Scallops and Flutes

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by

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Summary:

Flow markings in cave channels are divided into scallops - irregular concavities - and the special case of flutes - regular, periodic, parallel crested patterns transverse to the flow direction. It is shown that flutes arise under conditions of long periods of constant flow velocity (solution rate) and that all stable solution flutes have a universal profile, downstream direction of propagation and Reynolds number of formation. A quantitative relation between flute period and flow velocity is proposed, the period being inversely proportional to velocity.

Scallop patterns appear to be primarily due to variable flow conditions and are therefore less useful than flutes in estimating past flow conditions in caves, except for flow direction. In both cases flow markings are only observed in channels carrying relatively rapid flow and are readily obscured, removed or prevented from forming by a variety of factors.

Introduction:

Flow markings in the form of concavities, or ridges predominantly transverse to the flow direction, are known to be produced during the solution or erosion of submerged limestone surfaces in caves. They are generally known in England as "scallops" and in America as "flutes", but have also been termed grooves, pockets, ripples and pits. Similar forms produced by free-surface flow, producing "valleys" parallel to the flow direction, are properly "grooves" or lapies grooving. The former variety are the subject of this study. For the moment I shall refer to any of this type of flow marking as "scallops".

Scalloped limestone surfaces are found in surface streams as well as in caves, and were first remarked upon (Lugeon, 1915) as a mechanism of surface stream erosion. This theme was later taken up in more detail by Maxson and Campbell (1935) and again by Maxson (1940). Bretz (1942) observed the form on the floor, walls and ceilings of caves and suggested their use, now common, in determining the direction of cave stream flow. This idea was developed in more detail by Coleman (1949), who suggested the term "scallop". Scalloped surfaces have also been observed on surface

ice and in ice caves (Sharp, 1947; Leighly, 1948).

The dominant form of scallops may be described as "interrupted concavities", usually covering a surface in a pattern which, while appearing largely random, exhibits a degree of uniformity in size, shape and spacing. The observable complexity of the forms produced, depending on stream and surface configuration and other factors, led Maxson and Campbell to classify flow markings into sweep, undulation, pocket, spiral, pothole and fret flutes (scallops).

The ridges or crests between the concavities vary from sharp to smooth. If a section is taken through crests which lie transverse to the flow direction it is almost always found that the lee slope is steeper than the streamward slope. This property provides an indication of flow direction in cave passages no longer carrying streams. A less common form of scallop possesses nearly parallel crests and a relatively constant distance between crests. These appear to be a simplification of the irregular scallop pattern.

Abrasion of limestone by suspended sediments in a stream was favored as the essential erosive agent by Maxson and Campbell and Maxson, but Bretz and Coleman observed that scallops on the walls and ceilings of cave passages often have chert and fragile silicified fossil inclusions standing out in relief from the surface. Therefore solution must also play a major part. That scallops are usually (though not always) found on limestone supports this contention. It is more evident in the scalloping of ice that solution (sublimation) or melting (Heat Transfer) are the only factors.

All of these authors believed that the character of the flow is primarily responsible for the sizes and forms of scallops, the rock or ice character being secondary. It is generally affirmed that the flow itself is turbulent and hence carries eddying or vortex currents. Maxson suggested that the salient edges of the boulders in streams, on which he found his examples of scallops, produced a turbulent wake containing trailing vortices which eroded scallops, but he was unable to explain the rough periodicity of the patterns. However he remarked that, once formed, the scallops may "fix" the position of vortices supplied by the turbulent flow. Maxson and Campbell give a figure showing hypothetical attached vortices behind scallop crests. Bretz and Coleman went further in suggesting self-stabilizing interaction of vortices and scallops. Leighly suggested a rather different mechanism - natural cellular convection - for scallops he observed on the ceiling of ice shelters. This would require negligible air velocities.

Maxson attempted the only theoretical consideration to date of the problem by introducing the Reynolds number of the flow producing the scallops, basing it on the dimensions of the boulder on which the scallops occurred. He used this viewpoint because of his belief that the vortices induced by the salients of the boulder were the active agents. This attempt was unsuccessful.

As solution proceeds on a scalloped surface the pattern propagates into the wall. In addition it probably also propagates parallel to the flow. Bretz, observing the steep lee slopes and believing that this indicated that solution rate is there maximum, concluded that the pattern propagates upstream. We shall see later that the reverse is true. The author (1959), Glennie (1963) and Eyre (1963) have recently suggested an inverse relation between the size of scallops and the velocity of the flow forming them. This is a question of great importance in interpreting the past hydrology of a cave system.

Illustrations of typical scallop patterns may be found in Maxson, Bretz and Coleman, and in many cave photographs in general publications. That particular form of marking where the crests are nearly parallel to one another for distances greater than the distance between crests, referred to hereafter as flutes, are rarer and less frequently illustrated. In Plates 1 - 7 are shown flutes, occurring usually in association with scallops, on rock and ice. Scallops and flutes on ice are usually an order of magnitude larger than those forming on rock. Flutes, being of simpler geometry, appear to be more readily subject to theoretical analysis than scallops and therefore the greater part of this study is devoted to flutes.

Similar wave phenomena occur throughout nature. Sand dunes (Scheidegger, 1961), submarine sand ripples and waves (Jordan, 1962; Thomas, 1964) and water waves (Ursell, 1956) are examples, but all differ significantly from the scallop phenomenon. On sand dunes and waves material is removed, transported, and redeposited - by air or by water. Only energy is transferred in the case of water waves. However these and scallops are all similar in that the controlling factor is the interaction between the flow of a fluid and the response of a modifiable surface. The shallow pits found on some meteorites may be more analogous to the flow markings discussed here (Williams, 1959, 1963).

I will show there, among other things, that all stable flutes have the same profile and this profile propagates into the rock and downstream; that the distance between crests of flutes (their period) is strictly inversely proportional to flow velocity and that a quantitative relation relating

flute period, velocity and water properties may be obtained from measurements. The probable connection between flutes and scallops will also be discussed.

Hydrodynamic Interaction:

Scallops and flutes are the consequence of the interaction of fluid flow and rate of solution of a soluble surface, or rate of combined solution and erosion. The removal of material at the surface develops the concavity. This may establish a new boundary for the flow which in turn modifies the flow pattern and the rate of removal of material. If these two interacting processes can come into an equilibrium so that the surface form is no longer modified, but propagates unchanged into (and along) the surface, we have stable scallops or flutes. The existence of flute patterns with nearly uniform periods and individual profiles suggests that a flute pattern may become stable. The situation in the case of scallops is less clear as the pattern is irregular. It is possible that within a scallop pattern, growth, coalescence and initiation of concavities proceed simultaneously, as suggested by Yeh (Davies, 1963) and stability in the above sense may not be possible.

The flow pattern over a stable flute pattern must itself be periodic with the same period as the flutes as otherwise the regularity of the pattern would be lost. On the other hand the flow pattern over any particular flute must be affected by the previous flutes. As apparently regular patterns of only a few periods are observed, it is likely that the flow pattern over a particular flute is nearly completely determined by its own profile and that of the previous period only.

Why this may be true is explained by considering the flow in more detail. At the initial crest of a particular flute a disturbance and separation, or at least the formation of a lee wake, occurs. (Goldstein, 1938). This will strongly modify the flow pattern directly behind a crest. This distrubed flow subsequently joins the main external flow to pass over the next crest. Therefore while details of the flow at a crest is determined by the previous flute, the flow pattern within a concavity is primarily a consequence of the lee wake of each initial crest.

If the resulting modified flow pattern within a concavity establishes a solution rate profile which maintains the flute profile, the pattern will remain stable. There is no restriction in this on whether the flow is laminar or turbulent. If the velocity is high enough so that separation does occur at the crest, an eddy or vortex will form behind the crest and over some portion of the flute profile on the lee side of a

crest will be a reversed flow. Such a situation is nearly always hydro-dynamically unstable and turbulence will develop and diffuse into the main stream flow. Therefore the flute pattern will itself cause turbulence and its attendant fluctuating velocities. Since this is generated at the wall it has the greatest intensity there and turbulent fluctuations in the main stream would have only a secondary influence near the wall. Consequently it is not correct to say that turbulence is responsible for the development of scallops and flutes; rather it is the reverse. It will be shown that the flow velocity required for flute stability is apparently high enough that we may restrict ourselves to the turbulent situation.

At high enough flow velocities in a smooth channel, turbulence will also develop, subjecting the wall to fluctuating velocities and fluc-tuating local solution rates. However the frequencies of these fluctua-tion are so high compared to the rate of retreat of the wall that only average solution rates are important. Any irregularity in the surface, no matter how small, will initiate a lee wake and the modified average flow pattern will with time cause a change in the shape of the surface. Therefore any soluble surface is initially unstable with respect to the development of scallops or flutes. The details of the development pro-cess are not known at present so we must restrict ourselves to questions of the stability and propagation of only stable flutes.

In the lee wake of a crest the flow velocities are on the average smaller than in the main stream. With forward flow over a crest and reversed flow just behind it, there must exist some point on the lee side at which the average velocity is zero. In the vicinity of this point there should be a minimum in the rate of solution. The streamward side of a crest extends into the main flow and is subject to the direct impingement of the highest velocities, and we would expect a maxi-mum in solution rate somewhere on this side. This is similar to the behavior in the distribution of solution or mass transfer about a sphere, the minimum occurring in the vicinity of the separation point (Hsu and Sage, 1957; Garner and Suckling, 1958). We shall see later that this conclusion is found experimentally and is also entirely consistent with a steeper lee slope.

Due to the high turbulence levels induced by a scalloped wall, the exchange of solvent between the surface and main stream is very rapid. At the surface a saturation condition may exist but the aver-age concentration in the fluid must fall very rapidly with distance from the surface to the main stream concentration. This means that the concentration boundary layer at the wall is very thin compared to typical scallop dimensions and therefore the concentration gradients

at a point in the profile have a negligible affect on the solution rate at nearby points on the profile - that is, the local hydrodynamic conditions may be considered as controlling the solution rate profile along the surface.

If abrasion due to suspended sediments is also important, it will be most rapid where the flow impinges most directly on the surface - again on the streamward side of crests. This will modify the erosion pattern produced by solution but will not change the general conclusions above. Of course larger sediment particles may be trapped in floor scallops by gravity and by moving in response to the eddying lee wake flow erode the base of the concavity. Under some conditions this may lead to higher removal rates at the bottom of concavities and the development of the familiar pot-holes. This affect is absent in wall or ceiling scallops.

Deposition of fine clay on the surface will also modify the solution process, by locally decreasing the rate of solution. We would expect such deposition to occur to the greatest extent on the lee slopes where the flow velocities are lowest, but what actually occurs will depend on many factors including surface roughness, type and quantity of suspended material, orientation of the surface and the flow conditions. Such deposition would probably be irregular and thus introduce irregularities into the process, leading to scallops rather than flutes. Even if regular over a flute pattern, clay deposition would be distributed over a flute profile, modifying its form and dimensions. The subsequent analysis applies primarily to stable, sediment-free, solution flutes.

Dimensions of Stable Flutes:

A characteristic dimension of a stable flute pattern is its crest-to-crest wave length, or period, usually of the order of 5 to 15 centimeters. If the pattern is stable the period is a dependent variable completely determined by the independent variables consisting of flow conditions, fluid properties, and rock properties (This analysis applies equally to ice flutes but will be phrased in terms of the more familiar water-rock situation). To simplify the theoretical consideration of the problem the flow conditions will be taken to be characterized by the average velocity of flow in the channel, U, and a channel dimension H. The only fluid properties which can be involved are the density, ρ, and viscocity, μ. The solubility of the rock and saturation conditions of the water would only be important in regard to the rate of propagation of a flute pattern but

could not themselves alter the pattern of local dissolution rate established by the flow phenomena. However the diffusivity, $\underline{D}$, of the solute ions in solution may affect the pattern of local solution rates through the concentration boundary layer. Assuming that solution alone is important and that the rock is homogeneous, the stable flute period, $\underline{L}$, is a function of $\underline{U}$, $\underline{H}$, ρ, μ, and $\underline{D}$ only. We may write: (Bird, Stewart, Lightfoot, 1960)

$$\underline{L} = f(\underline{U}, \ \underline{H}, \ \rho, \ \mu, \ \underline{D}) \tag{1}$$

From dimensional analysis it is necessary then that

$$\frac{\rho U L}{\mu} = f(\frac{L}{H}, \ \frac{\mu}{D\rho}) \tag{2}$$

The ratio on the left is the Reynolds number for the flow, based on the flute period. On the right appear a length ratio and the Schmidt number, relating the diffusivities of momentum and matter.

As already discussed, in the presence of the highly disturbed flow region near a fluted surface the flow disturbance produced by other surfaces of the channel may be expected to be of less importance, except to the extent that other surfaces determine the average flow velocity in the region of the surface upon which the flutes form. This may be seen, for example, in Eldons Cave, Massachusetts, U.S.A. (Perry, 1946) where flutes follow the surface around corners parallel to the flow direction without change of period or profile. As long as L/H is small, equation (2) should not depend upon it to an important degree.

Likewise, the Schmidt number, which controls the relative thickness of the mass and momentum transfer boundary layers, should not be important when it is large and the concentration boundary layer is extremely thin. If the function in equation (2) is independent, or extremely weakly dependent upon its arguments, it must be a constant and we may write

$$\frac{\rho U L}{\mu} = N_f^* \tag{3}$$

The dimensionless number N_f^* is the stable flute Reynolds number. It differs in derivation and principle from the Reynolds number introduced by Maxson. This relation states that the flute period is inversely proportional to flow velocity but also depends on the fluid properties. The dependence on velocity has been suggested empirically by Glennie (1963) and Eyre (1963). If the value of N_f^* can be

found, flow velocities can be determined from measurements of stable flute period and knowledge of water properties.

Two sets of flutes have been measured in an attempt to estimate N_f^*. In Boydens Cave, Calif., U.S.A. (Halliday, 1962) flutes occur on active and fossil stream side walls. Measurements made on an active stream section (Plate 4) found a mean flute period of 6.5 cm. Stream velocity was estimated to be 50 cm./sec. At the cave temperature (8 °C) the value of the kinematic viscosity of water, μ/ρ , is 0.0138 cm^2/sec. Therefore if these flutes are stable under these conditions the value of N_f^* is 23,500.

The second flute set comes from an entirely different environment. In the Eisriesenwelt, Salzburg, Austria, one wall of the Mörkdom is entirely ice, past which flows a steady stream of air towards the entrance. (Czoernig-Czernhausen, 1926). Flutes and scallops have developed in this ice either by sublimation of the ice into the air stream, or by melting due to transfer of heat from the slightly warmer (1° C) air (Plate 6). In either case equation (2) must also apply, except that if heat transfer is important the Prandtl number ($C_p\mu/k$, where C_p is the specific heat the k the thermal conductivity of air) replaces the Schmidt number.

The mean flute period was found to be 137 cm. (over twenty times that in the rock-water system) and the measured air velocity was 21 cm/sec. The kinematic viscosity of air at 1 atm. and 1° C is 0.133 cm^2/sec. If these are stable conditions the flute Reynolds number is 21,600.

These two estimates are within 5% of a mean value of 22,500. The agreement may be partly fortuitous as the water velocity was only estimated and there is no assurance that the velocities represent the conditions under which the flutes formed. However as the data come from the radically different systems of water-rock and air-ice, it is likely that the dependence in equation (3) on kinematic viscosity is correct, and with some confidence the dependence on velocity. Equation (3) is plotted in Figure 1 as L versus U, for the water-rock system at various temperatures and the air-ice system at 0° C, using $N_f^* = 22,500$. This value is, of course, subject to correction by further observations or experiment.

This analysis states that constant water velocity and temperature during solution are sufficient to produce a flute pattern of

Fig. 2.

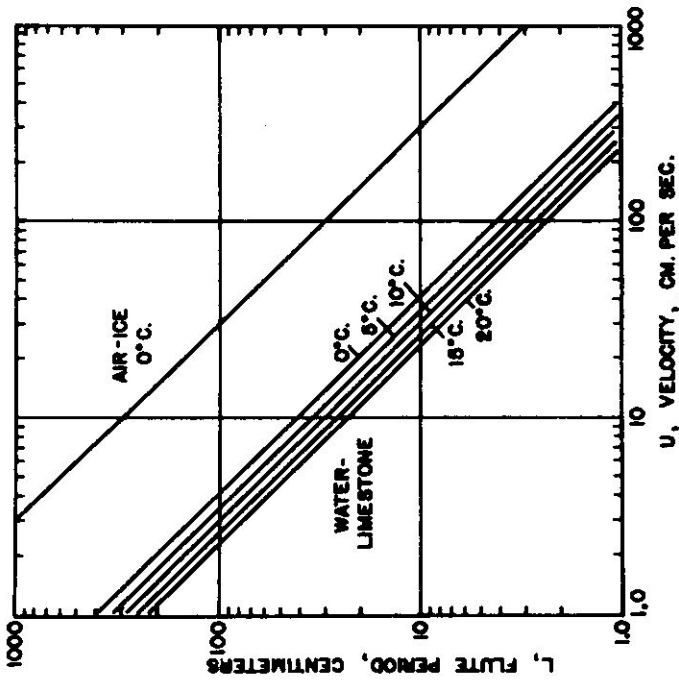Overall and local geometry of stable flute profiles

Fig. 1.

Relation between stable flute period and fluid velocity for the water-limestone and ice-air systems. From equation 3, using $N_f^* = 22,500$.
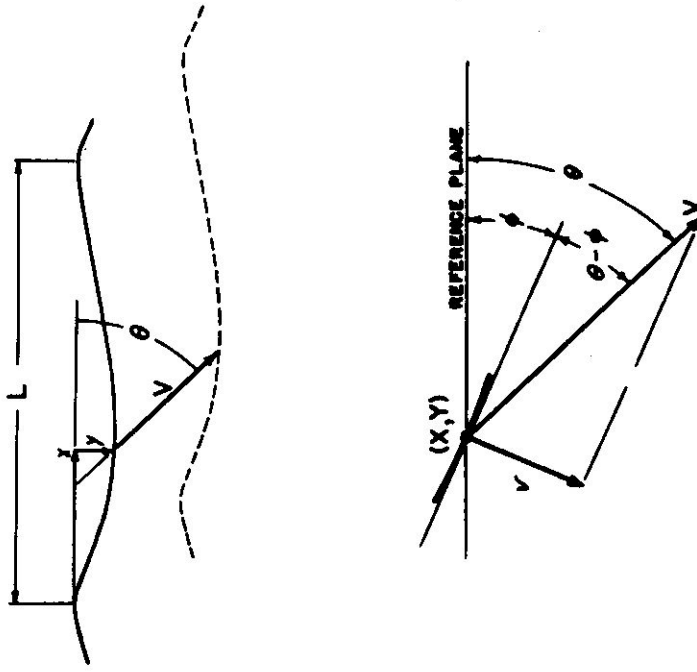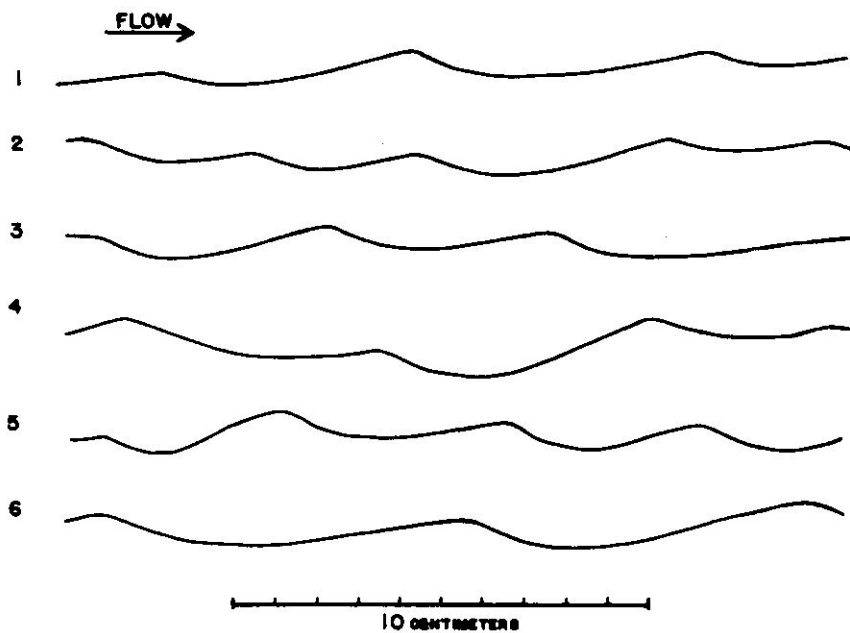
FLOW →

1

2

3

4

5

6

10 CENTIMETERS

FIG. 3

Fig. 3.

Profiles copies from flutes in Boyden's Cave, Calif., U.S.A. All
located close to the present stream level except No. 4. No. I
shown also in Plate 4 and No. 6 in Plate 5.

constant period. Flutes are usually observed high on the walls and on the ceilings of caves which flood completely and thereby provide a constant overall water pressure head to produce constant flow rate through the terminal constrictions. The flutes in the early passages of Eldons Cave and in Polnagollum Cave, Co. Clare, Ireland (Plate 1) (Collingridge, 1962) may be typical of this situation. Simultaneous abrasion by suspended sediments may alter N_f^* and confuse the interpretation of flute period, but this may leave its own evidence, which will be discussed later.

The causes for the loss of the parallel crest structure of flutes to produce the somewhat randomly placed concavities of scallops are not known at present. However scallop patterns require many more length dimensions to characterize them than a single period and therefore other variables must enter into their determination. The pattern of scallops is partly random and would require a statistical description. This suggests that the additional variables may also be statistical variables associated with fluctuating flow conditions during scallop formation. Scallops are also associated with variable flow conditions by Maxson and Coleman. Nevertheless a scallop pattern might remain statistically stable even though individual scallops change with time, if the statistical flow properties are constant.

This idea gains support from a consideration of what would happen to regular flute patterns under fluctuating flow conditions. As the flutes would continually be attempting to adjust to the stable period a variety of periods would develop and superimpose in a complicated manner. This requires the junction of flute crests to form a scallop configuration. If this interpretation is correct it will be much more difficult to determine flow conditions from scallop size than from flute size although a rough inverse dependence on average flow velocity might be a valid assumption.

The flute profile $y(x)$, defined in **Figure 2**, is a dimension which must also be functionally related to the same variables as the period and also x. Writing this dependence in terms of a dimensionless distance between crests,

$$y\left(\frac{x}{L}\right) = f\left(U, H, \rho, \mu, D, \frac{x}{L}\right) \tag{4}$$

we obtain by dimensional analysis,

$$\frac{\rho y U}{\mu} = f\left(\frac{x}{L}, \frac{UH}{\mu}, \frac{\mu}{D\rho}\right). \tag{5}$$

If the dependence on the channel Reynolds number and Schmidt number may be neglected for the reasons already given in regard to L, we may substitute equation (3) and obtain

$$\frac{y}{L} = \frac{1}{N_f^*} f\left(\frac{x}{L}\right) \tag{6}$$

and conclude that all stable flute profiles must be geometrically similar.

In Figure 3 are shown six portions of flute patterns from Boydens Cave, Calif. We see a superficial resemblance of flute profiles, but irregularities prevent adequate comparison on the basis of equation (6). The observed irregularity is not unexpected as the necessary conditions for the exact validity of equation (6) would be rarely encountered. But with sufficient data from flute patterns of different average period, a statistical comparison with equation (6) should be possible. If stable solution flute profiles are not all geometrically similar, a dependence on channel size would have to be considered. If scallop patterns do result from statistical fluctuations in flow velocity a similar test is not possible due to the simultaneous effects of many, and unknown, additional variables describing the flow variations. However if two scallop patterns are shown to be statistically similar, even if the average size of scallops are different, the statistical properties of scallop profiles might be compared to find if they obey the scaling law of equation (3).

In the preceeding it is implicit that the boundary layer thickness also scales with period. For a fluted surface in an infinite fluid the boundary layer would of course grow without limit. Therefore the finite channel size enters in limiting boundary layer growth and establishing a mean velocity in the channel and eventually a uniform pressure gradient along a uniform channel. Since the stable flute Reynolds number is apparently so large, the channel Reynolds number must be in the range from about 200,000 to 2,000,000, and the channel flow is fully turbulent. Consequently the mean velocity and the maximum velocity of the main stream flow are not very different, and the apparent boundary layer thickness will be constant and scale with flute period.

Rate of Solution and Friction Factor:

The average mass transfer coefficient over a full flute period, k (cm/sec), is related to the flow conditions and flute period. Writing

$$k = f(U, H, \rho, \mu, D, L) \tag{7}$$

PLATE 2

Flute patterns crossing impurity bands in marble. Eldon's Cave, Mass., U.S.A. Total height about five feet.
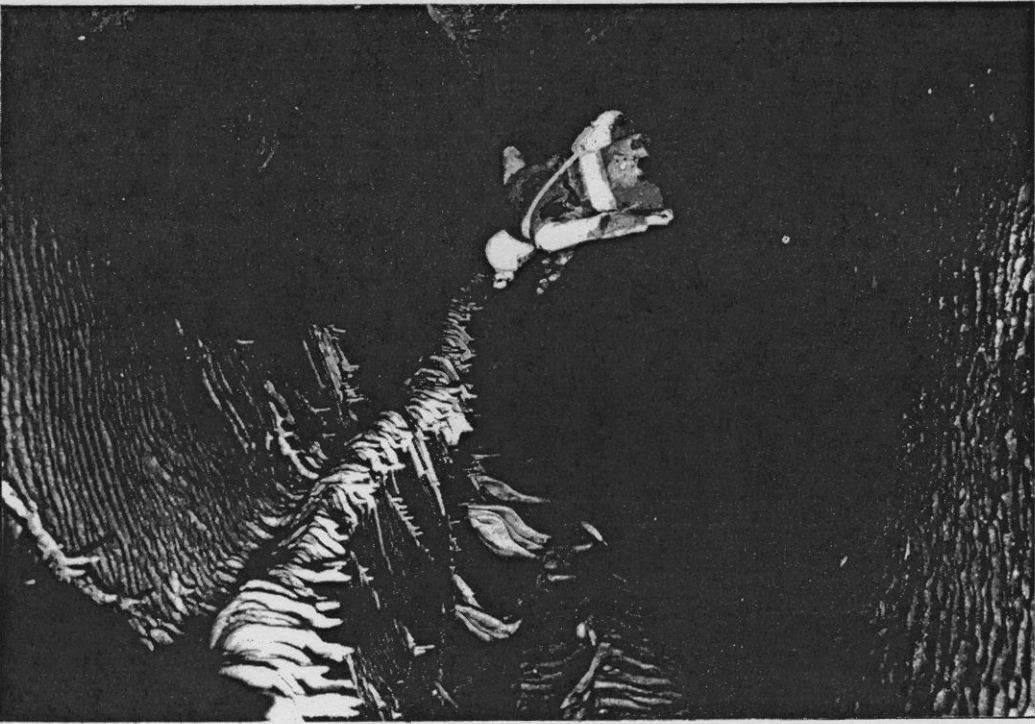

PLATE I

Scallops and flutes in Poulnagollum Cave, Ireland. Note extensive development of flute pattern on the ceiling, as compared with the floor. Reproduced with permission of Univ. Bristol Speleological Society (O.C. Lloyd).
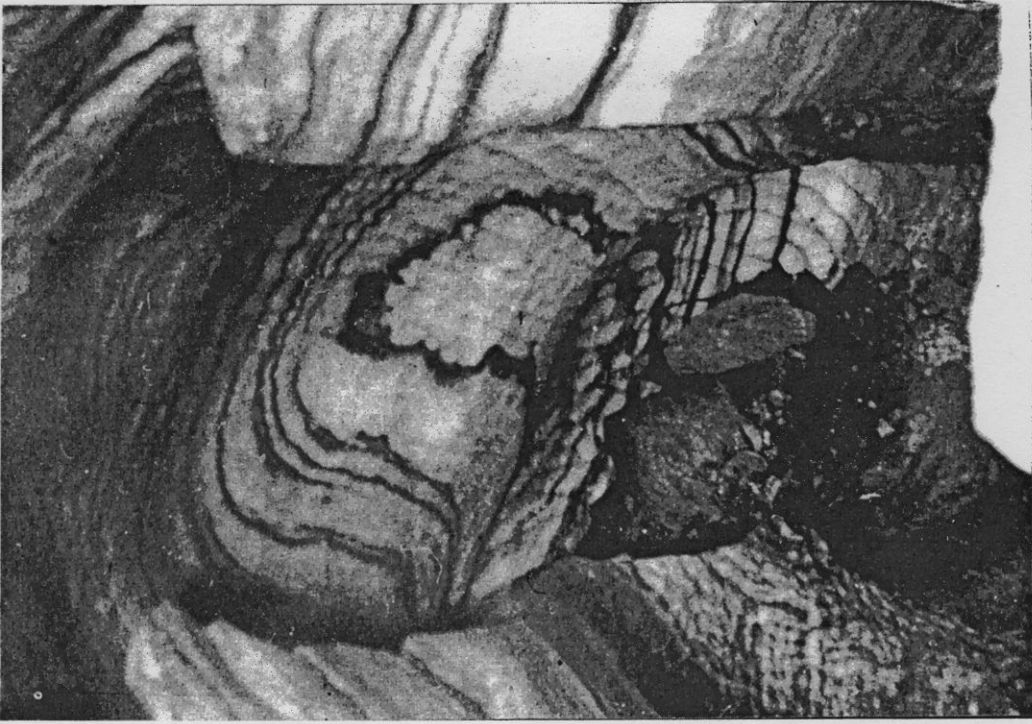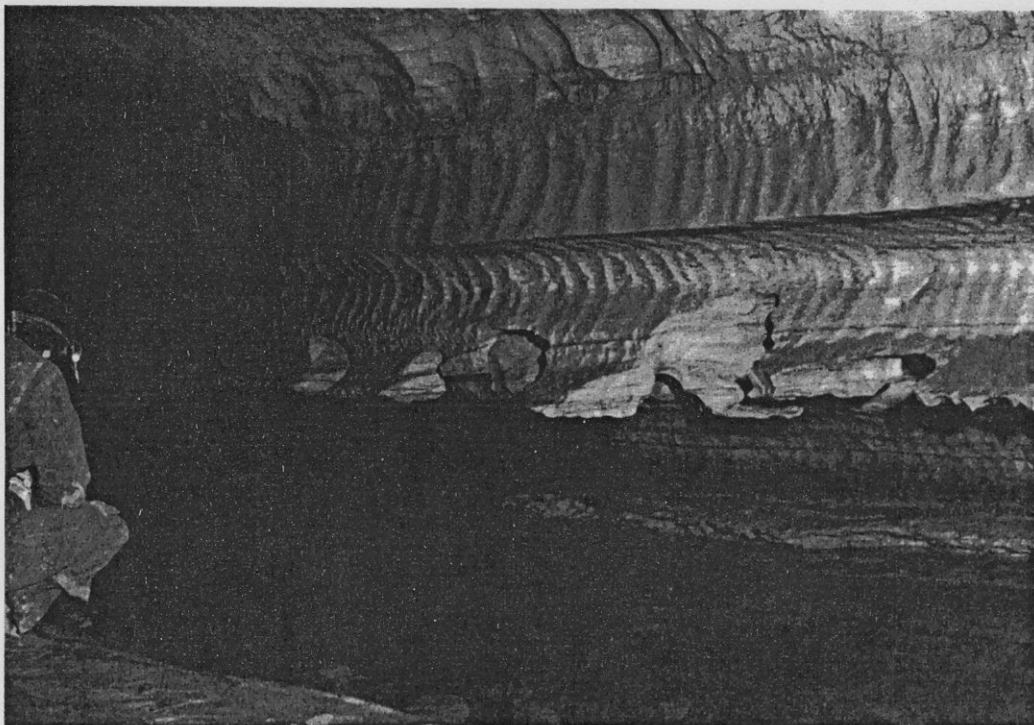
PLATE 3

Flute patterns in B & B Cave, Ind., U.S.A. (G. F. Jackson).
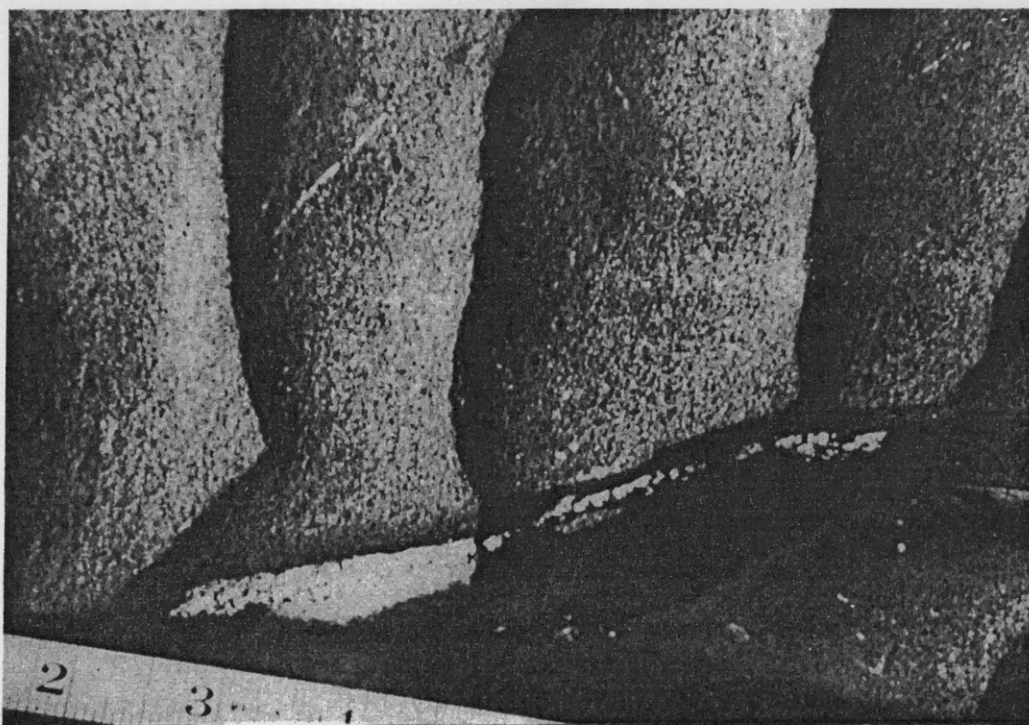
PLATE 4

Flute pattern at stream level in Boyden's Cave, Calif., U.S.A. Flow is from left to right. Profile shown in Figure 6, No. 1.

PLATE 5

Flute pattern on pendant in Boyden's Cave, Calif., U.S.A. Flow from right to left.

PLATE 6

Flutes on ice wall in the Eisriesenwelt, Salzburg, Austria. Height of wall is about two meters. a Air flow from right to left.

PLATE 8

Water tunnel in the Laboratorium voor Fysische Technologie, Technische Hogeschool, Eindhoven, Holland (J.A.H. Berghs).

PLATE 7

Seven meter high scalloped ice wall in the Eisriesenwelt.

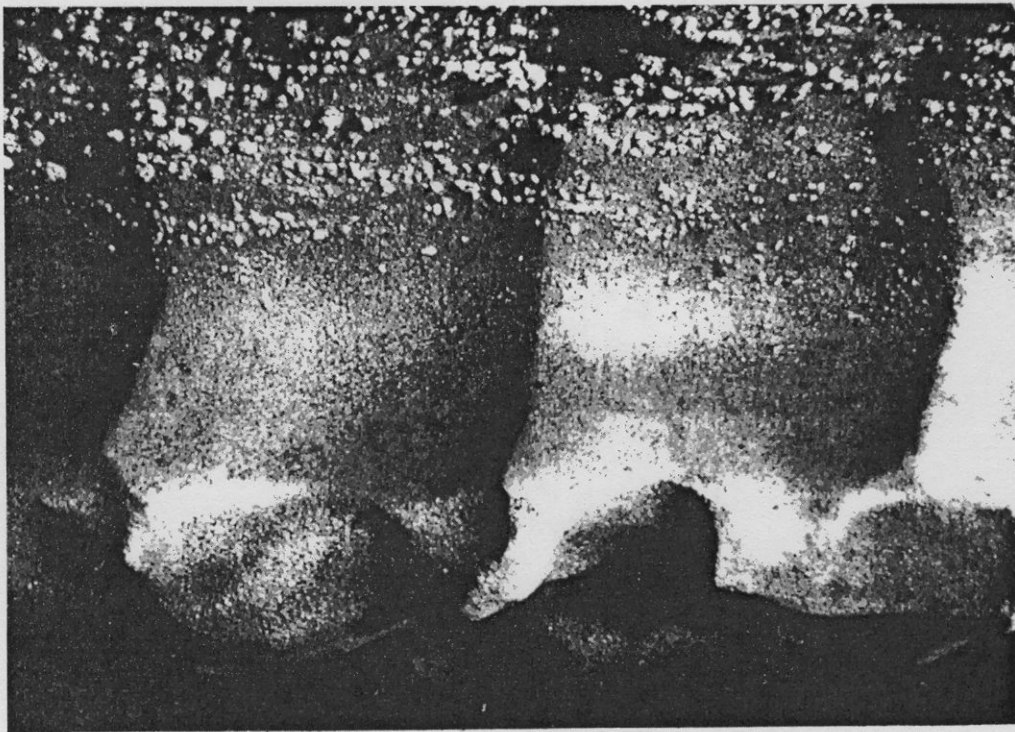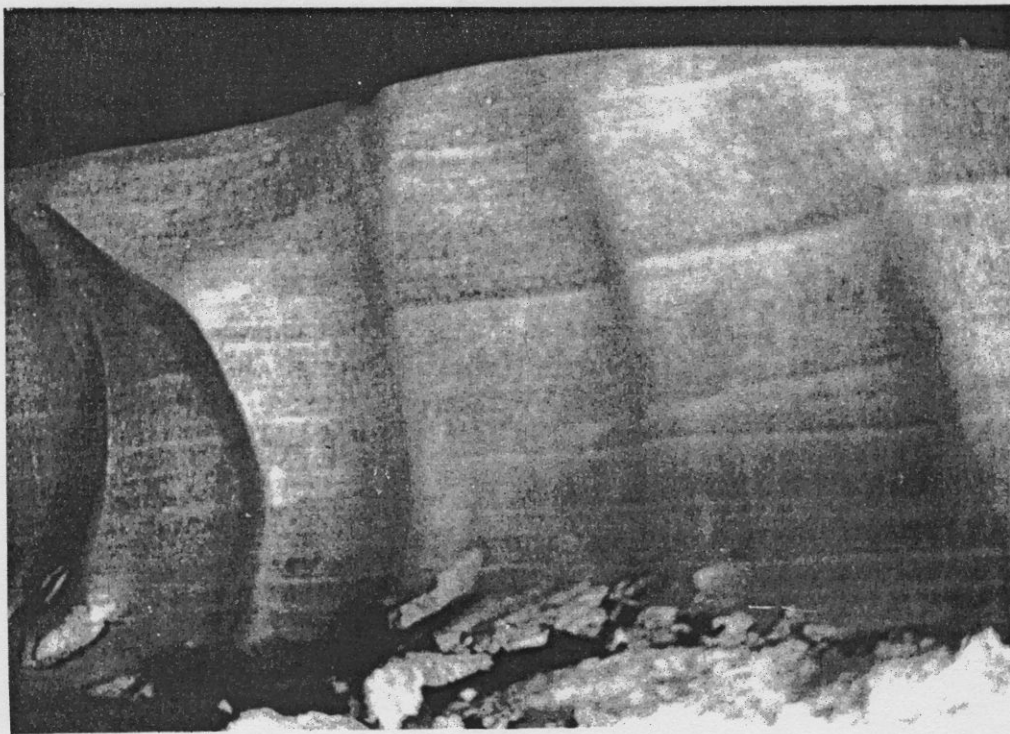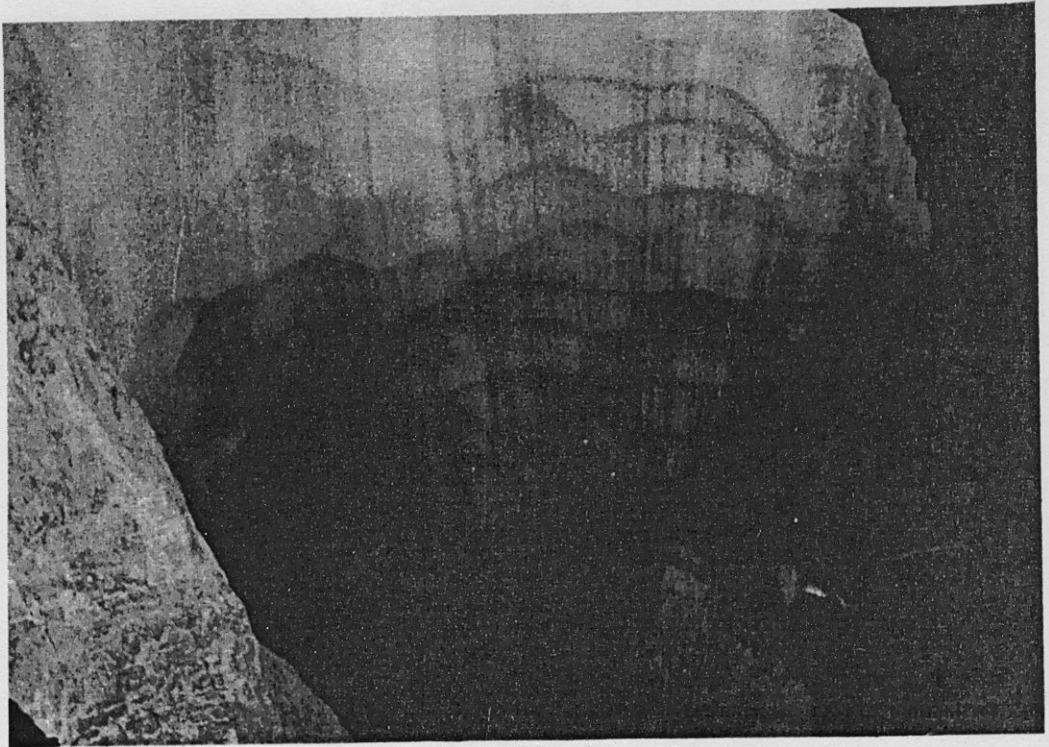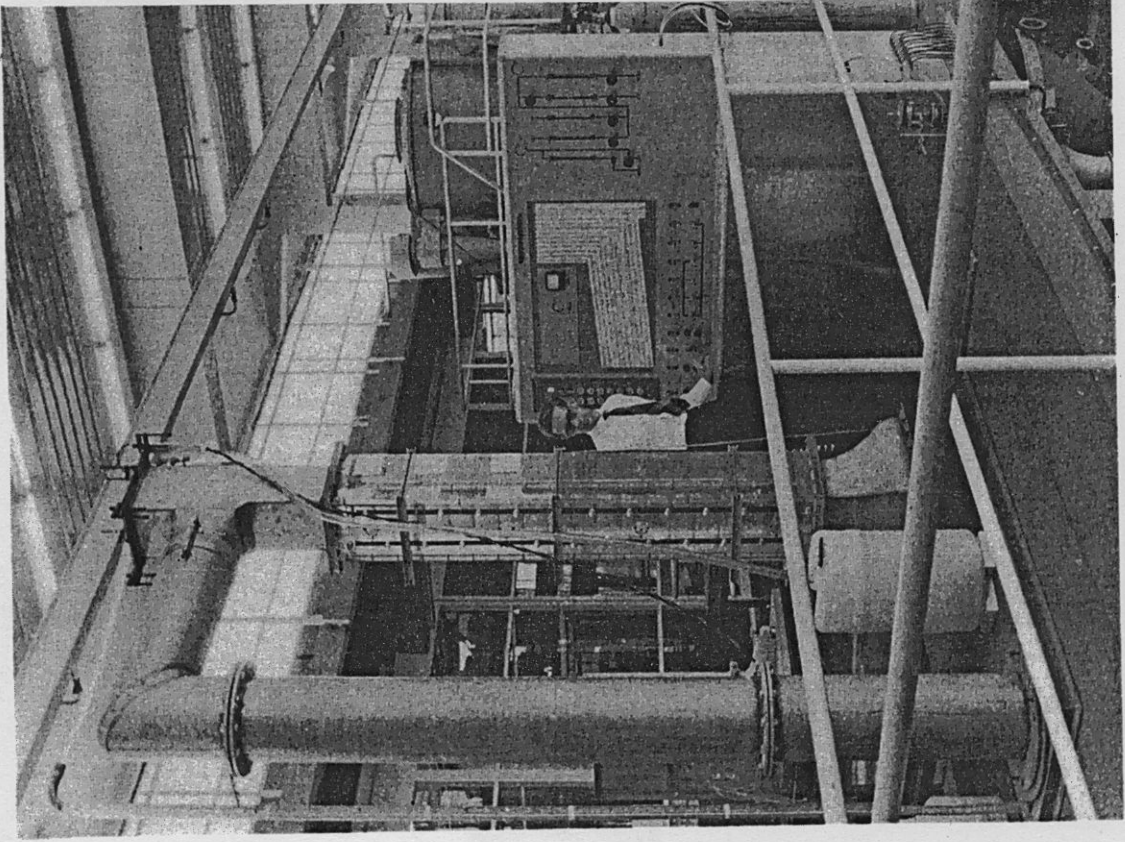where now, since all stable flutes are similar, we need only introduce the period $\underline{L}$, dimensional analysis gives

$$\frac{kL}{D} = f\left(\frac{L}{H}, \frac{\mu}{D\rho}, \frac{\rho UL}{\mu} \right) \tag{8}$$

The dimensionless ratio on the left is the overall average Sherwood number on the flute pattern, $\underline{N}_s$. Again neglecting the channel influence we obtain

$$\underline{N}_s = f\left(\frac{\mu}{D\rho}, \underline{N}_f \right) \tag{9}$$

We cannot omit the Schmidt number here as the concentration boundary layer does influence the mass transfer rate perpendicular to the surface. For a stable flute pattern $\underline{N}_s = N_f^*$ so that for a given fluid-surface system $\underline{N}_s = N_s^*$, a constant. Therefore the rate of solution of the pattern is inversely proportional to flute period and hence proportional to stream velocity <u>when stability is attained</u>. This need not be true for changing velocities over a fixed flute pattern. It means that the flute pattern will evolve with time to establish a constant Sherwood number. We may also conclude that stable flute patterns with small period are propagating more rapidly than those with longer periods if all other factors (including solvent saturation) are the same. It also means that shorter flute periods should be more common than longer periods.

The friction factor for flow over a surface is defined as

$$\underline{f} = \frac{\tau}{\frac{1}{2} \rho U^2} \tag{10}$$

where τ is the average shear stress on the surface (including form drag). It is necessary dimensionally that

$$\underline{f} = f(N_f) \tag{11}$$

over stable profiles, and hence the friction factor is a constant when $N_f = N_f$, for any period. Since we know the flow for flute stability is turbulent we may use the fact that the friction factor for flow over a rough surface is independent of flow velocity to conclude that the friction factor is independent of the evolution of stable flute patterns between two stability conditions. However it may vary while the pattern re-adjusts to a new stability condition. We may also conclude that the pressure drop in a fluted channel (running full) varies as the square of velocity.

There is some evidence (Schlichting, 1936; Wiederhold, 1949; Seiferth and Krüger, 1950) that flute-like roughness in pipes produces an anomolous friction-factor relation - the pressure drop being higher than expected from an equivalent roughness correlation - although this seems to occur at lower values of N_f than required for flute stability.

Motzfeld (1937) has studied experimentally average flow patterns over various wavy walls, and Benjamin (1959) has considered the problem theoretically. These studies have been restricted to flow and pressure phenomena and do not extend to the soluble surface situation. In the present work pressure loss phenomena are not involved, but it would appear that flutes and scallops may play some role, as yet unknown, in the general cave passage development and competition problem.

Geometry and Stability Conditions:

We may establish the relation between local solution rate and flute profiles from a geometric argument. Consider the profile shown in Figure 2. If the profile is stable, it propagates without change of shape to a new position at a velocity $\underline{V}$. The angle of propagation, measured from a plane parallel to the flute system, is $\underline{\theta}$. At point $\underline{x}$ the slope (angle) of the surface is $\underline{\phi}$. The normal velocity at that point is $\underline{v}$. This is proportional to the rate of solution at that point. By resolution of velocities we obtain

$$\frac{v}{V} = \sin(\theta - \phi) = \sin\theta\,\cos\phi\; -\; \sin\phi\,\cos\theta \qquad (12)$$

Every point of equal slope has equal solution rate and, in particular, the top of the crest and the bottom of the concavity must have equal solution rate. This vertical rate of solution, expressed as a velocity $\underline{v}_o$, occurs where the slope of the profile is zero, and $\underline{v}_o = \underline{V}\sin\theta$. Therefore

$$\frac{v}{v_o} = \cos\phi\; -\; \sin\phi\;\operatorname{ctn}\theta \qquad (13)$$

or, in terms of local slope, since

$$\cos\phi = \frac{1}{\sqrt{1 + y'^2}} \qquad (14)$$

$$\sin \phi = \frac{y'}{\sqrt{1 + y'^2}}$$

where y' is the derivative of y with respect to x, we have

$$\frac{v}{v_o} \sqrt{1 + y'^2} = 1 - y' \, \text{ctn} \, \theta \tag{15}$$

The left hand term, plotted against y', should be a straight line with slope ctn θ if the profile is stable. From this θ may be determined, if v may be measured on a stable profile. This is a test for flute stability. This relation also shows that, given a stable profile, there may be many solution rate profiles which are able to maintain it, corresponding to the different values θ may have. Thus we are not able to determine the solution rate profile given only the stable flute profile.

To all appearances flute profiles are quite regular. That is, they have one maximum (the crest), one minimum and a smooth profile between except for surface roughness due to rock granularity. The crests are relatively sharp, but whether they may be real surface slope discontinuities, or only locations where the slope changes rapidly, is not obvious. We will begin an analysis of details of a stable flute profile by considering a regular smooth profile with the following properties (Figure 4).

1.   The profile has two points, A and B, where $\phi = 0$.

2.   Between A and B on the lee slope ϕ is everywhere positive finite and on the streamward slope is everywhere negative finite.

3.   Between A and B on the lee slope is one inflection point (1), where the curvature changes sign (and is therefore zero) and where ϕ_1 is a maximum for the profile.

4.   Between B and A on the streamward slope is one inflection point (2), where the curvature is also zero and ϕ_2 is a minimum (negative) for the profile.

From the profile stability relation (equation 12), since v must be everywhere positive,

$$\phi_1 \leqslant \theta \leqslant \pi + \phi_2 \tag{16}$$

At point $\underline{1}$, ϕ_1 is the maximum slope and therefore $\phi_1 < \theta$. At point $\underline{2}$ $\phi_2 = -|\phi_2|$ is the minimum slope and hence $\theta \leq \pi - |\phi_2|$. ($\phi_2$ is inherently negative. It is convenient to write its absolute value in these expressions.) Therefore θ must lie within the crest angle, defined as the angle between tangents to points $\underline{1}$ and $\underline{2}$. This is also necessary as otherwise a point on the profile would somewhere propagate out of the profile, which is impossible.

Differentiating equation $\underline{12}$ with respect to distance along the surface, $\underline{s}$, we obtain

$$\frac{1}{V}\frac{\partial v}{\partial s} = -\cos(\theta - \phi) \frac{\partial \phi}{\partial s} \tag{17}$$

Where the derivative on the left is zero we have critical points of the solution rate profile; maxima, minima or inflection points. The derivative on the right is the curvature of the surface. The critical points of $\underline{v}$ occur where

$$\frac{\partial \phi}{\partial s} = 0 \tag{18a}$$

or

$$\theta - \phi = \tfrac{1}{2}\pi \tag{18b}$$

The condition (18b) is where the surface and the direction of propagation are perpendicular (if there is such a point). Then from equation $\underline{12}$ we find $\underline{v} = \underline{V}$ and the solution rate is there maximum. To determine the nature of the other critical points we differentiate again with respect to $\underline{s}$ and obtain

$$\frac{1}{V}\frac{\partial^2 v}{\partial s^2} = -\cos(\theta - \phi)\frac{\partial^2 \phi}{\partial s^2} - \sin(\theta - \phi)\left(\frac{\partial \phi}{\partial s}\right)^2 \tag{19}$$

A variable, at a point where the first derivative is zero, is a maximum if the second derivative is negative and a minimum if the second derivative is positive. At the critical points $\underline{1}$ and $\underline{2}$:

1. At ϕ_1 ;

$$\frac{\partial \phi}{\partial s} = 0 \tag{20a}$$

$$\frac{\partial^2 \phi}{\partial s^2} < 0 \tag{20b}$$

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and $\underline{v}_1$ is a maximum if, a) $\theta - \phi_1 > \frac{1}{2}\pi$ and a minimum if, b) $\theta - \phi_1 < \frac{1}{2}\pi$.

   2. At ϕ_2;

$$\frac{\partial \phi}{\partial s} = 0 \qquad\qquad\qquad (21a)$$

$$\frac{\partial^2 \phi}{\partial s^2} > 0 \qquad\qquad\qquad (21b)$$

and $\underline{v}_2$ is a minimum if, a) $\theta + |\phi_2| > \frac{1}{2}\pi$ and a maximum if, b) $\theta + |\phi_2| < \frac{1}{2}\pi$.

   Now if θ lies in the interval

$$\tfrac{1}{2}\pi - |\phi_2| < \theta < \phi_1 + \tfrac{1}{2}\pi \qquad\qquad (22)$$

the direction of propagation must be perpendicular to the surface at two points in each period, at which $\underline{v} = \underline{V}$ (maxima), while there will be minima in $\underline{v}$ at both points $\underline{1}$ (case 1b above) and $\underline{2}$ (case 2a above). The maxima and minima in $\underline{v}$ will, of course, alternate along the profile.

   There are many pairs of points with equal solution rate (equal ϕ) and two solution rate maxima are imaginable, but to have the maxima identical ($\underline{v} = \underline{V}$) would appear to be too fortuitous to be likely, and therefore θ probably does not lie in the interval of relation (22). The alternatives are that θ lies in the interval (from relations (16) and (22))

$$\phi_1 \leqslant \theta < \tfrac{1}{2}\pi - |\phi_2| \qquad\qquad (23)$$

or

$$\phi_1 + \tfrac{1}{2}\pi < \theta \leqslant \pi - |\phi_2| \qquad\qquad (24)$$

when, in either case, the direction of propagation is nowhere perpendicular to the surface, and each period has but one maximum and one minimum in solution rate.

   If θ lies in the interval (23), the propagation direction is downstream and $\underline{v}_2 > \underline{v}_1$. If θ lies in the interval (24), the propagation direction is upstream and $\underline{v}_1 > \underline{v}_2$. It has already been argued that $v_2 > v_1$ so on that basis we expect downstream propagation.

   The ratio of the solution rates at the two critical points is, from equation (12),

$$\frac{v_1}{v_2} = \frac{\sin(\theta - \phi_1)}{\sin(\theta + |\phi_2|)} \qquad (25)$$

Now the crest angle is

$$\Psi = \pi - \phi_1 - |\phi_2| \qquad (26)$$

and writing the angle between ϕ_1 and θ as $\Psi_1 = \theta - \phi_1$, equation (25) becomes

$$\frac{v_1}{v_2} = \frac{\sin(\Psi_1)}{\sin(\pi - (\Psi - \Psi_1))} = \frac{\sin(\Psi_1)}{\sin(\Psi - \Psi_1)} \qquad (27)$$

Thus the ratio of the solution rates at 1 and 2 is the ratio of the sines of the angles into which the propagation direction divides the crest angle. The direction of propagation of a stable flute pattern is determined entirely by the ratio of solution rates at the points where the maximum and minimum occur. If the ratio is small, the direction of propagation lies close to ϕ_1, the lee slope.

A portion of equation (27) is plotted in Figure 10 as Ψ_1 versus Ψ for various values of $\underline{v}_1 / \underline{v}_2$. The propagation direction lies in the interval (23)(downstream) if Ψ_1 and Ψ for a stable flute lie to the right of the dotted line. (For a stable flute pattern there is only one correct point in this figure). An average of eleven crest angles from the best developed crests in Figure 3 gives $\Psi = 144 \pm 6^{\circ}$. For v_1 / v_2 less than 0.8 propagation is downstream, and there is but one point of maximum solution rate.

The rate of solution would have to be continuous at the crest if the crest were absolutely sharp (a cusp) with a discontinuity in ϕ at that point. If the sharp crest were equivalent to coincidence of the inflection points 1 and 2, the direction of propagation must bisect the crest angle. This gives a minimum solution rate at the cusp and a maximum somewhere within the concavity and, from usual flute profiles, an upstream propagation. The first consequence contradicts our expectation and therefore stable flute profiles can not have a cusp. A possible exception is when abrasion is also important, in which case there may be a discontinuity of material removal rate across the crest and an attendant discontinuity in slope.

Fig. 4.
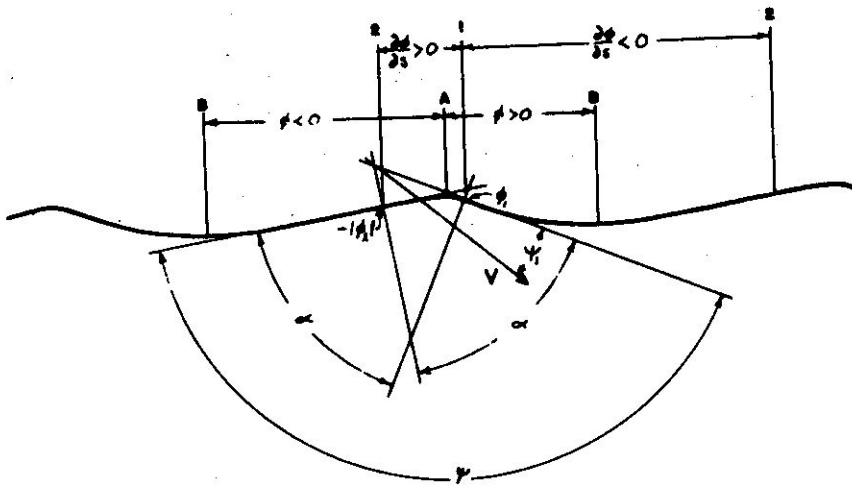
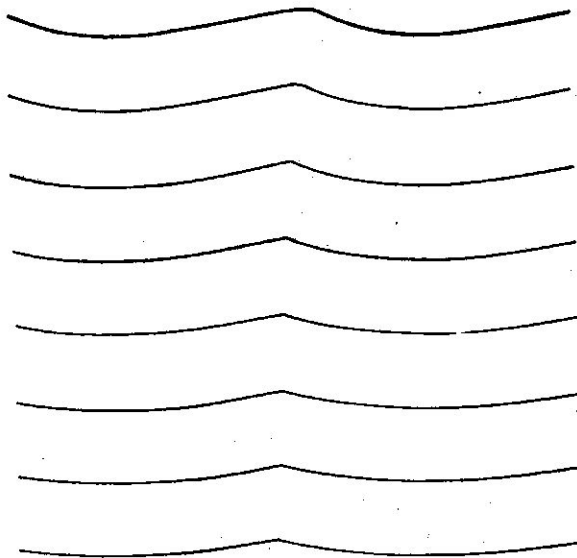Crest and critical point geometry of a stable flute profile.

Fig. 5.

Profile reduction of flutes by uniform solution. The vertical scale, representing the distance the wall has retreated, by solution, from the original position (top), is the true scale.
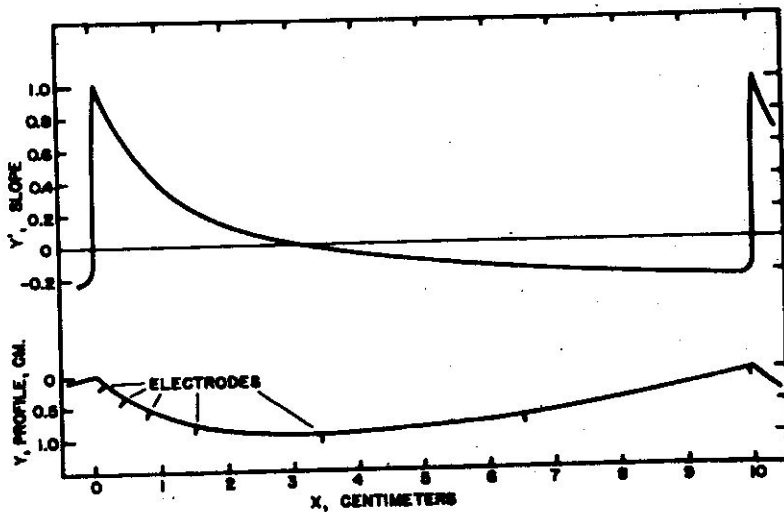

Fig. 6.

Profile and local slope (as the derivative of the profile) of the test profile used in polarographic mass transfer measurements.

Lange (1959) has shown that uniform solution rate over a surface produces, with time, a rounding of all internal (concave) corners and a sharpening of all external (convex) corners. All convex surfaces will eventually form cusps. This is shown in Figure 5 for uniform solution of an initially smooth, stable flute profile.

This is not a contradiction of the previous conclusions. Uniform solution will eventually remove all protuberances and produce a flat surface. Therefore the intermediate cuspate surface is not stable. However such a cuspate surface could be maintained if the solution rate were non-uniform. The required profile of solution rate must have a minimum at the cusps and a maximum within the concavity. Such a solution rate profile cannot be produced by flow parallel to the surface and transverse to the crests. However it could be produced by flow parallel to the crests if the crests are not submerged, or only submerged at intervals. This is the situation in lapies grooving and cusps should not be unexpected in such cases. Cusps on what otherwise appear to be a stable flute pattern must therefore indicate intervals of uniform solution (ponded water), periods of lapies production superimposed on the flute process, or simultaneous removal of surface by abrasion, or possibly modification of the process by sediment deposition.

It may, at first thought, appear strange that the bottoms of the concavities do not represent points of maximum solution rate. This is related to the downstream propagation of the pattern. The streamward slope of the crest dissolves most rapidly to become, with time, the concavity bottom and thereafter, with further solution, to be modified and included in the lee-slope of the preceding crest.

Measurements:

In order to test the above conclusions which were arrived at by deduction and to determine the mass transfer properties of a stable flute profile, measurements were made on a flute pattern. The 25 x 25 cm water tunnel in the Department of Physical Technology, Technische Hogeschool, Eindhoven, The Netherlands, was used for this purpose. The water tunnel installation is shown in Plate 8.

It would be desirable to generate a flute pattern by actual solution of a soluble surface but the available equipment was not suitable for this. Instead, the mass transfer profiles were measured on a fixed flute pattern machined into Perspex. The flute profile chosen was copied from that shown as No. 1 in Figure 3.

This was reproduced in Perspex as shown in <u>Figure 6</u>. After machining, the profile was measured with sufficient accuracy so that its slope profile could be used in the stability test (equation <u>15</u>).

When this profile was adopted it had not yet been recognised that a stable flute pattern cannot have cusps. Nevertheless measurements on it provide useful conclusions even though we might have predicted that it is not a stable profile.

Mass transfer rates on this insoluble profile were determined with the polarographic technique described by Reiss and Hanratty (1962), Jottrand and Grunchard (1962) and earlier workers. It consists of carrying out the cathodic reduction reaction

$$Fe(CN)_6^{-3} + e^- \rightleftharpoons Fe(CN)_6^{-4} \tag{28}$$

on an electrode flush with the surface, under conditions such that the ferricyanide ion concentration at the electrode is zero. The current flowing to the electrode is then a direct measure of the turbulent diffusion rate of ferricyanide ions to the electrode.

Seven 0.5 mm nickel electrodes were mounted flush with the surface at the points indicated in <u>Figure 6</u>. These were polished and cleaned just prior to use. The electrode voltage was set at a level where the current was independent of the voltage but other cathodic reactions did not take place. This is about -1.5 volts with respect to the anode - the stainless steel tunnel sections above the Perspex tunnel. The electrode current was measured with a sensitive recording microammeter. Sources of error included slight imperfections in the electrode surfaces, slight variability in exposed electrode area and polarization and fouling difficulties. Individual measurements are probably within only 15% of the correct value which prevents one from obtaining fine details of the mass transfer profiles.

The solution used was 0.7 N KOH (to minimize transference effects); 0.01 molar potassium ferricyanide and 0.01 molar potassium ferrocyanide. At 20° C., where all experiments were conducted, the solution properties are: $\rho = 1.036$ g/cm^3; $\mu = 1.10$ cp; and $\underline{D} = 0.62 \times 10^{-5}$ cm^2/sec.

Average velocities were determined by pressure drop measurements in the converging venturi section ahead of the tunnel section. Flute Reynolds numbers were computed on the basis of

average tunnel velocity. Sixteen identical flute periods preceeded the measuring period in order to establish a nearly stabilized boundary layer flow. One flute period followed the measuring period.

Average electrode current over the test period was computed from individual measurements and the known profile. These are shown as a function of flute Reynolds number in Figure 7. The ordinate has units of microamperes, an arbitrary scale. The slope of the line shown is 0.50. This is the value usually estimated from mass transfer measurements on spheres, cylinders and in packed beds (Linton and Sutherland, 1960; Thoenes, 1958) where separated turbulent flows occur. Nothing unusual is apparent in the vicinity of the expected stability Reynolds number, $N_f^* = 22,500$.

The measured mass transfer profiles are shown in Figure 8. Within the accuracies of the measurements, they vary only in magnitude, but not form, with N_f. When plotted in the test for profile stability we find that the chosen profile is not a stable profile at any of the test Reynolds numbers (Figure 9). However using a least-squares regression we obtain, from equation 15, $\theta = 71^{\circ} \pm 2^{\circ}$

Also from the measured mass transfer profiles we may estimate the ratio of minimum to maximum v to be about 0.45. This ratio should not be very sensitive to profile details. The region corresponding to the measured average Ψ of flutes ($144^{\circ} \pm 6^{\circ}$) and this value of v_1/v_2 is encircled in Figure 10. We find that hydrodynamic arguments and these estimates of θ and v_1/v_2 from experiments agree to confirm downstream propagation of a flute pattern. Finding the stable profile with the above methods would be a trial and error procedure. From the measurements we may conclude:

1. N_s is proportional to $N_f^{\frac{1}{2}}$ for a fixed pattern.

2. The mass transfer profile is weakly dependent upon N_f.

3. The mass transfer profile is strongly dependent on the flute profile, especially along the lee slope.

A large variation in instantaneous local mass transfer rate was observed in these experiments. At high frequencies this amounted to a maximum of nearly 100% of the average transfer rate. The largest fluctuations occurred between the third and sixth electrodes (counted from the lee side of the crest) with considerably smaller fluctuations just on the two sides of the crest. This turbulence in mass transfer

rate is of course a consequence of the turbulence in the flow.

The weak dependence of solution rate profile on flow rate and the strong dependence on flute profile means that flutes will form much more rapidly than they will adjust to a regular (stable) pattern. Consequently patterns intermediate between scallops and stable flutes may occur, even at constant flow rate, during the development of the stable pattern. A stable flute pattern should be a quite rare occurrence in nature.

These measurements also demonstrate that it is not possible to determine N_f^* by this technique. Not only do we not know the stable profile in advance but the weak dependence of solution rate profile on N_f demands extreme accuracy in determining the transfer rate profile in order to distinguish stability from instability.

Shaw and Hanratty (1964) have recently demonstrated that polarographic transfer measurements with extended electrode surfaces yield the same result as with point (wire end) electrodes. This would be a consequence of the mass transfer boundary layer being orders of magnitude smaller than the single electrode size. We may estimate the mass transfer coefficient in fluted conduits from the water tunnel measurements.

The ferricyanide ion flux to the electrodes is given in gram-moles/cm^2 sec by

$$n = \frac{i}{FA} \tag{29}$$

where i is the measured electrode current, A the electrode area and F, Faraday's number. In a run at $N_f = 22,500$ (Series I) an average (over all electrodes) current of 6.5×10^{-6} amperes was measured. The electrode area is 1.96×10^{-3} cm^2. Therefore the flux n was 3.4×10^{-8} g-mol/cm^2 sec.

Defining a transfer coefficient k by

$$n = k(c - c_o) \tag{30}$$

where c and c_o are the ferricyanide molar concentrations in the bulk flow and at the electrode surface respectively, and since c_o is zero under the measuring conditions while c was 0.0101 molar (10^{-5} mol/cm^3), we find $k = n/c = 3.4 \times 10^{-3}$ cm/sec.
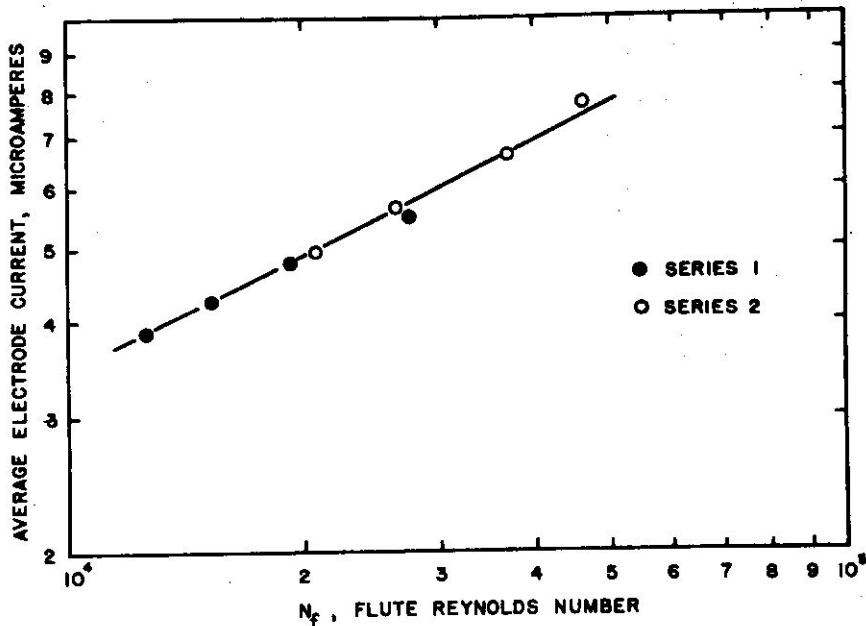
Fig. 7.

Mean measured electrode currents as a function of flute Reynolds number. Series I data have been corrected to the electrolyte composition of series 2.
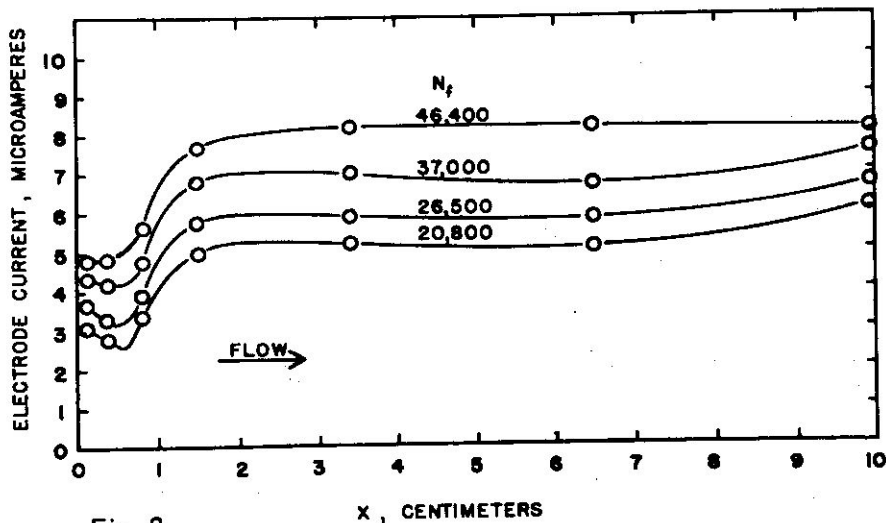
Fig. 8.

Measured electrode current profiles on perspex model. The lines drawn connect related points and may not represent correct values between electrode locations.
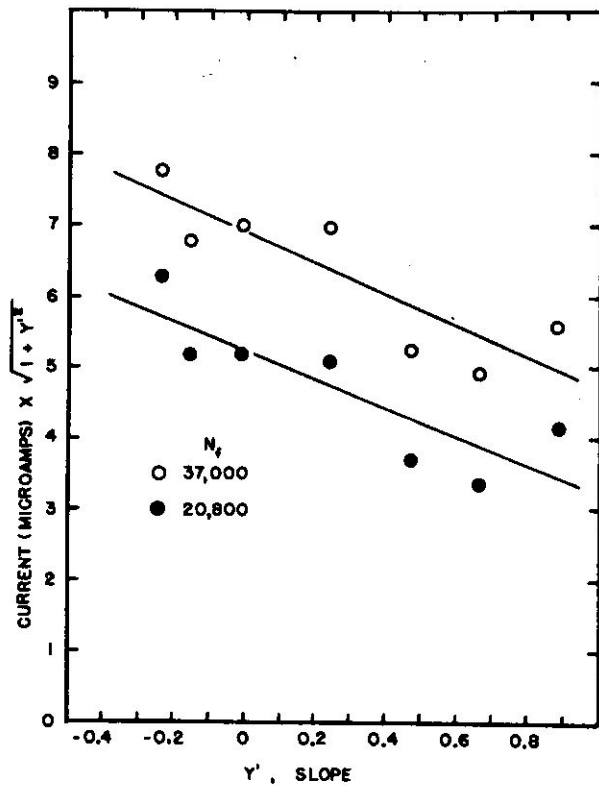
Fig. 9.

Test for profile stability based on equation 15.
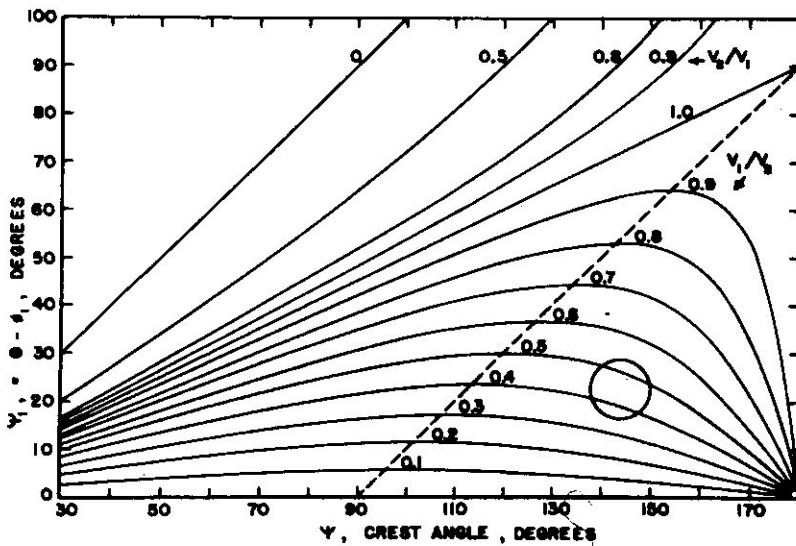
Fig. 10.

Partial plot of equation 27, relating crest angle, propagation angle relative to the maximum lee slope and ratios of critical point solution rates. The circle indicates the region, determined from measured crest angles and solution rate ratios, where it is believed the point representing stable flute geometry lies.

The Sherwood number is defined as (based on flute period)

$$N_s = \frac{kL}{D} \qquad (31)$$

The solute diffusivity $\underline{D}$ may be estimated for the water tunnel experiments from the value given by Shaw, Reiss and Hanratty (1963) by means of the Stokes-Einstein relation. This gives $\underline{D} = 0.62 \times 10^{-5}$ cm^2/Sec. Since the test flute period was 10 cm., $N_s^* = 5550$ (this is an estimate of the stable flute Sherwood number as the experiment was performed at the estimated stable flute Reynolds number.)

From equation (9), the measured dependence of N_s on $N_f^{\frac{1}{2}}$, and the Schmidt number dependence we would expect for mass transfer through a boundary layer on a fixed surface (Bird, Stewart and Lightfoot, 1960), we may write a correlation for the Sherwood number in the form

$$N_s = c_1 N_f^{\frac{1}{2}} \quad Sc^{1/3} \qquad (32)$$

where c_1 is a constant and $Sc = \mu / \rho D$. The constant may be evaluated from N_s^*, N_f^* and $Sc = 1710$ for the ferricyanide solution. The result is $c_1 = 3.07$.

This relation applies to the transfer coefficient over stable flute profiles of constant period with varying stream velocity. It is interesting to compare it with the equivalent expression for flow in packed beds with (say) 50% void space. With Reynolds and Sherwood numbers based on particle diameter and the average interstitial velocity the correlation of Thoenes and Kramers (1958) becomes

$$N_s = 1.0 N_p^{\frac{1}{2}} \quad Sc^{1/3} \qquad (33)$$

We conclude that the fundamental processes are similar for varying flow rate in a fixed geometry.

Flutes, however, tend to adjust their period so that N_f^* is equal to N_f. Hence in an equilibrated fluted conduit equation (32) becomes

$$N_s^* = 465 \, Sc^{1/3} \qquad (34)$$

In an approximately circular conduit of diameter $\underline{d}$, the Reynolds and Sherwood numbers are defined as

$$Sh = \frac{kd}{D} \tag{35}$$

and

$$Re = \frac{Ud\rho}{\mu f} \tag{36}$$

from which it follows that

$$Sh = \frac{N_s}{N_f} \; Re \tag{37}$$

Therefore, in an equilibrated conduit, from equation (34) and $N_f^* = 22,500$

$$Sh = 0.0207 \; Re \; Sc^{1/3} \tag{38}$$

The Chilton-Colburn mass transfer analogy to the Colburn heat transfer correlation (Bird, Stewart and Lightfoot, 1960) for flow in a smooth circular pipe is

$$Sh = 0.023 \; Re^{0.8} \; Sc^{1/3} \tag{39}$$

Equation (38) gives a considerably larger transfer coefficient at high conduit Reynolds numbers than does (39) for smooth pipes, as would be expected in a "rough" tube.

Either relation (31)= or (38) may be used to estimate transfer coefficients in fluted conduits depending on whether it is believed the flow and flutes are equilibrated (38) or not (32). We cannot at present give similar relations for scalloped conduits. Of course the appropriate physical constants (especially D) must be used.

On the Absence of Scallops or Flutes:

Having concluded that solution of a soluble wall is always structurally unstable and scallops or flutes will develop, we must account for the quite common absence of flow markings in cave channels. This discussion must be partly conjectural as we only have direct information about the presence and behavior of flow markings. However a number of possible causes of absence of apparent flow markings may be suggested.

1.   If irregularities in cave channel surfaces, arising from other causes, are of a size smaller than the nominal period of the flute or

scallop pattern which should develop, flow markings will be obscured. Factors such as variable rock properties, close fracturing and sediment imposed irregularities are among possible causes of inherent channel irregularity.

2.   When the flow velocity is such that the expected scale of flow markings is of the same order of magnitude as the channel size, the mechanism of flow marking may alter ($L \sim H$). Furthermore, such large scallops or flutes tend toward being indistinguishable from gross channel irregularities, turns, etc. We may make a rough estimate of the flow rate below which flow markings will apparently be absent by assuming the limit is in the vicinity of $L = H$, defining H as the diameter of a circle having the same area as the channel cross section. The volumetric flow through the channel is then

$$Q = \frac{\pi \, UH^2}{4} \tag{40}$$

Substituting for $\underline{U}$ from equation ($\underline{3}$) and letting $L = H$, we obtain

$$Q = \frac{\pi \nu \, N_f^* H}{4 \rho} \tag{41}$$

which for water at $10\,^{\circ}C$ and $N_f^* = 22{,}500$, becomes (with Q in liters/sec., and H in meters)

$$Q = 23.2H \tag{42}$$

Alternatively, if we substitute $\underline{H}$ for $\underline{L}$ in Figure 1, we obtain a plot for the approximate limiting velocity, below which flow markings will not be apparent.

We see that for a reasonable traversable channel ($H \sim 1$ meter) the minimum flow rate is quite significant (3 cm/sec, equivalent to 23.2 liters/sec or 368 gallons (U.S.A.)/min). For a larger channel the flow required to produce observable flow markings is proportionately larger in volume (though smaller in velocity). Therefore we would not expect to observe flutes or scallops in channels enlarged by slow circulation in sub-water table channels. In fact, meandering of such channels may be, in part, an expression of such large scale "flow markings".

On the basis of the assumption that flow markings will not be evident when their period approaches the conduit size the smallest possible conduit Reynolds number for the existence of periodic flow markings is about the same as the stable flute Reynolds number. However the range of conduit Reynolds number from 2000 to 22,500

also represents turbulent flow. Since flow in this range will not produce observable flutes we see that while flow markings are concurrent with turbulent flow, turbulent flow may not produce observable markings.

3.   Stability, statistical or periodic, of flow markings requires that the rate of removal of material at the crest, v_c, be equal to that at the bottom of the concavity, v_b. If for any reason $v_c < v_0$ the concavity will deepen, while if $v_0 < v_c$, the relief of the surface will decrease.

If sediment deposits on the concavity bottom but not on the crest, the pattern will decrease in relief, conceivably as far as planation of the surface. Even with sediment over the whole surface it would probably be thinner at the crests, giving a higher solution rate there. Therefore flow markings should not develop beneath a sediment layer and pre-existing patterns should be slowly removed.

It has been suggested that abrasion by suspended sediments may cause true cusps to form due to a discontinuity in material removal rate at the crest. A stable pattern could result so long as v remains only a function of slope, ϕ. However if v becomes a function of the curvature of the surface also, the situation changes. This may occur when rapid streams with heavy sediment loads remove material by chipping and gouging of the surface. The crests, being mechanically weaker, would be more rapidly eroded causing v_c to become greater than v_0. Therefore we expect strong mechanical erosion to remove flow markings.

Weathering of a scalloped or fluted surface will also remove the patterns. Weathering agents which penetrate the surface and weaken the rock structure will penetrate furthest at the protuberent crests. Subsequent corrosion or erosion will than be facilitated at the crests as opposed to the concavity bottoms, again making v_c greater than v_0.

4.   Uniform solution of a patterned surface also removes the pattern (Figure 5). This may occur during periods when the velocity is very small. It might be thought of as a preliminary step toward case 2 above.

5.   Flaking, exfoliation and similar processes will remove developing or pre-existing flute or scallop patterns.

Other Forms of Scallops:

A variety of other mechanisms may cause scallop-like concavities on cave surfaces. These have not been much investigated but a few causes may be surmised.

If a limestone contains chert nodules or similar inclusions, which may be weathered away or mechanically removed faster than the retreat of the limestone walls, depressions will be produced. At low flow velocities, these may expand and coalesce to give sharp crested but shallow ceiling or wall concavities. These would be a member of the class of modified irregularities caused by near uniform solution of initially larger depressions (Lange, 1959).

Ceiling concavities might also develop at low flow rates, under totally submerged conditions, by a natural convection mechanism. Any initial ceiling concavity may be the site for a natural convection "cell" in which the slightly denser solution at the surface, caused by limestone solution, flows away from the surface at the crests and is replaced by fresh solvent at the center. The higher solute concentration and lower velocity along the perimeter of the concavity would cause a lower solution rate there than in the center, and the concavity could deepen. The extent to which this occurs in caves is not known nor are we able as yet to ascribe ceiling "pockets" to this, or another mechanism, with assurance.

It is the author's opinion that it is unwise to ascribe large flute and scallop forms to the same mechanism responsible for shorter period patterns unless other features of true flutes and scallops - periodicity, asymmetry rounded crests, etc. - are also present and consistent with the likely channel flow conditions.

Conclusions:

The conclusions reached in the course of this study are summarized here:

1.   Scallops develop as a consequence of the interaction of the flow of a solvent with a soluble surface, irregularities being amplified by the induced flow pattern at the surface.

2.   Regular period scallops with parallel crests, called flutes, are a special case produced by constant flow conditions.

3.   All flutes generated by solution alone are similar, propagate into the wall and downstream at the same angle (about 71° from the horizontal) and have rounded crests.

4.   The ratio of minimum to maximum solution rate on a stable flute pattern is about one half, while all points of equal slope have equal solution rates.

5.   The stable flute Reynolds number, based on flute period, flow velocity and the fluid properties, is a universal constant, as are also the Sherwood number and friction factor over stable flute patterns. As a consequence small flutes (and scallops) tend to dominate due to their higher rate of solution.

6.   A proposed value of the stable flute Reynolds number is $N_f^* = 22,500$.

7.   The flute generating flow is always turbulent as a consequence of the flow conditions apparently required for flute stability, although turbulent flow characteristics are not directly related to flute form.

8.   Variable flow conditions, sediment deposition and simultaneous abrasion may all modify stable solution flute patterns to produce modified flute patterns or scallop patterns.

9.   The interpretation of scallop patterns in terms of the determining flow conditions is more complicated than for flute patterns due to the many unknown additional significant factors. At present it may only be suggested that mean scallop pattern period is roughly related to some mean flow conditions in a manner similar to flute relations.

10.   Flutes and scallops will be absent or obscured in channels carrying flows giving stable mean periods larger than channel dimensions; if sediment deposition is extensive; if abrasion is an important factor; if inherent channel irregularities are smaller than the expected flute or mean scallop periods; or if penetrant weathering or surface flaking are important processes.

        Flutes are the most useful form of flow marking for the interpretation of past flow conditions. However this limits us to cases where flow was constant, an unusual and rare circumstance. In addition we may only obtain useful observations from channels which carried relatively large flows. The utility of flutes and scallops in determining previous rapid flow directions is important, but a large class of possible previous flow conditions either produce no apparent flow markings or their quantitative interpretation is obscured by irregularities in the patterns.

        It is possible that with sufficient study of scallop patterns their statistical properties may be interpreted in terms of the responsible flow conditions, but as we have now no conceptions of the relations between varying flow and scallop pattern statistics it appears that an initial experimental program is necessary.

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SCALLOPS AND FLUTES

ERRATA

(Trans. Cave Res. Grp., $\underline{7}$ (2) 121-160 (1966))

p. 139, line 1:   "equals" sign missing

, eqn. $\underline{16}$: $\emptyset_1 \leqq \theta \leqq \pi + \emptyset_2$

p. 140, line 1:   .... and therefore $\emptyset_1 < \theta$.

, line 2: $\emptyset_2 = -|\emptyset_2|$ in ..... and hence $\theta \leqq \pi - |\emptyset_2|$.

p. 141, eqn. $\underline{24}$: $\emptyset_1 + \frac{1}{2}\pi < \theta \leqq \pi - |\emptyset_2|$

p. 142, eqn. $\underline{25}$:

$$\frac{V_1}{V_2} = \frac{\sin(\theta - \emptyset_1)}{\sin(\theta + |\emptyset_2|)}$$

p. 143, fig.:   "α" designates the ranges of θ given by eqns. (23) and (24)

p. 146, line 4 $\underline{up}$: $D = 0.62 \times 10^{-5}$ cm^2/sec

p. 151, line 10:   .... except for mass transfer $\underline{through}$ a ....

p. 152, line 15:   Either relation $(\underline{32})$ or $(\underline{38})$ may ....

, line 17:   .... equilibrated $(\underline{38})$ or not ....

p. 148:   I have misinterpreted the work of Shaw and Hanratty (1964). Isolated point electrodes will give $\underline{high}$ values for transfer coefficients compared to the correct value. Consequently the coefficients in eqns. (32), (34) and (38), as well as the "stable" Sherwood number, are probably two to five times too large (estimated). The functional dependencies, however, remain the same.

The dependence of transfer rate on $N_c^{1/2}$ (fixed profile) implies that mass transfer is controlled by a $\underline{laminar}$ $\underline{boundary\ layer}$. It would appear that the separated flow from a crest reattaches within the profile, initiating essentially laminar boundary layers downstream and upstream (in the lee vortex) from that point.

R. L. Curl

NOTE: March 2009

In this paper the characteristic flute Reynolds Number was based on the mean channel velocity. Blumberg and Curl (1974) instead based it upon the *friction velocity* Reynolds Number, which is a constant independent of the channel size or configuration. The channel Reynolds Number was then related to this by means of the *universal law of the wall* for turbulent flow. A consequence is that transport properties over equilibrated flutes and scallops vary with the channel Reynolds Number.

Application of this for estimating paleo channel flow from flute or scallop sizes was represented by Curl (1974).

Paul N. Blumberg and Rane L. Curl, "Experimental and theoretical studies of dissolution roughness", Journal of Fluid Mechanics, Vol. 65, Part 4, October 1974, pp 735-751.

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