CASE STUDIES IN PROCESS CAPABILITY MEASUREMENT

Steven J. Littig
and
C. Teresa Lam
Department of Industrial and Operations Engineering
The University of Michigan
Ann Arbor, MI 48109-2117

Technical Report 93-5

January 1993

CASE STUDIES IN PROCESS CAPABILITY MEASUREMENT

Steven J. Littig
Research Assistant

C. Teresa Lam
Assistant Professor

Industrial and Operations Engineering Department
University of Michigan
Ann Arbor, MI 48109

SUMMARY

Throughout the automotive industry, the operational definition of quality has taken the form of process capability indices. These indices are fairly well defined for the standard case of a univariate process characteristic with symmetric bilateral tolerances. However, for other nonstandard tolerancing situations process capability indices are poorly defined and potentially misleading measures of process quality. In this paper, we describe methods for computing process capability indices for three nonstandard design tolerances: unilateral tolerances, nonsymmetric bilateral tolerances, and bivariate circular tolerances. Examples will be given to show that these new indices consistently convey the ability of a process to meet design specifications.

INTRODUCTION

Process capability indices have become widely used in American and Japanese industries to quantify process quality. In fact both Ford Motor Company, in its Quality System Standard Q-101 Manual, and General Motors Corporation, in its Manufacturing Systems Qualification Manual GM 1735, require computation and documentation of process quality for all manufacturing processes throughout their organizations.

In Ford's Q-101 Manual, they specifically outline the corporate requirements regarding process capability. For example, while process capability "does not apply to processes evaluated using attributes data", it "does apply to all critical and significant characteristics evaluated using variables data". Furthermore, the following specific numerical requirements for C_p and C_{pk} are detailed in Q-101. Ford's requirement for process capability is that C_p and C_{pk} equal at least 1.33 and that the producer pursue process improvement to increase capability. Based on the actual C_p and C_{pk}, the following responses are required: For unstable processes and those with a C_p or C_{pk} less than 1.0, **containment actions** (typically 100% inspection or testing) **must be implemented immediately and sustained**. For processes with C_p and C_{pk} levels above 1.0 but less than 1.33, containment is required unless otherwise stated on an approved control plan. Process improvement to reach 1.33 is mandatory. Therefore, both C_p and C_{pk} must be at least 1.0 for "all characteristics using variables data" and attempts to improve them to 1.33 or greater are mandated by corporate decree. This perception that 1.0 is marginally acceptable and 1.33 or better must be achieved for all processes is very much ingrained in the minds of managers and engineers throughout the organization.

The goals of 1.0 and 1.33 were computed from the case of a normally distributed univariate process characteristic toleranced with symmetric bilateral tolerances. ANSI (1982) defined a nonconforming unit as any unit that falls outside the tolerance zone. Since C_p is a one-to-one function of the potential proportion nonconforming, each C_p value can be translated to a specific defect rate (Boyles (1991)). For example a C_p of 1.0 indicates a potential proportion nonconforming of 2700 parts per million (ppm). However, C_{pk} is not a one-to-one function of the actual proportion nonconforming, but it does provide an upper and lower bound on the actual proportion nonconforming (Boyles 1991). This fact is fairly surprising to many engineers and managers since C_{pk} is usually treated as a direct function of the actual proportion nonconforming.

Since C_{pk} has been shown to be an inconsistent measure of the actual proportion nonconforming, the goals of 1.0 and 1.33 are somewhat arbitrary for C_{pk} even in the standard symmetric bilateral tolerance situation. When you begin to apply that goal to other "critical and significant characteristics" that are not the standard symmetric univariate case, the interpretation becomes even more arbitrary. In order to solve this problem, Lam and Littig (1992) have recommended the direct use of the actual and potential proportion nonconforming as process performance measures. However, to satisfy common corporate requirements, these values will generally have to be expressed in terms of capability indices. In particular, if p^* and p are the potential and actual proportion nonconforming of a process, they define

$$C_{p^*} = \frac{1}{3}\Phi^{-1}\left(1 - \frac{p^*}{2}\right) \qquad \text{and} \qquad C_{pp} = \frac{1}{3}\Phi^{-1}\left(1 - \frac{p}{2}\right) \tag{1}$$

where $\Phi(x)$ is the integral of a standardized normal density from minus infinity to any real number x. The definition of C_{p^*} is chosen such that $C_{p^*} = C_p$ for the standard case of a univariate normal distribution with a symmetric bilateral tolerance. They recommend that C_{p^*}, C_{pp} and the k (Kane (1986)) indices should be reported in order to consistently, and effectively, communicate the ability of a process to meet design tolerances.

In this paper we will show that by using C_{pp} in place of C_{pk}, especially for nonstandard tolerance situations, interpretation and logical decision making will be improved. In this paper we consider three examples: the case of unilateral tolerances (used to measure flatness and surface finish for example), nonsymmetric bilateral tolerances (often used to tolerance assembly fit characteristics), and bivariate circular tolerances (used for hole location and two dimensional characteristics).

TEXT

In this section, the three nonstandard tolerance cases will be discussed. For each case we will first discuss the specific tolerance and why it is used. Capability indices currently being used for these situations will then be presented. A simple example will demonstrate why this definition is ineffective and misleading. Finally we will discuss a new definition to measure capability in these cases and demonstrate the logical interpretation by applying the new definition to the previous example.

Unilateral Tolerances

A common nonstandard case is the situation of a process characteristic toleranced with a unilateral tolerance. Generally in this situation, the nominal target value is set at zero and a single upper specification limit is defined. By the ANSI definition, any characteristic that lies above the upper limit (USL) is considered to be nonfunctional.

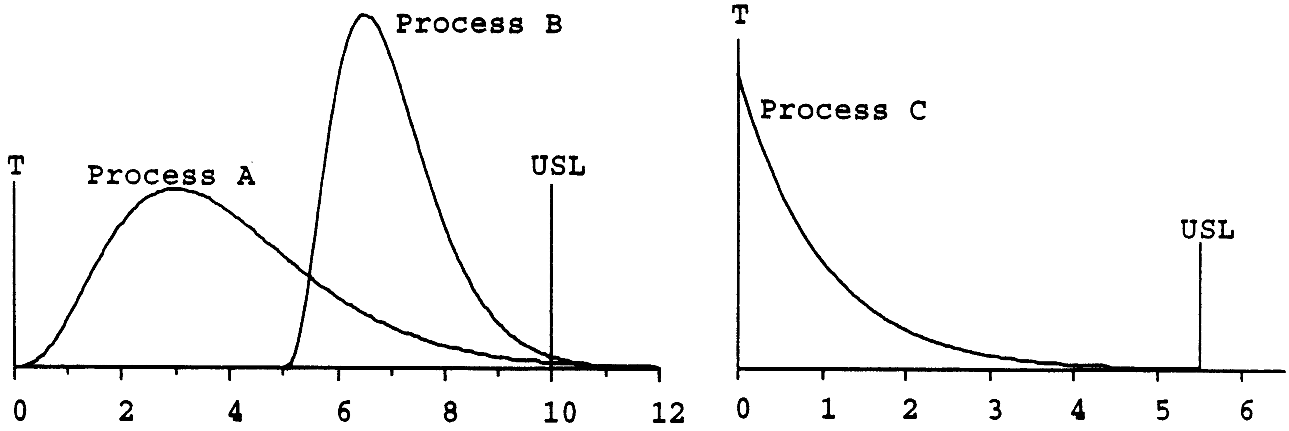

Figure 1: A comparison between 3 different processes over a unilateral tolerance

Currently for unilateral tolerances, Q-101 states that C_p does not exist since the process mean cannot be centered at the target. This is due to the fact that if the mean of a normal distribution were centered at the target value of zero, some process characteristics would have negative values. Since this is physically impossible C_p is not reported. Therefore, C_{pk} is the only index that is reported in this situation and it is defined identically to the CPU index (Kane (1986)) for the bilateral specification case with the target value centered between the specification limits.

The problem with defining process capability for unilateral tolerances in this manner stems largely from the fact that the underlying process distribution is nonnormal and usually skewed to the right. Since the traditional capability indices implicitly assume a normally distributed process, calculation of the indices when normality is clearly not present can lead to misleading and incomplete results.

Consider the two processes presented in Figure 1. Process A follows a three parameter gamma distribution (Johnson and Kotz (1970)) with shape parameter $\alpha = 4$, scale parameter $\beta = 1$, and a minimum value $\gamma = 0$, and process B follows a gamma distribution with $\alpha = 4$, $\beta = 1/2$, and $\gamma = 5$. Since C_p is not reported for unilateral tolerances, and they both have a C_{pk} value of 1.0, an engineer would logically assume that the two processes are identical and equally capable. However, it can be clearly seen that Process B has a distinct location bias and much smaller variance than Process A. Therefore if the location bias for Process B can be corrected, it would be far more potentially capable than Process A. However, this information is never reported using the current system.

Aside from this problem, there is a major discrepancy with the definition of C_{pk} in the unilateral case. We have already shown that many engineers and managers judge the capability of a process by the magnitude of the process capability index. An index value of 1.33 or greater usually indicates that the process is conforming appropriately to the specification limits. However, the unilateral definition of capability can be extremely misleading. Examine process C in Figure 1. This process is exponentially distributed with a C_{pk} value of 1.5 and therefore is presumably capable of meeting the design specifications. However, the defect rate for the process is over 4000 defects per million! Since a capability of 1.5 in the case of a normal distribution with bilateral specifications implies a defect rate of less than 7 units per million, this is a dangerously misleading result.

The basic problem in measuring capability for unilateral specifications is the implied assumption of normality. In fact, a characteristic with a unilateral tolerance is much better modeled by a gamma distribution. Using a three parameter gamma distribution has several primary advantages:

1. The gamma distribution has a distinct minimum value $\gamma \geq 0$. This correlates well with the physical process since the process cannot produce any parts below zero. Furthermore, this minimum does not have to be a zero. For example in Figure 1, Process B is a Gamma Distribution with a minimum value of 5. This minimum value allows us to define a C_{p*} index since we could presumably shift the process so at least one part could be produced at the target value.
2. The gamma distribution is very flexible in shape and form. This allows the distribution to more closely model the sample data. As discussed in Lehrman (1991), exponential and chi-squared distributions (both special cases of gamma distributions) are among the commonly used frequency distributions in modeling data from production.
3. Proportion nonconforming calculations (and thus C_{p*} and C_{pp} values using Equation (1) above) will be far more accurate in reflecting the ability of the process to produce parts which conform to tolerance.

Thus the new proposed method for computing capability for unilateral tolerances is to use a three parameter gamma distribution. Compute the proportion of this distribution that falls outside the tolerance range. This is the actual proportion nonconforming. Compute C_{pp} using Equation (1) above. To compute C_{p*}, shift the gamma distribution so that the minimum value is at the target location. Compute the potential proportion nonconforming and use Equation (1) to compute C_{p*}. Also, it is useful to report the k index and in this case, $k = \gamma/USL$. k here is a measure of the deviation of the minimum value from the target. $k = 0$ means that the process is centered so that at least some parts are produced at the target value.

Applying this method to the previous examples demonstrates the consistency of this approach. In the first example, Process A will have a k value of 0, a C_{p*} and C_{pp} value of 0.85. Process B will have a k value of 0.5, C_{p*} of 1.55 and a C_{pp} of 0.85. It is then clear that while both processes currently have the same C_{pp} value, process B has the potential to greatly outperform Process A. Process C in Figure 1 would have a k value of 0, a C_{p*} and a C_{pp} of 0.96 which is a more accurate representation of the actual conformance of the process to the tolerance.

Nonsymmetric Bilateral Tolerances

Another common situation is that of nonsymmetric design tolerances where the process target is not centered between the upper and lower design specifications. This is generally used for assembly fit process characteristics where deviation in one direction is less acceptable than deviation in the other. The current method for calculating capability in these cases was described by Kane (1986). In particular, he defined

$$k = \frac{|T-\mu|}{\min\{T-LSL, USL-T\}}, \qquad C_p = \min\left\{\frac{USL-T}{3\sigma}, \frac{T-LSL}{3\sigma}\right\}, \tag{2}$$

and

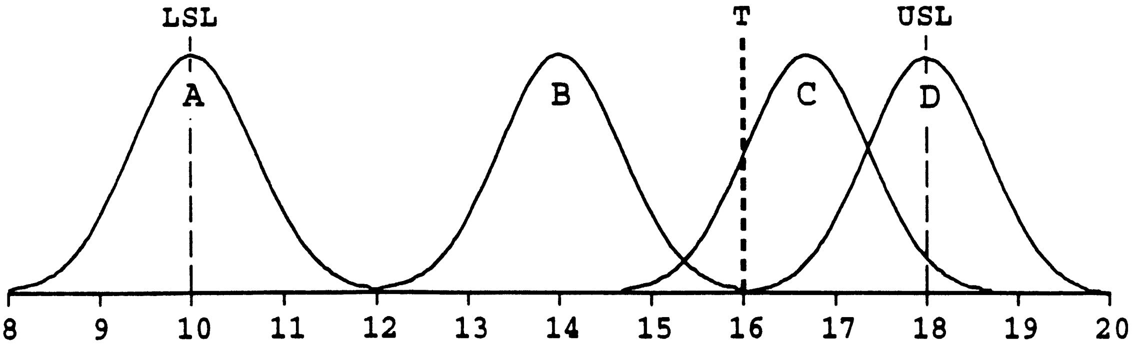

Figure 2: A comparison between 4 different processes over a nonsymmetric tolerance

$$C_{pk} = \max\left\{\min\left\{\frac{T\text{-}LSL}{3\sigma}\left(1-\frac{|T-\mu|}{T\text{-}LSL}\right),\frac{USL\text{-}T}{3\sigma}\left(1-\frac{|T-\mu|}{USL\text{-}T}\right)\right\},0\right\} = \max\{C_p(1-k),0\}$$

where μ and σ are the process mean and process standard deviation respectively. This method has some fairly substantial flaws primarily because it ignores the fact that deviation on one side is more allowable than on the other. The problem with this method is highlighted in Figure 2. Note that all processes in this example have a standard deviation of 0.67. Under the current method for capability computation, processes A, B, and D all have a $C_p = 1.0$ and a $C_{pk} = 0$. Therefore a simple examination of the capability indices by an engineer would indicate that all three processes are identical. This is clearly not the case. Examine A, B, and D by sight and ask yourself which of the three processes is currently best conforming to the tolerance. It is obvious that process B is the best of the three. While the means for both processes B and D are both located 3 standard deviations from the target, process B is clearly preferable to process D since the intent of the design engineer is to penalize deviation toward the lower specification less than deviation toward the upper specification. Since the deviations are penalized equally in both directions in computing C_{pk}, this important fact is ignored.

A factor which makes calculation and interpretation of capability indices difficult for bilateral nonsymmetric tolerances is the fact that the primary intent of the design engineer is not to simply minimize the proportion of nonconforming units. Instead, the engineer wants to minimize proportion nonconforming under the additional constraint that the majority of units be produced near the target value. With this in mind, observe processes B and C which have identical standard deviations. The mean of process B is 2 units below the target value of 16. Since the lower specification limit is set at 10, the deviation of the mean below the target for process A has consumed a third of the allotted lower tolerance. Similarly for process C, we can see that the mean of 16.67 indicates that the deviation of the mean above the target value has consumed a third of the allotted upper tolerance. Although the mean for process B has deviated from the target three times more than the mean for process C, proportionally they have identical deviations since deviation in the lower direction is three times more acceptable than deviation in the upper direction. Therefore, it seems reasonable to assume that both processes B and C are currently equally capable. This can be achieved through the use of the following approach.

Define $d_1=\max\{(T-LSL)/(USL-T),1\}$ and $d_2=\max\{(USL-T)/(T-LSL),1\}$, i.e., d_1 or d_2 compare the allowable deviation below the target and the allowable deviation above the target. Assuming that the underlying process is normally distributed, we define the modified potential proportion nonconforming, p^*, and the modified actual proportion nonconforming p as follows:

$$p^* = \Phi\left(-\frac{USL-T}{d_2\sigma}\right) + \Phi\left(-\frac{T-LSL}{d_1\sigma}\right)$$

and

$$\begin{aligned} p &= \Phi\left(-\frac{USL-\max\{T,\mu\}}{d_2\sigma} - \frac{\max\{T-\mu,0\}}{d_1\sigma}\right) + \Phi\left(-\frac{\min\{T,\mu\}-LSL}{d_1\sigma} - \frac{\max\{\mu-T,0\}}{d_2\sigma}\right) \\ &= \Phi\left(-3C_{p*}(1-k_N)\right) + \Phi\left(-3C_{p*}(1+k_N)\right) \end{aligned}$$

where $k_N = \max\{(T-\mu)/(T-LSL),(\mu-T)/(USL-T)\}$. Note that C_{p*} defined by Equation (1) using the modified potential proportion nonconforming p^* is the same as C_p defined by Kane (1986) (Equation (2) above). Also, the modified potential proportion nonconforming is weighted such that in the centered case ($\mu = T$), the modified proportion nonconforming in the upper and lower tails is the same.

Using these definitions, we recompute the capability for the examples presented in Figure 2. In this case, all indices will still have a $C_{p*} = 1.0$. However, processes A and D now have $C_{pp} = 0.225$ and $k_N = 1$, and processes B and C have a $C_{pp}=0.76$ and $k_N = 0.33$. Therefore, processes B and C are correctly identified as currently being equivalent. Note that the C_{pp} for A and D is not equal to zero using this method since only 50%, and not 100%, of the parts produced do not conform to the tolerance. Note that the k index defined in Kane (1986) is equal to 1 for both processes B and D even though process B is a better process. Hence, the k_N index defined here is a more consistent measure than the k index for deviation of the mean from the target for nonsymmetric tolerances.

Bivariate Characteristics

The third situation we present is the case of a bivariate process characteristic. One example is a location characteristic for the center of a drilled hole. The target in this case will be both an x and y coordinate and the tolerance zone is generally circular or elliptical in nature. Currently, there are two methods for compressing the two dimensional data down to one dimension so that the traditional capability indices C_p and C_{pk} can be applied. **Part center radial deviation** measures the radial deviation from a reference point on the plane outside of the specification zone (for example, the part center) and **true position deviation** measures the absolute deviation from the target location. It is not difficult to see from Figure 3 that even though sample A is preferable to sample B, the current methods would both report an estimated C_p value of 1.0. This shows that compressing the two dimensional data into one dimension results in an inherent loss of information and can produce misleading representations of the true bivariate capability.

The solution to this problem is to compute the potential proportion nonconforming p^* and the actual proportion nonconforming p over the circular or elliptical tolerance zone. In the case when the underlying process is bivariate normal, a computational procedure is given in Littig, Lam and Pollock (1992) to compute p^* and p efficiently. Once p^* and p are available, C_{p*} and C_{pp} can then be computed using Equation (1) above. We can also compute the k index for a bivariate characteristic with process mean (μ_1,μ_2) over a circular tolerance zone centered at (0,0) with radius r. In particular, $k = \sqrt{\mu_1^2+\mu_2^2}/r$. For a more detailed discussion of computing process capability for bivariate and mul-

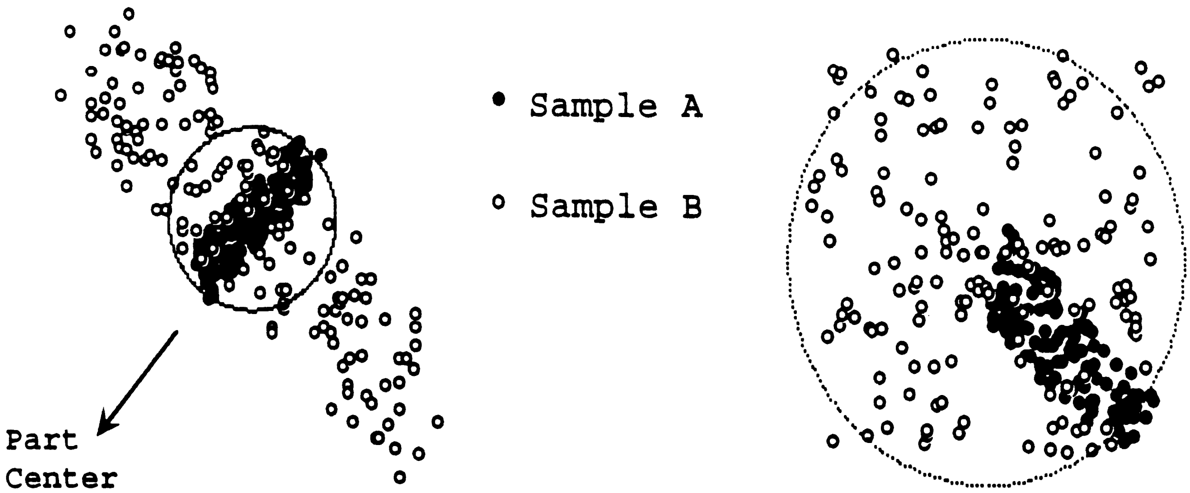

Figure 3: Current methods to compute process capability for a bivariate characteristic over a circular tolerance zone

tivariate characteristics over complex tolerance regions, see Littig, Lam and Pollock (1992).

CONCLUSION

In this paper we have presented process capability indices for three non-standard tolerance situations. The overall intent of our research is to develop a comprehensive and complementary set of capability measurements for all possible tolerancing situations. Since this method relies on a computer to calculate the indices, software has been developed and implemented for this purpose. Research continues on extending this approach to other cases and determining the statistical properties of the estimators presented here.

BIBLIOGRAPHY

ANSI Standard Y14.5M. Dimensioning and Tolerancing, ASME, 1982.

Boyles, R. A., "The Taguchi Capability Index", Journal of Quality Technology, Volume 23, page 17-26, 1991.

Ford Worldwide Quality System Standard Q101, Ford Motor Company, 1990

Johnson, N. L. and Kotz, S., Continuous Univariate Distributions - 1, Wiley-Interscience, 1970.

Kane, V. E., "Process Capability Indices", Journal of Quality Technology, Volume 18, page 41-52, 1986.

Lam C. T. and Littig, S. J., "A New Standard in Process Capability Analysis", Technical Report 92-23, University of Michigan.

Lehrman, K. -H., "Capability Analyses and Index Interval Estimates: Quantitative Characteristics (Variables)", Quality Engineering, Volume 4, Number 1, page 93-130, 1991-1992.

Littig, S. J., Lam, C. T. and Pollock, S. M., "Process Capability Measurements for a Bivariate Characteristics over an Elliptical Tolerance Zone", Technical Report 92-42, University of Michigan.