Stochastic Models of Cavern Development

by RANE L. CURL

ABSTRACT—A population of caves evolves from a population of cave precursors consisting of joint systems of different complexity, which are subject to the invasion of solvent water whose source, composition, and availability vary in space and time. Although phenomenological theories have had considerable success in the identification and explanation of the succession of geomorphic processes responsible for cave development, these processes also produce manifestations in a cave population related to processes of a random or stochastic nature.

Stochastic models have been constructed to mathematically reproduce the evolution of a particular population manifestation, namely the distribution of cave lengths. Intuitively "simple" mechanisms for the rate of cave growth and decay have been used for this purpose. The theories provide a quantitative description of the evolution of cave length distributions and, conversely, some attributes of cave precursors which would lead to present-day length distributions. An estimate of the length distribution of all caves more than 100 feet long in West Virginia is used for these comparisons.

The complexity of the evolutionary process of cave populations and the urge to select evolutionary mechanisms which are subjectively simple as well as mathematically tractable are perhaps contradictory; but stochastic-process concepts are essential for a more quantitative understanding of the cavern cycle, and simple models may serve as a point of departure.

INTRODUCTION

A stochastic process is one containing events attributable to chance or randomness. The word stochastic is preferred to the other terms, as it implies the presence of both deterministic and indeterministic aspects to a process. The presence of random elements in geomorphic processes is the rule rather than the exception, though for many descriptive purposes it has not been necessary to give specific consideration to those aspects of land forms which are the product of these random elements. Some examples of the latter are the distribution and amounts of rainfall; variations in rock structure and composition; and location and type of vegetation cover. Geomorphic processes which reflect the influence of random elements are drainage patterns; meander development; stream piracy; and the development of terrain in general. Thornbury (1954, p. 114) writes: "Insequent valleys are those whose courses are controlled by factors which are not determinable. They show no apparent adjustment to structure or initial slopes and seemingly developed where they are by chance. This undoubtedly was not so but the controlling factors escape detection". It appears that insequent valleys are the result of processes in which the determinant features are more obscure than the chance features. Thornbury's reservation on the role of random elements probably arises from a desire to relate specific forms to specific processes but his phrasing might also be applied to another random process as in the toss of dice, for there too "controlling factors escape detection".

When we say that the cause of a particular landscape form is a certain process, we have stated a theory for the evolution of that form. When such a theory is made quantitative using mathematics it is often called a model because the theory then reproduces in its symbolism some behavior of the physical process. A model is necessarily a simplification of the real process. We choose events in constructing a model which are in our view simple, and the form of a model will be conditioned by the ideas already held and the understanding we have already gained. It is therefore not surprising that a model must often be modified or discarded because it can no longer, without contradiction, include all observations or because its bases are found to be not so "simple" as first supposed. A stochastic model applied to cave development must be constructed from the knowledge used in existing genetic theories.

The author has already proposed a stochastic model for the evolution of cave entrances (Curl, 1958). Stochastic models have for some time found application in a number of fields. Neyman and Scott (1959) in reviewing some of these wrote, "A few simple chance mechanisms may combine to reproduce many manifestations of a complex phenomenon.." Terrain analysis studies ~~are~~ the results of the stochastic processes of terrain development with the aid of statistical methods.

In the subsequent sections *cave, cave population,* the *distribution of cave lengths,* and possible processes in a stochastic model of cavern development will be defined, discussed, and applied to the evolution of cave length distributions. The derivation of the models will use the Davis (1930) two-cycle theory of cave origin which implies that cave growth, transition to vadose conditions, and finally decay, are consecutive and non-overlapping epochs. One-cycle caves (after Davis) will not be considered.

Each epoch including growth, transition, and decay, will be considered in turn, after which a composite model will be given for West Virginia caves. In particular an attempt will be made to explain the length distribution of all caves in West Virginia which is shown in figure 1. The discussion will be directed toward seeking reasonable assumptions concerning the quantitative aspects of the particular processes. Because of lack of space the mathematical development of each case has been omitted and only the results presented.

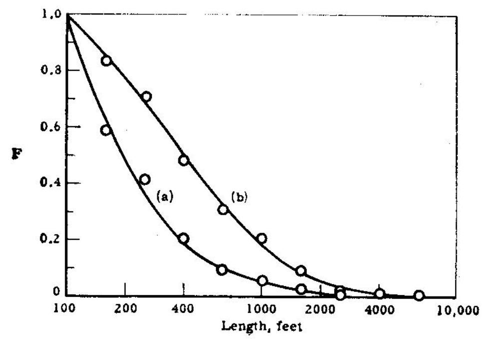

Figure 1
Distribution function of length for caves in West Virginia, where **F** is the fraction of caves longer than a given length: **(a)** all caves longer than 100 feet; **(b)** all such caves with only one entrance.

Cave Populations

A *cave population* is an ensemble of caves related by proximity or other features useful for classification. When Davis urged the collection of better and more extensive data on certain cave features he had already implicitly used the idea of populations of caves developing by similar means and possessing similar genetic characteristics. From the limited data then available he sought to deduce population characteristics which should be observed. The existence of more extensive data today permits the population view of certain cave feature origin presented here.

The individual cave is the element or member of the cave population. It may be a single solution cavity or a fragment of an originally larger system. It does not matter whether or not it has an entrance. The fundamental basis for identifying a cave as such is the size of a human; if a different scale of measurement were used there might be no "caves", or, at the other extreme, all caves might be one cave. A solution cavity in limestone is a cave if we can get into it. Caves too narrow to traverse could be included in a cave population, if necessary, by imagining ourselves to be smaller than we are. (However it is our misfortune from the standpoint of understanding cave origin that we are unable, except in rare instances, to study small passages of the primitive cave system.)

The length of a cave is the distance we may enter into the cave, assuming that we may gain entrance. A definable length exists because caves tend to develop upon joint systems which contribute their linear structure to subsequent passages, and cave terminations are usually quite final. At one end an entrance terminates an enterable cave and at the other, or others, there may be a blank wall, flowstone, impassable breakdown, a filled passage, or a passage smaller than our measurement basis—the size of a man.

Caves which have an impassable but observable connection will be considered as two caves. A corollary is that an unenterable section of a known cave is a second cave without an entrance. Some such distinction must always be made although where we choose to draw the line may vary with circumstances.

In the previous study on cave entrances, the length distribution of one entrance caves shown in figure 1, curve *b* was introduced empirically. It was thought then that the distribution of length must also be a product of a stochastic geomorphic process. However it is the population of *all* caves which is of interest in a theory of cavern development, not just those which happen to possess entrances, so the result derived by the earlier methods for all caves, shown in figure 1, curve *a*, will be used to represent the present circumstances in the state of West Virginia.

Any actual cave population is finite and therefore has a largest member, a deepest member, etc. It is convenient for the purpose of discussing generalized cave populations to overlook this fact and consider the existing population as a sample of caves from an infinite population. Distribution functions of length as in figure 1, when applied to a finite population, may yield a fraction of a cave as the number longer than some length, which just means that there is a small likelihood of that length occurring in samples of caves of the observed number chosen from the infinite "parent" population.

Growth Epoch

A growth-population consists of growing and maturing cavern passages. Little is known in detail of this process. It is probably at the end of this epoch that caves have their largest size and greatest extent. It is also at this time that we can identify, at least in principle and retrospect, the parts of the primitive network which were responsible for the structure of individual caves.

The primitive system may be enlarged either continuously or discontinuously. The former means that enterable passages in the system remain always in connection and cave length grows from some single unit to include eventually the utilizable (though not the available) phreatic network. Discontinuous means that the primitive system evolves to passable size in a number of sections which may in some cases coalesce (producing a discontinuous increase in length and the loss of a "cave") before the subaerial stages commence; each forms a cave which would be associated with a larger—though intraversable—basic system. This does not include caves which are separated from a continuous growth cave by later modification. In this treatment of growth the continuous model will be used although Davis (1930) implied his preference for the discontinuous mode in writing of the "integration of small systems into few systems of larger extent".

Assumption 1. — The number of caves in a growing population remains constant.

The continuous linear extension of a cave during the growth epoch occurs at a rate which depends at least on the following circumstances: (1) The availability of water which varies from place to place, and in time, due to differences in surface drainage patterns and fluctuations in climatic conditions. Water availability is also likely to change with the size which a cave has attained, a larger system being able to divert surface drainage underground over a wider area. (2) The solvent power of water (carbon dioxide content and initial approach to saturation) will vary with time and source. (3) Corrasive power of subsurface drainage will vary with time and source. (4) The properties of the limestone in which the cave is developing are determined beforehand but our lack of knowledge of variations of the properties and their significance in cavern development requires that they be considered as stochastic variables. (5) The configuration of the primitive network influences the rate as well as the possible extent of growth.

Together these factors cause a distribution in the rate of cave enlargement. They may be divided into those that are dependent upon cave length and those that are independent of length. The first factor cited above is of the former type. The fourth and fifth may be dependent upon length because the cave could encounter, sequentially, structures of different properties and thereby have variations in its rate of growth. However, variations in structural properties may be included with temporal variations when the nature of the variability in the structures encountered does not itself depend on length.

In figure 2 three possible histories of the length of a cave are shown which, it is assumed, ended growth with a particular length at a particular time. Curve *a* results from growth at a constant rate while in curve *b* the rate of growth increased with length and in this case is proportional to length. For curve *c* time-dependent factors entered in such a way that a relatively slow rate changed subsequently to a more rapid rate. Since a larger cave may receive more solvent water it should be expected that big caves tend to grow bigger and faster, though some big caves will suffer setbacks in growth and some little caves will exhibit growth spurts. Members of a cave population would start at different lengths, grow at different rates, and end at different lengths, rather than the illustrative situation in figure 2.

Assumption 2.—The Rate of Growth of Length of a Cave is Proportional to the Length Already Attained, All Other Things Being Constant, While the Proportionality Constant Varies Stochastically with Time for Each Cave.

The rate of growth varies from cave to cave and the distribution at any instant must be complex and changing in a complex manner. However only a suitable average over the whole epoch of growth rate need be considered, as in figure 2 where all three histories have an identical average rate of growth over the period. The distribution of this average rate of growth should be more stable. Specific assumptions about the epoch-average growth rate distributions will be made in connection with examples.

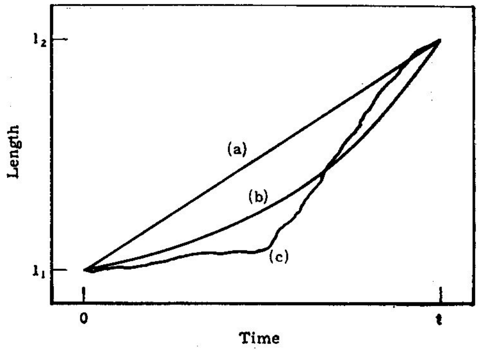

Figure 2
Mode of growth: **(a)** constant; **(b)** exponential; **(c)** possible real case.

The growth epoch starts with the caves existing at the end of the primitive stage though perhaps it is not realistic to distinguish between these stages as no special event marks the change. The treatment of the subsequent processes must be a study of the transformations of earlier populations until more is known about the origins of solutional openings in limestone. A distribution of time varying rates of growth acting on an initial distribution of cave lengths constitutes a stochastic process for the growth epoch. Each cave will increase in length, some slowly and some rapidly, and a new distribution of lengths will evolve. Caves with lengths in some range are produced by others growing into that range, and removed by growing beyond. The manner in which this evolution takes place will depend upon the superimposed growth rate distribution. A mathematical statement of this process would make an accounting of the numbers of caves entering and leaving the above range of lengths, and equate this to the rate of increase of the number of caves in the

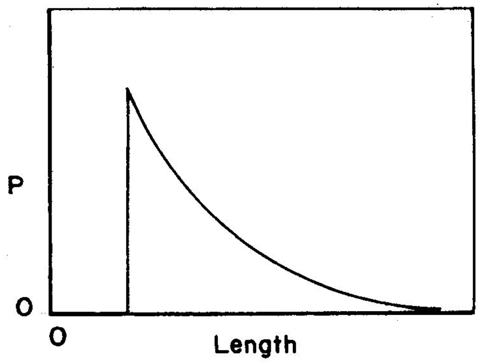

Figure 3
Exponential growth of uniform population where **P** is the relative frequency of caves of given length.

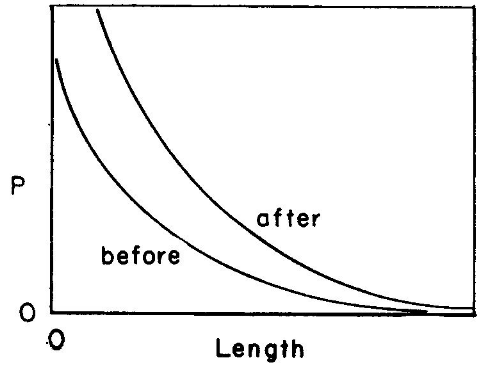

Figure 4
Invariant growth population showing new distribution after some time has passed.

range. The assumptions which determine the process have already been stated. The resulting transformation equation for growth is given in the Appendix. Examples follow:

Figure 3 shows the relative frequency of caves, *P,* (the higher the curve the more frequent are caves near that length) plotted versus length, for the case when all the caves originally had the same length and the epoch-average growth rate is exponentially distributed.

If the relative frequencies of caves are initially distributed as shown in figure 4, then after some time, for *any* distribution of the growth rate parameter, the new distribution will be as shown. This case is particularly interesting because the *form* of the distribution does not change. If instead we plotted these curves as in figure 1, as the fraction of caves longer than each length for caves over 100 feet long, the curve would remain always the same during growth. There would of course be more caves longer than 100 feet, but the fractions of them would remain distributed in the same way. As this is true no matter what the epoch-average growth rate distribution is, it will be called an *invariant growth population.*

The assumptions that have been made in the above model for a stochastic growth process are of course restrictive. Also, while the evolution of one distribution (length) has been "explained", other distributions were introduced for this purpose — those for initial lengths and epoch-average growth rates. It is believed, however, that the introduction of the latter is a useful step in understanding the growth process.

Transition Epoch

The two-cycle cave origin theory states that a regional uplift, or river downcutting, brings caves above the zone of saturation following the period of enlargement of a cave system to its largest extent by solution and corrosion. It is believed (Davies, 1958, p. 27) that cave fill, collapse (breakdown), and the development of entrances occurs during this time. Water barriers may also be left in a cave while elsewhere the surface may dissect a cave into several fragments.

A characteristic modification of this transition epoch will be assumed to be the dividing of a cave into smaller caves, or fragments. A large cave may be expected to suffer more divisions than a small cave, but some caves will be undivided and others highly fragmented. Variations in the frequency and location of modification by fragmenting during transition identifies this as a stochastic geomorphic process, and associated population manifestations should be expected. Other events do, of course, occur during transition. There may be considerable loss of cave passage by filling or total removal, but here the specific effect of fill on length distributions will not be treated except later in regard to the decay epoch.

Fragmenting may be either length-preserving or length-destroying. The former is an idealization of those divisions of a cave system which destroy little net length. The latter case, which is the real situation, may be considered as superimposed decay.

Assumption 3.—Cave Fragmenting is Length Preserving.

Random fragmenting is assumed for the lack of a better hypothesis. *Random* is used to mean that divisions of the caves following the growth epoch are equally likely to occur at any point in a cave but have some average frequency of occurrence per foot of cave (much less than one).

Assumption 4.—Caves are Divided into Fragments by a Random Process in which Divisions are Equally Likely to Occur Anywhere, but an Average Frequency per Unit Length Exists (Poisson Process).

During transition some caves would not be divided at all and these would join the new population unchanged in length or number. Other caves would be divided once or more into two or more fragments. These would join the new population as shorter and more numerous caves. This process would transform the initial population to a more numerous population with a new distribution of lengths. The transformation equation for transition is given in the Appendix. Short caves can arise by either never growing very big, or by being fragments of larger caves. Both types must exist, although they are all reported as individual caves. But a cave which terminates in a short distance by breakdown or fill is probably a fragment and is likely to continue beyond (or rather, there is probably a second cave beyond).

Table 1 indicates how often caves of different lengths will be divided for a case when divisions occur at the average frequency of, for example, 0.0005 per foot of cave. Values are given for the percent of caves of a given length which will receive one or more divisions.

TABLE 1

Length (feet)	200	800	2000	4000
Percent divided (approx.)	10	32	63	86

Figures 5 and 6 show the effects of fragmenting various initial populations with random divisions. In figure 5 all caves had the same length to start. Some remain undivided. In figure 6 the general shape (exponential) of the curve representing the relative number of caves has not been altered by the transition process, although the relative number of shorter caves has increased at the expense of longer caves.

Only one model for transition modification of a cave population based on a reasonable mechanism for a change which might occur during this stage has been considered in detail. We know that caves are often terminated by breakdown and other passage closures, and it is possible to get beyond such blocks often enough to support the belief that beyond many barriers which stop exploration now there is more cave passage. This is justification for the belief that caves are interrupted and that it is permissible to consider the process as stochastic. The stochastic process of cave fragmenting tends to produce an apparent upper limit on the size of caves. By measurements of length alone it is not possible to distinguish between an upper limit on length imposed by limitations of growth or subsequent fragmenting. A comparative study of the nature of cave terminations is necessary to help decide this question.

Decay Epoch

The decay process is like the growth process; caves are changing length as part of a stochastic geomorphic process. However the agents are now not those of enlargement, but of weathering, erosion, weakening, collapse, and fill. They are primarily surface agents which act on the evolution of entrances and the filling or cutting back of points in the cave system through which they have access. It is likely that caves decay inward from points of surface intersection while the internal cave passages are relatively protected and static. If this is the

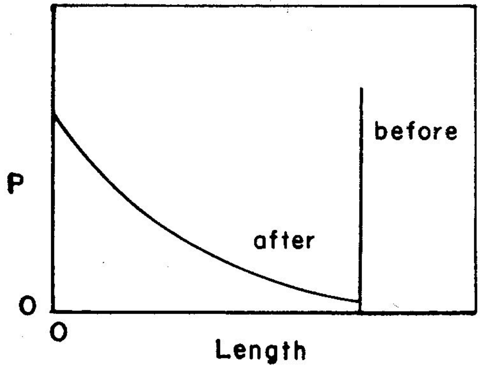

Figure 5
Transition of uniform population (**before**) to fragmented population (**after**).

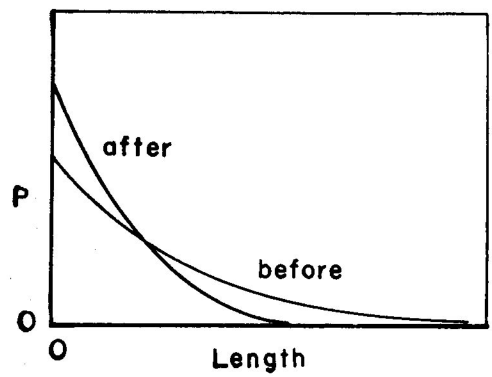

Figure 6
Transition of exponential population (**before**) to fragmented population (**after**).

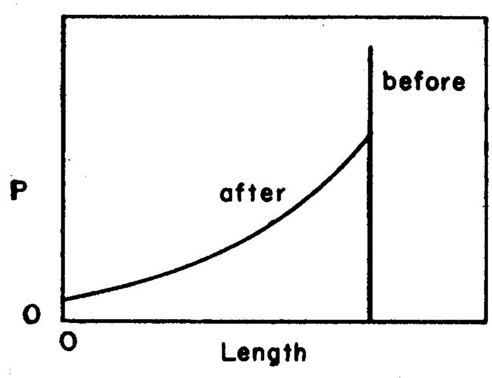

Figure 7
Exponential decay of a uniform population.

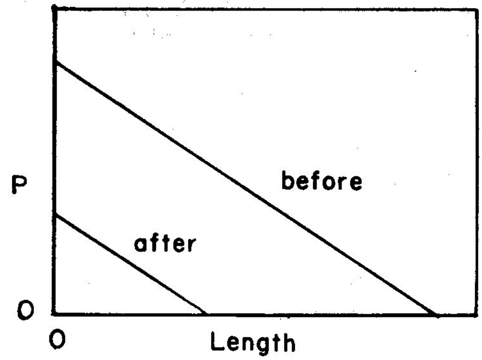

Figure 8
Invariant decay population.

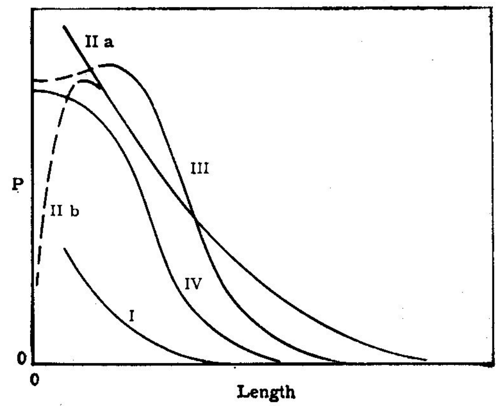

Figure 9
Evolution of a cave population, where **P** is the relative frequency of caves of given length: (**I**) start; (**II**) after growth; (**III**) after transition; (**IV**) after decay.

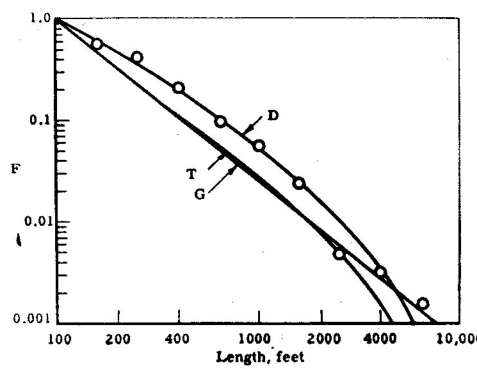

Figure 10
Distribution function of length for all caves in West Virginia longer than 100 feet: (**G**) growth population (invariant); (**T**) after transition; (**D**) after transition and decay.

case, the points of division in the transition epoch, together with the original "ends" of the cave, could be the locations from which decay proceeds. Additional intersections might be produced as the surface above is eroded with fragmenting and decay proceeding simultaneously. Such overlapping of processes will not be considered here. Since the processes occurring at divisions and ends can hardly be influenced by many of the interior passages of a cave, an appropriate guess would be that the rate of decay of a cave does not depend on its length although the rate may vary with other geomorphic influences. An epoch-average decay rate for each cave is also applicable here.

Assumption 5.—The Rate of Decay of a Cave is Independent of Length but Varies with Time.

A distribution of time-varying rates of decay acting on an initial distribution of cave lengths constitutes a stochastic process for the decay epoch. Each cave will decrease in length and eventually reach zero length. A decaying cave population moves toward a final state in which all caves have zero length. Since not only erosional removal of rock but also fill and collapse are included in decay, some zero-length caves would still be detectable. Later events may rejuvenate such caves but this complexity is not being considered. The transformation equation for the decay epoch is given in the Appendix. Examples follow:

If all caves are initially of a single length and the epoch-average decay rate is exponentially disturbed, then the population decays are as shown in figure 7. Some caves already have zero length.

If the initial relative frequency of different lengths is as shown in figure 8, then the population evolves without a change in the straight line form of the distribution no matter what the decay rate distribution. This is therefore, by analogy to the growth case, called an *invariant decay population.*

A constant rate of decay (or growth) produces simply a fixed displacement of the previous figures to the right or left depending on whether the process is growth or decay.

The conclusion from this discussion is that decay, being independent of cave length, would cause the greatest relative changes to occur in short caves.

Evolution of Cave Populations

Hypothetical Population

All of the previous models may be summarized by showing the effects each has during the evolution of a cave population. Figure 9 shows the evolutionary sequence for the length distribution of a hypothetical population. An initial population *I* has mostly short caves. Growth extends this distribution to greater length, eventually to give population *IIa.* Since it is likely that the shortest members of this population are also too narrow to enter, a modified population *IIb* is shown to represent enterable caves only. Transition modifies this by transforming long caves to shorter fragments to give population *III.* A number of short but enterable caves have also been formed and the distribution is raised near the origin. This population then decays and the distribution sinks toward zero length *IV.* Sometime during this epoch the relative number of caves of some length may increase for a while, but eventually all are gone.

Lengths of Caves in West Virginia

We return at last to a consideration of the phenomena which prompted that which is presented here: the observed distribution of the lengths of caves in West Virginia shown in figure 1b for all caves over 100 feet long and with one entrance. It has been recognized that cave entrances are accidents, and to explain the lengths of one-entrance caves requires both an explanation for the lengths of all caves, and an explanation of the way entrances are distributed among caves. The latter step was taken in a previous paper (Curl, 1958). The same data, modified slightly by the inclusion of a recently discovered cave (Culverson Creek Cave) were used to compute the distribution of length for all caves in West Virginia shown in figure 1a.

The data of figure 1a have been replotted in figure 10 using logarithmic coördinates. An "invariant" length distribution has been assumed for the growth population because the mathematical development showed that this is a distribution of great generality obtained even with a variety of quite different assumptions about initial length and growth rate distributions. An "invariant" population plots as a straight line in figure 10, and does not change during the growth epoch.

The transition epoch transforms curve *G* to *T* by causing a relative decrease in the frequency of long caves. In the case shown, the average distance between divisions (if all the caves were strung out end to end) is 2000 feet (0.0005 divisions per foot of cave). The subsequent epoch of decay is particularly hard on short caves which also constitute a large proportion of the population. A process of decay with an assumed constant loss of 60 feet from every cave produces the transformation to *D*. Many caves will have decayed below 100 feet and be no longer represented in the population now shown.

The parameters were chosen to make the final curve *D* agree with the characteristics of the data. Considerable flexibility existed in producing a good correlation since the two processes of transition and decay affect the long and short caves respectively, most strongly. However it was still necessary that the direction of the effects of transition and decay be in accord with the long and short cave properties of the data before a model could be used. This, then, is a possible model for the evolution of the West Virginia cave population.

Conclusions

All geomorphic processes, and in particular the processes of cavern development, are stochastic processes by virtue of elements of chance or randomness which enter into them. In the evolution of a cave population certain manifestations may reflect the operation of relatively simple chance mechanisms and a study of these can be useful in gaining a better understanding of the basic processes.

Stochastic models for the growth, transition, and decay epochs of the cave population of West Virginia, based on a two-cycle geomorphic history, have been proposed and compared with data. Modern knowledge on cave development has been used to guide the choice of assumptions. An "invariant" growth population was found which turned out to be very similar to the data, and the subsequent predictions of the effects of simple transition and decay models improved the correlation. The numerical values for the transition fragmenting frequency, 0.0005 per foot, and the decayed length, 60 feet, for West Virginia caves, seem reasonable in the absence of better data to check them.

The alternative to a stochastic model is to maintain that every cave is unique and that no processes may be identified as acting in common upon all caves. On inspection many cave features can be ascribed to very particular circumstances for that cave, but to criticize stochastic models in the light of such observations is to claim that a prediction of a cave feature (length) could have been made. This is more than any present theory attempts to do and is irrelevant, if not impossible, according to stochastic theories.

More observational information on the mechanisms of cave growth, the nature and causes of cave interruptions, and the effects of surface degradation on cave modification will be needed before the validity of the models presented herein can be finally ascertained.

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RICHARD R. ANDERSON, *Bell Telephone Laboratories*: You assume that the rate of growth is dependent on the length. I would think it would be more closely related to the number of terminations.

AUTHOR: The number of terminations to a cave is statistically related to its length. In addition, length is more readily obtained from data. The fact that great simplifications have some utility implies that a great many other variables "average out" in some sense, or they don't enter into population manifestations in the same way they do in individual caves.

ALAN D. HOWARD, *Yale University*: Let's suppose we consider the breaking up of caves as the dropping of bombs in a random manner. If there are two types of caves, those with a single long passage and those with a maze pattern, doesn't it seem more likely that you will produce more fragments from the single-passage cave than from the network cave?

AUTHOR: A maze cave is a "multiple-connected" cave and a linear cave is "simple-connected". In the latter you can take a ball of string and run it around, tying it together anyway you want, and still be able to pull the mesh from the cave. As you observe, if a multiple-connected cave is divided, it is still possible to have just one cave. My models apply only to simple-connected caves (or passages) but these constitute the majority. Again, many of the peculiarities of the members of these populations are submerged in the averages which are taken.

WILLIAM B. WHITE, *Pennsylvania State University*: I would like to return to your statement that the rate of increase of cave length is proportional to the length, which would mean that the length of a cave increases exponentially with time up to the end of growth. What transition is necessary to terminate growth to keep the length of the cave from going to infinity?

AUTHOR: Yes, the length of the cave would go to infinity in time, so we must consider the reasons why caves are relatively short. We could say that geologic control is the answer, but this is a weak argument because there is always much jointed limestone that doesn't have any caves in it. I have solved the problem by assuming growth of all caves in a population to cease at the same time. If it didn't, the processes of growth, transition, and decay would overlap and considerably confuse the picture. If anyone works out an alternative model, I would be quite interested. I chose to take the simplest case.

ANDERSON: This is presumably a continuous process. I would think there are caves being both created and destroyed now. Do you believe that these processes must happen at different times?

AUTHOR: The different populations could certainly exist simultaneously, even in a relatively small area.

ANDERSON: Then did you say the ones you were studying were all of the same population?

AUTHOR: I have confidence in feeling that they are. In the previous paper on cave entrances, I considered a basic parameter of these populations for all caves (with entrances) over 100 feet long, over 500 feet long, and over 1000 feet long. If anything is going to be different about different groups of caves, these ought to have included different types of caves. This basic population parameter turned out to be the same for all these groups of caves in West Virginia.

HOWARD: Have you done anything with smaller groups within the larger total to see if over a smaller areal range there might be significant variation?

AUTHOR: Only in the earlier paper. Too few data is the difficulty if small groups are concidered. If a way could be found to look at individual caves for properties which would include or exclude them from the local homogeneous population then something might be said about smaller groups. It is not at all obvious yet what these properties would be.

APPENDIX

The following equations were derived from the assumptions given in the text and used in applying each model:

GROWTH:

$$P_2(L) = \int_0^\infty \exp(-\mathbf{u}t) P_1[L \exp(-\mathbf{u}t)] P(\mathbf{u}) d\mathbf{u}$$

where $P_1[L \exp(-\mathbf{u}t)]$ is the probability density distribution of initial lengths evaluated at $L \exp(-\mathbf{u}t)$; $P(\mathbf{u})$ the distribution of the epoch-average growth rate $\mathbf{u}$ (and $dL/dt = uL$); and $P_2(L)$ the resulting length distribution. Statistical independence of $\mathbf{u}$ and initial length has been assumed.

TRANSITION:

$$P_3(L) = \frac{e^{-IL}}{1+I\mathbf{l}} \left\{ P_2(L) + I \int_L^\infty [2 + I(l-L)] P_2(l) dl \right\}$$

where $P_2(l)$ is the length distribution at the end of growth; I the average number of divisions per foot of cave; $\mathbf{l}$ the average length of the cave in the population prior to fragmenting; and $P_3(L)$ the probability density distribution of L resulting from transition.

DECAY:

$$P_4(L) = \int_0^\infty P_3(L + \mathbf{v}t) P(\mathbf{v}) d\mathbf{v}$$

where the terms are as defined in the growth case except applied now to the population before and after (some) decay. The distribution of the epoch-average rate of cave decay is $P(\mathbf{v})$ (and $dL/dt = -v$). Statistical independence of $\mathbf{v}$ and length has been assumed.