MHD turbulence in a channel with spanwise field

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We study turbulent channel flow of an electrically conducting liquid with a homogeneous magnetic field imposed in the spanwise direction. The Lorentz force is modelled using the quasistatic approximation. Direct and large-eddy simulations are performed for hydrodynamic Reynolds numbers Re=10000 and Re=20000 and the Hartmann number varying in a wide range. The main effect of the magnetic field is the suppression of turbulent velocity fluctuations and momentum transfer in the wall-normal direction. Comparing the results from direct and large-edddy simulations we show that the dynamic Smagorinsky model accurately reproduces the flow transformation.

1 Introduction

A channel flow in a purely spanwise magnetic field can be viewed as a generalized model of flows in the presence of a magnetic field with non-zero component parallel to solid walls. Such flows can be found in numerous metallurgical and materials processing applications, e.g. the electromagnetic flow control in continuous casting [1,2] and growth of large silicon crystals. Another area of applications is the liquid metal cooling blanket for fusion reactors [3]. Instability and transition to turbulence in sidewall boundary layers (in respect to which the magnetic field is spanwise) is one of the possible ways to achieve the desired intensification of heat and mass transfer. Lee & Choi [4] have shown that a spanwise magnetic field can lead to substantial turbulent drag reduction in a channel flow. Recently, we have studied transition to turbulence in this configuration at subcritical Reynolds numbers [5]. Here we extend these numerical studies to turbulence at supercritical Reynolds numbers. On the one hand, we examine the flow transformation caused by the spanwise magnetic field. The changes of turbulence statistics, wall friction, and coherent structures are documented. On the other hand, we examine the performance of the classical and dynamic Smagorinsky models for the large-eddy simulation (LES) of channel flow with spanwise magnetic field by comparison with direct numerical simulations (DNS).

2 Computational model

We consider the flow of an incompressible electrically conducting fluid in an infinite plane channel between insulating walls located at $z = \pm D$ with a magnetic field B_0 in the spanwise direction. The flow is driven by a pressure gradient in the streamwise direction such that the volume flux Q remains constant. We apply the quasi-static approximation, whereby the fluctuations of the magnetic field arising due to the fluid motion adjust instantaneously to the velocity fluctuations and are much weaker than the imposed magnetic field. The governing equations reduce to the Navier-Stokes system with the additional Lorentz force. The length and velocity scales are the laminar centerline velocity U and the channel half width D. The unit of time is D/U. Nondimensional parameters are the Reynolds number $Re = UD/\nu$ and the Hartmann number

$$Ha = B_0 D \sqrt{\frac{\sigma}{\rho\nu}}. \tag{1}$$

For the numerical simulations we use a pseudo-spectral Fourier-Chebshev method, where the flow field is represented by velocity potentials complying with the incompressibility constraint. A description of the algorithm and the corresponding flow solver is given elsewhere [5]. The algorithm is parallelized using domain decomposition in the streamwise direction. Fourier transforms are performed locally by transposition of the data array across the processors.

3 Results

Table 1 lists the parameters of the DNS and LES runs. The Hartmann number was increased until the turbulence could no longer be sustained. For the two values of Re, the mean friction coefficient c_f at the largest Ha decreased by about 30% (relative to its value at $Ha = 0$). At the same time, the mean velocity profile becomes steeper upon increasing Ha, as seen in Fig. 1 for $Re = 10^4$. One can also see that the logarithmic region is lost at $Ha > 0$ as a result of the magnetic damping of the turbulence. Correspondingly, the Reynolds stresses decrease with increasing Ha. The same observations apply for

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Re	Re_Q	Ha	DNS: $N_x \times N_y \times N_z$	LES: $N_x \times N_y \times N_z$	$L_x \times L_y \times L_z$
10000	6.67×10^3	0, 10, 20, 30	$256 \times 256 \times 256$	$64 \times 64 \times 64$	$2\pi \times \pi \times 2$
20000	1.33×10^4	0, 20, 30, 40	$512 \times 512 \times 256$	$128 \times 128 \times 128, 64 \times 64 \times 64$	$2\pi \times \pi \times 2$

Table 1 Parameters of DNS and LES. The parameters L_i denote the dimensions of the computational domain and N_i the corresponding numbers of collocation points. Re_Q is the flux-based Reynolds number.
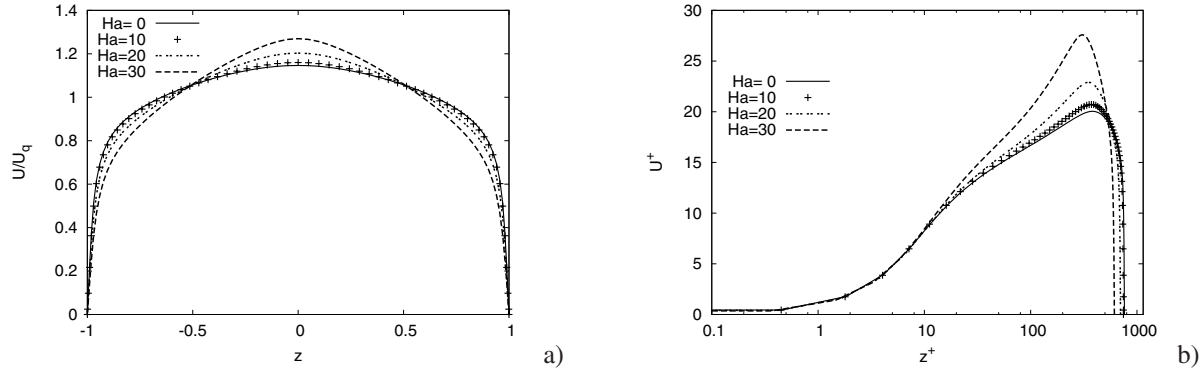

Fig. 1 Effect of the magnetic field on the flow properties. Results of DNS are shown for $Re = 10^4$. *(a)* Mean velocity profiles in global units. *(b)* Mean velocity profiles in friction units.
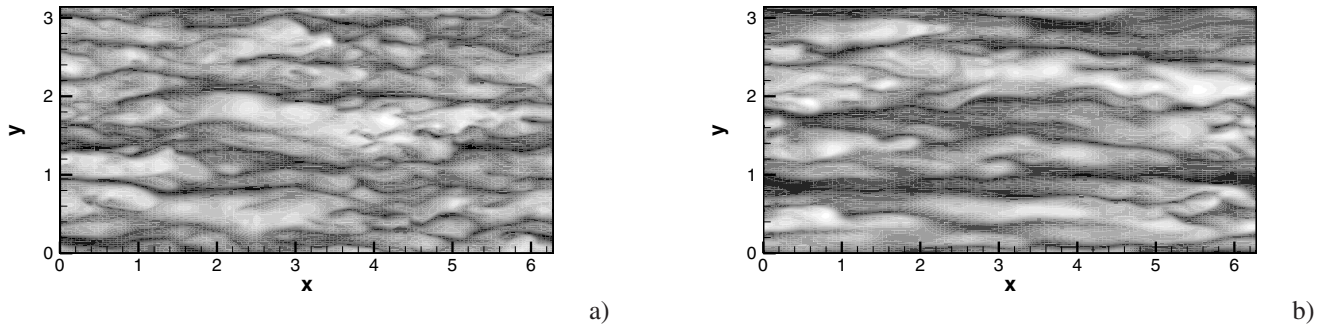

Fig. 2 Effect of magnetic field on coherent structures of the flow at $Re = 10^4$: Contour plots of the streamwise velocity fluctuations (from DNS) at $Ha = 0$ *(a)* and at $Ha = 30$ *(b)*.

$Re = 2 \times 10^4$. Flow structures in the near-wall region are visualized in Fig. 2 by contours of streamwise velocity perturbations at $z = 0.95D$. The fields are normalized by their rms values averaged over the horizontal plane. Upon increasing Ha, the near-wall streaks become more stable (less susceptible to small-scale perturbations) and wider in the spanwise direction. These observations are also in agreement with the earlier results by Lee and Choi [4]. For one-point correlations, the comparison of LES with DNS shows generally good agreement for the dynamic Smagorinsky model, whereas the classical Smagorinsky model deviates from DNS for larger values of Ha. Velocity gradient statistics are less well reproduced by the LES.

References

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