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# Physics in a Toy Boat

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A method of propulsion commonly used in toy boats consists of a shallow chamber, covered by a thin diaphragm, which is connected to the water astern of the boat by pipes. Filling the chamber with water and then heating it's base leads to vigorous self-induced vibration of the diaphragm and water column in the pipes with resulting forward motion of the boat. The paper describes the mechanisms of self-induced vibration and of propulsion. It is shown that this inexpensive toy demonstrates a number of physical principles and provides opportunities for further research.

THERE is a child's toy known as the putt-putt boat, which demonstrates a remarkable number of physical principles. For the benefit of those who have not experimented with such a boat, its operation may be described with respect to Fig. 1. A thin diaphragm E covers a shallow chamber A. From the base of the chamber, a pipe, or usually two pipes, lead to the rear of the boat at C. If the chamber and pipes are filled with water and a heat source B, such as a candle, is applied then after a short time the diaphragm E and water in the pipes begin to vibrate. At the same time the boat moves forward, making characteristic putt-putt noises.

The first discussion of the boat in the scientific literature appears to have been by Baker[1] in 1933. More recently, the writers[2] have discussed his conclusions and, in particular, have examined the nonlinear aspects of the boat's behavior. It was shown that Baker's explanation of the method of propulsion was substantially correct, but that his explanation of the mechanism of vibration could not apply to boats of type shown in Fig. 1. Very recently, it has been brought to the writers' attention that two letters on the putt-putt boat have appeared in the *American Journal of Physics*.[3,4] The first of these references incorrectly describes the mechanism of propulsion, while the second is a more complete discussion in which some of the conclusions are similar to those obtained independently in reference 2. Despite the amount of attention this small boat has received, none of the references appear to be suitable for an elementary, yet fairly complete, explanation of the boat's main features. The present article is an attempt to supply this information.

As a starting point, it is best to refer to an experiment in which the chamber and pipes are made out of glass and their contents observed during vibration. If heat is applied to the base of the chamber, shortly thereafter steam begins to form in the chamber and moves down a short distance into the pipes. At this stage, the diaphragm and the water in the pipes begin to vibrate. The motion of water in the two pipes is in phase and is usually approximately sinusoidal. Despite the presence of steam in the pipes, there is normally quite a lot of water left in the shallow chamber, particularly around the rim. Excessive heating drives all water from the chamber and may even drive water completely out of the tubes. Under these conditions, vibration no longer occurs. The mean water level in the pipes is normally lowered by increased heating and

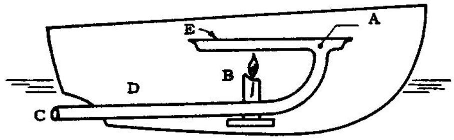


FIG. 1. Schematic drawing of putt-putt boat.

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[1] J. G. Baker, Trans. ASME 55, AMP-55-2, 5 (1933).
[2] I. Finnie and R. L. Curl, *Proc. Internat. Union of Theor. and Appl. Mechanics. Symposium on Non-Linear Oscillations; Kiev, U.S.S.R., September 1961.* (to be published by Acad. of Sciences of Ukrainian SSR)
[3] J. S. Miller, Am. J. Phys. 26, 199 (1958).
[4] R. S. Mackay, Am. J. Phys. 26, 583 (1958).

raised closer to the chamber if the pipes are cooled. During vibration, the top of the water column often splashes into the chamber. Under this condition, continuous oscillation has been observed for durations of well over an hour. However, even without the return of water to the chamber, vibration may also occur for extended periods. In this case, the chamber first becomes dry and then, with the cessation of steam generation to maintain the pressure, suddenly fills completely with water to start another spell of vibration.

## MECHANISM OF PROPULSION

The forward propulsion which results from alternating flow in the pipes is perhaps the most intriguing aspect of the boat's behavior. The superficial reasoning, based on symmetry, that if the boat moves forward during outflow it should move backward during inflow, is clearly wrong. That outflow produces a thrust is a familiar experience. That inflow of the same magnitude does not produce an equal and opposite thrust is less frequently encountered.

During outflow, water leaves the tubes as a jet which, because of viscosity, is separated from the surrounding water by a thin highly turbulent layer. During the inflow part of the cycle, there is no longer jet flow, but rather the water flows from all around into the tube, as if the end of the tube were a "sink." In either case, the motion of the boat may be described by the basic equation of variable mass dynamics as written for rectilinear motion of a rocket[5]

$$m\ddot{x}+D(\dot{x})=F-\partial^2(mh)/\partial t^2, \qquad (1)$$

where $m$ is the mass of boat and contents at time $t$, $x$ is the coordinate locating the exit plane of the tubes, $h$ is the distance, in the $x$ direction, from the exit plane to the center of mass of the boat, $D(\dot{x})$ is the drag force on the hull in the $x$ direction, and $F$ is the combined effect, in the $x$ direction, of the force due to the pressure distribution over the hull and the momentum flux across its boundaries. For the simple rocket problem, in which pressures are equal all over the surface and the jet velocity is constant over the cross section, the term $F$ is merely the product of the mass rate of flow and the jet velocity relative to the rocket. However, in this present case, the evaluation of $F$ presents more difficulty. For this reason, it is convenient to express $F$ in terms of a surface vector integral which is simply the resolution of all surface pressure and momentum stresses expressed as a vector. The value of this formulation will appear shortly. Thus,

$$\mathbf{F}=\mathbf{i}\int_S\{\rho\mathbf{U}(\mathbf{U}\cdot\mathbf{n})+\mathbf{n}p\}dS, \qquad (2)$$

where $\mathbf{i}$ is the unit $x$ coordinate vector, $\rho$ is the density of the fluid, $p$ is the pressure at the surface, $\mathbf{U}$ is the water velocity vector at the surface, and $\mathbf{n}$ is the unit normal vector at the surface (+ inward) and it should be noted that the surface includes the exit plane of the tubes as well as the wetted area of the hull.

It is interesting to note that many treatments of variable mass dynamics are incomplete in that they do not contain the last term of Eq. (1). If the present problem were one of transient acceleration, this term would have to be considered. However, in the present case it may be eliminated if it is assumed that the boat is operating in a "steady state," though oscillatory, manner. Averaging Eq. (1) over one complete cycle, after substituting for $F$, assuming all cycles are identical, and using the notation

$$\frac{1}{T}\int_0^T Gdt=[\![G]\!]$$

leads to

$$[\![m\ddot{x}+D(\dot{x})]\!]$$
$$=\mathbf{i}\int_S\{\rho[\![\mathbf{U}(\mathbf{U}\cdot\mathbf{n})]\!]+\mathbf{n}[\![p]\!]\}dS-\frac{1}{T}\frac{\partial(mh)}{\partial t}\bigg]_0^T. \qquad (3)$$

The last term in Eq. (3) is equal to zero as $\partial(mh)/\partial t$ has the values at the end of the cycle that it had at the beginning.

It is convenient now to consider the flow as the sum of intake and exhaust flows, $\mathbf{U}=\mathbf{U}_i+\mathbf{U}_e$ and $p=p_i+p_e$, such that $\mathbf{U}_i$ and $p_i$ are zero during exhaust and $\mathbf{U}_e$ and $p_e$ are zero during intake. Then the surface integral may be divided into the sum of two integrals, in each of which only $\mathbf{U}_i$ and $p_i$ or $\mathbf{U}_e$ and $p_e$ enter, although the

[5] J. L. Meriam, J. Eng. Educ. 51, 241 (1960).

averages are still to be taken over a complete cycle. The integral is readily evaluated, at least approximately, during exhaust as flow is occurring only in a direction normal to the exit plane of the tubes and the pressure in the jet of water as it leaves the boat is essentially equal to the static pressure of the surrounding water. Thus,

$$\mathbf{i}\int_S \{\rho[\![\mathbf{U}_e(\mathbf{U}_e\cdot\mathbf{n})]\!]+\mathbf{n}[\![p_e]\!]\}dS \simeq \mathbf{i}\rho\int_S [\![\mathbf{U}_e(\mathbf{U}_e\cdot\mathbf{n})]\!]dS=\rho[\![U_e^2]\!]a,$$

where $a$ is the total cross-sectional area at exit from the tubes and $U_e^2$ is an average over the cross section.

During inflow, the fluid velocity at entrance to the tubes will not be normal to the tube exit plane and pressures will vary over the boat and exit plane surfaces due to flow. As a starting point, the equation of motion of an incompressible fluid may be written as[6]

$$\partial\mathbf{U}/\partial t+(\mathbf{U}\nabla)\mathbf{U}=-\nabla(\Omega+p/\rho)+\mu(\nabla^2\mathbf{U})/\rho, \quad (4)$$

where $\mu$ is the dynamic viscosity, and $\Omega$ is the gravitational potential. This equation can now be integrated over all the volume of fluid outside the boat and its exit planes and averaged over the period of one cycle.

As these operations may be done in either order, one obtains after rearranging,

$$\rho\int_V [\![(\mathbf{U}\nabla)\mathbf{U}]\!]dV+\int_V [\![\nabla p]\!]dV=-\rho\int_V \mathbf{U}\Big]_0^T \times dV-\rho\int_V \nabla\Omega dV+\mu\int_V [\![\nabla^2\mathbf{U}]\!]dV. \quad (5)$$

Since $\mathbf{U}$ is the same at the ends of a cycle, the volume integral of $\mathbf{U}(T)-\mathbf{U}(0)$ vanishes. Also, applying Gauss' theorem to convert volume integrals to surface integrals leads to

$$\int_S \{\rho[\![\mathbf{U}(\mathbf{U}\cdot\mathbf{n})]\!]+\mathbf{n}[\![p]\!]\}dS=-\rho\int_S \mathbf{n}\Omega dS-\mu \times\int_V [\![\nabla^2\mathbf{U}]\!]dV-\int_V [\![\mathbf{U}(\nabla\mathbf{U})]\!]dV, \quad (6)$$

which is just the integral in Eq. (3). Since the flow is incompressible, the last term in Eq. (6) vanishes, as then $\nabla\mathbf{U}=0$, while the gravitational potential $\Omega$ produces no thrust when resolved in the $x$ direction. Moreover, during inflow, the flow will be approximately irrotational and, hence, the second integral on the right hand side of Eq. (6) is zero or very small. Thus,

$$\int_S \{[\![\mathbf{U}_i(\mathbf{U}_i\cdot\mathbf{n})]\!]+\mathbf{n}[\![p_i]\!]\}dS\simeq 0,$$

so that $[\![m\ddot{x}+D(\dot{x})]\!]=\rho[\![U_e^2]\!]a$ and the boat moves forward.

It is interesting also to note that

$$\rho[\![U_e^2]\!]a\simeq-\mu\int_V [\![\nabla^2\mathbf{U}_e]\!]dV$$

even though this integral could not be evaluated explicitly in the actual eddying flow during outflow. A small contribution must also come from this term during inflow as the eddies produced on outflow will not have completely subsided.

An alternative explanation may be given for the inflow part of the cycle which is considerably less rigorous, but may be physically more understandable than the vector manipulation just discussed. As before, it is assumed that water flows from all around into the pipes. On entrance to the tubes, a *vena contracta* will be formed after which parallel, though turbulent, flow will be established. This situation, at least up to the *vena contracta*, is just the classical Borda mouthpiece and it may be shown that the area of the *vena contracta* is half that of the entry plane.[6] It is interesting to note that Borda,[7] in 1766, derived this result and by experiment obtained the value (1/1.942). The *average* velocity $V_C$ at the *vena contracta* is thus just twice the average velocity $V_B$ at a section $B$ in the parallel flow region just following the *vena contracta*. Between these two regions, there will presumably be a loss in static pressure due to the sudden increase in flow area. This loss is shown in almost any fluid mechanics or hydraulics text to be $\rho(V_C-V_B)^2/2$ and since $V_C=2V_B$ the loss in pressure is $\rho V_B^2/2$. Adding this term to the pressure deficit due to velocity given by Bernoulli's

[6] L. M. Milne-Thomson, *Theoretical Hydrodynamics* (MacMillan and Company Ltd., London, 1960), 4th edition.

[7] J. C. Borda, Mém. Acad. Sci. (Paris), p. 579 (1766).

equation and eliminating, as before, the acceleration term leads to the result that the static pressure in the parallel flow region following the *vena contracta* is less than that of the water outside the boat, at the same level, and at rest by $\rho V_B^2$.

Considering the momentum flux and pressure at section $B$, ignoring drag forces inside the pipe between section $B$ and the exit plane and assuming the pressures elsewhere on the boat's surface are not influenced by flow leads again to the conclusion that inflow has no influence on propulsion. In considering this alternative explanation, it should be noted that the previous analysis is rigorous and involves only the flow external to the boat. Hence any conclusion based on the existence of a *vena contracta must* conform to the no-thrust result. It is possible to imagine placing objects in the entry tube which make the consideration of a *vena contracta* impossible, but the net thrust or intake must still be zero with external irrotational flow.

## MECHANISM OF SELF-EXCITED VIBRATION

By observing the glass model, mentioned earlier, it is clear that steam is being generated in the chamber and condensed in the pipes. These processes are undoubtedly time varying and quite complicated. However, as an approximation, the influence of pressure changes will be neglected and the rate of steam generation will be taken as a constant while the rate of steam condensation will be taken as proportional to the area of the available condensing surface. Thus, $\dot{S}=k_1-k_2y$, where $S$ is the amount of steam in the chamber and pipes at a given time, $k_1$, $k_2$ are constants and $y$ is the distance from the base of the chamber to the top of the water column.

During periods of relatively steady oscillation, the mean level of water in the pipes will be determined by the relative magnitudes of $k_1$, and $k_2$, the steam generation and condensation parameters.

If the chamber is completely filled with water before heating, the mean water level during operation is only slightly below the base of the chamber. This is due probably to the very large heat transfer coefficients which characterize the condensing of pure vapors. However, as the presence of noncondensible gases is known to greatly decrease the rate of condensation of steam, one would expect the mean level of water to be lowered by the addition of air to the chamber. In fact, this is observed and indeed the performance of the boat is usually improved by the presence of small amounts of air in the chamber. This may be due to the fact that it is possible to have larger amplitudes of vibration of the water column when the mean level is lower.

If the assumption as to steam generation and condensation is accepted, the physical basis for the self-excitation is fairly clear. Water moving down the tubes by increasing the condensation rate decreases the chamber pressure which in turn draws the water up into the tubes. Since the chamber pressure changes due to condensation will lead the motion of the water by approximately 90°, i.e., in phase with its velocity, energy will be fed into the vibrating system. As a result, the amplitude of vibration will increase until limited by nonlinear terms or by a physical change in the problem such as water splashing back into the chamber. There is still, of course, a question of whether or not vibration will start in a given system. This is answered by considering the linear problem of very small oscillations and the equation for the motion $y$ of the water column may be written as

$$\ddot{y}+C_1\dot{y}+\omega_2 y=C_2\int_0^t k_2 y dt,$$

where $C_1$ and $C_2$ are constants, and $\omega$ is the frequency of small amplitude vibration of the water

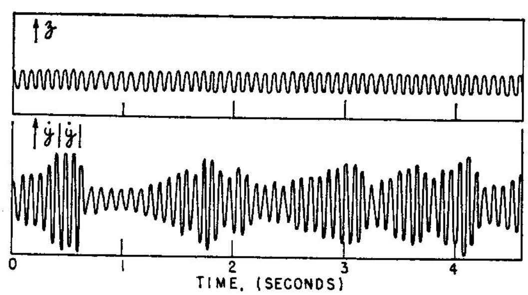


FIG. 2. Observations on a model with 2-in.-diam×0.002-in. diaphragm and 6-in.-long×0.18-in.-diam pipes. Water level outside pipes was 1 in. below the base of chamber. Heat input 60 W. Top record shows frequency and phase of $z$, the diaphragm deflection, but waveform is distorted by measuring device. Bottom record shows $\dot{y}|\dot{y}|$ as measured by Pitot tube.

column for the idealized situation in which the linear damping and steam condensation terms are absent (i.e., the water column mass with the vapor and thin diaphragm considered as a spring). Unless the condensing term is very large, the initial frequency of oscillation will be approximately equal to $\omega$ and the condition for initiation of self-excitation is then $k_2C_2 > \omega^2 C_1$. That is, for a given system, the condensation coefficient must be greater than some multiple of the damping term due to pipe flow. The inequality also shows that self-excitation should occur more readily when the system has a low natural frequency. This explains one role of the thin diaphragm. That this is its primary role is demonstrated by the fact that the chamber and diaphragm may be replaced by a metal bellows to make a perfectly operable, though noiseless, putt-putt boat.[2]

## NONLINEAR ASPECTS

The boat has several interesting nonlinear aspects which can be discussed qualitatively. This appears to be more instructive than attempting to develop equations which are amenable only to computer solution.[2]

Apart from the condensation and generation of steam, the most obvious nonlinearity lies in the diaphragm. Since the diaphragm becomes considerably stiffer with increasing deflection, one would expect the frequency of vibration to increase with increasing amplitude and this is observed. However, in attempting to predict this effect, it was found that the deflections of the thin diaphragms used in experiments were in

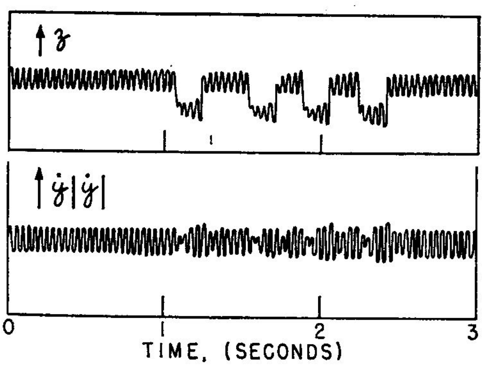


FIG. 3. Observations on a glass model with $1\frac{1}{2}$-in.-diam $\times$0.002-in. diaphragm and 6-in. long$\times$0.18-in.-diam pipes. A considerable amount of air was introduced into the chamber. Comments on Fig. 2 on $\dot{y}$ and $z$ measurements apply here also.

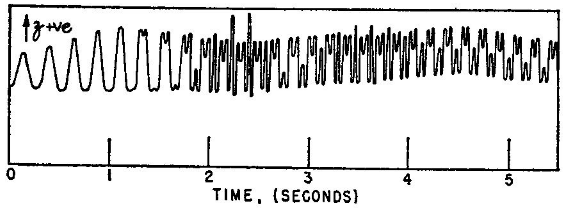


FIG. 4. Observations on a model with 6-in.-diam$\times$0.007-in. diaphragm and 5.75-in.-long$\times$0.43-in.-diam pipes. Deflection measured with wire resistance strain gauge attached to diaphragm.

serious disagreement with the predictions of elastic theory. This is due apparently to the difficulty of producing a geometrically perfect thin diaphragm. These nonuniformities may also provide an explanation of the noise generated since they allow a coupling of the basic motion to higher modes of vibration which produce the characteristic noise.

In setting up the equation of motion for water in the pipes, the damping term involves the resistance to oscillating flow in the pipes. However, in this system there is another important source of damping due to the loss at entry to the pipes discussed earlier. This contributes a damping term proportional to $(\dot{y} - |\dot{y}|)^2$. From both analog tests and experiment, it appears that this half-cycle damping may lead to very unusual waveforms at large amplitudes if the length/diameter ratio of the pipes is small.[2]

Other unusual and fairly repeatable wave forms are produced when the water level is very near either the top or the bottom of the tubes. Figs. 2, 3, and 4 illustrate a few of the waveforms which have been observed.

## CONCLUSIONS

From a purely practical point of view, the putt-putt method of propulsion is unattractive, since for a typical case, the ratio of useful propulsion work-to-work dissipated as inlet losses may be only about 0.1. Quite apart from this, the maximum thermodynamic efficiency of the mechanism must also be very low since steam is generated and condensed at nearly the same temperature.

However, the system demonstrates a large number of physical principles and offers a number of interesting problems for future exploration.