Deducing Flow Velocity in Cave Conduits from Scallops*

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ABSTRACT

Flowing water in caves frequently forms dissolution patterns, called scallops, on limestone surfaces. It has long been known that scallops may be used to indicate past flow direction. More recently, it has been learned that information about flow velocity may also be obtained from them.

The basic hydrodynamic phenomena that control the characteristic dimensions of scallops have been deduced from experiments in their generation on soluble surfaces and are summarized here. Relations are developed for estimating the average flow rate in conduits, given certain dimensional information about scallops and about the conduit.

Introduction

In Fig. 1 is shown the form of flow markings, or scalloping, that develops in limestone caves and in caves in ice. Flow markings have been the subject of a number of recent studies (Curl, 1966; Allen, 1971; Goodchild and Ford, 1971; Blumberg, 1970; Blumberg and Curl, 1974) that have considered them from geological and hydrodynamic viewpoints. Because almost nothing about this cave phenomenon has appeared in the American speleological literature, the purpose of this paper is to review some recent theoretical and experimental findings and to extend them to practical use in deducing "paleo-hydrologic" conditions in cave systems.

The Scalloping Process

The basic setting for the production of scallops is the turbulent flow of a solvent over a soluble surface. In nature, this occurs most frequently with water dissolving limestone or with air "dissolving" ice (exaporation being completely analogous to the dissolution process). In either case, surface irregularity may lead to the flow situation shown in Fig. 2, in which Blumberg (1970) has observed the following features: At the crest of an irregularity (Point 1), the main flow *separates,* that is, it forms a "jet" above a region of slower, recirculating flow. Within a short distance, this jet flow becomes strongly irregular and itself becomes turbulent (Point 2). Because the turbulence thereby produced causes mixing between the fluid in the lee eddy (Point 3) and the jet, fluid is entrained out of the lee eddy, causing the jet to turn toward the surface and *reattach* at Point 4. Some of the fluid then enters the lee eddy region and the rest flows onward along the surface.

In the vicinity of reattachment (Point 4), where the turbulent jet flow impinges most directly upon the surface, the rate of solution (or evaporation) is the highest. One consequence of this is that the scallop pattern moves *downstream* as it is dissolved further into the wall. This has been observed in all experimental simulations of scallop development. The characteristic asymmetry of scallop profiles, from which the direction of flow may be deduced, is also indicated in Fig. 2.

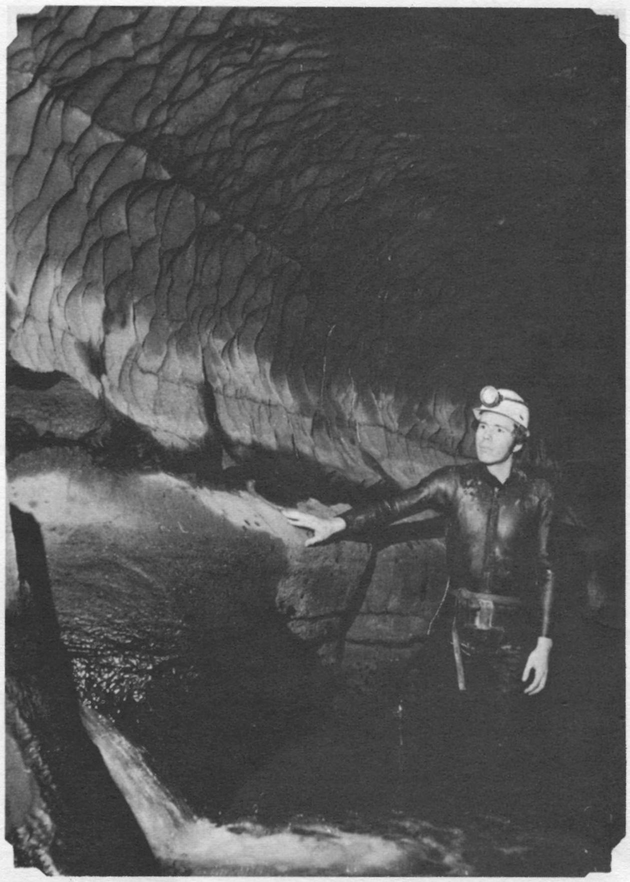

Fig. 1. Scalloping in Little Neath River Cave, South Wales. Photo by P. A. Standing.

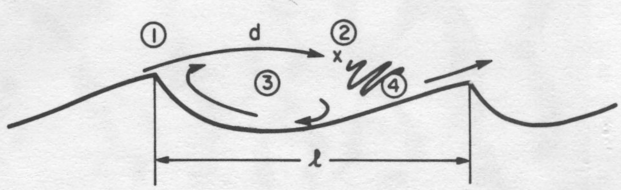

Fig. 2. Fluid motion in the vicinity of a scallop. Point 1: flow separation at crest. Point 2: transition of laminar shear layer to turbulence. Point 3: recirculating flow in lee eddy. Point 4: jet reattachment region.

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The hydrodynamic processes in the vicinity of scallops are also responsible for the well-known inverse relationship between the size of scallops and the velocity of the flowing fluid (water or air) that produced them. This inverse relationship is produced by phenomena associated with the *free laminar shear layer* (between Points (1) and (2)) that separates the outer rapid turbulent flow from the slow, recirculating flow in the lee eddy (Point 3). It has been found that a free laminar shear layer undergoes transition to turbulence in a distance d that is determined by the density, ρ, viscosity μ, velocity U of the jet, and by the level of initial turbulence in the jet. This implies (see Blumberg and Curl, 1974) that there is a characteristic Reynolds Number for transition, $Re_t = \rho U d/\mu$ (See Table 1), which should depend only on the nature of the outer turbulent flow. Experiments with laminar jets producing a free laminar shear layer have given a value $Re_t = 30{,}000$. This value should be smaller when scalloping occurs and there is a turbulent outer flow.

The characteristic scaling of scallop size with the reciprocal of velocity is a consequence of the above phenomenon. If, for example, the scallop is too small (or the velocity too low for that scallop size), transition to turbulence (Point 2) will occur further along the scallop and reattachment will impinge on the next crest. The higher solution rate at that point will reduce that crest and, in effect, lengthen the scallop. On the other hand, if the scallop is too large (or the velocity too high for that scallop size), transition and reattachment will occur sooner. In this case, the distance between the reattachment (Point 4) and the next crest will be increased and an irregularity in this region could be the origin of a new scallop, thereby reducing the average scallop size.

There are two important consequences of this mechanism. First, because the characteristic scaling of scallop size is the result of a purely hydrodynamic mechanism, we do not expect the molecular diffusivity of the dissolving material to play an important role. Second, the scaling mechanism acts *longitudinally* (in the flow direction) and, therefore, scallop dimensions in that direction most directly reflect the scaling mechanism, as contrasted with scallop depth or width that are the consequence of secondary flow mechanisms. These aspects will be discussed in a more quantitative form in the following sections.

Various other features of scallop development and hydrodynamics, such as the rate of solution, the direction of propagation of the pattern (downstream, at about 60° into the wall), the wall friction, and the profile of individual depressions, are treated in detail in Blumberg and Curl (1974). Their experimentally developed scalloping is shown in Fig. 3. For our present purposes, we need only to review the nature of turbulent flow in the vicinity of the rough wall and the interaction of this with the roughness caused by dissolution of the surface.

Turbulent Flow Near a Rough Wall

Experiments on flow through artificially roughened conduits have shown that a moderately good approximation to the average velocity profile near such a rough wall is given by Prandtl's "universal velocity distribution law"

$$u/v^* = 2.5 \ln \frac{y}{L} + B_L \tag{1}$$

(Schlichting, 1968), where u is the average flow velocity at distance y from the wall, L is some characteristic dimension of the roughness, and v^* is the friction velocity $\sqrt{\tau/\rho}$ where τ is the average shear stress at the wall and ρ the fluid density. The "roughness" constant B_L depends only on the nature (geometry) of the wall roughness.

Still following Prandtl (as presented by Schlichting, 1968), we assume that Equation (1) applies everywhere

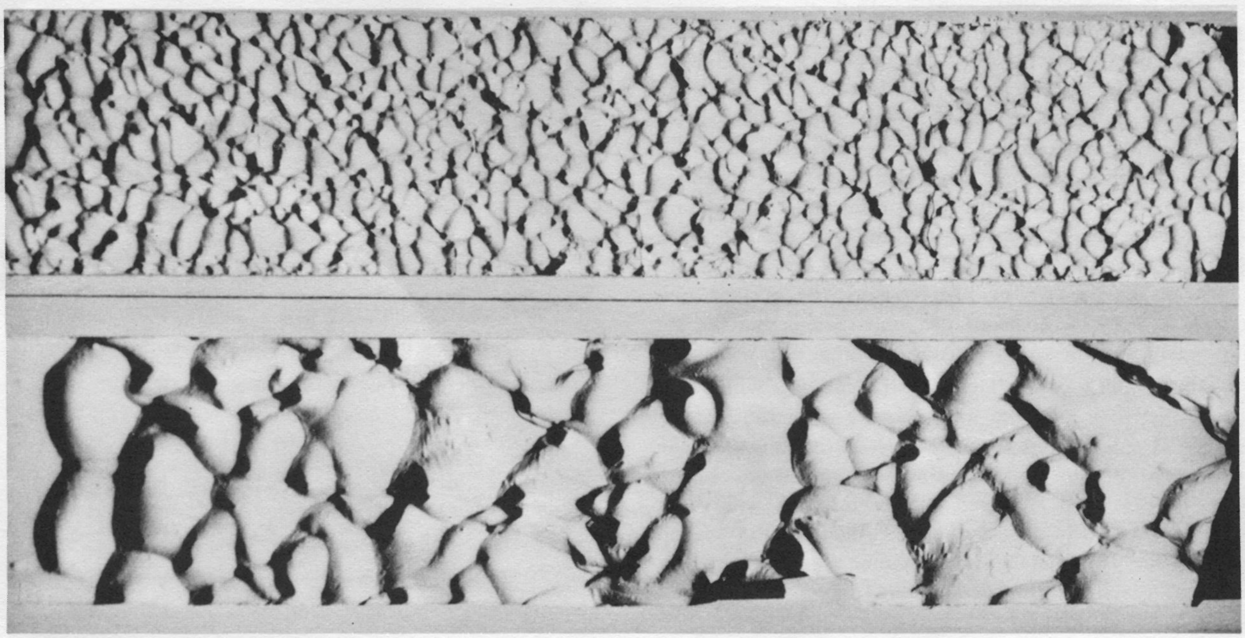

Fig. 3. Artificially produced scallops on plaster of paris, blocks are 76 cm long and 15 cm wide. The larger scallops developed at $u\ (\overline{L}_{32}) = 40$ cm/sec and 16 °C., $\overline{L}_{32} = 6.0$ cm. The smaller scallops developed at $u\ (\overline{L}_{32}) = 90.7$ cm/sec and 33 °C., $\overline{L}_{32} = 1.7$ cm. The values of Re_L are both close to 21,000. Flow (and illumination) was from left to right.

in a rough conduit and that we therefore may average u over the cross section of a conduit by appropriately integrating Equation (1) from the wall ($y=0$) to the center ($y=D/2$). D is the diameter of a circular conduit, or the width between two parallel walls. The result, for the *average velocity u*, is

$$\bar{u} = v^* \left[2.5 \left(\ln \frac{D}{2L} - 3/2\right) + B_L\right] \tag{2}$$

for the circular conduit, and

$$\bar{u} = v^* \left[2.5 \left(\ln \frac{D}{2L} - 1\right) + B_L\right] \tag{3}$$

for the parallel walls. It remains to relate v^*, L, and B_L to the scalloping phenomenon.

Characteristic Scallop Size

The following section follows Blumberg and Curl (1974):

Imagine that, in a soluble conduit, we impose a fixed pressure drop or, more particularly, an average wall shear stress τ. This is equivalent to imposing a value of the friction velocity v^*, given the fluid (water or air) with which we are dealing. It is the nature of turbulent flow near a wall that the velocity profile depends primarily upon the wall roughness and shear stress. That is, the flow near the wall is not "aware" of the conduit size except as it affects τ.

As the walls dissolve, scalloping will develop with some characteristic dimension L. This characteristic dimension will depend upon v^* and the fluid properties (density ρ and viscosity μ) and, possibly, upon the molecular diffusivity $\mathcal{D}$ of the solute (calcium bicarbonate or water vapor). We may express the dependence by writing

$$L = f\,(v^*, \rho, \mu, \mathcal{D}) \tag{4}$$

Nondimensionalizing this general statement, we conclude that

$$Re^* = \frac{L v^* \rho}{\mu} = f\left(\frac{\rho \mathcal{D}}{\mu}\right) \tag{5}$$

that is, that the Reynolds number based on the friction velocity and the scallop size Re^*, depends, at most, on the Schmidt number $Sc = \rho \mathcal{D} / \mu$.

Observations of the phenomenon in nature, the results of experiments, and the earlier comment on the role of molecular diffusivity all suggest that the dependence of Re^* on Sc is very weak (see also Wigley [1972]). If it is negligible, Re^* must be a universal constant.

In the above, it was not presumed which dimension (L) of scalloping was being considered. It may have been an average scallop length, or width, or depth, or any other composite dimension. Since all must scale according to Equation (5), ideal scalloping must also have a "universal shape" (albeit of the nature of a random-pattern), varying only in size with changing conditions. As turbulent velocity profiles have been found to be similar over similar roughness, we deduce that B_L is also a universal constant for scalloping.

The choice of a characteristic dimension L for a scallop pattern is rather arbitrary. Goodchild and Ford (1971) used the number-mean maximum length of each depression. They also provided evidence, however, that the average size of "scallops" depends to some extent on the material being dissolved—air bubbles (in plaster) and insoluble inclusions (such as fossils in limestone) creating the conditions for the development of smaller scalloping. In addition, scalloped surfaces exhibit a number of small depressions that appear to be related to the intersections of the rims of the depressions. Consequently, a better method of deriving the average size would suppress the importance of the smaller features, especially if comparison should be made with the regular, periodic, two-dimensional flow markings (flutes) that sometimes appear.

We will choose here the definition

$$\bar{L}_{32} = \frac{\sum l_i^3}{\sum l_i^2} \tag{6}$$

where l_i is the largest *longitudinal* (parallel to the flow) dimension of the *ith* scallop. (This average is called a "Sauter-mean".)

Using this definition for L in Equations (1) through (5), Blumberg and Curl (1974) found from their experiments $Re^* = 2200$ and $B_L = 9.4$. The product of these is (from Equations [1] and [5] $Re_L = \rho u(\bar{L}_{32})\,\bar{L}_{32}/\mu = 21{,}000$.

This is a Reynolds number based on the fluid velocity at a distance from the wall equal to the chosen characteristic dimension $L = \bar{L}_{32}$.

Scallop—Conduit Reynolds Number

The Reynolds number based on mean scallop size and average fluid velocity in a conduit is

$$\overline{Re}_L = \frac{\rho \bar{u} \bar{L}_{32}}{\mu} \tag{7}$$

By multiplying Equations (2) and (3) by $\rho L/\mu$ and using $L = \bar{L}_{32}$, we obtain

$$\overline{Re}_L = Re^* \left[2.5 \left(\ln \frac{D}{2\bar{L}_{32}} - 3/2\right) + B_L\right] \tag{8}$$

for the circular conduit and

$$\overline{Re}_L = Re^* \left[2.5 \left(\ln \frac{D}{2\bar{L}_{32}} - 1\right) + B_L\right] \tag{9}$$

for the case of parallel walls.

If Re^* and B_L are known, and if D and $\bar{L}_{32}$ are measured for the particular situation, it is possible to calculate $\overline{Re}_L$.

Then, the average fluid velocity under which the scallops were developed may be found if values for μ/ρ are known or can be guessed (for water at 10°C, $\mu/\rho = 0.013$ cm²/sec; for air at 0°C, $\mu/\rho = 0.132$ cm²/sec).

Using the values for Re^* and B_L given earlier, Equations (8) and (9) are plotted in Fig. 4.

Fig. 4 shows that scalloping of a given size in a large conduit represents a larger mean flow velocity than it would in a small conduit. This is to be expected, once we accept the inverse relationship between scallop size and some near-wall velocity ($u[\bar{L}_{32}]$, say), as this velocity will be lower in the larger conduit for the same mean conduit velocity.

The accuracy of Fig. 4 for estimating prior flow conditions depends on several factors. It is useful to enumerate them here.

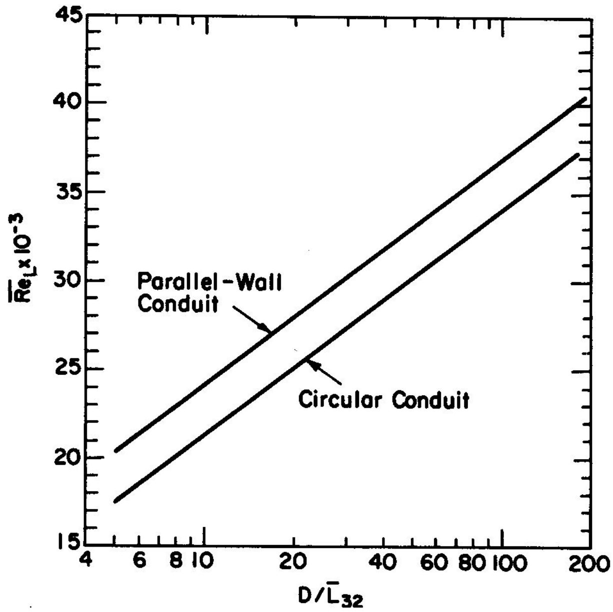

Fig. 4. The predicted relation between the (mean velocity in conduit) x (mean scallop size) Reynolds number ($\overline{Re}_L$) and the ratio of conduit diameter or width to mean scallop size ($D/\overline{L}_{32}$).

1. Equation (1) is an adequate approximation to the turbulent flow velocity profile across a rough conduit. There are alternative velocity profile expressions (see Schlichting, 1968) and some fussing has been done with the constants 2.5 and B_L (for various types of roughness).

Given, however, other, greater, sources of error, Equation (1) should be adequate at high Reynolds numbers based on conduit diameter, $\overline{Re}_{\mathcal{D}} = \rho\bar{u}D/\mu$. (See Table 1)

2. The constants Re^* and B_L that have been used here will be subject to some revision when experiments are conducted in longer conduits than anyone has as yet used. It is estimated, however, that they are now known with sufficient accuracy that Fig. 4 is correct within about ± 15%. The values used here differ considerably from an equivalent estimate made by Goodchild and Ford (1971). The exact reason for this is not yet known.

3. The conduit must be of regular cross section and must be sufficiently long and straight for almost fully developed flow to be established. The cross section need not be either circular or parallel-walled (the effect will be a relation lying between those for the circular and parallel-walled channels in Fig. 4), but it should be unchanging for some distance. In curving conduits, the velocity is greatest near the inner wall at the beginning of the turn and near the outer wall at the end of the turn (there also may be a reverse flow on the inner wall near the end of the turn). Therefore, Equations (2) and (3) are highly approximate in other than regular, straight conduits.

4. The flow must have been at a constant velocity (actually, at constant $\rho\bar{u}/\mu$) throughout the period of final development of the scallop pattern. This is unlikely to be true in any given case but, since scallop patterns develop most rapidly at high velocities, they tend to reflect the past history of the higher velocity flows in a given conduit.

5. The dissolution process should be dominated by diffusional mass-transfer, not by a chemical rate-limiting step at the surface. For pure calcite, it has been shown (Curl, 1968) that at low dissolution rates the process is controlled by the diffusional transfer of Ca^{++} (plus HCO_3^-) between the surface and the bulk solution, while at high transfer rates it is controlled by the diffusional transfer of H_2CO_3 (not CO_2) to the surface. There was predicted to be very little effect of solvent motion on the rate of dissolution in an *intermediate calcite dissolution regime.* Is scalloping in this regime?

The intermediate regime may be defined approximately (and non-dimensionally) by

$$1 < \frac{1}{h}\sqrt{\mathcal{D}k_2} < 100 \qquad (10)$$

where h is the mass transfer coefficient for H_2CO_3, and k_2 the rate constant for the homogeneous reaction step $H_2CO_3 \rightarrow CO_2 + H_2O$. In addition, the mass transfer coefficient on a scalloped surface (measured by Blumberg [1970] and reported by Blumberg and Curl [1974]) is

$$\frac{h\overline{L}_{32}}{\mathcal{D}} = 112\, Sc^{1/3}. \qquad (11)$$

Eliminating h between Equations (10) and (11), we obtain

$$112\, Sc^{1/3}\sqrt{\frac{\mathcal{D}}{k_2}} < \overline{L}_{32} < 1.12 \times 10^4\, Sc^{1/3}\sqrt{\frac{\mathcal{D}}{k_2}}\,. \qquad (12)$$

At 10°C, $k_2 = 3.45$ sec^{-1}, $\mathcal{D} = 1.43 \times 10^{-5}$ cm^2/sec and $Sc = 914$ (Curl, 1968). These give $2.2 < \overline{L}_{32} < 220$ cm. *This range includes most natural occurrences of scallops on limestone.*

It appears that, in turbulent (rapidly fluctuating) flows over microscopically rough surfaces, a flow velocity effect on the calcite dissolution rate is not fully suppressed. This problem has not been studied, but something can be said about its effect on scallop dimensions and geometry.

It was found by Blumberg (1970) that, although flutes of a dimension not matching (in terms of Re^*) the adjacent flow velocity still retained their *shape,* the direction of propagation was changed. As the velocity was doubled, the downstream propagation angle increased from 60° to 75° into the surface. The local average rate of dissolution remained consistent with the original geometry. It may be demonstrated that, if the actual local rate of dissolution were to vary as, say, $h^{0.5}$, due to kinetic phenomena, rather than being directly proportional to h (as would be the case outside the intermediate calcite dissolution regime, or if the substrate were gypsum), *the profile of a flute would show little change,* although the angle of propagation would steepen to near 75°. In addition, the characteristic dimension L (or $\overline{L}_{32}$ in particular) still should be determined mostly by hydrodynamics and, therefore, be largely independent of the additional kinetic phenomena.

This matter requires further study.

6. A multitude of factors that can modify or obscure scallop patterns have been omitted from the foregoing discussion. These have been discussed in some detail elsewhere (Curl, 1966) and include close jointing or fracturing, a heavy bed load, deposition of clay, and numerous insoluble inclusions in limestone.

Acknowledgement

Measurements of the dimensions of the scallops in Fig. 3 were taken by Mr. Barry I. Hollander, as part of a senior research project in the Department of Chemical Engineering, University of Michigan.

Table 1. Definitions of Reynolds numbers

Characteristic length	Characteristic Velocities: "Near wall" U	Friction v^*	At $y=\overline{L}_{32}$ $u(\overline{L}_{32})$	Average conduit, $\overline{u}$
Distance to transition, d	Re_t	—	—	—
Average scallop size, L_{32}	—	Re^*	Re_L	$\overline{Re}_L$
Conduit diameter or width, D	—	—	—	$\overline{Re}_D$

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Wigley, T.M.L. (1972)—Analysis of Scallop Patterns by Simulation under Controlled Conditions: A Discussion: *Jour. Geol.* 80:121-122.

Note added in proof:

An additional important general reference on flow markings is:

Allen, J.R.L. (1971)—Transverse Erosional Marks of Mud and Rock: Their Physical Basis and Geological Significance: *Sedimentary Geol.* 5:165-388.

The evolution of scalloping from an initially flat surface is treated in some detail. It was observed in experiments that the mean dimension of scalloping decreased with time and appeared to be approaching a limit. An estimate of the limiting value of Re_L (based on $\overline{L}_{32}$), is 25,000, although Allen did not measure $u(\overline{L}_{32})$ or v^*. Of particular importance, however, is the demonstration that the average scallop size is evolutionary between equilibrium conditions and, therefore, the patterns observed in nature may not represent "steady" conditions. The patterns also evolve from defects, which can affect their shapes and average dimensions.

A recent analysis of ripples on the underside of ice covers on rivers is given by: Ashton, G.D. and J.F. Kennedy (1972): *Proc. Am. Soc. Civil Eng., Hydraulics Div.* 98 (HY9):1603-1624.

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Curl, R. L. (1974)—Deducing Flow Velocity in Cave Conduits from Scallops: *NSS Bull.* 36(2):1-5.

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p. 3, Eqn (6).

$$\overline{L}_{32} = \frac{\sum_i l_i^3}{\sum_i l_i^2} \qquad (6)$$

7 lines further: . . . $Re_L = \rho\, \overline{u}(\overline{L_{32}})\overline{L}_{32}/\mu = \ldots$

Eqn (8). omit parenthesis after B_L.

4 lines further: . . . to calculate $\overline{Re}_L$.

p. 4, col. 1, line 8, . . . $\overline{Re}_D = \rho\overline{u}D/\mu$.

line 9, . . . constants Re^* and . . .

p. 5, col. 2, line 11 *up*: . . . measure $u(\overline{L_{32}})$ or . . .

col. 1, under Characteristic Velocities (Table 1)

Average
conduit, $\overline{u}$

Literature Cited:

Blumberg, P. N. and R. L. Curl (1974)—Experimental and Theoretical Studies of Dissolution Roughness: *Journal of Fluid Mechanics.* 65:735-ff.