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THE EFFECTS OF SHEAR DEFORMATION AND AXIAL FORCE IN BATTENED AND LACED STRUCTURAL MEMBERS

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A	Area. Integration constant. Left end of structural member
A_b	Total cross-sectional area of batten or strut elements in one unit panel
A_c	Cross-sectional area of one of two longitudinal elements in a built-up structural member (half of total cross-sectional area of longitudinal elements in a symmetrical section)
A_d	Total cross-sectional area of diagonal elements acting within the unit-panel length "a"
B	Integration constant. Right end of structural member
$C_{11} \ldots C_{22}$	Slope-deflection constants
E	Modulus of elasticity
F_y	Minimum yield point of the type of steel being used
F.S.	Factor of safety
G	Shear modulus
I	Moment of inertia of the entire longitudinal cross-section (excluding web material)
I_b	Moment of inertia of the battens of cross-sectional area A_b
I_c	Moment of inertia of the longitudinal element of cross-sectional area A_c
L	Overall length of structural member with end stay plates
M	Bending moment
M_a, M_b	End moments of structural member
M_{fa}	Fixed end moment at left end of the member
M_o	Moment loading

M_s	Bending moment due to external loading (excluding axial force) in a simple beam
M_x	Bending moment at a distance x from left end of the member
P	Axial load in the structural member
$P_{cr,f}$	Critical load of a column which is fixed at both ends
$P_{cf,f-h}$	Critical load of a column which is fixed at one end and hinged at the other end
$P_{cr,h}$	Critical load of a column which is hinged at both ends
P_e	Euler load for a column without stay plates, $P_e = \frac{\pi^2 EI}{L^2}$
P_i	P_i, for i = 1, 2, are the axial forces of the longitudinal sub-elements as shown in Figure 2-5(b). These axial forces will vary from panel to panel along the length of the member, will be different on the two sides of longitudinal elements
V, V_1, V_2	Constant shear forces
V_a	Shear force at right end of member
V_b	Shear force at left end of member, total stresses of strut elements as shown in Figure 2-3(b)
V_d	Total stresses of diagonal elements as shown in Figure 2-3(b)
V_s	Shear force due to external loading (excluding axial force) in a simple beam
V_x	Shear force at a distance x from left end of the member
W	Concentrated load
Z	Semi-rigid connection constant
a	A unit-panel length in the lacing or batten arrangement of a built-up structural member
b	Distance between the axis of the two longitudinal elements of a built-up structural member

k	Nondimensional parameter, Equations (3-27) and (3-40)
k^*	$k^* = (1 - \delta_1 - \delta_2)k$
k_o	Nondimensional parameter, Equation (2-26)
ℓ	Length of a member, effective length of a structural member between end stay plates
m	Height-length ratio between stay plates, $m = \frac{b}{\ell}$
n	Number of panels, $n = \frac{\ell}{a}$
$p(x)$	Distributed load per unit length
r_{12}, r_{21}	Carry-over factors
r_b, r_c	Radius of gyration of batten, longitudinal elements, respectively, where $r_b^2 = I_b/A_b$, $r_c^2 = I_c/A_c$
s	Slope of the lacing elements (diagonals) with respect to batten elements, $s = \xi_a \frac{a}{b}$
$s_{opt.}$	Optimum slope of s, for which μ is a minimum value
w	Uniformly distributed load per unit length
x	Coordinate axis coinciding with the axis of the structural member in the undeflected rate
y	Deflection of the structural member
α	Load parameter, $\alpha = P/P_e$
γ	Shear angle
δ_1, δ_2	Length factors of stay plates
δ	Length factor of stay plates, where $\delta = \delta_1 = \delta_2$
$\Delta_1 \ldots \Delta_4$	Shear displacements

η, η_b, η_c	Shear shape factors
Θ	Joint rotation
Θ_a, Θ_b	End rotations of the structural member
μ	Nondimensional shear flexibility parameter
$\mu*$	$\mu* = (1 - \delta_1 - \delta_2)^2\mu$
ξ_a, ξ_b	Connection factors of longitudinal, strut elements, respectively
ρ	Length factor
σ_i	Actual stresses in the longitudinal sub-elements, where $\sigma_i = P_i/A_c$, for i = 1, 2
ϕ	Rotation of cross-section per unit length due to bending moment alone
ϕ_{A12}	Coefficient, Equations (3-66), (3-80), and (3-92)
ϕ_{B12}	Coefficient, Equations (3-67), (3-81), and (3-93)
ϕ_d	Amplification factor, Equations (2-24), (2-25), (2-28), and (2-29)
ϕ_m	Amplification factor, Equations (2-31) to (2-34)
ψ	Member rotation

ABSTRACT

This dissertation presents a theoretical analysis of the elastic behavior of both the battened and laced structural member considering the effects of axial load, shear deformation, and connection rigidity of sub-elements, and the overall effects of axial load, moment gradient, and effective shear deformation of the complete member. The analytical solution is used to obtain modified slope-deflection equations to generalize a relation between applied forces and joint displacements.

Types of the structural members may be catalogued here according to three different web configurations such as solid, battened, and laced. The arrangements of web elements are assumed to be the same throughout the effective length of the battened and laced structural members between end rigid stay plates.

A nondimensional parameter, the shear flexibility, is defined so as to characterize the shear flexibility of the structural members and to take account of the effects of axial force, local joint connections, and local connection flexibility of the battened members.

The fundamental linear second-order differential equation for the deflection curve of the structural member which includes the effect of shear deformation has been derived. The general solutions of this differential equation are of a fundamentally different nature for the cases of no axial force, compression, or tension axial force. By application of the natural boundary conditions to the general solution of deflected shape of the structural member, the solutions are set up in the forms of slope-deflection equations. In the

evaluation of the fixed end moments for a concentrated load, the reciprocal theorem is applied so as to make use of the deflection curves of the members which have been previously defined in the case of the homogeneous solution. From this basic expression one can derive fixed end moments for any combination of concentrated loads by simple summation or for continuously distributed loads by integration.

Elastic buckling loads for structural members with rigid stay plates and constant shear flexibility have been evaluated for the cases of a column with hinged ends, a column with one end fixed and the other hinged, and a column with both ends fixed.

Finally, numerical examples of beam and frame analyses are presented to provide a comparison with the ordinary beam and frame theories which neglect the effects of shear deformation and axial force.

CHAPTER I

INTRODUCTION

In addition to the deflection due to elongation and compression of fibers from bending moment, there is a further deformation due to shear and axial force and consequent strains in a beam. This is not usually considered in the analysis and design of frames made up of structural members of solid cross-section for which the influence of shear deformation is usually very small. This is due to the fact that the shear deformation is resisted in solid structural members by a continuous web which participates uniformly in the transmission of the shearing forces. The distortion caused by the shearing stresses in such a case is relatively small except for very short members. However, the conditions are different in battened or laced built-up structural members, in which case the contribution of shear deformation to the total deflection may be appreciable. By neglecting the deformation due to shear, errors of considerable magnitude may be introduced in frame analysis.

Many studies have been made by different investigators, such as Engesser (1891),[1,2] Muller-Breslau (1910),[3] Timoshenko (1936),[4] Amstutz and Stüssi (1941),[6] Pippard (1948),[9,14] Takekazu (1951),[12] Bleich (1952),[13] Jones (1952),[15] Koenigsberger and Mohsin (1956),[16,23] and Tomayo (1965)[22] to determine the critical loads as well as frame behaviors of built-up columns as affected by shear. More recently, Williamson and Margolin (1966)[24] have studied the effect of shear deformation on shears and moments in laced guyed towers. Glauser (1967) has studied the shear effect here at The University of Michigan, but has not considered the effect of axial force or shear in the local sub-element.

A nondimensional parameter of shear flexibility μ will be introduced to characterize the shear flexibility of battened and laced structural members. The parameter μ will be evaluated so as to take account of the effects of axial force, local eccentric joint connections, and local connection flexibility[7,8,10] of batten structural members. The effect of rigid stay plates[11] at the ends of the structural member will be considered in the evaluation of the deflection curve for the member.

The purpose of this thesis is to develop a reasonably accurate yet comparatively simple evaluation of the shear flexibility μ for a wide variety of cases, and to generalize the modified slope-deflection equations[5,8] and elastic stability for the battened and laced structural members.

The shear flexibility parameter μ, limitation of the maximum local slenderness ratio, slope-deflection constants and carry-over factors, fixed-end moments, and critical loads of the built-up structural members will be evaluated for structural design office use.

It is a further purpose to develop criteria[18] to guide the designer to a decision whereby he might with reasonable accuracy neglect the effect of shear and axial force, either in a battened sub-element, or in an entire member.

CHAPTER II

THE SHEAR FLEXIBILITY OF STRUCTURAL MEMBERS

2.1. DEFINITION OF THE SHEAR FLEXIBILITY

A nondimensional parameter μ is introduced to characterize the shear flexibility of structural members for a wide variety of particular battened or lacing arrangement. Consider an idealized element of the structural member whose length is defined by "a." An equilibrium state of the free element is shown in Figure 2-1. In general, this element is acted on by the axial force P, the shearing force V, and the bending moment M. The length of the uniform cross-section of a structural member between rigid stay plates at the ends of the member will be denoted as "ℓ." As Figure 2-1 shows, we separate the state of deformation into two parts: (a) bending, and (b) shear, and for each of these the axial force will be considered simultaneously. The state of bending causes

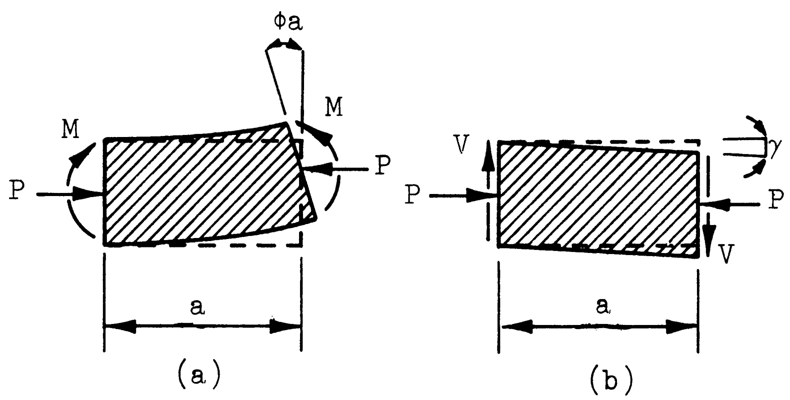

Figure 2-1. Element of length "a" of a structural member under (a) bending, and (b) shear.

a rotation Φa of the cross-section while the state of shear causes the deformation γ as shown. The shear flexibility is defined as the ratio of the changes in slope in length "a" due to shear deformation and due to the bending rotation, which is equal to $\gamma/\Phi a$. Under bending alone the change in slope in length "a" is

$$\Phi a = \frac{Ma}{EI}$$

While under shear alone, the change in slope is

$$\gamma = \frac{\eta V}{AG}$$

where

EI is the flexural rigidity of the structural member,

AG is the shear rigidity of the structural member,

η is the shear shape factor.

Thus the ratio

$$\frac{\gamma}{\Phi a} = \frac{\eta EI}{aAG} \frac{V}{M} \tag{2.1}$$

It is desirable to define a parameter which will be equal to the foregoing ratio for a specific relationship between V and M and having the dimension 1/L. In this study V/M will be taken as a/ℓ^2 which is the same as that adopted by Washio,[5] Takekazu,[12] Glauser. Other investigators Maugh,[8] Gere,[21] Williamson and Margolin[24] have used ratios of V/M differing only by a numerical coefficient.

In evaluating the shear flexibility parameter μ, the effects of shear deformation as well as axial force on the behavior of the member sub-element will be considered. It should be noted that, with shear deformation considered, the deformed cross-section of the member is no longer a plane perpendicular to the tangent of the deflected axis.

2.2. THE SHEAR FLEXIBILITY OF LACED STRUCTURAL MEMBERS

Types of laced structural members may be catalogued according to five different lacing configurations as shown in Figure 2-2. They are parts of the laced structural members which consist of two main longitudinal elements, lacing

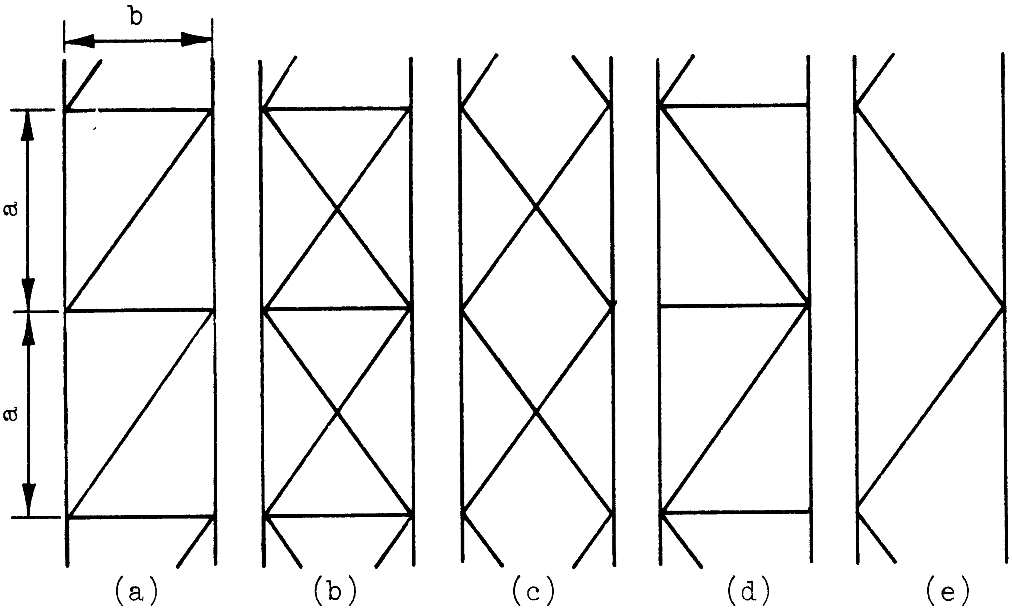

Figure 2-2. Five different lacing configurations for the laced structural members.

(diagonal) elements, with or without strut (transverse) elements. The two longitudinal elements are connected in one, two (or more) planes by the lacing bars and strut elements which serve as the web of the member. The assumption of an equivalent solid member not requiring a consideration of shear deformation can be made if the properties and geometric configuration of the sub-elements are such as to make the shear parameter μ relatively small. The effect of shear also depends greatly on the moment gradient, which is a measure of the shear to moment ratio.

The two main longitudinal elements are assumed to form a symmetrical section. A_c is the cross-sectional area of one of the two longitudinal elements. The properties of both the diagonal and strut elements, are the same throughout the length ℓ of the member. A_d is the total cross-sectional area of all diagonal elements within one panel length "a." A_b is total cross-sectional area in one unit of strut elements.

The force equilibrium condition of a typical panel is shown in Figure 2-3(b), assuming hinges at the ends of all strut and diagonal elements. Then we can obtain forces in the elements in terms of shearing force, where

$$V_1 = V + V_2 = \frac{1 + \xi_a}{2\xi_a} V \quad (2\text{-}2)$$

$$V_2 = \frac{1 - \xi_a}{2\xi_a} V \quad (2\text{-}3)$$

$$V_b = - V_1 - V_2 = - \frac{V}{\xi_a} \quad (2\text{-}4)$$

$$V_d = \frac{1}{\xi_a} \sqrt{1 + \left(\frac{\xi_a a}{b}\right)^2} V \quad (2\text{-}5)$$

The additional slope γ of the deflection curve due to shear V will now be determined. Firstly, consider the shear displacement Δ_1, as shown in

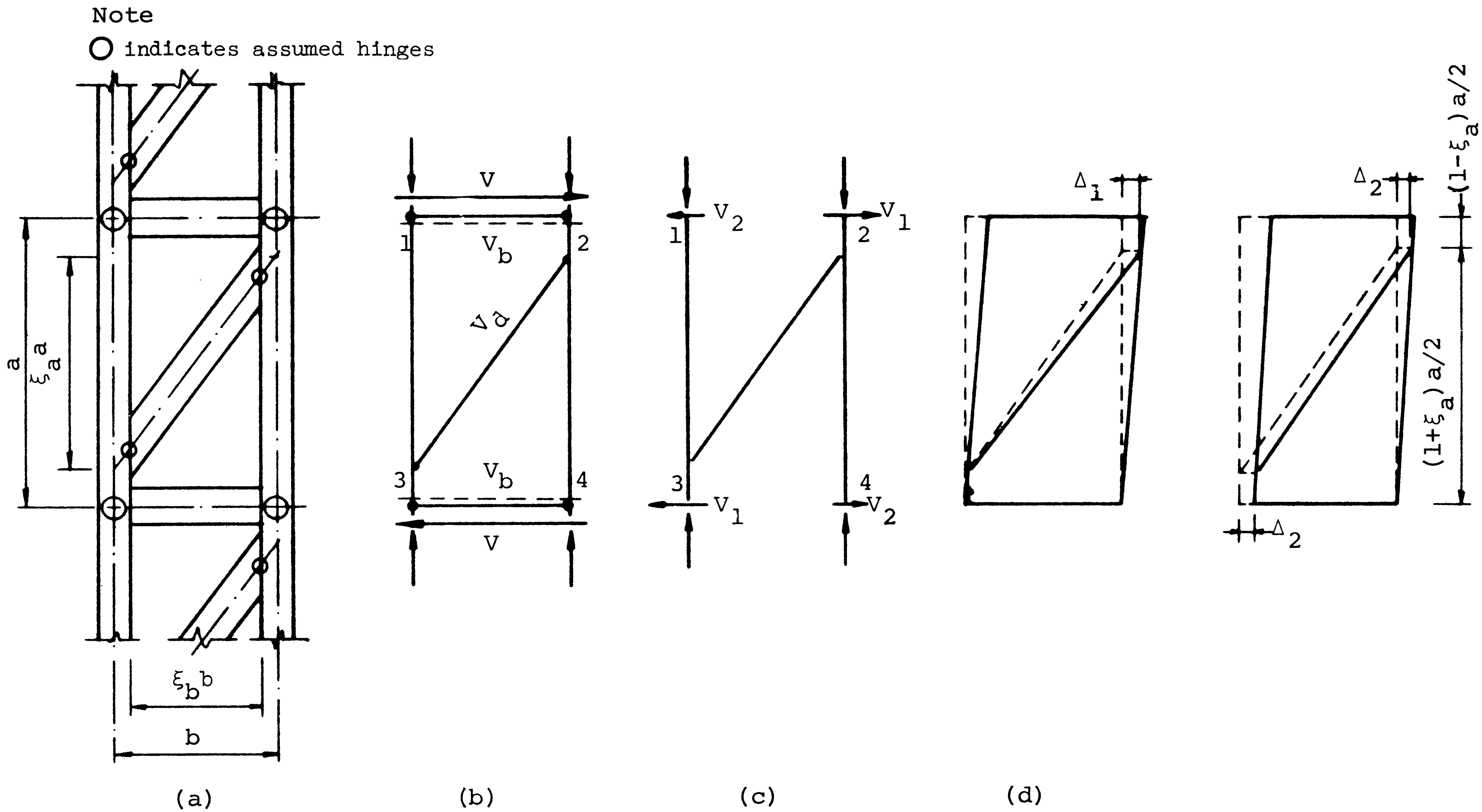

Figure 2-3. Typical panel of laced structural member; (a) geometric configuration, (b) equilibrium relation, (c) equilibrium of internal forces, (d) shear deformation due to the lengthening and shortening of the diagonal elements, and (e) shear deformation due to the shortening of the strut elements.

Figure 2-3(d), due to the lengthening and shortening of the lacing elements in each panel. Then we obtain

$$\Delta_1 = \frac{\xi_b b}{\xi_a}\left[1 + \left(\frac{\xi_a a}{b}\right)^2\right]^{3/2} \frac{V}{EA_d} \tag{2-6}$$

Secondly, consider the shear displacement Δ_2, as shown in Figure 2-3(e), due to the shortening of the strut elements in each panel. Then we have

$$\Delta_2 = \frac{\xi_b}{\xi_a} \frac{Vb}{EA_b} \tag{2-7}$$

where ξ_a and ξ_b are the connection eccentricity factors. They are dependent on the local joint geometry. A_b is the total cross-sectional area in one unit (panel) of strut elements.

Therefore, the total angular rotation γ caused by the shearing force V alone is

$$\gamma = \frac{\Delta_1 + \Delta_2}{1/2\ (1 + \xi_a)a}$$

$$\gamma = \frac{2\xi_b}{1 + \xi_a} \frac{V}{EA_d}\left\{\frac{b}{\xi_a a}\left[1 + \left(\frac{\xi_a a}{b}\right)^2\right]^{3/2} + \frac{b}{\xi_a a}\frac{A_d}{A_b}\right\} \tag{2-8}$$

The state of bending causes a rotation Φa of the cross-section due to moment M, as shown in Figure 2-1(a), while the state of shear causes the shear deformation γ.

$$\Phi a = \frac{V\ell^2}{EI} \tag{2-9}$$

where I = moment of inertia of the member which is excluding web elements

$$I = \frac{1}{2}\left[1 + \left(\frac{2r_c}{b}\right)^2\right] b^2 A_c \qquad (2\text{-}10)$$

r_c and A_c are the radius of gyration and the cross-sectional area of one of two longitudinal elements in a built-up structural member (half of total cross-sectional area of longitudinal elements in a symmetrical section), respectively. ℓ is the length of that portion of a member which has uniform panel arrangements with the same repeating properties of the web elements throughout.

For laced structural members in general,

$$\left(\frac{2r_c}{b}\right)^2 << 1 \quad ,$$

then

$$I = \frac{1}{2} b^2 A_c \qquad (2\text{-}11)$$

Then we get

$$\phi a = 2\left(\frac{\ell}{b}\right)^2 \frac{V}{EA_c} \qquad (2\text{-}12)$$

From the definition (2-1), we can now obtain the following expression for the shear flexibility parameter μ of laced structural members as follows:

$$\mu = \frac{\xi_b}{1 + \xi_a} \left(\frac{b}{\ell}\right)^2 \frac{A_c}{A_d} \left\{ \frac{b}{\xi_a a} \left[1 + \left(\frac{\xi_a a}{b}\right)^2\right]^{3/2} + \frac{b}{\xi_a a} \frac{A_d}{A_b} \right\} \qquad (2\text{-}13)$$

If we change variables,

s = slope of the diagonal elements with respect to strut elements.

$$s = \frac{\xi_a a}{b} \tag{2-14}$$

$$m = \frac{b}{\ell} \tag{2-15}$$

then the expression for μ becomes

$$\mu = \frac{\xi_b}{1 + \xi_a} m^2 \frac{A_c}{A_d} \left[\frac{(1 + s^2)^{3/2}}{s} + \frac{A_d}{A_b} \frac{1}{s} \right] \tag{2-16}$$

Moreover, if we introduce a new variable n,

$$n = \frac{\ell}{a} \tag{2-17}$$

Then Equation (2-13) may be transformed to the expression:

$$\mu = \frac{\xi_b m^2}{(1 + \xi_a)\xi_a} \frac{A_c}{A_d} \left\{ mn \left[1 + \left(\frac{\xi_a}{mn} \right)^2 \right]^{3/2} + mn \frac{A_d}{A_b} \right\} \tag{2-18}$$

It is noted that the last term of Equations (2-13), (2-16), and (2-18) represent the contribution of the strut elements, A_b. This term should be omitted whenever strut elements, as shown in Figures 2-2(c) and 2-2(e), are missing, or whenever the strut elements, as shown in Figure 2-2(b) and 2-2(d), do not take part in the transmission of the shearing force of the structural members.

We observe in Equation (2-16) that the value of the shear flexibility parameter μ becomes infinite as the slope of diagonal elements approaches either zero or infinity. Thus there are optimum values of s which will minimize the shear flexibility parameter, μ. It is, therefore, important to know this geometric arrangement of diagonal elements.

Let $s_{opt.}$ represent the value for which the correspondent value of shear flexibility μ becomes a minimum value. It implies that the first partial derivative of μ with respect to s is equal to zero. Therefore, the minimized condition is:

$$(1 + s^2_{opt.})^{1/2}(2s^2_{opt.} - 1) - \frac{A_d}{A_b} = 0 \qquad (2\text{-}19)$$

The optimum slope $s_{opt.}$ of the diagonal elements is plotted in Figure 2-4 as a function of the area ratio A_d/A_b. It is noted that the minimum slope is $\sqrt{2}/2$, or 35.27 degrees for the case where strut elements are missing or stress-free and thereafter increases with the ratio A_d/A_b.

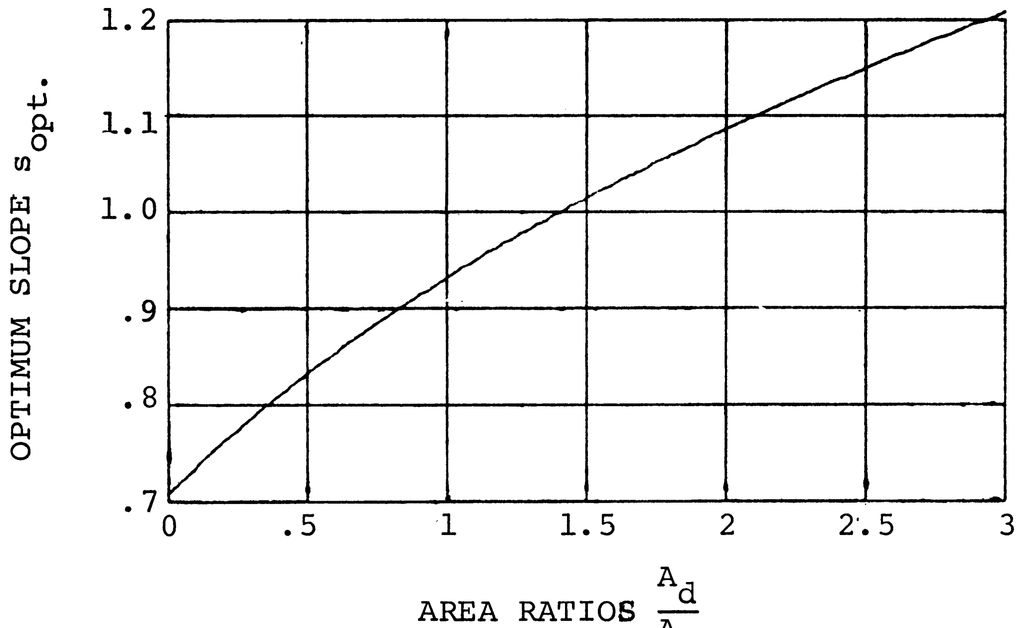

Figure 2-4. The optimum arrangement of the diagonal elements.

If we substitute the value A_d/A_b from Equation (2-19) into (2-16), and restrict the value s to $s_{opt.}$, an expression for the minimum value of shear flexibility is obtained for the laced structural members:

$$\mu_{min.} = \frac{3\xi_b}{1 + \xi_a} m^2 \, s_{opt.} (1 + s^2_{opt.})^{1/2} \frac{A_c}{A_d} \qquad (2\text{-}20)$$

The values of the shear flexibility parameter μ, or $\mu_{min.}$ of the laced structural members can be simply obtained from the Equations (2-13), (2-16), (2-18), or (2-20). The effect of rigid stay plates at the ends of the members will be introduced later.

2.3. THE SHEAR FLEXIBILITY OF BATTENED STRUCTURAL MEMBERS

Figure 2-5(a) shows the basic elements of the battened structural member, which consist of two main longitudinal elements joined by batten elements. The two longitudinal elements are assumed to form a symmetrical section and connected, in one or two (or more) planes of batten elements, by means of rigid or semi-rigid joint connections. The batten elements serve as the web of the member, transmitting shear force in the member by virtue of their own shear and moment resistance, in combination with the local bending of the longitudinal elements.

The geometric arrangement of the battened structural members is based on the typical unit of length a, and width b. The properties of the battened structural members are characterized by the moment of inertia of the batten and longitudinal elements, and the semi-rigid connection constant which vanishes for a perfectly rigid joint connection.

As we know, the battened structural members are highly redundant structures whose exact solution may be extremely laborious. We may assume, however, that the points of inflection in the batten elements of a symmetric structural system are at the midpoint, and that for the longitudinal elements with adequate

Note

○ indicates assumed hinges

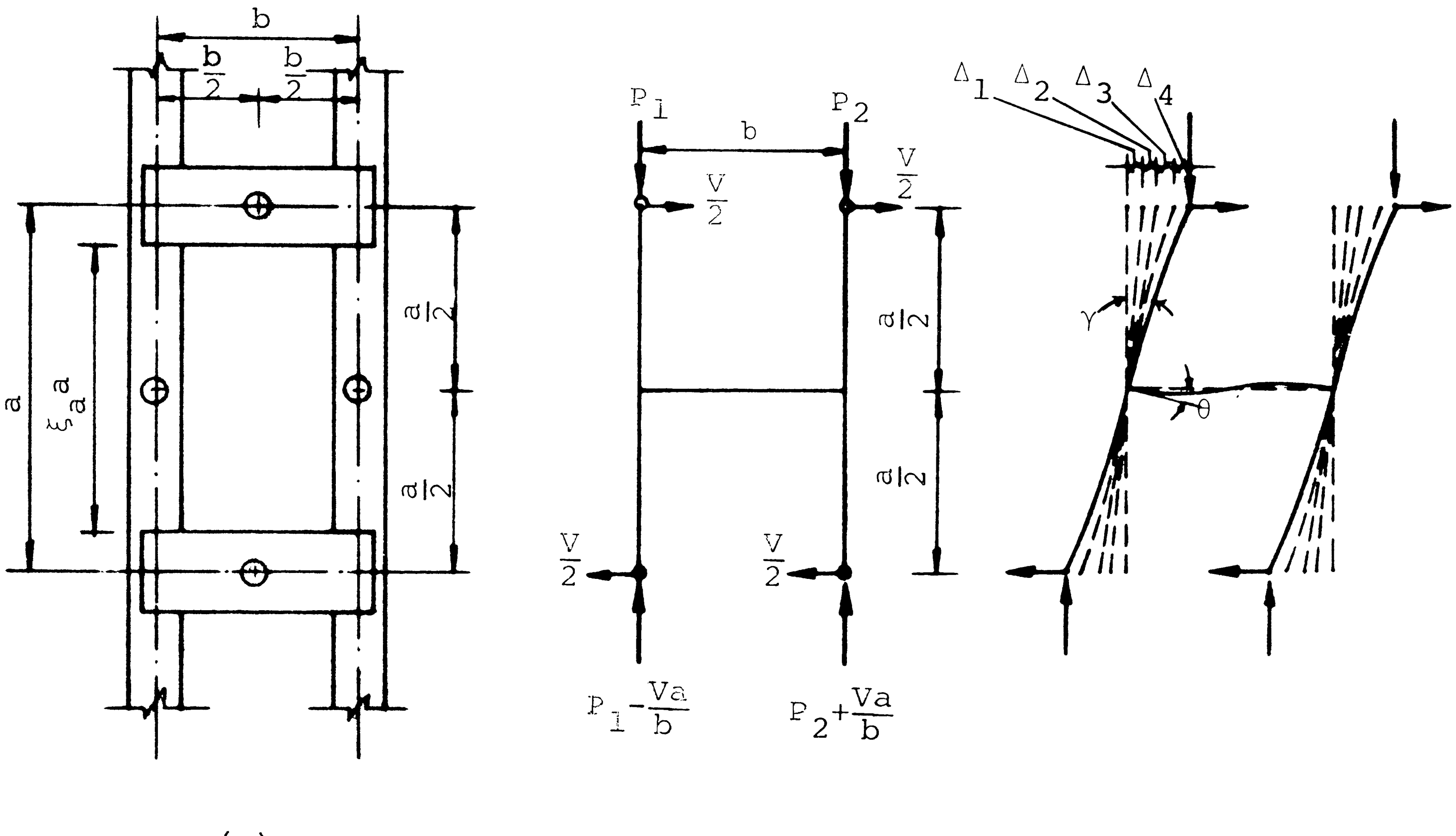

Figure 2-5. Typical panel of battened structural member; (a) the geometrical configuration, (b) force equilibrium, and (c) shear deformation.

shear resistance, this is also approximately true. When hinges at the mid-points of all elements are assumed the battened structural member becomes statically determinant.

To determine the additional slope γ of the deflection curve due to shear force V, we first consider the lateral displacement Δ_3, as shown in Figure 2-5(c). This is due to the shear force V/2 with the axial force P_i as follows:

$$\Delta_3 = \frac{(\xi_a a)^3 V}{48 EI_c} \phi_d \tag{2-21}$$

Where

$$P_1 = \frac{P}{2} + \frac{M_x}{b} \tag{2-22}$$

$$P_2 = \frac{P}{2} - \frac{M_x}{b} \tag{2-23}$$

$$\phi_d = \frac{3}{k_o^2}\left(\frac{\tan k_o}{k_o} - 1\right) \quad \text{...Amplification factor} \tag{2-24}$$

or

$$\phi_d = 1 + \frac{2}{5} k_o^2 + \frac{17}{105} k_o^4 + \frac{62}{945} k_o^6 + \ldots \tag{2-25}$$

$$k_o^2 = \frac{P_i\left(\frac{\xi_a}{2} a\right)^2}{EI_c} = \frac{\sigma_i}{4E}\left(\frac{\xi_a a}{r_c}\right)^2 \tag{2-26}$$

$$\sigma_i = \frac{P_i}{A_c} \tag{2-27}$$

M_x is the bending moment at a distance x from end of the member.

P_i, for i = 1, 2, are the axial forces of the longitudinal sub-elements as shown in Figure 2-5(b). These axial forces will vary from panel to panel along the length of the member and will be different on the two sides.

I_c = the moment of inertia of one of the two longitudinal elements in a battened structural member.

E = modulus of elasticity.

If P is tension, then

$$\phi_d = \frac{3}{k_o^2}\left(1 - \frac{\tanh k_o}{k_o}\right) \tag{2-28}$$

or

$$\phi_d = 1 - \frac{2}{5} k_o^2 + \frac{17}{105} k_o^4 - \frac{62}{945} k_o^6 + \ldots \tag{2-29}$$

Second, consider the lateral displacement Δ_1, as shown in Figure 2-5(c), due to the angular rotation at the end of the batten elements as follows:

The angular of rotation θ, as shown in Figure 2-5(c), at each end of the batten elements is:

$$\theta = \frac{abV}{12\ EI_b} \phi_m \tag{2-30}$$

where

$$\phi_m = \frac{\tan k_o}{k_o} \quad \ldots\text{Amplification factor} \tag{2-31}$$

or

$$\phi_m = 1 + \frac{1}{3} k_o^2 + \frac{2}{15} k_o^4 + \frac{17}{315} k_o^6 + \ldots \tag{2-32}$$

I_b = the total moment of inertia in one unit panel of the batten elements.

In case of tension P, then

$$\phi_m = \frac{\tanh k_o}{k_o} \qquad (2\text{-}33)$$

or

$$\phi_m = 1 - \frac{1}{3} k_o^2 + \frac{2}{15} k_o^4 - \frac{17}{315} k_o^6 + \ldots \qquad (2\text{-}34)$$

Then we can obtain

$$\Delta_1 = \frac{a}{2}\,\Theta$$

$$\Delta_1 = \frac{a^2 b\ V}{24\ EI_b}\,\phi_m \qquad (2\text{-}35)$$

Third, consider the lateral displacement Δ_2, as shown in Figure 2-5(c), due to the shear deformation of the batten and longitudinal elements is

$$\Delta_2 = \frac{aV}{2}\left(\frac{a\eta_b}{bGA_b} + \frac{\xi_a \eta_c}{2GA_c}\right) \qquad (2\text{-}36)$$

where

η_b and η_c are shear shape factors of the individual batten and longitudinal elements, respectively, which depend on the shape of the individual element cross-sections.

G = shear modulus

ξ_a = connection factor

Fourth, consider the lateral displacement Δ_4, as shown in Figure 2-5(c), due to a semi-rigid connection.

$$\Delta_4 = \frac{1}{4} a^2 Z \, V \, \phi_m \tag{2-37}$$

Equation (2-37) is restricted to small deformation due to connection non-linearity and implies the condition:

$$\frac{aZP_i}{4} \ll 1 \tag{2-38}$$

Z = semi-rigid connection constant,(7,8,10)

i.e.: Z = 0 for rigid connection,

Z = ∞ for hinged connection.

From these results, the overall shear deformation due to shear force V is given by:

$$\gamma = \frac{\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4}{a/2}$$

$$\gamma = V\left[\frac{ab}{12\,EI_b}\,\phi_m + \frac{a\eta_b}{bGA_b} + \frac{\xi_a\eta_c}{2GA_c} + \frac{\xi_a^3\,a^2}{24\,EI_c}\,\phi_d + \frac{aZ}{2}\,\phi_m\right] \tag{2-39}$$

From Equations (2-9) and (2-10) we have a moment rotation ϕa due to bending alone. This leads again, according to the definition (2-1), to the following expression for the shear flexibility parameter μ of battened structural members:

$$\mu = \left[\frac{1}{\left(\frac{\ell}{r_c}\right)^2} + \left(\frac{b}{2\ell}\right)^2\right]\left[\frac{ab}{6r_b^2}\,\frac{A_c}{A_b}\,\phi_m + 5.2\,\frac{a}{b}\,\eta_b\,\frac{A_c}{A_b} + 2.6\xi_a\eta_c + \frac{\xi_a^3}{12}\left(\frac{a}{r_c}\right)^2\phi_d + aA_cEZ\,\phi_m\right] \tag{2-40}$$

Again if we introduce new variables $n = \ell/a$ and $m = b/\ell$ we have a modified expression for μ:

$$\mu = \left[\frac{1}{\left(\frac{\ell}{r_c}\right)^2} + \left(\frac{m}{2}\right)^2\right]\left[\frac{m}{6n}\left(\frac{\ell}{r_b}\right)^2 \frac{A_c}{A_b}\phi_m + \frac{5.2\eta_b}{mn}\frac{A_c}{A_b} + 2.6\xi_a\eta_c + \frac{\xi_a^3}{12n^2}\left(\frac{\ell}{r_c}\right)^2\phi_d + \frac{\ell A_c}{n} EZ\,\phi_m\right] \quad (2\text{-}41)$$

It is noted that the last term of Equations (2-40) and (2-41) represents the contribution of the semi-rigid connections. This term will vanish in case of perfectly rigid connection. Welded connections designed for full moment and joint shear may be assumed to provide full continuity, i.e., they are then termed "rigid."

We observe by Equation (2-41) that the value of the shear flexibility parameter μ becomes infinite either as the number of panels n approaches zero, or as the semi-rigid constant Z approaches infinity, and the value of μ approaches zero as n approaches infinity which leads to the case of the solid structural member. Then from Equation (2-41) we have

$$\lim_{n\to\infty} \mu = 2.6\eta_c\left[\frac{1}{\left(\frac{\ell}{r_c}\right)^2} + \left(\frac{m}{2}\right)^2\right] \quad (2\text{-}42)$$

The values of the shear flexibility parameter μ of battened structural members can be obtained from the Equations (2-40) and (2-41). The effect of rigid stay plates at the ends of the members will be introduced later.

In an actual situation, the axial forces P_i (for i = 1, 2, as expressed in Equations (2-22) and (2-23)) of the longitudinal sub-elements, as shown in Figure 2-5(b), will vary from panel to panel along the length of the member, and will be different on the two sides of the longitudinal elements, due to the bending moment M_x. However, it is desired that the shear flexibility μ as a constant value for the entire length ℓ of the member in this analysis. To permit the assumption of constant shear flexibility μ, we should determine a correct limit on the ratio of the local slenderness a/r_c, so that the influence of the axial forces P_i will be small. P_i (for i = 1, 2) are caused by the axial force P, the bending moment M_x, or by these in combination, so as to develop the maximum allowable axial stress, in any one of the longitudinal sub-elements under any condition of external loading. The expressions for the amplification factors ϕ_d and ϕ_m, in Equations (2-25), (2-29), (2-32), and (2-34), respectively, are functions of the axial forces P_i or the axial stresses σ_i, and are approximately equal to 1 when the nondimensional parameter k_o is less than 1/3. Therefore, the analysis is applicable and of sufficient accuracy in the range of $k_o \leq 1/3$, so that the shear flexibility parameter μ will be nearly a constant value through the entire length ℓ of the battened structural member. Therefore, we have the condition:

$$\phi_d = \phi_m \doteqdot 1 \quad , \text{ for } k_o \leq 1/3 \qquad (2\text{-}43)$$

This approximation may cause maximum localized errors of the amplification factors ϕ_d, ϕ_m, not more than 4.7%.

From Equation (2-26) and the condition (2-43), we obtain the limiting ratio of local slenderness to be:

$$\frac{a}{r_c} = \frac{2k_o}{\xi_a}\sqrt{\frac{E}{\sigma_i}} \tag{2-44}$$

The maximum local slenderness ratio a/r_c will be obtained from Equation (2-44), by simply considering the factor of safety, F.S., which is defined by the applicable specification. Then the limiting condition, for the battened structural members, shall be such that the ratio of local slenderness is defined by:

$$\frac{a}{r_c} < \frac{2}{3\xi_a}\sqrt{\frac{E}{(F.S.)\sigma_i}} \tag{2-45}$$

where

$(F.S.)\sigma_i \leq F_y$

F_y is specified minimum yield point of the type of structural steel to be used.

This limitation of the maximum local slenderness ratio a/r_c with different connection factors ξ_a is provided as shown in Figure 2-6.

It is noted that the upper bound value (for rigid connections) of the non-dimensional parameter k_o is $\pi/2$, for which the amplification factors Φ_d and Φ_m become infinity. Then the battened structural member will collapse due to local (premature) failure. Therefore, we can conclude that if the local slenderness ratio a/r_c reaches $\pi/\xi_a\sqrt{E/(F.S.)\sigma_i}$, premature local failure will occur. The proposed limitation provided by Equation (2-45) should provide an adequate safety against this possibility.

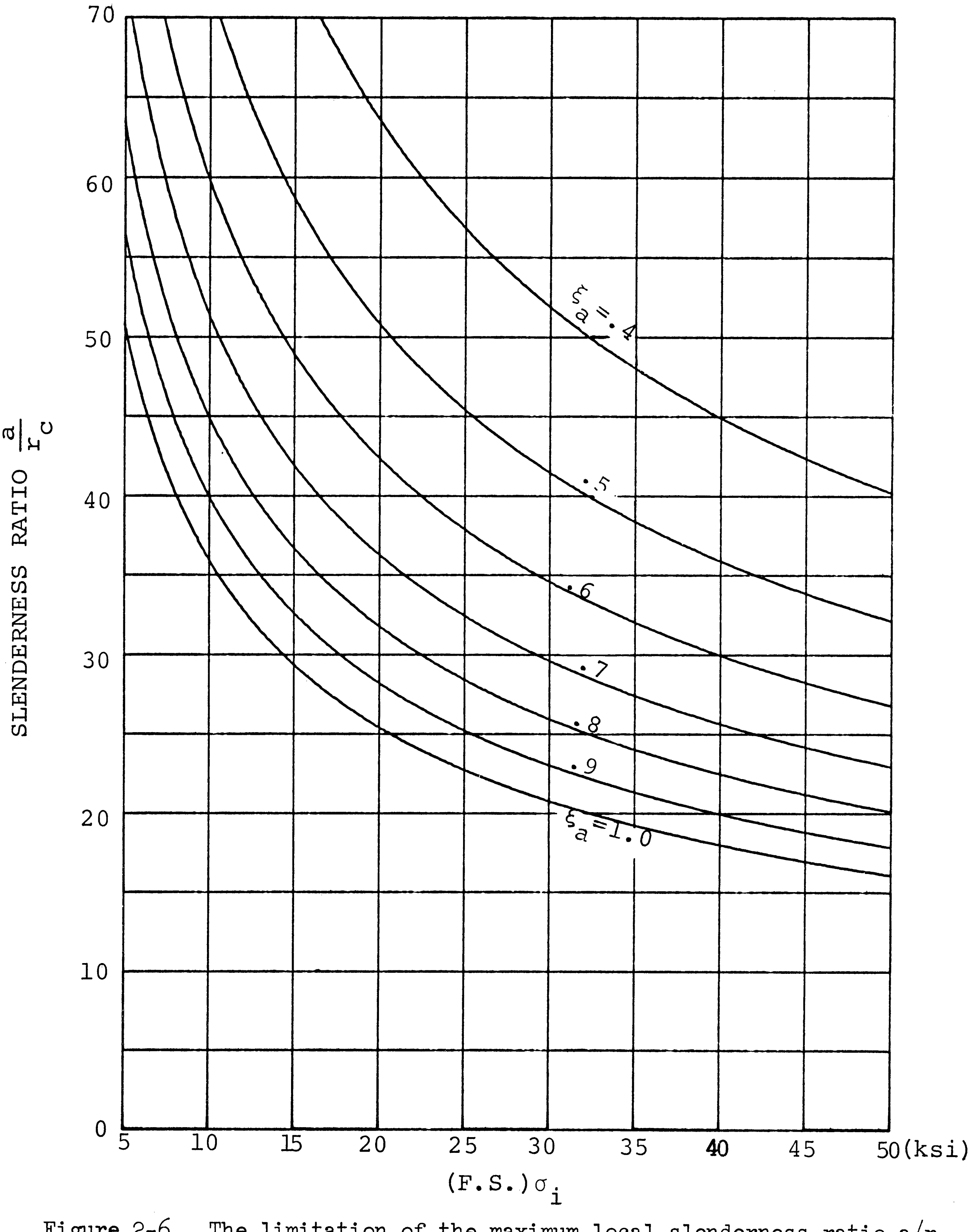

Figure 2-6. The limitation of the maximum local slenderness ratio a/r_c of the battened structural member with different connection factors ξ_a.

For the case of k_o in the range of $1/3 < k_o < \pi/2$, the effect of axial load in the longitudinal panel elements should be considered. The shear flexibility parameter μ will vary from panel to panel along the length of the member. A trial method may be used to obtain an average parameter μ for the entire length of member. Alternatively, each panel may be considered individually and a numerical method utilized, but this is outside the scope of this dissertation.

CHAPTER III

THE STIFFNESS PROPERTIES AND FIXED END MOMENTS OF STRUCTURAL MEMBERS WITH RIGID STAY PLATES AND CONSTANT SHEAR FLEXIBILITY

3.1. THE SHEAR FLEXIBILITY OF SOLID-WEB STRUCTURAL MEMBERS

The shear flexibility parameters, μ, for battened and laced structural members have been established. Although the influence of shear deformations on the properties of structural member with solid webs is usually small, except for very short members, we need to relate the built-up structural member to an equivalent continuous solid member as a function of the parameter μ. The relationship will be clear when the shear flexibility of a solid structural member (again characterized by the parameter μ) is given by the following expression:

The angular rotation γ caused by shearing force V in a solid member is:

$$\gamma = \frac{\eta V}{AG} \tag{3-1}$$

where

η = shear shape factor of the solid structural member.

A = the cross-sectional area of the solid structural member.

The bending rotation Φa caused by the bending moment (again $V\ell^2/a$) alone is:

$$\Phi a = \frac{\frac{V\ell^2}{a}}{EI}\, a = \frac{V\ell^2}{EI} \tag{3-2}$$

where

I = the moment of inertia of the solid structural member.

According to definition (2-1), the shear flexibility parameter μ of the solid structural members is, therefore, obtained as follows:

$$\mu = \frac{\eta EI}{AG\ell^2} \tag{3-3}$$

where

E and G are the modulus of elasticity and the shear modulus of the material, respectively, for structural steel, the value of the ratio E/G is equal to 2.6.

It is noted that the parameter μ of the solid structural members is identical to the parameter μ of battened structural members when the value n (number of panels) approaches infinity in Equation (2-42).

A relation for the shear flexibility of a structural member with rigid stay plates will be developed. Let L be the total length of a structural member with the length of rigid stay plates included. Then we have the geometric relation as shown in Figure 3-1. The structural member which has constant shear flexibility and bending stiffness is attached to supports A and B by means of rigid stay plates of length $\delta_1 L$ and $\delta_2 L$ at each end of the members.

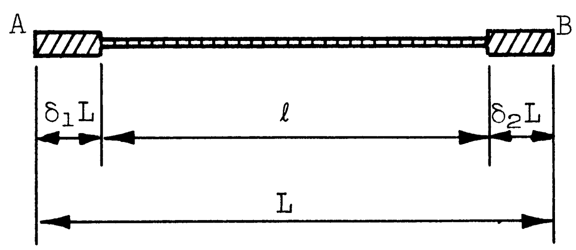

Figure 3-1. The structural member with rigid stay plates at the ends of the member.

From Figure 3-1, we have

$$L = \ell + \delta_1 L + \delta_2 L$$

$$\therefore \ell = (1 - \delta_1 - \delta_2)L \tag{3-4}$$

where

δ_1 and δ_2 are nondimensional factors relating the length of the rigid stay plates to the total length L of the member.

From Equations (3-3) and (3-4), we obtain

$$\mu = \frac{1}{(1 - \delta_1 - \delta_2)^2} \frac{\eta EI}{AGL^2} \tag{3-5}$$

It should be remembered that the application of μ by Equation (3-5) is only applicable in the region ℓ.

3.2. FUNDAMENTAL DIFFERENTIAL EQUATIONS

The contribution of the bending moment M_x and the shear force V_x to the deformation of an infinitesimal element dx, cut out from the structural member, is illustrated in Figure 3-2.

The unit angle change ϕ due to bending moment and the angular rotation γ due to shear force, as shown in Figure 3-2, are given by the well-known formulas:

$$\phi = -\frac{M_x}{EI} \tag{3-6}$$

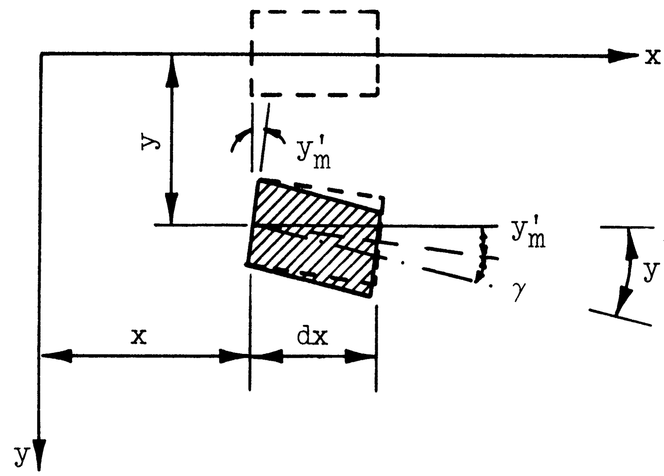

Figure 3-2. The deformation of infinitesimal element dx, cut out from the structural member, due to shear force.

and

$$\gamma = \frac{\eta V_x}{AG} \tag{3-7}$$

The additional slope (the shear strain γ) of the deflection curve due to the shear force V_x is seen from the detail of cut out section. Adding this shear rotation γ to the slope y'_m, due to bending moment M_x only, gives the total slope:

$$y' = y'_m + \gamma \tag{3-8}$$

Differentiating with respect to x produces

$$y'' = y''_m + \gamma' \tag{3-9}$$

where

$$y''_m = \phi = - \frac{M_x}{EI}$$

Equations (3-6), (3-7), and (3-9) lead immediately to the fundamental linear second-order differential equation for the deflection curve of the structural member, i.e.,

$$y'' + \frac{M_x}{EI} - \left(\frac{\eta V_x}{AG}\right)' = 0 \qquad (3\text{-}10)$$

Consider now Figure 3-3 which presents the structural member A-B subjected to an arbitrary external loading p(x) in a general condition.

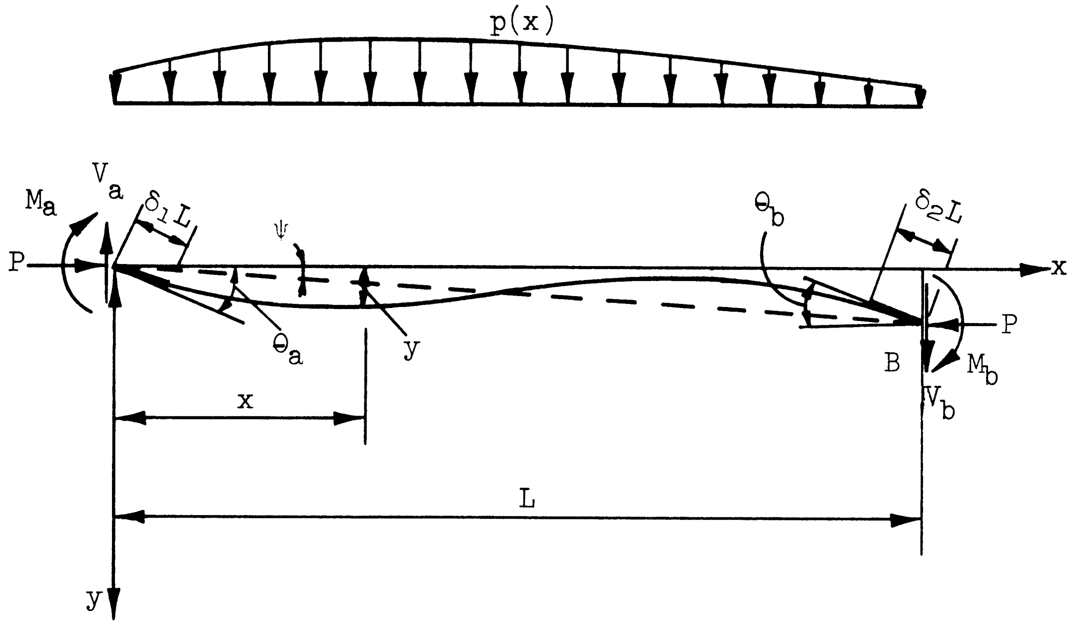

Figure 3-3. The structural member A-B subjected to an arbitrary external loading p(x) and joint displacement.

Let M_s and V_s be the moment and shear force due to an arbitrary external loading p(x) on a simply supported beam of length L, when the axial load P is not acting. Then the total bending moment M_x, and the total shear force V_x, at a distance x from the support A in the member A-B, are simply given by:

$$M_x = M_s + M_a + V_a x + Py \qquad (3\text{-}11)$$

$$V_x = V_s + V_a + Py'_m \tag{3-12}$$

where

M_a and V_a are the end bending moment and shear force, respectively.

From Equation (3-8) is derived

$$y'_m = y' - \gamma$$

Substituting y'_m into Equation (3-12) which becomes

$$V_x = \frac{1}{1 + \frac{\eta P}{GA}} (V_s + V_a + Py') \tag{3-13}$$

Substituting the expressions for M_x and V_x from Equations (3-11) and (3-13), respectively, into Equation (3-10), we obtain

$$\frac{1}{1 + \frac{\eta P}{GA}} y'' + \frac{P}{EI} y = - \frac{1}{EI} (M_s + M_a + V_a x) + \left(\frac{V_s + V_a}{\frac{GA}{\eta} + P}\right)' \tag{3-14}$$

where

$$\delta_1 L \leq x \leq (1 - \delta_2)L$$

It is noted that Equation (3-14) illustrates a most important property of this kind of the second-order theory. As long as the axial load P is kept constant, then the effects of end moments, end-shear forces and the external loading p(x) can be superimposed. The ordinary beam theory as modified to include shear can still be applied since the effect of axial load P appears here only in a modification of the stiffness properties of the structural member.

We now develop relations for the stiffness properties of the structural member. For convenience we let the external loading p(x) be equal to zero. Then from the equilibrium conditions we obtain:

$$M_s = V_s \equiv 0 \tag{3-15}$$

$$V_a = V_b \tag{3-16}$$

$$V_b = - P\psi - \frac{M_a + M_b}{L} \tag{3-17}$$

where ψ is the member rotation.

Substitute these values of M_s, V_s, V_a, V_b from the above expressions into Equations (3-11) and (3-13) for the bending moment and shear which becomes:

$$M_x = M_a - P\psi x - (M_a + M_b) \frac{x}{L} + Py \tag{3-18}$$

$$V_x = \frac{1}{1 + \frac{\eta P}{GA}} \left(- P\psi - \frac{M_a + M_b}{L} + Py'\right) \tag{3-19}$$

Then the fundamental differential Equation (3-14), for the flexible part, leads to:

$$\frac{1}{1 + \frac{\eta P}{GA}} y'' + \frac{P}{EI} y = \frac{1}{EI} [P\psi x + (M_a + M_b) \frac{x}{L} - M_a] \tag{3-20}$$

The general solutions of this differential equation are of a fundamentally different nature for the cases when the axial load P is greater than zero (compression axial force), equal to zero, and less than zero (tension axial force).

The general solutions of the differential Equation (3-20), and their first derivatives are of interest and are given by the following expressions:

(1) Case P = 0 (No axial force)

$$y = (M_a + M_b)\frac{L}{EI}\frac{x^3}{6L^2} - M_a\frac{L}{EI}\frac{x^2}{2L} + Ax + BL \qquad (3\text{-}21)$$

$$y' = (M_a + M_b)\frac{L}{EI}\frac{x^2}{2L^2} - M_a\frac{L}{EI}\frac{x}{L} + A \qquad (3\text{-}22)$$

(2) Case P > 0 (Compressive axial force)

$$y = AL\cos k\frac{x}{L} + BL\sin k\frac{x}{L} + \psi x + \frac{M_a + M_b}{P}\frac{x}{L} - \frac{M_a}{P} \qquad (3\text{-}23)$$

$$y' = -Ak\sin k\frac{x}{L} + Bk\cos k\frac{x}{L} + \psi + \frac{M_a + M_b}{PL} \qquad (3\text{-}24)$$

(3) Case P < 0 (Tension axial force)

$$y = AL\cosh k\frac{x}{L} + BL\sinh k\frac{x}{L} + \psi x - \frac{M_a + M_b}{P}\frac{x}{L} + \frac{M_a}{P} \qquad (3\text{-}25)$$

$$y' = Ak\sinh k\frac{x}{L} + Bk\cosh k\frac{x}{L} + \psi - \frac{M_a + M_b}{PL} \qquad (3\text{-}26)$$

where

$$k = L\sqrt{\frac{P}{EI}\left(1 + \frac{\eta P}{GA}\right)} \qquad (3\text{-}27)$$

A and B are integration constants.

Four boundary conditions are available for the evaluation of the integration constants A, B, and either the end-moments M_a, M_b, or the end-rotation Θ_a, Θ_b of the structural members. The four boundary conditions are:

(i) $x = \delta_1 L$, $\qquad y = \Theta_a \delta_1 L \qquad (3\text{-}28)$

(ii) $x = \delta_1 L$, $\qquad y' = \left(1 + \frac{\eta P}{GA}\right)\Theta_a - \frac{\eta}{GA}\left(P\psi + \frac{M_a + M_b}{L}\right) \qquad (3\text{-}29)$

Similarly,

$$\text{(iii)} \quad x = (1 - \delta_2)L, \qquad y = (\psi - \theta_b \delta_2)L \tag{3-30}$$

$$\text{(iv)} \quad x = (1 - \delta_2)L, \qquad y' = (1 + \frac{\eta P}{GA})\,\theta_b - \frac{\eta}{GA}\,(P\psi + \frac{M_a + M_b}{L}) \tag{3-31}$$

3.3. THE MODIFIED SLOPE-DEFLECTION EQUATIONS

By application of the four boundary conditions (3-28), (3-29), (3-30), (3-31) to the general solution of deflected shape of the structural member, the coefficients for the modified slope-deflection equations may be determined. The equations are:

$$M_a = \frac{EI}{L}\,[C_{11}\theta_a + C_{12}\theta_b - (C_{11} + C_{12})\,\psi] \tag{3-32}$$

$$M_b = \frac{EI}{L}\,[C_{21}\theta_a + C_{22}\theta_b - (C_{21} + C_{22})\,\psi] \tag{3-33}$$

Or, to show the relation to the moment-distribution procedure, they may be written:

$$M_a = C_{11}\,\frac{EI}{L}\,[\theta_a + r_{12}\theta_b - (1 + r_{12})\,\psi] \tag{3-34}$$

$$M_b = C_{22}\,\frac{EI}{L}\,[r_{21}\theta_a + \theta_b - (1 + r_{21})\,\psi] \tag{3-35}$$

where C_{11}, C_{12}, C_{21}, and C_{22} are the well known slope-deflection constants which depend upon the properties of the structural member and, in addition, are functions of the axial load P. r_{12} and r_{21} are carry-over factors in the direction from point A to B and from point B to A, respectively, as shown in Figure 3-3, and therefore are defined by:

$$r_{12} = \frac{C_{12}}{C_{11}} \tag{3-36}$$

$$r_{21} = \frac{C_{21}}{C_{22}} \tag{3-37}$$

For convenience, the dimensionless axial load parameter α introduced as:

$$\alpha = \frac{P}{P_e} \tag{3-38}$$

where P_e is the Euler load for a column without stay plates, i.e.,

$$P_e = \frac{\pi^2 EI}{L^2} \tag{3-39}$$

The parameter k which has been defined in Equation (3-27), now can be transformed in terms of nondimensional parameters μ and α in the Equations (3-5) and (3-38), respectively. Thus we have

$$k^2 = \pi^2\alpha(1 + \pi^2\alpha\mu^*) \tag{3-40}$$

where

$$\mu^* = (1 - \delta_1 - \delta_2)^2\mu \tag{3-41}$$

For convenience, let

$$k^* = (1 - \delta_1 - \delta_2)k \tag{3-42}$$

Then, the four boundary conditions (3-28), (3-29), (3-30), and (3-31) lead to a system of four simultaneous linear equations for the joint rotations Θ_a and Θ_b; the member rotation ψ; the integration constants A, B; and the member end-moments M_a and M_b. Now it will also be transformed in terms of non-dimensional parameters μ, δ_1, δ_2, α, and k as follows:

(1) Case $\alpha = 0$ (No Axial Force)

$$\begin{bmatrix} \frac{\delta_1^3}{6} - \frac{\delta_1^2}{2} & \frac{\delta_1^3}{6} & \delta_1 & 1 & -\delta_1 & 0 & 0 \\ \frac{(1-\delta_2)^3}{6} - \frac{(1-\delta_2)^2}{2} & \frac{(1-\delta_2)^3}{6} & 1-\delta_2 & 1 & 0 & \delta_2 & -1 \\ \mu^* + \frac{\delta_1^2}{2} - \delta_1 & \mu^* + \frac{\delta_1^2}{2} & 1 & 0 & -1 & 0 & 0 \\ \mu^* + \frac{(1-\delta_2)^2}{2} - (1-\delta_2) & \mu^* + \frac{(1-\delta_2)^2}{2} & 1 & 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \frac{M_a L}{EI} \\ \frac{M_b L}{EI} \\ A \\ B \\ \Theta_a \\ \Theta_b \\ \psi \end{bmatrix} = 0 \qquad (3\text{-}43)$$

(2) Case $\alpha > 0$ (Compressive Axial Force)

$$\begin{bmatrix} -\frac{1-\delta_1}{\pi^2\alpha} & \frac{\delta_1}{\pi^2\alpha} & \cos k\delta_1 & \sin k\delta_1 & -\delta_1 & 0 & \delta_1 \\ -\frac{\delta_2}{\pi^2\alpha} & \frac{1-\delta_2}{\pi^2\alpha} & \cos k(1-\delta_2) & \sin k(1-\delta_2) & 0 & \delta_2 & -\delta_2 \\ \mu^* + \frac{1}{\pi^2\alpha} & \mu^* + \frac{1}{\pi^2\alpha} & -k\sin k\delta_1 & k\cos k\delta_1 & -1-\pi^2\alpha\mu^* & 0 & 1+\pi^2\alpha\mu^* \\ \mu^* + \frac{1}{\pi^2\alpha} & \mu^* + \frac{1}{\pi^2\alpha} & -k\sin k(1-\delta_2) & k\cos k(1-\delta_2) & 0 & -1-\pi^2\alpha\mu^* & 1+\pi^2\alpha\mu^* \end{bmatrix} \begin{bmatrix} \frac{M_a L}{EI} \\ \frac{M_b L}{EI} \\ A \\ B \\ \Theta_a \\ \Theta_b \\ \psi \end{bmatrix} = 0 \qquad (3\text{-}44)$$

(3) Case $\alpha < 0$ (Tension Axial F

$$\begin{bmatrix} -\frac{1-\delta_1}{\pi^2\alpha} & \frac{\delta_1}{\pi^2\alpha} & \cosh k\delta_1 & \sinh k\delta_1 & -\delta_1 & 0 & \delta_1 \\ -\frac{\delta_2}{\pi^2\alpha} & \frac{1-\delta_2}{\pi^2\alpha} & \cosh k(1-\delta_2) & \sinh k(1-\delta_2) & 0 & \delta_2 & -\delta_2 \\ \mu^* + \frac{1}{\pi^2\alpha} & \mu^* + \frac{1}{\pi^2\alpha} & k\sinh k\delta_1 & k\cosh k\delta_1 & -1-\pi^2\alpha\mu^* & 0 & 1+\pi^2\alpha\mu^* \\ \mu^* + \frac{1}{\pi^2\alpha} & \mu^* + \frac{1}{\pi^2\alpha} & k\sinh k(1-\delta_2) & k\cosh k(1-\delta_2) & 0 & -1-\pi^2\alpha\mu^* & 1+\pi^2\alpha\mu^* \end{bmatrix} \begin{bmatrix} \frac{M_a L}{EI} \\ \frac{M_b L}{EI} \\ A \\ B \\ \Theta_a \\ \Theta_b \\ \psi \end{bmatrix} = 0 \qquad (3\text{-}45)$$

To formulate the modified slope-deflection equations, the sets of four linear nonhomogeneous equations (3-43), (3-44), and (3-45), in four unknowns M_a, M_b, A, and B will be solved by Cramer's Rule in terms of Θ_a, Θ_b, and ψ. Then, the solutions will be set up in the forms of Equations (3-32), (3-33), (3-34), and (3-35).

Where

(1) Case $\alpha = 0$

$$C_{11} = 4\left[\frac{3\mu^* + \delta_1^2 + \delta_1(1 - \delta_2) + (1 - \delta_2)^2}{(1 - \delta_1 - \delta_2)^3(12\mu + 1)}\right] \tag{3-46}$$

$$C_{12} = C_{21} = 2\left[\frac{(1 - \delta_2)^2(1 + 2\delta_2) - \delta_1^2(3 - 2\delta_1) - 6(1 - \delta_1 - \delta_2)^3\mu}{(1 - \delta_1 - \delta_2)^4(12\mu + 1)}\right] \tag{3-47}$$

$$C_{22} = 4\left[\frac{3\mu^* + (1 - \delta_1)^2 + (1 - \delta_1)\delta_2 + \delta_2^2}{(1 - \delta_1 - \delta_2)^3(12\mu + 1)}\right] \tag{3-48}$$

$$r_{12} = \frac{1}{2}\left[\frac{(1 - \delta_2)^2(1 + 2\delta_2) - \delta_1^2(3 - 2\delta_1) - 6(1 - \delta_1 - \delta_2)^3\mu}{(1 - \delta_1 - \delta_2)(3\mu^* + \delta_1^2 + \delta_1(1 - \delta_2) + (1 - \delta_2)^2)}\right] \tag{3-49}$$

$$r_{21} = \frac{1}{2}\left[\frac{(1 - \delta_2)^2(1 + 2\delta_2) - \delta_1^2(3 - 2\delta_1) - 6(1 - \delta_1 - \delta_2)^3\mu}{(1 - \delta_1 - \delta_2)(3\mu^* + (1 - \delta_1)^2 + (1 - \delta_1)\delta_2 + \delta_2^2)}\right] \tag{3-50}$$

(2) Case $\alpha > 0$

$$C_{11} = \frac{1 - k^* \cot k^* + [\mu^* + \delta_1(1 - \delta_2)]\ \pi^2\alpha}{(1 - \delta_1 - \delta_2)\left[\frac{2}{k^*}(1 + \pi^2\alpha\mu^*)\tan\frac{k^*}{2} - 1\right]} \tag{3-51}$$

$$C_{12} = C_{21} = \frac{k \csc k^* - 1 - k(\delta_1 + \delta_2)\cot k^* + (\delta_1\delta_2 - \mu^*)\ \pi^2\alpha}{(1 - \delta_1 - \delta_2)\left[\frac{2}{k^*}(1 + \pi^2\alpha\mu^*)\tan\frac{k^*}{2} - 1\right]} \tag{3-52}$$

$$C_{22} = \frac{1 - k^* \cot k^* + [\mu^* + (1 - \delta_1)\delta_2]\ \pi^2\alpha}{(1 - \delta_1 - \delta_2)\left[\frac{2}{k^*}(1 + \pi^2\alpha\mu^*)\tan\frac{k^*}{2} - 1\right]} \tag{3-53}$$

$$r_{12} = \frac{k \csc k^* - 1 - k(\delta_1 + \delta_2)\cot k^* + (\delta_1\delta_2 - \mu^*)\ \pi^2\alpha}{1 - k^* \cot k^* + [\mu^* + \delta_1(1 - \delta_2)]\ \pi^2\alpha} \tag{3-54}$$

$$r_{21} = \frac{k \csc k^* - 1 - k(\delta_1 + \delta_2)\cot k^* + (\delta_1\delta_2 - \mu^*)\ \pi^2\alpha}{1 - k^* \cot k^* + [\mu^* + (1 - \delta_1)\delta_2]\ \pi^2\alpha} \tag{3-55}$$

(3) Case $\alpha < 0$

$$C_{11} = \frac{k^* \coth k^* - 1 - [\mu^* + \delta_1(1 - \delta_2)]\ \pi^2\alpha}{(1 - \delta_1 - \delta_2)\left[1 - \frac{2}{k^*}(1 + \pi^2\alpha\mu^*)\tanh\frac{k^*}{2}\right]} \tag{3-56}$$

$$C_{12} = C_{21} = \frac{1 - k \,\mathrm{csch}\, k^* + k(\delta_1 + \delta_2)\coth k^* + (\mu^* - \delta_1\delta_2)\ \pi^2\alpha}{(1 - \delta_1 - \delta_2)\left[1 - \frac{2}{k^*}(1 + \pi^2\alpha\mu^*)\tanh\frac{k^*}{2}\right]} \tag{3-57}$$

$$C_{22} = \frac{k^* \coth k^* - 1 - [\mu^* + (1 - \delta_1)\delta_2]\ \pi^2\alpha}{(1 - \delta_1 - \delta_2)\left[1 - \frac{2}{k^*}(1 + \pi^2\alpha\mu^*)\tanh\frac{k^*}{2}\right]} \tag{3-58}$$

$$r_{12} = \frac{1 - k \,\mathrm{csch}\, k^* + k(\delta_1 + \delta_2)\coth k^* + (\mu^* - \delta_1\delta_2)\ \pi^2\alpha}{k^* \coth k^* - 1 - [\mu^* + \delta_1(1 - \delta_2)]\ \pi^2\alpha} \tag{3-59}$$

$$r_{21} = \frac{1 - k \,\mathrm{csch}\, k^* + k(\delta_1 + \delta_2)\coth k^* + (\mu^* - \delta_1\delta_2)\ \pi^2\alpha}{k^* \coth k^* - 1 - [\mu^* + (1 - \delta_1)\delta_2]\ \pi^2\alpha} \tag{3-60}$$

The numerical values of the stiffness constant C_{11} and its corresponding carry-over factor r_{12} are presented in Figures 3-4 for the special (symmetrical) case of $\delta_1 = \delta_2 = \delta$.

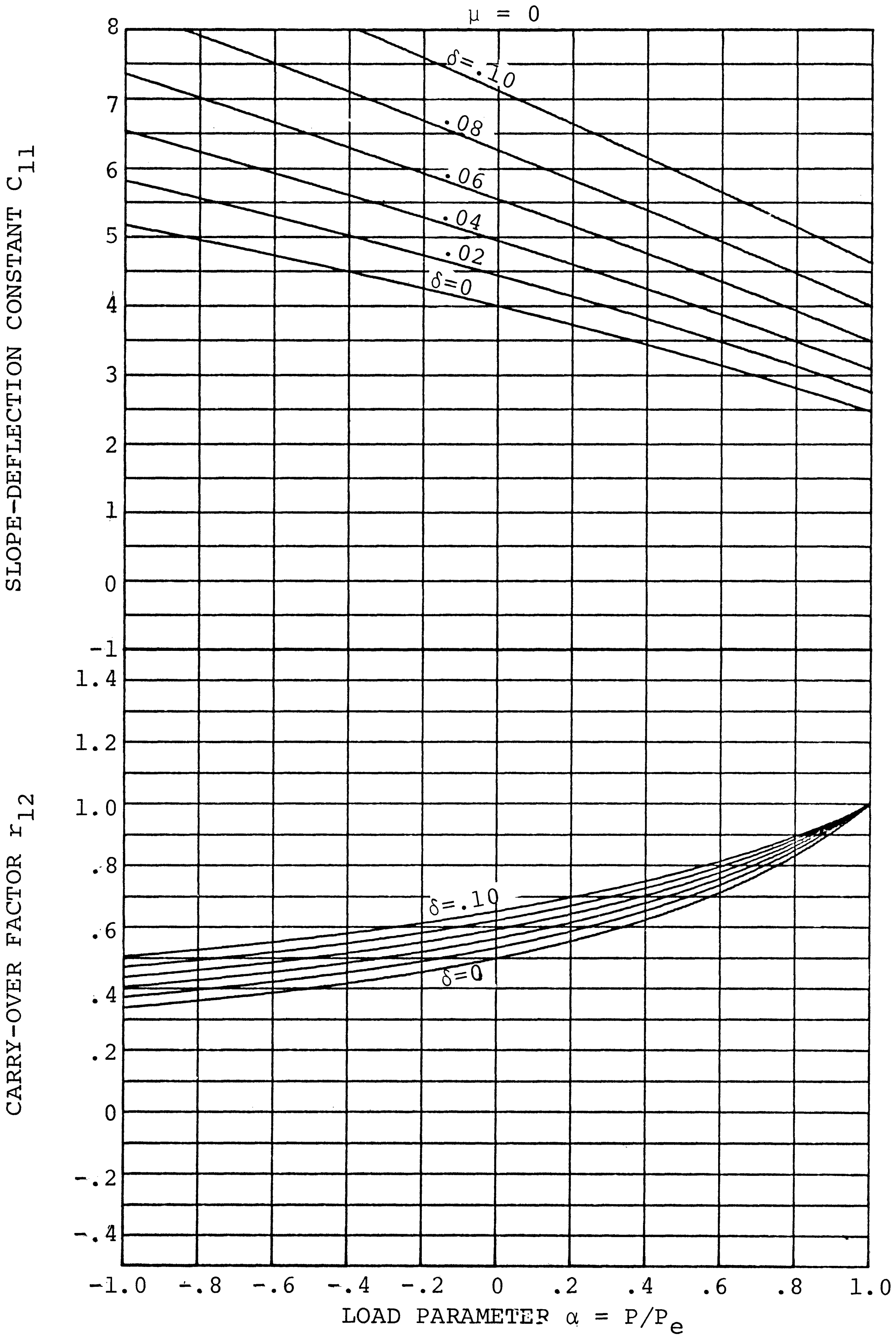

Figure 3-4. The slope-deflection constant C_{11} and carry-over factor r_{12} of structural members with different rigid stay plates δ and constant shear flexibility μ.

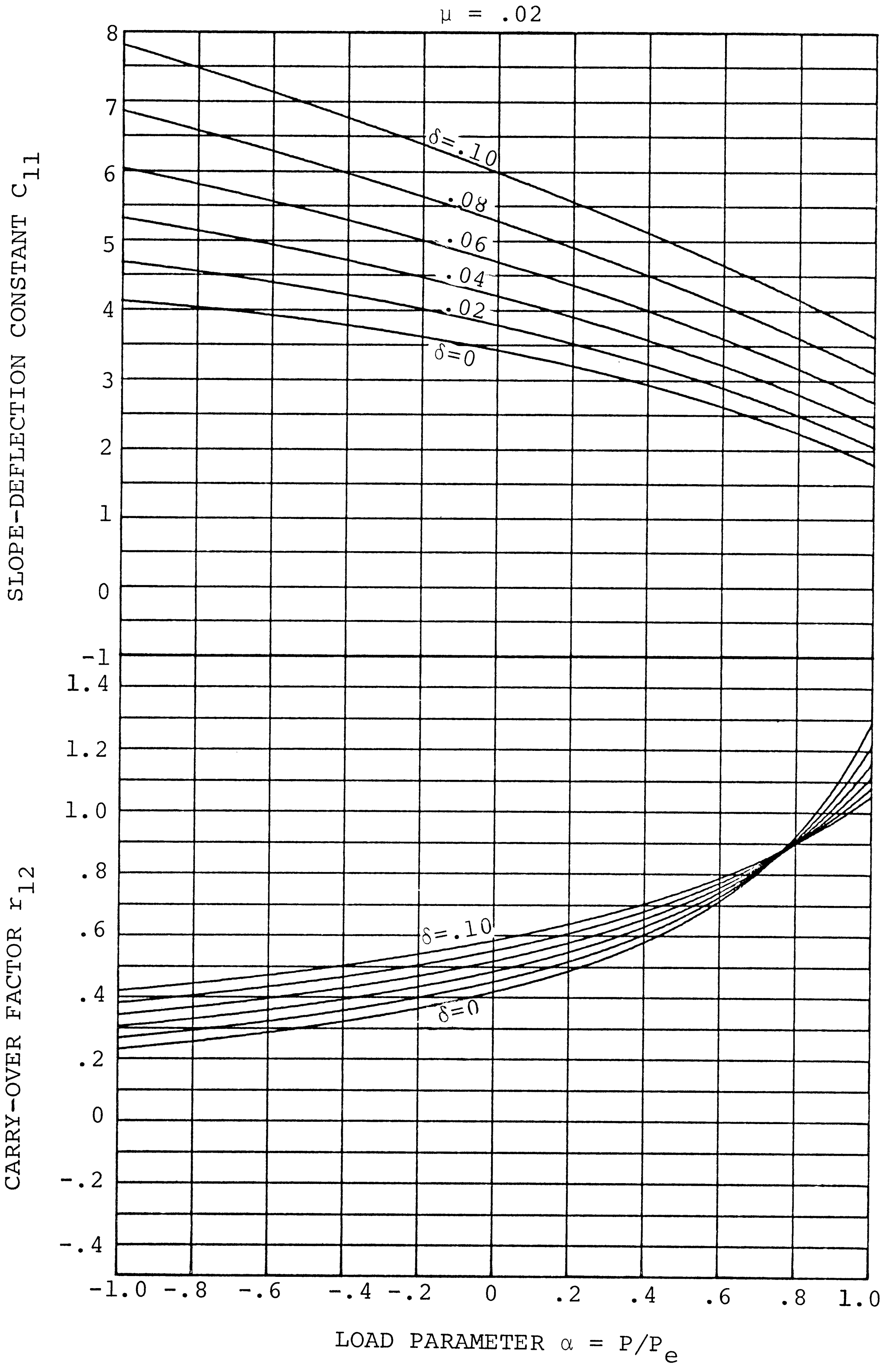

Figure 3-4 (Continued).

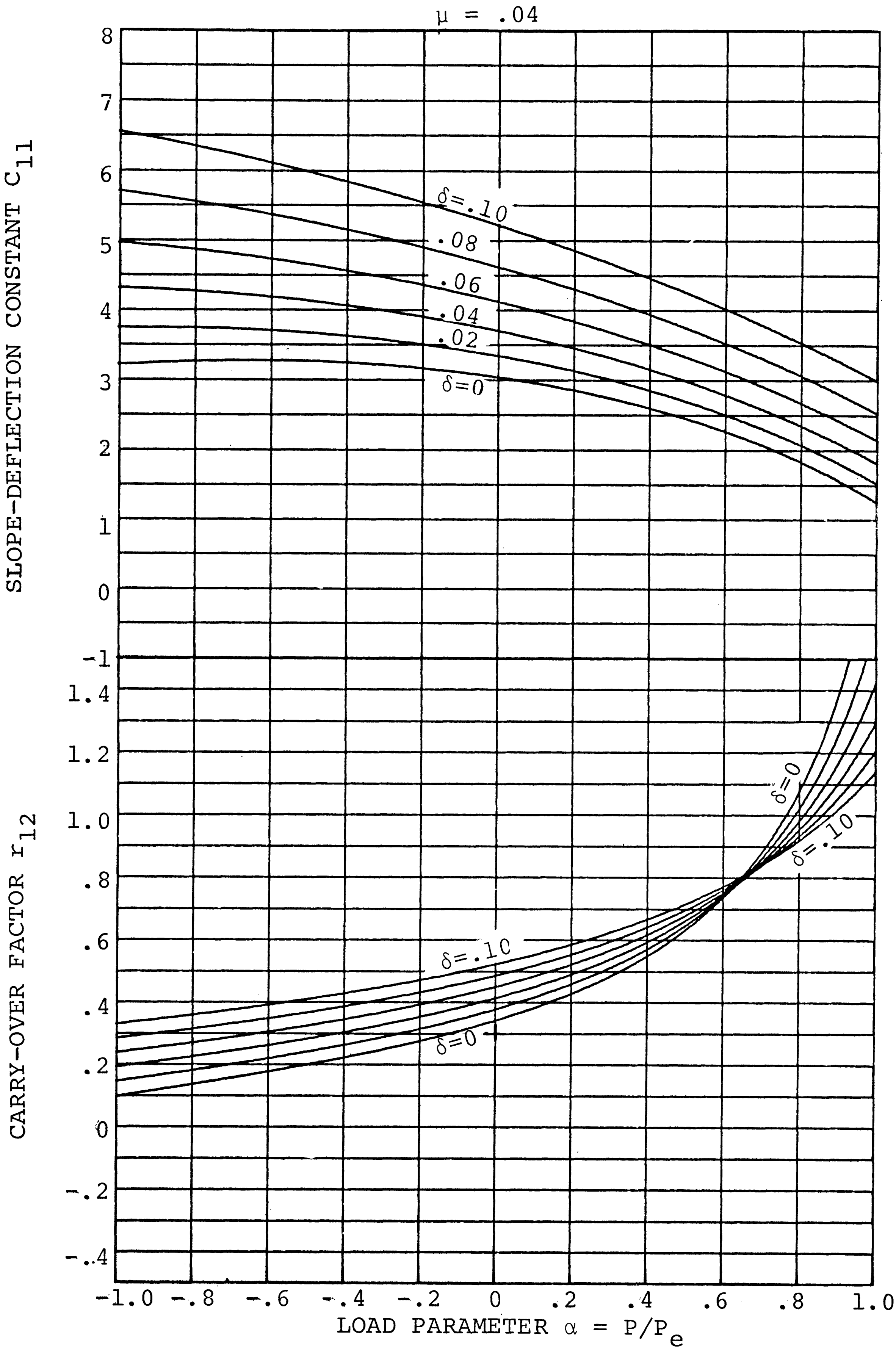

Figure 3-4 (Continued).

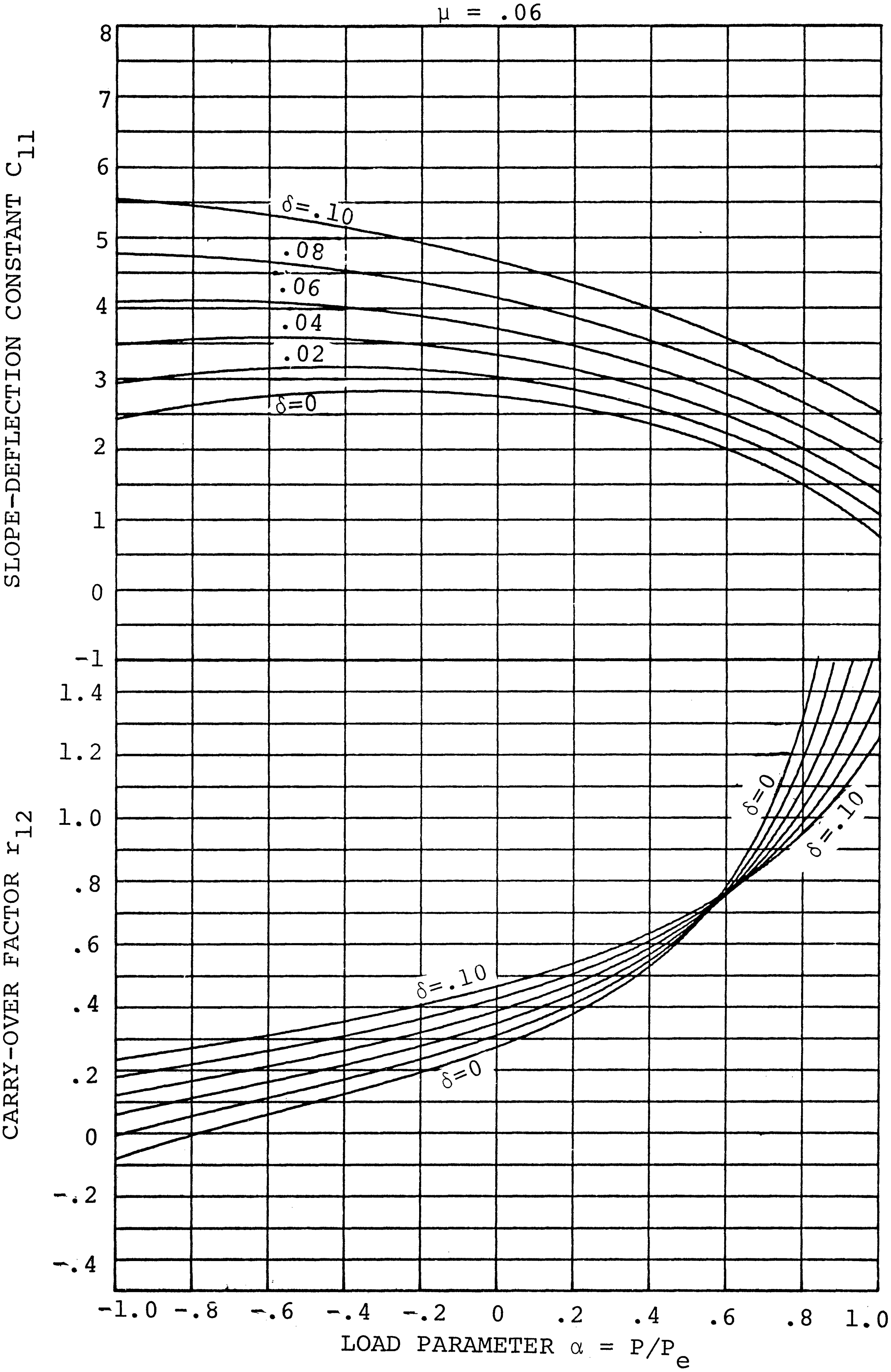

Figure 3-4 (Continued).

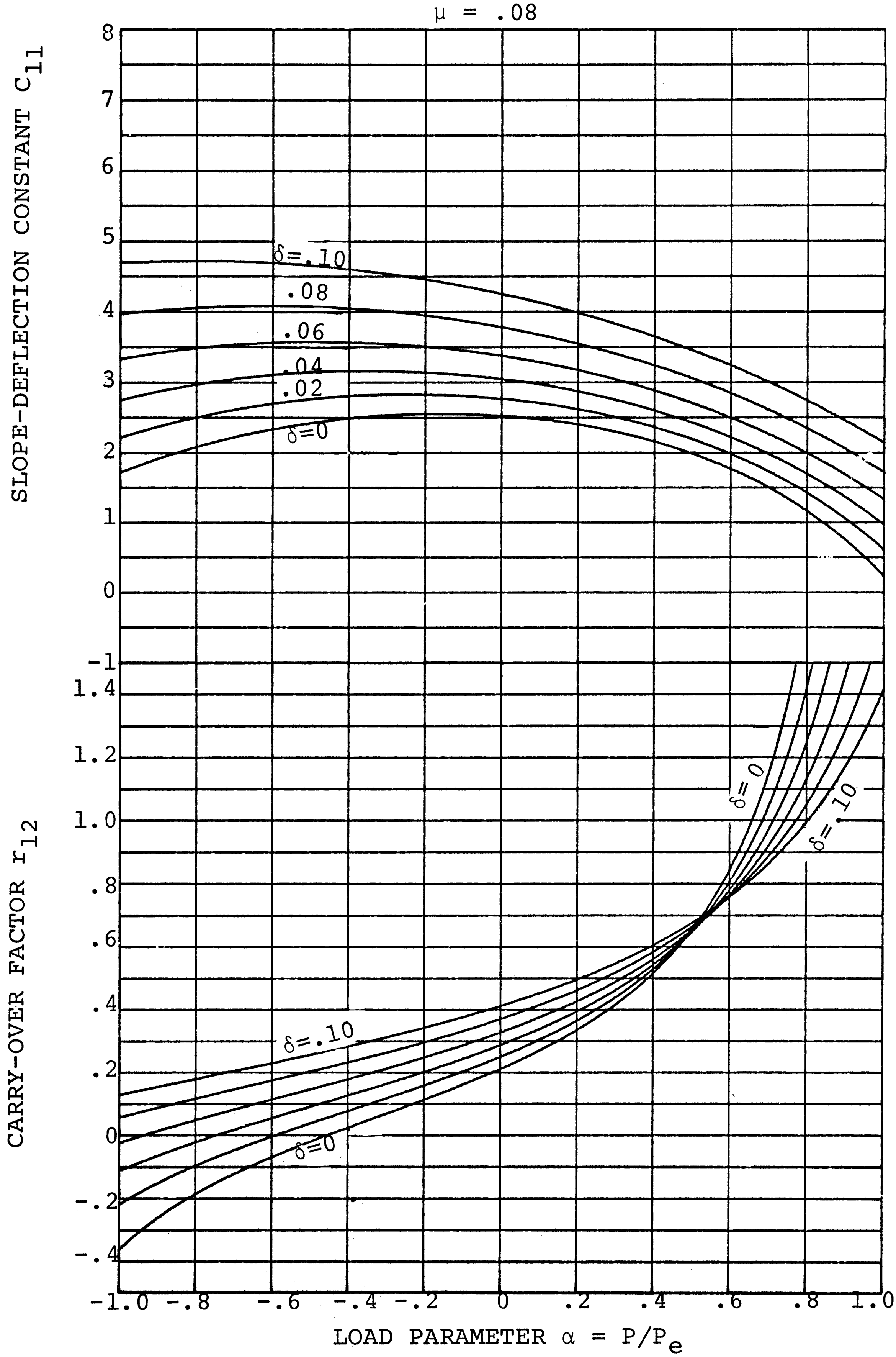

Figure 3-4 (Continued).

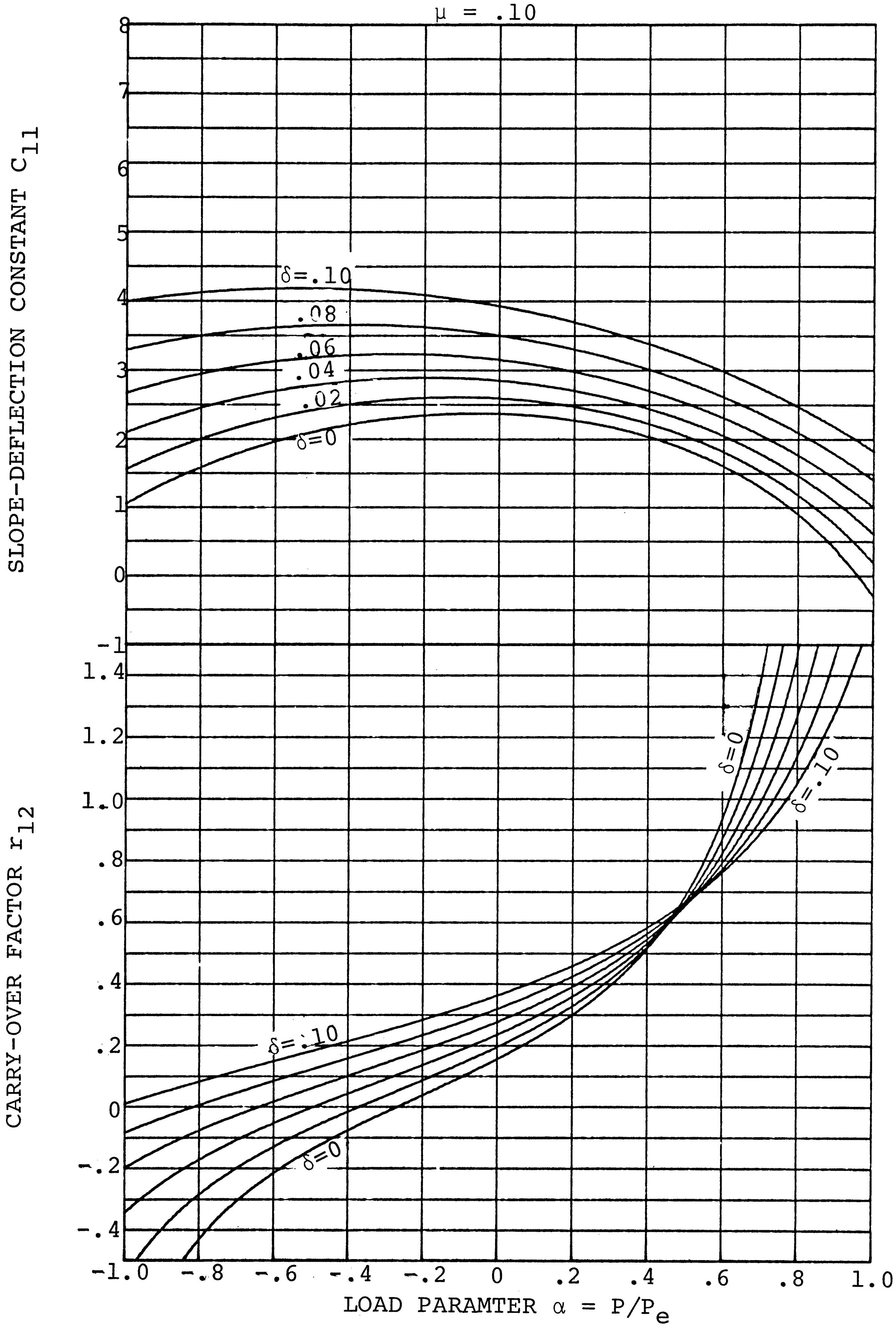

Figure 3-4 (Continued).

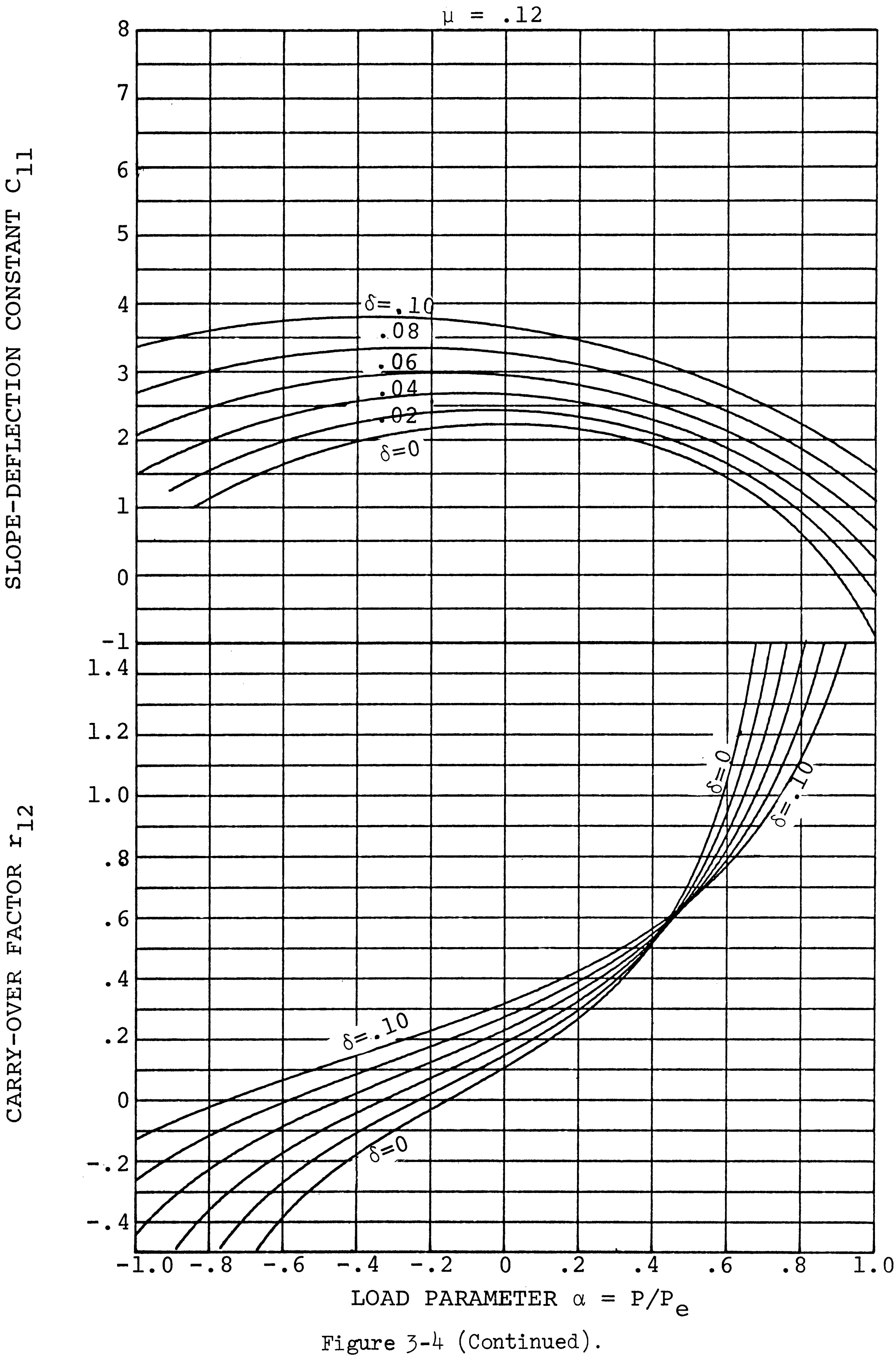

Figure 3-4 (Continued).

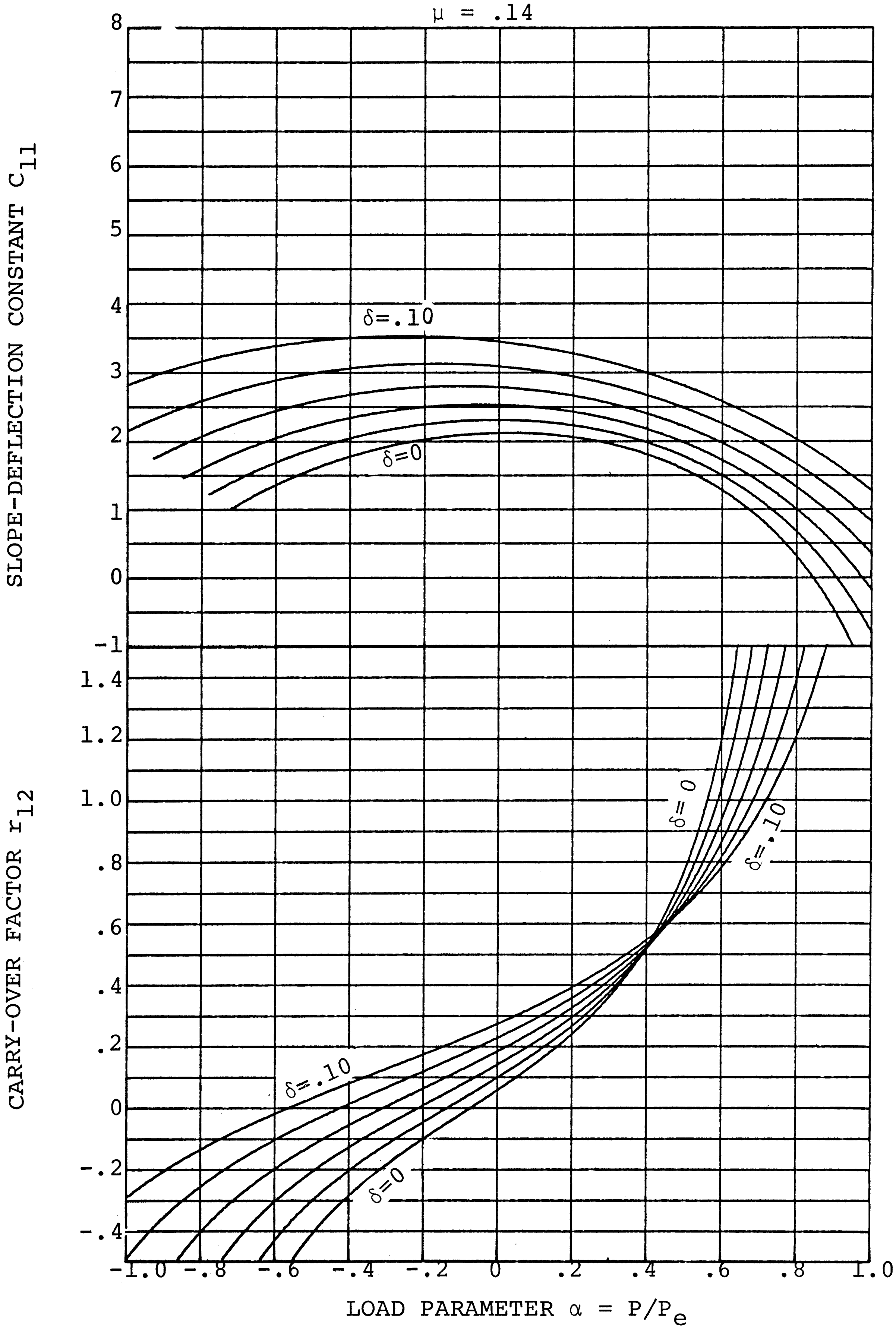

Figure 3-4 (Continued).

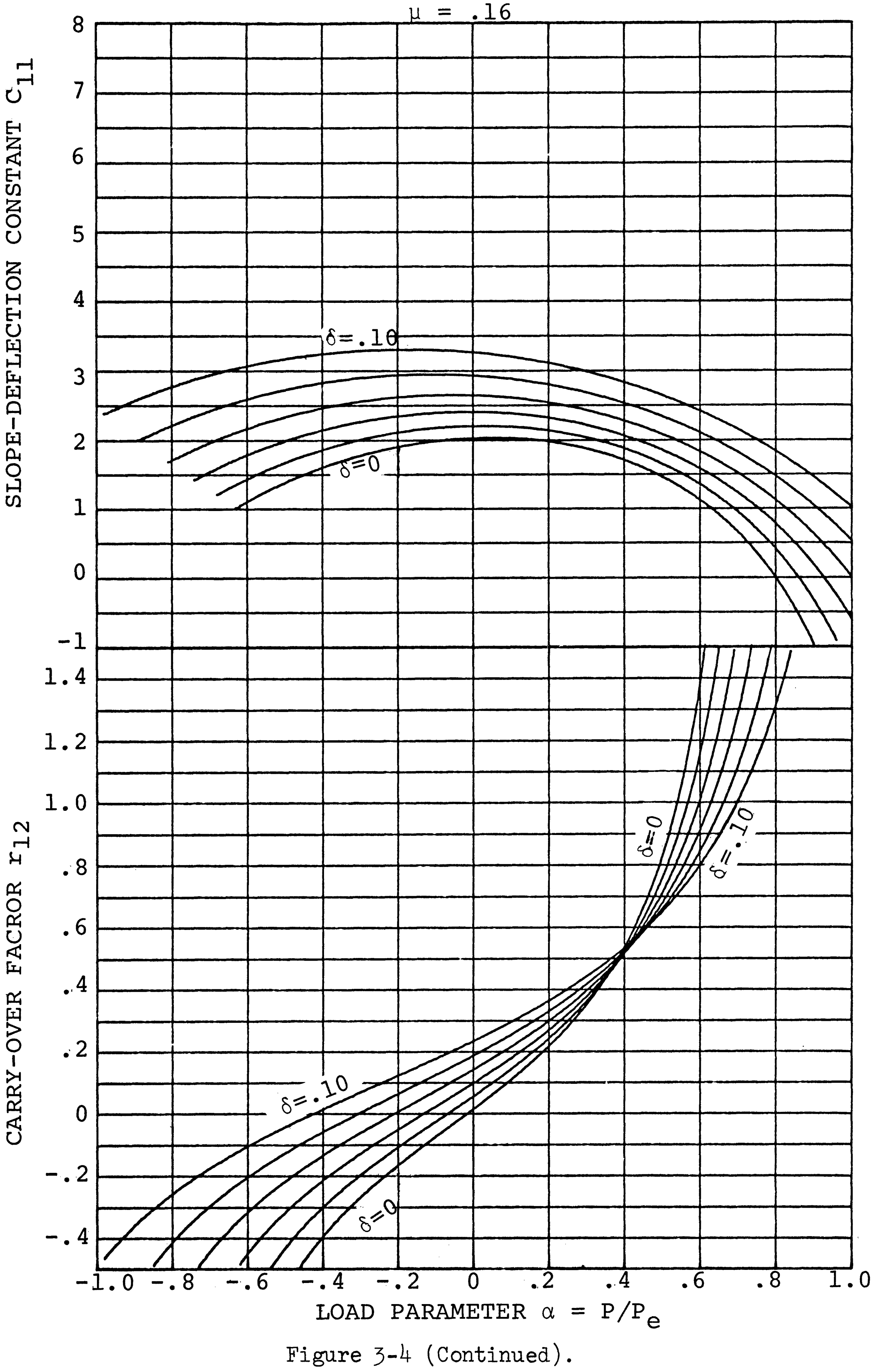

Figure 3-4 (Continued).

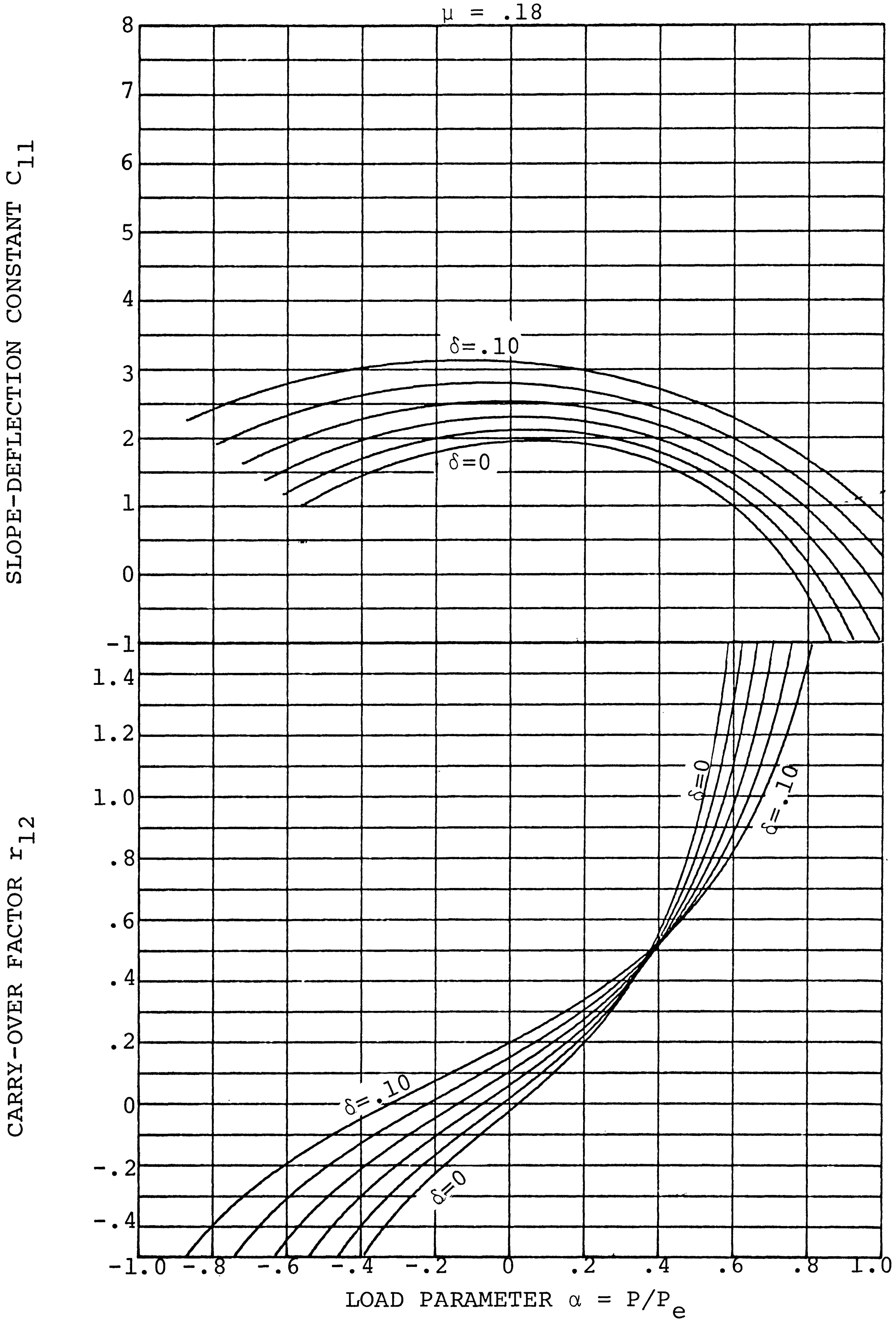

Figure 3-4 (Continued).

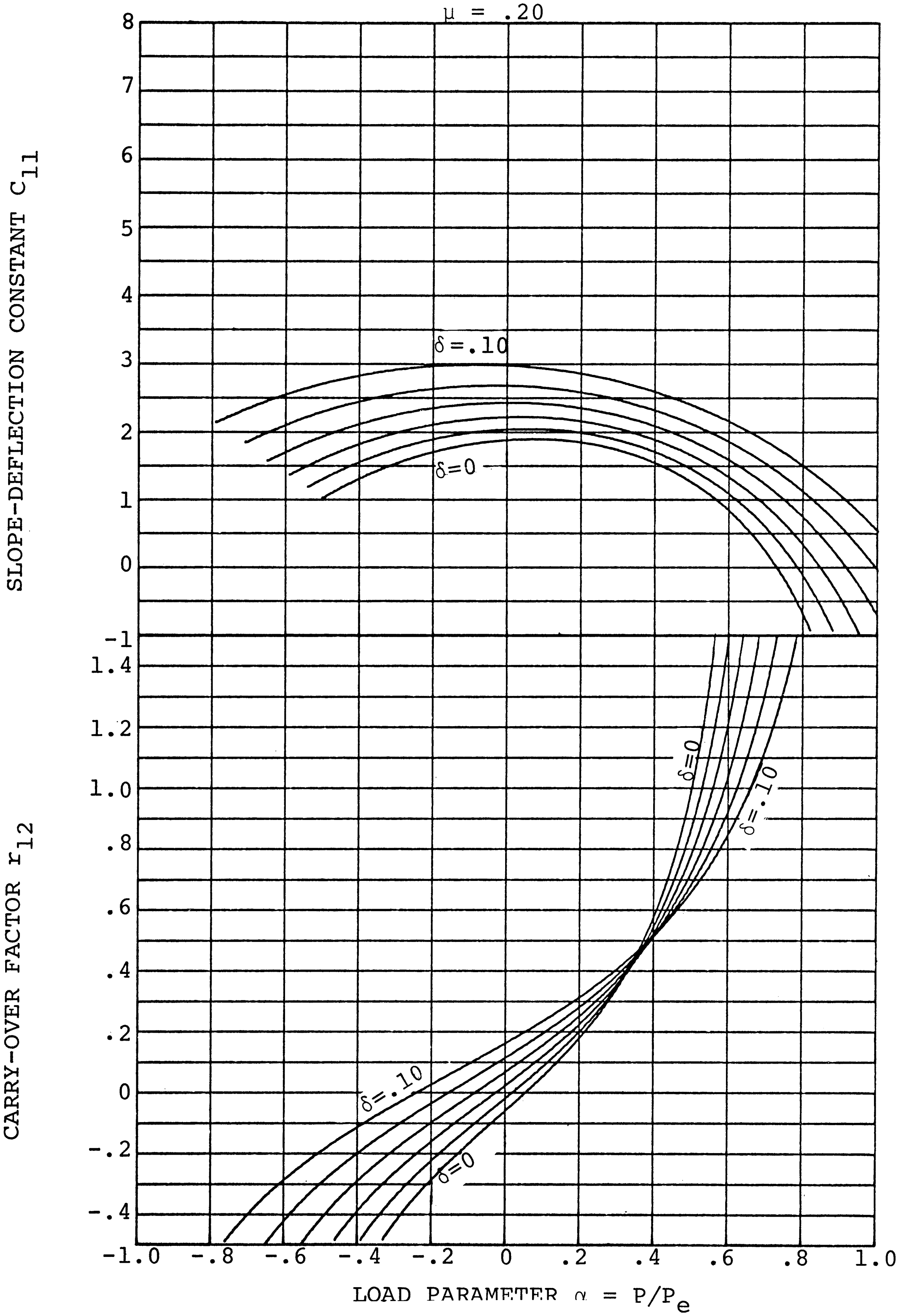

Figure 3-4 (Continued).

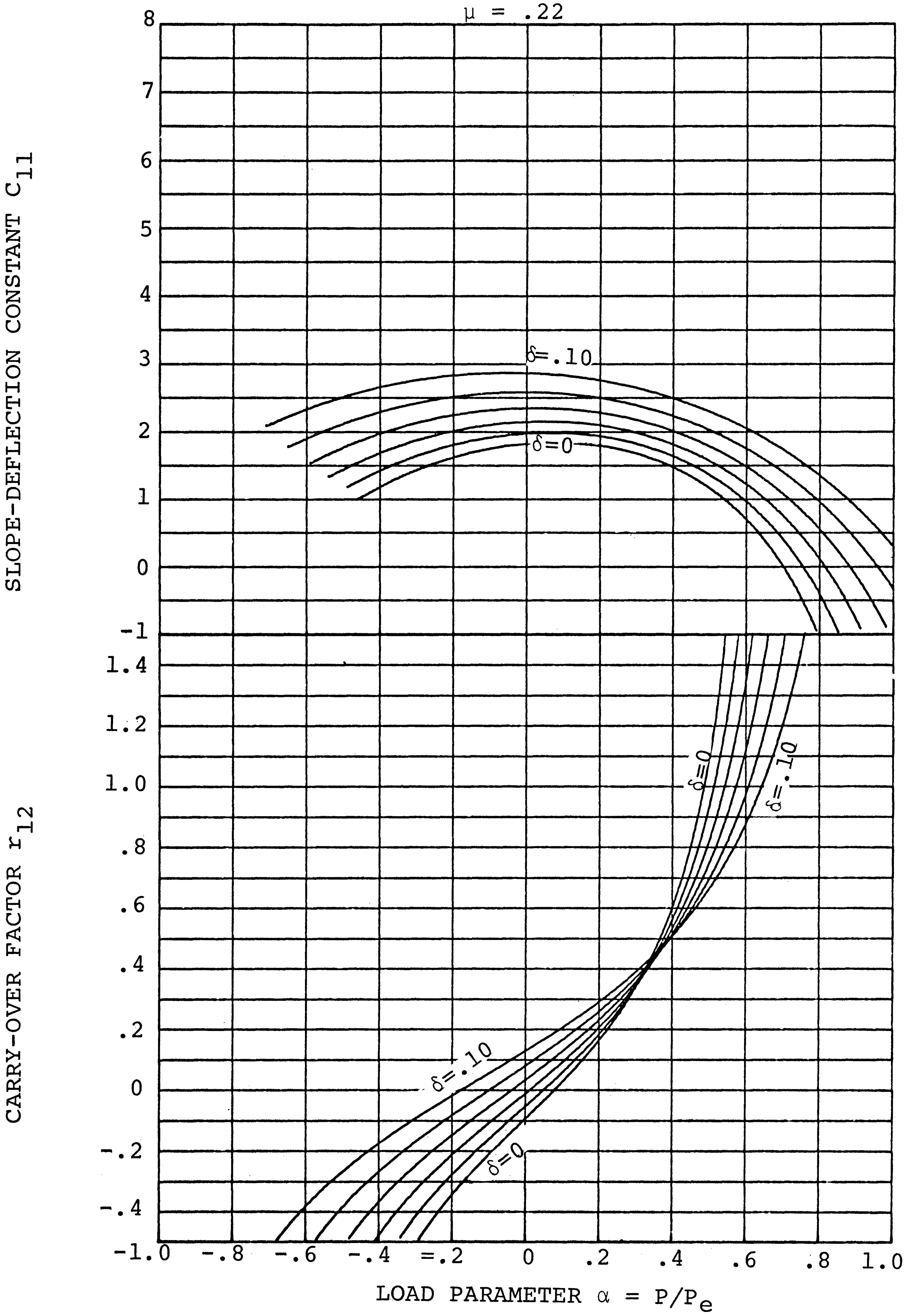

Figure 3-4 (Continued).

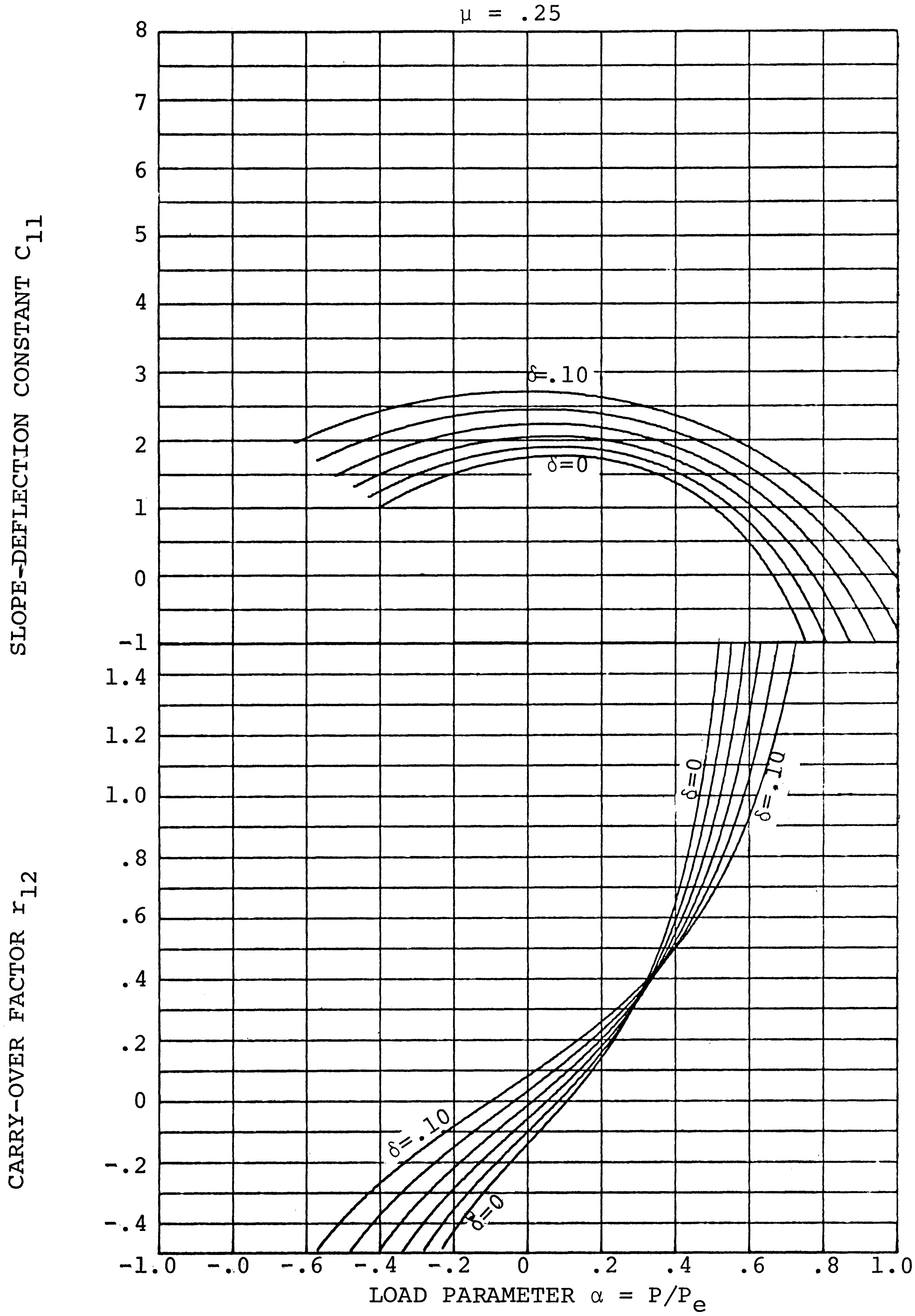

Figure 3-4 (Continued).

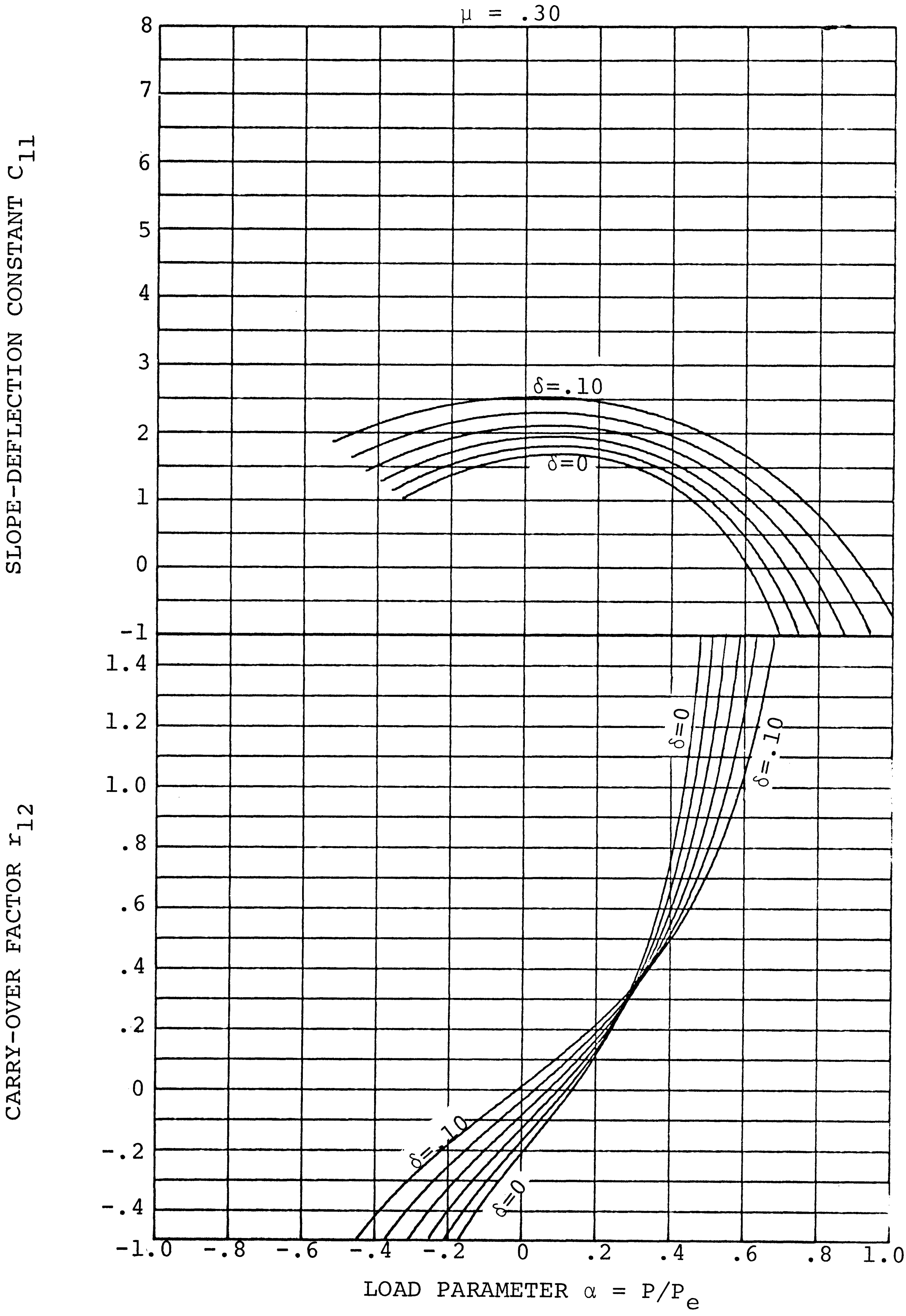

Figure 3-4 (Continued).

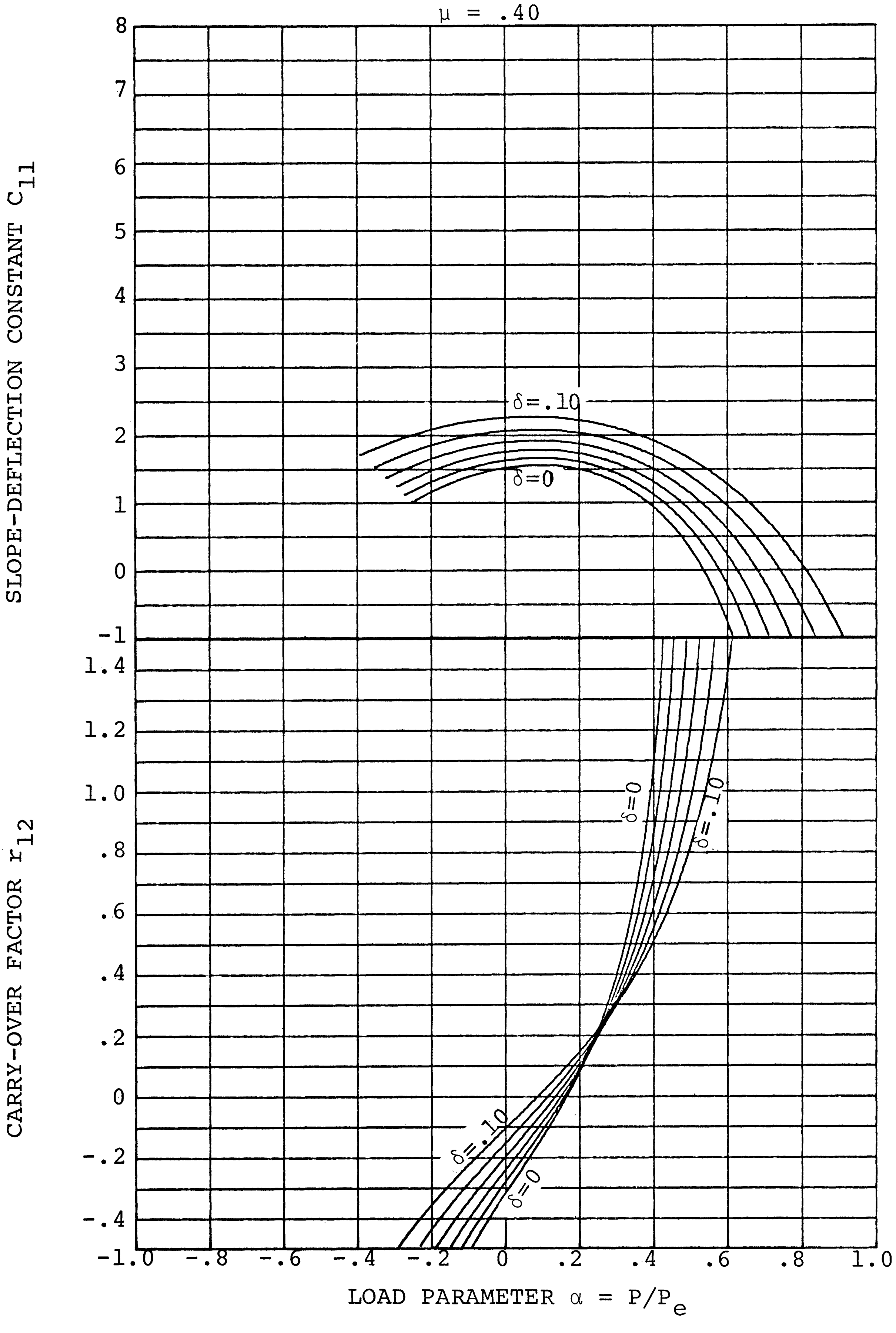

Figure 3-4 (Continued).

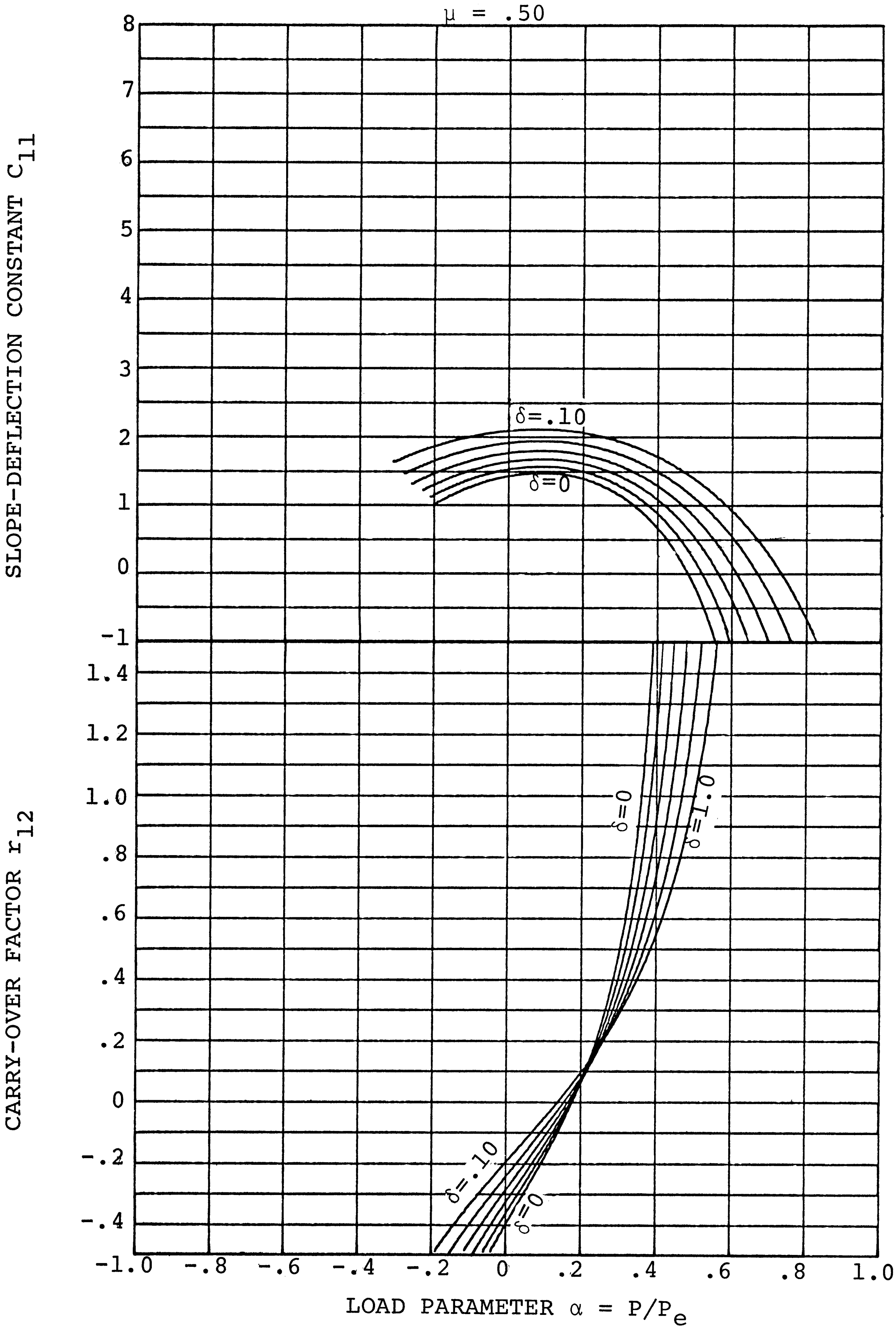

Figure 3-4 (Concluded).

3.4. FIXED END MOMENTS

In the evaluation of fixed end moments we can apply the reciprocal theorem and use the deflection curves of the previous presentation. Consider the two sets of force systems which are acting on the same structural member A-B as shown in Figure 3-5.

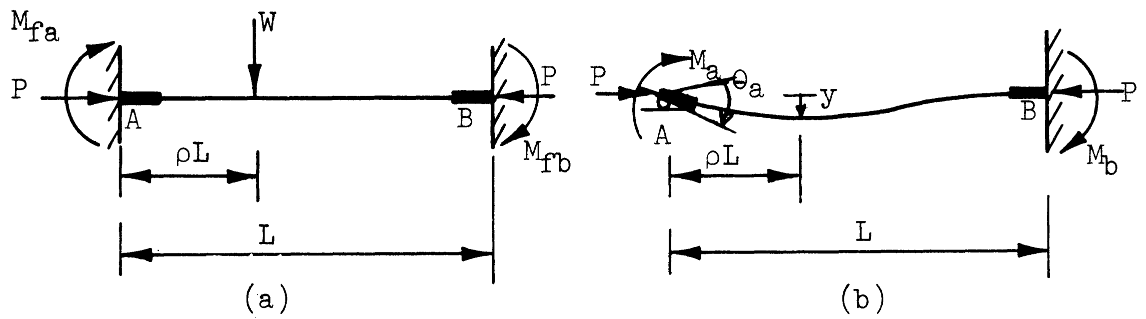

Figure 3-5. Two sets of force systems acting on the same structural member.

According to the reciprocal theorem, we have the relation:

$$M_{fa}\Theta_a + Wy + M_{fb}\cdot 0 = M_a\cdot 0 + M_b\cdot 0 \quad ,$$

then

$$M_{fa} = -W\frac{y}{\Theta_a} \tag{3-61}$$

where

M_{fa} = the fixed end moment at left end, as shown in Figure 3-5(a).

W = an arbitrary concentrated force acting on a structural member, a distance ρL from its left end.

y = deflection of the structural member, a distance ρL from its left end, caused by end moment M_a.

ρ = nondimensional length factor which prescribes the location of the concentrated load W.

Θ_a = end rotation caused by end moment M_a.

To determine the deflection curves of the structural members in terms of the member properties and end moments M_a and M_b we again consider the sets of four linear nonhomogeneous equations (3-43), (3-44), (3-45) in four unknowns, two integration constants A and B; two end rotations Θ_a, Θ_b.

We apply Cramer's Rule again to solve for A, B in terms of M_a, M_b, and ψ, thus obtain:

(1) Case $\alpha = 0$

$$A = \left[- (\delta_1 + \delta_2)\ \mu^* + \frac{(1 - \delta_2)^3}{3} + \delta_2(1 - \delta_2) + \delta_1^2\left(\frac{1}{2} - \frac{\delta_1}{3}\right)\right]\frac{M_a L}{EI}$$

$$+ \left[- (\delta_1 + \delta_2)\ \mu^* - \frac{(1 + 2\delta_2)}{6}(1 - \delta_2)^2 - \frac{\delta_1^3}{3}\right]\frac{M_b L}{EI} + \psi \tag{3-62}$$

$$B = \delta_1\left[\mu^* - \delta_1\left(\frac{1}{2} - \frac{\delta_1}{3}\right)\right]\frac{M_a L}{EI} + \delta_1\left[\mu^* + \frac{\delta_1^2}{3}\right]\frac{M_b L}{EI} \tag{3-63}$$

$\because M_b = r_{12}M_a$ and $\psi \equiv 0$, then

$$A = \phi_{A12}\frac{M_a L}{EI} \tag{3-64}$$

$$B = \phi_{B12}\frac{M_a L}{EI} \tag{3-65}$$

where

$$\phi_{A12} = - (1 + r_{12})(\delta_1 + \delta_2)\ \mu^* + \delta_2(1 - \delta_2) + \frac{(1 - \delta_2)^2}{6}\left[2(1 - \delta_2) - r_{12}(1 + 2\delta_2)\right]$$

$$+ \delta_1^2\left[\frac{1}{2} - \frac{\delta_1}{3}(1 + r_{12})\right] \tag{3-66}$$

$$\phi_{B12} = \delta_1 \left\{ (1 + r_{12})\, \mu^* - \delta_1 \left[\frac{1}{2} - \frac{\delta_1}{3}(1 + r_{12}) \right] \right\} \tag{3-67}$$

The deflection curve y and the rotation θ_a caused by end moment M_a are:

$$y = \left(\frac{1 + r_{12}}{6}\rho^3 - \frac{\rho^2}{2} + \phi_{A12}\rho + \phi_{B12}\right) \frac{M_a L^2}{EI} \tag{3-68}$$

$$\theta_a = \frac{1}{C_{11}} \frac{M_a L}{EI} \tag{3-69}$$

Then, from Equation (3-61) we can obtain the fixed end moment M_{fa}, as shown in Figure 3-6 caused by concentrated load W:

$$M_{fa} = - WL\, C_{11}\left(\frac{1 + r_{12}}{6}\rho^3 - \frac{\rho^2}{2} + \phi_{A12}\rho + \phi_{B12}\right) \tag{3-70}$$

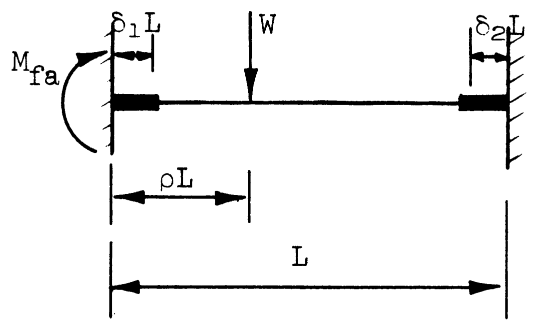

Figure 3-6. Fixed end moment M_{fa} for concentrated load W.

From this basic equation (3-70), we can derive fixed end moment for any combination of concentrated loads by simple summation and for continuously distributed loads by integration.

As examples, the fixed-end moments M_{fa} for uniformly distributed load w and for moment loading M_o are determined as follows:

(i) Uniformly distributed load w.

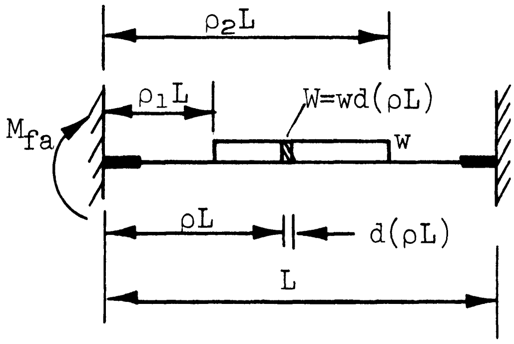

Figure 3-7. Fixed end moment M_{fa} for uniformly distributed loading w.

$$M_{fa} = - wL^2 C_{11} \int_{\rho_1}^{\rho_2} \left(\frac{1 + r_{12}}{6} \rho^3 - \frac{\rho^2}{2} + \phi_{A12}\rho + \phi_{B12}\right) d\rho \qquad (3\text{-}71)$$

For fully loading, $\rho_1 = 0$, $\rho_2 = 1$

$$M_{fa} = - \frac{1}{2} w(\delta_1 L)^2 - wL^2 C_{11} \int_{\delta_1}^{1-\delta_2} \left(\frac{1 + r_{12}}{6} \rho^3 - \frac{\rho^2}{2} + \phi_{A12}\rho + \phi_{B12}\right) d\rho + 0$$

$$M_{fa} = - wL^2 C_{11} \left\{ \frac{\delta_1^2}{2C_{11}} + \frac{1 + r_{12}}{24} \left[(1 - \delta_2)^4 - \delta_1^4\right] - \frac{1}{6}\left[(1 - \delta_2)^3 - \delta_1^3\right] \right.$$
$$\left. + \frac{\phi_{A12}}{2} \left[(1 - \delta_2)^2 - \delta_1^2\right] + \phi_{B12}(1 - \delta_1 - \delta_2) \right\} \qquad (3\text{-}72)$$

(ii) Moment loading M_o:

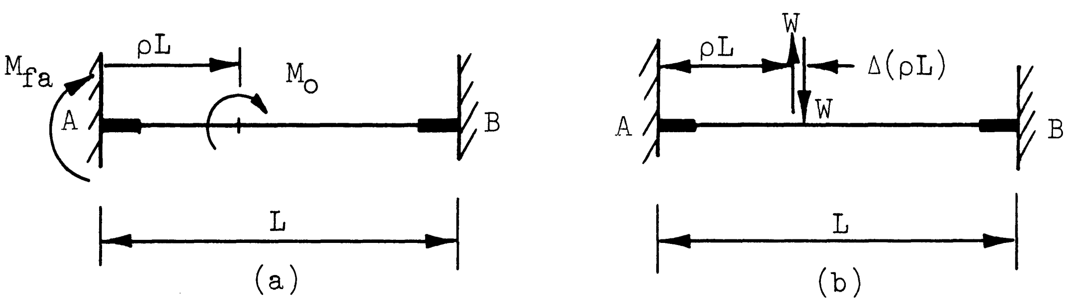

Figure 3-8. Fixed end moment M_{fa} for the moment loading M_o.

Consider the condition as shown in Figure 3-8(a) and (b), they are identical if $\Delta(\rho L)$ is approaching to zero. Then

$$M_o = \lim_{\Delta(\rho L) \to 0} W \, \Delta(\rho L) \quad ,$$

then

$$M_{fa} = \lim_{\Delta\rho \to 0} - WL \, \Delta(\rho L) \, C_{11} \, \frac{f(\rho + \Delta\rho) - f(\rho)}{L \, \Delta\rho}$$

$$M_{fa} = - M_o \, C_{11} \, f'(\rho) \tag{3-73}$$

where

$$f(\rho) = \frac{1 + r_{12}}{6} \rho^3 - \frac{\rho^2}{2} + \phi_{A12}\rho + \phi_{B12} \tag{3-74}$$

$$f'(\rho) = \frac{d}{dx} f(\rho) = \frac{1 + r_{12}}{2} \rho^2 - \rho + \phi_{A12} \tag{3-75}$$

(2) Case $\alpha > 0$

Similarly we can obtain

$$A = \frac{[\delta_2 k \cos k(1 - \delta_2) + (1 + \pi^2\alpha\mu^*) \sin k(1 - \delta_2)] \frac{M_a L}{EI} + [(1 + \pi^2\alpha\mu^*) \sin k\delta_1 - \delta_1 k \cos k\delta_1] \frac{M_b L}{EI}}{\pi^2\alpha\{k(\delta_1 + \delta_2) \cos k^* + [1 + (\mu^* - \delta_1\delta_2)\pi^2\alpha] \sin k^*\}} \quad (3\text{-}76)$$

$$B = \frac{[\delta_2 k \sin k(1 - \delta_2) - (1 + \pi^2\alpha\mu^*) \cos k(1 - \delta_2)] \frac{M_a L}{EI} - [(1 + \pi^2\alpha\mu^*) \cos k\delta_1 + \delta_1 k \sin k\delta_1] \frac{M_b L}{EI}}{\pi^2\alpha\{k(\delta_1 + \delta_2) \cos k^* + [1 + (\mu^* - \delta_1\delta_2)\pi^2\alpha] \sin k^*\}} \quad (3\text{-}77)$$

$$\because M_b = r_{12} M_a \quad , \; P = \alpha \frac{\pi^2 EI}{L^2} \quad ,$$

then

$$A = \phi_{A12} \frac{M_a L}{EI} \quad (3\text{-}78)$$

$$B = - \phi_{B12} \frac{M_a L}{EI} \quad (3\text{-}79)$$

where

$$\phi_{A12} = \frac{[1 + \pi^2\alpha\mu^*][\sin k(1 - \delta_2) + r_{12} \sin k\delta_1] + k[\delta_2 \cos k(1 - \delta_2) - r_{12}\delta_1 \cos k\delta_1]}{\pi^2\alpha\{k(\delta_1 + \delta_2) \cos k^* + [1 + (\mu^* - \delta_1\delta_2)\pi^2\alpha] \sin k^*\}} \quad (3\text{-}80)$$

$$\phi_{B12} = \frac{[1 + \pi^2\alpha\mu^*][\cos k(1 - \delta_2) + r_{12} \cos k\delta_1] + k[r_{12}\delta_1 \sin k\delta_1 - \delta_2 \sin k(1 - \delta_2)]}{\pi^2\alpha\{k(\delta_1 + \delta_2) \cos k^* + [1 + (\mu^* - \delta_1\delta_2)\pi^2\alpha] \sin k^*\}} \quad (3\text{-}81)$$

Then the deflection curve y and end rotation Θ_a caused by end moment M_a are:

$$y = (\phi_{A12} \cos k\rho - \phi_{B12} \sin k\rho + \frac{1 + r_{12}}{\pi^2\alpha} \rho - \frac{1}{\pi^2\alpha}) \frac{M_a L^2}{EI} \quad (3\text{-}82)$$

$$\theta_a = \frac{1}{C_{11}} \frac{M_a L}{EI} \tag{3-83}$$

Then, the fixed end moment M_{fa} for concentrated load W with compression axial force P is expressed in the following equation (3-84), as shown in Figure 3-9.

$$M_{fa} = - WL\ C_{11}(\phi_{A12} \cos k\rho - \phi_{B12} \sin k\rho + \frac{1 + r_{12}}{\pi^2\alpha} \rho - \frac{1}{\pi^2\alpha}) \tag{3-84}$$

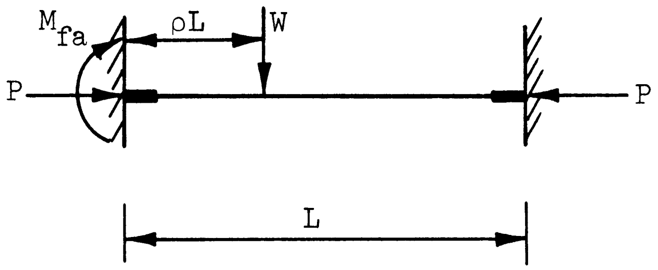

Figure 3-9. Fixed end moment M_{fa} for concentrated load W with compression axial force.

Similarly, the fixed-end moments M_{fa} for uniformly distributed load w and for moment loading M_o are determined as follows:

(i) Uniformly distributed load w (as shown in Figure 3-10):

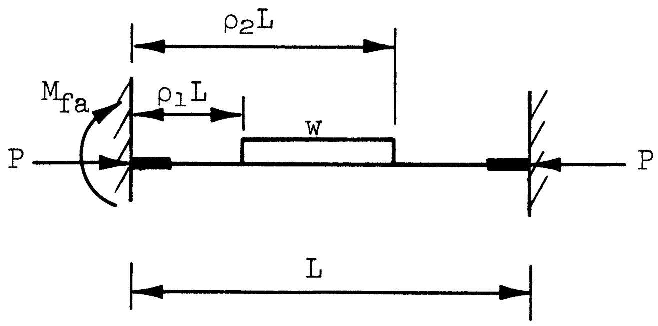

Figure 3-10. Fixed end moment M_{fa} for uniformly distributed load w with compression axial force.

$$M_{fa} = -wL^2 C_{11} \int_{\rho_1}^{\rho_2} (\phi_{A12} \cos k\rho - \phi_{B12} \sin k\rho + \frac{1 + r_{12}}{\pi^2\alpha} \rho - \frac{1}{\pi^2\alpha}) d\rho \tag{3-85}$$

For fully loading, $\rho_1 = 0$, $\rho_2 = 1$, then

$$M_{fa} = -wL^2 C_{11} \left\{ \frac{\delta_1^2}{2C_{11}} + \frac{\phi_{A12}}{k} [\sin k(1 - \delta_2) - \sin k\delta_1] + \frac{\phi_{A12}}{k} [\cos k(1 - \delta_2) - \cos k\delta_1] + \frac{1 + r_{12}}{2\pi^2\alpha} [(1 - \delta_2)^2 - \delta_1^2] - \frac{1}{\pi^2\alpha}(1 - \delta_1 - \delta_2) \right\} \tag{3-86}$$

(ii) Moment loading M_o (as shown in Figure 3-11):

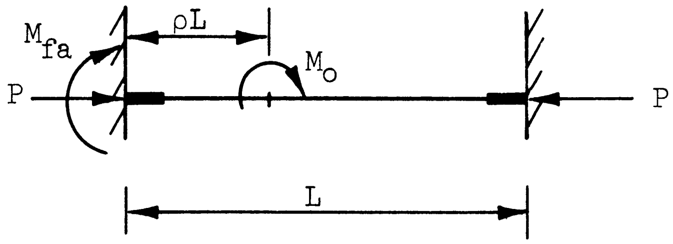

Figure 3-11. Fixed end moment M_{fa} for the moment loading M_o with compression axial force.

$$M_{fa} = -M_o C_{11} k(-\phi_{A12} \sin k\rho - \phi_{B12} \cos k\rho + \frac{1 + r_{12}}{\pi^2\alpha k}) \tag{3-87}$$

(3) Case $\alpha < 0$:

$$A = \frac{[k\delta_2 \cosh k(1-\delta_2) + (1+\pi^2\alpha\mu^*) \sinh k(1-\delta_2)] \frac{M_a L}{EI} + [(1+\pi^2\alpha\mu^*) \sinh k\delta_1 - k\delta_1 \cosh k\delta_1] \frac{M_b L}{EI}}{\pi^2\alpha\{k(\delta_1+\delta_2) \cosh k^* + [1 + (\mu^* - \delta_1\delta_2)\pi^2\alpha] \sinh k^*\}} \tag{3-88}$$

$$B = -\frac{[k\delta_2 \sinh k(1-\delta_2) + (1+\pi^2\alpha\mu^*) \cosh k(1-\delta_2)] \frac{M_a L}{EI} + [(1+\pi^2\alpha\mu^*) \cosh k\delta_1 - k\delta_1 \sinh k\delta_1] \frac{M_b L}{EI}}{\pi^2\alpha\{k(\delta_1+\delta_2) \cosh k^* + [1 + (\mu^* - \delta_1\delta_2)\pi^2\alpha] \sinh k^*\}} \tag{3-89}$$

$$M_b = r_{12} M_a \quad , \quad P = -\alpha \frac{\pi^2 EI}{L^2} \quad ,$$

then

$$A = \phi_{A12} \frac{M_a L}{EI} \tag{3-90}$$

$$B = -\phi_{B12} \frac{M_a L}{EI} \tag{3-91}$$

where

$$\phi_{A12} = \frac{(1+\pi^2\alpha\mu^*)\,[\sinh k(1-\delta_2) + r_{12} \sinh k\delta_1] + k[\delta_2 \cosh k(1-\delta_2) - r_{12}\delta_1 \cosh k\delta_1]}{\pi^2\alpha\{k(\delta_1+\delta_2) \cosh k^* + [1 + (\mu^* - \delta_1\delta_2)\pi^2\alpha] \sinh k^*\}} \tag{3-92}$$

$$\phi_{B12} = \frac{(1+\pi^2\alpha\mu^*)\,[\cosh k(1-\delta_2) + r_{12} \cosh k\delta_1] + k[\delta_2 \sinh k(1-\delta_2) - r_{12}\delta_1 \sinh k\delta_1]}{\pi^2\alpha\{k(\delta_1+\delta_2) \cosh k^* + [1 + (\mu^* - \delta_1\delta_2)\pi^2\alpha] \sinh k^*\}} \tag{3-93}$$

Then the deflection curve y and end rotation Θ_a caused by end moment M_a are:

$$y = (\phi_{A12} \cosh k\rho + \phi_{B12} \sinh k\rho + \frac{1 + r_{12}}{\pi^2\alpha} \rho - \frac{1}{\pi^2\alpha}) \frac{M_a L^2}{EI} \tag{3-94}$$

$$\Theta = \frac{1}{C_{11}} \frac{M_a L}{EI} \tag{3-95}$$

Then

$$M_{fa} = - WL\, C_{11}(\phi_{A12} \cosh k\rho - \phi_{B12} \sinh k\rho + \frac{1 + r_{12}}{\pi^2\alpha} \rho - \frac{1}{\pi^2\alpha}) \tag{3-96}$$

Similarly, the fixed-end moments M_{fa} for uniformly distributed load w and for moment loading M_o are determined as follows:

(i) Uniformly distributed load w:

$$M_{fa} = - wL^2\, C_{11} \int_{\rho_1}^{\rho_2} (\phi_{A12} \cosh k\rho - \phi_{B12} \sinh k\rho + \frac{1 + r_{12}}{\pi^2\alpha} \rho - \frac{1}{\pi^2\alpha}) d\rho \tag{3-97}$$

For full loading, $\rho_1 = 0$, $\rho_2 = 1$, then

$$M_{fa} = - wL^2\, C_{11} \left\{ \frac{\delta_1^2}{2C_{11}} + \frac{\phi_{A12}}{k} [\sinh k(1 - \delta_2) - \sinh k\delta_1] - \frac{\phi_{B12}}{k} [\cosh k(1 - \delta_2) - \cosh k\delta_1] + \frac{1 + r_{12}}{2\pi^2\alpha} [(1 - \delta_2)^2 - \delta_1^2] - \frac{1}{\pi^2\alpha} (1 - \delta_1 - \delta_2) \right\} \tag{3-98}$$

(ii) Moment loading M_o:

$$M_{fa} = - M_o\, C_{11}\, k(\phi_{A12} \sinh k\rho - \phi_{B12} \cosh k\rho + \frac{1 + r_{12}}{\pi^2\alpha k}) \tag{3-99}$$

Fixed end moment M_{fa} of structural members for a uniform load w over full-length L with different rigid plates δ (where $\delta = \delta_1 = \delta_2$) and constant shear flexibility μ is presented in terms of wL^2 as shown in Figure 3-12.

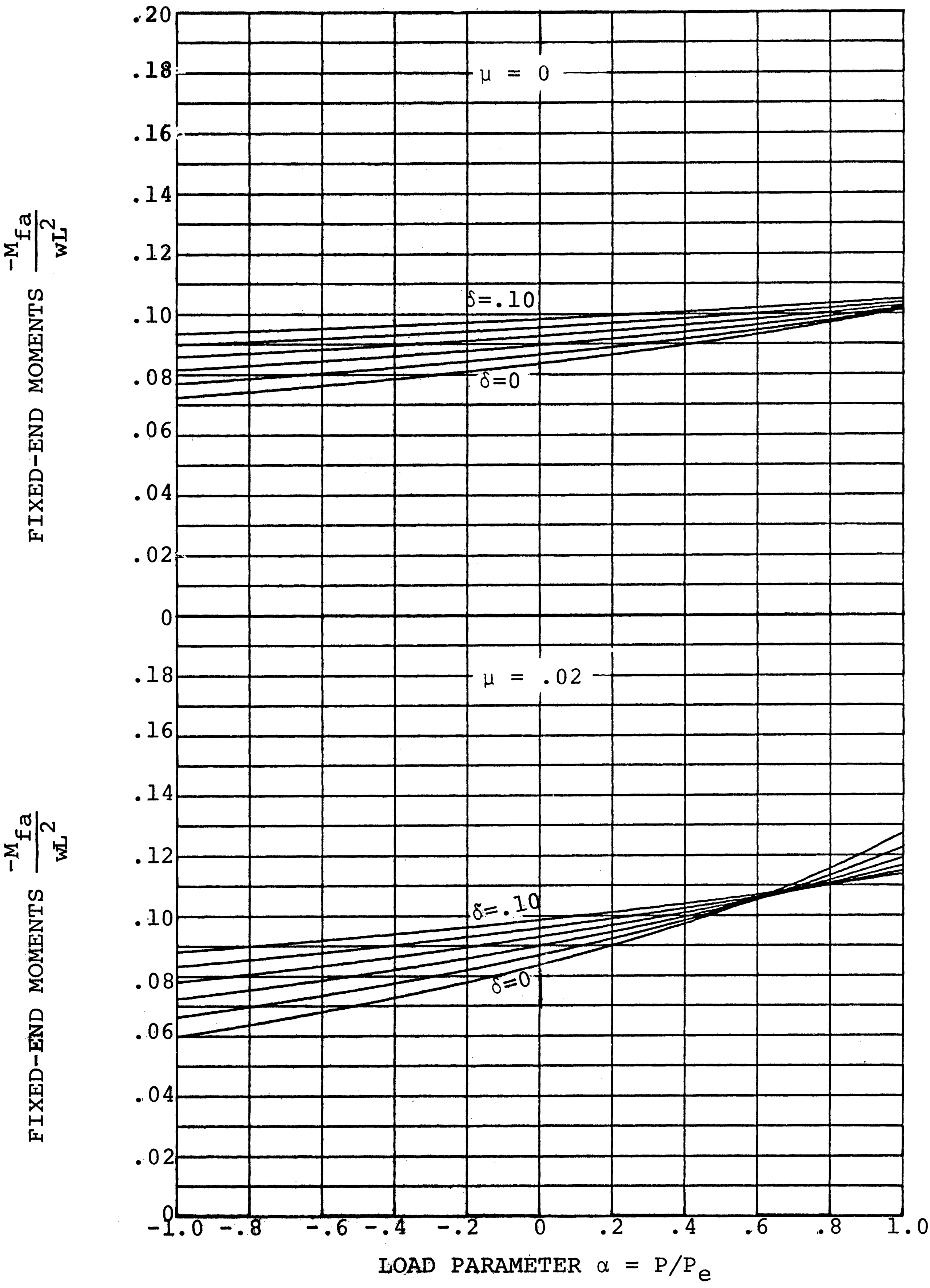

Figure 3-12. Fixed end moment M_{fa} of structural members for a uniform load w over full-length L, with different rigid stay plates δ, and constant shear flexibility μ.

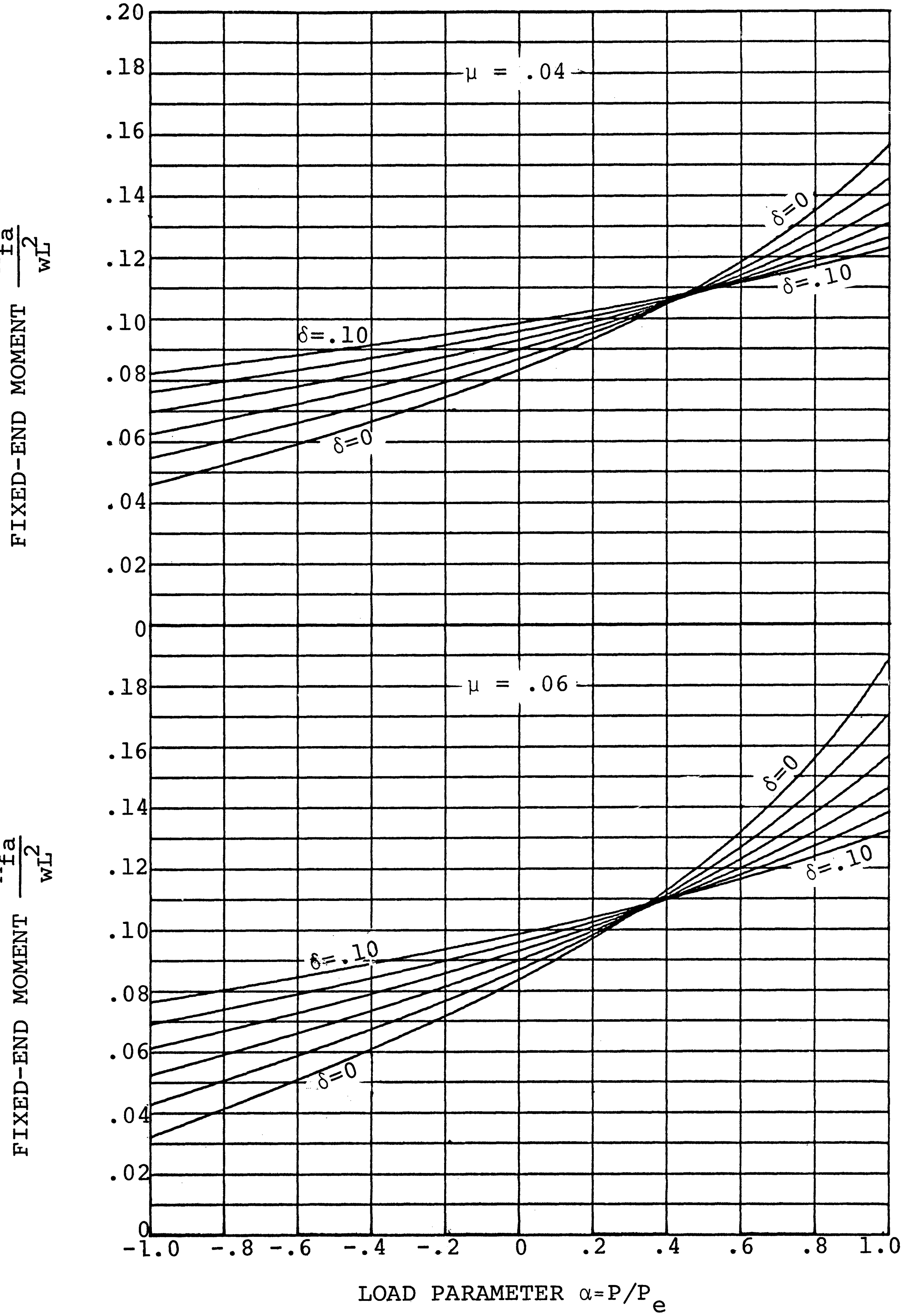

Figure 3-12 (Continued).

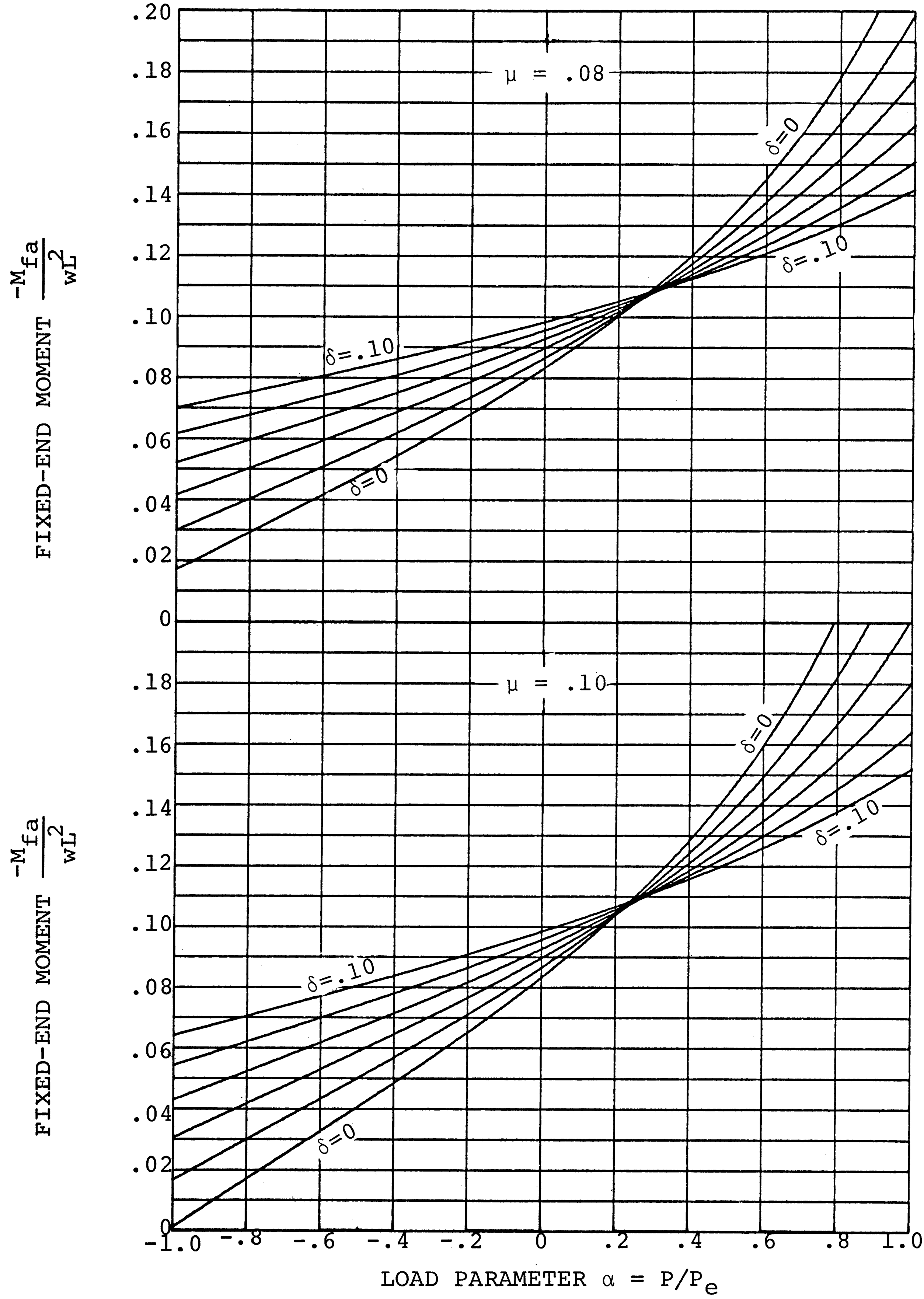

Figure 3-12 (Continued).

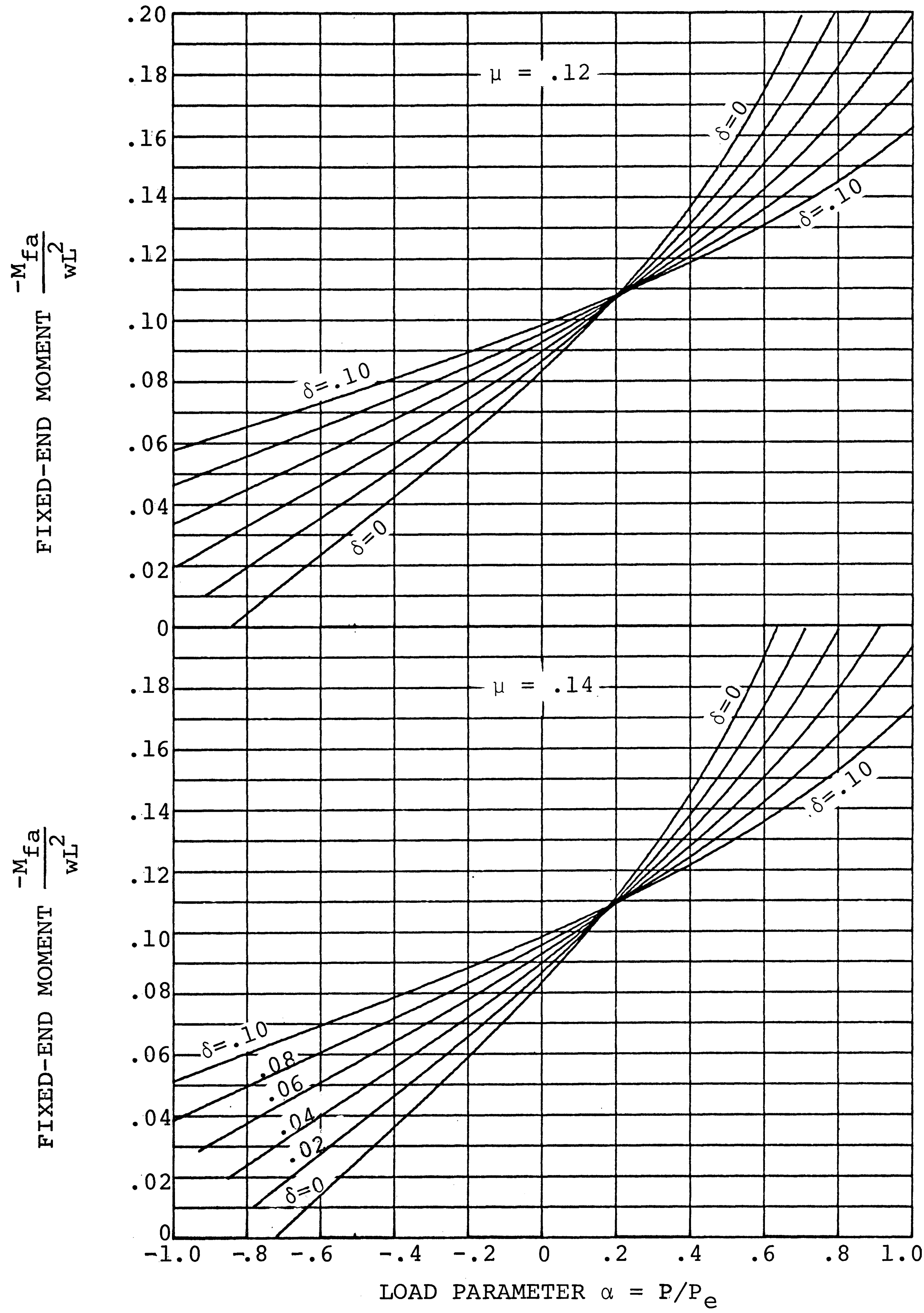

Figure 3-12 (Continued).

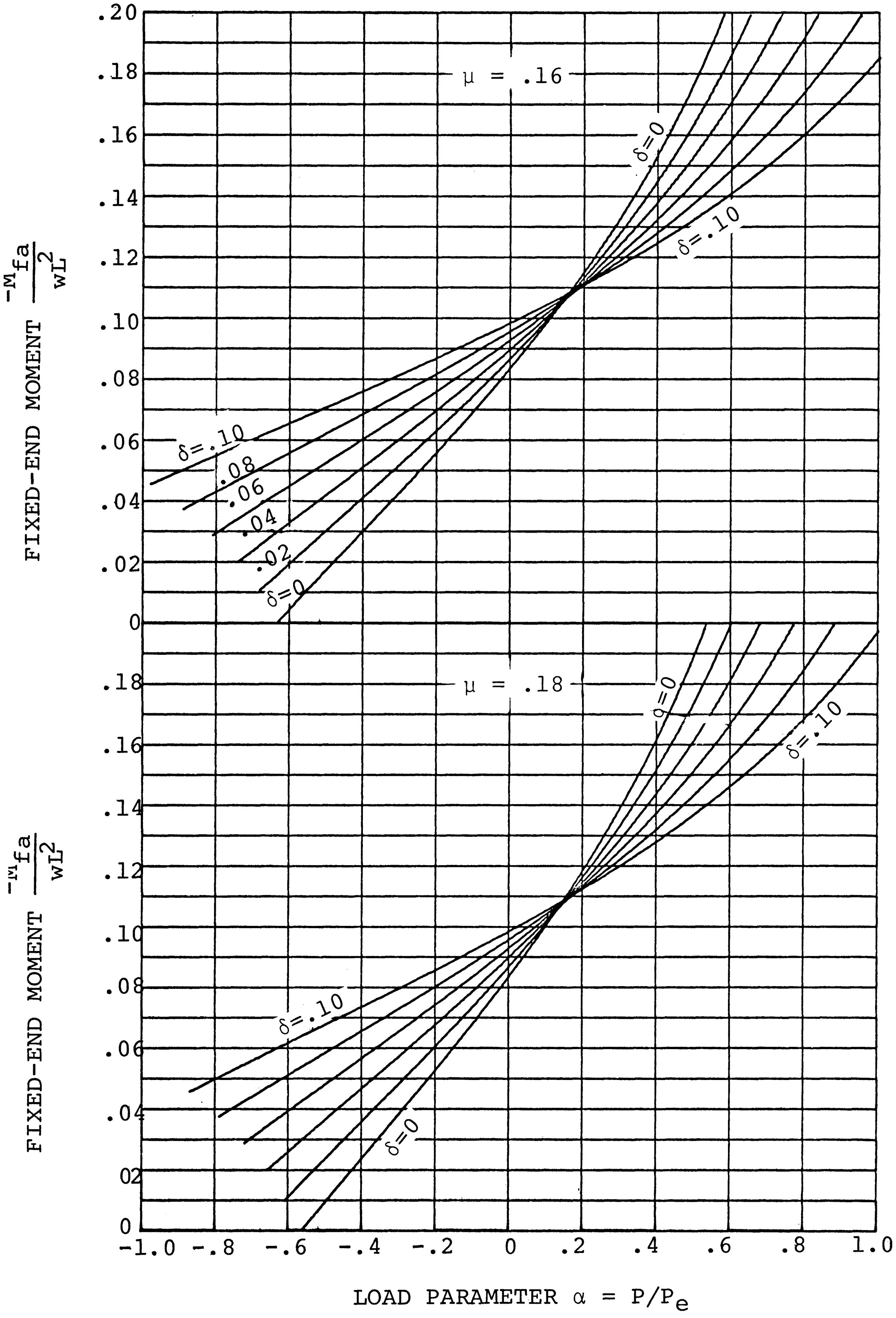

Figure 3-12 (Continued).

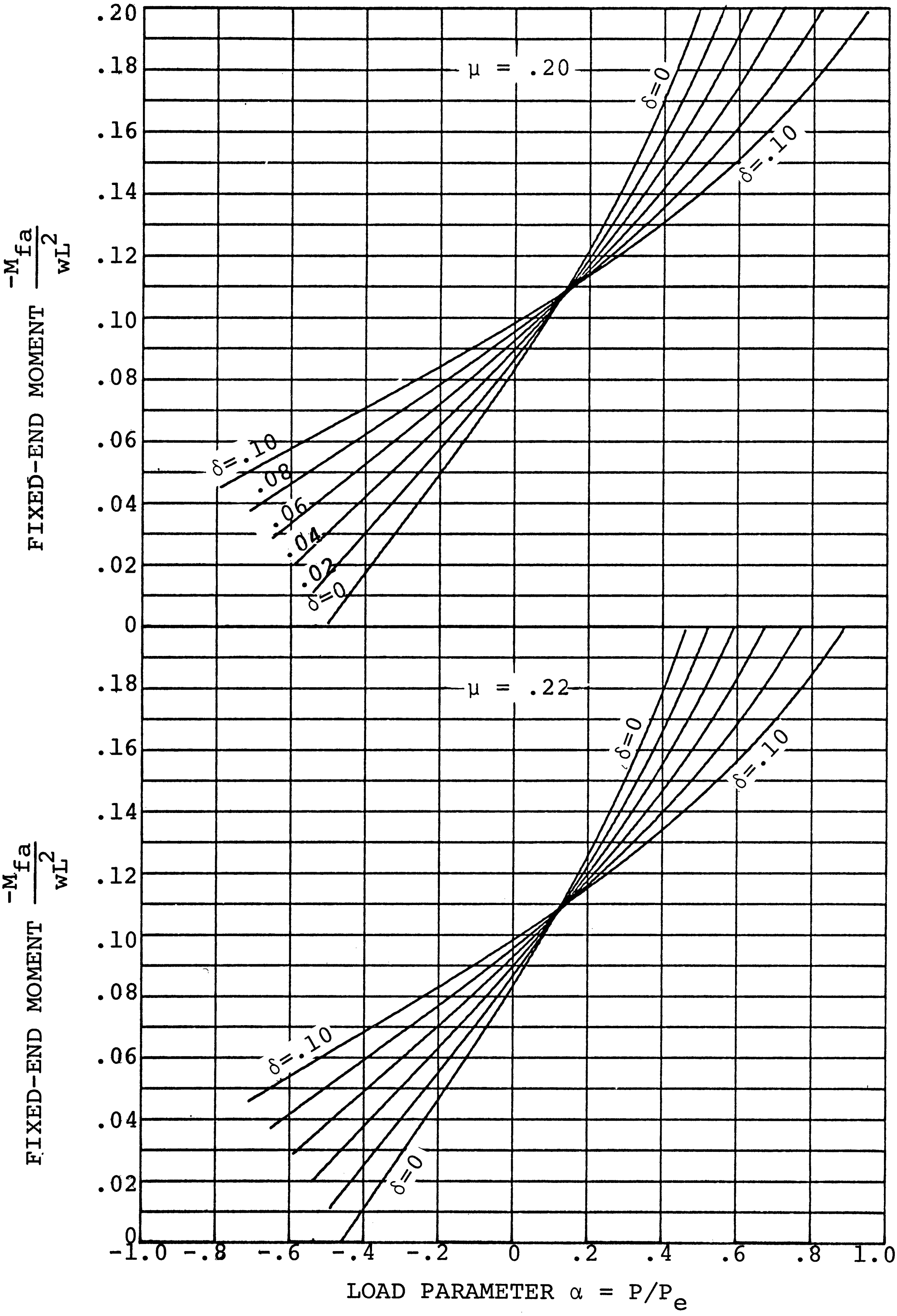

Figure 3-12 (Continued).

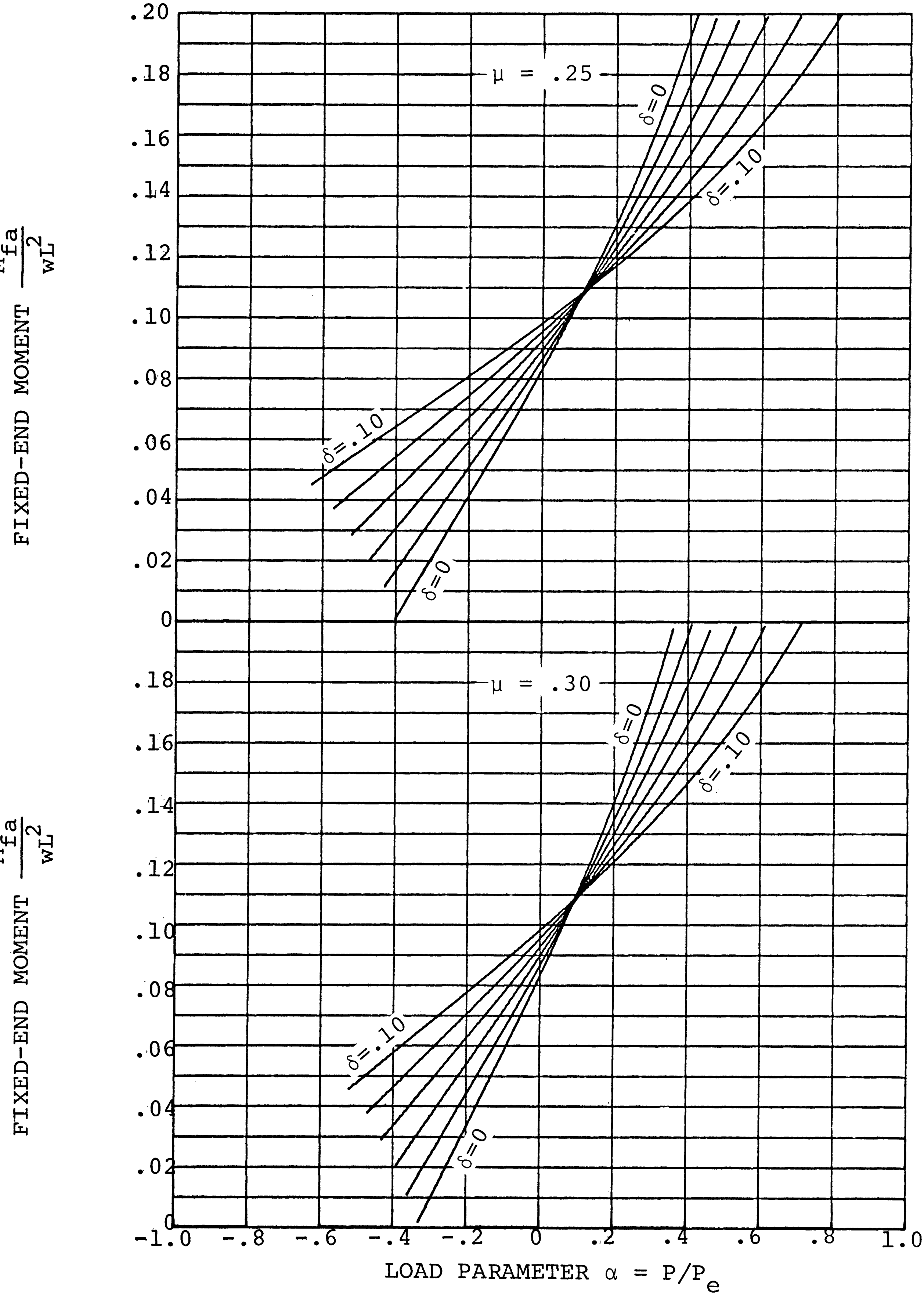

Figure 3-12 (Continued).

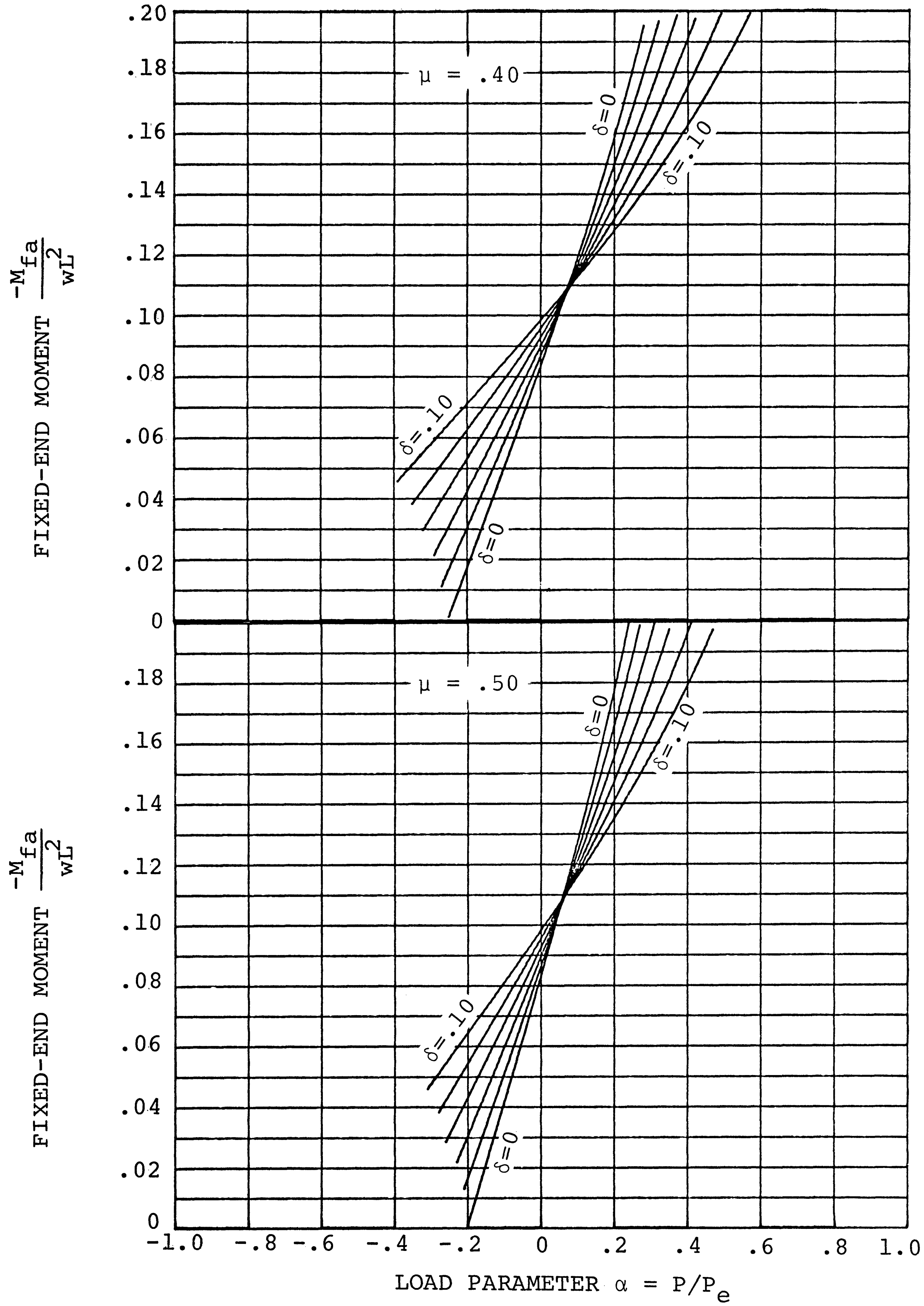

Figure 3-12 (Concluded).

CHAPTER IV

ELASTIC BUCKLING OF BATTENED OR LACED STRUCTURAL MEMBERS WITH RIGID STAY PLATES AND CONSTANT SHEAR FLEXIBILITY

The lowest elastic buckling loads will be evaluated for different end-conditions of built up columns, as determined by the eigenvalues of certain submatrices of the system of Equation (3-44).

4.1. COLUMN WITH HINGED ENDS

The critical load $P_{cr,h}$ of the column which is hinged at both ends causes the matrix (3-44) with the unknowns Θ_a, Θ_b, A, B to be singular, since the natural boundary conditions give $M_a = 0$, $M_b = 0$, and $\psi = 0$. Then

$$\begin{bmatrix} \cos k\delta_1 & \sin k\delta_1 & -\delta_1 & 0 \\ \cos k(1 - \delta_2) & \sin k(1 - \delta_2) & 0 & \delta_2 \\ - k \sin k\delta_1 & k \cos k\delta_1 & -1 - \pi^2\alpha\mu^* & 0 \\ - k \sin k(1 - \delta_2) & k \cos k(1 - \delta_2) & 0 & -1 - \pi^2\alpha\mu^* \end{bmatrix} = 0$$

Then we have the buckling condition:

$$\tan k^* + \frac{k(\delta_1 + \delta_2)}{1 + \pi^2\alpha\,(\mu^* - \delta_1\delta_2)} = 0 \qquad (4\text{-}1)$$

The lowest root of Equation (4-1) yields the critical load $P_{cr,h}$.

4.2. COLUMN WITH ONE END FIXED AND THE OTHER HINGED

The critical load $P_{cr,f-h}$ of a column which is fixed at one end and hinged at the other causes the matrix (3-44) with the unknowns M_a, A, B, Θ_b

to be singular, since the natural boundary conditions give $M_b = 0$, $\Theta_a = 0$, and $\psi = 0$.

$$\begin{bmatrix} -\frac{1-\delta_1}{\pi^2\alpha} & \cos k\delta_1 & \sin k\delta_1 & 0 \\ -\frac{\delta_2}{\pi^2\alpha} & \cos k(1-\delta_2) & \sin k(1-\delta_2) & \delta_2 \\ \mu^* + \frac{1}{\pi^2\alpha} & -k\sin k\delta_1 & k\cos k\delta_1 & 0 \\ \mu^* + \frac{1}{\pi^2\alpha} & -k\sin k(1-\delta_2) & k\cos k(1-\delta_2) & -1-\pi^2\alpha\mu^* \end{bmatrix} = 0$$

Then we have the buckling condition:

$$\tan k^* - \frac{k^*}{1 + \pi^2\alpha[\mu^* + (1-\delta_1)\delta_2]} = 0 \qquad (4\text{-}2)$$

The lowest root of Equation (4-2) yields the critical load $P_{cr,f\text{-}h}$.

4.3. COLUMN WITH FIXED ENDS

The critical load $P_{cr,f}$ of the column, which is fixed at both ends, again causes the matrix (3-44) with the unknowns M_a, M_b, A, B to be singular, since the natural boundary conditions give $\Theta_a = 0$, $\Theta_b = 0$, and $\psi = 0$. Then

$$\begin{bmatrix} -\frac{1-\delta_1}{\pi^2\alpha} & \frac{\delta_1}{\pi^2\alpha} & \cos k\delta_1 & \sin k\delta_1 \\ -\frac{\delta_2}{\pi^2\alpha} & \frac{1-\delta_2}{\pi^2\alpha} & \cos k(1-\delta_2) & \sin k(1-\delta_2) \\ \mu^* + \frac{1}{\pi^2\alpha} & \mu^* + \frac{1}{\pi^2\alpha} & -k\sin k\delta_1 & k\cos k\delta_1 \\ \mu^* + \frac{1}{\pi^2\alpha} & \mu^* + \frac{1}{\pi^2\alpha} & -k\sin k(1-\delta_2) & k\cos k(1-\delta_2) \end{bmatrix} = 0$$

Then we have the buckling condition:

$$\sin\frac{k^*}{2}\left[\frac{k^*}{2(1+\pi^2\alpha\mu^*)}\cos\frac{k^*}{2} - \sin\frac{k^*}{2}\right] = 0 \qquad (4\text{-}3)$$

There are two solutions in Equation (4-3), one of them is the symmetrical buckling condition, and the other corresponds to the antisymmetric buckling pattern. We are interested here only in the symmetrical buckling shape, since the critical value is always smaller than for the case of antisymmetry. Therefore, we consider only the symmetrical buckling condition which is:

$$\sin \frac{k^*}{2} = 0 \tag{4-4}$$

For the lowest root of this equation gives:

$$k(1 - \delta_1 - \delta_2) - 2\pi = 0 \tag{4-5}$$

Equation (4-5) yields the critical load $P_{cr,f}$.

The critical loads of columns with different end conditions, constant shear flexibility μ, and rigid stay plates δ, are provided in Figure 4-1.

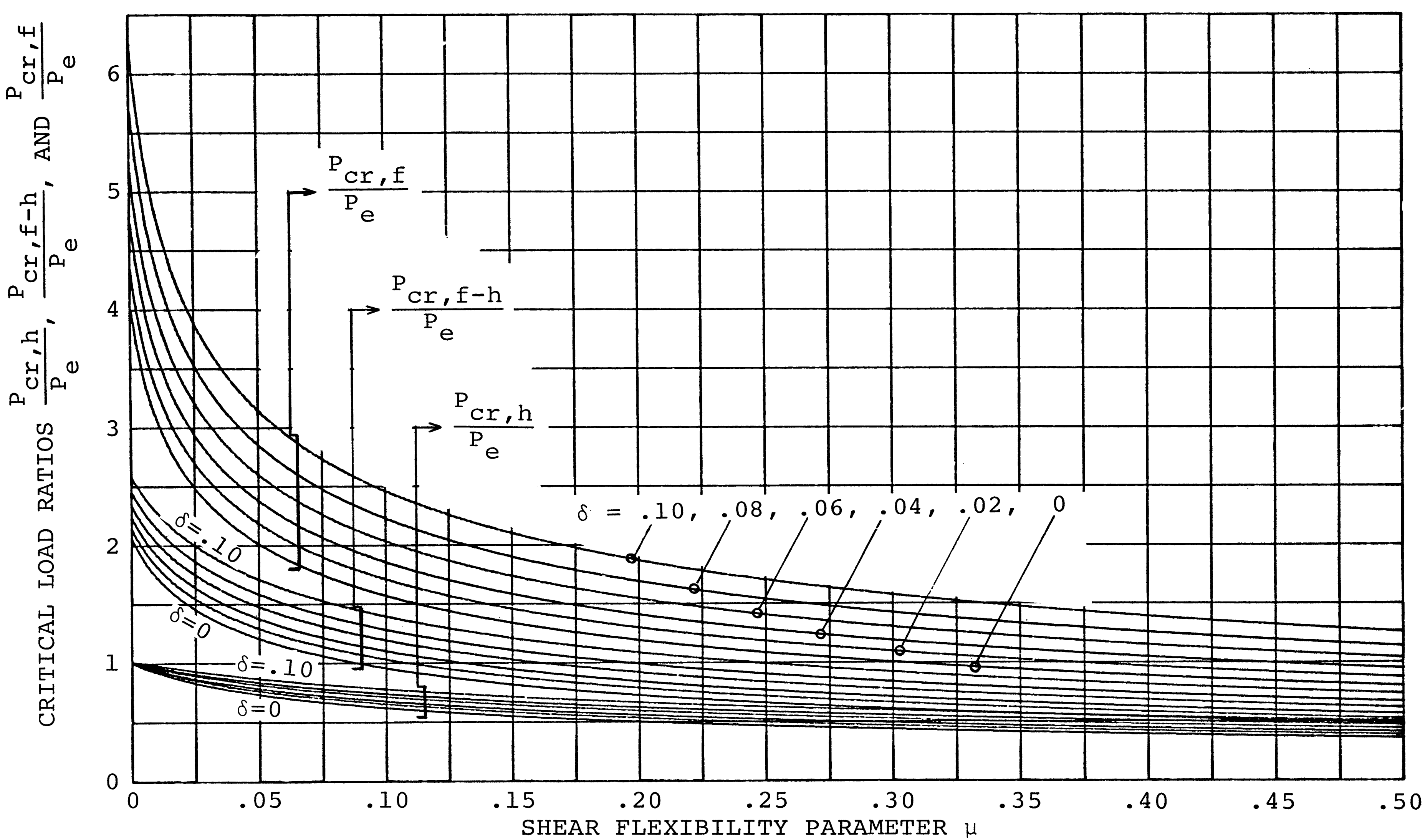

Figure 4-1. The critical loads of columns with different end conditions, constant shear flexibility μ, and rigid stay plates δ.

CHAPTER V

NUMERICAL EXAMPLES OF BEAM AND FRAME ANALYSES

5.1. A BEAM ANALYSIS

A structural member 0-1-2, as shown in Figure 5-1(1), is composed of two parts 0-1 and 1-2 with fixed ends at 0 and 2. Part 0-1 is solid-web member, with assumed shear flexibility equal to zero. Part 1-2 is a battened structural member with rigid joint connections (Z = 0), and with appreciable shear flexibility. However, parts 0-1 and 1-2 are assumed to have the same nominal flexural rigidity EI, and length L.

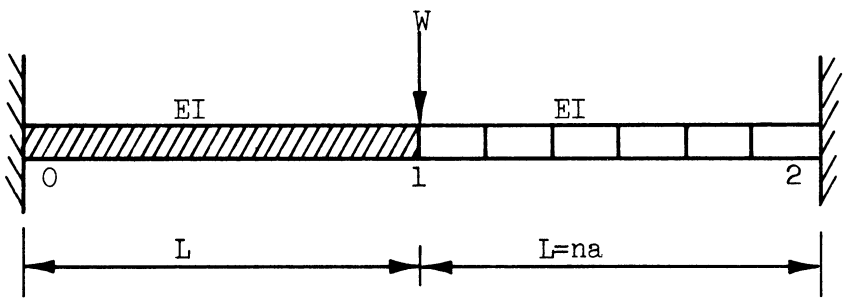

Figure 5-1(1). A structural member 0-1-2.

As shown in Figure 5-1(2), the battened structural member 1-2 is composed of the following elements:

longitudinal elements: 14WF43

batten elements: 2-9[15

The geometrical arrangement of the battened structural member is:

a = 30 in.

b = 20 in.

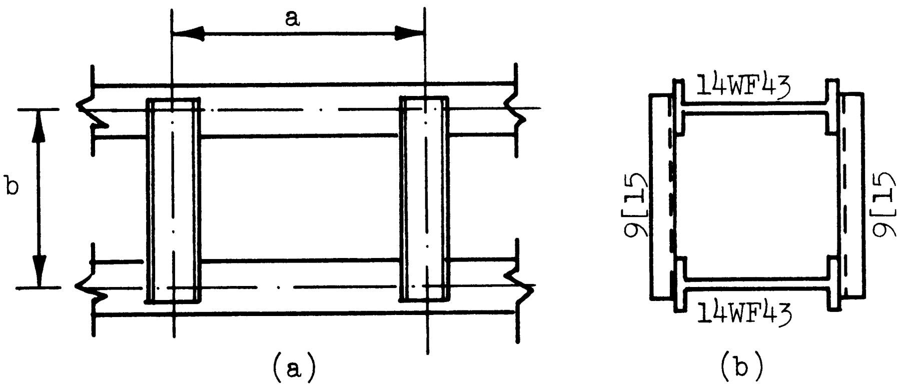

Figure 5-1(2). The battened structural member 1-2;
(a) geometrical arrangement and (b) cross-section.

Four cases will be compared here for n = 2, 6, and 10 as follows:

Case 1. Results are obtained considering the effect of shear deformation.

Case 2. Results are obtained considering the effect of shear deformation but neglecting the effect of the local connection factor (i.e., $\xi_a = 1$).

Case 3. Results are compiled from STRESS computer program by MIT.

Case 4. Results are obtained from ordinary beam theory (neglecting the effect of shear deformation).

The shear flexibility μ, stiffness C_{11}, carry-over factor r_{12}, of the structural member 1-2, and bending moments M_o, M_1, and M_2 due to concentrated load W are tabulated as follows:

Case	n	μ	C_{11}	r_{12}	$\frac{M_o}{WL}$	$\frac{M_1}{WL}$	$\frac{M_2}{WL}$
	2	1.030	1.22	-.63	-.553	.250	.053
1	6	.114	2.26	.11	-.323	.250	-.176
	10	.041	3.00	.334	-.279	.250	-.221
	2	1.457	1.16	-.72	-.593	.250	.093
2	6	.162	2.02	.01	-.347	.250	-.152
	10	.058	2.76	.27	-.290	.250	-.209
	2		1.46	-.56	-.503	.258	.021
3	6		2.54	.15	-.308	.251	-.189
	10		3.22	.35	-.272	.250	-.227
4			4.00	.5	-.250	.250	-.250

The bending moment diagrams are presented in Figure 5-1(3).

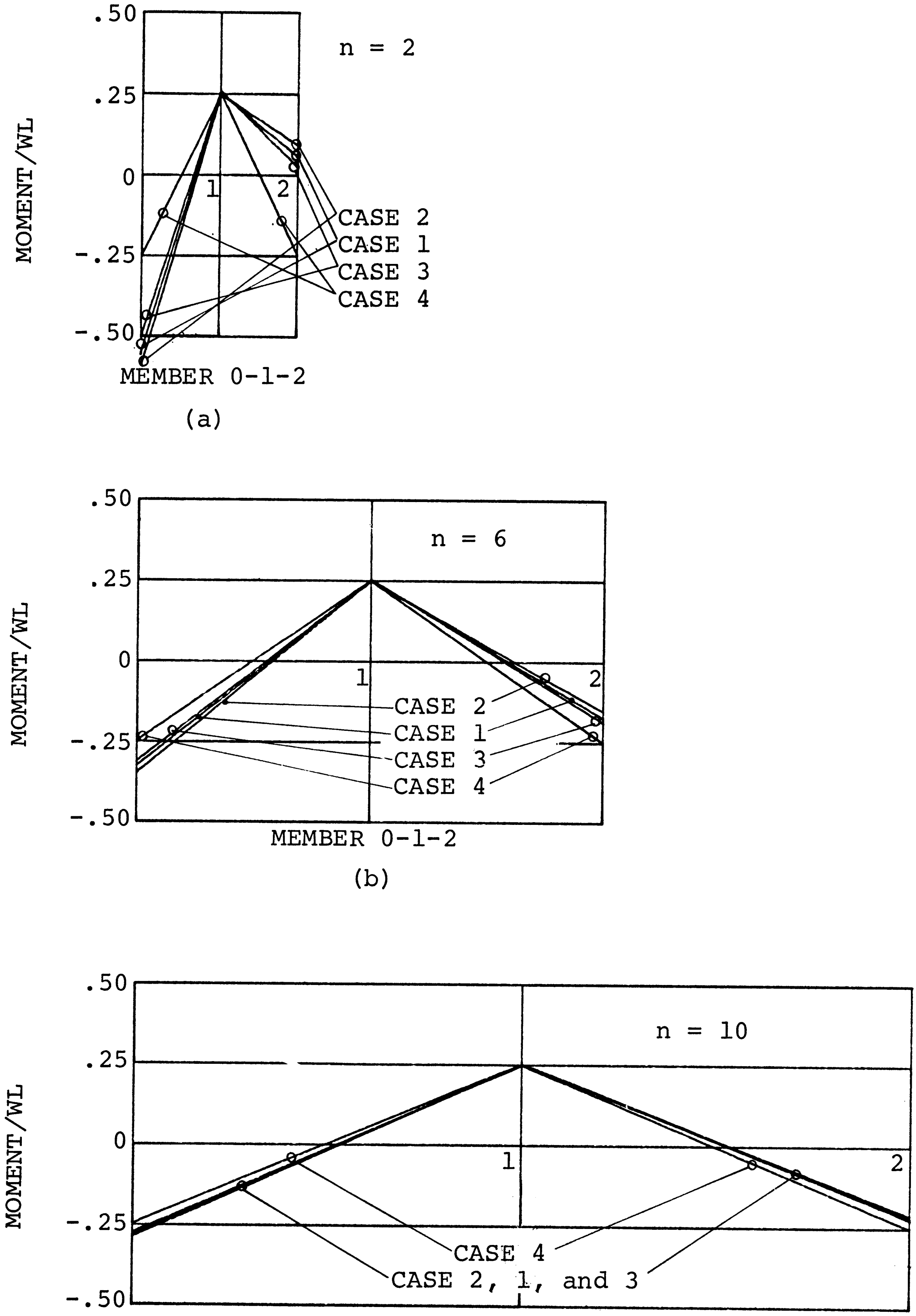

Figure 5-1(3). Moment diagrams for the Cases 1, 2, 3, and 4 for various numbers of panels.

5.2. A FRAME ANALYSIS

A two-bay symmetrical frame structure, as shown in Figure 5-2(1), is composed of:

Two-type 1, laced structural member DE and EF to be used as roof truss beams. The two longitudinal elements (10WF45) are connected, in two planes by the diagonal and strut elements (9[15).

Two-type 2, battened structural members AD and CF to be used as exterior built-up columns.

One-type 3, battened structural member BE to be used as an interior built-up column.

The type 2 and type 3 battened structural members which consist of two main longitudinal elements (10WF45) with batten elements (9[20). The two longitudinal elements are connected in two planes of batten elements, by means of perfectly rigid joint connection (Z = 0).

The geometrical arrangements and elements' properties of the frame structure are given in Table 5-2(1).

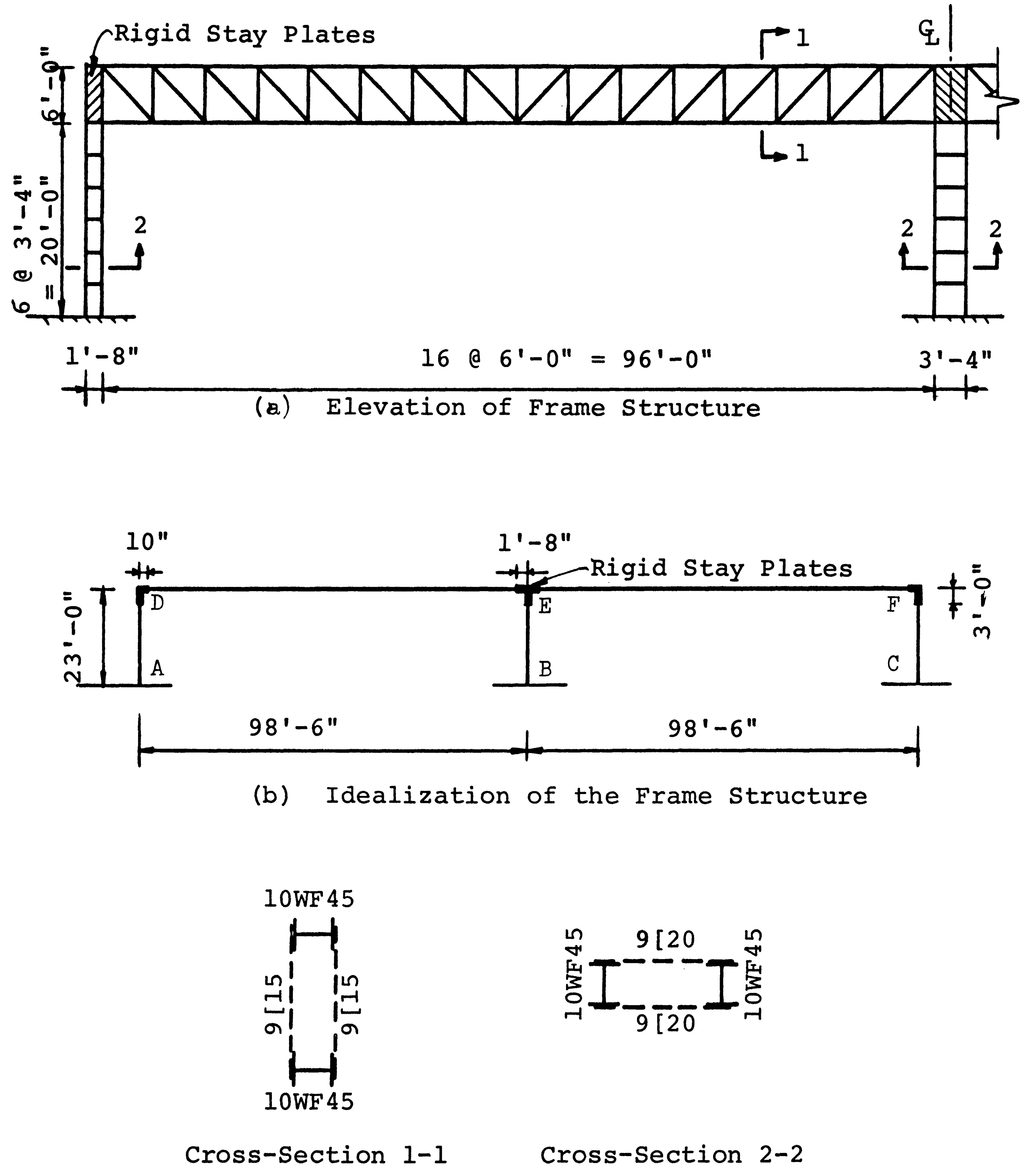

Figure 5-2(1). A two-bay symmetrical frame structure.

TABLE 5-2(1)

THE GEOMETRIC ARRANGEMENTS AND ELEMENTS' PROPERTIES OF THE FRAME STRUCTURE

Member	Type*	n	a in.	b in.	L	I in.^{4}	Long. Element				Batten Element				Diag. Element		μ
							Size	A_c in.^{2}	r_c in.	η_c	Size	A_b in.^{2}	r_b in.	η_b	Size	A_d in.^{2}	
DE, EF	1	16	72	72	98'-6"	34,580	10W45	13.24			9[15	8.78			9[15	8.78	.0113
AD, CF	2	6	40	20	23'-0"	2,766	10W45	13.24	2.0	1.32	9[20	11.72	3.22	1.45			.0898
BE	3	6	40	40	23'-0"	10,746	10W45	13.24	2.0	1.32	9[20	11.72	3.22	1.45			.3910

*Type 1 is laced structural member (where $\xi_a = .875$, $\xi_b = .944$).
Types 2 and 3 are battened structural members (where $\xi_a = .775$).

Consider three loading conditions as shown in Figure 5-2(2):

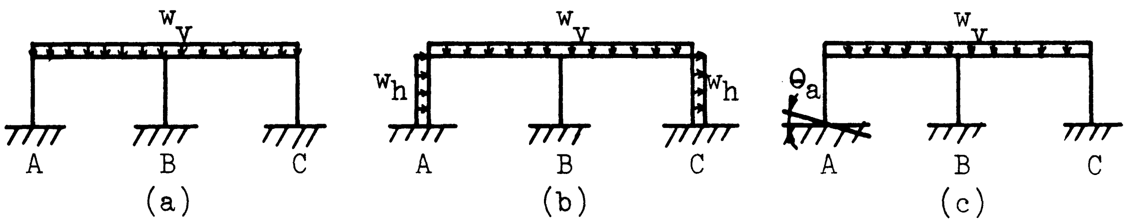

Figure 5-2(2). Loading conditions. (a) Loading 1, dead load (w_v) only. (b) Loading 2, dead load (w_v) + wind load (w_h). (c) Loading 3, dead load (w_v) + appreciable footing rotation θ_a, where w_v = 1.25 Kips per ft, w_h = .50 Kips per ft, and θ_a = .001 radians (+).

The end moments will be determined simply by using the slope-deflection equations, and the equilibrium conditions of each joint and the whole frame structure in the following two cases.

Case 1. Considering the effects of shear deformation and axial force.

Case 2. Neglecting the effects of shear deformation and axial force (ordinary frame analysis).

The solution of the end bending moments, and the moment diagrams of the framed structure are presented in Table 5-2(2) and Figure 5-2(3), respectively. The corresponding errors which result if the effects of shear deformation and axial force are neglected, are also indicated in percentages.

TABLE 5-2(2)

THE END MOMENTS (KIP-FT) OF THE FRAME STRUCTURE, AND ITS CORRESPONDENT ERRORS (%) WHICH OCCUR IN THE EFFECTS OF SHEAR DEFORMATION AND AXIAL FORCE ARE NEGLECTED

Loading	Case	AD		DE		EB		EF		FC	
		M_{AD}	M_{DA}	M_{DE}	M_{ED}	M_{EB}	M_{BE}	M_{EF}	M_{FE}	M_{FC}	M_{CF}
1	1	67.78	263.75	-263.75	1,418.19	0	0	-1,418.19	263.75	-263.75	-67.78
	2	128.95	257.90	-257.90	1,387.03	0	0	-1,387.03	257.90	-257.90	-128.95
	Error	90.2	-2.2		-2.2	0.0	0.0	-2.2	-2.2		90.2
2	1	11.22	247.47	-247.47	1,444.76	-57.42	-60.52	-1,387.34	280.71	-280.71	-124.54
	2	77.78	249.30	-249.30	1,417.47	-60.88	-84.08	-1,356.59	266.50	-266.50	-180.12
	Error	593.2	.7		-1.9	6.0	38.9	-2.2	-5.1		-44.0
3	1	668.76	310.44	-310.44	1,496.31	-258.20	-258.79	-1,238.11	435.83	-435.83	-263.88
	2	995.73	517.63	-517.63	1,478.65	-399.74	-596.26	-1,078.91	431.17	-431.17	-350.68
	Error	48.9	66.7		-1.2	54.8	130.4	-12.9	-1.1		32.9

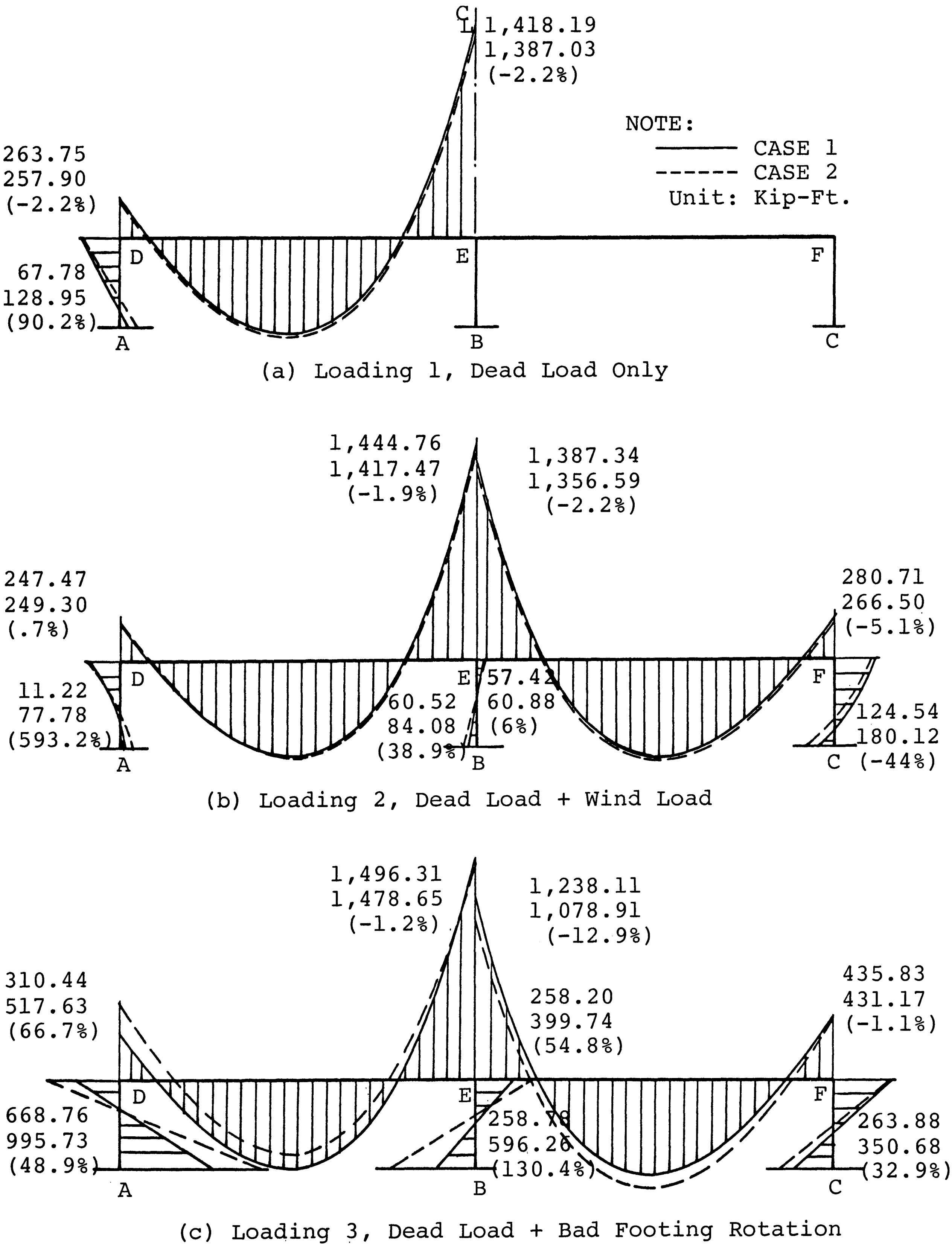

Figure 5-2(3). Moment diagrams of the frame structure for the loadings 1, 2, and 3.

CHAPTER VI

SUMMARY AND CONCLUSION

6.1. SUMMARY

This dissertation presents a theoretical analysis of the behavior of battened and laced structural members for an ideal perfectly elastic material and for small deformations. The analytical solution is made by application of classical procedures and modified slope-deflection equations are developed to generalize a relation between applied forces and joint displacements. If the displacements of the joints in a structure are known, it is a comparatively easy matter to obtain the bending moments and shear forces at any location of the structure.

A nondimensional parameter μ for the shear flexibility is defined as the ratio of the change in slope in a unit panel length due to shear deformation to the change in slope due to the bending rotation for a relationship between shear and moment. The parameter μ is calculated with consideration of the effects of axial force, local shear deformation, local joint eccentricity, and local connection flexibility of battened structural members.

Types of the structural members may be catalogued here according to three different web configurations such as solid, battened, and laced structural members. The battened and laced structural members are built up from two (or more) main longitudinal elements which are assumed to form a symmetrical section. The two longitudinal elements are connected in one, two (or more) planes by the diagonal and strut elements or batten elements. The arrangements of web elements are assumed the same throughout the effective length of the flexible portion of the structural member between end rigid stay plates. The shear flexibility parameter is evaluated in terms of the properties of the composed elements, the geometric arrangements, the local joint connection factors, and the axial force.

The optimum slope of the diagonal element is evaluated for the laced structural member to minimize the influence of shear deformation. In an actual situation, the axial forces of the longitudinal sub-elements will vary from panel to panel along the length of the members, and also will be different on the two sides of the longitudinal elements due to the lateral external loadings. However, ultimately the analysis assumes the shear flexibility μ as a constant value for the entire length of the structural member. To permit the assumption of constant shear flexibility, a limit on the ratio of the local slenderness has been determined for the battened structural members, so that the influence of the actual axial forces will be relatively negligible. The upper bound of the premature local failure is also investigated for the battened structural members.

The fundamental linear second-order differential equation for the deflection curve of the structural member has been derived for which shear deformation is considered. As long as the axial load is kept constant, then the effects of end moments, end-shear forces and external loading can be superimposed. The ordinary beam theory can still be applied since the effect of axial force appears here only in a modification of the stiffness properties of the structural members. The general solutions of this differential equation are of a fundamentally different nature for the cases when the axial force is equal to zero, greater than zero (compression axial force), and less than zero (tension axial force). By application of the natural boundary conditions to the general solution of deflected shape of the structural member, the solutions are set up in the forms of slope-deflection equations. Therefore the slope-deflection constants are obtained in terms of the shear flexibility parameter, the length factors of stay plates, and effect of axial force.

In the evaluation of the fixed end moments for a concentrated load, the reciprocal theorem is applied so as to make use of the deflection curves of structural members which have been previously defined for the homogeneous solution (no external lateral loading). From this basic expression one can derive fixed end moments for any combination of concentrated loads by simple summation and for continuously distributed loads by integration.

Elastic buckling of the structural members with rigid stay plates and constant shear flexibility have been evaluated for the cases of a column with hinged ends, a column with one end fixed and the other hinged, and a column with fixed ends.

Finally, numerical examples of beam and frame analyses are presented to provide a comparison with analyses using the ordinary beam and frame theories which neglect the effects of shear deformation and axial force.

6.2. CONCLUSIONS

1. The property of the shear flexibility parameter μ

   (a) For the laced structural members:

(i) The parameter μ becomes infinite as the slope of diagonal elements approaches either zero or infinity.

(ii) The optimum slope of the diagonal elements is $\sqrt{2}/2$ whenever strut elements are missing, or whenever the strut elements do not take part in the transmission of the shearing force of the structural members. When stressed strut elements are part of the system, the optimum slope increases with the ratio of cross-sectional area of the diagonal elements to the strut elements.

(iii) The parameter μ increases with increase of the height-length ratio of the member, and with the ratio of cross section of the longitudinal elements to the diagonal elements.

(b) For the battened structural members:

(i) The parameter μ becomes infinite either as the number of panels approaches zero, or as the semi-rigid constant approaches infinity (i.e., hinged connection). The parameter μ approaches zero as the number of panels approaches infinity.

(ii) The parameter μ is inversely proportional to the ratio of slenderness of the member, and proportional to the height-length ratio.

2. If the local slenderness ratio a/r_c reaches $\pi/\xi_a \sqrt{E/(F.S.)\sigma_i}$, premature local failure will occur. The proposed limitation $a/r_c < 2/3\xi_a \sqrt{E/(F.S.)\sigma_i}$ should provide adequate safety against this possibility.

3. For tension axial force, when

$$\alpha\mu = -\frac{1}{\pi^2(1 - \delta_1 - \delta_2)^2}$$

then the equivalent flexural rigidity of the structural member becomes infinite, and therefore leads to a trivial solution of the deflection curve of a structural member. Beyond this condition, the general solution of the deflection curve is not applicable.

4. The slope-deflection constant C_{11} and carry-over factor are almost linearly proportional to the axial load parameter α in the range of $|\alpha| < .15$ and $\mu < .10$. Beyond this range, however,

   (i) The constant C_{11} will decrease as the compression axial force increases, and may either decrease or, for very small values of μ, increase as the tension force increases.

   (ii) The carry-over factor will increase rapidly with increasing compression axial force. The increase is approximately proportional to the parameter α for small values of α.

5. For a structural member of normal design proportions with a certain constant shear flexibility μ there exists an axial load parameter α, for which the carry-over factor is very nearly independent of the effect of the end rigid stay plates.

6. Effect of shear deformation varies with the magnitude of the axial force, properties of the bracing systems, and the height-length ratio of the member.

7. For a symmetrical structural member with uniformly distributed loading, the fixed-end moments are independent of the effect of the shear flexibility when there is no axial force.

8. The influence of the end rigid stay plates is of considerable significance since large increases in the carrying capacity can be achieved by very short end rigid stay plates.

9. The influence of the shear deformation may be neglected in the range of $\mu < .004$, and $|\alpha| < .50$, for which the errors of the stiffness constant C_{11}, carry-over factor, and fixed-end moments will be not more than 5%. However, beyond this range large errors will be introduced in the analysis if account is not taken for the influence of shear deformation.

APPENDIX

TABLE 1

THE CONSTANT SHEAR FLEXIBILITY PARAMETER μ FOR LACED STRUCTURAL MEMBERS

(1) VALUES OF $\mu(1+\xi_a)/\xi_b$ FOR THE CASE OF NO STRUT ELEMENTS

A_c/A_d	ℓ/b \ $\xi_a a/b$ =	.4	.6	.8	1.0	1.2	1.4	1.6
2	4	.3904	.3304	.3282	.3536	.3970	.4547	.5248
	6	.1735	.1469	.1458	.1571	.1765	.2021	.2332
	8	.0976	.0826	.0820	.0884	.0993	.1137	.1312
	10	.0625	.0529	.0525	.0566	.0635	.0728	.0840
	12	.0434	.0367	.0365	.0393	.0441	.0505	.0583
	14	.0319	.0270	.0268	.0289	.0324	.0371	.0428
	16	.0244	.0207	.0205	.0221	.0248	.0284	.0328
	18	.0193	.0163	.0162	.0175	.0196	.0225	.0259
	20	.0156	.0132	.0131	.0141	.0159	.0182	.0210
4	4	.7808	.6608	.6563	.7071	.7940	.9094	1.0495
	6	.3470	.2937	.2917	.3143	.3529	.4042	.4665
	8	.1952	.1652	.1641	.1768	.1985	.2273	.2624
	10	.1249	.1057	.1050	.1131	.1270	.1455	.1679
	12	.0868	.0734	.0729	.0786	.0882	.1010	.1166
	14	.0637	.0539	.0536	.0577	.0648	.0742	.0857
	16	.0488	.0413	.0410	.0442	.0496	.0568	.0656
	18	.0386	.0326	.0324	.0349	.0392	.0449	.0518
	20	.0312	.0264	.0263	.0283	.0318	.0364	.0420
6	4	1.1713	.9913	.9845	1.0607	1.1911	1.3641	1.5743
	6	.5206	.4406	.4375	.4714	.5294	.6063	.6997
	8	.2928	.2478	.2461	.2652	.2978	.3410	.3936
	10	.1874	.1586	.1575	.1697	.1906	.2183	.2519
	12	.1301	.1101	.1094	.1179	.1323	.1516	.1749
	14	.0956	.0809	.0804	.0866	.0972	.1114	.1285
	16	.0732	.0620	.0615	.0663	.0744	.0853	.0984
	18	.0578	.0490	.0486	.0524	.0588	.0674	.0777
	20	.0469	.0397	.0394	.0424	.0476	.0546	.0630
8	4	1.5617	1.3217	1.3126	1.4142	1.5881	1.8188	2.0991
	6	.6941	.5874	.5834	.6285	.7058	.8083	.9329
	8	.3904	.3304	.3282	.3536	.3970	.4547	.5248
	10	.2499	.2115	.2100	.2263	.2541	.2910	.3358
	12	.1735	.1469	.1458	.1571	.1765	.2021	.2332
	14	.1275	.1079	.1072	.1154	.1296	.1485	.1714
	16	.0976	.0826	.0820	.0884	.0993	.1137	.1312
	18	.0771	.0653	.0648	.0698	.0784	.0898	.1037
	20	.0625	.0529	.0525	.0566	.0635	.0728	.0840
10	4	1.9521	1.6521	1.6408	1.7678	1.9851	2.2735	2.6238
	6	.8676	.7343	.7292	.7857	.8823	1.0104	1.1661
	8	.4880	.4130	.4102	.4419	.4963	.5684	.6560
	10	.3123	.2643	.2625	.2828	.3176	.3638	.4198
	12	.2169	.1836	.1823	.1964	.2206	.2526	.2915
	14	.1594	.1349	.1339	.1443	.1620	.1856	.2142
	16	.1220	.1033	.1026	.1105	.1241	.1421	.1640
	18	.0964	.0816	.0810	.0873	.0980	.1123	.1296
	20	.0781	.0661	.0656	.0707	.0794	.0909	.1050

TABLE 1

THE CONSTANT SHEAR FLEXIBILITY PARAMETER μ
FOR LACED STRUCTURAL MEMBERS
(2) VALUES OF $\mu(1+\xi_a)/\xi_b$ FOR A_d/A_b = .5

A_c/A_d	ℓ/b \ $\xi_a a/b$ =	.4	.6	.8	1.0	1.2	1.4	1.6
2	4	.5467	.4346	.4063	.4161	.4491	.4953	.5638
	6	.2430	.1931	.1806	.1849	.1996	.2219	.2506
	8	.1367	.1086	.1016	.1040	.1123	.1248	.1410
	10	.0875	.0695	.0650	.0666	.0719	.0799	.0902
	12	.0607	.0483	.0451	.0462	.0499	.0555	.0626
	14	.0446	.0355	.0332	.0340	.0367	.0408	.0460
	16	.0342	.0272	.0254	.0260	.0281	.0312	.0352
	18	.0270	.0215	.0201	.0205	.0222	.0247	.0278
	20	.0219	.0174	.0163	.0166	.0180	.0200	.0226
4	4	1.0933	.8692	.8126	.8321	.8982	.9987	1.1277
	6	.4859	.3863	.3611	.3698	.3992	.4439	.5012
	8	.2733	.2173	.2031	.2080	.2246	.2497	.2819
	10	.1749	.1391	.1300	.1331	.1437	.1598	.1804
	12	.1215	.0966	.0903	.0925	.0998	.1110	.1253
	14	.0893	.0710	.0663	.0679	.0733	.0815	.0921
	16	.0683	.0543	.0508	.0520	.0561	.0624	.0705
	18	.0540	.0429	.0401	.0411	.0444	.0493	.0557
	20	.0437	.0348	.0325	.0333	.0359	.0399	.0451
6	4	1.6400	1.3038	1.2189	1.2482	1.3473	1.4980	1.6915
	6	.7289	.5794	.5417	.5547	.5988	.6658	.7518
	8	.4100	.3259	.3047	.3120	.3368	.3745	.4229
	10	.2624	.2086	.1950	.1997	.2156	.2397	.2706
	12	.1822	.1449	.1354	.1387	.1497	.1664	.1879
	14	.1339	.1064	.0995	.1019	.1100	.1223	.1381
	16	.1025	.0815	.0762	.0780	.0842	.0936	.1057
	18	.0810	.0644	.0602	.0616	.0665	.0740	.0835
	20	.0656	.0522	.0488	.0499	.0539	.0599	.0677
8	4	2.1867	1.7383	1.6251	1.6642	1.7964	1.9973	2.2553
	6	.9719	.7726	.7223	.7397	.7984	.8877	1.0024
	8	.5467	.4346	.4063	.4161	.4491	.4953	.5638
	10	.3499	.2781	.2600	.2663	.2874	.3196	.3608
	12	.2430	.1931	.1806	.1849	.1996	.2219	.2506
	14	.1785	.1419	.1327	.1359	.1466	.1630	.1841
	16	.1367	.1086	.1016	.1040	.1123	.1248	.1410
	18	.1080	.0858	.0803	.0822	.0887	.0986	.1114
	20	.0875	.0695	.0650	.0666	.0719	.0799	.0902
10	4	2.7334	2.1729	2.0314	2.0803	2.2455	2.4967	2.8191
	6	1.2148	.9657	.9029	.9246	.9980	1.1096	1.2530
	8	.6833	.5432	.5079	.5201	.5614	.6242	.7048
	10	.4373	.3477	.3250	.3328	.3593	.3995	.4511
	12	.3037	.2414	.2257	.2311	.2495	.2774	.3132
	14	.2231	.1774	.1658	.1698	.1833	.2038	.2301
	16	.1708	.1358	.1270	.1300	.1403	.1560	.1762
	18	.1350	.1073	.1003	.1027	.1109	.1233	.1392
	20	.1093	.0869	.0813	.0832	.0898	.0999	.1128

TABLE I

THE CONSTANT SHEAR FLEXIBILITY PARAMETER FOR LACED STRUCTURAL MEMBERS

(3) VALUES OF $\mu(1+\xi_a)/\xi_b$ FOR $A_d/A_b = 1.0$

A_c/A_d	$\xi_a a/b$ = / ℓ/b	.4	.6	.8	1.0	1.2	1.4	1.6
	4	.7029	.5388	.4844	.4786	.5012	.5440	.6029
	6	.3124	.2394	.2153	.2127	.2228	.2418	.2680
	8	.1757	.1347	.1211	.1196	.1253	.1360	.1507
	10	.1125	.0862	.0775	.0766	.0802	.0870	.0965
2	12	.0781	.0599	.0538	.0532	.0557	.0604	.0670
	14	.0574	.0440	.0395	.0391	.0409	.0444	.0492
	16	.0439	.0337	.0303	.0299	.0313	.0340	.0377
	18	.0347	.0266	.0239	.0236	.0248	.0269	.0298
	20	.0281	.0216	.0194	.0191	.0200	.0218	.0241
	4	1.4058	1.0775	.9688	.9571	1.0024	1.0880	1.2058
	6	.6248	.4789	.4306	.4254	.4455	.4835	.5359
	8	.3515	.2694	.2422	.2393	.2506	.2720	.3014
	10	.2249	.1724	.1550	.1531	.1604	.1741	.1929
4	12	.1562	.1197	.1076	.1063	.1114	.1209	.1340
	14	.1148	.0880	.0791	.0781	.0818	.0868	.0984
	16	.0879	.0673	.0606	.0598	.0626	.0680	.0754
	18	.0694	.0532	.0478	.0473	.0495	.0537	.0595
	20	.0562	.0431	.0388	.0383	.0401	.0435	.0482
	4	2.1088	1.6163	1.4532	1.4357	1.5036	1.6319	1.8087
	6	.9372	.7183	.6459	.6381	.6683	.7253	.8039
	8	.5272	.4041	.3633	.3589	.3759	.4080	.4522
	10	.3374	.2586	.2325	.2297	.2406	.2611	.2894
6	12	.2343	.1796	.1615	.1595	.1671	.1813	.2010
	14	.1721	.1319	.1186	.1172	.1227	.1332	.1476
	16	.1318	.1010	.0908	.0897	.0940	.1020	.1130
	18	.1041	.0798	.0718	.0709	.0743	.0806	.0893
	20	.0844	.0647	.0581	.0574	.0601	.0653	.0723
	4	2.8117	2.1550	1.9376	1.9142	2.0048	2.1759	2.4116
	6	1.2496	.9578	.8612	.8508	.8910	.9671	1.0718
	8	.7029	.5388	.4844	.4786	.5012	.5440	.6029
	10	.4499	.3448	.3100	.3063	.3208	.3481	.3858
8	12	.3124	.2394	.2153	.2127	.2228	.2418	.2680
	14	.2295	.1759	.1582	.1563	.1637	.1776	.1969
	16	.1757	.1347	.1211	.1196	.1253	.1360	.1507
	18	.1388	.1064	.0957	.0945	.0990	.1075	.1191
	20	.1125	.0862	.0775	.0766	.0802	.0870	.0965
	4	3.5146	2.6938	2.4221	2.3928	2.5059	2.7199	3.0145
	6	1.5621	1.1972	1.0765	1.0635	1.1138	1.2088	1.3398
	8	.8787	.6734	.6055	.5982	.6265	.6800	.7536
	10	.5623	.4310	.3875	.3828	.4010	.4352	.4823
10	12	.3905	.2993	.2691	.2659	.2784	.3022	.3349
	14	.2869	.2199	.1977	.1953	.2046	.2220	.2461
	16	.2197	.1684	.1514	.1495	.1566	.1700	.1884
	18	.1736	.1330	.1196	.1182	.1238	.1343	.1489
	20	.1406	.1078	.0969	.0957	.1002	.1088	.1206

TABLE I

THE CONSTANT SHEAR FLEXIBILITY PARAMETER μ FOR LACED STRUCTURAL MEMBERS

(4) VALUES OF $\mu(1+\xi_a)/\xi_b$ FOR A_d/A_b = 1.5

A_c/A_d	$\xi_a a/b$ = ℓ/b	.4	.6	.8	1.0	1.2	1.4	1.6
2	4	.8592	.6429	.5625	.5411	.5533	.5886	.6420
	6	.3819	.2857	.2500	.2405	.2459	.2616	.2853
	8	.2148	.1607	.1406	.1353	.1383	.1472	.1605
	10	.1375	.1029	.0900	.0866	.0885	.0942	.1027
	12	.0955	.0714	.0625	.0601	.0615	.0654	.0713
	14	.0701	.0525	.0459	.0442	.0452	.0481	.0524
	16	.0537	.0402	.0352	.0338	.0346	.0368	.0401
	18	.0424	.0317	.0278	.0267	.0273	.0291	.0317
	20	.0344	.0257	.0225	.0216	.0221	.0235	.0257
4	4	1.7183	1.2858	1.1251	1.0821	1.1065	1.1772	1.2839
	6	.7637	.5715	.5000	.4809	.4918	.5232	.5706
	8	.4296	.3215	.2813	.2705	.2766	.2943	.3210
	10	.2749	.2057	.1800	.1731	.1770	.1884	.2054
	12	.1909	.1429	.1250	.1202	.1229	.1308	.1427
	14	.1403	.1050	.0918	.0883	.0903	.0961	.1048
	16	.1074	.0804	.0703	.0676	.0692	.0736	.0802
	18	.0849	.0635	.0556	.0534	.0546	.0581	.0634
	20	.0687	.0514	.0450	.0433	.0443	.0471	.0514
6	4	2.5775	1.9288	1.6876	1.6232	1.6598	1.7659	1.9259
	6	1.1456	.8572	.7500	.7214	.7377	.7848	.8559
	8	.6444	.4822	.4219	.4058	.4150	.4415	.4815
	10	.4124	.3086	.2700	.2597	.2656	.2825	.3081
	12	.2864	.2143	.1875	.1804	.1844	.1962	.2140
	14	.2104	.1574	.1378	.1325	.1355	.1442	.1572
	16	.1611	.1205	.1055	.1014	.1037	.1104	.1204
	18	.1273	.0952	.0833	.0802	.0820	.0872	.0951
	20	.1031	.0772	.0675	.0649	.0664	.0706	.0770
8	4	3.4367	2.5717	2.2501	2.1642	2.2131	2.3545	2.56[illegible]
	6	1.5274	1.1430	1.0001	.9619	.9836	1.0464	1.1412
	8	.8592	.6429	.5625	.5411	.5533	.5886	.6420
	10	.5499	.4115	.3600	.3463	.3541	.3767	.4108
	12	.3819	.2857	.2500	.2405	.2459	.2616	.2853
	14	.2805	.2099	.1837	.1767	.1807	.1922	.2096
	16	.2148	.1607	.1406	.1353	.1383	.1472	.1605
	18	.1697	.1270	.1111	.1069	.1093	.1163	.1268
	20	.1375	.1029	.0900	.0866	.0885	.0942	.1027
10	4	4.2959	3.2146	2.8127	2.7053	2.7664	2.9431	3.2098
	6	1.9093	1.4287	1.2501	1.2023	1.2295	1.3081	1.4266
	8	1.0740	.8037	.7032	.6763	.6916	.7358	.8024
	10	.6873	.5143	.4500	.4328	.4426	.4709	.5136
	12	.4773	.3572	.3125	.3006	.3074	.3270	.3566
	14	.3507	.2624	.2296	.2208	.2258	.2403	.2620
	16	.2685	.2009	.1758	.1691	.1729	.1839	.2006
	18	.2121	.1587	.1389	.1336	.1366	.1453	.1585
	20	.1718	.1286	.1125	.1082	.1107	.1177	.1284

TABLE I

THE CONSTANT SHEAR FLEXIBILITY PARAMETER μ FOR LACED STRUCTURAL MEMBERS

(5) VALUES OF $\mu(1+\xi_a)/\xi_b$ FOR A_d/A_b = 2.0

A_c/A_d	$\xi_a a/b$ = ℓ/b	.4	.6	.8	1.0	1.2	1.4	1.6
	4	1.0154	.7471	.6407	.6036	.6054	.6333	.6810
	6	.4513	.3320	.2847	.2682	.2690	.2815	.3027
	8	.2539	.1868	.1602	.1509	.1513	.1583	.1703
	10	.1625	.1195	.1025	.0966	.0969	.1013	.1090
2	12	.1128	.0830	.0712	.0671	.0673	.0704	.0757
	14	.0829	.0610	.0523	.0493	.0494	.0517	.0556
	16	.0635	.0467	.0400	.0377	.0378	.0396	.0426
	18	.0501	.0369	.0316	.0298	.0299	.0313	.0336
	20	.0406	.0299	.0256	.0241	.0242	.0253	.0272
	4	2.0308	1.4942	1.2813	1.2071	1.2107	1.2665	1.3620
	6	.9026	.6641	.5695	.5365	.5381	.5629	.6053
	8	.5077	.3735	.3203	.3018	.3027	.3166	.3405
	10	.3249	.2391	.2050	.1931	.1937	.2026	.2179
4	12	.2256	.1660	.1424	.1341	.1345	.1407	.1513
	14	.1658	.1220	.1046	.0985	.0988	.1034	.1112
	16	.1269	.0934	.0801	.0754	.0757	.0792	.0851
	18	.1003	.0738	.0633	.0596	.0598	.0625	.0673
	20	.0812	.0598	.0513	.0483	.0484	.0507	.0545
	4	3.0463	2.2413	1.9220	1.8107	1.8161	1.8998	2.0430
	6	1.3539	.9961	.8542	.8047	.8071	.8444	.9080
	8	.7616	.5603	.4805	.4527	.4540	.4749	.5108
	10	.4874	.3586	.3075	.2897	.2906	.3040	.3269
6	12	.3385	.2490	.2136	.2012	.2018	.2111	.2270
	14	.2487	.1830	.1569	.1478	.1483	.1551	.1668
	16	.1904	.1401	.1201	.1132	.1135	.1187	.1277
	18	.1504	.1107	.0949	.0894	.0897	.0938	.1009
	20	.1219	.0897	.0769	.0724	.0726	.0760	.0817
	4	4.0617	2.9883	2.5626	2.4142	2.4214	2.5331	2.7241
	6	1.8052	1.3282	1.1390	1.0730	1.0762	1.1258	1.2107
	8	1.0154	.7471	.6407	.6036	.6054	.6333	.6810
	10	.6499	.4781	.4100	.3863	.3874	.4053	.4358
8	12	.4513	.3320	.2847	.2682	.2690	.2815	.3027
	14	.3316	.2439	.2092	.1971	.1977	.2068	.2224
	16	.2539	.1868	.1602	.1509	.1513	.1583	.1703
	18	.2006	.1476	.1266	.1192	.1196	.1251	.1345
	20	.1625	.1195	.1025	.0966	.0969	.1013	.1090
	4	5.0771	3.7354	3.2033	3.0178	3.0268	3.1663	3.4051
	6	2.2565	1.6602	1.4237	1.3412	1.3452	1.4073	1.5134
	8	1.2693	.9339	.8008	.7544	.7567	.7916	.8513
	10	.8123	.5977	.5125	.4828	.4843	.5066	.5448
10	12	.5641	.4150	.3559	.3353	.3363	.3518	.3783
	14	.4145	.3049	.2615	.2463	.2471	.2585	.2780
	16	.3173	.2335	.2002	.1886	.1892	.1979	.2128
	18	.2507	.1845	.1582	.1490	.1495	.1564	.1682
	20	.2031	.1494	.1281	.1207	.1211	.1267	.1362

TABLE II

THE CONSTANT SHEAR FLEXIBILITY PARAMETER μ FOR BATTENED STRUCTURAL MEMBERS. THE LONGITUDINAL AND BATTEN ELEMENTS ARE WF SHAPES AND CHANNELS RESPECTIVELY.
THE VALUES OF μ FOR η_c=2.0, η_b=1.5, ξ_a=.85, Z=0

(1) r_b/r_c = 1.0, A_c/A_b = .5

ℓ/r_c	ℓ/a = / ℓ/b	6	8	10	12	14	16	18
	6	.2289	.1674	.1344	.1140	.1002	.0903	.0829
	8	.1212	.0884	.0710	.0603	.0531	.0480	.0442
	10	.0768	.0560	.0450	.0382	.0337	.0305	.0281
	12	.0544	.0396	.0318	.0271	.0239	.0216	.0199
80	14	.0415	.0302	.0243	.0206	.0182	.0165	.0151
	16	.0334	.0244	.0196	.0166	.0146	.0132	.0122
	18	.0280	.0204	.0164	.0139	.0123	.0111	.0102
	20	.0242	.0177	.0142	.0120	.0106	.0096	.0088
	6	.3218	.2304	.1815	.1514	.1312	.1167	.1058
	8	.1650	.1175	.0924	.0771	.0669	.0596	.0542
	10	.1015	.0720	.0566	.0472	.0410	.0366	.0333
	12	.0698	.0495	.0389	.0324	.0282	.0251	.0229
100	14	.0518	.0367	.0288	.0240	.0209	.0186	.0170
	16	.0406	.0288	.0226	.0189	.0164	.0146	.0133
	18	.0332	.0235	.0185	.0154	.0134	.0119	.0109
	20	.0280	.0198	.0156	.0130	.0113	.0101	.0092
	6	.4357	.3076	.2394	.1975	.1693	.1491	.1340
	8	.2190	.1534	.1189	.0980	.0840	.0740	.0666
	10	.1320	.0920	.0711	.0586	.0502	.0443	.0399
	12	.0891	.0619	.0478	.0393	.0337	.0297	.0268
120	14	.0649	.0450	.0347	.0286	.0245	.0216	.0195
	16	.0500	.0346	.0267	.0220	.0188	.0166	.0150
	18	.0401	.0278	.0214	.0176	.0151	.0133	.0120
	20	.0333	.0230	.0178	.0146	.0125	.0111	.0100
	6	.5705	.3991	.3079	.2520	.2144	.1876	.1675
	8	.2830	.1959	.1503	.1227	.1043	.0912	.0815
	10	.1684	.1158	.0885	.0721	.0612	.0535	.0478
	12	.1122	.0768	.0585	.0476	.0404	.0353	.0316
140	14	.0807	.0551	.0419	.0340	.0289	.0253	.0226
	16	.0613	.0418	.0318	.0258	.0219	.0191	.0171
	18	.0486	.0331	.0251	.0204	.0173	.0151	.0135
	20	.0398	.0271	.0206	.0167	.0142	.0124	.0111
	6	.7261	.5046	.3870	.3149	.2666	.2320	.2061
	8	.3569	.2451	.1866	.1513	.1278	.1111	.0987
	10	.2104	.1433	.1086	.0877	.0740	.0643	.0571
	12	.1389	.0940	.0710	.0572	.0482	.0418	.0372
160	14	.0990	.0667	.0502	.0404	.0340	.0295	.0262
	16	.0746	.0501	.0377	.0303	.0255	.0221	.0196
	18	.0586	.0393	.0295	.0237	.0199	.0173	.0154
	20	.0475	.0319	.0239	.0192	.0161	.0140	.0124

TABLE II

THE CONSTANT SHEAR FLEXIBILITY PARAMETER μ FOR BATTENED STRUCTURAL MEMBERS. THE LONGITUDINAL AND BATTEN ELEMENTS ARE WF SHAPES AND CHANNELS RESPECTIVELY.
THE VALUES OF μ FOR $\eta_c=2.0$, $\eta_b=1.5$, $\xi_a=.85$, Z=0

(2) $r_b/r_c = 1.0$, $A_c/A_b = 1.0$

ℓ/r_c	ℓ/b \ ℓ/a =	6	8	10	12	14	16	18
	6	.3618	.2671	.2141	.1804	.1572	.1401	.1272
	8	.1874	.1381	.1108	.0935	.0815	.0729	.0662
	10	.1177	.0866	.0695	.0587	.0512	.0458	.0417
	12	.0831	.0612	.0491	.0414	.0362	.0324	.0295
80	14	.0636	.0468	.0376	.0317	.0277	.0247	.0225
	16	.0515	.0379	.0304	.0257	.0224	.0200	.0182
	18	.0434	.0320	.0257	.0216	.0189	.0169	.0153
	20	.0378	.0279	.0224	.0189	.0164	.0147	.0133
	6	.5124	.3733	.2958	.2467	.2128	.1881	.1693
	8	.2554	.1853	.1467	.1223	.1056	.0935	.0843
	10	.1545	.1118	.0884	.0737	.0637	.0564	.0509
	12	.1054	.0762	.0602	.0502	.0434	.0385	.0347
100	14	.0780	.0563	.0445	.0371	.0321	.0285	.0257
	16	.0611	.0442	.0349	.0291	.0252	.0223	.0202
	18	.0501	.0362	.0286	.0239	.0206	.0183	.0165
	20	.0424	.0307	.0243	.0202	.0175	.0155	.0140
	6	.6969	.5035	.3961	.3280	.2812	.2471	.2211
	8	.3391	.2435	.1910	.1580	.1354	.1191	.1067
	10	.2001	.1431	.1120	.0926	.0794	.0698	.0626
	12	.1333	.0950	.0743	.0614	.0526	.0463	.0415
120	14	.0964	.0686	.0536	.0443	.0380	.0334	.0300
	16	.0739	.0526	.0411	.0339	.0291	.0256	.0230
	18	.0593	.0422	.0329	.0272	.0233	.0205	.0184
	20	.0492	.0350	.0274	.0226	.0194	.0170	.0153
	6	.9152	.6576	.5147	.4243	.3622	.3168	.2824
	8	.4382	.3124	.2435	.2003	.1708	.1494	.1332
	10	.2544	.1803	.1401	.1151	.0981	.0858	.0765
	12	.1667	.1176	.0912	.0748	.0637	.0558	.0498
140	14	.1186	.0835	.0646	.0530	.0451	.0395	.0352
	16	.0895	.0629	.0487	.0399	.0339	.0297	.0265
	18	.0707	.0496	.0384	.0314	.0268	.0234	.0209
	20	.0578	.0406	.0314	.0257	.0219	.0191	.0171
	6	1.1672	.8354	.6516	.5355	.4556	.3974	.3532
	8	.5527	.3920	.3042	.2492	.2117	.1845	.1640
	10	.3172	.2234	.1726	.1411	.1197	.1043	.0927
	12	.2053	.1439	.1108	.0904	.0767	.0668	.0593
160	14	.1443	.1007	.0775	.0631	.0535	.0465	.0414
	16	.1077	.0750	.0576	.0469	.0397	.0345	.0307
	18	.0841	.0584	.0448	.0365	.0309	.0269	.0239
	20	.0680	.0472	.0362	.0294	.0249	.0217	.0192

TABLE II

THE CONSTANT SHEAR FLEXIBILITY PARAMETER μ FOR BATTENED STRUCTURAL MEMBERS. THE LONGITUDINAL AND BATTEN ELEMENTS ARE WF SHAPES AND CHANNELS RESPECTIVELY.
THE VALUES OF μ FOR η_c=2.0, η_b=1.5, ξ_a=.85, Z=0

(3) r_b/r_c = 1.0, A_c/A_b = 1.5

ℓ/r_c	ℓ/b \ ℓ/a =	6	8	10	12	14	16	18
	6	.4947	.3667	.2938	.2469	.2141	.1900	.1715
	8	.2537	.1878	.1505	.1266	.1099	.0977	.0883
	10	.1585	.1173	.0940	.0791	.0687	.0611	.0553
	12	.1119	.0828	.0664	.0558	.0485	.0432	.0391
80	14	.0857	.0634	.0508	.0428	.0372	.0330	.0299
	16	.0695	.0515	.0413	.0347	.0301	.0268	.0242
	18	.0589	.0436	.0349	.0294	.0255	.0227	.0205
	20	.0514	.0381	.0305	.0257	.0223	.0198	.0179
	6	.7029	.5162	.4102	.3420	.2945	.2596	.2328
	8	.3458	.2531	.2009	.1675	.1444	.1274	.1144
	10	.2075	.1516	.1202	.1002	.0864	.0763	.0686
	12	.1409	.1028	.0815	.0680	.0586	.0518	.0466
100	14	.1041	.0759	.0602	.0502	.0433	.0383	.0344
	16	.0817	.0596	.0472	.0394	.0340	.0300	.0270
	18	.0670	.0489	.0388	.0323	.0279	.0246	.0222
	20	.0569	.0415	.0329	.0275	.0237	.0209	.0188
	6	.9580	.6994	.5528	.4586	.3931	.3450	.3081
	8	.4592	.3335	.2630	.2180	.1869	.1641	.1467
	10	.2682	.1941	.1529	.1266	.1086	.0954	.0853
	12	.1775	.1282	.1008	.0835	.0716	.0629	.0563
120	14	.1278	.0922	.0725	.0600	.0514	.0452	.0405
	16	.0979	.0706	.0554	.0459	.0394	.0346	.0310
	18	.0785	.0566	.0444	.0368	.0315	.0277	.0248
	20	.0652	.0470	.0369	.0306	.0262	.0230	.0206
	6	1.2598	.9161	.7215	.5967	.5099	.4461	.3972
	8	.5935	.4288	.3366	.2779	.2373	.2076	.1850
	10	.3404	.2448	.1917	.1581	.1349	.1181	.1052
	12	.2211	.1585	.1239	.1021	.0871	.0762	.0679
140	14	.1564	.1119	.0873	.0719	.0613	.0537	.0478
	16	.1177	.0840	.0656	.0540	.0460	.0403	.0359
	18	.0928	.0662	.0516	.0425	.0362	.0317	.0282
	20	.0758	.0541	.0422	.0347	.0296	.0259	.0231
	6	1.6082	1.1662	.9163	.7560	.6446	.5628	.5002
	8	.7486	.5389	.4217	.3471	.2956	.2580	.2293
	10	.4240	.3035	.2367	.1945	.1655	.1443	.1283
	12	.2718	.1937	.1507	.1237	.1051	.0917	.0815
160	14	.1897	.1348	.1047	.0858	.0729	.0636	.0565
	16	.1408	.0998	.0774	.0634	.0539	.0470	.0417
	18	.1096	.0775	.0601	.0492	.0418	.0364	.0324
	20	.0884	.0625	.0484	.0396	.0337	.0293	.0261

TABLE II

THE CONSTANT SHEAR FLEXIBILITY PARAMETER μ FOR BATTENED STRUCTURAL MEMBERS. THE LONGITUDINAL AND BATTEN ELEMENTS ARE WF SHAPES AND CHANNELS RESPECTIVELY.
THE VALUES OF μ FOR η_c=2.0, η_b=1.5, ξ_a=.85, Z=0

(4) r_b/r_c = 1.0, A_c/A_b = 2.0

ℓ/r_c	ℓ/a = / ℓ/b	6	8	10	12	14	16	18
	6	.6275	.4664	.3736	.3133	.2711	.2398	.2157
	8	.3200	.2375	.1903	.1597	.1383	.1225	.1104
	10	.1994	.1480	.1185	.0995	.0863	.0765	.0689
	12	.1407	.1044	.0836	.0702	.0609	.0540	.0486
80	14	.1078	.0800	.0641	.0538	.0466	.0413	.0373
	16	.0876	.0650	.0521	.0437	.0379	.0336	.0303
	18	.0743	.0552	.0442	.0371	.0321	.0284	.0256
	20	.0651	.0483	.0387	.0325	.0281	.0249	.0224
	6	.8934	.6591	.5245	.4373	.3762	.3310	.2963
	8	.4362	.3209	.2551	.2127	.1831	.1613	.1445
	10	.2605	.1913	.1520	.1268	.1092	.0962	.0863
	12	.1765	.1295	.1029	.0858	.0739	.0651	.0584
100	14	.1303	.0956	.0759	.0633	.0545	.0481	.0431
	16	.1022	.0750	.0596	.0497	.0428	.0377	.0338
	18	.0839	.0616	.0489	.0408	.0351	.0310	.0278
	20	.0713	.0524	.0416	.0347	.0299	.0263	.0236
	6	1.2192	.8952	.7095	.5892	.5051	.4429	.3952
	8	.5792	.4235	.3350	.2781	.2383	.2091	.1867
	10	.3363	.2452	.1937	.1607	.1377	.1209	.1080
	12	.2216	.1613	.1273	.1056	.0905	.0794	.0710
120	14	.1593	.1158	.0913	.0757	.0649	.0570	.0509
	16	.1219	.0885	.0698	.0579	.0496	.0436	.0389
	18	.0977	.0710	.0560	.0464	.0398	.0349	.0312
	20	.0812	.0590	.0465	.0386	.0331	.0290	.0259
	6	1.6045	1.1746	.9283	.7690	.6576	.5753	.5121
	8	.7487	.5452	.4297	.3555	.3039	.2658	.2367
	10	.4265	.3093	.2433	.2011	.1718	.1503	.1339
	12	.2756	.1994	.1566	.1293	.1104	.0966	.0861
140	14	.1943	.1402	.1100	.0908	.0776	.0679	.0605
	16	.1459	.1052	.0825	.0680	.0581	.0508	.0453
	18	.1148	.0827	.0648	.0535	.0457	.0400	.0356
	20	.0938	.0676	.0529	.0437	.0373	.0326	.0291
	6	2.0493	1.4970	1.1809	.9766	.8337	.7282	.6472
	8	.9445	.6858	.5392	.4451	.3796	.3314	.2945
	10	.5308	.3835	.3008	.2479	.2112	.1844	.1639
	12	.3382	.2435	.1906	.1569	.1336	.1166	.1036
160	14	.2350	.1688	.1319	.1085	.0923	.0806	.0716
	16	.1740	.1247	.0973	.0800	.0681	.0594	.0528
	18	.1351	.0967	.0754	.0620	.0527	.0460	.0409
	20	.1089	.0778	.0607	.0499	.0424	.0370	.0329

TABLE II

THE CONSTANT SHEAR FLEXIBILITY PARAMETER μ FOR BATTENED STRUCTURAL MEMBERS. THE LONGITUDINAL AND BATTEN ELEMENTS ARE WF SHAPES AND CHANNELS RESPECTIVELY.
THE VALUES OF μ FOR η_c=2.0, η_b=1.5, ξ_a=.85, Z=0

(5) r_b/r_c = 1.5, A_c/A_b = .5

ℓ/r_c	ℓ/a =	6	8	10	12	14	16	18
	ℓ/b							
	6	.1704	.1236	.0993	.0848	.0752	.0684	.0634
	8	.0961	.0696	.0560	.0478	.0424	.0386	.0358
	10	.0637	.0462	.0371	.0317	.0281	.0255	.0237
	12	.0466	.0338	.0272	.0232	.0205	.0187	.0173
80	14	.0364	.0265	.0213	.0181	.0160	.0146	.0135
	16	.0299	.0217	.0175	.0149	.0131	.0119	.0110
	18	.0254	.0185	.0149	.0127	.0112	.0101	.0093
	20	.0223	.0162	.0130	.0111	.0098	.0088	.0081
	6	.2312	.1624	.1272	.1061	.0924	.0827	.0756
	8	.1264	.0886	.0693	.0578	.0503	.0451	.0413
	10	.0814	.0570	.0446	.0372	.0324	.0290	.0266
	12	.0580	.0406	.0318	.0265	.0231	.0207	.0189
	14	.0442	.0310	.0243	.0203	.0176	.0158	.0144
	16	.0354	.0249	.0195	.0163	.0141	.0127	.0116
	18	.0294	.0207	.0162	.0135	.0118	.0105	.0096
	20	.0252	.0177	.0139	.0116	.0101	.0090	.0082
	6	.3058	.2102	.1614	.1325	.1136	.1004	.0907
	8	.1638	.1120	.0858	.0703	.0603	.0533	.0482
	10	.1035	.0706	.0540	.0443	.0380	.0336	.0304
	12	.0724	.0494	.0378	.0309	.0265	.0235	.0212
120	14	.0543	.0370	.0283	.0232	.0199	.0176	.0159
	16	.0427	.0292	.0223	.0183	.0157	.0139	.0126
	18	.0349	.0239	.0183	.0150	.0129	.0114	.0103
	20	.0294	.0202	.0155	.0127	.0109	.0096	.0087
	6	.3942	.2668	.2021	.1638	.1389	.1215	.1087
	8	.2082	.1398	.1054	.0853	.0722	.0632	.0566
	10	.1298	.0868	.0653	.0528	.0447	.0391	.0350
	12	.0897	.0599	.0450	.0363	.0307	.0269	.0241
140	14	.0664	.0443	.0333	.0269	.0227	.0199	.0178
	16	.0516	.0345	.0259	.0209	.0177	.0155	.0139
	18	.0417	.0279	.0210	.0169	.0143	.0125	.0112
	20	.0347	.0232	.0175	.0141	.0120	.0105	.0094
	6	.4962	.3322	.2491	.2000	.1680	.1458	.1295
	8	.2595	.1720	.1282	.1026	.0860	.0745	.0662
	10	.1603	.1057	.0785	.0626	.0525	.0455	.0404
	12	.1097	.0721	.0534	.0426	.0357	.0309	.0274
160	14	.0805	.0528	.0391	.0312	.0261	.0226	.0201
	16	.0620	.0407	.0302	.0240	.0201	.0174	.0155
	18	.0497	.0326	.0242	.0193	.0161	.0140	.0124
	20	.0410	.0269	.0200	.0159	.0133	.0115	.0102

TABLE II

THE CONSTANT SHEAR FLEXIBILITY PARAMETER μ FOR BATTENED STRUCTURAL MEMBERS. THE LONGITUDINAL AND BATTEN ELEMENTS ARE WF SHAPES AND CHANNELS RESPECTIVELY.
THE VALUES OF μ FOR η_c=2.0, η_b=1.5, ξ_a=.85, Z=0

(6) r_b/r_c = 1.5, A_c/A_b = 1.0

ℓ/r_c	ℓ/b \ ℓ/a =	6	8	10	12	14	16	18
	6	.2449	.1794	.1440	.1220	.1071	.0963	.0882
	8	.1373	.1005	.0807	.0684	.0600	.0540	.0495
	10	.0914	.0670	.0538	.0455	.0400	.0360	.0329
	12	.0676	.0495	.0398	.0337	.0295	.0265	.0243
80	14	.0535	.0393	.0315	.0267	.0234	.0210	.0192
	16	.0445	.0327	.0262	.0222	.0194	.0174	.0159
	18	.0383	.0282	.0226	.0191	.0167	.0150	.0136
	20	.0340	.0250	.0201	.0169	.0148	.0132	.0120
	6	.3312	.2374	.1871	.1561	.1352	.1202	.1089
	8	.1781	.1274	.1003	.0837	.0725	.0645	.0585
	10	.1144	.0817	.0643	.0537	.0465	.0414	.0376
	12	.0818	.0584	.0460	.0384	.0333	.0296	.0269
100	14	.0628	.0450	.0354	.0296	.0256	.0228	.0206
	16	.0508	.0364	.0287	.0239	.0207	.0184	.0167
	18	.0426	.0306	.0241	.0201	.0174	.0155	.0140
	20	.0368	.0265	.0209	.0174	.0151	.0134	.0121
	6	.4371	.3087	.2402	.1982	.1699	.1496	.1345
	8	.2287	.1606	.1247	.1028	.0881	.0777	.0698
	10	.1430	.1003	.0777	.0640	.0549	.0484	.0436
	12	.0999	.0700	.0542	.0447	.0383	.0338	.0304
120	14	.0750	.0526	.0408	.0336	.0288	.0254	.0229
	16	.0594	.0417	.0324	.0267	.0229	.0202	.0181
	18	.0489	.0344	.0267	.0220	.0189	.0166	.0150
	20	.0415	.0292	.0227	.0187	.0161	.0142	.0127
	6	.5625	.3931	.3031	.2480	.2110	.1846	.1648
	8	.2886	.2002	.1537	.1255	.1067	.0933	.0834
	10	.1772	.1224	.0938	.0765	.0650	.0569	.0508
	12	.1216	.0838	.0642	.0523	.0444	.0389	.0347
140	14	.0899	.0620	.0474	.0386	.0328	.0287	.0257
	16	.0701	.0483	.0370	.0302	.0256	.0224	.0200
	18	.0569	.0393	.0301	.0245	.0208	.0182	.0163
	20	.0476	.0329	.0252	.0206	.0175	.0153	.0137
	6	.7073	.4906	.3758	.3056	.2585	.2250	.1999
	8	.3579	.2459	.1873	.1518	.1282	.1115	.0990
	10	.2169	.1481	.1124	.0910	.0767	.0667	.0592
	12	.1469	.1000	.0758	.0612	.0516	.0448	.0398
160	14	.1073	.0729	.0552	.0446	.0376	.0326	.0290
	16	.0826	.0562	.0425	.0343	.0289	.0251	.0223
	18	.0663	.0451	.0341	.0276	.0232	.0202	.0179
	20	.0549	.0373	.0283	.0229	.0193	.0167	.0149

TABLE II

THE CONSTANT SHEAR FLEXIBILITY PARAMETER μ FOR BATTENED STRUCTURAL MEMBERS. THE LONGITUDINAL AND BATTEN ELEMENTS ARE WF SHAPES AND CHANNELS RESPECTIVELY.
THE VALUES OF μ FOR η_c=2.0, η_b=1.5, ξ_a=.85, Z=0

(7) r_b/r_c = 1.5, A_c/A_b = 1.5

ℓ/r_c	ℓ/b \ ℓ/a =	6	8	10	12	14	16	18
	6	.3193	.2352	.1886	.1592	.1390	.1242	.1130
	8	.1785	.1314	.1054	.0890	.0777	.0695	.0632
	10	.1192	.0878	.0704	.0594	.0519	.0464	.0422
	12	.0886	.0653	.0523	.0442	.0385	.0344	.0313
80	14	.0706	.0521	.0417	.0352	.0307	.0274	.0248
	16	.0590	.0436	.0350	.0295	.0256	.0229	.0207
	18	.0512	.0379	.0303	.0256	.0222	.0198	.0179
	20	.0457	.0338	.0271	.0228	.0198	.0176	.0159
	6	.4311	.3123	.2471	.2061	.1780	.1577	.1422
	8	.2299	.1662	.1314	.1096	.0947	.0839	.0758
	10	.1473	.1064	.0841	.0702	.0606	.0538	.0485
	12	.1055	.0763	.0603	.0503	.0435	.0385	.0348
100	14	.0814	.0589	.0466	.0388	.0336	.0297	.0268
	16	.0661	.0479	.0379	.0316	.0273	.0242	.0218
	18	.0558	.0405	.0320	.0267	.0231	.0204	.0184
	20	.0485	.0352	.0279	.0233	.0201	.0178	.0160
	6	.5684	.4071	.3190	.2638	.2261	.1989	.1782
	8	.2935	.2093	.1636	.1352	.1159	.1020	.0915
	10	.1826	.1299	.1015	.0838	.0719	.0632	.0567
	12	.1273	.0906	.0707	.0584	.0501	.0441	.0395
120	14	.0958	.0682	.0533	.0440	.0377	.0332	.0298
	16	.0761	.0542	.0424	.0350	.0300	.0264	.0237
	18	.0629	.0449	.0351	.0290	.0249	.0219	.0196
	20	.0536	.0383	.0300	.0248	.0213	.0187	.0168
	6	.7309	.5193	.4041	.3322	.2832	.2477	.2209
	8	.3690	.2605	.2020	.1657	.1411	.1235	.1102
	10	.2247	.1580	.1223	.1002	.0853	.0746	.0666
	12	.1536	.1078	.0833	.0683	.0581	.0509	.0454
140	14	.1134	.0796	.0615	.0504	.0429	.0375	.0335
	16	.0885	.0622	.0481	.0394	.0335	.0293	.0262
	18	.0720	.0506	.0392	.0321	.0273	.0239	.0213
	20	.0605	.0426	.0329	.0270	.0230	.0201	.0180
	6	.9185	.6489	.5025	.4111	.3490	.3042	.2703
	8	.4564	.3197	.2463	.2010	.1704	.1484	.1318
	10	.2735	.1906	.1464	.1193	.1010	.0879	.0781
	12	.1841	.1279	.0981	.0798	.0676	.0588	.0522
160	14	.1341	.0930	.0713	.0580	.0491	.0427	.0379
	16	.1032	.0716	.0549	.0446	.0378	.0329	.0292
	18	.0829	.0575	.0441	.0359	.0303	.0264	.0235
	20	.0687	.0478	.0366	.0298	.0252	.0219	.0195

TABLE II

THE CONSTANT SHEAR FLEXIBILITY PARAMETER μ FOR BATTENED STRUCTURAL MEMBERS. THE LONGITUDINAL AND BATTEN ELEMENTS ARE WF SHAPES AND CHANNELS RESPECTIVELY.
THE VALUES OF μ FOR η_c=2.0, η_b=1.5, ξ_a=.85, Z=0

(8) r_b/r_c = 1.5, A_c/A_b = 2.0

ℓ/r_c	ℓ/b \ ℓ/a =	6	8	10	12	14	16	18
80	6	.3938	.2911	.2333	.1964	.1709	.1521	.1378
	8	.2197	.1623	.1301	.1096	.0954	.0849	.0770
	10	.1469	.1086	.0871	.0733	.0638	.0568	.0514
	12	.1095	.0810	.0649	.0546	.0475	.0423	.0383
	14	.0876	.0649	.0520	.0437	.0380	.0338	.0305
	16	.0736	.0545	.0437	.0367	.0319	.0283	.0256
	18	.0641	.0475	.0381	.0320	.0278	.0246	.0222
	20	.0574	.0425	.0341	.0286	.0248	.0220	.0198
100	6	.5311	.3873	.3071	.2561	.2209	.1951	.1755
	8	.2816	.2050	.1624	.1354	.1169	.1033	.0930
	10	.1803	.1311	.1039	.0866	.0748	.0661	.0595
	12	.1293	.0941	.0746	.0622	.0537	.0474	.0427
	14	.1000	.0728	.0577	.0481	.0415	.0367	.0330
	16	.0815	.0594	.0471	.0393	.0339	.0299	.0269
	18	.0690	.0504	.0400	.0333	.0287	.0254	.0228
	20	.0602	.0440	.0349	.0291	.0251	.0221	.0199
120	6	.6996	.5056	.3977	.3294	.2824	.2481	.2220
	8	.3584	.2579	.2025	.1676	.1437	.1263	.1131
	10	.2221	.1596	.1252	.1036	.0888	.0781	.0699
	12	.1548	.1111	.0872	.0721	.0618	.0544	.0487
	14	.1166	.0838	.0657	.0544	.0466	.0410	.0367
	16	.0928	.0667	.0524	.0434	.0372	.0327	.0293
	18	.0769	.0554	.0435	.0360	.0309	.0271	.0243
	20	.0657	.0474	.0373	.0309	.0264	.0232	.0208
140	6	.8992	.6456	.5051	.4163	.3553	.3108	.2770
	8	.4495	.3208	.2502	.2059	.1756	.1536	.1370
	10	.2721	.1936	.1507	.1239	.1057	.0924	.0824
	12	.1855	.1318	.1025	.0843	.0718	.0628	.0560
	14	.1370	.0973	.0757	.0622	.0530	.0464	.0414
	16	.1070	.0760	.0591	.0486	.0414	.0363	.0323
	18	.0872	.0620	.0483	.0397	.0338	.0296	.0264
	20	.0733	.0522	.0407	.0335	.0285	.0250	.0223
160	6	1.1297	.8073	.6292	.5167	.4395	.3834	.3407
	8	.5548	.3935	.3054	.2502	.2126	.1853	.1647
	10	.3301	.2331	.1804	.1476	.1253	.1092	.0970
	12	.2213	.1559	.1204	.0984	.0835	.0728	.0647
	14	.1609	.1131	.0874	.0714	.0605	.0527	.0469
	16	.1238	.0871	.0672	.0549	.0466	.0406	.0361
	18	.0995	.0700	.0540	.0442	.0375	.0326	.0290
	20	.0826	.0582	.0449	.0367	.0312	.0272	.0241

TABLE II

THE CONSTANT SHEAR FLEXIBILITY PARAMETER μ FOR BATTENED STRUCTURAL MEMBERS. THE LONGITUDINAL AND BATTEN ELEMENTS ARE WF SHAPES AND CHANNELS RESPECTIVELY.
THE VALUES OF μ FOR $\eta_c=2.0$, $\eta_b=1.5$, $\xi_a=.85$, Z=0

(9) $r_b/r_c = 2.0$, $A_c/A_b = .5$

ℓ/r_c	ℓ/a =	6	8	10	12	14	16	18
	ℓ/b							
	6	.1500	.1082	.0870	.0745	.0664	.0607	.0566
	8	.0873	.0631	.0507	.0434	.0386	.0353	.0329
	10	.0591	.0427	.0343	.0294	.0261	.0238	.0221
	12	.0438	.0317	.0255	.0218	.0194	.0176	.0164
80	14	.0347	.0251	.0202	.0172	.0153	.0139	.0129
	16	.0287	.0208	.0167	.0143	.0126	.0115	.0106
	18	.0245	.0179	.0143	.0122	.0108	.0098	.0090
	20	.0216	.0157	.0126	.0107	.0095	.0086	.0079
	6	.1995	.1386	.1081	.0903	.0788	.0708	.0650
	8	.1129	.0784	.0611	.0511	.0445	.0400	.0368
	10	.0744	.0517	.0404	.0337	.0294	.0264	.0242
	12	.0539	.0375	.0293	.0245	.0213	.0191	.0176
100	14	.0416	.0290	.0227	.0189	.0165	.0148	.0136
	16	.0336	.0235	.0184	.0154	.0134	.0120	.0110
	18	.0281	.0197	.0154	.0129	.0112	.0100	.0092
	20	.0242	.0170	.0133	.0111	.0097	.0087	.0079
	6	.2604	.1761	.1342	.1098	.0941	.0834	.0756
	8	.1445	.0975	.0742	.0607	.0520	.0461	.0418
	10	.0925	.0631	.0480	.0393	.0337	.0298	.0271
	12	.0665	.0450	.0343	.0280	.0240	.0213	.0193
120	14	.0505	.0342	.0261	.0213	.0183	.0162	.0147
	16	.0402	.0273	.0208	.0171	.0146	.0129	.0117
	18	.0331	.0225	.0172	.0141	.0121	.0107	.0097
	20	.0280	.0191	.0146	.0120	.0103	.0091	.0082
	6	.3325	.2205	.1651	.1330	.1124	.0983	.0881
	8	.1820	.1202	.0897	.0722	.0610	.0533	.0478
	10	.1163	.0767	.0572	.0460	.0389	.0340	.0305
	12	.0818	.0540	.0403	.0324	.0274	.0239	.0215
140	14	.0613	.0405	.0303	.0244	.0206	.0180	.0161
	16	.0482	.0319	.0239	.0192	.0163	.0142	.0127
	18	.0393	.0261	.0195	.0157	.0133	.0116	.0104
	20	.0329	.0219	.0164	.0132	.0112	.0098	.0088
	6	.4157	.2718	.2008	.1598	.1336	.1156	.1027
	8	.2254	.1465	.1077	.0855	.0714	.0618	.0548
	10	.1427	.0925	.0679	.0539	.0449	.0389	.0345
	12	.0994	.0644	.0473	.0375	.0313	.0271	.0240
160	14	.0740	.0480	.0352	.0279	.0233	.0202	.0179
	16	.0577	.0374	.0275	.0218	.0182	.0158	.0140
	18	.0466	.0303	.0223	.0177	.0148	.0128	.0114
	20	.0387	.0252	.0186	.0148	.0123	.0107	.0095

TABLE II

THE CONSTANT SHEAR FLEXIBILITY PARAMETER μ FOR BATTENED STRUCTURAL MEMBERS. THE LONGITUDINAL AND BATTEN ELEMENTS ARE WF SHAPES AND CHANNELS RESPECTIVELY.
THE VALUES OF μ FOR η_c=2.0, η_b=1.5, ξ_a=.85, Z=0

(10) r_b/r_c = 2.0, A_c/A_b = 1.0

ℓ/r_c	ℓ/b \ ℓ/a =	6	8	10	12	14	16	18
	6	.2040	.1487	.1194	.1015	.0895	.0810	.0746
	8	.1197	.0874	.0702	.0596	.0525	.0475	.0437
	10	.0822	.0601	.0482	.0410	.0360	.0325	.0299
	12	.0621	.0454	.0365	.0309	.0272	.0245	.0225
80	14	.0500	.0366	.0294	.0249	.0218	.0196	.0180
	16	.0420	.0308	.0247	.0209	.0183	.0165	.0151
	18	.0365	.0269	.0215	.0182	.0159	.0143	.0130
	20	.0326	.0240	.0192	.0163	.0142	.0127	.0116
	6	.2678	.1898	.1491	.1244	.1080	.0964	.0878
	8	.1511	.1071	.0841	.0702	.0609	.0544	.0495
	10	.1003	.0712	.0559	.0467	.0405	.0361	.0329
	12	.0735	.0522	.0411	.0343	.0297	.0265	.0241
100	14	.0575	.0410	.0322	.0269	.0233	.0208	.0189
	16	.0471	.0337	.0265	.0221	.0192	.0171	.0155
	18	.0400	.0286	.0226	.0188	.0163	.0145	.0131
	20	.0349	.0250	.0197	.0165	.0143	.0127	.0115
	6	.3462	.2405	.1857	.1527	.1309	.1155	.1042
	8	.1900	.1316	.1015	.0834	.0715	.0632	.0570
	10	.1231	.0853	.0658	.0541	.0463	.0409	.0369
	12	.0882	.0612	.0472	.0388	.0333	.0294	.0265
120	14	.0676	.0470	.0363	.0299	.0256	.0226	.0204
	16	.0543	.0379	.0293	.0241	.0207	.0182	.0164
	18	.0453	.0317	.0245	.0202	.0173	.0153	.0137
	20	.0388	.0272	.0211	.0174	.0149	.0131	.0118
	6	.4391	.3005	.2290	.1863	.1581	.1383	.1237
	8	.2362	.1609	.1223	.0993	.0842	.0737	.0659
	10	.1502	.1022	.0776	.0630	.0534	.0467	.0418
	12	.1058	.0720	.0547	.0444	.0377	.0330	.0295
140	14	.0799	.0544	.0414	.0336	.0285	.0250	.0223
	16	.0633	.0432	.0329	.0268	.0227	.0199	.0178
	18	.0520	.0356	.0272	.0221	.0188	.0164	.0147
	20	.0440	.0302	.0231	.0188	.0160	.0140	.0125
	6	.5464	.3699	.2792	.2251	.1896	.1646	.1462
	8	.2897	.1947	.1464	.1177	.0990	.0859	.0763
	10	.1818	.1218	.0914	.0734	.0617	.0535	.0475
	12	.1264	.0847	.0635	.0510	.0429	.0372	.0330
160	14	.0943	.0632	.0474	.0381	.0320	.0278	.0247
	16	.0739	.0496	.0372	.0299	.0252	.0218	.0194
	18	.0601	.0404	.0304	.0244	.0206	.0178	.0159
	20	.0503	.0339	.0255	.0206	.0173	.0150	.0133

TABLE II

THE CONSTANT SHEAR FLEXIBILITY PARAMETER μ FOR BATTENED STRUCTURAL MEMBERS. THE LONGITUDINAL AND BATTEN ELEMENTS ARE WF SHAPES AND CHANNELS RESPECTIVELY.
THE VALUES OF μ FOR η_c=2.0, η_b=1.5, ξ_a=.85, Z=0

(11) r_b/r_c = 2.0, A_c/A_b = 1.5

ℓ/r_c	ℓ/a = / ℓ/b	6	8	10	12	14	16	18
	6	.2580	.1892	.1518	.1285	.1127	.1012	.0926
	8	.1521	.1117	.0896	.0758	.0664	.0596	.0545
	10	.1054	.0775	.0621	.0525	.0460	.0412	.0376
	12	.0804	.0591	.0474	.0401	.0350	.0313	.0285
80	14	.0653	.0481	.0386	.0325	.0284	.0254	.0231
	16	.0554	.0409	.0328	.0276	.0241	.0215	.0195
	18	.0485	.0359	.0287	.0242	.0211	.0188	.0170
	20	.0436	.0323	.0259	.0218	.0189	.0169	.0153
	6	.3360	.2410	.1900	.1585	.1373	.1220	.1105
	8	.1893	.1357	.1070	.0893	.0773	.0687	.0623
	10	.1262	.0906	.0715	.0596	.0516	.0459	.0415
	12	.0931	.0670	.0529	.0441	.0382	.0339	.0307
	14	.0734	.0529	.0418	.0349	.0302	.0267	.0242
	16	.0607	.0438	.0346	.0289	.0250	.0221	.0200
	18	.0519	.0376	.0297	.0248	.0214	.0190	.0171
	20	.0456	.0330	.0261	.0218	.0188	.0167	.0150
	6	.4320	.3048	.2371	.1956	.1677	.1477	.1328
	8	.2355	.1658	.1288	.1062	.0910	.0802	.0721
	10	.1526	.1074	.0835	.0688	.0590	.0520	.0467
	12	.1098	.0774	.0602	.0496	.0425	.0375	.0337
120	14	.0846	.0598	.0465	.0384	.0329	.0290	.0260
	16	.0685	.0485	.0378	.0312	.0267	.0236	.0211
	18	.0575	.0408	.0318	.0263	.0225	.0198	.0178
	20	.0496	.0353	.0276	.0228	.0195	.0172	.0154
	6	.5457	.3805	.2930	.2396	.2038	.1783	.1592
	8	.2905	.2016	.1548	.1264	.1075	.0940	.0840
	10	.1842	.1276	.0980	.0800	.0680	.0595	.0531
	12	.1299	.0901	.0692	.0565	.0480	.0420	.0375
140	14	.0984	.0683	.0525	.0429	.0365	.0319	.0285
	16	.0783	.0545	.0420	.0343	.0292	.0255	.0228
	18	.0648	.0452	.0348	.0285	.0242	.0212	.0189
	20	.0551	.0385	.0297	.0243	.0207	.0181	.0162
	6	.6771	.4679	.3576	.2904	.2456	.2136	.1898
	8	.3541	.2430	.1850	.1499	.1265	.1100	.0977
	10	.2208	.1511	.1148	.0929	.0784	.0682	.0606
	12	.1534	.1049	.0797	.0645	.0544	.0473	.0420
160	14	.1146	.0784	.0596	.0482	.0407	.0354	.0314
	16	.0901	.0617	.0470	.0380	.0321	.0279	.0248
	18	.0735	.0505	.0385	.0312	.0263	.0229	.0203
	20	.0619	.0426	.0325	.0264	.0223	.0194	.0172

TABLE II

THE CONSTANT SHEAR FLEXIBILITY PARAMETER μ FOR BATTENED STRUCTURAL MEMBERS. THE LONGITUDINAL AND BATTEN ELEMENTS ARE WF SHAPES AND CHANNELS RESPECTIVELY.
THE VALUES OF μ FOR η_c=2.0, η_b=1.5, ξ_a=.85, Z=0

(12) r_b/r_c = 2.0, A_c/A_b = 2.0

ℓ/r_c	ℓ/a = / ℓ/b	6	8	10	12	14	16	18
	6	.3120	.2297	.1842	.1555	.1358	.1215	.1106
	8	.1846	.1360	.1090	.0920	.0803	.0718	.0653
	10	.1286	.0948	.0760	.0641	.0559	.0499	.0453
	12	.0986	.0728	.0584	.0492	.0428	.0382	.0346
80	14	.0806	.0596	.0477	.0402	.0350	.0311	.0282
	16	.0687	.0509	.0408	.0343	.0298	.0265	.0240
	18	.0605	.0449	.0359	.0302	.0262	.0233	.0210
	20	.0547	.0405	.0325	.0273	.0237	.0210	.0189
	6	.4042	.2922	.2310	.1927	.1665	.1476	.1332
	8	.2275	.1644	.1299	.1084	.0937	.0830	.0750
	10	.1522	.1101	.0870	.0726	.0627	.0556	.0502
	12	.1128	.0817	.0646	.0539	.0466	.0412	.0372
100	14	.0893	.0649	.0513	.0428	.0370	.0327	.0295
	16	.0742	.0540	.0427	.0356	.0308	.0272	.0245
	18	.0638	.0465	.0368	.0307	.0265	.0234	.0211
	20	.0562	.0411	.0326	.0271	.0234	.0207	.0186
	6	.5178	.3692	.2886	.2385	.2045	.1799	.1614
	8	.2810	.1999	.1561	.1290	.1106	.0973	.0873
	10	.1822	.1296	.1012	.0836	.0717	.0631	.0566
	12	.1314	.0936	.0731	.0604	.0518	.0456	.0409
	14	.1016	.0726	.0568	.0469	.0402	.0354	.0317
	16	.0826	.0591	.0463	.0383	.0328	.0289	.0259
	18	.0696	.0499	.0391	.0324	.0278	.0244	.0219
	20	.0603	.0434	.0340	.0282	.0241	.0212	.0190
	6	.6524	.4604	.3570	.2929	.2495	.2183	.1947
	8	.3447	.2422	.1874	.1535	.1307	.1144	.1021
	10	.2181	.1531	.1183	.0969	.0825	.0722	.0644
	12	.1540	.1081	.0836	.0685	.0583	.0510	.0455
140	14	.1169	.0822	.0636	.0521	.0444	.0388	.0347
	16	.0934	.0658	.0510	.0418	.0356	.0312	.0278
	18	.0775	.0547	.0425	.0348	.0297	.0260	.0232
	20	.0662	.0468	.0364	.0299	.0255	.0223	.0199
	6	.8078	.5659	.4360	.3558	.3016	.2626	.2334
	8	.4184	.2913	.2236	.1820	.1541	.1342	.1192
	10	.2599	.1804	.1383	.1125	.0952	.0828	.0736
	12	.1804	.1252	.0959	.0780	.0660	.0574	.0510
160	14	.1349	.0936	.0718	.0584	.0494	.0430	.0382
	16	.1063	.0739	.0567	.0461	.0391	.0340	.0302
	18	.0870	.0606	.0466	.0379	.0321	.0280	.0248
	20	.0734	.0513	.0394	.0321	.0272	.0237	.0211

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This dissertation presents a theoretical analysis of the elastic behavior of both the battened and laced structural member considering the effects of axial load, shear deformation, and connection rigidity of sub-elements, and the over-all effects of axial load, and effective shear deformation of the complete member. The analytical solution is used to obtain modified slope-deflection equations to generalize a relation between applied forces and joint displacements. A nondimensional parameter, the shear flexibility, is defined so as to characterize the shear flexibility of the members and to take account of the effects of axial force, local joint connections, and local connection flexibility of the battened members. The fundamental linear second-order differential equation for the deflection curve of the member which includes the effect of shear deformation has been derived. By application of the boundary conditions to the general solution of deflected shape of the member, the solutions are set up in the forms of slope-deflection equations. In the evaluation of the fixed end moments for a concentrated load, the reciprocal theorem is applied so as to make use of the deflection curves which have been previously defined in the case of the homogeneous solution. Elastic buckling loads for the structural members have been evaluated for the cases of a column with hinged ends, a column with one end fixed and the other hinged, and a column with both ends fixed. Finally, numerical examples of beam and frame analyses are presented to provide a comparison with the ordinary beam and frame theories which neglect the effects of shear deformation and axial force.

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