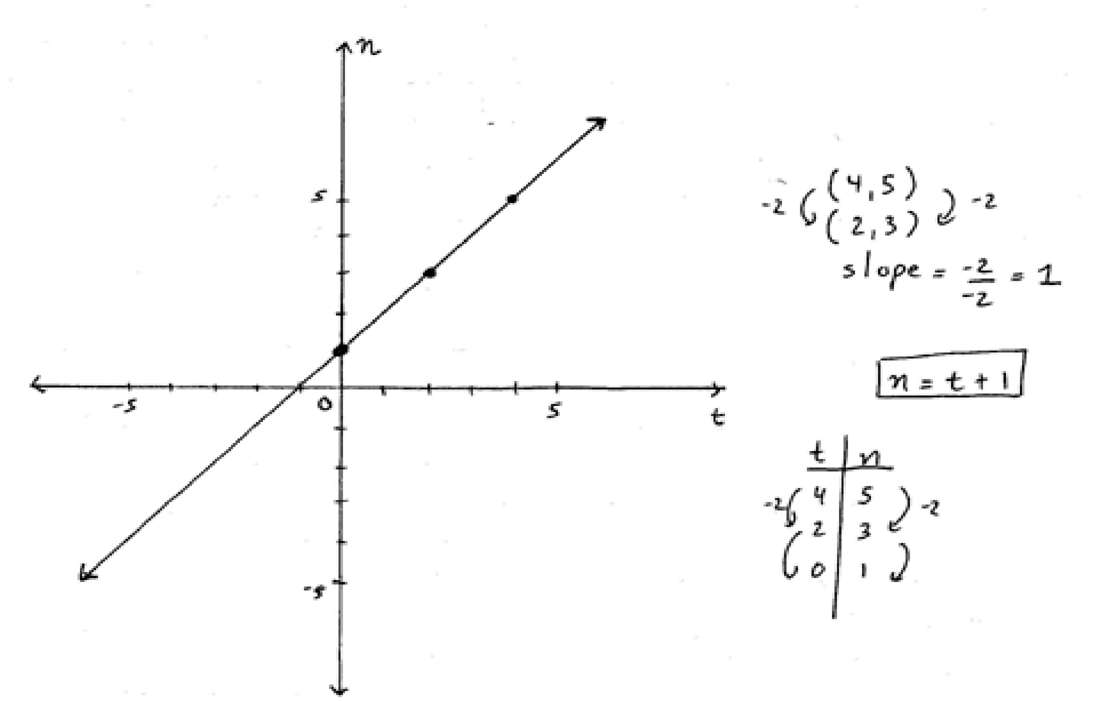
283 Kiara: -1/2t + 7 ½.

Once I got them to focus on the -1^{st} term, Ashley and Kiara were able to determine the 0^{th} term and Kiara provided the algebraic representation (line 283). I still find it remarkable that my students can make errors with basic operations, but can find the algebraic representation.

The y-intercept (Day 61). We were still using the 0^{th} term, because I felt that this terminology gave added meaning when we graphed the line. I continued to use the 0^{th} term until we graphed the linear equation and I could connect the origin as part of the 0^{th} term. Figure 4-32 shows the graph of when I transitioned to the *y*-intercept.

Figure 4- 32. Showing the *y*-intercept on a graph.

Work



The excerpt below was how I introduced the *y*-intercept.

484 Teacher: Up. Right? That's a positive slope. Where is zero term?

485 Female: Zero?

486 Teacher: Right there. We also call it the...*y*-intercept if you remember your term from last year.

As I pointed to the 0^{th} term, I stated that this was also called the *y*-intercept. Switching to slope was much easier than switching from the 0^{th} term to the *y*-intercept. I believed the 0^{th} term held more meaning than the *y*-intercept. The 0^{th} term could be used in contextual situation and graph. The *y*-intercept, on the other hand, was tied to the graph and forced all linear equations to be written with the same output variable. I wished I spent more time thinking about the terminologies when I switched to slope and *y*-intercept.

Slope-y-intercept (Day 61). I gave them another task from Larson to see if students could switch to the technical term of slope and *y*-intercept. I used the task below from Larson:

Find the slope and *y*-intercept.

a) $y = 2x + 1$ (p. 363)

The discussion that took place is in the excerpt show the difficulty of transferring to the standard variables.

312	Teacher:	…So the book doesn't use jump or does it use zero term, they use the term slope and *y*-intercept, ok. So number thirteen they wrote $y = 2x + 1$. Who can tell me what the slope is?
313	Kiara:	$2x + 1$.
314	Alonzo:	The 2.
315	Christian:	$3x$?
316	Teacher:	What number is it, the 2 or the 1?
317	Mickey:	The 2.
318	Christian:	1
319	Teacher:	And the *y*-intercept is the?
320	Mickey:	1
321	Teacher:	1
322	Christian:	Oh. So you take the one with the variable?

During this episode, a student was able to make this leap, but for others the *x* and *y* were foreign to their understanding. Christian's statement (line 322) was the kind of thinking that I wanted to avoid in the class. She wanted to equate the slope to the coefficient with the variable *x*. While this method worked for this equation, it would not work for others.

I decided to rewrite the algebraic representation with *n* and *t* to see if this would clear up the confusion. Figure 4-33 shows what I wrote on the overhead.

Figure 4- 33. Work and discourse on slope and *y*-intercept.

Work
$y = 2x + 1$ $n = 2t + 1$

This excerpt below was our continued discussion about slope and *y*-intercept.

326 Teacher:		Well just like we do with *n* and *t*, right? If I would have written it like this, (pause) could you tell me where the slope is?
327 Christian:		The slope would be 2.
328 Teacher:		And the zero term is?
329 Christian:		1
330 Teacher:		1, it's the same thing except we use *y*'s and *x* for this one.
331 Christian:		I get it now. It's like *x* is turned into a term without saying it.

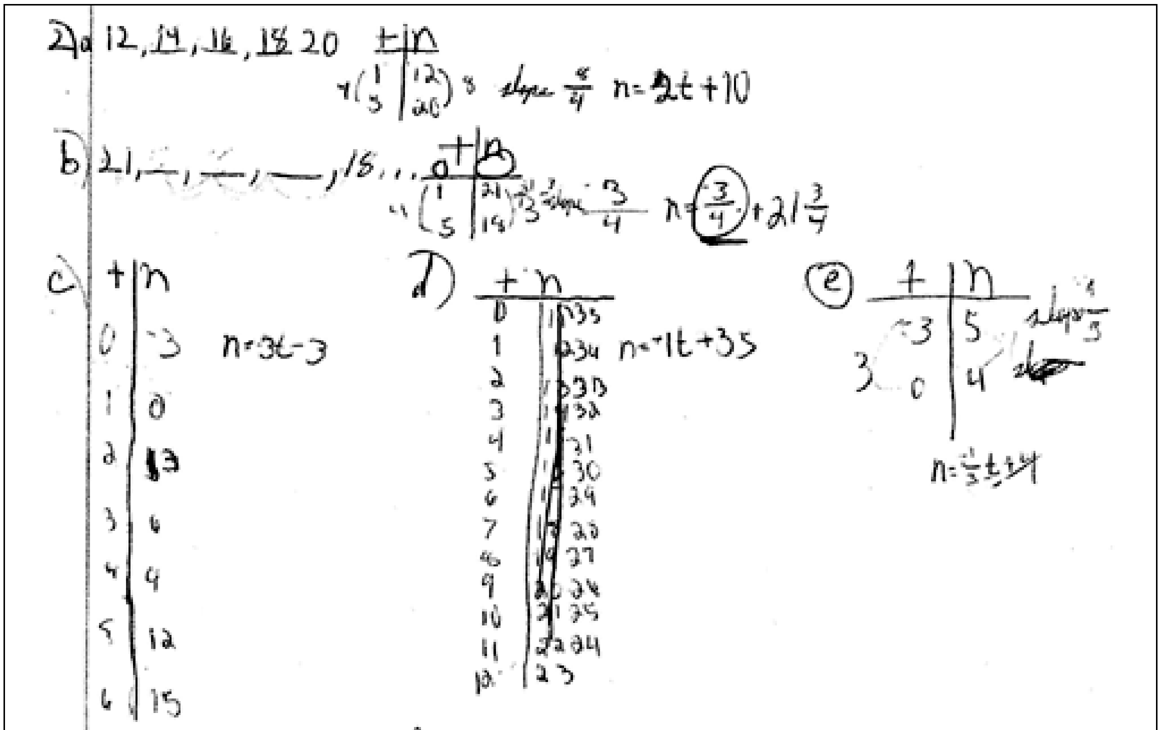
I wanted students to seamlessly move between the variables because it was a characteristic of the variable that we hadn't touched on. Being flexible with the variable would be useful in future mathematics classes. This ability to transfer between variables was no small feat.

Students' Work (Day 64). This assessment was given December 22nd, the Friday before Christmas break. This is significant for JHS because students decided to show up for class during 6th hour to take this assessment[8] instead of leaving early for Christmas break. This was my way of countering the "social" culture of JHS. From the start of the school year, I had tried to separate the school from the classroom environment. Twenty students out of 35 students showed up to take their assessment.

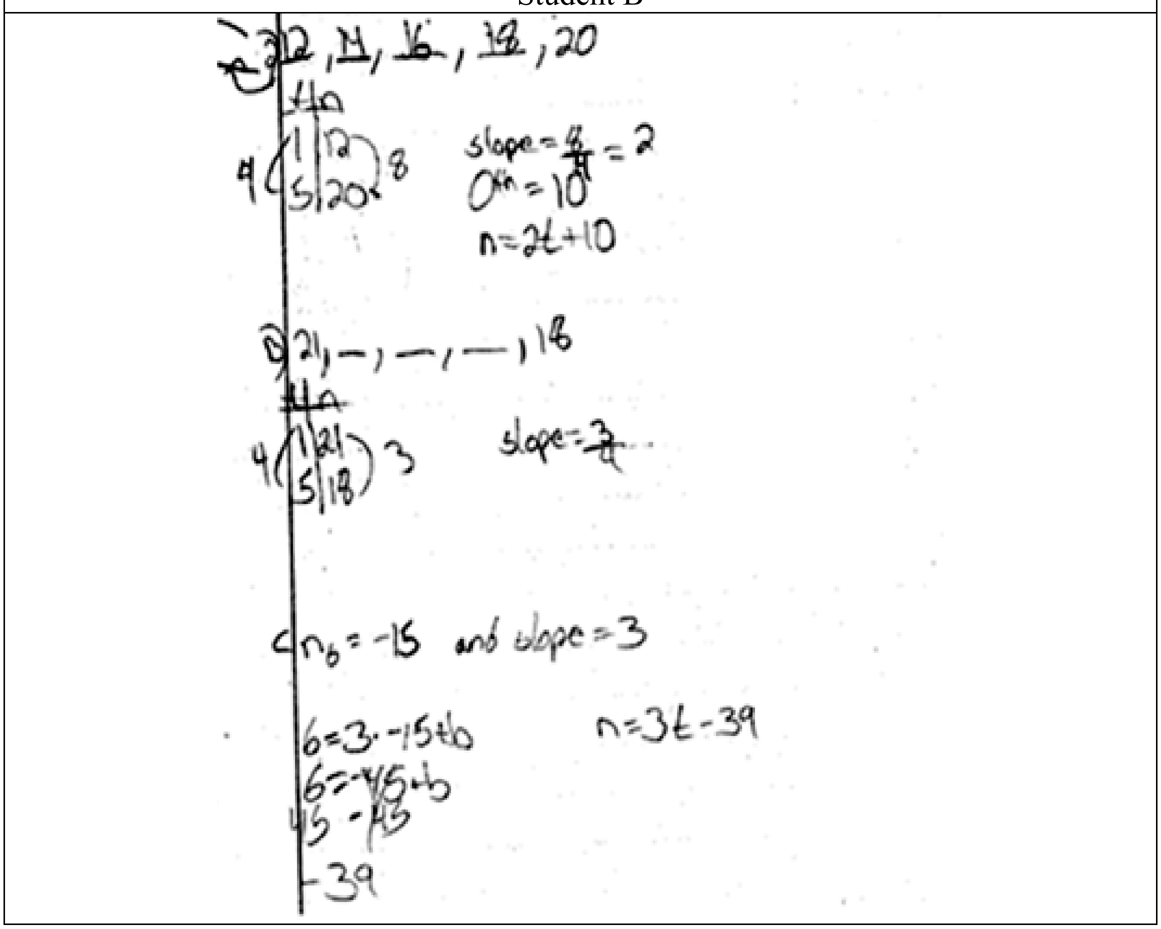
The objectives were to be able to determine the slope and the algebraic representation for each of the sequences. Students could use tables or the algebraic formula to obtain the slope and algebraic representation. Students had made arithmetical errors in our class and I made sure to include these on this assessment. Figure 4-34 shows the task and work of two students.

Figure 4- 34. Task and the work of two students.

Task
For the following linear sequences: (1) make a table; (2) find the slope; and (3) the algebraic representation. a), 12, ___, ___,___, 20,… b) 21, ___, ___, ___, 18,…. c) $n_6 = -15$ and slope = 3 d) $n_{12} = 23$ and slope = -1 e) $n_{-3} = 5$ and $n_0 = 4$
Student A


2) a) 12, 14, 16, 18, 20
t | n
1 | 12
5 | 20
slope 8/4 n = 2t + 10
b) 21, __, __, __, 18, ...
t | n
1 | 21
5 | 18
slope -3/4 n = -3/4 + 21 3/4
c) t | n
0 | -3
1 | 0
2 | 3
3 | 6
4 | 9
5 | 12
6 | 15
n = 3t - 3
d) t | n
n = -1t + 35
e) t | n
-3 | 5
0 | 4
slope

Student B


a) 12, 14, 16, 18, 20
t | n
1 | 12
5 | 20
slope = 8/4 = 2
0th = 10
n = 2t + 10
b) 21, __, __, __, 18
t | n
1 | 21
5 | 18
slope =
c) n6 = -15 and slope = 3
6 = 3 · -15 + b
6 = -45 + b
45 - 45
-39
n = 3t - 39

For part a), students could determine the slope with a table or guess the slope and fill in the missing numbers. Once they found the slope, they could apply the procedure for finding the 0^{th} term. Students A and B made a table to find the slope and wrote the algebraic representation and correctly simplified the fraction.

For part b), using a table would be easier than guessing the slope. Finding the 0^{th} term required the addition of whole number and fraction. Student A made a table and found the slope (with the correct sign) and 0^{th} term to write the algebraic representation. Student B made a table but made a mistake determine the sign for the output and did not find the algebraic representation. This student may not fully understand the significance of the arrows.

For part c), students could make table to find the 0^{th} term and write the algebraic representation or apply the slope-intercept form to find the algebraic representation. Student A wrote the 6^{th} term incorrectly but applied the slope correctly. Student B used the slope-intercept form but incorrectly substituted the values. I may need to think of a different way to approach the slope-intercept form.

For part d), I reversed the sign for the slope and students could use a table or apply the slope-intercept form to get the algebraic representation. Student A initially made an error with the jump, but corrected it by rewriting the numbers in the table correctly. This student was able to self-correct the mistake which is a key to understanding algebra.

For part e), students needed to see that the 0^{th} term was given and they needed only to find the slope to be able to write the algebraic representation. Student A found the slope with the table and wrote the algebraic representation.

Student B, however, did not do parts d) or e) and incorrectly did part b). This student knew parts of the process for doing algebra, but could not connect it all. This is the dilemma. Do I continue moving forward with the lesson, or do I take time to fix Student's B errors? With about five days in the semester, I decided not to teach any new material until the second semester. Pushing forward would accomplish nothing for my students.

Moving to Standard Usage. My students' weaknesses with arithmetic showed up again when we did slope. Students were able to practice the procedures learned from the second cardmarking to help write the slope-intercept form. I began this section worried that my students would not be able to transfer their terminologies of "jump" and "0^{th} term" to that of "slope" and "*y*-intercept." In prior years, I was so worried that I used the variables *x* and *y* in place of *t* and *n* to make the transfer to the slope-intercept more seamless. By taking this approach, students needed to be able to associate *x* for the term and *y* for the number. After a suggestion from a colleague, I decided that the transfer to the technical terminologies would occur later. I incorporated *t* and *n* into my classroom until the last week in December. As a teacher, I knew that this needed to be done, but I did not know when and how to do this. Moses (2001) tackled this issue of local and technical terminology within a class period. I could not change to standard terminologies, because these local terms, "jump" and "0^{th} term," provided meaning prior to the introduction of slope and *y*-intercept.

During our first lesson with slope, I had a conversation with Ashley about her prior experience with slope:

418 Teacher: So you learned it by formula or what'd did you do? Do you remember?

419 Ashley: Same way, algebraic. We had, we had to find, sometimes we had to find what the slope was, sometimes the slope was already there and we had to find (inaudible). It was difficult last year.

420 Teacher: It seemed difficult last year.

421 Ashley: Yeah

422 Teacher: Well you have the second time around. …

I wondered if algebra was easier now because this was her second time with algebra or because of my approach. Internally, I felt pretty good that she had found my algebra class easy, because I felt that my class was difficult.

Chapter 5: Writing Linear Equations (Day 65)

A friend asked me, “Does it work?” I wished I could have presented some statistical measure of pre- and post- tests or higher scores on a standardized test. For teachers, “Does it work?” often means “Does it stick with the students?” or “Do students show results on standardized tests?” Students being able to recall and know when to apply “keep it-change it-flip it” told me that this “teaching device” did stick with them when I looked through their assessments. Abandoning it would be the same as another teacher abandoning my use of “jump” and “0^{th} term.”

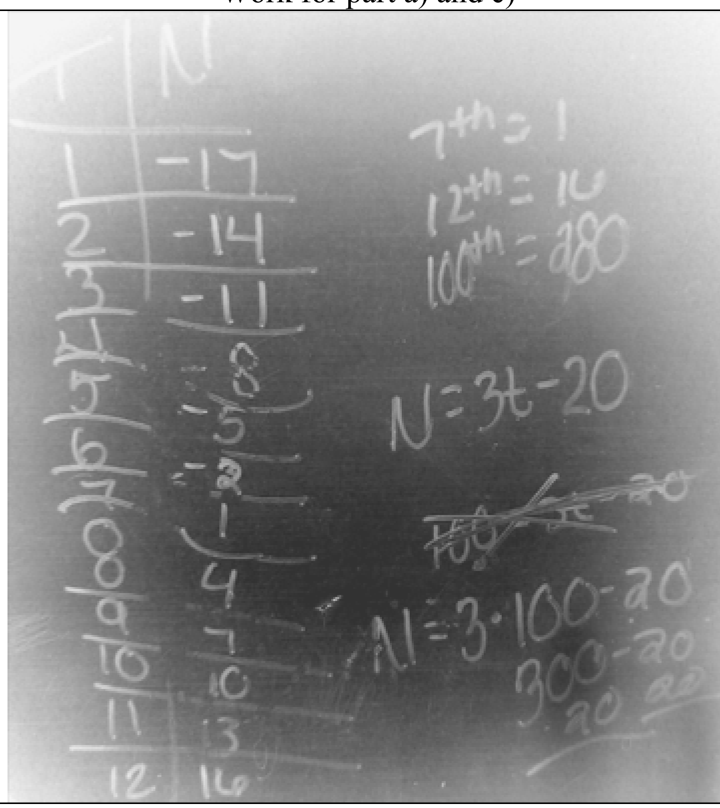
In this section, I present our work from January 3^{rd}, which was the first day we met as a class after Christmas break. Final exams were scheduled for January 5^{th} and 8^{th}; therefore, I decided to give them a review of work for the past semester. I worried that the time off might have lead to forgetfulness and off-task behavior. I wrote in my journal:

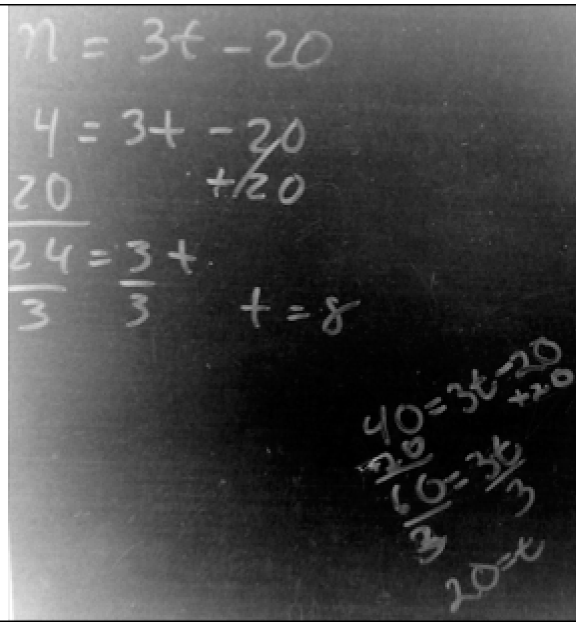
> The new semester does not begin until January 21^{st}. More weeks of silliness, why do I put myself through this? Journal entry: January 3^{rd}, 2007.

In this section, I present four review tasks: numerical pattern, sum of numerical patterns, a contextual, and a geometric pattern. The objectives for the numerical pattern task were to write variable expressions, evaluate, write and solve linear equations. With the sum of sequence, I wanted to students to evaluate, write and solve a linear equation for the sum of two numerical patterns. For the contextual task, my objective was to see if my students could apply their algebra knowledge to a contextual situation. The goal of the geometric pattern task was to find and write the algebraic representation for geometric shapes. These review tasks provide a glimpse of the type of tasks used and the procedures used in the first semester of algebra. We would finish our day working with *Cognitive Tutor: Algebra 1.*

Numerical Pattern (Day 65). The numerical pattern task was a review of a task we did in September. Students volunteered and I sat and watch them do the work. Figure 4-35 shows the task and student work.

Figure 4- 35. Task and students' work.

Task
Given -17, -14, -11, -8, -5, …. a) Find the 7^{th}, 12^{th}, and 100^{th} term. b) What term is equal to 4 and 40? c) What is the algebraic?
Work for part a) and c)

Work for part b)

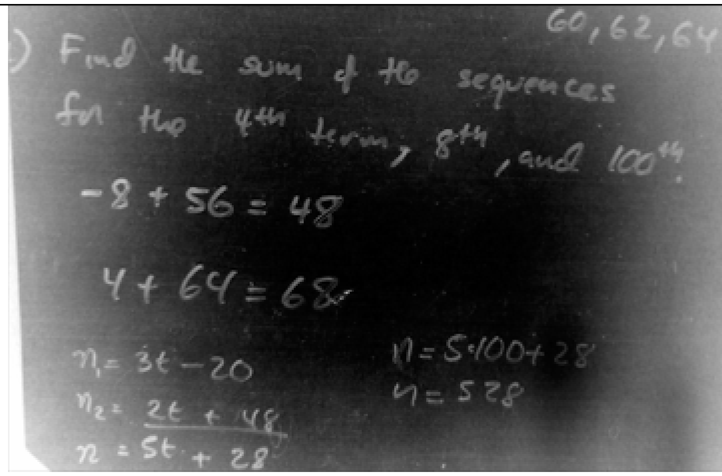


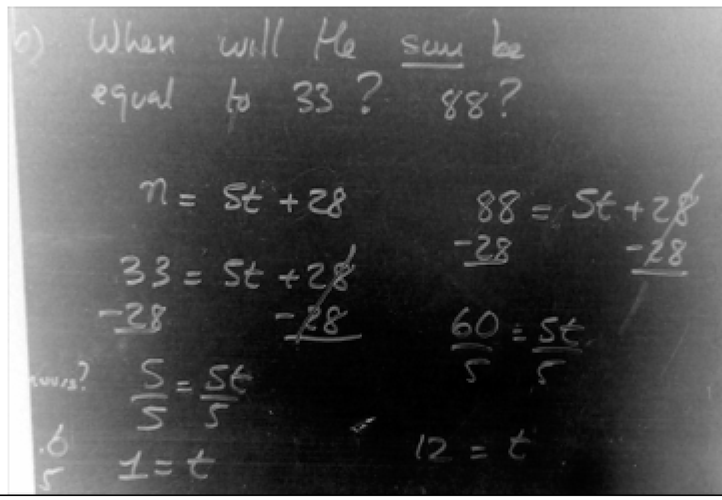
This task had the usual questions concerning term, number, and algebraic representation. The student who presented on the overhead decided that doing a table was much easier than applying the algebraic representation. In finding the 100th term, she had substituted incorrectly into the number instead of the term. I asked students how to solve part b) and I heard students telling me to substitute the value into *t* which was incorrect. Some students were able to handle this shift in question, while others continued to make the same mistake. I attributed this to forgetfulness rather than lack of understanding by this student, but I could be wrong.

Sum of Sequences (Day 65). For the second task, the objective was the sum of the sequences and solving for certain terms. Figure 4-36 presents the task and work.

Figure 4- 36. Task and work for sum of sequences.

Task
Given -17, -14, -11, -8, -5, ….and 50, 52, 54, 56, 58 a) Find the sum of the sequences for the 4th, 8th, and 100th b) When will the sum the sum be equal to 33? 88? c) What is the algebraic for the sum?
Work





Instead of writing two new linear sequences, I borrowed the last sequence and gave them another one. Parts a), b), and c) were similar to the last task. I made the mistake of forgetting to ask when and where the sequences met. Students took the role of teacher while I sat back and followed their lead. For this task, I felt more confident that students had remembered how to do this task.

Contextual task (Day 65). Numerical pattern sequences drive the curriculum, but we also did work with contextual task. Students had difficulties with the form $y = b - mx$. I saw this in their assessments and in the computer lab and decided to write another task in same form. Figure 4-37 shows the task and the work determining the distance.

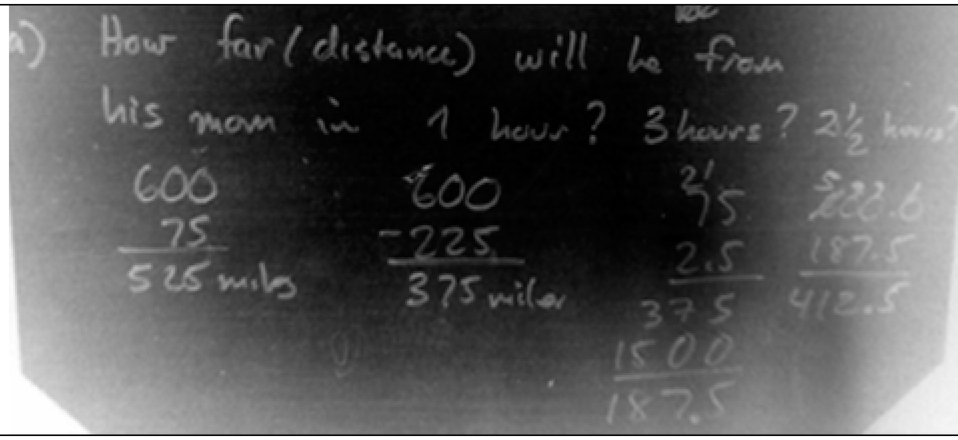
Figure 4- 37. Task and work.

Task

Marshal needs to drive 600 miles to visit his mother. Assume that he drives 75 mph and starts at 8 AM.

a) How far (distance) will he be from his mother in 1 hour? 3 hours? 2.5 hours?
b) When will Marshall reach his mother house?

Work



I stood by the overhead as students directed me toward the solution. The excerpt below was our discussion for finding the distance for 3 hours.

200 Shantae: I found it.

201 Teacher: Alright.

202 Christian: 485

203 Teacher: So how do we get this answer here?

204 Several: You take 75 and multiply by 3 …

205 Breanna: your gonna get 225 and take that away from 600

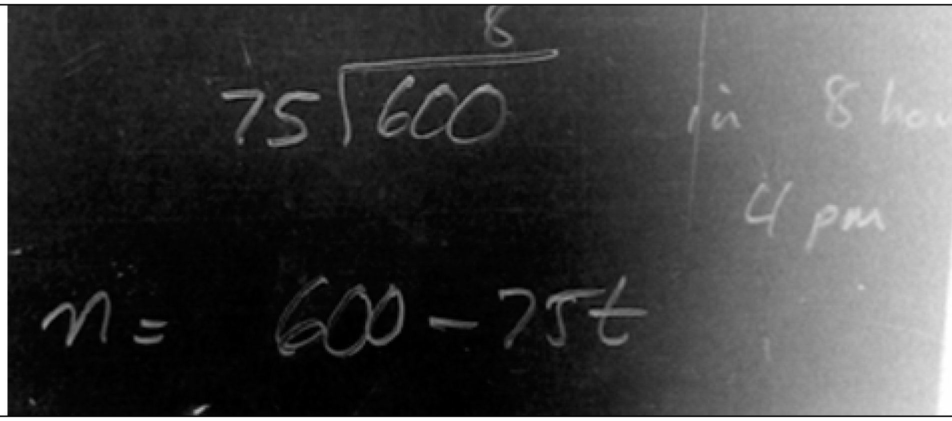
This excerpt showed that students could explain well how to solve this task. There were students who don't normally participate who voiced their comments about the task.

We finished part a) and were now working on part b) of the task. Figure 4-38 contains the task and work.

Figure 4- 38. Task, work, and discourse on how to find the time.

Task
Marshal needs to drive 600 miles to visit his mother. Assume that he drives 75 mph and starts at 8 AM. b) When will Marshall reach his mother house?

Work



This discussion in the excerpt below highlights a few students ability to solve this task using arithmetic instead of algebra.

220 Teacher: How do we do this one?

221 Breanna: 75 divided by 600

222 Teacher: 600 divided by 75. And our answer comes out to?

223 Alonzo: 8 hours

224 Teacher: In 8 hours, so what’s the time?

225 Jay: 4 o’clock

226 Teacher: Algebraic representation

227 Christian: *n*= 75 no 600- 75

228 Jay: *t*

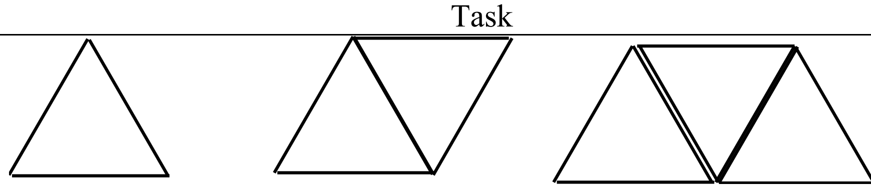
I wasn’t quite sure how students would solve this task. They could substitute the value into the algebraic representation and solve or they could divide to find the number of hours. I posed the question (line 221) and Breanna stated the operations incorrectly (line 221). I might have focused too much with the operation with integers and overlooked students’ difficulties with fractions. I saw difficulties on assessments and in class, all from different students. I wrote down the calculation (see work) to show what 600 divided by 75 looked like. I thought that Breanna was the only one who did this correctly, but Alonzo stated the answer (line 223) as I posed the question (line 222). I asked for the time and after a few seconds Jay correctly states the time (line 225). I moved toward the algebraic representation (line 226) and Christian began the algebraic representation incorrectly and then restated her response (line 227) with Jay helping out (line 228). They were able to see the algebraic representation for a contextual situation.

They could do the algebra but stumbled on the arithmetic. I gave my students opportunities to work with me on arithmetic after class and only seven took up this offer. I was also confident that students had not forgotten how to do this task.

Geometric task (Day 65). I worked with geometric task the first two weeks of the semester, because of the time constrain. I wasn't quite sure if students would remember how to do a task like this and they struggled with these tasks on assessments. Figure 4-39 is the task and work on how we arrived at the number of lines for the 8th figure.

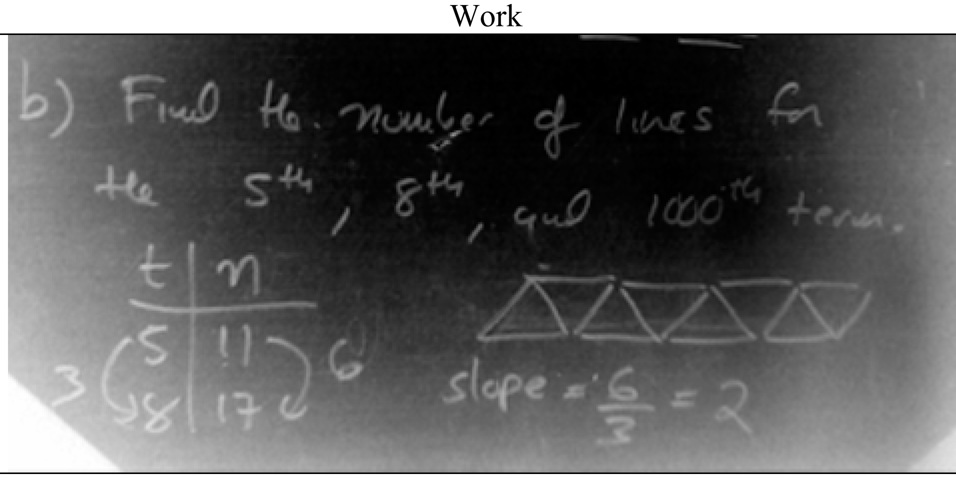
Figure 4- 39. Geometric task, work, and discourse.

Task



a) Draw the fourth and fifth term
b) Find the number of lines for the 5th, 8th, and 1000th term.

Work



This discussion came after a conversation about finding the number of lines for the 8th term. The excerpt below shows how we arrived at finding the algebraic representation for the term and number of lines needed.

240 Teacher: That 15..17 how do we find the jump? 11 to 17 is

241 Several: 6

242 Teacher: 5 to 8 is

243 Jayvon: 3

244 Teacher: My slope is?

245 Jayvon: ½

246 Several: 6 over 3

247 Teacher: Which is

248 Several: 3, ½, 2

249 Teacher: So if my jump is 2 or the slope is 2 what is my 0th term?

250 Breanna: One, one

251 Teacher: So what is my algebraic?

252 Jayvon: $n = 2t+1$

Once again, students could not simplify 6 divided by 3. On day 58, they made the mistake with 4 divided by -8. Internally, I grew frustrated with these errors. I have shown in class when the simplification is whole, fractional, and negative and asked them on assessments. As we were nearing the last five days of class, I showed my frustration by not explaining it anymore. This was not acceptable to be making this type of error. Although they were able to write the algebraic representation, I worried that I had not done enough with fractions.

Mistakes. As I listened to the audio tape, I was quite proud of how confident they sound solving these tasks. They still made errors substituting the correct values and continued to struggle with basic simplification with fractions and this included some of my top students, but I felt that they could write the algebraic representation for a variety of different situations. When I gave them the geometric task, some replied that this was easy, but I had not done any geometric pattern since September. It wasn't that they couldn't do algebra, but it was their arithmetic skills that continued to hinder them. Perhaps, I should have done more arithmetic review, but I didn't really have any extra instruction time.

I still had a few more topics to teach, but the school administration had scheduled final exams for this class January 8th, even though the start of the new semester was January 22nd. After the final exam, I would not see students until the start of the second semester. This meant that I would push some of the algebra topics such as standard form and point-slope into the second semester. I was worried about connecting standard form and point-slope to our current approach to algebra. I didn't want these topics to be seen as something extra.

Use of resources

In my classroom, I had a classroom set of algebra textbooks by Larson (2004), Foster (1998), and Brown (1992) and a classroom set of TI-83 plus graphing calculator. I also had access to the computer lab with *Cognitive Tutor: Algebra 1*. When I wrote my lessons, I began the class with my own task and transitioned to the textbooks for procedural work such as operations with real numbers, solving linear equations, and slope problems and I used *Cognitive Tutor: Algebra 1* for contextual tasks and the solving of linear equations. The textbook allowed students to work individually or in groups and freed me to help students individually. Without Larson, I would have to find an algebra textbook to obtain tasks for practice and homework.

By teaching the writing of linear equations on the first week of school, students were able to transition to *Cognitive Tutor: Algebra 1.* As mentioned before, I tried unsuccessfully teaching algebra with Larson. Using *Cognitive Tutor: Algebra 1* as the main curriculum was also problematic, because I was never sure if the server and computers worked from day to day. With only twenty-eight computers with 35 students on my roster, I would only bring students into the computer if I had enough working computers. The authors for *Cognitive Tutor: Algebra 1* approach algebra through multiple representations. Students learn to fill in tables and write algebra representation for a context. Later units incorporate the graph and the solving of linear equations. Students enjoyed working in the computer lab, because it allowed students the freedom to work at their own pace. Without *Cognitive Tutor: Algebra 1,* I would need to do more contextual tasks and graphing in the classroom. I understood that each of these resources offered different opportunities for student learning (Cohen, Raudenbush, & Ball, 2003).

Discussion

The National Mathematics Advisory Panel (2008) commissioned a survey to determine the obstacles facing Algebra teachers. NMAP found, “The survey revealed that teachers rate their students’ background preparation for Algebra 1 as weak. The three areas in which teachers report their student to have the poorest preparation are rational numbers, word problems, and study habits” (National Mathematics Advisory Panel, 2008 p. 9). These algebra teachers list these factors: unmotivated students, mixed-ability groupings, lack of family support, and making mathematics assessable and comprehensible as major challenges they face.

I am in agreement with their observations but would add the school environment as another factor influencing algebra at my school. In this chapter, I presented a classroom where students and teacher worked together on the task. I have also provided examples where students questioned the “known” mathematics, chose their own problem-solving strategies, and helped build the mathematics in the classroom. I could not have developed TGT algebra without the participation with the students. Creating a curriculum tailored to students required student participation.

In order to teach in the inner-city, I created a classroom environment to allow my students and me the time and space to do algebra and lessen the negative aspects of the school environment. Students felt comfortable to voice their understanding even when it was incorrect. This gave me another lens to view the students.

I began building a classroom environment with the students on the first day of algebra by doing a variety of algebra tasks to show what was expected from them cognitively and behaviorally. By beginning with a numerical pattern task, students could use arithmetic. I introduced local terms, jump and 0^{th} term, which proved to be useful for contextual and geometric tasks. I could foreground and background parts of the tasks to fit the specific algebra standard we were working on. With numerical pattern tasks, I placed conceptual understanding first and developed procedural fluency later. Designing my own tasks gave me the flexibility to look for connections between the algebra topics between the sections in a chapter and between chapters. This freedom allowed me to think about ordering of the different algebra topics and also create a new connection between multiplication and division and the iterative notation. The method used to find the 0^{th} term allowed us to move to the addition and subtraction of real numbers. These inverse relationships allowed us another way to talk about the solving of linear equations and when we transitioned into slope. I held tightly to Chantel's use of the "0^{th} term" for "*y*-intercept."

I learned that I could not force students to do algebra. I had to create tasks that pushed students out of their comfort zone with arithmetic. Once this was done, some students chose algebra when needed and some students used algebra exclusively. Students' prior knowledge made it difficult to re-teach the addition and subtraction of integers and the solving of linear equations by the cover-up method. My mistake was the tasks used didn't create the tension needed to move them toward algebra.

With each passing day, I became more mentally and physically fatigued and it affected my teaching. There were many better teaching decisions that I could have taken, but lacked the clarity to see in the moment of teaching. Why didn't I push my students more with fractions, cover-up method, iterative notation, ratio problems, and slope just to name a few topics? In the midst of teaching, trying to balance the needs of my students and trying to fulfill the requirements of the district mandated curriculum often forced me to make these curricular decisions. I realized the importance of solving linear equations for future mathematics classes. Thus, I dedicated 31 days for that chapter. As the teacher, I also felt that students needed time to reflect on the material and since I would not be following them to the 10th grade, I needed to provide all of my students the basic algebra tools necessary to be successful.

I could lessen the negative aspects of the school environment, but could not completely remove it from my classroom. I have seen growth in my students in the classroom and on their assessments. Students volunteered to do the tasks, but more importantly, were more involved during discussions. There were episodes, where I sat back and allowed them space to argue about a task. The assessments showed that they were not bound by procedures. They could adapt and become flexible with the algebra. Students could change from local terms to technical terms. Students could do algebra with weak prior knowledge in arithmetic.

CHAPTER 5

PREPARATION FOR TEACHING

For this chapter, I return to the last question: What does a week of teaching look like using this approach? Instead of looking at instructional challenges throughout the semester in the classroom like I did in Chapter 4, I want to highlight how I approached the teaching of algebra for a week to reveal the decisions in analyzing assessment, writing lesson plans, and enacting the lesson plans.

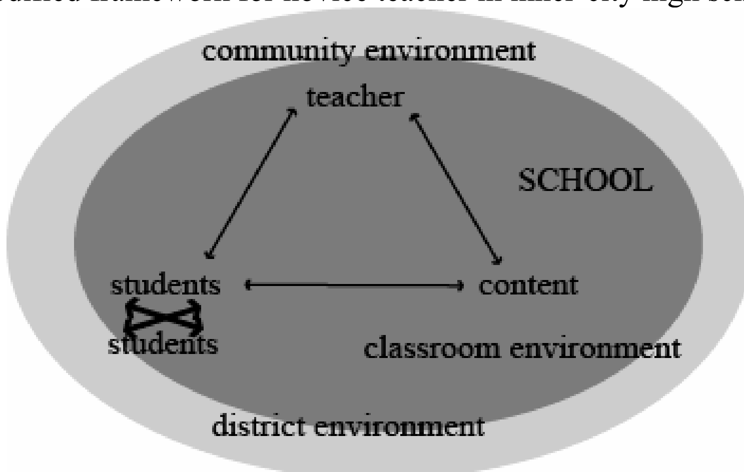
In the first part of this chapter, I provide the background information of how I came to establish a classroom environment. I want to show that in order to teach the content I had to contend with different elements inside the instructional triangle. In other words, I could not have created TGT algebra with a numerical pattern approach without first dealing with the school and classroom environment. In order to show this, I used the instructional triangle to map out my teaching career at Jefferson High School (JHS) that began in the fall of 1993 and ended in the fall of 2006.

The second part deals with the daily chores of teaching I did in the fall of 2006. I want to show through a teaching cycle of assessment-lessons-enactments how I approach each week of teaching. I want to show how I analyzed an assessment and how it affected my lesson writing and the enactment of teaching. In order to show this, I will look at a one-week interval of teaching that begins with an assessment on Friday and ends with another assessment on the following Friday. In this section, I will present a flow chart of how I analyzed assessments and when I am in the midst of teaching. In the last section of this chapter, I will present a discussion of this chapter.

Changing Focus

Teaching in the inner-city required that I understood how each element inside the instructional triangle framework of Cohen, Raudenbush, and Ball (2003) interacted with the other. I needed to continually shift my attention to this framework in order to teach. Figure 5-1 shows how I envisioned my own teaching in 1993.

Figure 5- 1. Modified framework for novice teacher in inner-city high school.



In this figure "school" is capitalized to illustrate how dominating the school environment had been on the classroom environment. Fights, fire alarms, or bake sales in the school dominated the students' topics of conversation and overwhelmed the learning experience when I taught in 1993. The students' interactions with one another were much stronger than my interaction with the students; I spent a lot of time trying to get control of the classroom. Students didn't care what I was teaching. It wasn't the lack of resources (e.g., textbooks, graphing calculators, and other teachers) that prevented me from teaching, but a lack of uninterrupted time in the classroom to develop a lesson and an inability to use these resources. I had problems with tardiness and truancy. During academic years 1993-1994 and 1994-1995, I hid in the classroom.

Minimizing the School Environment

My first step was to learn how to minimize or buffer the influences of the school environment. I realized that it was not possible to implement and enforce the classroom rules – administrative support was simply too inconsistent Thus, I focused a lot of my energy on truancy and absenteeism, which I felt had the most bearing on the structure in my classroom. These two issues allowed me to buffer some of the negative effects of the school environment.

The term "buffering" is most often associated with the principal and parents (DiPaola & Tschannen-Moran, 2005) or teacher and parents (Aldridge, 1998). Ogawa (1998) wrote, "…research consistently demonstrates that teachers expect principals to shield them from undue parental influence and that principals do perform this functions" (p. 11). I had no issues with the parents and have always had support from them during our conversations on the phone or during Parent-Teacher conferences. The obstacle to my teaching was buffering the school environment from my classroom.

Tardiness directly effected the classroom environment. Late students took instructional time away from those who were already in the classroom because I had to deal with late students. Allowing late students into the classroom also sent a message that being late was acceptable at JHS. Since JHS did not enforce the tardy policy, I decided to create my own tardy policy by not allowing late students into the classroom without a pass. This was tested early in the fall semester in 1995 when I asked a student to get a pass. She returned back to the class banging on the door for me to let her in. Upon opening the door, she handed me the pass and proceeded to take a seat. I stated that having a pass did not entitle her to be rude to me and the class. I sent her back out of the classroom. A few minutes later, she returned with a staff member who asked me why I sent her out of the classroom. I told her that her banging on the door had disrupted the classroom and her behavior was unacceptable. The staff member apologized and escorted her out of the classroom. After our incident, I never had anymore disruptions from her. I felt for the first time at JHS that I was in control of the classroom.

Tardiness occurred also during our first year implementing the Small Learning Community for the 9th graders in 2002. As a group of 9th grade teachers, we decided to implement our own tardy policy by having teachers who were not teaching that hour be in charge of writing passes and documenting late students. By doing this, we could identify late students and demonstrate to the other students that there was a repercussion for being late. To my surprise within the first week of teaching, the principal knocked on my door and ordered me to allow late students into the classroom. We abandoned the tardy policy during the first week of school.

During the academic year 2006, as a way of encouraging students to be on time, the introductory task or warm-up task was timed and graded. The setting up of the classroom environment occurred on the first day of algebra. As mentioned before in Chapter 4, I established the classroom environment with the following introductory task:

Warm-Up: Given the following pattern 3, 6, 9, 12, 15…

a) Find the 8th term.
b) Find the 10th term.
c) Find the 15th term.
d) What is the term when the number is 21?
e) What is the rule for this pattern? *(algebraic representation)*

This task established my mathematical expectation for my students. Students needed to present for this graded task in order to understand the objective of the lesson. This introductory task was a higher-level demand task; therefore, students needed time to determine how to solve the task and get settled down. If I gave them a lower-level demand tasks, then students who finished early would bother the other students. If the tardy policy could not be enforced, then as the teacher I needed to construct another structure to serve as a rationale.

Even with a graded introductory task, students came to class late although to a lesser extent than before because their friends also came to class late. In this except below, I was beginning to go over a new lesson the iterative notation, but had to allow to two students to enter class.

1	Teacher	Please don't be late. Separate. One sit over here and one sit on the other side. All right. (pause) All right, here we go. So because this is not linear, Aubrey.

This occurred on the 22nd instructional day and I still had students who came late to class. I reprimanded the two students and I would prefer not to suspend them, because they would miss more instructional days. I provided this example to show that our original tardy policy, which was to close the door and not allowed students into the classroom, was quite effective, but this had to be abandoned because of the strain it created for the school administration. The use of a graded introductory task helped increase attendance in my class but did not achieve the same effect as closing the door when the late bell rang.

Not surprisingly, truancy was another major issue at JHS. Nationally, major inner-city school districts have a larger proportion of students who chronically skip school (Meredith, 1999). As a way of reducing truancy for my classroom, I created m own classroom rule in fall of 1996 that fixed the assessment date on Friday for each week. I alternated between individual and group assessment. The individual assessment allowed students to demonstrate their understanding and group assessment allowed students to teach each other on a graded assessment, which allowed students to try on the teacher role. By fixing the assessment on Friday, students needed to show up Friday, but also be present Monday through Thursday for the lessons in order to do well on the assessment. I modified the content on the assessment accordingly whenever we had an interruption during the week. I made it harder to make up an assessment by not allowing students to make up an assessment during class time. Students who missed an assessment needed to come during their lunch hour or after school.

Every year, the Homecoming Dance fell on a Friday, which was also my assessment day. Jefferson students saw this a “right” to skip this day because it had become a tradition. I have seen students who were habitually absent from my class show up at this dance. Every year, I asked my 9th graders to come to class before going to the dance. During the fall of 2006, I said the following:

79	Teacher	This week is individual and please, I know that we have Homecoming this week, take your test then go have fun at Homecoming.

I tried to reason with them but only 11 students showed up for that Homecoming assessment. It was difficult to teach in a “social” high school and changing students’ attitude about the purposes of school was a struggle. I could not compete against Homecoming, but I had better attendance with half-days that landed on Fridays and in the Friday before the Christmas break. Teaching at JHS meant trying different ideas to see how it could improve my classroom environment. Not all of the ideas were successful.

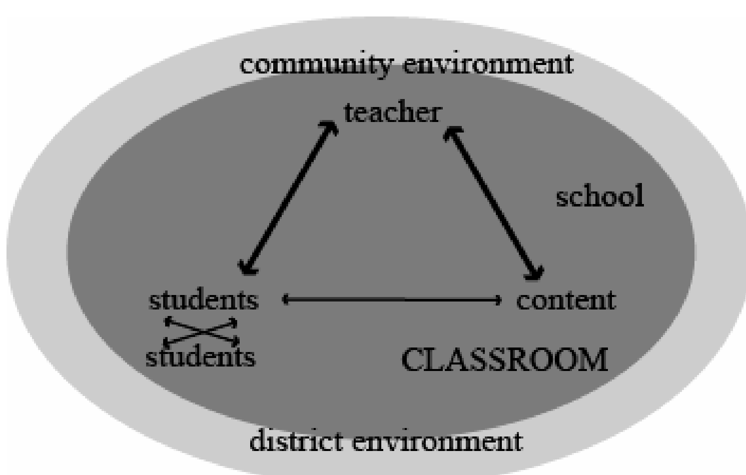
Other policies such as the dress code and cell phones were not strongly enforced at JHS. I now found I didn’t have the energy and means to enforce these policies. I would ask students to comply and if that didn’t occur then I would report students to the administration. Most students complied. Students knew from my actions that tardiness and absenteeism were the two major policies that I valued most, but as my structure and instructional practices improved, so did their discipline and learning.

Setting Up a Classroom Environment

Once I was able to buffer the school environment, I had fewer disruptions in the classroom and I was able to focus my attention on building rapport with my students. In the fall of 1996, I was teaching two regular Algebra 1 classes, two Basic Algebra 1 classes, and one Remedial Algebra class. Each class had a separate textbook and required a separate lesson plan. I could not see any mathematical or behavioral differences between my regular and basic algebra students. I began questioning how students were selected for these classes. With my basic algebra students, I had a difficult time motivating them. I decided to use the Brown (1992), which was the Algebra 1 textbook. I found that those basic algebra students were more responsive to my directions and realized that my basic algebra students wanted another opportunity to prove that they could do "regular" algebra. Prior to this realization, I had lowered my expectations on their assessments thinking that I could build their self-esteem. When I raised my expectations, they raised theirs, and I felt better as a teacher. This occurred in 1996, when I realized that I needed to try different ideas to improve the classroom environment.

Instead of using the district's curriculum guide for pacing, I divided the objectives into weekly lessons and moved through the curriculum. I used Brown (1992), which approached the teaching of algebra through symbolism. I attributed students' lack of success on my lack of pedagogical content knowledge for algebra. Figure 5-2 is a pictorial representation of teaching when I had better control of the classroom during academic year 1996-1997.

Figure 5- 2. Instructional triangle with a focus on classroom environment.



In spite of this, I taught tasks that I enjoyed as a student, but I still had bored students in my classroom. I rationalized that these 9th graders could not see the benefit of mathematics for future classes. In the fall of 1997-1998, more veteran staff members retired, I was moved to teach the advanced mathematics classes (e.g., Algebra 3, Algebra 4, Pre-Calculus, and Calculus). I applied these classroom structures to my juniors and seniors. I was able to teach, but students still had difficulties applying algebra skills. My students had difficulties following the mathematics on the blackboard and seemed stuck on substitutions and simplifications. Even though these juniors and seniors were taking advanced mathematics classes, they didn't have mental stamina and lacked the algebra skills. I began thinking that I needed to build their algebra skills in the 9th grade.

During the year of this study, I established the classroom environment with the first introductory task. I used this task to establish the mathematical expectation but also the behavioral expectation for the classroom. I return back to an incident that occurred with a student on that day in excerpt below:

3 Teacher: Mickey, put the hood down man or whatever you got there….

I didn't know how Mickey would react to my request, because I really didn't know my students on the third day of semester. Even though this was a minor issue with the dress code, not dealing with it might send a signal that I was lax with the rules. I have also shown in Chapter 4 that I allowed students to take the lead in the classroom. Students participated in the classroom discussion about the mathematics and 30% of the students' actions were comments about the task. Students making comments about the task meant that they had less time to be off-task. As the semester progressed, students volunteered to grade and to work out the task on the overhead. By allowing students to take on these responsibilities, the students and I helped shape the classroom environment. This appeared when the "bad class" occurred and a Kiera was able to help establish the control in the classroom.

Students-Content

I left JHS at the end of 1998 to continue my PhD studies. I returned to JHS in the spring of 2001 as a substitute teacher. I began class with an introductory task and gave assessment every Friday. It took me a week to establish my classroom environment and I did not have any major management issues. I asked my curriculum leader to assign me to 9^{th} grade algebra in the fall of 2001 because I still had hopes of teaching algebra more effectively. I assumed that my second attempt at teaching algebra would allow me to help my students understand algebra better, because I had better control of classroom and was more confident with algebra content. But when I returned to JHS, the district had replaced the textbook by Brown with Foster (1998). I found the tasks were overly complicated. For example,

Solve $\frac{2.405}{3.67} = \frac{g}{1.88}$ (Foster et al., 1998 p. 199)

The decimal numbers complicated the objective of the task, which was to solve proportions. I decided to keep a classroom set of Brown for extra practice. It was during this academic year 2001-2002 that I had a conversation with a female student about algebra that I realized that my assumption about algebra was quite wrong. She found algebra stupid; the way things were going, I agreed with her characterization. I still had more work to do.

Even though I had more control of the classroom, stronger understanding of the content, and better rapport with my students, I was still unable to make the content relate better to my students and the interactions between students and content remained constricted. Some students would participate, but their algebra understanding did not seem robust.

In 2004, the district mandated curriculum now used the textbook authored by Larson, which was similar to the two previous district mandated textbooks. I realized that the interaction between the students and Larson had reached its full curriculum potential.

In other words, I did not believe that I could increase the interactions between the content and students with Larson. Ben-Peretz (1975) wrote:

> Three factors—materials, analysis, and interaction between materials and user—are significant in shaping curriculum potential… And student questions, relationships between materials and experience, associations deriving from unplanned classroom situations, and innovative uses of materials arising from particular conditions may all yield a broad spectrum of curriculum potential ideas. (p. 154)

In other words, no matter how I presented Larson to my students, the algebra that I envisioned (e.g., questioning their understanding, choosing problem solving strategies, and helping build the mathematics) could never be realized with Larson. I decided to find a different approach to the teaching of algebra. I wrote the lesson in Figure 5-3 for my 9th graders in the fall of 2004 and it was the first lesson where Larson did not take the lead in the instruction.

Figure 5- 3. Lesson for September 2nd, 2004

Date: September 2, 2004 Algebra 1

Objective: Students will operate on integers using the TI-Graphing calculator.

Warm-Up: Given the following pattern 3, 6, 9, 12, …, find the following:

a) The fifth term.
b) The sixth term.
c) The 10th term.
d) The 30th term.
e) What is the rule?

TI-Graphing: With the TI-83, we can do the same problem by doing:

Enter 3	3
Hit enter	
+3	Ans + 3

By hitting the enter button we can find the next term. This gets at the formula NEXT = NOW + 3 (iterative). Another way to get the same pattern is seeing that is you multiply the term you are looking for without needing to find all the terms before hand: Output = 3 times the number you want (recursive).

Y=
Enter y= 3x
2nd Graph gets us the Table
Make sure you set the TableSet 2nd Window
Tablestart (TblStart) is 1
Use the arrow buttons to find the term you are looking for.

Discussion: Students could express the pattern as NEXT=NOW+3 or output=3 times the Term.

Questions: What do you see about the rule NEXT =NOW+3 and Output = 3 times Term? Will the rules NEXT = NOW+3 and Output = 3 time Term always work with any term we start with? When will the rule work? Can you find some other rules?

Journal: Tell me everything you know.

This lesson showed my first ideas in implementing a numerical pattern approach to the teaching of algebra. Instead of trying to find the slope-intercept form of the equation, I decided to teach the iterative notation. Since the academic year 2004-2005, I have made a lot of changes to the numerical pattern approach to the teaching of algebra.

Discussion

I traced out my teaching career when I first walked into JHS in 1993 to the year I conducted my research in 2006. I did this to show that in order to teach in an inner-city classroom I had to continually try different ideas to improve the classroom environment. If I hadn't tried to improve my working condition, then my teaching career would have been quite short and I would have pursued a different career path. I tried to improve my working environment in a systematic order and did this in the following order: (1) minimizing the school environment, (2) setting up the classroom environment, and (3) reexamining interaction between student and content. In the next section, I provide more detail on how I approach the content.

Teaching Cycle

Before a teacher steps into the classroom, he brings his prior experience in how he learned the topic and how he has taught this particular topic to the next enactment of teaching of this topic (Zaharlick & Green, 1991). The Mathematics Learning Study Committee (National Research Council, 2001) categorizes this as *planning*. As a novice teacher in 1993, I planned lessons at the end of the day, but found that I was choosing tasks merely to fill the class period, by listing "activities, page numbers in the textbook or the teacher's guide, and perhaps a few words about concepts to be covered" (McCutcheon, 1980). In the fall of 1996, I shifted to planning during the weekend, which allowed me to look at a week's worth of lessons and determined what algebra topics to focus on. During the planning phase, I had time to reflect on how students interacted with the content by looking at their assessments.

The teaching cycle begins when Friday's assessments has been graded. I looked for uptakes and errors made by the students. I would write the lessons for the following week and determine what to reteach and what new standards to teach. During the enactments of teaching, I would follow the lesson but would make changes accordingly to students' inputs. I did this for the whole semester.

In the next section, I present a weekly teaching cycle. I do this to illustrate how I was able to tailor my lessons to the needs of my students and to balance this with the need to teach the algebra curriculum. I selected a weekly teaching cycle that began with a group assessment on October 13th and ended with an individual assessment on October 20th.

Group Assessment

Group assessments allowed pairs of students to work together. Since this was a graded assignment, I allowed students to choose their partner. In order to make sure that both students worked on the same task, I handed out individual task to groups. When they finished a task, I would give them another task. This ensured that students would not work out a task individually. I wanted both minds to focus on a single task. Working with a partner, I believed, was less stressful than an individual assessment. It was also possible for an absent student to get caught up by working with a partner and also allowed students to try on the teacher role.

Prior to the October 13th group assessment, we had been working on the addition and subtraction with real numbers. Figure 5-4 was the assessment from October 13th. The main objective of the assessment was the operations with real numbers with addition and subtraction which was question 1. A secondary objective was the contextual task in the form $y = b - mx$ in question 2. The third question utilized matrices to teach addition and subtraction. I felt compelled to teach this because I had seen it on numerous standardized tests and in Larson. I didn't understand how students could see the importance of matrices when used to teach arithmetic tasks. Question 4 was another contextual task and I included a distracter, 4×6, into the task. Question 5 and 6 focused on an understanding of the operations and I wanted students to show me by writing, integer tiles, or a number line how they understood the arithmetic tasks.

Figure 5- 4. Group assessment for October 13th.

1. Find the first five terms and algebraic representation for the following:
   a) 3rd term is -15 and jump is -13
   b) 3rd term is 12.35 and jump is 4.56
   c) 3rd term is $2\frac{3}{4}$ and the jump is $\frac{3}{4}$
   d) 3rd term is $\frac{3}{4}$ and the jump is $\frac{2}{7}$

2. Jimmy borrows $250 from his mother and promises to pay his mother $2 a week.
   a) What is the 0th term and jump?
   b) What is the algebraic representation for the amount money owed to his mother?
   c) How much does he owe 1 month from now?
   d) When will he finish paying his mother?

3. Evaluate
   a) $\begin{bmatrix} 11 & -8 \\ 2 & -5 \end{bmatrix} + \begin{bmatrix} -5 & 2 \\ 6 & -8 \end{bmatrix}$ b) $\begin{bmatrix} 2.1 & 3.6 \\ 2.56 & 5 \end{bmatrix} + \begin{bmatrix} 5.6 & 2.25 \\ 6 & 8.2 \end{bmatrix}$
   c) $\begin{bmatrix} 11 & -8 \\ 2 & -5 \end{bmatrix} - \begin{bmatrix} -5 & 2 \\ 6 & -8 \end{bmatrix}$
   d) $2\frac{1}{2} \div \frac{1}{4}$ (Bonus: Solve this problem by using box diagrams.)

4. Jefferson High School is getting ready for the Homecoming. Jamie decided to print some 4×6 photo, which costs $3 each and she would like to give an additional $5 she gives to her brother for helping her. Jamie has $50 to spend.
   a) Make an input and output table
   b) Find the algebraic representation for the total cost and the amount of photos she prints.
   c) If she has $50 to spend, how many photos can she print?
   d) Find the cost for 60 photos.

5. Please explain in separate paragraphs how to solve the following problems and then solve them:
   a) $2 + (-5)$ b) $(-2) + 5$ c) $(-2) + (-5)$
   d) $-2 - 5$ e) $-2 - (-5)$

6. Please explain in separate paragraphs how to solve the following problems and then solve them:
   a) $2 \bullet (-5)$ b) $(-2) \bullet 5$ c) $(-2) \bullet (-5)$
   d) $(-2) \div 4$ e) $(-4) \div (-2)$

In answering the first question, some groups wrote the arithmetic tasks and could determine the correct answers. I also had groups who could not correctly determine the correct results. Most groups could do part b) of the first task. For part c), I had groups who could add and simplify $2\frac{6}{4}$ into $3\frac{2}{4}$ and groups who left it as $2\frac{6}{4}$. This assessment suggested that most groups struggled with the addition and subtraction of fractions with different denominators and that I would need to do more work with fractions.

For the second question, some groups used $250 as their 3rd term and were able to make the table with this information. I needed to use terms other than the 3rd term in my lessons and assessments, because students were beginning to associate the 3rd term as the *y*-intercept. For part b), I had groups who could not determine correctly the sign for the slope or the sign for the constant term. Most groups skipped part c) and for part d) of the task, I was looking an answer in weeks, but a group used the fact that $8 was equivalent to one month and determined the answer in number of month. I was quite happy because this group was quite flexible in how they approached this task.

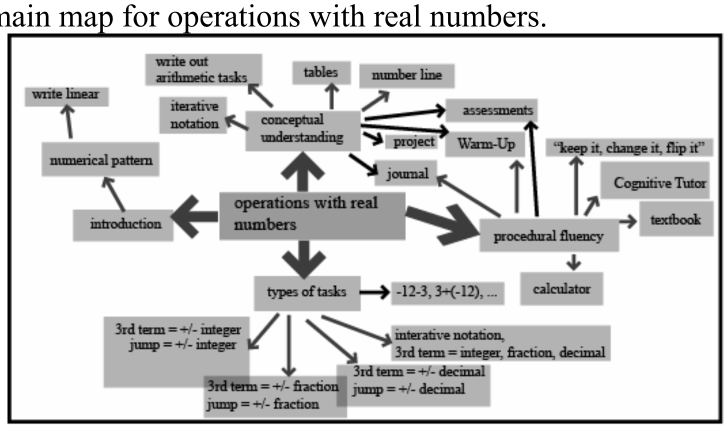
In question 3, most groups had no difficulties determining how to obtain the correct answers and I decided not to write anymore questions with matrices. In question 4, most groups ignored the distractor. Prior teaching with *Cognitive Tutor* had shown that students would parse through the contextual situation and apply numbers without much thought (Schoenfeld, 1988). The common error made was to ignore the constant term, which was the cost for her helper. I believed that these errors made in the writing of an algebraic representation could be fixed with *Cognitive Tutor: Algebra 1*. For part c), groups used the table made in part a) for the number of photos and cost in order to solve part c).

In question 5, the groups used integer chips to demonstrate addition and subtraction and in question 6, groups wrote the rules for multiplication and division with signed numbers. I wanted students to use contextual examples I used in prior lessons and should have stated this in the task. I would write a few more arithmetic tasks in the upcoming lessons.

Domain Map and Flow Chart. After four years of teaching algebra using a numerical pattern approach, I have created tasks and developed ideas about how to approach and build conceptual understanding and procedural fluency. After looking at the assessment I thought about how best to adjust my lesson plans for the following week. These assessments addressed many standards. Thus, I needed to decide which standards would be taught or re-taught. Figure 5-5 is the domain map for Chapter 2, which is on the operations with real numbers. I used the domain map when analyzing assessments and when in the midst of teaching. The domain map contains the following four elements: (1) the introduction, (2) conceptual understanding, (3) procedural fluency, and (4) the types of tasks.

The domain map is my way of organizing the targeted standards in the chapter. For each standard, I wrote an introductory task that connected prior standards to the new standard. I used the TGT tasks to further build conceptual understanding and other resources such as Larson and *Cognitive Tutor* to build procedural fluency. The domain map also includes a type of tasks used.

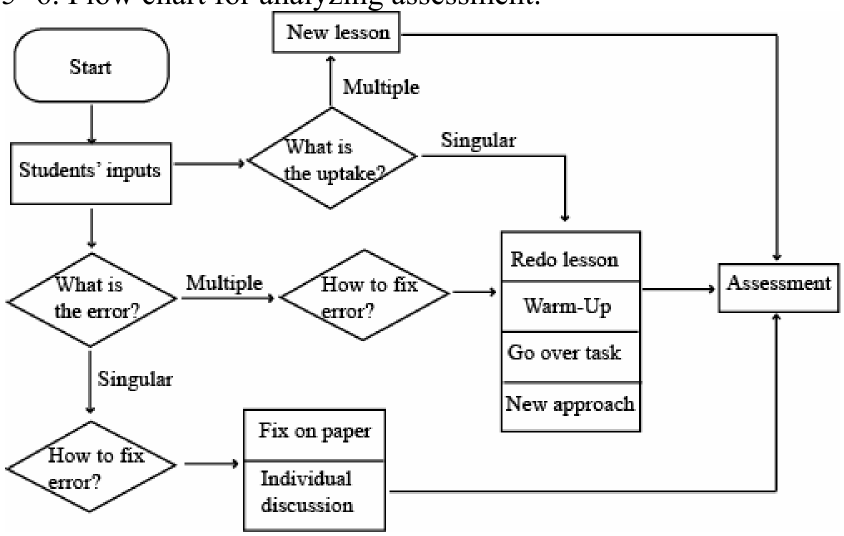
Figure 5- 5. Domain map for operations with real numbers.



From this domain map, a journal could be used to build procedural fluency and conceptual understanding.

Figure 5-6 is a flow chart of my thinking process as I graded and analyzed the assessment. As I graded student's works, I looked at the errors and uptakes. If multiple students made the same error, then I need to look at how to fix this serious error, which could mean a re-doing of the lesson. If a few students made an error, then the fix need not be so drastic. I might simply write another similar task in the next lesson. If multiple students had uptakes, then I will not ask this task for the next assessment, but may write a similar task in future assessment. If only a few students had uptakes, then I continually keep teaching the same standard and may use a different approach.

Figure 5- 6. Flow chart for analyzing assessment.



I think I had been using a rudimentary flow chart in the fall of 1996, when I moved the assessment date to every Friday. I used this decision making process to determine when to move forward or to remain in the same algebra curriculum. Since I gave a weekly assessment, I was able to make minute changes in the algebra curriculum.

The district pacing chart allotted one week for the addition and subtraction of real numbers. I had already used up those days and I needed to rethink about using integer tiles and the use of programs for the graphing calculators to help my students get a grasp with addition and subtraction. I decided that I needed to embed addition and subtraction tasks in future assessments.

Lesson Writing

The challenge for me was to balance the errors made in Friday's assessment and move forward in the algebra curriculum. I usually used Monday thru Wednesday to introduce a new lesson and reserved Thursday to review before the next assessment. To address the errors made on the first question in the last assessment, I wrote another similar task for Thursday's lesson. For the errors made with the contextual tasks for question 2 and 4, I brought students into the computer lab and helped them individually. For the simple addition and subtraction tasks, I would write a few more questions for Thursday and encouraged the students to work with the graphing calculators.

I decided that the objective for the following week would be to tackle the operations with multiplication and division, which was the last topic for this chapter. Larson approached the multiplication of signed numbers by providing rules for each case. I had used these rules for the past two years but was never satisfied with this approach so I kept looking for an algebraic way to teach multiplication and division. I needed to finish this topic because we were heading into our state mandated testing period and with a different teaching schedule, those instructional days were not conducive to teaching.

My approach to the teaching multiplication and division of real numbers would be similar to the addition and subtraction of real numbers. I wanted students to write the arithmetic tasks when they solved the algebraic task. This would allow students to see the inverse nature of multiplication and division. So far, we had been working with linear patterns and the use multiplication and division would change the nature of the linear pattern to an exponential pattern. I had been reluctant to do this, because I wanted everything for the first semester to work with linear functions. I didn't want to do any unnecessary digression.

In the fall 2004, while perusing through the National Council of Teachers of mathematics [NCTM]'s *Principles and Standards for School Mathematics* (PSSM)

(2000), I encountered the iterative function: "Another new type of representation that teachers might wish to introduce their students to is a NOW-NEXT equation, which can be used to defined relationships among variables iteratively" (p. 285). Figure 5-7 was my first attempt at embedding the iterative function into a lesson plan. The complete lesson is in Figure 5-3.

Figure 5- 7. Lesson plan incorporating the iterative function.

September 2, 2004
Objective: Students will operate on integers using the TI-Graphing calculator.

Warm-Up:	Given the following pattern 3, 6, 9, 12, …, find the following:		
	a)	The fifth term.	
	b)	The sixth term.	
	c)	The 10th term.	
	d)	The 30th term.	
	e)	What is the rule?	
TI-Graphing:	With the TI-83, we can do the same problem by doing:		
	Enter 3		3
	Hit enter		
	+3		Ans + 3
	By hitting the enter button we can find the next term. This gives us the formula NEXT = NOW + 3		

I enacted this lesson on September 2nd, 2004 and implemented the graphing calculator to show how one number connected to the next number in the linear pattern. It appeared that the students loved the iterative form of the formula, because it was easy to understand. I realized only later that students preferred the iterative form of the function and fought me when I tried to teach the slope-intercept form for the function. Thus, I abandoned the iterative notation for the academic year 2005-2006.

In the fall of 2006, I thought more about the connection between multiplication and division. My mind went back to the iterative notation, but not in the NOW-NEXT form. I would use the iterative notation from my days working with sequences and series. Instead of introducing the iterative function with addition and subtraction, the introduction of the iterative form made more sense with the multiplication and division of real numbers. I would begin with the table and provide them with integers. Students would fill in the table and we write the iterative function. I wanted to minimize the students' difficulties with fractions; consequently, I chose the multiplication table for two. By doing this, I hoped to keep the students focused on the mathematics concept and less on the numbers. Figure 5-8 shows the lesson written for the following Monday after grading and analyzing the group assessment for October 13th.

Figure 5- 8. Lesson for October 16th, 2006.

October 16th, 2006
Objective: Multiplication and division with introduction of distributive property

Warm-Up: Given the following table, fill in the missing values and describe the pattern.

T	n
1	
2	
3	2
4	4
5	8
6	
7	

Discussion: What is the rule for this pattern? In order to get the 6th and 7th term we need to multiply by 2, but in order to get the 1st and 2nd term we need to divide by 2. Is this pattern linear?

In order to get the next term, you multiply by 2.
$U_{n+1} = 2U_n$ with $U_3 = 2$

T	n
1	
2	
3	9
4	3
5	1
6	
7	

What is the rule here? In order to get the next term, you divide by 3.
$U_{n+1} = \frac{1}{3}U_n$ with $U_3 = 9$
Notice that in the last week's problem, we worked with addition and subtraction. Now for these patterns, we work with multiplication and division.

Find the first five terms if the 3rd term is 10 and the rule is $U_{n+1} = 2U_n$.
Find the first five terms if the 3rd term is 10 and the rule is $U_{n+1} = -2U_n$.

Oral Quiz

1. $4 \bullet (-9)$
2. $6 \bullet (-13)$
3. $(-8) \bullet (-5)$
4. $(-2) \bullet (-7)$
5. $-11 \bullet 8$
6. $-24 \div 6$
7. $6 \div 24$
8. $6 \div (1 \div 2)$
9. $(1 \div 2) \div 6$
10. $(1 \div 2) \div (1 \div 6)$

Let's focus on understanding what is meant by division. For our exam, the problem was $2\frac{1}{2} \div \frac{1}{4}$, once again if you plan on doing mathematics well, try to understand the problem and less on the procedures.

Try: a) $6 \div (1 \div 2)$ b) $(1 \div 2) \div 6$ c) $(1 \div 2) \div (1 \div 6)$

Journal: Explain in short sentences how you would solve the following $2\frac{3}{4} \div \frac{1}{4}$

Homework:

The warm-up or introductory task was a PWC task, but I included PWOC tasks at the end of class to build procedural fluency. I used small number in the introductory task to allow students the opportunity to solve this task. My objectives for this task were to show how the operations multiplication and division were inverse operations and to introduce the iterative notation. Since this was a new task I created, I was unsure how to write the iterative notation and defaulted to how I learned the iterative notation. I also decided to review division of fractions, which was on the last assessment and this would lead to the journal entry for that day. I left homework blank depending on how students' reacted to the lesson. This lesson plan had enough structure for me to get at the objective but enough freedom to allow me to introduce changes as needed.

Enactment of the Lesson

> Today, I'm attempting to teach multiplication and division with tables. I will introduce the iterative notation. With iterative notation, the objective is to get them to understand the notation. Journal entry: October 16th, 2006

Figure 5-9 shows the actual enactment of the lesson that took place on October 16th. I have included task numbers on the side to show the difference between the lesson written and lesson enacted.

Figure 5- 9. Instructional practice for October 16th.

<table>
<tr><th>Task</th><th>Description of tasks</th></tr>
<tr><td>1</td><td>Warm-Up: Given the following table, fill in the missing values and describe the pattern.
<table>
<tr><td>t</td><td>n</td></tr>
<tr><td>1</td><td></td></tr>
<tr><td>2</td><td></td></tr>
<tr><td>3</td><td>2</td></tr>
<tr><td>4</td><td>4</td></tr>
<tr><td>5</td><td>8</td></tr>
<tr><td>6</td><td></td></tr>
<tr><td>7</td><td></td></tr>
</table></td></tr>
<tr><td>2</td><td>Given the following table, fill in the missing values and describe the pattern.
<table>
<tr><td>t</td><td>n</td></tr>
<tr><td>1</td><td></td></tr>
<tr><td>2</td><td></td></tr>
<tr><td>3</td><td>9</td></tr>
<tr><td>4</td><td>3</td></tr>
<tr><td>5</td><td>1</td></tr>
<tr><td>6</td><td></td></tr>
<tr><td>7</td><td></td></tr>
</table></td></tr>
</table>

<table>
<tr><td>3</td><td>Given the following table, fill in the missing values and describe the pattern.
<table>
<tr><th>t</th><th>n</th></tr>
<tr><td>1</td><td></td></tr>
<tr><td>2</td><td></td></tr>
<tr><td>3</td><td>5</td></tr>
<tr><td>4</td><td>25</td></tr>
<tr><td>5</td><td>125</td></tr>
<tr><td>6</td><td></td></tr>
<tr><td>7</td><td></td></tr>
</table></td></tr>
<tr><td>4</td><td>Find the first six terms if $n_3 = 1$ and $n_{t+1} = 3n_t$</td></tr>
<tr><td>5</td><td>Find the first six terms if $n_4 = 24$ and $n_{t+1} = 2n_t$</td></tr>
<tr><td>6</td><td>Find the first six terms if $n_3 = 10$ and $n_{t+1} = -2n_t$</td></tr>
<tr><td>7</td><td>Find the first six terms if $n_3 = 1$ and $n_{t+1} = \frac{1}{3}n_t$</td></tr>
<tr><td>8</td><td>Oral quiz
1. $4 \bullet (-9)$ 2. $6 \bullet (-13)$
3. $(-8) \bullet (-5)$ 4. $(-2) \bullet (-7)$
5. $(-11) \bullet 8$ 6. $-24 \div 6$
7. $6 \div 24$ 8. $6 \div (1/2)$
9. $(1/2) \div 6$ 10. $2\,{}^1\!/_2 \div {}^1\!/_4$</td></tr>
<tr><td>9</td><td>Journal: Given $n_{t+1} = 2n_t$ and $n_3 = 12$. Explain how you would find the 1st six terms?</td></tr>
</table>

During my enactment of this lesson in 3rd hour, Sheryl couldn't understand why we were using *n* for the term now when we used it for the variable *n* for number. I was careless in how I chose my notation and luckily Sheryl spoke up. She suggested that it should be $U_{t+1} = 2U_t$. In my journal, I wrote that she was right about the notation but I modified this notation to $n_{t+1} = 2n_t$ when I enacted the lessons with the other algebra classes.

I played the lesson in my mind before the actual enactment in the classroom, but I was unprepared to how best to teach the iterative function. Since this was my first time using this task, I was unsure how to guide students to the iterative notation and decided to tell them. In the excerpt below, I began explaining the iterative notation for the first task.

5	Teacher	I haven't looked at it yet, I will in a sec. All right, here we go. So because this is not linear I have to describe this pattern algebraically differently. So let me give it to you the first time and see what's, Alonzo, you really need to focus, this is something new again, all right? So here we go. *n*. *t* + 1 and we said this is all being multiplied by? $2n_t$. Ok. Copy it down and I know you're going to

> ask me what does all this stuff means? So *n* is the number here, right? Your *t* represents your term. What's the difference between this *t* here and this *t* + 1 here?

After my explanation in line 5, a student asked for another similar task and I wrote second task. During this second task, Kiera said the following when I asked how to divide by fractions.

32 Kiara Keep it, change it, flip it.

This was unexpected by me and I decided to follow her lead to see if she had fully understood her statement. I elevated her method by using it in the classroom. Students still struggled with the writing of the iterative notation specifically with the subscript and I moved to the third task in the lesson, which was not part of the written lesson. As I finished the task, an intercom message interrupted the lesson. I took this time to remind my students about Friday's assessment, which coincided with Homecoming.

I continued with the lesson and gave them the third task not from the lesson. I had fewer questions about the notation and felt confident that I could push the iterative notation. In the original plan, I was supposed to move toward the oral quiz with multiplication and division of real numbers. I decided to change the lesson by giving the students the iterative notation in task 4 and have students generate a similar table like the first task.

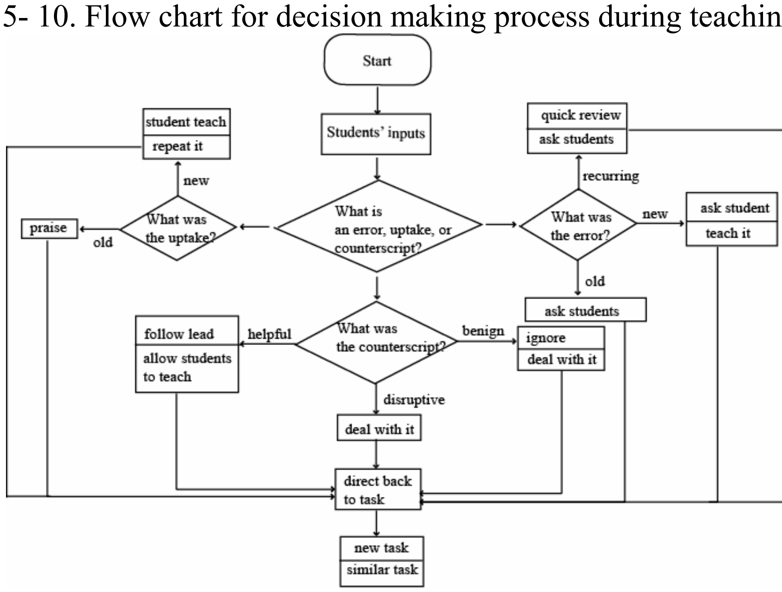
Students had no difficulties with filling in the table and Alonzo volunteered to do task 5 and was at the overhead explaining how to fill in the table. For task 7, I decided to include negative numbers. I went back to the lesson and gave them the oral quiz (task 8). I changed the journal question (task 9) to the iterative notation, because this was the focus for our lesson.

<u>Flow Chart for Classroom</u>. As illustrated with previous excerpts, it was difficult to get students to discover the iterative notation and I had to teach it. I felt that understanding the notation was more important than its discovery. When I moved to the

second task, Kiera provided a technique for dividing fractions. I repeated her technique and allowed Kiera to teach it to the class. I decided to create another task to allow students the opportunity to write the iterative notation. Up to this point, the numbers used have been small. I took another digression from the lesson plan by giving students the iterative notation and from students' positive reaction; I decided to give another similar task (task 3).

In the classroom, I must decide how best to respond to the errors, uptakes, or counterscripts made by my students[1]. As I listened to our discussion, I didn't hear any uptakes by the students. I decided that I needed to teach them how to write the iterative notation. When Kiera told the class about her technique, I considered it as a counterscript that was helpful for the classroom and decided to follow her lead. When I noticed students making errors in their notation, I decided to create another task. Figure 5-10 is a flow chart that represents of my decision-making process during teaching.

Figure 5- 10. Flow chart for decision making process during teaching.



[1] Errors are mistakes made by the students. Uptakes are actions that demonstrate an understanding, much harder to determine. The student's counterscripts are moves made by the students unplanned by the teacher.

The structure of my lesson plans allowed for digressions from the written lesson plan. I built in more time for focusing on students' needs and inputs. Such a digression occurred when a student asked me about changing the notation for the iterative notation and I had to incorporate this into the lesson in the moment of teaching. Another difference between the planned and enacted lesson was giving students the iterative notation and having them make a table. This was important because students needed to see the iterative notation from different perspectives. I often changed my lesson plan before and during class if I felt the tasks weren't going well, which could be characterized by a lack of participation by the students. These changes could be a simple change in the numbers used in a task or the ordering of the tasks.

Reflections. After the initial mistake of the notation with the iterative form, I thought the introductory task in Figure 5-9 would connect well with the tasks of Chapter 1, where students learned to write linear function. The primary objective of the lesson was a review multiplication and division of real numbers and the secondary objective was to learn to write the iterative form. By introducing the iterative form at this point, the students can concentrate on the arithmetic tasks. This was my first time teaching the iterative form with multiplication and division; therefore, getting students to discover the iterative form needed a lot of work. I also needed to spend time discussion whether this function was linear.

Students had difficulties understanding the iterative form of the numerical pattern and also the writing of the iterative form of the numerical pattern. My belief is the subscript caused the confusion. For the iterative form, the Writing Group for PSSM (2000) recommends using NOW-NEXT form. If I chose to use this form, I would still need to introduce the subscript for an initial value, which the Writing Group for PSSM forgot to include.

The objectives for this lesson were not the iterative form, but multiplication and division of real numbers. The iterative form connected the arithmetic to the algebra. To reduce the confusion between divisions with fraction, I should probably use the fraction bar in place of the division symbol. My students' understanding of division by a fraction hinged upon them remembering the "Keep it-Change it-Flip it" rule they learned in the middle school. What should I do as a teacher? The students seemed to be very flexible with this rule and would I do more damage to their understanding by throwing this rule out and re-teaching the lesson on division by a fraction? I was taught to multiply by the reciprocal. I might do an explanation of this rule when we move to the solving of linear equation.

Individual Assessment

An individual assessment was the most stressful assessment for my students. I gave each student a copy of the assessment and they worked on the assessment for the whole hour. During this time period, I checked their work in their notebooks. In this section, I want to show what an assessment looked like to illustrate how I moved through the algebra curriculum.

This was an individual assessment and occurred one week after the October 13th group assessment. The main objective for this assessment (Figure 5-11) was the operation with multiplication and division. A secondary objective was the introduction to the solving of linear equations. Since I wasn't happy with the result with addition and subtraction from the last assessment, I wrote question 2. Question 3 was another contextual task in the form $y = mx + b$. Question 4 contains the typical PWOC tasks for the solving of linear equations. Question 5 is the solving of linear equations with numerical patterns. For question 6, students needed to write out the contextual situation for a given equation.

Figure 5- 11. Individual assessment for October 20th.

1. Find the first five terms and algebraic representation for the following:
   a) $n_{t+1} = 3n_t$ and $n_3 = 12$
   b) $n_{t+1} = 2n_t$ and $n_3 = 9$
   c) $n_{t+1} = \frac{2}{3}n_t$ and $n_3 = 30$
   d) $n_{t+1} = .25n_t$ and $n_3 = 8$

2. Determine the first five terms and the algebraic representations for the following linear pattern.
   a) 3^{rd} term = 9 and jump = -4
   b) 3^{rd} term = -12 and jump = -3
   c) 5^{th} term = $5\frac{4}{5}$ and jump = $\frac{1}{5}$
   d) 1^{st} term = $\frac{2}{3}$ and jump = $\frac{1}{4}$

3. Lincoln's student population in September was 1200 students. Every week, 10 new students enroll into Lincoln. Find the following:
   a) Make a table for the students' population for the first five weeks.
   b) Find the algebraic representation for the number of weeks and the student population
   c) What would be the population in 10 weeks?
   d) When will the population of Lincoln be 1275?

4. Solve
   a) $x + 4 = -100$
   b) $2x - 4 = 12$
   c) $\frac{x-3}{10} = 2$
   d) $\frac{2}{3}(x-3) = 8$
   e) $\frac{3}{4}x - 6 = 12$

5. Given the following linear pattern 9, 12, 15, 18, 21,... Find the following:
   a) Find the 22^{nd}, 35^{th}, and 123^{rd} term.
   b) What term is equal 51and 141?
   c) Find the algebraic representation.

   Given the following linear pattern $3\frac{1}{2}, 4, 4\frac{1}{2}, 5, 5\frac{1}{2}, \ldots$
   a) Find the 10^{th}, 12^{th}, and 100^{th} term.
   b) What term is equal to 10 and 20?
   c) Find the algebraic representation.

6. Write a scenario (Story problem) for the following problem
   a) $n = 3t + 100$
   b) $n = 200 - 4t$

As I graded the first question, I noticed students' difficulties with the division of real numbers. A number of students could do the more difficult manipulation as in part c) but had difficulties with part a). Students may have understood the procedure but may not understand the results of their calculations. Students also had difficulties with decimals in part d).

In question 2, many students wrote out the arithmetic tasks and were able to correctly calculate the tasks. Those students, who didn't write out the arithmetic tasks, made mistakes by reversing the directions for the number for part a) and part b). Many students had fewer difficulties with the adding and subtracting of fractions in part c) and part d). Some students did better on fractions than the last assessment. I also incorporated the 1st and 5th term into the task, so that students didn't always focus on the 3rd term. I felt the review we did on Thursday helped.

When I first introduced contextual tasks on the third day of algebra, students used the variables *n* and *t*. But as students became more comfortable some students used the variables *w* and *s,* for the number weeks and the number of students respectively. For question 3, most students were able to make the table but some students made errors in placing the initial value of 1200 as the first term or the third term in the table. The use of the algebra software *Carnegie Tutor: Algebra 1* would continue to give students more confidence with the choosing of the variables and the writing the algebraic representation.

For question 4, the difficulties occurred in parts d) and e). In part d), many students tried to work with the number inside the variable and for part e) they worked on the wrong side of the equation. I believed my students did not truly understand what it meant to solve a linear equation. They looked at solving linear equation as a set of procedures to be learned and applied the undoing operations method incorrectly.

For question 5, many students were able to write the correct algebraic representation and were able to solve part b) with algebra. Question 6 was my students' first encounter with this type of tasks and I was happy to see that most students were able to write a scenario for each of the algebraic representations and some included what the variables stood for.

<u>Reflections</u>. While students could apply the procedures for division of fractions, they struggled with simple division like in part a) for the first task. I saw improvements in

the addition and subtraction of fractions. Students struggled with the solving of linear equations and during the week leading up to this assessment, I had taught lots of procedural steps without any conceptual development. During the year of this study, I began with the cover-up approach to the solving of linear equations but I ended up emphasizing the undoing operations approach. I chose this approach for the following three reasons: (1) I was taught in this way, (2) the textbook along with the computer program approached this topic in similar manner, and (3) my students were familiar with this approached from their middle school experiences. If the students had done well with this assessment, would I even think about another approach for this week? I feel as though I took a wrong turn with the start of this chapter. I needed to go back and think about what it meant to solve an equation. The calculator programs along with more linear equations containing fractions should help my students' continued struggles with fractions and decimals. With their poor results on the solving of linear equation, I would redo the lesson and approach it with the cover-up method.

Summary. The teaching cycle allowed me to divide the algebra curriculum into weekly segments. This teaching cycle also allowed me the opportunity to rest on Friday as the students worked on the assessment. By doing this, I was able to focus all of my energy on planning teaching for Monday through Thursday. When I began teaching in 1993, I believed that I could plan lessons after school but I found that the pressure of daily planning forced me to write a lesson that had no coherence from one lesson to another. I was unable to picture the whole algebra curriculum. Teaching in the inner-city high school drained my physical and mental energy and I lacked the creative energy needed to plan daily.

Taking time on the weekend to plan allowed me readjust where my class was in the algebra curriculum and allowed me to see where we were heading. I used my energy to make minor adjustment in the lesson plan when I taught. The weekly assessments allowed me to review prior tasks and introduce new tasks. I did this because it allowed students more opportunities to address their prior misunderstanding.

Even after a decade of teaching, I continued to write and modify my lesson plans, because each year contained a different group of students with different set of skills.

Discussion

In the first half of this chapter, I looked at how my teaching evolved as I was handled the obstacles that impeded my ability to teach in the classroom. Drawing from the discussion above, I provide a pictorial representation for my teaching in Figure 5-12. The four major time points are: (1) minimizing the negative aspect of the school environment, (2) setting up a classroom environment, (3) building interaction with students and content, and (4) rethinking the content.

Figure 5- 12. Phases of my teaching career.

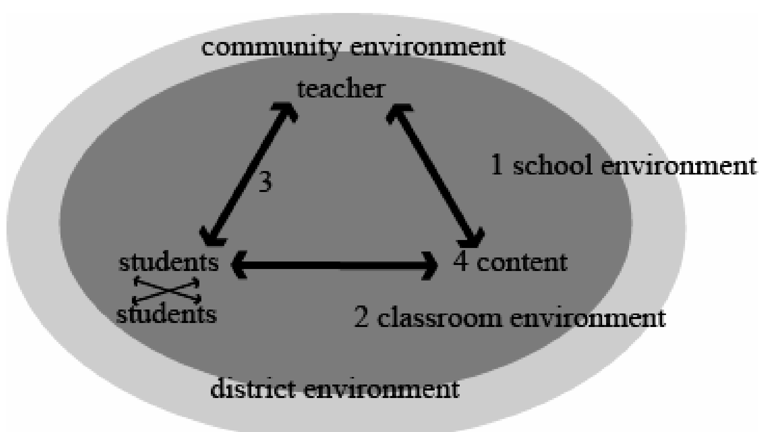


Figure 5-12 shows the process of changing focus needed to teach in the inner-city classroom. As a novice teacher, it would not have been possible for me to teach algebra without understanding the school and classroom environment. Without establishing a good rapport with the students, it would also be difficult to write assessments and lesson plans. These four time points can serve as a guide for novice and experienced teachers who wish to understand their own teaching. What I have learned from my own experience is that in order to thrive in an inner-city environment, I had to continually try different things, such as a new management policy or a new task. This, I believe, kept my teaching from being stagnant.

In the second part of this chapter, I looked at the teaching cycle. I could not have arrived at this point if I didn't have an established classroom environment. I moved my lesson writing to the weekend so that I could use the results from the assessments to write the lesson plans. Depending on the results, I would reteach the standard, find a different approach to reteach the standard, or teach a new standard for the following week of lesson.

I created an introductory task that connected prior standards to the new standard. This allowed students to review past standards and to see how the new standard connected with old standards. This introductory task provided structure in the classroom, because it allowed students time to settle down, begin their work, and provided a transition to the new standard. After each lesson enactment for a given day, I may adjust the numbers in the tasks, change the ordering of the tasks, or write more tasks. Thus, I was able to tailor my lesson plans for individual classes for a given day, but also from day to day. Every Thursday night, I reflected on what we did in the past week and used the last assessment as a guide to write the assessment for Friday, which was another opportunity to tailor the mathematics for my students.

CHAPTER 6

IMPLICATIONS

This dissertation is not the first to look at teaching in a difficult working environment and it won't be the last. I want this dissertation to be more than a case study for teaching algebra in the inner-city classroom. I want this dissertation to build upon those other researchers who have also dedicated their time and energy into teaching in difficult situations for different topics. For this last chapter, I draw upon research to provide some commonalities or themes that run across this research and my dissertation. By doing this, I provide a template, or blueprint, for those wishing to work in a complex working environment.

In Chapter 1, Lee (2001; 2007) showed how English teachers tailored the content to help struggling readers with literacy. The content allowed students opportunities to demonstrate, question, and voice their understanding of the literature. With these students' input, teachers were able to respond accordingly. I took Lee's Cultural Modeling Project framework and incorporated it with Cohen, Raudenbush, and Ball's (2003) instructional triangle. The enhanced framework in Figure 6-1 shows how the content, students, and teacher interacted with each other[1].

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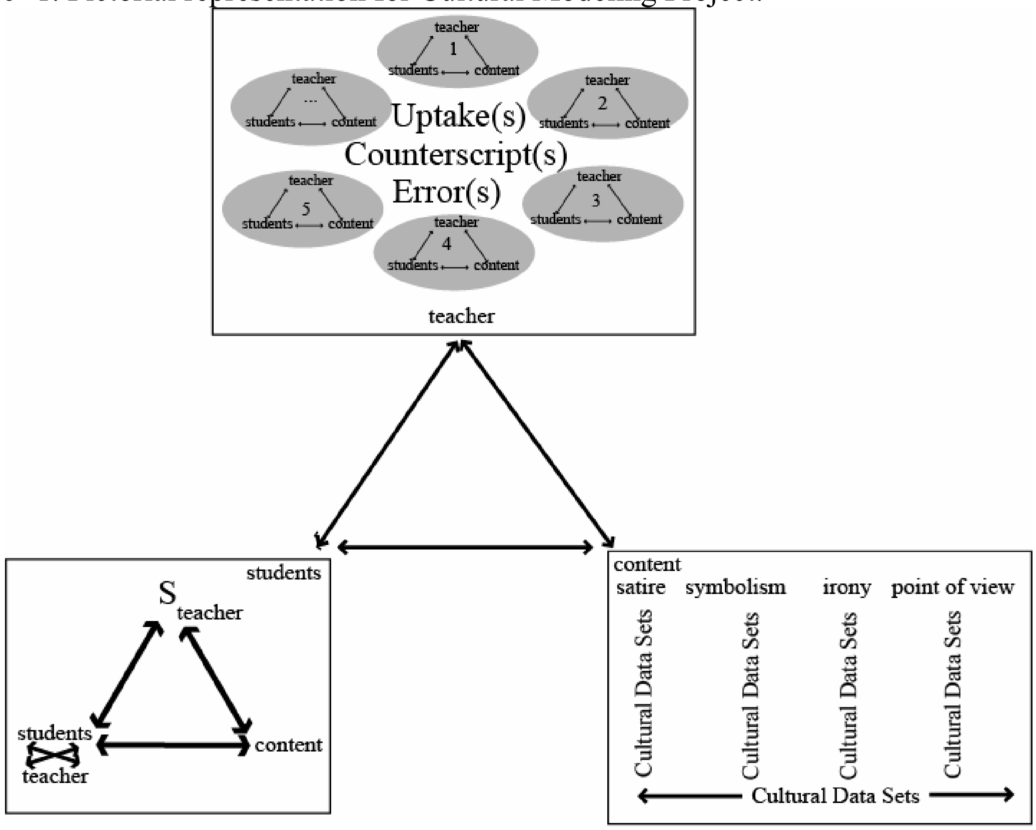
[1] This figure is the same as Figure 1-2.

Figure 6- 1. Pictorial representation for Cultural Modeling Project.



Using this enhanced framework, I adapted it to my work teaching algebra. Figure 6-2 shows the enhanced instructional triangle for TGT algebra. In Chapter 3, I showed I was able to teach a new standard by foregrounding the new standard and backgrounding the prior standards. By doing this I was able to weave the standards throughout the first semester of algebra. I was able to teach new standards while reviewing prior standards. The pictorial representation of the content is in the bottom right corner of Figure 6-2. The arrows within the content box represent the interconnections between the four major standards for algebra.

In Chapter 4, I provided evidence that students questioned their “known” mathematics, chose appropriate problem solving strategies, and contributed to the building and teaching of TGT tasks in the classroom. By taking on these roles, students assume the teacher role. A pictorial representation of a student taking on the teacher role is in bottom left hand corner of Figure 6-2. Evidence in Chapter 4 also showed that I communicated with the students when in the process of solving mathematics. By not dominating the conversation, I listened to my student input and responded accordingly to keep the lesson moving forward.

Figure 6- 2. Enhanced framework for TGT algebra.

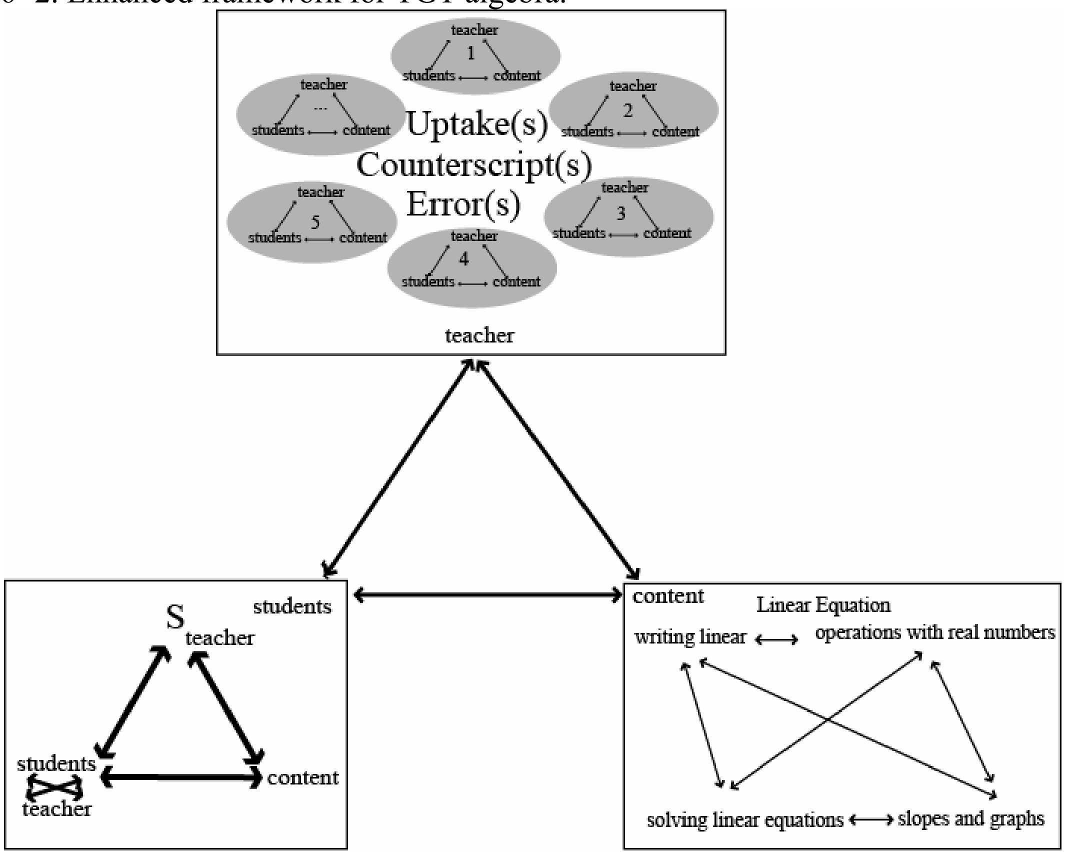


Figure 6-1 and Figure 6-2 show commonalities that exist in Lee's work on literacy and my work with algebra. I draw upon these commonalities to present three themes that a teacher needs to tackle in order to work in these difficult teaching environments: (1) reconceptualizing the content, (2) using students as resources, and (3) managing the chaos of a classroom. Although I will look at the three themes individually, they are dependent upon each other. In other words, I could not reconceptualize the content without my students' input and I could not have managed the chaos without developing a content specific to my students. I also revisit other researchers presented in Chapter 1.

In the last section of this chapter, I look at the implications for implementation of a numerical pattern approach for my district. I had given my lesson plans to other teachers, but never felt that those students received the full potential of the lesson. In Chapter 5, I showed a lesson and its enactment and found that the enactment of teaching was different from the lesson. Thus, in this chapter, I provide some ideas on how to implement TGT using a numerical pattern approach in an algebra classroom.

Reconceptualizing the Content

In order to begin changing instruction, a person must realize that the current state is not acceptable. A person must recognize that a problem exists. This may not be easy. For a teacher in an inner-city classroom it is always easier to blame the low test scores on family situations, school environment, or lack of resources, just to name a few. This could be a reason why there is a lack of wholesale change in inner-city schools. Recognizing that the problem may lie with the content requires the teacher to question his or her prior experiences in learning and teaching the content. The obstacle in reconceptualizing the content as the answer may not be obvious and may lie outside of the teacher's experiences.

Reconceptualizing the content begins with recognizing the problem. The next step is to search for a solution to the problem and could entail the following: reading literature, attending professional meetings and conferences, talking to peers, and even talking to the students. The last aspect for reconceptualizing the content is the restructuring of the content to fit the needs of the students. Each of these ideas will be further developed.

Recognizing the Problem

Each of the researchers discussed in the first chapter recognized that a disconnect existed between the students and content. Moses (2001) recognized the problem by the high number of students taking remedial algebra in college; Lee (2001; 2007) saw it on the below average reading levels for her students; Chazan (2000) saw it on the bored faces of his students; Lensmire (1994) saw it on the written pages of students' work; and Gutstein (2003) realized that the textbook did not tackle social justice. In the instructional triangle, this disconnect, or problem, could be due to the teacher, students, or content or a combination of these. I came to discover the problem through a discussion with a student about algebra.

Lee, Chazan, and I were experienced teachers teaching in difficult teaching environment and could have easily blamed the problem on the students or school environment. Lee recognized that her students exhibited the interpretative skills necessary for reading through their interactions with their peers. Thus, Lee focused her attention to the content and chose content that highlighted these interpretative skills. I had few management issues in the classroom and through a process of elimination, I recognized that the problem was with the content more so than with the students. Lensmire and Moses did not write about their difficulties with classroom management, but they did focus their writing on the content. All of us realized that a problem existed with the content and each of us took a different approach to the problem.

Finding an Approach

Lee (2001; 2007) and the English teachers selected texts that connected to the students while Chazan (2000) selected mathematical tasks appropriate to his students. Gutstein (2003) created mathematical tasks to supplement the district mandated curriculum. Lensmire (1994) allowed the students to choose the writing topic. My path for finding an approach mirrored closely to Chazan (2000), Moses (2001), and Lee (2001; 2007).

Chazan and Moses investigated the different algebra approaches and determined the algebra content before choosing an approach. I had a variety of different approaches to teaching algebra and chose the approach I felt most appropriate for my students. Similar to Moses and Chazan, I investigated and tried different approaches to the teaching of algebra before choosing the numerical pattern approach. In choosing this approach, I augmented the work of English & Warren (1999) by showing how numerical pattern, an elementary approach to algebra, could be used to introduce algebra standards but to also teach other algebra standards. I chose this approach to connect arithmetic with algebra. Thus, I provided opportunities for my students to master arithmetic while learning algebra. Since this was an elementary approach to the teaching of algebra, I had to create mathematical tasks that drove the curriculum and used the textbooks and *Cognitive Tutor* to supplement my approach. In other words, whereas, Gutstein (2003) used his projects to supplement the district mandated curriculum, I used the district mandated resources to supplement my approach.

Restructuring the Content

Moses (2001) and Lensmire (1994) used the students to determine the content. Gutstein (2003), Lee (2001; 2007), Chazan (2000), and I restructured the content by selecting and choosing texts or mathematical tasks. I took a further step to show how I restructured the content by changing the order of how the standards were taught. By introducing the standard for writing linear representation first, I was able to connect this standard with standards such as evaluating variable expressions, operating with real numbers, writing and solving linear equations, and determining the slope. This foregrounding and backgrounding of the standards allowed for the interweaving of the standards and created an algebra, which I feel, connected to my students.

In order to make these connections between standards, I had to determine which standards to add and remove. I moved the standard for writing the iterative form of the function from the second semester to the first semester. I did this so that students could see the inverse relationships between multiplication and division. I moved the absolute value function from the first semester to the second semester. I believed students would have deeper understanding of the absolute value function after encountering the linear functions. This restructuring took into account the pacing of the district mandated curriculum. All of these decisions were made to create a tighter connection between the standards. Fortunately, my restructuring of the content did not raise any objections from the curriculum leader, which was not the case for Moses (2001).

This restructuring of the content is an ongoing process. I am continually looking for new connections between the standards. Lee (2001; 2007) added and removed texts yearly to teach a particular interpretative problem. Lensmire on the other hand needed to restructure his approach in order to change the content.

Using Students as Resources

The discussion about resources has often looked at the lack of resources in a school district, but the discussion has now shifted to how to use the resources effectively (Cohen et al., 2003):

> Researchers report that schools and teachers with the same resources do different things, with different results for learning. The differences depend on the use of resources; access creates opportunities for resource use, but resources are only used by those who work in instruction. (p. 119)

In these discussions, educational resources are often associated as concrete items that can be purchased, such as computers, books, teacher training, graphing calculators, computer labs, libraries, and such (Cohen et al., 2003), but my focus is on how researchers use students as resources by determining the problem, fostering the classroom environment, and tailoring the content.

Determining the Problem

The researchers mentioned above determined that a problem existed between the students and content through the students. The students' actions or lack of action allowed the teacher to gauge students' interactions with content. Thus, the teacher had to act accordingly. Lee (2001; 2007) removed texts that did not connect to the students. Lensmire (1994) modified the protocols for writer's workshop after reading students' works. Moses (2001) recognized the importance of peers in shaping attitudes about algebra. Gutstein (2003) recognized the lack of mathematics in his projects from his students. Chazan (2000) realized that the functional approach to the teaching of algebra needed to be modified from his students' works.

I developed a classroom in which students felt comfortable enough to question the mathematics. Through interactions with the students, I was able to determine the mathematical obstacles they faced, such as *y*-intercept and writing of the iterative form of the function needed to be modified. In the classroom and on assessments, student input allowed me to determine the errors which affected how I planned my lessons.

The development of the numerical pattern approach could not have progressed without the aid of students. Student input allowed me to question my own understanding of algebra and how I taught algebra. I learned that students needed to be convinced of the effectiveness of algebra and my job was to create tasks that moved students from arithmetic to algebra. When I began this section, it was a student that pushed me to rethink my own understanding of algebra. Without her input, where would I be now as an algebra teacher? What would this dissertation look like?

Fostering the Classroom Environment

In order to build a classroom environment, Lensmire (1994) and I had to establish classroom environments that allowed students opportunities to participate. Lensmire instilled new rules to the writer's workshop and I had to set up structures to allow students and me the opportunity to build a classroom environment. Once these basic structures were in place, my students allowed members of the classroom to participate. Thus, students understood that participation in the classroom was expected.

Moses (2001) incorporated students' participation as part of the process in learning algebra. Moses realized that he could use students to help change students' negative attitudes toward mathematics as evidence by students seeking help from the teacher so they could keep up with their peers.

Lee (2001; 2007), Lensmire(1994), Gutstein(2003), Moses and I used the content to build a classroom environment. Our students felt comfortable to question, share, and teach other members of the classroom which included the teacher. In my classroom, students suggested different algorithms to a particular task. Most of these algorithms did not have a strong mathematical foundation and were often procedural in nature. Still, I decided to use some of them and abandoned others in order to show that I valued their ideas and prior mathematical experiences. I wanted to build a classroom where all members had a voice in the classroom discussions.

During assessments, I allowed students to choose their approach to particular tasks. I believed this allowed them to explore their understanding of arithmetic and algebra. Gutstein (2003) looked beyond the classroom and developed students who used mathematics to examine their communities and place in society. I wanted students to be flexible problem solvers.

Tailoring the Content

All of the researchers used students input to tailor their content approaches. Moses (2001) and Lensmire (1994) allowed students to choose the content. Lensmire had to modify this in order to prevent chaos in the classroom. Moses provided an example where teachers used the bus system instead of the trains to teach operations with integers. Lee (2001; 2007) and I used our teaching experience to move, foster, or stop the classroom conversation. This allowed us to target a specific aspect of the content in the classroom. I did this by changing the ordering of the tasks or the numbers in the tasks. I also reflected after each lesson and tried to improve it for the next class and did this for the whole semester.

Management of Chaos

The management of chaos can take on many forms. It could be in the classroom, in the district, from the students, or in the content. In an inner-city classroom, managing the chaos can refer to classroom management and to the "buffering" of the school environment. Chaos could refer to their interactions with their peers or with the teacher. Their actions can help or hinder the instructional practice. In this section, I will focus on the chaos in the content and in the classroom.

Approach to the Content

In the case of literature, Lee (2001; 2007) chose texts not often found in a traditional literature class. Before the start of the semester, members of the Cultural Modeling Project talked about the texts and how students might respond to them. This allowed each member to foresee any interpretive obstacles in the classroom before the enactment. Cultural Modeling Project members brought in rap music, lyrics, and non-canonical texts to focus students to a particular interpretative problem. The selection of the content helped build and foster the classroom environment. Lensmire (1994) provided the approach but it was the students who chose the content. This created chaos in the content and chaos in the classroom environment.

In the case of high school algebra, the chaos occurred because there was no consensus on what to teach and how to teach algebra (RAND Mathematics Study Panel, 2003). Moses (2001) chose modeling reality as his approach. Chazan (2000) chose functions, and I chose a numerical pattern approach. Moses also encountered teachers who did not believe in his modeling reality approach to the teaching of algebra and teachers who were uncomfortable with algebra. Similarly, I helped the other two teachers with the numerical pattern approach to algebra and one teacher expressed a better understanding of algebra. Doing this, I was able to gain further insight into how to teach algebra and professional development.

Chazan tried to define the algebra content. I on the other hand decided to make the study of linear equations as the focus for the first semester of algebra. Once I made this decision, it made it easier for me to determine the standards to target and I believed made algebra more accessible to the students. I wasn't teaching a new standard from week to week. Gutstein (2003) avoided the issue of content by supplementing the content with projects. This allowed him to create projects that focused on other objectives, which were not always mathematical. Gutstein worried that the time spent with his own project would take instructional time from the district mandated curriculum. I also worried that I would not be able to target the entire district mandated curriculum.

Moses (2001) encountered a different type of chaos. Cambridge School District used "ability grouping" in their mathematics classes, which was counter to the beliefs in the Algebra Project. Parents and Moses had to convince the district to stop using "ability grouping."

Different from the other researchers, I did worry that the district might not approve of my approach to the teaching of algebra. The district curriculum guide (Norde et al., 2006) contained the individual lessons along with pacing. Thus, I believed the district wanted all of their students to follow the district mandated curriculum. I, on the other hand, felt that the district mandated curriculum did not connect to my students; thereby, I chose to reduce the chaos in the classroom and worry less about the district.

In the Classroom

It took me three years to learn how to create a classroom environment that allowed me to teach and students to participate, reflect, and question. I set up structures in the classroom to minimize the negative school and classroom environment. Lee's (2001; 2007) ability to listen and connect to the students allowed her to manage the chaos and teach in the classroom. In the midst of teaching, Lee was able to handle the students' off-task behaviors. For me, I was able to set structures by implementing a graded introductory task and a weekly assessment. These simple structures buffered the negative effects of the school. Since I wrote the lesson plans, I had freedom to modify or throw out a particular task. This allowed me to gauge the classroom environment. There were occasions when I increased or decreased the cognitive demands in the class through tasks (refer to Chapter 5). Thus, I had to constantly gauge the classroom environment to minimize the chaos. I believed students were interested in the tasks I created in my classroom.

The chaos in Lensmire's (1994) class remained hidden in the classroom and appeared in the writing and presentation of students' work. Lensmire developed a close rapport with all students and was able to diffuse many of the situations in the classroom by talking with the feuding parties. These issues between students also occurred in my classroom and similar to Lensmire, Chazan (2000), Lee, and me and we were able to diffuse most of these issues.

Moses (2001) could not get his daughter to take algebra in the 8th grade. He had to convince her peers first. My experience was slightly different. My students wanted to learn algebra and I felt that they wanted a challenge. Thus, they were often quiet whenever we go over a new task. During the enactments, there were times when I could not get the students to determine the correct response and I ended up telling them the answer. I did this for the first encounter with the writing of a linear representation and the iterative form of the function. I rationalized that since these conceptual leaps also occurred in the development of algebra. Gutstein (2003) also resorted to telling his students the mathematics in order to keep the lesson moving in the classroom.

Discussion

Each of researchers tackled the problem by examining the content and deciding how to reconceptualize the content to fit with his or her students. Moses created the Algebra Project process that allowed students to draw, explain, and present their mathematical representation of reality. In Lee's work, Cultural Modeling Project members used students' prior knowledge of African American Vernacular English to connect the text to the students and members were able to teach the interpretative problems in reading. Students played an integral role in leading and questioning the discussion. Lensmire used his rapport with students to discover socio-economic and gender classification and reconceptualized Writing Workshop. Gutstein presented students' work, writing, and anecdotal evidences to show his students' newfound understanding and appreciation of mathematics. Chazan recognized that he needed to tailor the algebra curriculum to his students.

I chose a conceptualization of algebra from the elementary school, because I recognized my students' weakness with arithmetic. Thus, I decided to use arithmetic as the bridge to teaching algebra. Doing this allowed my students opportunities to review arithmetic concepts but also to learn algebra. I worried that students and other teachers might view this approach as not rigorous enough. All of us chose conceptualizations that went against the common approach for that subject matter. We used our prior experience with students to determine the appropriate approach to the content.

The management of chaos can take on many forms. Moses had to deal with the district and teachers. Gutstein had to determine how to fit his project with the district mandated curriculum. Lee managed the chaos in the classroom by developing a rapport with her students that allowed her to manage the multiparty overlapping talk. Lensmire had to manage students' socio-economic and gender differences. Chazan had to manage the content. Creating lesson plans allowed me to tailor the classroom environment for my students. This allowed me to control the chaos in the classroom and buffer the school environment. Chazan, Moses, and I struggled trying to determine to the best algebra approach. In the end, Chazan chose functions. Moses chose modeling reality and I chose numerical patterns. Instead of allowing these obstacles to deter us, all of us found ways to get around and overcome these obstacles.

These three themes are present in all these research and cut across a variety of different subjects: literature, writing, middle-school mathematics, and high school algebra. All of this research can be looked at as a case study specific to a particular topic, but when all of this research, including mine is analyzed, it revealed that there are commonalities that transcend subject matter. This implies that for those wishing to embark on research projects in teaching, irrespective of the subject matter, need to reconceptualize the content, use students as resources, and manage the chaos.

Into the Classroom

My dissertation showed that it was possible to teach inner-city 9th graders algebra that had high cognitive demands, connections between standards, and depth for certain standards in a classroom that allowed students to question their “known” mathematics, choose appropriate problem solving strategies, and contribute to the building and to the teaching of algebra in the classroom. I have included the tasks used for the 23 classroom enactments in the appendix, but I do not believe that merely teaching these tasks would achieve the same results for others. I allowed teachers at JHS to use my lesson plans, but never felt the teachers understood the nuances of each lesson. Using the tasks in order does not take into account the differences in the classroom environment and the amount of changes made by me in the classroom. These lesson plans were written for a particular group of students tailored to the errors made in the classroom and on an assessment.

I have shown in Chapter 5 how the written lesson was very different to the enacted lesson. The tasks in the lesson plan built a structure and flow in the classroom and served as a rough guide to what I want to achieve for that day. The lesson plans connected prior lessons and future lessons. Thus looking at week’s worth of lesson would not give a complete picture.

Martin (2000) studied the implementation of Moses’ Algebra Project in a school district. He found that students didn’t connect to the curriculum and some students questioned the “mathematics” in the Algebra Project. I would further add that I believe the teachers in this high school never believed in the curriculum and the Algebra Project never had a chance. In the next section, I consider different components that I believe are necessary to implement this curriculum into the classroom.

Teacher Development

The first component requires that teachers understand the rationale behind the numerical pattern approach. This means that teachers need to reconceptualize their understanding of algebra and a component for a lack of students’ success is a disconnect between the content and the students. Here is the first Michigan Department of Education's standard that all 9^{th} graders encounter in algebra class.

> A1.1.1: Give a verbal description of an expression that is presented in symbolic form, write an algebraic expression from a verbal description, and evaluate expressions given values of the variables.

I want to focus on the highlighted text above, because this is how most textbooks approach the teaching of algebra. When I read the highlighted text, I no longer read it as an objective, but turn this statement into the following question. Why would you evaluate a variable expression with a given value? For students, the answer to this question is their rationale for doing algebra. The rationale has to be more than "because the teacher asked me to do it." The turning of each Michigan standards into a question leads me to the creation of numerical pattern tasks. Thus, understanding a numerical task requires analyzing each part of the task. I first presented this task in Chapter 3:

> Given the following pattern 3, 6, 9, 12, 15…
> a) Find the 8th, 10th, and 100th term
> b) What term is equal to 51?
> c) What is the rule for this pattern?

For part a), the question is: why would you evaluate an equation? For part b), the question is: why would write and solve an equation? And for part c), the question is: why would you write a linear representation? The answer is the writing of a linear representation in part c) makes it easier to solve part a) and part b). Once a teacher understands the rationale for the construction of this task, it is now possible for the teacher to create, tailor, and modify the task for their students within the classroom.

This allows for a close tailoring of the task to fit the needs of the students. Thus, following my tasks will not be successful unless the teacher understood the objectives of the tasks and is able to adapt these tasks accordingly in the classroom. I have tried to lay out my rationale in Chapter 3 of how I moved from chapter to chapter for the first semester. Teachers would need to understand how each of those tasks was created so that teachers can modify and create their own tasks. I believe that my understanding of algebra has grown by my use of the numerical pattern approach.

Curriculum Guide

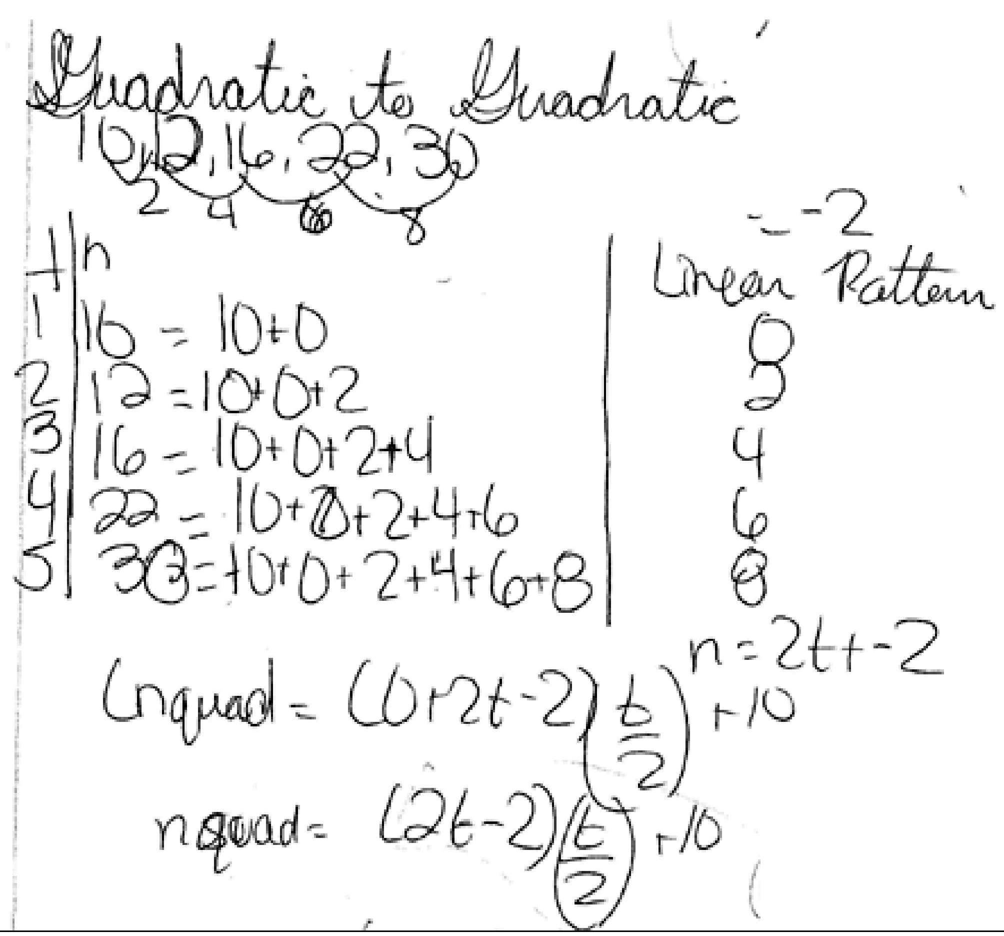
For TGT, I made two curricular decisions: (1) the major topics of algebra; and (2) the ordering of topics. I had to identify the major topics for algebra because it was not possible to teach all of the topics mandated by the district. The major topics of algebra were the writing of linear equation, operations with real numbers, solving linear equations, and slope. I chose these major topics because I knew that students needed to have a strong foundation in order to take the more advanced mathematics classes. My second decision was to change the ordering of the topics. I moved the writing of linear equations, which was normally the last chapter for the first semester, to the first chapter. I did this so I could take full advantage of a student's ability to write linear equations with topics such as the operation with real numbers and solving of linear equations.

I decided to move absolute values and exponents to second semester. I did this because I wanted students to understand linear equations fully before studying the absolute value functions and exponents. I also moved the summation of functions and the iterative notations to the first semester, because introducing these topics connected well to the other algebra topics. There was another topic, functional notation, $f(x)$, which I was not able to teach in the first semester. I couldn't determine the appropriate spot in the algebra curriculum. On the other hand, the introduction of the iterative notation was appropriate because it connected multiplication and division together. I didn't like introducing extraneous mathematics topics to my students.

Even though this dissertation covered the first semester of algebra, I continued to use numerical pattern tasks for quadratics. Larson and *Cognitive Tutor* approach quadratics with formulas, which was for me, too procedural. Using numerical patterns, I believed, pushed my understanding of quadratics. I had done a lot of thinking on how to connect the algebra experience of the first semester to the second semester. I found a way of connecting linear and quadratic functions. Prior to this, I approached quadratics by the multiplication of two linear patterns, but I wasn't happy with the transition. Inspired by Professors Bass and Kollar, I decided to work with numerical patterns that were quadratics. Figure 6-3 shows the work of a group of my students finding the quadratic equations for a numerical pattern they created.

Figure 6- 3. Students' work with quadratics.

Task
Determine the equation for 10, 12, 16, 22, 30, …



This figure shows that this group could apply our procedures for the writing of linear equations to write quadratic equations. This group did not simplify their quadratic equation. In this simplified form, $n = (t)(t-1)+10$ we could find the vertex, the axis of symmetry, and move the discussion to transformations. With this approach, the need to manipulate symbolically becomes more important.

I have shown that it was possible to use a numerical pattern approach for the first semester of algebra and provided a hint that it could be used for the second semester of algebra. In order to teach using a numerical pattern approach, the teacher must make curricular decisions, by deciding how to order and prioritize the algebra topics.

The National Mathematics Advisory Panel (NMAP) (2008) took a step by defining the major topics for algebra. Figure 6-4 is a list of the major topics recommended by the NMAP.

Figure 6- 4. List of major topics for algebra (National Mathematics Advisory Panel, 2008 p. 16).

Algebra topics
Symbols and Expressions • Polynomial expressions • Rational expressions • Arithmetic and finite geometric series **Linear Equations** • Real numbers as points on the number line • Linear equations and their graphs • Solving problems with linear equations • Linear inequalities and their graphs • Graphing and solving systems of simultaneous linear equations **Quadratic Equations** • Factors and factoring of quadratic polynomials with integer coefficients • Completing the square in quadratic expressions • Quadratic formula and factoring of general quadratic polynomials

- Using the quadratic formula to solve equations

Functions

- Linear functions
- Quadratic functions—word problems involving quadratic functions
- Graphs of quadratic functions and completing the square
- Polynomial functions (including graphs of basic functions)
- Simple nonlinear functions (e.g., square and cube root functions; absolute value; rational functions; step functions)
- Rational exponents, radical expressions, and exponential functions
- Logarithmic functions
- Trigonometric functions
- Fitting simple mathematical models to data

Algebra of Polynomials

- Roots and factorization of polynomials
- Complex numbers and operations
- Fundamental theorem of algebra
- Binomial coefficients (and Pascal's Triangle)
- Mathematical induction and the binomial theorem

Combinatorics and Finite Probability

- Combinations and permutations, as applications of the binomial theorem and Pascal's Triangle

The list presents a two-year course in algebra, which is the basic requirement for high school algebra. Figure 6-5 is the list of algebra topics that could be used for the first of year of algebra. I selected these topics in Figure 6-5 by taking into account my students' prior mathematical experiences and the time frame at my high school. My list represents what I would like to accomplish for a year of algebra.

Figure 6- 5. List of major algebra topics for Algebra 1 class.

Algebra topics for Algebra 1
Symbols and Expressions • Polynomial expressions

- Arithmetic and finite geometric series

Linear Equations

- Real numbers as points on the number line
- Linear equations and their graphs
- Solving problems with linear equations
- Linear inequalities and their graphs
- Graphing and solving systems of simultaneous linear equations

Quadratic Equations

- Factors and factoring of quadratic polynomials with integer coefficients
- Completing the square in quadratic expressions
- Quadratic formula and factoring of general quadratic polynomials
- Using the quadratic formula to solve equations

Functions

- Linear functions
- Quadratic functions—word problems involving quadratic functions
- Graphs of quadratic functions and completing the square
- Simple nonlinear functions (e.g., square and cube root functions; absolute value; rational functions; step functions)

Algebra of Polynomials

- Fundamental theorem of algebra

A close look at these topics divides the first year algebra into a study of linear function for the first half of the semester and the study of quadratics for the second half of the year. For me, I would not include the square root, cube root, and rational functions, but included the absolute value and step functions.

Use of Resources

Using the numerical pattern approach required that I looked at the resources available. I had a classroom set of Larson and I also had access to *Cognitive Tutor: Algebra 1*. As mentioned before, it was never the lack of resources at JHS that prevented me from teaching algebra. Perhaps, it was too many resources (e.g., two algebra curricula, graphing calculators, communication hardware for graphing calculator, technology for polling students) fighting for instructional time. I had to determine which of these resources to use in the classroom.

I used the district mandated pacing chart as a guide of what I needed to cover in algebra class. I took Larson's objectives and wrote them into my lesson plans and assigned homework. The algebra objectives were the same but I approached these objectives with numerical pattern tasks. Thus, I was able to say to school administration that I taught the district mandated algebra. As mentioned before, the high number of lower-level demand tasks in Larson combined with few connections between the chapters made it the perfect resource for procedural fluency. These tasks in Larson allowed me to assign students work in class but also for homework. I made these curricular decisions because I now understood Larson's approach to algebra. I knew how Larson introduced a topic and the type of tasks found in Larson. Thus, I introduced higher level demand tasks and used Larson's task to build procedural fluency.

With the *Cognitive Tutor: Algebra 1,* I was never confident that the computer lab would be operable and the accompanying textbook repeated the same information in the computer lab. *Cognitive Tutor: Algebra 1* had great contextual tasks for linear equation, but not such much for quadratic functions. The approach of quadratic was through the formulas: quadratic formula, vertex formula, and axis of symmetry formula. With each visit to the computer lab, students worked on different topics in the computer program. Students enjoyed working at their own pace, but as the teacher, determining what algebra topic to teach for the week and how to write assessments proved to be daunting. I had no idea what my role as the teacher was with *Cognitive Tutor.* I still believed that I could do a better job teaching algebra than *Cognitive Tutor,* because I was able to tailor the tasks for my students. When I was in the computer lab, I felt more like a computer technician than a teacher. Since *Cognitive Tutor* took a secondary role in my classroom, I was able to use *Cognitive Tutor* without any conflicts because students could apply the procedures and terminologies used with numerical patterns tasks for contextual tasks. This allowed students opportunities to write algebraic representations for contextual situations.

I also had a set of graphing calculators and hardware to communicate between graphing calculators. I used the graphing calculators with a software program to help students build their arithmetic skills. I also used them when we did work on graphing. Other than these two occasions, I used the graphing calculator sparingly, but I allowed students to use their own calculators. I wanted my students to be flexible with arithmetic and using the calculator everyday would undermine the objective. Plus, the distributing and collecting of graphing calculator took a lot of instructional time. In summary, the teacher must understand the strengths and weaknesses of each resource and chose the best resource for the particular lesson.

Administration

I worked in a difficult environment where the school administration sent mixed messages to the students and teachers through their haphazard enforcements of school policies. In Chapter 5, I showed how as a teacher I was able to buffer my classroom from the negative effects of the school environment. I also showed in Chapter 4 that these buffers were temporary and were breached during the “bad class” episode. The physical and mental toll required to teach in this environment were tremendous. I believed that the school administration tried its best to educate the students that walked through its door and I could not have conducted this research without their support. But the school administration with the help of the district needs to determine what policies can be enforced in these high schools and provide the proper support. The district also needs to determine what innovative teachers are doing in the classroom and use them as resources for professional development and the building of a stronger teaching staff.

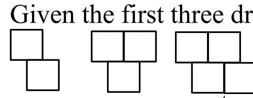
Discussion

It has taken me four years to develop a numerical approach to the teaching of algebra for the first semester of algebra. Each year, I was able to learn something new about this approach along with my students. Through this journey, I believed that I became a better teacher. In Chapter 3, I took a stance in how to teach and what to teach for the first semester of algebra. I targeted a smaller set of MDE standards than the district mandated standards, but did it a way so that these standards had connections between standards and depth. In Chapter 4, I showed that I able to establish a classroom environment to allow students the opportunity to explore and question algebra they were learning. I also showed that it was possible to teach algebra to students with weak arithmetic skills. In Chapter 5, I showed how I was able to buffer the chaotic school environment to allow me to teach algebra to my students. In this section, I have shown that it was possible to teach algebra using numerical patterns with the Larson and *Cognitive Tutor.* Implementation requires teacher development to understand the numerical pattern approach, a retooling of the curriculum, and an analysis of the available resources and how best to use them.

APPENDIX

APPENDIX

TGT TASKS

Tasks	Tasks
September 18th Introduction to algebra Given the linear pattern 3, 6, 9, 12,15… Find the following: a) Find the 8th term b) Find the 10th term c) Find the 15th term d)What term is equal to 51? e) What is the rule? __________ Given the linear pattern 4, 8, 12, 16,20… Find the following: a) Find the 60th term b) Find the 132nd term c) What term is equal to 256? d)What term is equal to 28? e) What is the algebraic representation? __________ Given $n = 5t$, find the first five numbers. Given $n = 5t + 2$, find the first five numbers. __________ Paul makes \$6.25 per hour. a) How much money will he make working 3 hours? b) In 5 hours? c) In 5.5 hours? d) Find the algebraic. __________ (Larson et al., 2004) p. 6 Evaluate the expression when $y = 5$. 6. $\frac{24}{y}$ 7. $y+19$ 8. $y-2$ 9. $y \div 3$ 10. $27-y$ 11. $2.5y$ 12. $3.2+y$ __________ In Exercises 17 and 18, you want to hike a round-trip distance of 10 miles from the Hioutchi Information Center along the Little Bald Hills Trail and back. Calculate how long it will take if	September 25th Introduction to algebra Given the linear pattern 2, 11, 20, 29, 38… a) Find the algebraic representation b) Find the 12th term c) Find the 30th term d) What term is equal to 83? __________ Given the first three drawings, find the perimeter:  a) 1st, 2nd, and 3rd b) 5th and 50th c) What is algebraic? d) 100th, 1 millionth e) 1 trillion __________ Find the first five integer values for a) $n = t^2 + 1$ b) $n = 3t^2$ c) $n = 2t^2 + 1$ (Larson et al., 2004) page 19 4, 5, 6, 7, 8, 10 Evaluate the expression for the given value of the variable. 4. $x^4 - 3$ when $x = 2$ 5. $5 \bullet 6y$ when $y = 5$ 6. $a^3 + 10a$ when $a = 3$ 7. $\frac{16}{x} - 2$ when $x = 4$ 8. $\frac{22}{x} \div 2 + 16$ when $x = 11$ 10. $(x+5) \div 4$ when $x = 9$ __________ Journal: Describe how to find the value for $t = 5$ for $n = 2t^2 - 3$

you hike at a rate of 1.25 miles per hour. 17. Write a verbal model, provide labels, and write an algebraic model. 18. Show unit analysis.	
October 2nd Evaluating Variable expression Given $a = 2$, $b = 3$, and $c =$ -2. Find a) $\frac{ab}{c}$ b) $a^2 + 2b^3$ c) $abc + c$ d) Make two variable expressions equal to 6. ——— (Larson et al., 2004) page 21 50, 51, 52 In Exercises 50 and 51, use the table below. It shows the admission prices for the California State Fair in 1998. Suppose a family of 2 adults, 1 senior, and 3 children go to the State Fair. The children's ages are 13 years, 10 years, and 18 months. 50. Write an expression that represents the admission price for the family. 51. Evaluate the expression you wrote in Exercise 50. ——— 52. The area A of a trapezoid with parallel bases of lengths b_1 and b_2 and height h is $A = \frac{1}{2}h(b_1 + b_2)$. Find the area of the trapezoid whose height is 2 meters and whose bases are 6 meters and 10 meters. ——— Translating variable expression 1.eleven decreased by the quantity of four times a number 2. four increased by the quantity of four times a number 3. 4 times the quantity a number minus 11. 4. 4 times a number decrease by 11. 5.A number increase by 10 is 24 6.20 divided by a number is less than or equal to 2 ——— (Larson et al., 2004) page 35 13-21odd 13. Nine more than a number 15. Three more than half of a number 17. Quotient of a number and two tenths 19. Two cubed divided by a number 21. Five squared minus a number	October 3rd Translating variable expression Given the following table for the state fair, find the following: Tickets — Price Seniors ($age \ge 65$) — \$6.50 Adults — \$9.00 Teens (age < 18) — \$5.50 a) find the price for 2 seniors, 2 adults, and 3 teens b) Find the price 1 adult and 5 teens c) If you have \$50, how many teens go to the state fair? d) What is the algebraic representation for the cost for the number of teens? ——— (Larson et al., 2004) page 36 30, 32, and 34 30. A number t increased by the sum of seven and the square of another number s is 10. 32. Fourteen plus the product of twelve and a number y is less than or equal to fifty. 34. Seventy divided by the product of seven and a number p is equal to one. ——— Given that 3rd term is 10 and the jump is 5 find the first five terms and the algebraic representation for that. 3rd term = 10 and jump = -5 3rd term = -10 and jump = 5 ——— Journal: How would find the first five terms and algebraic if the 3rd term is -2 and the jump is 5.

October 9th
Review of integer operations and wringing algebraic representations

Given the 3rd term is 5 and the jump is 2, find the following:

a) Find the first five terms
b) Find the 100th term
c) Find algebraic representation for the linear pattern

Given 3rd term = 1/3 and jump = ¼

a) Find the first five terms and algebraic

Given 3rd term = 1/12 and jump is ¼

a) Find the first five terms and algebraic

Given 3rd term = ½ and jump is 1/3

a) Find the first five terms and algebraic

Given 3rd term = 5.5 and jump is 0.25

a) Find the first five terms and algebraic

Oral Quiz

a) 2-4
b) 19-12
c) 7-5
d) 7-27
e) 0-8
f) 0-(-6)
g) -16-0
h) -8-2
i) 3 – (-1)

(Brown et al., 1992) Page 61, 1 - 6, page 92 23-26

1. $25-213$ 2. $154-281$
3. $39-(-32)$ 4. $47-(-49)$
5. $-19-(-3)$ 6. $-25-(-9)$

(Brown et al., 1992) page 92 23-26

23. $\frac{2}{3}-\frac{1}{6}$ 24. $\frac{5}{7}-\frac{2}{3}$
25. $\frac{3}{4}-\frac{15}{21}$ 26. $\frac{3}{4}-\frac{5}{12}$

October 16th
Introduction to iterative form of the function and review of fractions

Given the following table, fill in the missing values and describe the pattern.

T	n
1	
2	
3	2
4	4
5	8
6	
7	

Given the following table, fill in the missing values and describe the pattern.

T	n
1	
2	
3	
4	3
5	9
6	27
7	

Given the following table, fill in the missing values and describe the pattern.

T	N
1	
2	
3	5
4	25
5	125
6	
7	

Find the first six terms if $n_3 = 1$ and $n_{t+1} = 3n_t$
Find the first six terms if $n_4 = 24$ and $n_{t+1} = 2n_t$
Find the first six terms if $n_3 = 10$ and $n_{t+1} = -2n_t$
Find the first six terms if $n_3 = 1$ and $n_{t+1} = \frac{1}{3}n_t$

Oral quiz
1. $4 \bullet (-9)$ 2. $6 \bullet (-13)$ 3. $(-8) \bullet (-5)$ 4. $(-2) \bullet (-7)$
5. $(-11) \bullet 8$ 6. $-24 \div 6$ 7. $6 \div 24$ 8. $6 \div (1/2)$
9. $(1/2) \div 6$ 10. $2\frac{1}{2} \div \frac{1}{4}$

Journal: Given $n_{t+1} = 2n_t$ and $n_3 = 12$. Explain how you would find the 1st six terms?

October 18th Distributive property and more work with fractions Find the 0th through 6th term for $n_3 = 36$ and $n_{t+1} = \frac{2}{3}n_t$. Find the 0th through 6th term for $n_3 = 72$ and $n_{t+1} = \frac{3}{2}n_t$. Oral quiz Use distributive property 1. $2(x+4)$ 4. $x(-x^2 - 2x)$ 5. $(x+2)(x+3)$ Extra $(x-3)(x-4)$	October 19th Review of fractions, decimals, and integers Find the 0th through 6th term for a) $n_3 = -36$ and $n_{t+1} = \frac{2}{3}n_t$. Find the first five terms and algebraic. b) 3rd term = 4 and jump is 3 c) 3rd term = 4 and jump is -3 d) 2nd term= 2.75 and jump is 1.2 e) 3rd term= $\frac{2}{3}$ and jump is $\frac{2}{3}$ f) 3rd term= $\frac{1}{6}$ and jump is $\frac{2}{3}$ $\frac{2}{3}$ Oral quiz 1. $2-(-3)$ 2. $3-5$ 3. $-4-6$ 4. $5-(-7)$ Computer Lab
October 23rd Solving linear equation Given $n_3 = 12$ and $n_{t+1} = n_t + 2$, Find the 0th through 5th term. Given the linear pattern 6, 8, 10, 12, 14… a) Find the 12th, 17th, and 10,000th term. b) What term is equal to 24? 58? And 123,456? Given 100, 98, 96, 94, 92… a) What term is equal to 50, -52, and -2? Computer Lab	October 24th Solving linear equation Find the 0th term through the 5th term. a) Given $n_{t+1} = 3n_t$ and $n_3 = 18$ b) 3rd term = 18 and jump is 3 Given $10\frac{1}{2}, 11, 11\frac{1}{2}, 12, 12\frac{1}{2}, \ldots$ a) What term is equal 15, 22, and 30? Solve $20 = \frac{2}{3}t - 10$ Solve $15 = \frac{3}{2}t - 15$ Journal: How do you solve $5 = \frac{3}{2}t - 15$?
October 25th Solving linear equation Given 7, 10, 13, 16, 19… a) What term is equal to 40? 55? And 142? (Brown et al., 1992) Solve $-6 + \frac{y}{5} = 5$	October 30th Solving linear equation Given $2\frac{1}{4}, 2\frac{2}{4}, 2\frac{3}{4}, 3, 3\frac{1}{4} \ldots$ a) Find the algebraic b) Find the 12th term, 100th term, 16th term. c) What term is equal to 8? Find the 0th through 5th term for $n_{t+1} = 3n_t$ and

Solve $\frac{7}{8}x+2=9$ Solve $\frac{x+5}{3}=7$ Solve $\frac{x-5}{4}=8$ Solve $\frac{3}{5}(x+2)=12$ Solve $21=\frac{3}{2}(x-2)$ Calculator exercises	$n_3=12$ Find the 0th through 5th term for $n_{t+1}=\frac{2}{3}n_t$ and $n_3=30$ Find the 0th through 5th term for 1st term = $\frac{2}{3}$ and jump $\frac{1}{4}$. Solve $2x=16$ Solve $2x=10$ Solve $2x+3=15$ Solve $2(x+6)=18$ Solve $\frac{2}{3}x+1=16$ Solve $\frac{2x+3}{5}=10$ (Larson et al., 2004) p. 135 and p. 142 Solve $2x+7=15$
October 31st Solving linear equation (30 minute class) Given the $n_4=72$ and $n_{t+1}=\frac{2}{3}n_t$. Find the 0th through 5th term. (Larson et al., 2004) p 148 Solve 24. $22=18-\frac{1}{4}x$ 23. $3-\frac{3}{4}x=-6$ Journal: How do you solve $\frac{4}{3}x+3=23$? 25 Solve $8x-3x=10$ 26. $-7x+4x=9$	November 13th Solving linear equation with variables on same sides (30 minute class) Solve a) $\frac{2}{5}x-6=20$ b) $\frac{2}{5}(x-6)=20$ Given the following linear sequences 5, 7, 9, 11, 13,… and 4, 8, 12, 16, 20,. a) What is the sum of the sequences for the 5th term? 8th term? And 100th term? b) When will the sum of the sequences equal to 15, 33, and 69? c) What is the algebraic representation for the sum of both sequences?
November 14th Solving equations with variables on the same side (2nd hour) Given the following linear sequences 5, 7, 9, 11, 13… and 1, 4, 7, 10, 13,…. a) What is the sum of the sequences for the 5th term? 8th term? 100th term? b) What is the algebraic representation for the sum	November 15th Solving with variables on same side Solve $\frac{x}{5}-5=-1$ Solve $6(2-x)=18$ Solve $x+5x-5=1$ Solve $2-x=6$

of the sequences? c) When will the sum of the sequences equal to 16? 66? And 106? (Larson et al., 2004) p. 148 20. Solve $30 = 16 + \frac{1}{5}x$ 23. Solve $3 - \frac{3}{4}x = -6$ 26. $-7x + 4x = 9$	(Larson et al., 2004) p 148 Solve 26. $-7x + 4x = 9$ 35. $-10 = 4 - \frac{7x}{4}$ 23. $3 - \frac{3}{4}x = -6$ 32. $\frac{-4}{9}(2x - 4) = 48$ ____________ Find three consecutive integers whose sum is 33. Find three consecutive odd integers whose sum is 105.
November 16th Review for weekly assessment Solving linear equation Solve $-7(x - 3) = 49$ Solve $6x - 8x + 7 = 13$ ____________ (Larson et al., 2004) p. 148 Solve 27. $x + 5x - 5 = 1$ 33. $17 = 2(3x + 1) - x$ 36. $-10 = \frac{1}{2}x + x$ $2(3x - 1) + 2 = 18$ $\frac{2}{3}x = 15$ ____________ Find three consecutive even integers whose sum is 306. Find three consecutive odd integers whose sum is 45. ____________ Given two linear patterns 1, 4, 7, 10, 13… and 2, 5, 8, 11, 14… a) Find the sum of the sequence for the 8th term b) Find the sum of the sequence for the 10000000th term. c) When will the sum be 75?	November 20th Solving equation with variables on opposite side Given two linear patterns -10, -8, -6, -4, -2… and 4, 7 , 10, 13, 16… a) Find the sum of the sequence for the 8th term b) Find the sum of the sequence for the 10h term. c) When will the sum be 19? 50? ____________ Given two linear patterns 1, 3, 5, 7, 9…and 12, 13, 14, 15, 16… a) When and where will the patterns meet? Given two linear patterns -10, -8, -6, -4, -2…and 4, 7, 10, 13, 16… a) When and where will the patterns meet? ____________ (Brown et al., 1992) p. 118 Solve $5n = 2n + 6$ Solve $2b = 80 - 8b$ Solve $30 = 8 - 2x$ Solve $39c + 78 = 33c$
November 21st Solving equation with variables on both sides	November 28th Introduction to ratio Given two linear patterns 14, 15, 16, 17, 18… and 10, 13, 16, 19, 22…

(Brown et al., 1992) p. 118 8. Solve $51 = 9 - 3x$ 11. Solve $98 - 4b = -11b$ 14. $5p - 9 = 2p + 12$ 31. $\frac{1}{3}(12 - 6x) = 4 - 2x$ 33. $5(2 + n) = 3(n + 6)$ 35. $5u + 5(1 - u) = u + 8$ ――――――― Computer Lab	a) When and where will both patterns meet? ――――――― (Brown et al., 1992) page 295 Solve 4. $\frac{6n}{5} = \frac{3}{1}$ 8. $\frac{3}{8} = \frac{9}{4k}$ 26. $\frac{6x - 2}{7} = \frac{5x + 7}{8}$ (Brown et al., 1992) p 289 Write each ratio in simplest form. 1. 5:15 2. 18:24 3. 49:35 5. $4x : 6x$ 6. $20t : 35t$ 7. $\frac{\pi(3r)^2}{\pi r^2}$
December 4th Solving ratio problems Given two linear patterns 20, 18, 16, 14, 12… and 6, 5, 4, 3, 2, 1… a) Find algebraic representation for both patterns a) When and where will both patterns meet? ――――――― (Brown et al., 1992) p 289 Write each ratio in simplest form. 7. $\frac{64a^3b}{16ab^4}$ 8. $\frac{72rs^5}{12r^2s^2}$ p. 291 Solve 8. Concrete can be made by mixing cement, sand, and gravel in the ratio 3:6:9. How much gravel is needed to make 850 m^3 of concrete? 10. A new allow is made by mixing 8 parts of iron, 3 parts of zinc, and 1 part of tungsten. How much of each metal is needed to make 420 m^3 of the alloy? The ratio of boys to girls is 4:3 in my class. a) How many boys and girls are there if there are 28 students? b) If the boys outnumbered the girls by 5, then	December 5th Solving ratio problems The ratio of the number of marbles in Box A to that of Box B is 3:7. Box B has 12 more marbles than Box A. a) How many marbles are there altogether? b) If 3 marbles are moved from Box A to Box B, what will be the new ratio of the marbles in Box A to that of Box B? (Brown et al., 1992) p. 296 1. Six oranges cost $.99. How much do ten oranges cost? 3. Maria drove 111 mi in 3h. About how far could she drive in 5 h? 7. At a fixed interest rate, an investment of $40000 earns $210. How much do you need to invest at the same rate to earn $336? ――――――― Computer Lab

<table>
<tr><td>how many boys and girls are there?</td><td></td></tr>
<tr><td>
December 6th
Review for test: Percents and equation

Solve $\frac{3x-5}{2}=\frac{2x-5}{3}$
Write 65% as a decimal and fraction.

The population at this school declined from 1200 to 1000 students. What is the percent change?

(Brown et al., 1992) p. 319
7. Yvonne paid $11,448 for a new automobile. This amount included the 6% sales tax. What was the price of the automobile without the tax?

8. Emily Ling is a real estate broker who earns a 12% commission on each house she sells. If she earned $21,600 on the same of house, what was the original price?

9. At the Runner's Shop anniversary sale, running shoes were on sale at 15% discount. If Alonzo paid $35.70 for a pair of running shoes, what was the original price?

Computer Lab
</td><td>
December 11th
Introduction to slope

Solve
a) $\frac{3}{4}=\frac{7}{x}$ b) $\frac{6x+18}{5}=2$
c) $\frac{2x-3}{3}=\frac{2x+2}{4}$

The ratio of ducks to chickens is 5:2. Find the following:
a) If the total number of ducks and chickens is 98, then how many of each kind do I have?
b) Find the number of ducks, if there are 48 chickens.
c) If the ducks outnumber the chickens by 30, then how many ducks and chickens do I have?

Given the following linear pattern 2, _, 12, 17, 22… Find the following:
a) The missing number
b) the jump
c) the 0th term
d) algebraic representation

Given the following linear pattern 3, _, _, 24,31…
Find the following:
a) The missing numbers
b) the jump
c) the 0th term
d) algebraic representation

Given the following linear pattern 5, _, _, 11,_…
Find the following:
a) The missing numbers
b) the jump
c) the 0th term
d) algebraic representation

Given the following linear pattern _,3, _,, 11,_,_…
Find the following:
a) The missing numbers
b) the jump
c) the 0th term
d) algebraic representation

Given the following linear pattern 12, _, 30,_ ,_…
Find the following:
a) The missing numbers
b) the jump
c) the 0th term
d) algebraic representation
</td></tr>
</table>

	Given the following linear pattern 100, _, _, _, 116… Find the following: a) The missing numbers b) the jump c) the 0th term d) algebraic representation Journal: Given the following linear pattern 3, _, 3, _, _ , 18… how would you find the algebraic representation? ——————— Computer Lab
December 13th More work with slope The ratio of coffee to tea is 4 to 1. find the following: a) The total number of drinkers is 55. b) The number of coffee drinkers is 24, how many tea drinkers are there? c) Coffee drinkers outnumber tea by 30, how many of each kind are there? ——————— (Larson et al., 2004) p. 230 Find the slope and algebraic representation 12. (6,9), (4,3) 13. (7,4),(-1,8) 15. (1,1),(4, -3) Find the slope, algebraic, and graph (video) 20. (4,5), (2,3) 21. (1,5),(5,2) ——————— Computer Lab	December 19th Given the following linear pattern _, 3, _, _, 4 … Find the following: a) The missing numbers b) the jump c) the 0th term d) algebraic representation Given the following linear pattern -12, _, _, _, 4… Find the following: a) The missing numbers b) the jump c) the 0th term d) algebraic representation Given $n_3 = 12$ and slope = $\frac{2}{3}$ a) the 0th term b) algebraic representation ——————— (Brown et al., 1992) p. 393 Find the slope and algebraic representation 5. (-5,3), (6,5) 9. (4,8), (1,3) Find the slope and *y*-intercept for each line. 13. $y = 2x + 1$ 14. $y = 3x - 2$ 17. $3x + 2y = 6$ ——————— Computer Lab

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