Deducing Flow Velocity in Cave Conduits from Scallops

Rane L. Curl**

ABSTRACT

Flowing water in caves frequently forms dissolution patterns, called scallops, on limestone surfaces. It has long been known that scallops may be used to indicate past flow direction. More recently, it has been learned that information about flow velocity may also be obtained from them.

The basic hydrodynamic phenomena that control the characteristic dimensions of scallops have been deduced from experiments in their generation on soluble surfaces and are summarized here. Relations are developed for estimating the average flow rate in conduits, given certain dimensional information about scallops and about the conduit.

INTRODUCTION

In Fig. 1 is shown the form of flow markings, or scalloping, that develops in limestone caves and in caves in ice. Flow markings have been the subject of a number of recent studies (Curl, 1966; Allen, 1971; Goodchild and Ford, 1971; Blumberg, 1970; Blumberg and Curl, 1974) that have considered them from geological and hydrodynamic viewpoints. Because almost nothing about this cave phenomenon has appeared in the American speleological literature, the purpose of this paper is to review some recent theoretical and experimental findings and to extend them to practical use in deducing "paleo-hydrologic" conditions in cave systems.

THE SCALLOPING PROCESS

The basic setting for the production of scallops is the turbulent flow of a solvent over a soluble surface. In nature, this occurs most frequently with water dissolving limestone or with air "dissolving" ice (evaporation being completely analogous to the dissolution process). In either case, surface irregularity may lead to the flow situation shown in Fig. 2, in which Blumberg (1970) has observed the following features: At the crest of an irregularity (Point 1), the main flow separates, that is, it forms a "jet" above a region of slower, recirculating flow. Within a short distance, this jet flow becomes strongly irregular and itself becomes turbulent (Point 2). Because the turbulence thereby produced causes mixing between the fluid in the lee eddy (Point 3) and the jet, fluid is entrained out of the lee eddy, causing the jet to turn toward the surface and reattach at Point 4. Some of the fluid then enters the lee eddy region and the rest flows onward along the surface.

In the vicinity of reattachment (Point 4), where the turbulent jet flow impinges most directly upon the surface, the rate of solution (or evaporation) is the highest. One consequence of this is that the scallop pattern moves downstream as it is dissolved further into the wall. This has been observed in all experimental simulations of scallop development. The characteristic asymmetry of scallop profiles, from which the direction of flow may be deduced, is also indicated in Fig. 2.

---

** Department of Chemical Engineering, The University of Michigan, Ann Arbor, Mich. 48104.

THE NSS BULLETIN, 1974, 36(2):1-5
The hydrodynamic processes in the vicinity of scallops are also responsible for the well-known inverse relationship between the size of scallops and the velocity of the flowing fluid (water or air) that produced them. This inverse relationship is produced by phenomena associated with the free laminar shear layer (between Points (1) and (2)) that separates the outer rapid turbulent flow from the slow, recirculating flow in the lee eddy (Point 3). It has been found that a free laminar shear layer undergoes transition to turbulence in a distance $d$ that is determined by the density, $\rho$, viscosity $\mu$, velocity $U$ of the jet, and by the level of initial turbulence in the jet. This implies (see Blumberg and Curl, 1974) that there is a characteristic Reynolds Number for transition, $Re_t = \rho Ud/\mu$ (See Table 1), which should depend only on the nature of the outer turbulent flow. Experiments with laminar jets producing a free laminar shear layer have given a value $Re_t = 30,000$. This value should be smaller when scalloping occurs and there is a turbulent outer flow.

The characteristic scaling of scallop size with the reciprocal of velocity is a consequence of the above phenomenon. If, for example, the scallop is too small (or the velocity too low for that scallop size), transition to turbulence (Point 2) will occur further along the scallop and reattachment will impinge on the next crest. The higher solution rate at that point will reduce that crest and, in effect, lengthen the scallop. On the other hand, if the scallop is too large (for the velocity too high for that scallop size), transition and reattachment will occur sooner. In this case, the distance between the reattachment (Point 4) and the next crest will be increased and an irregularity in this region could be the origin of a new scallop, thereby reducing the average scallop size.

There are two important consequences of this mechanism. First, because the characteristic scaling of scallop size is the result of a purely hydrodynamic mechanism, we do not expect the molecular diffusivity of the dissolving material to play an important role. Second, the scaling mechanism acts longitudinally (in the flow direction) and, therefore, scallop dimensions in that direction most directly reflect the scaling mechanism, as contrasted with scallop depth or width that are the consequence of secondary flow mechanisms. These aspects will be discussed in a more quantitative form in the following sections.

Various other features of scallop development and hydrodynamics, such as the rate of solution, the direction of propagation of the pattern (downstream, at about 60° into the wall), the wall friction, and the profile of individual depressions, are treated in detail in Blumberg and Curl (1974). Their experimentally developed scalloping is shown in Fig. 3. For our present purposes, we need only to review the nature of turbulent flow in the vicinity of the rough wall and the interaction of this with the roughness caused by dissolution of the surface.

**Turbulent Flow Near a Rough Wall**

Experiments on flow through artificially roughened conduits have shown that a moderately good approximation to the average velocity profile near such a rough wall is given by Prandtl's "universal velocity distribution law"

$$u/\nu^* = 2.5 \ln \frac{y}{L} + B_L$$

(Schlichting, 1968), where $u$ is the average flow velocity at distance $y$ from the wall, $L$ is some characteristic dimension of the roughness, and $\nu^*$ is the friction velocity $\sqrt{\tau/\rho}$ where $\tau$ is the average shear stress at the wall and $\rho$ the fluid density. The "roughness" constant $B_L$ depends only on the nature (geometry) of the wall roughness.

Still following Prandtl (as presented by Schlichting, 1968), we assume that Equation (1) applies everywhere.
in a rough conduit and that we therefore may average \( u \) over the cross section of a conduit by appropriately integrating Equation (1) from the wall (\( y=0 \)) to the center (\( y=D/2 \)). \( D \) is the diameter of a circular conduit, or the width between two parallel walls. The result, for the average velocity \( \bar{u} \), is

\[
\bar{u} = u^* \left[ 2.5 \left( \ln \frac{D}{2L} - 3/2 \right) + B \frac{BL}{D} \right]
\]

(2)

for the circular conduit, and

\[
\bar{u} = u^* \left[ 2.5 \left( \ln \frac{D}{2L} - 1 \right) + B \frac{BL}{D} \right]
\]

(3)

for the parallel walls. It remains to relate \( u^* \), \( L \), and \( B_L \) to the scalloping phenomenon.

**Characteristic Scallop Size**

The following section follows Blumberg and Curl (1974):

Imagine that, in a soluble conduit, we impose a fixed pressure drop or, more particularly, an average wall shear stress \( \tau \). This is equivalent to imposing a value of the friction velocity \( u^* \), given the fluid (water or \( \omega \)) with which we are dealing. It is the nature of turbulent flow near a wall that the velocity profile depends primarily upon the wall roughness and shear stress. That is, the flow near the wall is not “aware” of the conduit size except as it affects \( \tau \).

As the walls dissolve, scalloping will develop with some characteristic dimension \( L \). This characteristic dimension will depend upon \( u^* \) and the fluid properties (density \( \rho \) and viscosity \( \mu \)) and, possibly, upon the molecular diffusivity \( D \) of the solute (calcium bicarbonate or water vapor). We may express the dependence by writing

\[
L = f \left( u^*, \rho, \mu, D \right)
\]

(4)

Nondimensionalizing this general statement, we conclude that

\[
Re^* = \frac{Lo^* \rho}{\mu} = f \left( \frac{\rho D}{\mu} \right)
\]

(5)

that is, that the Reynolds number based on the friction velocity and the scallop size \( Re^* \), depends, at most, on the Schmidt number \( Sc = \rho D / \mu \).

Observations of the phenomenon in nature, the results of experiments, and the earlier comment on the role of molecular diffusivity all suggest that the dependence of \( Re^* \) on \( Sc \) is very weak (see also Wigley [1972]). If it is negligible, \( Re^* \) must be a universal constant.

In the above, it was not presumed which dimension \( (L) \) of scalloping was being considered. It may have been an average scallop length, or width, or depth, or any other composite dimension. Since all must scale according to Equation (5), ideal scalloping must also have a “universal shape” (albeit of the nature of a random-pattern), varying only in size with changing conditions. As turbulent velocity profiles have been found to be similar over similar roughness, we deduce that \( B_L \) is also a universal constant for scalloping.

The choice of a characteristic dimension \( L \) for a scallop pattern is rather arbitrary. Goodchild and Ford (1971) used the number-mean maximum length of each depression. They also provided evidence, however, that the average size of “scallops” depends to some extent on the material being dissolved—air bubbles (in plaster) and insoluble inclusions (such as fossils in limestone) creating the conditions for the development of smaller scalloping. In addition, scalloped surfaces exhibit a number of small depressions that appear to be related to the intersections of the rims of the depressions. Consequently, a better method of deriving the average size would suppress the importance of the smaller features, especially if comparison should be made with the regular, periodic, two-dimensional flow markings (flutes) that sometimes appear.

We will choose here the definition

\[
T_{32} = \frac{\sum i l_i^3}{\sum i l_i^2}
\]

(6)

where \( l_i \) is the largest *longitudinal* (parallel to the flow) dimension of the \( i \)th scallop. (This average is called a “Sauter-mean”.)

Using this definition for \( L \) in Equations (1) through (5), Blumberg and Curl (1974) found from their experiments \( Re^* = 2200 \) and \( B_L = 9.4 \). The product of these is (from Equations [1] and [5]) \( Re = \rho u \left( T_{32} \right) L_{32}/\mu = 21,000 \).

This is a Reynolds number based on the fluid velocity at a distance from the wall equal to the chosen characteristic dimension \( L = T_{32} \).

**Scallop—Conduit Reynolds Number**

The Reynolds number based on mean scallop size and average fluid velocity in a conduit is

\[
Re = \frac{\rho \bar{u}}{\mu} \frac{T_{32}}{L}
\]

(7)

By multiplying Equations (2) and (3) by \( \rho L^2 / \mu \) and using \( L = T_{32} \), we obtain

\[
Re = Re^* \left[ 2.5 \left( \ln \frac{D}{2T_{32}} - 3/2 \right) + B \frac{BL}{D} \right]
\]

(8)

for the circular conduit and

\[
Re = Re^* \left[ 2.5 \left( \ln \frac{D}{2T_{32}} - 1 \right) + B \frac{BL}{D} \right]
\]

(9)

for the case of parallel walls.

If \( Re^* \) and \( B_L \) are known, and if \( D \) and \( T_{32} \) are measured for the particular situation, it is possible to calculate \( Re = Re \).

Then, the average fluid velocity under which the scallops were developed may be found if values for \( \mu / \rho \) are known or can be guessed (for water at 10°C, \( \mu / \rho = 0.013 \text{ cm}^2 / \text{sec} \); for air at 0°C, \( \mu / \rho = 0.132 \text{ cm}^2 / \text{sec} \)).

Using the values for \( Re^* \) and \( B_L \) given earlier, Equations (8) and (9) are plotted in Fig. 4.

Fig. 4 shows that scalloping of a given size in a large conduit represents a larger mean flow velocity than it would in a small conduit. This is to be expected, once we accept the inverse relationship between scallop size and some near-wall velocity (\( u \left[ T_{32} \right] \), say), as this velocity will be lower in the larger conduit for the same mean conduit velocity.

The accuracy of Fig. 4 for estimating prior flow conditions depends on several factors. It is useful to enumerate them here.
5. The dissolution process should be dominated by diffusional mass-transfer, not by a chemical rate-limiting step at the surface. For pure calcite, it has been shown (Curl, 1968) that at low dissolution rates the process is controlled by the diffusional transfer of Ca++ (plus HCO$_3^-$) between the surface and the bulk solution, while at high transfer rates it is controlled by the diffusional transfer of H$_2$CO$_3$ (not CO$_3^-$) to the surface. There was predicted to be very little effect of solvent motion on the rate of dissolution in an intermediate calcite dissolution regime. Is scalloping in this regime?

The intermediate regime may be defined approximately (and non-dimensionally) by

$$1 < \frac{1}{h} \sqrt{D k_2} < 100$$

where $h$ is the mass transfer coefficient for H$_2$CO$_3$, and $k_2$ the rate constant for the homogeneous reaction step H$_2$CO$_3$ $\rightarrow$ CO$_3^-$ + H$_2$O. In addition, the mass transfer coefficient on a scalloped surface (measured by Blumberg [1970] and reported by Blumberg and Curl [1974]) is

$$\frac{hL_{32}}{D} = 112 \text{Sc}^{1/3}$$

Eliminating $h$ between Equations (10) and (11), we obtain

$$112 \text{Sc}^{1/3} \sqrt{\frac{D}{k_2}} < \frac{L_{32}}{k_2} < 1.12 \times 10^4 \text{Sc}^{1/3} \sqrt{\frac{D}{k_2}}$$

At $10^\circ$C, $k_2 = 3.45 \text{ sec}^{-1}$, $D = 1.43 \times 10^{-5} \text{ cm}^2/\text{sec}$ and $\text{Sc} = 914$ (Curl, 1968). These give $2.2 < L_{32} < 220 \text{ cm}$. This range includes most natural occurrences of scallops on limestone.

It appears that, in turbulent (rapidly fluctuating) flows over microscopically rough surfaces, a flow velocity effect on the calcite dissolution rate is not fully suppressed. This problem has not been studied, but something can be said about its effect on scallop dimensions and geometry.

It was found by Blumberg (1970) that, although flutes of a dimension not matching (in terms of $Re_*$) the adjacent flow velocity still retained their $shape$, the direction of propagation was changed. As the velocity was doubled, the downstream propagation angle increased from $60^\circ$ to $75^\circ$ into the surface. The local average rate of dissolution remained consistent with the original geometry. It may be demonstrated that, if the actual local rate of dissolution were to vary as, say, $h^{0.5}$, due to kinetic phenomena, rather than being directly proportional to $h$ (as would be the case outside the intermediate calcite dissolution regime, or if the substrate were gypsum), the profile of a flute would show little change, although the angle of propagation would steepen to near $75^\circ$. In addition, the characteristic dimension $L$ (or $L_{32}$ in particular) still should be determined mostly by hydrodynamics and, therefore, be largely independent of the additional kinetic phenomena.

This matter requires further study.

6. A multitude of factors that can modify or obscure scallop patterns have been omitted from the foregoing discussion. These have been discussed in some detail elsewhere (Curl, 1966) and include close jointing or fracturing, a heavy bed load, deposition of clay, and numerous insoluble inclusions in limestone.
ACKNOWLEDGEMENT

Measurements of the dimensions of the scallops in Fig. 3 were taken by Mr. Barry I. Hollander, as part of a senior research project in the Department of Chemical Engineering, University of Michigan.

Table 1. Definitions of Reynolds numbers

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>&quot;Near wall&quot; Friction At y=(L_{sl}) Average length</th>
<th>(U)</th>
<th>(v^*)</th>
<th>(u(\overline{L}_{sl})) conduit, (\overline{u})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance to transition, (d)</td>
<td>(Re_{f})</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Average scallop size, (L_{sl})</td>
<td>—</td>
<td>(Re^*)</td>
<td>(Re)</td>
<td>(\frac{Re}{L})</td>
</tr>
<tr>
<td>Conduit diameter or width, (D)</td>
<td>—</td>
<td>—</td>
<td>(\frac{Re}{L})</td>
<td>—</td>
</tr>
</tbody>
</table>

LITERATURE CITED


Note added in proof:

An additional important general reference on flow markings is:


The evolution of scalloping from an initially flat surface is treated in some detail. It was observed in experiments that the mean dimension of scalloping decreased with time and appeared to be approaching a limit. An estimate of the limiting value of \(Re_L\) (based on \(L_{sl}\)), is 25,000, although Allen did not measure \(u(\overline{L}_{sl})\) or \(v^*\). Of particular importance, however, is the demonstration that the average scallop size is evolutionary between equilibrium conditions and, therefore, the patterns observed in nature may not represent "steady" conditions. The patterns also evolve from defects, which can affect their shapes and average dimensions.


Manuscript received by the Editor 25 October 1973

Revised manuscript accepted 15 November 1973
ERRATA


**ERRATA**

\[
\sum_{i}^{12} L_{32} = \frac{1}{12} \sum_{i}^{12} L_{32} \tag{6}
\]

p. 3, Eqn (6).

7 lines further: \( Re_L = \frac{\bar{u}(L_{32})}{\bar{L}_{32}/\mu} \). 4 lines further: \( \ldots \) to calculate \( \bar{Re}_{L} \).

p. 4, col. 1, line 8, \( \ldots \bar{Re}_{D} = \frac{\bar{u}D}{\mu} \).

p. 4, col. 1, line 8, \( \ldots \bar{Re}_{D} = \frac{\bar{u}D}{\mu} \).

p. 5, col. 1, line 11 up: \( \ldots \) measure \( \bar{u}(L_{32}) \) or \( \ldots \)

col. 1, under Characteristic Velocities (Table 1) Average conduit, \( \bar{u} \)

Literature Cited: