LETTERS TO NATURE

Character of 'prolate' galaxies

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It is still not known whether elliptical galaxies are oblate, prolate or triaxial. The prototype 'prolate' galaxy NGC2685 has a rapid observed stellar rotation rate around the projected minor axis. We show here that a steady figure, exactly prolate model can have no such rotation, and that a nearly prolate model fails by a factor of 7. We suggest that the galaxy is oblate and that the gas disk is the wreck of a dwarf companion.

There has recently been much interest over the determination of the three-dimensional figure of elliptical galaxies. This interest can be classified into four areas—statistical studies of surface brightness[1-3], theoretical models[4,5], detailed surface photometry[6-8] and the study of the distribution of embedded gas[9]. The last approach is distinguished by its potential ability to make statements about individual galaxies.

In the absence of figure rotation (for example, the tumbling of a bar) a gaseous disk is thought to settle into the fundamental plane of a biaxial galaxy in a time scale given by

$$\tau \approx T\left(6J_2 \cos\theta \left|\frac{\mathrm{d}\ln T}{\mathrm{d}\ln r}\right| \frac{\Delta r}{r}\right)^{-1} \quad (1)$$

where τ is the settling time, T is the orbital time, J_2 is the quadrupole moment of the gravitational potential, θ is the angle between the disk plane and the fundamental plane of the galaxy, r is the disk radius and Δr is the characteristic excursion in radius of gas clouds in the disk[10]. That excursion can be related to the peculiar velocity Δv of the gas clouds through $\Delta r = \Delta v/K$ where K is the epicycle frequency. Specializing now to a logarithmic potential (flat rotation curve), we can use $K = \sqrt{2}v_c/r$ (where v_c is the circular velocity) to write

$$\tau \approx T\left(3\sqrt{2}J_2 \cos\theta \frac{\Delta v}{v_c}\right)^{-1} \quad (2)$$

Noting that J_2 is <0.5 even for very flat galaxies and that $\Delta v/v_c \approx 0.1$ (a reasonable assumption), we see that the settling time from an inclination of 60° or more is always $> \tau_0 \sim 9T$.

The fact that the characteristic settling time is less than the age of the Universe has been used to argue that NGC2685[11] is, in fact, a prolate object with a small pattern speed[12].

The next step is to construct a prolate stellar dynamical model for NGC2685 which is consistent with the observed stellar rotation. We attempted such a construction as part of a systematic study of prolate stellar systems using Schwarzschild's[4] self-consistent field method. That method chooses a specific potential, follows stellar orbits in that potential and finally adds up the time averaged stellar orbits to produce a density distribution corresponding to the original potential. We used a logarithmic potential with a flattening corresponding to an E5 galaxy[5].

Such a model must reproduce the large observed rotation rate about the projected minor axis. The observed rotation parameter v/σ is ~ 1 (ref. 13), while σ is ~ 100 km s^{-1}. (This dimensionless rotation rate is 3–10 times the normal observed value for an elliptical, and is much more consistent with an S0 disk). There is some evidence also for minor axis notation. If this is true, it would suggest that the galaxy is triaxial.

Because the figure is fixed, the observed rotation must be derived from orbits which stream through the figure. Such an orbit is shown in Fig. 1*a*. Maximizing the polar streaming velocity (using linear programming techniques) permits a maximum streaming velocity—corresponding to an observed v/σ of only 0.075 on the projected major axis. Moreover, this large streaming velocity can only be obtained if the y component l_y of the total angular momentum of each orbit is >0 for all $l_z = 0$ orbits (no orbit circulates in the opposite sense). Because these orbits are only neutrally stable against perturbations which cause rapid precession about the z axis, this configuration is implausible. Even if the orbits were stable, the model's v/σ completely fails to reproduce the observed one of unity.

A model more consistent with the observations can be constructed with a slightly triaxial figure. The orbit of Fig. 1*a* can easily be followed in a gravitational potential of the form

$$\Phi = \frac{1}{2}\ln\left(x^2 + \frac{y^2}{p^2} + \frac{z^2}{q^2}\right) \quad (3)$$

If $p < 1$ the orbit is stable but l_z is no longer conserved and the orbit is no longer required to reach $z = 0$ frequently (see Fig. 1*b*). Therefore a larger number of these orbits can be incorporated into the model. A nearly prolate model was constructed maximizing the influence of these streaming orbits with $p = 0.9$. Viewed from the x axis (edge on) its $v/\sigma = 0.1$. The maximum v/σ was achieved from $\sim 30°$ from the x axis in the x–z plane. From that vantage point $v/\sigma = 0.14$. Thus we cannot construct a nearly prolate model with no figure motion and large v/σ. We emphasize that this method maximizes v/σ within the set of all self-similar solutions of this general form.

The failure to construct a figure fixed prolate model with large v/σ indicates that the galaxy is not prolate. An alternative explanation for the gas ring is that it is the wreck of a dwarf

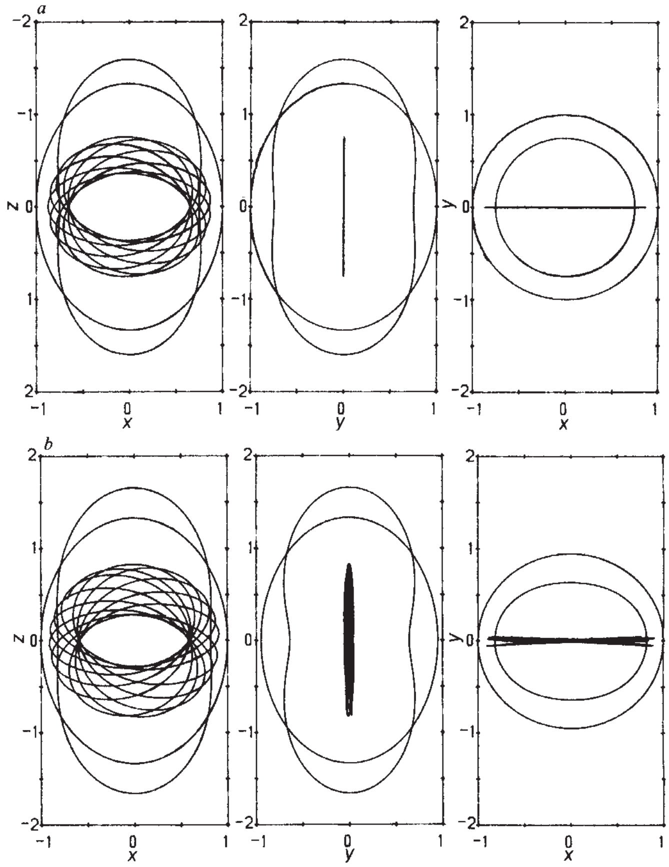

Fig. 1 *a*, Three orthogonal views of an orbit in a fixed biaxial prolate potential (with $p = 1$, $q = 1.333$, see equation (3)). The envelopes are cuts through the equipotential and equal-density surfaces. Note that the orbit streams through the galaxy maintaining a well defined circulation in the x–z plane. This orbit has no z angular momentum and is only neutrally stable. *b*, As *a* but with a triaxial potential with $p = 0.95$ and $q = 1.333$. This orbit also streams. It is a stable orbit with fluctuating z angular momentum.

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companion, which has not yet decayed to the fundamental plane of the galaxy.

Such a scenario does not, by itself, constrain the age of this gas disk. The disk is very inclined, and equation (2) suggests that for exact polar alignment ($\theta = 90°$), the disk lifetime is infinite.

In fact, equation (2) for the settling time also provides the means to calculate the time independent (or ensemble average) distribution of ring angles. From equation (2), $d\mu/dt \propto \mu^2$ where $\mu = \cos\theta$ is the cosine of the tilt angle ($\mu = 0$ for a polar ring). Because rings are inserted at some angle and then decay to the plane, they obey a continuity equation of the form

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial \mu}\left(\frac{d\mu}{dt} n\right) = \left(\frac{\partial n}{\partial t}\right)_i \quad (4)$$

where $n = n(\mu, t)$ is the chance of finding a disk at inclination μ at time t, and $(\partial n/\partial t)_i$ is the insertion rate. In steady state, $\partial n/\partial t = 0$. For insertion in strictly polar orbits it is easy to show that $n \propto \mu^{-2}$, while for an insertion rate independent of μ, $n \propto \mu^{-1}$ (to verify these results it is necessary to apply the appropriate boundary conditions at $\mu = 0$).

Note that the above results indicate a preference for disks in non-steady state but relatively long-lived nearly polar configurations regardless of the injection rate. The number of inclined disks will, however, be set by the ratio of the injection rate to the decay rate. Since the latter is $\approx \tau_0 \approx 10T$ ($\approx 10^9$ yr), a frequent incidence of tilted disks is required to demand an insertion time scale much shorter than the age of galaxies. However, these objects may be more common than we realize judging from a flurry of recent discoveries (ref. 14 and P. Schechter, unpublished data). Note that our Galaxy may be about to acquire such rings as the Magellanic clouds come apart[15].

Although we have presented a strong case for the view that this particular galaxy is axisymmetric with two long axes, we have not explicitly ruled out a slowly tumbling bar.

There is, however, an argument which suggests that slow figure motion does not alter the small values of v/σ derived above. Schwarzschild[16] and Miller and Smith[17] have constructed models of tumbling triaxial and biaxial prolate galaxies of very different rotation rates. In both studies the observable rotational velocity is never more than 1.5 times the pattern speed at any radius. Because the pattern speed must be small to avoid disrupting the polar rings in the galaxy under consideration if it is a tumbling bar[12], these models also indicate a small value of v/σ.

We conclude that it is easy to model NGC2685 as an oblate galaxy with polar rings of gas which are the wreck of a dwarf companion. It is impossible to construct a fixed figure prolate model which reproduces the observed stellar rotation rate. It is doubtful that a tumbling prolate model would resolve the discrepancy. It may be that a number of galaxies which have been identified as prolate objects because of embedded disks are in fact oblate objects.

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1. Marchant, A. B. & Olson, D. W. *Astrophys. J. Lett.* **230**, L157–L159 (1979).
2. Richstone, D. O. *Astrophys. J.* **234**, 825–828 (1979).
3. Olson, D. W. & de Vaucouleurs, G. *Astrophys. J.* **249**, 68–75 (1981).
4. Schwarzschild M. *Astrophys. J.* **232**, 236–247 (1979).
5. Richstone, D. O. *Astrophys. J.* **238**, 103–110 (1980).
6. Williams, T. B. & Schwarzschild, M. *Astrophys. J.* **227**, 56–63 (1979).
7. Williams, T. B. & Schwarschild, M. *Astrophys. J. Suppl.* **41**, 209–213 (1979).
8. Bertola, F. & Galleta, G. *Astr. Astrophys.* **77**, 363–365 (1979).
9. Bertola, F. & Galleta, G. *Astrophys. J. Lett.* **226**, L115–L118 (1978).
10. Tohline, J. E., Simonson, G. F. & Caldwell, N. *Astrophys. J.* **252**, 92–101 (1981).
11. Sandage, A. *The Hubble Atlas of Galaxies*, 7 (Carnegie Institution, Washington DC, 1961).
12. Tohline, J. E. & Durisen, R. H. *Astrophys. J.* **257**, 94–102 (1982).
13. Schechter, P. L. & Gunn, J. E. *Astr. J.* **83**, 1360–1362 (1978).
14. Hawarden, T. G., Elson, R. A. W. Longmore, A. J., Tritton, S. B. & Corwin, H. G. *Mon. Not. R. astr. Soc.* **196**, 747–750 (1981).
15. Tremaine, S. D. *Astrophys. J.* **203**, 72–74 (1976).
16. Schwarzchild, M. Preprint (Princeton University, 1982).
17. Miller, R. H. & Smith, B. F. *Astrophys. J.* **227**, 785–797 (1979).

Composition-dependent glass-transition temperatures and copolymers

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The glass transition is of primary relevance to the forming conditions and end-use properties of polymeric materials. Several polymers of industrial importance, most notably polyvinyl chloride (PVC), contain small molecules as processing lubricants (plasticizers) and are, therefore, solutions. Separately, economic and materials advantages have spurred the use of one-phase mixtures (miscible blends) of high polymers. The general theme of the compositional variation of glass-transition temperatures (T_g) includes, in addition, the variation of T_g with degree of polymerization, with cross-link density (network formation) and with copolymer composition. An attempt is made here to find a unified basis for the prediction of these several aspects of an identifiably single topic.

For solutions which are largely random and for phenomena which can be modelled in terms of such solutions, a recent thermodynamic theory for the compositional variation of glass-transition temperatures[1] has provided a method for calculating solution glass-transition temperatures, T_g, from pure-component glass-transition temperatures, T_{gi}, their corresponding glass-transition increments of heat capacity, ΔC_{pi}, and the mole or mass fractions, X_i, of these components. The equation for the glass-transition temperature of a c-component blend is, in the temperature-independent approximation for the ΔC_{pi},

$$\ln T_g = \frac{\sum_{i=1}^{c} X_i \,\Delta C_{pi} \ln T_{gi}}{\sum_{i=1}^{c} X_i \,\Delta C_{pi}} \quad (1)$$

Systems to which this entropic expression has been applied successfully include miscible blends of polymers[1–3] and plasticized straight-chain[4] and network[5] polymers. Additionally, the effect of degree of polymerization[6] and of cross-link density[7] on T_g have been accounted for by the theory.

For binary solutions, the derivative

$$\frac{d\ln T_g}{dX_2} = \frac{\Delta C_{p1}\Delta C_{p2} \ln T_{g2}/T_{g1}}{(X_1\Delta C_{p1} + X_2\Delta C_{p2})^2} \quad (2)$$

does not vanish (except in the trivial case) and, therefore, equation (1) predicts a monotonic dependence of T_g on overall composition. For near-random (and other) copolymers T_g can, however, manifest a compositional absolute extreme value. Thus, any model of copolymers as binary solutions cannot be generally acceptable. Nevertheless, as outlined below, it is possible to include copolymers of arbitrary ordering within the theory by recognizing formally that when differences between primary bonds must be considered in connection with the glass-transition properties of entities comprising a solution, the microscopic rather than overall composition determines the solution behaviour.

Sequence-distribution effects on T_g can be accounted for in the nearest-neighbour approximation by formal treatment of the general copolymer as a random solution of mer pairs, fractions f_{ii}, f_{jj}, f_{ij}. The particular version of equation (1) which then arises is

$$\ln T_g = \frac{f_{ii}\,\Delta C_{pii} \ln T_{gii} + f_{ij}\,\Delta C_{pij} \ln T_{gij} + f_{jj}\,\Delta C_{pjj} \ln T_{gjj}}{f_{ii}\,\Delta C_{pii} + f_{ij}\,\Delta C_{pij} + f_{jj}\,\Delta C_{pjj}} \quad (3)$$

The denominator of this relation is, as for equation (1), the solution transition increment of heat capacity, ΔC_p (ref. 1). For copolymers formed by continuous reaction of i and j monomers

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