

Tasks that embody knowledge, tasks that probe teaching

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My interest in mathematical tasks is based on two basic hypotheses (or points of departure). On the one hand a task has the chance to contain or embody mathematics (the doing of it) as intellectual work, as well as make use of mathematics (its achieved objects) as objects or tools of the work. In that sense a task is a representation of mathematical activity, embodied in the interactions among people and with cultural tools. Tasks that involve students in defining, conjecturing, representing, proving, are not only important insofar as they afford those students authentic experiences in the doing of mathematics. Tasks install a representation of mathematics in the public life of a new generation. Thus tasks don't just provide some cognitive or emotional benefits to individuals; they serve the cultural reproduction of mathematical practices.

On the other hand, when enacted in a classroom, a task may fit in customs of joint work or otherwise perturb this work. Surely, it can perturb the students' cognition (as when it creates cognitive conflict). But what is of special interest to me is the extent to which a task can perturb instruction (the system of customary transactions between teacher and student about academic work). A task can exert such perturbation by creating an opportunity for students to do some mathematical work that is not customary. In doing so, a task can be a probe on instruction: It can occasion responses and reactions from instruction that might help inform on the capacity of instruction to make room for such kind of mathematical work. A task can, in particular perturb the work of a teacher, bringing to the surface tensions that are customarily hidden. The pursuit of a task may require a teacher to manage those tensions, thus providing the opportunity to observe how the rationality of teaching works to create or preserve order. In this paper, I do an exercise of task analysis a priori, illustrating what one might learn about teaching by studying how a task probes the customary work of a real classroom.

Problem, Task, and Situation

In Herbst (2006) I proposed some distinctions between words to secure analytic leverage to examine the phenomena that happen in instruction apropos of tasks. I use *problem* to refer to the mathematical question whose answer calls for the development or the use of a mathematical idea. This usage draws from Brousseau (1983/1997) for whom a *problem* is a question whose answer hinges on bringing to bear a mathematical theory within which a concept, formula or method involved in answering the question is justified. **A problem is a representation of a piece of knowledge**, in that such problem points to a piece of knowledge, the knowledge that helps answer the question. In this paper we examine the so-called “angle bisectors problem”: what can one say about the angle bisectors of a quadrilateral? This problem refers to a collection

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of propositions about quadrilaterals, to a theory of quadrilaterals in Euclidean geometry.² When a question like that one is posed to a mathematically educated person it may identify tacitly what kinds of products, resources, and operations are involved in answering it. We know that a mathematician would not be content saying “there are four,” that they would not allow themselves to solely handle the question by drawing and looking at diagrams, that anything this mathematician “said” about the angle bisectors would at least *aspire* at being proved. Such mathematical behaviors could hardly be expected of artists, carpenters, or little children. Likewise, if a high school geometry class became occupied with that problem, if that problem became a part of practice of a school class, it would do so in a particular way: people would gravitate to do particular operations, use particular resources, and aim at particular goals. A problem’s actual or potential unfolding as a practical enactment in time and by a particular agent is what I call a *task*.

Building on Doyle’s (1988) work, I have used *task* to refer to the specific *units of meaning*, the goal oriented actions and interactions in an environment that constitute the epistemological context in which individuals get to think about the mathematical ideas at stake in a problem. A task is the system of dialectical interactions over time between a cognizing agent and a problem. The task can be modeled by referring to its *product* or *goal* (whose achievement marks the end of the task), its *resources* (the symbolic and material representations and the tools available, including for example the register and channel used to state the problem), and its *operations* (the ways of doing that are available). A task hosts or embodies a problem, gives the problem a possible life.

The third idea that is important to consider in the context of this work is that of “instructional situation.” Classroom encounters are not one-of-a-kind events, much in the same way that turns in talk are not unique. Every time somebody speaks to someone else they draw on an extant organization of social experience within which that conversation has at least a backdrop, if not a script (Goffman, 1997; see also Berne, 1996). I use the expression “instructional situation” to refer to the set of tacit agreements that teacher and students can default to when they (and in order to) undertake a task. Tasks (students’ work on problems) don’t occur in a vacuum. Indeed, the uptake and enactment of problems by students in school classrooms may serve different purposes and be labeled in different ways (Mason, 1999). These purposes and labels, though related to mathematics, are not so much related to the universe of possible epistemic actions that an individual might take in solving a particular problem, but rather related to the role that doing *that kind of work* customarily plays in fulfilling the curricular obligations included in the didactical contract of a class. In the US high school geometry class³ for example, some problems may fit an “exploration” (where students measure and manipulate diagrams to reach conclusions), others call for “construction” (where students use tools to make a diagram), others require “calculation” (where students use properties to set up and solve calculations to obtain dimensions of a figure), and yet others engage students in “doing proofs.” Each of them allows the teacher to lay a different kind of claim on students’ learning of the knowledge at stake. It is

² A mathematical treatment of this problem, including a proof of the fundamental theorem that angle bisectors of a quadrilateral intersect at a point if and only if the sums of the lengths of each pair of opposite sides of the quadrilateral are equal, can be retrieved from www.grip.umich.edu.

³ All instructional situations described in this paper are found in the US high school geometry class. No claim is being made that they exist in other courses of study or in other education systems.

conceivable that basically the same problem might elicit different kinds of thinking and acting on the part of students (i.e., might give rise to different mathematical tasks) according to the way in which the doing of such problem in the classroom is framed—that is depending on the instructional situation where the problem is posed. I use the word *situation* (after Goffman, yet not incompatibly with Brousseau’s use of *didactical situations*) to refer to the customary *units of work* or customary frames for classroom tasks.

I hypothesize that interaction in classrooms is organized around the performance of a number of instructional situations—those customary or default units of work. Original things may happen against the backdrop of those situations; when original things happen they often require negotiation; and, to be sure, some of those negotiations could lead to the establishment of new situations. Through a year of instruction a finite manifold of those instructional situations may get adapted from prior courses or developed anew to handle the teaching of the various ideas at stake. In the geometry class, “doing proofs,” “exploring a figure,” “calculating a measure,” “installing a theorem,” “constructing a figure,” and “doing proofs” are examples of those situations (see Herbst et al., in press; Herbst et al., in preparation; Herbst & Brach, 2006; Herbst & Nachlieli, 2007). Those situations are useful in geometry because they facilitate exchanges or transactions between, on the one hand, work that students do, and on the other hand, claims that the teacher can make on what has been taught and learnt. Each of those situations is like one marketplace in the larger economy of symbolic exchanges of the classroom: each of those situations makes room for students’ engagement in a kind of mathematical task and likewise each of those situations provides for the teaching of a particular kind of ideas (or the fulfillment of a particular kind of curricular obligation). A situation is thus a space for trade between work that classroom participants do (for example tasks that students engage in) and claims on curricular obligations that the teacher can make.

As argued in Herbst & Brach (2006) for the case of “doing proofs,” instructional situations host some tasks that are canonical or normal—tasks where everything that makes sense to do in response to them goes without saying in the context of that situation. For example, in a “proof” task it goes without saying that a student would justify all statements with a reason, but a task that relied on students drawing an auxiliary line would not be customary. Accordingly, one way to describe an instructional experiment where the angle bisectors problem was used is to say that one perturbs instruction by engaging the class in a task that is not customary in any one of the existing instructional situations. One does that in order to observe how instruction reacts to the perturbations induced by such task. One observes how a teacher and her students engage in two kinds of negotiation; each of those negotiations includes a particular kind of “repair” of the presumption that there is a fit between task and situation. One of those negotiations (negotiation of task) amounts to denouncing the task as unviable and engaging in transforming the task to fit the characteristics of the situation in which the class is operating. The other negotiation is a negotiation of the situation: in this negotiation the task is left untouched and what is negotiated are the norms that frame interaction about the task. We call “negotiation” the maneuvers made by participants to restore normalcy to their working relationship and we call “repair” the moves that they make to denounce that something is not according to norm. Quite often, repairs are among the initial moves of a negotiation. Those two negotiations, by the way are examples of what Brousseau would call negotiations of the didactical contract, in here referring to the

contract for a task or to the contract for a situation. We reserve the word “contract” by itself to the more general set of obligations that tie student and teacher to a course of studies.

Research Questions

Much current work on tasks is predicated on the presumption that engaging students in a task is beneficial to their learning. Scholars invested in the development of curriculum or in the creation of learning environments might operate on that presumption when they negotiate with a teacher the implementation of a task. In my work I investigate how a teacher handles that presumption, considering the hypotheses made above in regard to the teacher’s responsibility to operate a transaction between the work that the class does and the curricular obligations that the teacher needs to meet. In the specific case of a teacher who accepts to engage students in the angle bisectors problem, we ask first what is the instructional situation that can provide the initial context for students to work on that problem. In particular, in that situation

- 1) What kind of work does a class ordinarily do?, and
- 2) What curricular obligation(s) are usually at stake in that work?

In the specific case of the angle bisectors problem we ask how the angle bisectors problem probes that context, namely

- 3) How might the tasks that could possibly unfold from the angle bisectors problem conflict with the work the teacher would expect students to do in such situation?, and
- 4) How might the doing of such tasks challenge the teacher’s capacity to lay a claim on a curricular obligation?

Finally, we would also be interested in anticipating how a teacher (and her class) manages the perturbation that the emerging task imposes on that instructional situation, namely

- 5) How might a teacher manage the perturbation on the work of the class created by the tasks eventually associated to the angle bisectors problem? In particular, does the teacher engage in repairs that give evidence that instruction has been perturbed? Does the teacher promote a (a) negotiation of the task or (b) a negotiation of the situation? How so?
- 6) How might a teacher account for the time spent on the angle bisectors problem? Does the occasion create the need to “make up” objects of learning to give the experience some value? Is the experience written off as “time out”?

Our theoretical perspective would predict that those repairs would happen—we substantiate this below. We would predict that the decision to engage students in the angle bisectors problem can be viable on account of the existence of one or more default instructional situations within which the problem can be presented. But we predict also that such decision would be followed by a number of repairs whose purpose is to restore order. Some of those repairs will attempt to change the task into one that is more like those ordinarily hosted by the situation being played, whereas others will attempt to change to a different situation. Finally, we predict that the time spent will need to be accounted for in some way—either by the invention of an ad hoc stake, by the deliberate writing off of the time spent, or by identifying specific elements in the work done that can cash as the fulfillment of curricular obligations. This is the scope of the argument we make

below, showing that the theory (of classroom exchanges) allows one to identify a priori some phenomena of teaching.

What about the angle bisectors problem?

The problem we give to teachers for them to propose to their students is

“We know that angle bisectors of a triangle meet at a point. What about a quadrilateral?”

The problem is proposed as part of the unit on quadrilaterals, once students have studied the definitions of and some theorems about the special quadrilaterals. (But we have also run focus group sessions with teachers in which they are asked to envision this problem as the vehicle to teach the special quadrilaterals; see González and Herbst, 2007.).

We argue that, as proposed, a problem like that is ambiguous enough that it can fit in two of the instructional situations we have found in high school geometry classrooms (Herbst et al, in preparation): the problem can be posed inside a **situation of construction** as well as inside a **situation of exploration**. I describe those situations and their differences below. The teacher has the chance to choose the situation at the moment of designing how the problem will be presented to students. This moment involves making decisions about resources (representations and tools), even if she decides not to offer any other representation than the statement of the problem). Those decisions will shape the possible developments of work on the problem over time (the tasks) making it more like the tasks normally done in situations of construction or more like the tasks normally done in situations of exploration. Let’s be more specific.

In what situation might the angle bisectors’ problem initially fit?

We hypothesize the following resource variables being considered at the moment of the presentation of the problem to the class:

- (i) **Diagram** provided (5 values)
 - i. No diagram provided (ND)
 - ii. Diagram provided of a quadrilateral, four combinations of the following two independent variables
 - 1. With/without angle bisectors provided (WAB/WOAB)
 - 2. With/without unique intersection (WUI/WOUI) (see Figure 1)
- (ii) **Tools** provided
 - i. No tools (NT)
 - ii. Construction tools only (CT)
 - iii. Construction and measuring tools (MT)
 - iv. Dynamic Geometry software (DGS)

As we note above, by making those decisions at the onset of the problem the teacher establishes some possible paths for the problem to unfold over time: The decisions help shape what the task could be. The following table indicates how each combination of those variables turns the problem into a task closer to the canonical tasks of a situation of exploration or a situation of construction.⁴

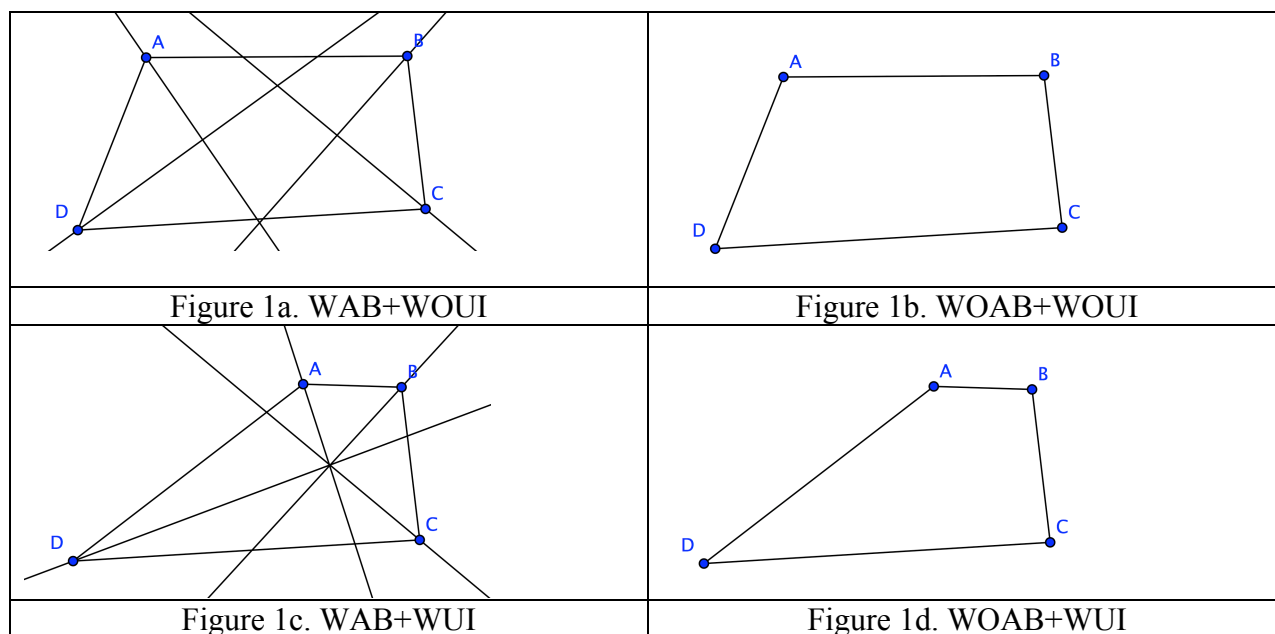
⁴ The value “none” for some cells in the table means that no existing situation could contain a task based on angle bisector’s problem with the corresponding assignments of Tools and Diagram.

Diagram/Tools	T1: NT	T2: CT	T3: MT	T4: DGS
D1: ND	None	Construction	Construction	Construction
D2: WAB+WUI	None	None	Exploration	Exploration
D3: WAB+WUI	Exploration	None	Exploration	Exploration
D4: WOAB+WUI	Exploration	Construction	Construction	Construction
D5: WOAB+WUI	Exploration	Construction	Exploration	Exploration

The table reports the relationship between the decisions the teacher might make in presenting the problem and the existing situation that would thus be evoked to provide context for the task of working on the angle bisectors problem. That process can be described generically as embedding the problem in an instructional situation by outlining a task that shares some aspects of the canonical task of one such situation.

Of course, the actual task that results may vary across choices made: while different combinations of choices yield the same situation (e.g., D3 and T1 yield an exploration just as much as D5 and T4 do), even at the onset one can anticipate the tasks to be different. The decision to give a quadrilateral that does not have unique intersections with its angle bisectors already drawn and no tools can enable a trivial exploration of the sort “how many intersections do they make” while the decision to give a quadrilateral that has unique intersection with its angle bisectors and measuring tools is more likely to enable an exploration of the metric properties of such kind of quadrilateral. The table shows that depending on what choices the teacher makes at the onset, the task may look more like **making** something (a diagram) or more like **saying** something (a statement on a figure).

Obviously, none of the possible tasks that result from the decisions made in the D and T variables *bound* the student to do things only one way; they don’t cleanly separate making diagrams from saying statements either. Some of those decisions, for example D5 and T3 would enable a bit of construction as part of the exploration. And students might always do other things than those they are enabled to by the teacher’s choices. For example, they could forgo any of the things provided, sketch another quadrilateral, freehand its angle bisectors, and deduce that pairs of angle bisectors make supplementary angles. I am not saying that such developments are impossible or even discouraged; I am saying that the teacher’s choices encourage looking at the problem as one like those other problems where students are expected to make something accurately (construct) or explore a given diagram. That encouragement comes at a cost, namely that doing something else, while entirely possible, is unexpected (in the sense that students can’t be held accountable for it) and thus might be a spontaneous source of perturbation on instruction (e.g., students might say “did you really expect us to do that?”).



The key distinction between the situations of construction (making a figure) and of exploration (stating about a figure) are proposed in terms of the register of the products required by the canonical tasks in that situation. In a situation of construction, the work to do includes as a main feature the production of a diagrammatic object. In a situation of exploration, the work to do includes as a main feature the production of a statement about abstract concepts. We include within the situations of exploration what is often labeled “conjecturing” by teachers, in which no tools other than visual inspection are involved and students are expected to produce a conjecture; explorations may also use instruments of measurement, mirrors, folding of paper, etc.

How could a task based on angle bisectors’ problem perturb the situation?

We now examine how a task based on the problem and presented in a given situation can perturb the normal characteristics of the situation. The case of the situation of construction with a given diagram of the quadrilateral is relatively straightforward: since the question is “what can be said” the task can easily become trivial (to show diagrammatically whether or not the angle bisectors meet at a point), especially if the diagram is provided. One would expect the teacher to issue that task to get students involvement creating one of the resources for a new task, and to transition next to a new situation (for example a situation of exploration) in which students would be asked to conjecture properties of the figure they made. If this were the plan, one double challenge from the first task comes in the form of the need for accuracy in the construction and the need for the whole task to take relatively short time. On the one hand the transactional value of that first task is low but on the other hand it has to be done carefully in order to facilitate the next task.

The construction tasks in which no diagram is given are more complex because they leave the students the choice of what quadrilateral to draw to begin with. We have used this task in multiple occasions and observed students engage in a range of behaviors, including choosing a special quadrilateral for which the angle bisectors will meet at a point, choosing a special quadrilateral for which students mistakenly take diagonals for angle bisectors, or choosing a

quadrilateral where students can show that the angle bisectors don't make a point. Our research group has created animations of classroom scenarios where two alternatives are pursued: in the movie "The Square" a teacher calls on a student who draws a square and its diagonals while in "The Kite" a teacher invites a student who had drawn a kite and its diagonals. Considering the work of the class as a whole, the yield of many different diagrams producing many different intersection figures is not too different from what we describe below—both in its affordances and in its challenges. The main challenge lies in the indefinite nature of the answer to the question—the answer becomes less definite the more time is spent on making constructions. One would expect this to challenge the extent to which the teacher can cash the work done for a particular curricular obligation, other than being able to construct different quadrilaterals.

Consider now the case when the problem is presented in a situation of exploration. For example a diagram like Figure 1a is given on the screen of a calculator equipped with DGS. We anticipate that the question "what can be said of the angle bisectors..." will be fast replaced to one of the following

P1--what can be said about the quadrilateral formed by the intersections of consecutive angle bisectors?

P2--what does it take to produce an interesting figure from the intersection of angle bisectors?

P3--what does it take to make the angle bisectors intersect at a point?

Tasks P1 and P3 focus the exploration on general statements about the figure. Ideally P1 could spur work oriented to stating the property that opposite angles of the quadrilateral formed by the intersections are supplementary, while P3 could spur work oriented to stating the property that if pairs of opposite sides of the given quadrilateral add to the same length, the angle bisectors meet at a point. The goal of the task in P1 is to describe the resulting quadrilateral, while the goal of the task in P3 is to describe the given quadrilateral. Operations available are similar across the two tasks—students can choose vertices to drag, and sides or angles to measure, they can look at the diagram, they can draw in auxiliary lines, and they can calculate with the available quantities. Some of those operations are more likely than others to be used. In particular, because the given quadrilateral and (except in the special cases of kite, rhombus, and square for P3) the quadrilateral that provides a unique point appear to perception so irregular, we anticipate that making measurements of segments or angles to verify what properties the figure have is unlikely to occur to students. Prior knowledge of the special quadrilaterals (what they are and what properties they have) will operate as an obstacle here in that students are likely to gravitate toward looking for the name of a shape as the answer to the question and will not naturally think of properties derived from operating on the measures. The problem thus challenges the situation of exploration in that the usual operations students do when they explore a figure are unlikely to include the operations they would need to do to the quadrilateral being explored in order to find a general statement. Rather, some of the operations they might do (dragging and measuring sides and angles) might lead them to state less than general statements: Both P1 and P3 could devolve into P2 and thus diverge from the general statements at stake.

Task P2 creates a very different kind of challenge. In here we anticipate students will drag the outside quadrilateral to look like a shape for which they have a name (parallelogram, trapezoid, kite, etc.) and observe what shape the angle bisectors make (which quadrilateral, or a point). The

feedback from what that figure appears to be may lead them to further drag the given quadrilateral. We expect students to find a host of correspondences between given quadrilaterals and intersection figures. The challenge of this task is in focusing on correspondences that attest to conceptually important properties. For example students might drag the given quadrilateral into a rhombus and notice that the diagonals of the rhombus are its angle bisectors, but they might also drag the given quadrilateral into a “dart” or “arrowhead.” The challenge posed by the task is not as much one of “can the students reach the goal?” as much as one of “can any goal reached by students count toward the procurement of an important mathematical stake?” I haste to say that most of these possible results from students’ explorations in P2 could be used to develop a valuable piece of mathematical work—but it would require from the teacher a key move of turning the situation from one of exploration to one of proving. Without that, the work could become one of offering a multiplicity of curiosities, thus challenge the situation of exploration not only in the sense that what ends up being explored may be a piece of geometric trivia (e.g., the angle bisectors of an isosceles trapezoid make a kite) but also that the class is occupied with a whole lot of things like that and the time needed to complete the exploration can expand too easily.

Management and accountability apropos of the angle bisectors’ problem

We are now in position to sketch our anticipation of what the teacher might do to handle the problems of management and accountability. These problems are represented in the following questions (listed before):

How might a teacher manage the perturbation on the work of the class created by the tasks associated to the angle bisectors problem?

How might a teacher account for the time spent on the angle bisectors problem?

We expect that, unattended, each of the tasks P1 and P3, can devolve into students’ ending the task claiming that the quadrilateral has nothing special or otherwise become P2. We anticipate that to keep the task focused on producing a general statement the teacher may manage that event by negotiating either the task or the situation. In particular,

The teacher may change the task into

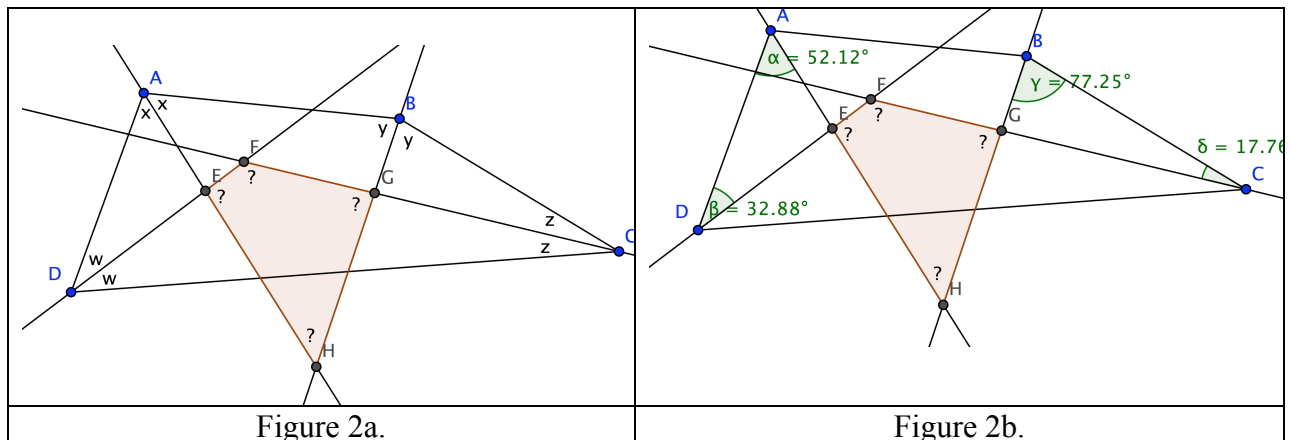
P4—“Measure this and that, do the following with those measurements, what do you notice?” (after either P1 or P3)

The teacher may change the situation into a situation of calculation

P5—Consider these to be the measures of the angles of the original quadrilateral, find the measures of the angles of the intersection quadrilateral (see Figures 2a and 2b for two possibilities after P1).

The teacher may change the situation into a situation of doing proofs

P6—Given these are the angle bisectors, prove $\angle FEH$ and $\angle FGH$ are supplementary (see Figure 2a, but give this figure without the angle variables, after P1; an analogous one could be proposed as a transformation of P3 into a proof exercise).



In the case of P2, we expect the teacher might repair the task by asking students to collect all different findings on a table and exploring whether any pattern is visible that describes, for example, when the angle bisectors meet at a point. Alternatively, we expect the teacher might repair the situation, turning it into a situation of doing proofs, for example by proposing

P7—Given a parallelogram and its angle bisectors, prove that the quadrilateral formed by its angle bisectors is a rectangle

Or

P8—Given ABCD is a kite BE bisects $\angle ABC$

Prove: DE bisects $\angle ADC$

(see Figure 3)

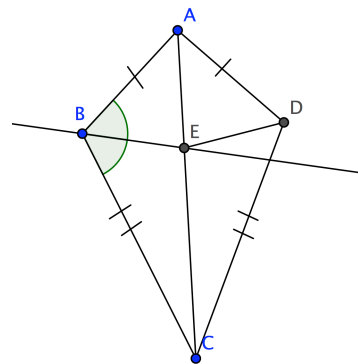


Figure 3.

This sketch of the kind of work that needs to be managed illustrates that the angle bisectors problem is likely to be a good tool to explore teaching. In particular, there exists a high chance that the teacher will negotiate a new situation in order to maintain the problem (in some version) on the floor. The second question becomes particularly important then: How might a teacher account for the time spent on the angle bisectors problem?

Our analysis above, while anchored on a DGS-based exploration could be reproduced for most other instrumentations. The interpretation of the problem according to task P2 and followed up by tasks like P7 or P8 is anticipated as one way in which the angle bisectors problem might find a stable existence in a geometry class. Such work might be accountable as a review of special

quadrilaterals and their properties. In the first task a broad range of quadrilaterals are surveyed tolerating some cranky ones like a “dart” and tolerating visual perception as the means of control in the benefit of broad recall of names and properties. This broad recall serves as motivation to later propose proof exercises that give students the opportunity to review definitions and properties of those quadrilaterals precisely. Likewise, the interpretation of the problem according to task P1 and followed by tasks P5 or P6 might be accountable as a serious application of the properties on the sum of the angles of a quadrilateral and of a triangle.

While the work done around the angle bisector’s problem might therefore cash as “review” or “application” of properties of quadrilaterals, it is clear that since such review could also be achieved through other tasks (e.g., worksheet with fill in the blanks for properties), the sustenance of the angle bisectors’ task would need to draw more justification from other aspects of the work done. That is, unless the experience with the problem could be billed as a case of something else that students are expected to learn in geometry, rather than just as review or application of what they have learned already about quadrilaterals, one could expect that this task would have little hope of survival.

Conclusion

This necessarily brief document outlines the kind of task analysis that I find helpful to do in my research. It hinges on differentiating three different constructs to talk about things that ordinarily are named “task” in the literature—the problem, the task, and the situation. That distinction helps map a set of phenomena in teaching—the negotiation of changes to the mathematical work (negotiations of task) and the negotiation of changes to the work environment that frames such mathematical work as valuable (negotiations of situation). The idea of this analysis is to suggest that task and situation are complementary mechanisms of classroom interaction, the existence of a situation can help a task become viable even if this task is not canonical, while the perturbations a task inflicts on instruction may lead to negotiations that expand the range of acceptable actions, eventually expanding the situation. Those two mechanisms operate at a surface level, they are fundamentally elements of a language of description of classroom interaction—a language that can help analyze the mathematics embedded in action as well as the mathematics at stake. Inasmuch as an analysis like this can help anticipate classroom events, it can help curriculum developers create and provide supports for teachers to manage the enactment of curricula and it can help classroom observers look at teaching with empathy.

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