6. Claoue-Long, J. C. & Nesbitt, R. W. *Nature* **313**, 247 (1985).
7. Huppert, H. E., Sparks, R. S. J. & Arndt, N. T. *Nature* **13**, 247-248 (1985).
8. Nisbet, E. G. *Nature* **309**, 14-15 (1984).
9. Claoue-Long, J. C., Thirlwall, M. F. & Nesbitt, R. W. *Nature* **307**, 697-701 (1984).
10. Bickle, M. J. *Nature* **312**, 702-703 (1984).
11. Compston, W., Williams, I. S., Campbell, I. H. & Gresham, J. J. *Earth planet. Sci. Lett.* (in the press).
12. Chauvel, C., Dupre, B. & Jenner, G. A. *Earth planet. Sci. Lett.* **74**, 315-324 (1985).
13. Huppert, H. E. & Sparks, R. S. J. *Earth planet. Sci. Lett.* **74**, 371-386 (1985).
14. Woolrich, P., Cowden, A. & Giorgetta, N. E. *Econ. Geol.* **76**, 1629-1644 (1981).
15. Groves, D. I., Barrett, F. N., Binns, R. A. & McQueen, K. G. *Econ. Geol.* **72**, 1224-1244 (1977).
16. Arndt, N. T. *Yb. Carnegie Instn. Wash* **75**, 555-562 (1976).
17. Craig, J. R. & Kullerud, G. *Econ. Geol. Monogr.* **4**, 344-358 (1969).
18. Ross, J. R. & Hopkins, G. M. F. *Economic Geology of Australia and Papua New Guinea, I. Metals* (ed. Knight, C. L.) 100-121 (Australian Institute of Mining and Metallurgy, Melbourne, 1975).
19. Bavinton, O. A. thesis, Australian National Univ. (1979).
20. Williams, R. K., Veeraburns, M. & Philbrook, W. O. *AIME Metall. Trans.* **3**, 255-260 (1972).
21. Jaeger, J. C. *Basalts: The Poldervaart Treatise on Rocks of Basaltic Composition* (eds Hess, H. H. & Poldervaart, A.) 503-536 (Wiley, New York, 1968).
22. Bickle, M. J. *Komatiites* (eds Arndt, N. T. & Nisbett, E. G.) 479-494 (Allen and Unwin, London, 1982).
23. Lesher, C. M. thesis, Univ. Western Australia (1983).
24. Groves, D. I., Lesher, C. M. & Gee, R. D. *Sulphide Deposits in Mafic and Ultramafic Rocks* (eds Buchannan, D. L. & Jones, M. J.) 1-13 (Institution of Mining and Metallurgy, London, 1984).
25. Groves, D. I., Barrett, F. M. & McQueen, K. G. *Can. Miner.* **17**, 319-336 (1979).
26. Ewers, W. E. & Hudson, D. R. *Econ. Geol.* **67**, 1075-1092 (1972).
27. Parker, P. thesis, Univ. Western Australia (1984).
28. Lesher, C. M., Lee, R. F., Groves, D. I., Bickle, M. J. & Donaldson, M. J. *Econ. Geol.* **76**, 1714-1728 (1981).

Estimating the error of age interpolation in sedimentary rocks

Catherine Badgley*, Lisa Tauxe† & Fred L. Bookstein‡

* Museum of Paleontology and ‡ Center for Human Growth and Development, University of Michigan, Ann Arbor, Michigan 48109, USA
† Scripps Institution of Oceanography, La Jolla, California 92093, USA

Magnetostratigraphic data can provide information on rates of sediment accumulation within a single sedimentation system over time spans from 10^4 to 10^6 yr. The short-term rate of sediment deposition varies with time; the apparent average rate over any longer interval also depends on the relative durations of periods of deposition, stasis (non-deposition), and erosion. While the average rate can be used to infer the time of occurrence of an event from its stratigraphical position, the inferred age has an uncertainty deriving from the variability in rate of sediment accumulation over all shorter timescales. We analyse here variability in sediment accumulation rates provided by the magnetostratigraphy of Miocene, Siwalik sediments from Pakistan. For long periods ($>10^6$ yr), sediment accumulation is approximately linear through time. Over short intervals (10^4–10^5 yr), however, there is considerable variability. To provide an error term for an absolute age interpolated between boundaries of polarity units, we use a resampling technique similar to the statistician's 'bootstrapping'. We illustrate this approach by estimating a standard error for the interpolated age of a biostratigraphical datum: the first appearance of hipparionine equids in the Siwalik sequence near the town of Khaur. The first appearance of "*Hipparion*" in the Khaur sequence is 9.22±0.09 Myr.

The assignment of absolute age estimates to sedimentary rocks requires either horizons susceptible to radiometric dating or access to the geomagnetic reversal timescale (GRTS). Fossils may indicate relative chronology in sedimentary rocks but do not convey information about absolute age. Thus, stratigraphical sections usually contain an arbitrary scatter of levels from which age information is available. Ages of all other levels are estimated indirectly by linear interpolation between levels of more accurately known age.

Linear interpolation of an estimated age within a stratigraphical section assumes that the sediment accumulation rate calculated for the longer, dated portion of the sequence also applies to much shorter time intervals. This is rarely the case. Unless sediment accumulates continuously and without erosion, sediment accumulation rate decreases as the time span of measurement is increased[1-3]. Thus it is necessary to consider when linear interpolation of an age estimate is justified, and how to assess the accuracy of the age estimate. Sadler[2] proposed replacing an interpolated age value by an age range based on an estimated incompleteness of the section. The completeness estimates[2,4] are based on the pattern of sediment accumulation rate versus time span for a large compilation of data from one class of sedimentary environments, such as fluvial systems. There is, however, no particular reason why any such variance should usefully model the variance within a sedimentation system.

A more direct approach is to exploit observable variability of sediment accumulation rates within a sedimentation system to build a statistical description of the relationship between sediment accumulation and time. The problem in age interpolation is how to estimate the proportion of time span present in a known proportion of sediment thickness from the interval of interpolation. One way to assess the standard error of such ages is to document this variability using intervals of a similar length for which age is not interpolated but is rather observed. Long magnetostratigraphic sections provide independent measurements of sediment accumulation rate for every discernible magnetic polarity interval. We demonstrate a method of 'resampling' a magnetostratigraphic section to generate multiple intervals that can be divided into shorter intervals of known time span and thickness. The original and resampled data together provide an estimate of the standard error about an interpolated age.

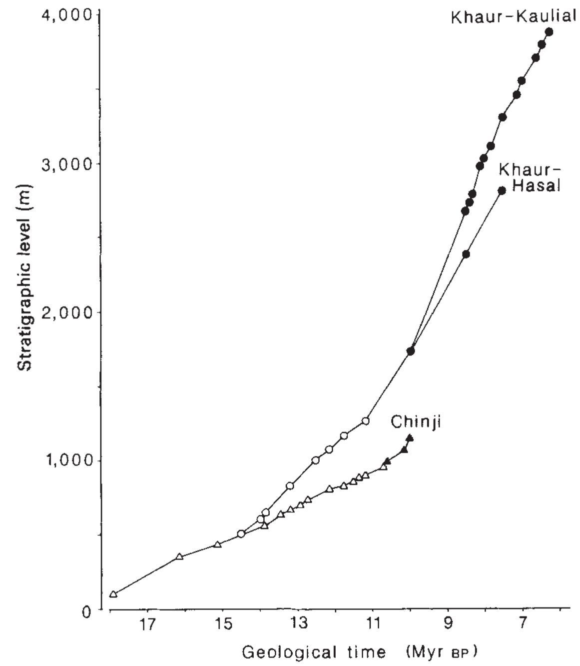

Fig. 1 Cumulative stratigraphic thickness over time in Siwalik sediments of northern Pakistan. Chinji and Khaur are two geographical areas in the same depositional system; Kaulial and Hasal are local section names[5]. Each data point demarcates a reversal of magnetic polarity. Triangles represent data from the Chinji area and circles data from the Khaur area. Solid symbols, strata older than 11 Myr; open symbols, strata younger than 11 Myr. A change in sediment accumulation rate occurred between 10 and 11 Myr, as indicated by the change in slope. Net sediment accumulation appears to be linear through time for the Chinji data older than 11 Myr and for the Khaur data younger than 11 Myr.

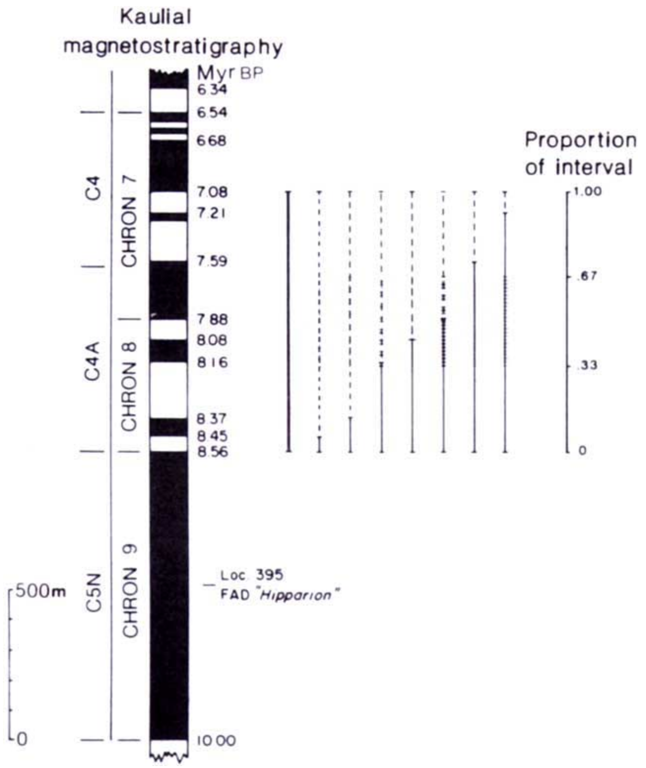

Fig. 2 Magnetostratigraphy from the Kaulial section in the Khaur area[5] illustrating the resampling method discussed. Absolute ages are indicated from the geomagnetic reversal timescale whenever a polarity interval is represented by at least three palaeomagnetic samples. Two timescales are used: that of Mankinen and Dalrymple[9] (right) and the new chron names (left) of Berggren *et al.*[8] given in the text. The first appearance datum (FAD) of "*Hipparion*" occurs within the long normal interval of Chron C5N. On the right, an interval is extracted from the left-hand record that is of about the same duration as Chron C5N. This long interval can be subdivided in seven different ways into complementary segment pairs of known thickness and time span; there result 14 calculations of proportion of sediment thickness and of time span. Of these segment pairs, three divide the long interval into segments between 33 and 67% (stippled band) of the whole. Ten intervals of durations between 1.2 and 1.8 Myr were located in Chrons C4 and C4A; the 104 segment pairs that can be formed within them provide the data in Fig. 3.

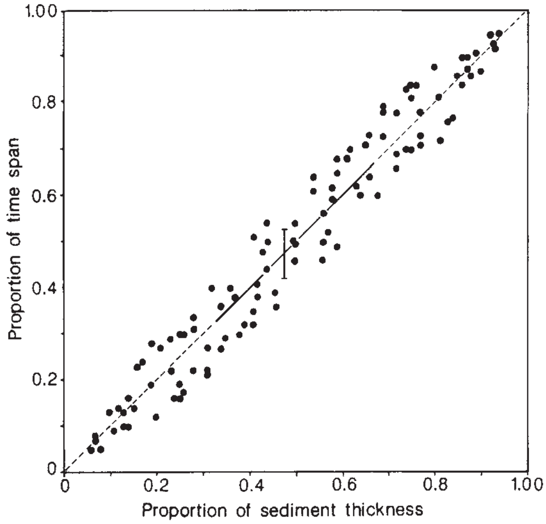

Fig. 3 Variability in proportion of time span in relation to proportion of sediment thickness from subsamples of 10 long intervals in the Kaulial palaeomagnetic section (Fig. 2). Five intervals are longer than the interval of interpolation (1.44 Myr) and five are shorter. Each subsample of a long interval gives rise to two complementary data points whose proportions along either axis sum to 100%. For this reason, the graph is unchanged by a 180° rotation about its centre. The standard error (vertical bar) of 0.06 (6%) was calculated for data in the region of the solid line ($n = 40$), corresponding to subsamples of $50 \pm 17\%$ of the whole interval. The value of $\pm 6\%$ appears characteristic of a considerably wider range of sediment proportions, perhaps 18–82%.

The data for demonstrating our approach are contained in two magnetostratigraphic studies of Siwalik sediments from Pakistan[5,6]. Siwalik rocks are a Neogene molassic sequence representing multiple fluvial systems on the southern margin of the Himalayas[7]. The pattern of Siwalik sediment accumulation through the middle to late Miocene in northern Pakistan is illustrated in Fig. 1. The Khaur sections exhibit higher sediment accumulation rates because they were closer than the Chinji sections to the inferred source area. In both areas, a change in sediment accumulation rate occurred between 10 and 11 Myr in response to an episode of Himalayan uplift[6]. This event, the major secular trend, constitutes a change in the controls of the sedimentation system. In each geographical location, sediment accumulation appears stationary on either side of this event. For the remainder of the discussion, we focus on a portion of the Khaur data.

Magnetostratigraphic data from the Khaur area are documented in 14 measured stratigraphic sections[5], each spanning part of a 2,100-m interval over a lateral extent of ~25 km. (Only two of these sections are shown in Fig. 1.) The sections are correlated by lithological marker horizons and magnetic polarity stratigraphy; sampling densities ranged from 1 in 3 m to 1 in 23 m. Altogether, the composite Khaur section represents 6.5–10.0 Myr, Chrons C4–C5N (terminology of Berggren *et al.*[8]). (We indicate also the GRTS of Mankinen and Dalrymple[9] in Fig. 2, for consistency with earlier studies. The choice of timescale does not affect the results of the technique presented here.) The longest continuous magnetostratigraphic section in deposits younger than 11 Myr, from Kaulial Kas (a local valley name), serves as the basis for the example discussed below (Fig. 2). The sampling density for this section ranged from 1 in 19 m to 1 in 23 m.

Siwalik sediments, rich in mammalian fossil remains, constitute the standard for South Asian, Neogene mammalian biostratigraphy[10,11]. The first appearance of the three-toed horse "*Hipparion*" in the Khaur Siwalik sequence is significant, both because it marks a rapid faunal turnover within the Siwalik mammalian fauna[10] and because it is used to link Miocene mammalian sites in the Old World[12]. "*Hipparion*" includes several hipparionine taxa not always represented by fossils diagnostic to the species level.) Since the first appearance of "*Hipparion*" in the Siwaliks occurs >2 Myr later than its first appearance in European and African sites[12–15], this event is also controversial. But it lies in the middle of the long normal interval of Chron C5N, thus being a genuine problem in age interpolation.

In the Kaulial section, the lowest occurrence of "*Hipparion*" is at the Harvard-Geological Survey of Pakistan Locality 395 (J. Barry, personal communication). Stratigraphically, Locality 395 lies 510 m above the base of the long normal unit, which is 940 m thick in Kaulial Kas (Fig. 2)[5]. A linear interpolation estimates this event to be 0.78 Myr younger than the age at the base of the long normal unit, or 9.22 Myr in absolute time. The "*Hipparion*" datum is near the middle of the long normal unit. To assess the expected error of this age estimate, we must know how the proportion of time span varies in relation to about half the sediment thickness for time intervals about the same length as the long normal period. Within the combined sequence of

Chrons C4-C4A, it is possible to delimit various intervals of durations between 1.2 and 1.8 Myr. These intervals can be subdivided into complementary pairs of segments, some representing about half of the full interval. For each segment, the discrepancy between proportion of time span and proportion of sediment thickness is directly observable. Thus, we use the pattern of sediment accumulation for a younger portion of the sediment regime to estimate the variance in an older part of the regime; the implied assumption of stationarity is justified by the linear pattern of cumulative stratigraphic thickness over the entire Kaulial sequence.

The resampling technique is illustrated in Fig. 2. The long interval shown as an example is 1.48 Myr in duration according to palaeomagnetic correlation. This interval contains nine short polarity intervals, of which the smallest, at 7.21 Myr, was represented by less than three samples. It was grouped with the next interval below it. (The same criterion was used to group together the four short intervals from 6.54 to 6.68 Myr in other samples.) The long interval may thus be broken into two pieces of known age in seven different ways. For each division, we calculated the proportion of total sediment thickness and proportion of total time span represented in each segment. In all, 10 long intervals, ranging from 1.25 to 1.77 Myr, were identified and subdivided in a similar fashion: these produced 104 segments, represented in Fig. 3.

For the "*Hipparion*" problem, the variance we need should correspond to a thickness proportion of ~0.50. We computed the mean sqaure

$$\frac{\Sigma\ (\text{time proportion} - \text{thickness proportion})^2}{n}$$

of time proportion about thickness proportion for thickness proportions between 0.33 and 0.67 (the solid line in Fig. 3). This quantity is equivalent to the variance in time proportion around a regression line on thickness proportion forced to have a slope of 1 and forced, also, to pass through the origin. For $n = 40$, within the central third of the range, the standard error of this discrepancy is 0.06. Thus, the standard error of the age estimate for the "*Hipparion*" datum should be taken as 6% of 1.44 Myr. By this approach, the age of the first appearance of "*Hipparion*" in the Khaur Siwalik sequence is 9.22 ± 0.09 Myr.

This method of generating a standard error is similar to bootstrapping, jackknifing, and other resampling techniques of modern statistical data analysis. One sample is used to generate many others through intensive resampling of the same data set[16,17]. The data of Fig. 3 are not statistically independent, even though that was assumed for the data of the original sample. In these circumstances, the computed value of 0.06 is a point estimate only, having no standard error itself.

The value 0.06 pertains to this particular situation, that is, the proportion of time span represented by 33-67% of sediment thickness for intervals of 1.2-1.8 Myr duration in Chrons C4 and C4A in the Kaulial section. The same standard error would apply to an interpolation problem concerning 55% of a polarity interval of 1.7-Myr duration, but not to 55% of an interval of 0.5-Myr duration. For this second case, it would be necessary to generate another sampling distribution. It is ultimately the pattern of the observed palaeomagnetic data which determines how the resampling technique can be applied. It would not be possible, in our example, to determine a standard error for intervals of 0.08 Myr duration, this being the shortest interval represented.

This standard error is not the only source of uncertainty in an interpolated age. Palaeomagnetic dates bear the limitations in time resolution of the GRTS, and the stratigraphic placement of reversal boundaries is partly a function of sample spacing. These factors contribute additional uncertainty to any palaeomagnetic age estimate (including age interpolations), although the error should not be a systematic one. The standard error calculated by this resampling method can be considered a minimum value attributable to variability in sediment accumulation rate.

Variation in sediment accumulation rates can be attributed to variation in instantaneous sedimentation rates and in completeness of stratigraphic sections. In a complete stratigraphic section with low variability of instantaneous sedimentation rates, linear interpolation provides the best estimate of age. But most stratigraphic sections cannot be considered complete. Even in systems such as lakes or deep ocean basins where deposition is virtually continuous, sediment is reworked by deep currents, turbidite processes, or dissolution. In fluvial systems, such as the Siwaliks, deposition in any one location is markedly discontinuous, even though deposition may occur most of the time somewhere in the system.

In incomplete sections, the results of linear interpolation are most plausible when gaps are relatively short and evenly distributed throughout the sedimentary record. But such convenient models cannot be assumed in general. The distribution of gaps relative to record must be interpreted in part by stratigraphic information from the section(s) under investigation. Fifty meters of laminated silt imply a markedly different distribution of gaps than 50 m of cross-bedded sandstone. When a pattern of sediment accumulation contains long, unevenly distributed discontinuities, linear age interpolation may be very prone to error; and whenever the magnitude of depositional and erosional events varies widely, the record is likely to be dominated by rare events of high magnitude[18].

These considerations argue against the computation of age-estimate errors according to any general stochastic model. In contrast, the resampling technique demonstrated here indicates the root-mean-square amount by which the interpolated age estimate differs from a 'known' value as a result of the variability in sediment accumulation rates specific to the particular sequence within which the interpolation is taking place. In this way, each system is characterized by its own pattern of variation in sediment accumulation rates, reflecting its own external factors, such as source-area uplift, or internal factors, such as local reworking of sediment.

We thank Gerald Smith, Jennifer Kitchell and Beth Dawson for constructive discussions. Karen Klitz drew Figs 1 and 2. This work was supported in part by NSF grant EAR 8305931 to C.B. and NSF grant OCE83-17546 to L.T.

Received 5 July; accepted 8 October 1985.

1. Schindel, D. E. *Palaeobiology* **6**, 408-426 (1980).
2. Sadler, P. M. *J. Geol.* **89**, 569-584 (1981).
3. Tipper, J. C. *Nature* **302**, 696-698 (1983).
4. Sadler, P. M. & Dingus, L. W. *Proc. 3rd N. Am. Paleontol. Conv.* **2**, 461-464 (1982).
5. Tauxe, L. & Opdyke, N. D. *Palaeogeogr. Palaeoclimatol., Palaeoecol.* **37**, 43-61 (1982).
6. Johnson, N. M., Stix, J., Tauxe, L., Cerveny, P. F. & Tahirkheli, R. A. K. *J. Geol.* **93**, 27-40 (1985).
7. Behrensmeyer, A. K. & Tauxe, L. *Sedimentology* **29**, 331-352 (1982).
8. Berggren, W. A., Kent, D. V., Flynn, J. & Van Couvering, J. A. in *Geochronology and the Geological Record* (ed. Snelling, N. J.) (Spec. Pap. Geological Society, London, in the press).
9. Mankinen, E. A. & Dalrymple, G. B. *J. geophys. Res.* **84**, 615-626 (1979).
10. Barry, J. C., Lindsay, E. H. & Jacobs, L. L. *Palaeogeogr. Palaeoclimatol. Palaeoecol.* **37**, 95-130 (1982).
11. Barry, J. C., Johnson, N. M., Raza, S. M. & Jacobs, L. L. *Geology* (in the press).
12. Bernor, R. L., Woodburne, M. O. & Van Couvering, J. A. *Geobios* **13**, 705-739 (1980).
13. Ginsburg, L., Janvier, P., Mornand, J. & Pouit, D. *C. r. Somm. Séanc. Soc. Géol. Fr.* 223-227 (1979).
14. Tiercelin, J.-J., Michaux, J. & Bandet, Y. *Bull. Soc. Géol. Fr.* **21**, 255-258 (1979).
15. Hussain, S. T. & Bernor, R. L. *Himalay. Geol.* **11**, 35-42 (1983).
16. Diaconis, P. & Efron, B. *Scient. Am.* **248** (5), 116-130 (1983).
17. Efron, B. & Tibshirani, R. *Tech. Rep.* **101** (Division of Biostatistics, Stanford University, 1985).
18. Crowley, K. D. *J. sedim. Petrol.* **54**, 127-136 (1984).