# THE UNIVERSITY OF MICHIGAN INDUSTRY PROGRAM OF THE COLLEGE OF ENGINEERING

DOSE RATES PRODUCED FROM GAMMA RAY SOURCES

J. G. LEWIS

#### PREFACE

This work was undertaken to provide a concise statement of certain methods and approximations useful in the estimation of physical dose rates in air in regions adjacent to sources of gamma radiation. The intent is primarily to provide working methods which yield approximate answers rapidly. To this end certain simplifying assumptions have been made and graphical methods of calculation have been included.

#### DOSE RATES PRODUCED FROM GAMMA RAY SOURCES

#### I. ABSTRACT

In this paper are presented some assumptions, methods, equations, and nomograms which have been employed to estimate physical dose rates in air due to sources of gamma radiation of selected shapes.

A method for estimating dose rates in air near hollow cylindrical sources is reviewed briefly, and it is shown how a general equation degenerates to simpler forms for selected special shapes of the sources and for locations of the point at which dose is calculated.

Nomograms are presented for the simplified estimation of dose rates due to hollow cylindrical sources and point sources.

## II. INTRODUCTION

The emphasis in this work is to summarize some methods for calculating problems which permit estimating gamma dose rates in configurations and under conditions frequently encountered in irradiation work. One purpose is to show the relationships in special cases which have been treated previously in calculating gamma dose rates. Another purpose is to present nomograms which permit rapid estimation of gamma dose rates in air. Rapid estimating procedures are of particular value to the engineer who does not have a wide background in nuclear studies, but who wishes to assess quickly the potentialities and requirements for radiation effects and experimental installations requiring the use of gamma radiation. To this end, it is suggested that the dose rates in air calculated from the charts presented here may be corrected for absorption of such things as pressure vessel walls, source containers, etc., by rough approximations taking account of exponential decrease in dose rates with thickness of thin layers of absorbing materials.

The use of sources of radiation has increased rapidly during the past few years because of the increased use of nuclear reactors. Reactors have made radiation available in a number of forms. Among these are beams of neutrons emitted directly from reactors, and gamma and beta radiation emitted during fission or resulting from delayed emission from the radioactive fission products. In addition nonactive materials may be placed in nuclear reactors and made radioactive by the absorption of neutrons. Cobalt-60 is produced in this manner, for chemical, radiographic, radiotherapeutic, and other biological work. Many other isotopes such as radioiodine, radiophosphorus, and radiogold, may be produced by neutron absorption for medical and other tracer work. Radiocesium and radiostrontium may be separated from nuclear fission products and employed in a variety of biological and chemical irradiations.

The nuclear radiation emitted by most nuclides of interest to irradiation work consists chiefly of electrons (beta particles) and electromagnetic emissions (gamma radiation). Pose rates due to beta radiation are not treated here. Dose rates due to gamma radiation are considered in this paper.

A knowledge of physical dose rates in air near sources of gamma radiation is of importance in handling and using many kinds of radioactive materials. Gamma radiation must be dealt with in such a way that personnel are not subjected to excessive exposures to the radiation. In addition, any applications of gamma radiation require some quantitative knowledge of the amounts of radiation received by a system of interest, to permit correlation of observed effects with the radiation.

Personnel protection requires shielding and problems encountered involve required thickness, shapes, and weights of the shielding materials. These in turn bring in consideration of absorption, scattering, dispersion, and other interactions of radiation and matter, e.g., as discussed by Moteff (7), and Goldstein and Wilkins (1). Applications of gamma radiation to irradiation of systems also involves these effects. However, the present discussion neglects such effects and concentrates upon the problem of rapid estimation of dose rates in air. Moderate thicknesses of air under ordinary conditions attenuate primary gamma radiation so slightly that the effect of air can be ignored in approximate calculations.

#### III. DOSE RATES IN AIR DUE TO SOURCES OF SELECTED SHAPES

In this discussion various equations are reviewed, relating the gamma dose rate to the characteristics of the source of radiation. The unit of gamma dose rate used is the "roentgen equivalent physical", often abbreviated as "rep", as discussed by Parker (8). The rep will be considered to correspond to the absorption of 93 ergs of energy from gamma radiation per gram of absorber (1 rep corresponds to about 6.4 x 10<sup>-12</sup> Btu/1b of absorber). The precise definition of units of dose rate is complicated by considerations such as atomic number of the absorber and energy spectrum of the gamma radiation, but these will not be considered further in this paper. The rep is only one of many units which have been suggested for the reporting of dose rates. See Siri (10).

A brief review of units and conversion factors is presented here for convenient reference:

It may be noted that curies, a commonly used measure of the radio-activity of a material, is analogous to current in ordinary electrical practice, in that it gives a measure of the number of nuclear emissions per unit time, without regard to the energy of each emission. Many gamma rays are emitted from decaying nuclei with characteristic energies, analogous to the spectral lines emitted by luminous bodies. The characteristic energy of emission of the gamma rays is often reported in millions of electron volts (mev). Consequently, the emission energy is analogous to voltage in electrical practice, and the product of curies and mev gives a measure of the power cutput of a source of radiation. A radioactive material often emits both beta and gamma rays of different energies and sometimes distributed among more than one decay scheme. Consequently, caution must be employed in drawing analogies such as these given above. Activities ensountered in practice may vary from microcuries (one microcurie equals  $10^{-6}$  curie) to megacuries (one megacurie equals  $10^{6}$  curie).

As an illustrative example, if a nuclide were emitting only one gamma of 1 mev per disintegration at a rate of one million curies, the power output of this source would be:

1 mev x 4.43 x 
$$10^{-20} \frac{\text{kwh}}{\text{mev}} \times 10^6 \text{ curies x 3.7 x } 10^{10} \frac{\text{disinteg.}}{\text{(sec.) (curie)}} \times 3600 \frac{\text{sec.}}{\text{hr}} = 3.9 \text{ kw.}$$

One million curies is a large source of radiation, and its power output of 5.9 kilowatts is not large compared with conventional sources of power. The relatively small power available from what are now regarded as large sources of radiation illustrates the need of utilizing either very large sources of radiation or using radiation for applications not suited to other sources of energy where power is the primary consideration.

It should be noted that the total power output of a radioactive source, 5.9 kilowatts in the example cited above, will not all be intercepted by a given specimen or absorber to be irradiated. This observation again leads to the discussion of dose rates, as follows.

The radiation power is distributed about a radiation source in complicated ways, which depend largely upon the shape of the source and the location of the absorber. If the activity of the source is uniformly distributed within the source and of uniform composition, then the estimation of radiation power absorbed at a point in the absorber (equivalent to estimation of the dose rate at the point in the absorber) becomes chiefly a problem in relating geometrical variables of source and absorber. The following discussion is to relate the geometrical and radiation variables concerned.

Lewis, et al. (4) have discussed equations for estimating dose rates near hollow cylindrical sources of gamma radiation. It is proposed in this section to discuss various forms into which the general equation may degenerate for certain restricted geometrical situations. Consider Fig. (1), a hollow cylindrical source having negligible wall thickness.

The dose rate I at point P may be expressed by Equation (1).

$$I = \frac{MC}{L(R+r)} [F (tan^{-1} \frac{Z_1}{|r-R|}, k) - F (tan^{-1} \frac{Z_1-L}{|r-R|}, k)]$$
 (1)

for  $Z_1 \ge 0$ ,  $R \ge 0$ , r > 0,  $R \ne r$ ,  $-\pi/2 < \tan^{-1} (Z_1-L)/|r-R| < \pi/2$ ,

$$0 \le \tan^{-1} \frac{Z_1}{|r-R|} < \pi/2; k = \frac{2\sqrt{Rr}}{R+r}$$
.

It should be noted here that the expressions

$$F \left( \tan^{-1} \frac{Z_1}{|r - R|}, k \right)$$
 (la)

and

F 
$$(\tan^{-1} \frac{Z_1 - L}{|r - R|}, k)$$
. (1b)

are geometrical functions of the bottom and top, respectively, of the hollow cylindrical radiation source as "seen" by the absorbing point at  $P(R,\theta_0,Z_1)$ , and hence may be considered as analogous to the geometrical factors encountered in radiant heat transfer, cf. McAdams (6). Expressions (la) and (lb) are "elliptic integrals of the first kind", tables of which appear in a number of sources, e.g., Jahnke and Emde (2). It is interesting to note that the elliptic integrals of the first kind shown in Equations (la) and (lb) degenerate to circular angles expressed in radians when k=0, corresponding in Equation (1) to the cases where R=0 or  $R\to\infty$ , i.e.,

$$\lim [F (\tan^{-1} \frac{Z_1}{|r - R|}, k)] = \tan^{-1} \frac{Z_1}{|r - R|} .$$
 (lc)

It is of further interest to note that the ellipse to which these functions refer may have a major axis of R + r, parallel to the base of the source, and a minor axis of R - r, with center located at  $(R + r, \Theta, \mathbb{T}_1)$ .

The following are special cases which may be developed from Equation (1) under the conditions noted:

A. Let R=0, i.e., let point P be on the axis of the source. Therefore, k=0, and consequently the elliptic integrals of the first kind degenerate to circular angles.

$$I = \frac{MC}{rL} \left( \tan^{-1} Z_{\underline{1}} / r - \tan^{-1} \frac{Z_{\underline{1}} - L}{r} \right) . \tag{2}$$

B. Let r = 0, i.e., let the cylindrical source degenerate to a line.

$$I = \frac{MC}{LR} \left( \tan^{-1} \frac{Z_1}{R} - \tan^{-1} \frac{Z_1 - L}{R} \right) . \tag{3}$$

C. Let  $Z_1 = L/2$ , i.e., let point P be always on the mid-plane of the cylindrical source.

$$I = \frac{2MC}{L(R+r)} \left[ F \left( \tan^{-1} \frac{L}{2|r-R|}, k \right) \right]. \tag{4}$$

D. Let L  $\rightarrow \infty,$  i.e., let the source become very long compared with r and R.

$$I = \frac{2MC}{L(R+r)} [K(k)]. \qquad (5)$$

E. Let R  $\to \infty$ , i.e., let R become very great compared with  $Z_1$ , r, and L. Then  $\lim_{R\to\infty} k=0$ , and the elliptic integrals again degenerate to circular angles, as in "A", above. For R >> r, L,  $Z_1$ , then  $\tan^{-1} |Z_1/| r-R|$  approaches  $|Z_1/| r-R|$  and  $|Z_1-L/| r-R|$  approaches  $|Z_1-L/| r-R|$  and the following relation results:

$$I = \frac{MC}{|r^2 - R^2|}$$

which can be further reduced to:

$$I = \frac{MC}{R^2} . (6)$$

That is, the source behaves as a point source when the receiving point is remote in a direction transverse to the axis of the source.

F. Let  $Z_1 \to \infty$ , i.e., let  $Z_1$  become very great compared with R, r, and L. For convenience, put Equation (1) in the form:

$$I = \frac{MC}{L (R + r)} [F (tan^{-1} \frac{R + r}{Z_1 - L}, k) - F(tan^{-1} \frac{R + r}{Z_1}, k)].$$
 (le)

An analysis similar to that of "E" results in the following:

$$I = \frac{MC}{Z_1^2} , \qquad (7)$$

which is also of the form for a point source.

### IV. SOME PROPERTIES OF RADIOACTIVE MATERIALS

Various methods are available for reporting the ionizing power of radioisotopes. In the discussion of Section III, the quantity "M" was introduced, related to the methods of Marinelli, et al. (5). The quantity M is the "roentgen equivalent physical" per hour produced in an absorber located one cm from a point source of radiation of one curie activity.

Moteff (7) has reported plots of the gamma radiation in mev per square centimeter per second required to produce a dose rate of one roentgen per hour as a function of the energy of the radiation. A similar relation appears in Figure 2, showing the M value plotted as a function of mev of the radiation, assuming 93 ergs absorbed per gram per rep. Figure 2 was plotted on the assumption of one gamma ray emitted per disintegration. If more or fewer are emitted in the average disintegration, the M value to be used with this paper should be corrected by simple addition of the contributions of each ray emitted.

#### V. GRAPHICAL ESTIMATION OF DOSE RATES

## FROM HOLLOW CYLINDRICAL SOURCES

Figure 3 is a nomogram which provides a graphical means of solving Equation (1), above. Equation (1) relates activity in curies, C, specific ionizing power of the radioactive material in the source, M, and the various dimensions of the system comprising source and absorbing point. The hollow cylindrical source configuration was chosen for solution since it is a shape frequently encountered in practice and also lends itself well to the approximation of other shapes. For instance, if the radius, r of the source becomes small compared to other dimensions, the cylinder approaches a rod or line source. If the height, L, becomes small compared to other dimensions, the cylinder approaches a ring source. If all dimensions of the source become small compared with the distance between the absorbing point and the source, the cylinder appears to approach a point source.

Provision is made in Figure 3 for direct solution with dimensions in either inches or centimeters. Scale factors are also introduced for taking account of ranges of variables greater than those scaled off directly on the chart. "Preferred ranges of variables" are indicated. If data are entered in these ranges, solutions for I, the dose rate, should not fall off the chart.

An illustrative example is solved below. See Figure 1 for meanings of dimensional symbols. Dashed index lines indicate the mode of solution in Figure 3. Take a source with the following characteristics:

$$M = 13,500 \frac{\text{rep at 1 cm}}{\text{hr x curie}} \text{ (cobalt-60)},$$

$$C = 3000 \text{ curies},$$

$$L = 10 \text{ inches},$$

$$R = 0.3 \text{ inches},$$

$$r = 4.7 \text{ inches},$$

$$Z_1 = 15.5 \text{ inches}.$$

(Absorbing point 0.3 inch from axis 15.5 inches above bottom of source.)

Compute R/r, here 0.0638; enter chart at upper right plot. Read up to curve and then to left, intersecting the index line at  $\alpha=28^{\circ}$  - 21'. (Note,  $\alpha=\sin^{-1}k=\sin^{-1}0.4749$ .) This operation identifies the proper  $\alpha$  line to use in the next step below. Next compute  $Z_1/|R-r|$ , 3.5227; and  $Z_1-L/|R-r|$ , (1.25). Enter both these values on the right side of the bottom

center plot. Project to the left to the tan-1 curve, and then project up to the plot above, intersecting the  $\alpha=28^\circ$  - 21' line found in the previous step. Project left from the  $\alpha=28^\circ$  - 21' line to the ordinate of the  $F(\emptyset,k)$  vs.  $\emptyset$  plot. Read  $F(\emptyset,k)$  for both  $Z_1/|R-r|$  and  $Z_1-I/|R-r|$ , where  $\emptyset$  is  $\tan^{-1} Z_1/|R-r|$  or  $\tan^{-1} Z_1-I/|R-r|$ . Subtract  $F(\tan^{-1} Z_1-I/|R-r|$ , k) from  $F(\tan^{-1} Z_1/|R-r|$ , k) giving  $\Delta=F(\emptyset_1,k)$  -  $F(\emptyset_2,k)$  = 1.3600 - 0.9207 = 0.4393.

This value of  $\triangle$ , 0.4393, is now transferred to the left scale of the ordinate line of the  $F(\emptyset,k)$  vs.  $\emptyset$  plot. The determination of  $\triangle$  (the term in brackets of Equation (1) ) just described is the most tedious part of the solution.

Next, the coefficient of the brackets in Equation (1) is combined with  $\Delta$  to find the dose rate, as follows:

In the lower left corner of the chart, locate L, 10 inches, and project through the (R+r) scale at 5.0 inches, to find L (R+r), 50 inches, on the L (R+r) scale. Now move to the M/10<sup>3</sup> scale, lower left of chart, and locate M/10<sup>3</sup> = 13.5 for cobalt-60. Project through the C scale at 3000 curies to find MC=40,500,000 in this example. Now connect A with L (R+r). Locate the point of intersection of the connecting line with the arbitrary index line. Connect this point on the arbitrary index line with the value of MC previously found, and project to the I scale, where the dose rate will be found (0.055) in this example), subject to correction of the decimal point. Multiply the value (0.055) read from the I scale by  $10^{[Q+S+\bar{J}-(N+P)]}$ , (In this example,  $10^{[3+0+3-(0+0)]} = 10^{(1)}$ ), the cumulative correction factor resulting from decimal point changes required to get all variables on scale.

The resulting dose rate is  $0.055 \times 10^6 = 55,000 \text{ rep/hr}$  in air at the location chosen.

Some precautions to observe are the following:

When R = 0, then k = 0. Consequently,  $\alpha = \sin^{-1}$ , k = 0, and one does not need to use the upper right-hand plot of k vs. R/r. One simply uses the  $\alpha = 0$  line in the plot of  $F(\emptyset,k)$  vs.  $\emptyset$ .

In determining  $\Delta = F(\phi_1, k) - F(\phi_2, k)$ , when  $0 = Z_1 \le L$ , that is, the absorbing point lies between the top and bottom altitudes of the source, then  $Z_1$  - L is negative. This makes the second term in brackets in Equation (1) negative. The subtraction of the negative second term then results in the addition of its absolute value to the first term in brackets.

The  $\Delta$  found by subtraction must be re-entered on the left side of the ordinate line of the F( $\emptyset$ , k) vs.  $\emptyset$  plot.

A record must be kept of the exponents, N, P, Q, S, used to put all values on scale and used in the final correction factor.

Answers may be read as described above if either inches or centimeters are used for dimensions, but the right scale of  $M/10^3$ , must be used for inches and the left scale for centimeters. Any other units may be used for dimensions of length L, R, r,  $Z_1$ , but the values of I calculated from the Figure 3 must then be corrected for the (length)2 factor in the denominator of 1M, e.g., if lengths used are in feet, and the  $M/10^3$  scale for inches is used in reading Figure 3, then values of I read from Figure 3 must be divided by  $(12)^2 = 144$ .

Figure 3 may give inaccurate results if the absorber is remote compared with source dimensions, since it will be observed that the various values of  $\alpha$  all give nearly the same (F ( $\emptyset$ ,  $\alpha$ ) for the small values of  $\emptyset$  which result in this case. If values of dose rate on the midplane of the cylinder are desired, and R is sufficiently great that the included angle subtended by the top and bottom of the source is equal to or less than 45°, then one may treat the source as a point with an error no greater than about 11%. See below for a graphical solution of the point source case.

## VI. GRAPHICAL ESTIMATION OF DOSE RATES FROM POINT SOURCE

Figure 4 is a nomogram for the estimation of dose rates from a point source of radiation. The symbols have the meanings given above, except that r,  $Z_1$ , and L have no application, and R is the distance from source to absorber.

The solution of a case is illustrated by dashed index lines.

The resulting value of dose rate read from Figure 4 is 10,900 rep/hr. Numerical calculation yields a value of:

$$I = \frac{MC}{R^2}$$
= (13,500)  $\frac{\text{rep at 1 cm}}{\text{hr curie}} \times \frac{\text{in.}^2}{6.45 \text{ cm}^2} \times \frac{3000 \text{ curies}}{(24)^2 \text{ in.}^2}$ 
= 10,899 rep/hr in air.

#### VII. ACKNOWLEDGEMENTS

The material contained in this paper was developed in part under the auspices of the Industry Program, College of Engineering, the University of Michigan.

The author wishes to thank the Engineering Research Institute, University of Michigan for permission to present this paper.

The author is grateful to H. A. Ohlgren, Professor, and J. J. Martin, Associate Professor, of the Department of Chemical and Metallurgical Engineering, the University of Michigan, for advice and encouragement in the preparation of this paper.

Mr. Richard Dilley, Mr. Gilbert Marcus, Mr. Howard Silver, and Mr. Wayne Thiessen have assisted the author in calculations, drafting, and criticism of the nomograms presented.

## VIII. DEFINITIONS OF SYMBOLS

I = dose rate, rep per hour,

A = area of source,

r = radius of source,

R = radial distance of point at which I is taken from axis of source,

 $\Theta$  = central angle from R to r,

Z = distance parallel to axis of source from base of source to element dA,

 $Z_1 = Z$ -coordinate of point at which I is taken,

 $k = 2 \sqrt{Rr}/(R + r),$ 

 $\alpha = \sin^{-1} k$ ,

 $F(\phi,k)$  = elliptic integral of first kind of modulus k and amplitude  $\phi$ ,

- K(k) = complete elliptic integral of first kind of modulus k,
  - C = total curies,
  - M = (roentgen equivalent physical at 1 cm)/(hr. x curie point source),
  - L = length of source.

## IX. BIBLIOGRAPHY

- 1. Goldstein, H., and Wilkins, J. E., NYO-3075 (1954).
- 2. Jahnke, E., and Emde, F., <u>Tables of Functions</u>, 4th ed., Dover, New York (1945).
- 3. Kaplan, W., Advanced Calculus, Addison-Wesley, Cambridge (1952).
- 4. Lewis, J. G., Nehemias, J. V., Harmer, D. E. and Martin, J. J., <u>Nucleonics 12</u>, No. 1, <u>40</u> (1954).
- 5. Marinelli, L. D., Quimby, E. H., and Hine, G. J., Am. J. Roentgenol. Radium Therapy 59, 260 (1948).
- 6. McAdams, W. H., Heat Transmission, 3rd Edition, McGraw-Hill, New York (1954).
- 7. Moteff, J., APEX-176 (1954).
- 8. Parker, H. M., Advances in Biological and Medical Physics, Vol. 1, Academic Press, New York (1948).
- 9. Sievert, R. M. Acta Radiologica I, 89 (1921).
- 10. Siri, W. E., <u>Isotopic Tracers and Nuclear Radiations</u>, McGraw-Hill, New York (1949).

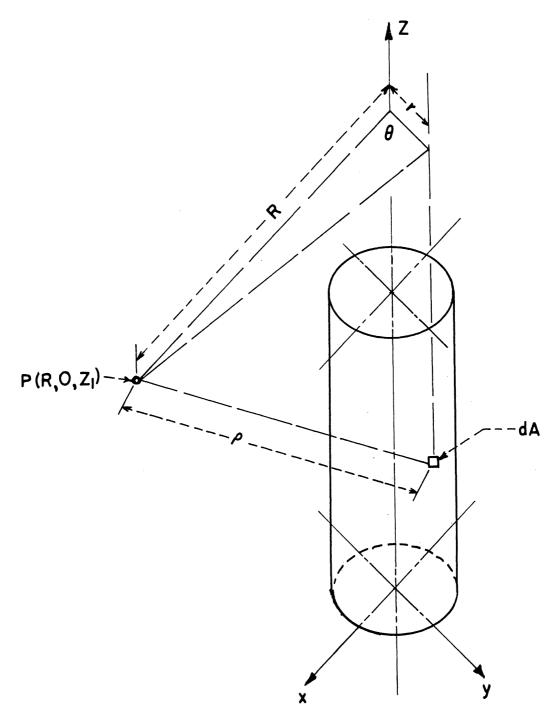


FIG. 1 - GEOMETRY OF CYLINDRICAL SOURCE HAVING NEGLIGIBLE WALL THICKNESS

FIG. 2 - DOSE FACTORS FOR GAMMA RADIATION AS FUNCTIONS OF ENERGY

