I. INTRODUCTION

Solution of complicated boundary value problems in the spectral domain is a very well known technique. Several years ago Itoh and Mittra used spectral analysis to solve planar geometry problems occurring in microwave integrated circuits, [1]. There have been numerous papers subsequently published utilizing this method, [2]. In this paper, Fourier analysis is used to formulate the solution for reflected and transmitted surface current densities in back to back shielded microstrip structures coupled by a transverse slot in their common wall. By applying Galerkin's procedure in the spectral domain on complementary field quantities, a solvable system of equations containing the desired reflection and transmission coefficient results.

II. THEORY

Application of the Galerkin method on the spectral domain electromagnetic quantities supported by the structure illustrated in Figure 1 requires a matrix equation which expresses electric and magnetic fields in terms of complement source densities. The appropriate expression is given in equation (1).

$$\begin{bmatrix} \tilde{E}_z^{(1a)}(\alpha, h_a, \beta) \\ \tilde{H}_z^{(1a)}(\alpha, 0, \beta) \\ \tilde{E}_z^{(1b)}(\alpha, -h_b, \beta) \end{bmatrix} = \begin{bmatrix} \tilde{G}_{11}(\alpha,\beta) & \tilde{G}_{12}(\alpha,\beta) & \tilde{G}_{13}(\alpha,\beta) \\ \tilde{G}_{21}(\alpha,\beta) & \tilde{G}_{22}(\alpha,\beta) & \tilde{G}_{23}(\alpha,\beta) \\ \tilde{G}_{31}(\alpha,\beta) & \tilde{G}_{32}(\alpha,\beta) & \tilde{G}_{33}(\alpha,\beta) \end{bmatrix} \begin{bmatrix} \tilde{K}_{z_a}(\alpha, h_a, \beta) \\ \tilde{M}_{x_a}(\alpha, 0, \beta) \\ \tilde{K}_{z_b}(\alpha, -h_b, \beta) \end{bmatrix} \tag{1}$$

Explicit expressions for the $\tilde{G}_{ij}(\alpha,\beta)$, along with an outline of their derivation are given in Appendix A. In this derivation, we have assumed narrow microstrip lines and a narrow coupling slot. These acceptable restrictions allow us to assume uni-directional surface current densities which have a Maxwellian distribution in the direction of their narrow dimension, [3]. Recent theoretical and experimental research by Dunleavy has shown that the narrow microstrip line current variation mentioned above is very accurate, [4]. Unfortunately, no such verification exists for a magnetic current on a narrow aperture. However, since a narrow microstrip line supporting an electric current and a narrow aperture supporting a magnetic current are eseentially dual quantities, [5], it seems that an assumed Maxwellian distribution for the aperture problem would be accurate.

III. SPECTRAL DOMAIN REPRESENTATION OF CURRENT DENSITIES

The detailed geometry of this problem is given in Figure 2. In the spatial domain the two electric surface current densities away from the discontinuities are written as:

$$K_{z_a}(x',h_a,z') = \left(e^{\gamma_{z_a}^{ms}z'} - Re^{-\gamma_{z_a}^{ms}z'}\right) g_a(x') \qquad z'\geq z_o$$

$$K_{z_b}(x',-h_b,z') = Te^{\gamma_{z_b}^{ms}z'} g_b(x') \qquad z'\leq -z_o \tag{2}$$

where

$$g_i(x') = \begin{cases} \dfrac{2}{\pi W_i} \left\{1 - \left[\dfrac{2(x'-x_i)}{W_i}\right]^2\right\}^{-1/2} & -\dfrac{W_i}{2} \le x'-x_i \le \dfrac{W_i}{2} \\ & (i = a,b) \\ 0 & \text{Otherwise} \end{cases} \tag{3}$$

It should be mentioned that $\gamma_{z_a}^{ms}$ and $\gamma_{z_b}^{ms}$ are known constants for their respective guiding structures. Their values may be obtained from the literature, [2]. In the vicinity of the discontinuities, all surface current densities in equation (1) must be expressed in terms of appropriate basis functions.

$K_{z_a}(x',h_a,z')$ and $K_{z_b}(x',-h_b,z')$ are written in the spatial domain as

$$K_{z_i}(x',h_i,z') = \sum_{n=1}^{N} I_{n_i}\, g_i(x')\, f_{n_i}(z') \quad i = a,b \tag{4}$$

Where the I_{n_i} are unknown amplitudes and

$$f_{n_i}(z') = \begin{cases} \dfrac{\sin\left[K(z'-z_n-1)\right]}{\sin(Kl_z)} & z_{n-1} \le z' \le z_n \\ \dfrac{\sin\left[K(z_{n+1}-z')\right]}{\sin(Kl_z)} & z_n \le z' \le z_{n+1} \\ 0 & \text{Otherwise} \end{cases} \tag{5a}$$

For $n \neq 1$

$$f_{1_i}(z') = \begin{cases} \dfrac{\sin\left[K\ (\pm z_o \mp l_z - z')\right]}{\sin(Kl_z)} & \pm z_o \mp l_z \le z' \le \pm z_o \\ 0 & \text{Otherwise} \end{cases} \tag{5b}$$

$$(\pm\mp)\quad i = \begin{pmatrix} a \\ b \end{pmatrix}$$

where $l_z = Z_{n+1} - Z_n = Z_n - Z_{n-1}$. The magnetic current on the coupling aperture is represented as

$$M_{x_a}(x', 0, z') = \sum_{n=1}^{M} V_n\ f_n(x')\ g_o(z') \tag{6}$$

where $f_n(x')$ and $g_o(z')$ are given as equations (7) and (8), respectively, and the V_m are unknown amplitude coefficients.

$$f_n(x') = \begin{cases} \dfrac{\sin\left[K(x' - x_{n-1})\right]}{\sin(Kl_x)} & x_{n-1} \le x' \le x_n \\ \dfrac{\sin\left[K(x_{n+1} - x')\right]}{\sin(Kl_x)} & x_n \le x' \le x_{n+1} \\ 0 & \text{Otherwise} \end{cases} \tag{7}$$

$$g_o(z') = \begin{cases} \dfrac{2}{\pi t}\left\{1 - \left[\dfrac{2z'}{t}\right]^2\right\}^{-1/2} & -\dfrac{t}{2} \le z' \le \dfrac{t}{2} \\ 0 & \text{Otherwise} \end{cases} \tag{8}$$

We also see that $l_x = x_{n+1} - x_n = x_n - x_{n-1}$ and $x_n = (n-1)\, l_x$. Under these conditions the requirement that the magnetic current is non-zero only on the coupling aperture is satisfied. The next step in the analysis is to obtain the Fourier transforms of (4) and (6). The transforms of interest are written as:

$$\tilde{K}_{z_a}(\alpha, h_a, \beta) = \iint_{-\infty}^{\infty} K_{z_a}(x', h_a, z')\, e^{j\alpha x'}\, e^{j\beta z'}\, dx' dz' \tag{9}$$

$$\tilde{K}_{z_b}(\alpha, -h_b, \beta) = \iint_{-\infty}^{\infty} K_{z_b}(x', -h_b, z')\, e^{j\alpha x'}\, e^{j\beta z'}\, dx' dz' \tag{10}$$

$$\tilde{M}_{x_a}(\alpha, 0, \beta) = \iint_{-\infty}^{\infty} M_{x_a}(x', 0, z')\, e^{j\alpha x'}\, e^{j\beta z'}\, dx' dz' \tag{11}$$

For $\tilde{K}_{z_a}(\alpha, h_a, \beta)$ we obtain

$$\tilde{K}_{z_a}(\alpha, h_a, \beta) = \int_{-\infty}^{\infty} g_a(x')\, e^{j\alpha x'}\, dx' \left[\sum_{n=2}^{N} I_{n_a} \int_{z_{n+1}}^{z_{n-1}} f_n(z')\, e^{j\beta z'} dz' + \int_{z_o - l_z}^{z_o} I_{1_a}\, f_1(z')\, e^{j\beta z'}\, dz' + \int_{z_o}^{\infty} \left(e^{\gamma_{z_a}^{ms} z'} - R e^{-\gamma_{z_a}^{ms} z'} \right) e^{j\beta z'}\, dz' \right]$$

$$\tilde{K}_{z_a}(\alpha, h_a, \beta) = I_1^a \left[I_2^a + I_3^a + I_4^a \right] \tag{12}$$

$$I_1^a = \int_{x_a - \frac{w_a}{2}}^{x_a + \frac{w_a}{2}} \frac{2}{\pi w_a} \left\{ 1 - \left[\frac{2(x' - x_a)}{w_a} \right]^2 \right\}^{-1/2} e^{j\alpha x'}\, dx' \tag{13}$$

$$I_2^a = \sum_{n=2}^{N} I_{n_a} \left[\int_{z_{n-1}}^{z_n} \frac{\sin K(z'-z_n-1)}{\sin(Kl_z)} e^{j\beta z'} dz' + \int_{z_n}^{z_{n+1}} \frac{\sin K(z_{n+1}-z')}{\sin(Kl_z)} e^{j\beta z'} dz' \right]$$

(14a)

$$I_3^a = \int_{z_o-l_z}^{z_o} I_{1_a} f_1(z') e^{j\beta z'} dz' \qquad (14b)$$

$$I_4^a = \int_{z_o}^{\infty} \left[e^{(\gamma_{z_a}^{ms} + j\beta) z'} - Re^{(-\gamma_{z_a}^{ms} + j\beta) z'} \right] dz' \qquad (15)$$

After some algebra I_1^a, I_2^a, and I_3^a are found to be:

$$I_1^a = e^{j\alpha x_a} J_o\left(\alpha \frac{w_a}{2}\right) \qquad (16)$$

$$I_2^a = \frac{j2\beta}{(K^2-\beta^2)} \sum_{n=2}^{N} I_{n_a} e^{j\beta z_n} \qquad (17a)$$

$$I_3^a = \frac{I_{1_a} e^{j\beta(z_o-l_z)}}{(K^2-\beta^2) \sin(Kl_z)} \left[K-e^{j\beta l_z} (j\beta \sin Kl_z - k\cos Kl_z) \right] \qquad (17b)$$

To evaluate I_4^a, some special considerations are necessary. I_4^a is written alternatively as

$$I_4^a = \int_{-\infty}^{\infty} u(z'-z_o) \left[e^{(\gamma_{z_a}^{ms} + j\beta)\ z'} - Re^{(-\gamma_{z_a}^{ms} + j\beta)\ z'} \right] dz' \tag{18}$$

and we assume a lossless system so that $\gamma_{z_a}^{ms}$ is a purely imaginary quantity, $\gamma_{z_a}^{ms} = j\ k_{z_a}^{ms}$. The Fourier transform of the product of two functions is equal to the convolution of the individual Fourier transforms of the two functions, [6]. Using this idea to evaluate (18) shows

$$I_4^a = F\left\{U(z'-z_o)\right\} * f\left\{ \left[e^{jk_{z_a}^{ms} z'} - Re^{-jk_{z_a}^{ms}} \right] \right\} \tag{19}$$

$$\int_{-\infty}^{\infty} U(z'-z_o)\ e^{j\beta z'} dz' = \left[\pi\delta(\beta) + \frac{1}{j\beta} \right] e^{j\beta z_o} \tag{20a}$$

$$\int_{-\infty}^{\infty} e^{jk_{z_a} z'} e^{j\beta z'} dz' = 2\pi\delta\left(\beta + k_{z_a}^{ms}\right) \tag{20b}$$

$$-R \int_{-\infty}^{\infty} e^{-jk_{z_a}^{ms} z'} e^{j\beta z'} dz' = -2\pi R\delta\left(\beta - k_{z_a}^{ms}\right) \tag{20c}$$

From equations (19) and (20) we express I_4^a as

$$I_4^a = 2\pi \int_{-\infty}^{\infty} e^{j\lambda z_o} \left[\pi\delta(\lambda) + \frac{1}{j\lambda}\right] \left[\delta\left(\beta-\lambda + k_{z_a}^{ms}\right) - R\delta\left(\beta-\lambda - k_{z_a}^{ms}\right)\right] d\lambda$$

$$= 2\pi \int_{-\infty}^{\infty} \left[\pi e^{j\lambda_o z_o}\, \delta(\lambda)\, \delta\left(\beta-\lambda + k_{z_a}^{ms}\right) + \frac{e^{j\lambda z_o}}{j\lambda}\, \delta\left(\beta-\lambda + k_{z_a}^{ms}\right)\right.$$

$$\left. -\pi R e^{j\lambda z_o}\, \delta(\lambda)\, \delta\left(\beta-\lambda - k_{z_a}^{ms}\right) - \frac{R e^{j\lambda z_o}}{j\lambda}\, \delta\left(\beta-\lambda - k_{z_a}^{ms}\right)\right] d\lambda$$

$$I_4^a = 2\pi^2\, \delta\left(\beta + k_{z_a}^{ms}\right) - 2\pi^2\, R\delta\left(\beta - k_{z_a}^{ms}\right)$$

$$+ 2\pi \left[\frac{-e^{j\left(\beta+k_{z_a}^{ms}\right) z_o}}{j\left(\beta+k_{z_a}^{ms}\right)} + R\, \frac{e^{j\left(\beta-k_{z_a}^{ms}\right) z_o}}{j\left(\beta-k_{z_a}^{ms}\right)}\right] \qquad (21)$$

Substituting (16), (17), and (21) into (12) shows:

$$\tilde{K}_{z_a}(\alpha, h_a, \beta) = -\frac{2\pi\, e^{j\alpha x_a}\, e^{-j\left(\beta+k_{z_a}^{ms}\right) z_o}}{j\left(\beta+k_{z_a}^{ms}\right)}\, J_o\!\left(\alpha\, \frac{W_a}{2}\right)$$

$$+ R\, \frac{2\pi\, e^{j\alpha x_a}\, e^{-j\left(\beta-k_{z_a}^{ms}\right)}}{j\left(\beta-k_{z_a}^{ms}\right)}\, J_o\!\left(\alpha\, \frac{W_a}{2}\right) + \frac{j2\beta e^{j\alpha x_a}\, J_o\left(\alpha\, \frac{W_a}{2}\right)}{\left(K^2-\beta^2\right)} \sum_{n=2}^{N} I_{n_a}\, e^{j\beta z_n}$$

$$+ I_{1_a}\, \frac{e^{j\alpha x_a}\, e^{j\beta(Z_o - l_z)}}{\left(K^2-\beta^2\right) \sin(k l_z)}\, J_o\!\left(\alpha\, \frac{W_a}{2}\right) \left[K - e^{j\beta l_z}\left(j\beta \sin K l_z - k \cos K l_z\right)\right]$$

$$+ 2\pi^2 \left[\delta\left(\beta+k_{z_a}^{ms}\right) - R\delta\left(\beta-k_{z_a}^{ms}\right)\right] e^{j\alpha x_a}\, J_o\!\left(\alpha\, \frac{W_a}{2}\right) \qquad (22)$$

In a completely analogous manner $\tilde{K}_{z_b}(\alpha,-h_b,\beta)$ is found to be

$$\tilde{K}_{z_b}(\alpha,-h_b,\beta) = -T \frac{2\pi\, e^{j\alpha x_b}\, e^{j\left(\beta+k_{z_b}^{ms}\right) z_o}}{j\left(\beta+k_{z_b}^{ms}\right)} J_o\left(\alpha \frac{W_b}{2}\right)$$

$$+ \frac{I_{1_b}\, e^{j\alpha x_b}\, e^{-j\beta(z_o-l_z)}\, J_o\left(\alpha \frac{W_b}{2}\right)}{\left(K^2-\beta^2\right)\sin(kl_z)} \left[K + e^{-j\beta l_z}\,(j\beta \sin Kl_z + k\cos Kl_z)\right]$$

$$+ \frac{j\beta\, e^{j\alpha x_b}\, J_o\left(\alpha \frac{W_b}{2}\right)}{\left(K^2-\beta^2\right)} \sum_{n=2}^{N} I_{n_b}\, e^{j\beta z_n} + 2\pi^2\, T\, e^{j\alpha x_b}\, J_o\left(\alpha \frac{W_b}{2}\right) \delta\left(\beta+k_{z_b}^{ms}\right) \tag{23}$$

The discretized microstrip currents near the discontinuity region are divided in 'N' partitions in both guides 'a' and 'b'. More severe disruption of the current occurs in guide 'a' due to the open end upon which K_{z_a} is incident. If 'N' partitions are required to accurately characterize K_{z_a} then the same number will represent K_{z_b} in its respective discontinuity region.

The Fourier transform of $M_{x_a}(x',0,z')$ is expressed from (6), (7), (8) and (11) as

$$\tilde{M}_{x_a}(\alpha,0,\beta) = \sum_{n=1}^{M} V_n \iint_{-\infty}^{\infty} f_n(x')\, g_o(z')\, e^{j\alpha x'} e^{j\beta z'} dx'\, dz' = I_4 \cdot I_5 \tag{24}$$

$$I_4 = \int_{-t/2}^{t/2} \frac{2}{\pi t} \left\{1-\left[\frac{2z'}{t}\right]^2\right\}^{-1/2} e^{j\beta z'} dz'$$

and clearly

$$I_4 = \int_{-t/2}^{t/2} \frac{2}{\pi t} \left\{1-\left[\frac{2z'}{t}\right]^2\right\}^{-1/2} \left[\cos\beta z' + j\sin\beta z'\right] dz'$$

$$= \int_{-t/2}^{t/2} \frac{2}{\pi t} \left\{1-\left[\frac{2z'}{t}\right]^2\right\}^{-1/2} \cos\beta z' \, dz'$$

Let

$$\frac{2z'}{t} = \cos\phi \Rightarrow dz' = \frac{-t}{2} \sin\phi \, d\phi$$

$$I_4 = \int_{-t/2}^{t/2} \frac{2}{\pi t} \frac{1}{\sin\phi} \cos\beta\left(\frac{t}{2}\cos\phi\right) - \frac{t}{2} \sin\phi \, d\phi$$

$$I_4 = \frac{1}{\pi} \int_0^{\pi} \cos\beta\left(\frac{t}{2}\cos\phi\right) d\phi = J_o\left(\beta \frac{t}{2}\right) \tag{25}$$

$$I_5 = \sum_{n=1}^{M} V_n \int_{X_{n-1}}^{X_{n+1}} f_n(x') \, e^{j\alpha x'} dx' \tag{26}$$

Substituting for $f_n(x')$ from equation (7) and evaluating the result yields

$$I_5 = \sum_{n=1}^{M} V_n \frac{j2\alpha \, e^{j\alpha X_n}}{(K^2-\alpha^2)} \tag{27}$$

Finally then, $\tilde{M}_{x_a}(\alpha, 0, \beta)$ is written as:

$$M_{x_a} = \frac{j2\alpha \, J_o\left(\beta \frac{t}{2}\right)}{(K^2 - \alpha^2)} \sum_{n=1}^{M} V_n \, e^{j\alpha x_n} \qquad (28)$$

Equations (22), (23), and (28) are very important quantities.

IV. FORMULATION OF THE MATRIX EQUATION

The results obtained in the previous section will allow us to solve equation (1). In the following discussion, spatial and/or spectral dependence of functions will be implied and not explicitly written. If there is any ambiguity, it will be clearly explained from equation (1):

$$\tilde{E}_z^{(1a)} = \tilde{G}_{11} \tilde{K}_{z_a} + \tilde{G}_{12} \tilde{M}_{x_a} \qquad (29)$$

$$\tilde{H}_z^{(1a)} = \tilde{G}_{21} \tilde{K}_{z_a} + \tilde{G}_{22} \tilde{M}_{x_a} \qquad (30)$$

$$\tilde{E}_z^{(1b)} = \tilde{G}_{32} \tilde{M}_{x_a} + \tilde{G}_{33} \tilde{K}_{z_b} \qquad (31)$$

Since, from Appendix A we know that $\tilde{G}_{13} = \tilde{G}_{23} = \tilde{G}_{31} = 0$.

Also, note that the surface current densities are expressed in terms of summations of appropriate basis functions as derived earlier. Before going any further with the analysis it will be convenient to write $\tilde{K}_{z_a}$, $\tilde{K}_{z_b}$, and $\tilde{M}_{x_a}$ as:

$$\tilde{K}_{z_a} = -\tilde{k}_o^a + R\ \tilde{k}_1^a + \sum_{n=1}^{N} I_{n_a}\ \tilde{K}_{z_a}^n \tag{32}$$

$$\tilde{K}_{z_b} = -\ T\ \tilde{k}_1^b + \sum_{n=1}^{N} I_{n_b}\ \tilde{K}_{z_b}^n \tag{33}$$

$$\tilde{M}_{x_a} = \sum_{n=1}^{M} V_n\ \tilde{M}_{x_a}^n \tag{34}$$

where $\tilde{k}_o^a$, $\tilde{k}_1^a$, and $\tilde{K}_{z_a}^n$ are easily deduced from (22). $\tilde{k}_1^b$, $\tilde{K}_{z_b}^n$ and $\tilde{M}_{x_a}^n$ are similarly obtained, respectively, from (23) and (28). We next multiply equation (29) by $\tilde{K}_{z_a}^q$ and integrate from $-\infty$ to ∞ with respect to α and β for different values of q. This shows:

$$\iint_{-\infty}^{\infty} \tilde{K}_{z_a}^q\ \tilde{E}_z^{(1a)} d\alpha\ d\beta = \iint_{-\infty}^{\infty} \tilde{K}_{z_a}^q\ \tilde{G}_{11}\ \tilde{K}_{z_a} d\alpha\ d\beta$$

$$+ \iint_{-\infty}^{\infty} K_{z_a}^q\ G_{12}\ \tilde{M}_{x_a} d\tilde{\alpha}\ d\tilde{\beta} \quad q = 1,2,...,N \tag{35}$$

but

$$\iint_{-\infty}^{\infty} \tilde{K}_{z_a}^q\ \tilde{E}_z^{(1a)} d\alpha\ d\beta = 0$$

Because, from Parseval's theorem

$$\iint_{-\infty}^{\infty} \tilde{K}_{z_a}^q\ \tilde{E}_z^{(1a)} d\alpha\ d\beta = \frac{1}{(2\pi)^2} \iint_{-\infty}^{\infty} K_{z_a}^q\ E_z^{(1a)} dx\ dz = 0$$

Since it is clear that $\tilde{K}_{z_a}^{q}$ and $\tilde{E}_z^{(1a)}$ are complementary in the x-direction. Using (32) and (34), equation (35) becomes:

$$\iint_{-\infty}^{\infty} \tilde{K}_{z_a}^{q} \tilde{G}_{11} \left[-\tilde{k}_o^{a} + R\,\tilde{k}_1^{a} + \sum_{n=1}^{N} I_{n_a} \tilde{K}_{z_a}^{n} \right] d\alpha d\beta + \iint_{-\infty}^{\infty} \tilde{K}_{z_a}^{q} \tilde{G}_{12} \sum_{n=1}^{M} V_n \tilde{M}_{x_a}^{n} \, d\alpha d\beta = 0 \qquad q = 1,2,\ldots,N$$

$$-\iint_{-\infty}^{\infty} \tilde{K}_{z_a}^{q} \tilde{G}_{11} \tilde{k}_o^{a} \, d\alpha d\beta + \iint_{-\infty}^{\infty} R\left(\tilde{K}_{z_a}^{q} \tilde{G}_{11} \tilde{k}_1^{a}\right) d\alpha d\beta + \sum_{n=1}^{N} I_{n_a} \iint_{-\infty}^{\infty} \tilde{K}_{z_a}^{q} \tilde{G}_{11} \tilde{K}_{z_a}^{n} \, d\alpha d\beta + \sum_{n=1}^{M} V_n \iint_{-\infty}^{\infty} \tilde{K}_{z_a}^{q} \tilde{G}_{12} \tilde{M}_{x_a}^{n} \, d\alpha d\beta = 0 \qquad q = 1,2,\ldots,N \tag{36}$$

Performing similar operations on equations (30) and (31) yields:

From (30)

$$-\iint_{-\infty}^{\infty} \tilde{M}_{x_a}^{q} \tilde{G}_{21} \tilde{k}_o^{a} \, d\alpha d\beta + R \iint_{-\infty}^{\infty} \tilde{M}_{x_a}^{q} \tilde{G}_{21} \tilde{k}_1^{a} \, d\alpha d\beta + \sum_{n=1}^{N} I_{n_a} \iint_{-\infty}^{\infty} \tilde{M}_{x_a}^{q} \tilde{G}_{21} \tilde{K}_{z_a}^{n} \, d\alpha d\beta + \sum_{n=1}^{M} V_n \iint_{-\infty}^{\infty} \tilde{M}_{x_a}^{q} \tilde{G}_{21} \tilde{M}_{x_a}^{n} \, d\alpha d\beta = 0 \qquad q = 1,2,\ldots,M \tag{37}$$

From (31)

$$- T \iint_{-\infty}^{\infty} \tilde{K}_{z_b}^{q} \tilde{G}_{33} \tilde{k}_1^{b} \, d\alpha d\beta + \sum_{n=1}^{M} V_n \iint_{-\infty}^{\infty} \tilde{K}_{z_b}^{q} \tilde{G}_{32} \tilde{M}_{x_a}^{n} \, d\alpha d\beta$$

$$+ \sum_{n=1}^{N} I_{n_b} \iint_{-\infty}^{\infty} \tilde{K}_{z_b}^{q} \tilde{G}_{33} \tilde{K}_{z_b}^{n} \, d\alpha d\beta = 0 \qquad q = 1,2,\ldots,N \tag{38}$$

Equations (36), (37), and (38) represent an inhomogeneous system of 2N+M equations containing 2N+M+2 unknowns. We get two additional relations by the requirement of electric surface current continuity at ($x=h_a$, $z=z_o$) and ($x=-h_b$, $z=-z_o$). Look at Figure 2.

From equations (2), (3), (4), and (5) we see

$$I_{1_a} = e^{jk_{z_a}^{ms} z_o} - Re^{-jk_{z_a}^{ms} z_o} \tag{39a}$$

$$I_{1_b} = T\, e^{-jk_z^{ms} z_o} \tag{39b}$$

Equations (39) represent the necessary relations for solving for R and T. We shall now express (36), (37), and (38) more compactly:

(36) becomes

$$S_q = R\, P_q + \sum_{n=1}^{N} I_{n_a} D_{qn} + \sum_{n=1}^{M} V_n G_{qn} \qquad q = 1,2,\ldots,N$$

where

$$S_q = \iint_{-\infty}^{\infty} \tilde{K}_{z_a}^q \; \tilde{G}_{11} \; \tilde{k}_o^a \; d\alpha d\beta$$

$$P_q = \iint_{-\infty}^{\infty} \tilde{K}_{z_a}^q \; \tilde{G}_{11} \; \tilde{k}_1^a \; d\alpha d\beta \tag{40}$$

$$D_{qn} = \iint_{-\infty}^{\infty} \tilde{K}_{z_a}^q \; \tilde{G}_{11} \; \tilde{K}_{z_a}^n \; d\alpha d\beta$$

$$G_{qn} = \iint_{-\infty}^{\infty} \tilde{K}_{z_a}^q \; \tilde{G}_{12} \; \tilde{M}_{x_a}^n \; d\alpha d\beta$$

(37) becomes

$$\$_q = R\; Q_q + \sum_{n=1}^{N} I_{n_a} \; L_{q_n} + \sum_{n=1}^{M} V_n \; F_{q_n} \qquad q = 1,2,\ldots,M$$

where

$$\$_q = \iint_{-\infty}^{\infty} \tilde{M}_{x_a}^q \; \tilde{G}_{21} \; \tilde{k}_o^a \; d\alpha d\beta$$

$$Q_q = \iint_{-\infty}^{\infty} \tilde{M}_{x_a}^q \; \tilde{G}_{21} \; \tilde{k}_1^a \; d\alpha d\beta \tag{41}$$

$$L_{qn} = \iint_{-\infty}^{\infty} \tilde{M}_{x_a}^q \; \tilde{G}_{21} \; \tilde{K}_{z_a}^n \; d\alpha d\beta$$

$$F_{qn} = \iint_{-\infty}^{\infty} \tilde{M}_{x_a}^q \; \tilde{G}_{21} \; \tilde{M}_{x_a}^n \; d\alpha d\beta$$

(38) becomes

$$0 = -\,T\,U_q + \sum_{n=1}^{N} I_{n_b}\,V_{qn} + \sum_{n=1}^{M} V_n\,X_{qn} \qquad q = 1,2,\ldots,N$$

where

$$U_q = \iint_{-\infty}^{\infty} \tilde{K}^q_{z_b}\,\tilde{G}_{33}\,\tilde{k}^b_1\,d\alpha d\beta$$

$$V_{qn} = \iint_{-\infty}^{\infty} K^q_{z_b}\,G_{33}\,\tilde{K}^n_{z_b}\,\tilde{d}\alpha d\tilde{\beta} \qquad (42)$$

$$X_{qn} = \iint_{-\infty}^{\infty} \tilde{K}^q_{z_b}\,\tilde{G}_{32}\,\tilde{M}^n_{x_a}\,d\alpha d\beta$$

V. DISCUSSION OF NUMERICAL SOLUTION

It is clear from the results of the last section that the problem under consideration is very difficult. Since no numerical results will be obtained, a discussion of how to do this is in order. Equations (40), (41), and (42) contain integrals which must be evaluated from $-\infty$ to ∞ with respect to two variables: α and β. A brief glance at Appendix A indicates that numerical integration of these terms will surely be required. Although it is not obvious, these integrals vary at worst as $\frac{1}{\alpha^{3/2}}$, $\frac{1}{\alpha^2}$, $\frac{1}{\beta^{3/2}}$, $\frac{1}{\beta^2}$ in one part or another. The squared terms will converge rapidly and the same can be said for the $\frac{1}{\alpha^{3/2}}$ and $\frac{1}{\beta^{3/2}}$ terms. Ultimately, one would need to program a computer to do these integrations to find how the convergence progresses. However, in [7], Mittra and Itoh discuss so called 'numerically efficient techniques' for solving boundary value problems. The approach used in this report closely resembles the algorithms discussed there. As a result, it would seem that satisfactory convergence is obtainable for the equations derived here. Besides having to evaluate the integrals mentioned, we must also solve a linear system of equations. There are many viable alternatives for this part of the problem and no further discussion is necessary.

VI. CONCLUSION

The solution for reflection and transmission coefficients in slot-coupled microstrip lines has been formulated in the spectral domain. Although no numerical results have been obtained, pretinent numerical considerations have been discussed.

REFERENCES

[1] T. Itoh and R. Mittra, "Spectral-Domain Approach for Calculating the Dispersion Characteristics of Microstrip Lines," IEEE Trans. Microwave Theory Tech., vol. MTT-21, No. 7, pp. 496-499, July 1973.

[2] T. Itoh, ed., "Planar Transmission Line Structures," IEEE Press, 1987.

[3] P. Katehi and N. Alexopoulos, "Frequency-Dependent Characteristics of Microstrip Discontinuities in Millimeter-Wave Integrated Circuits," IEEE Trans. Microwave Theory Tech., vol. MTT-33, No. 10, pp. 1029-1035, October 1985.

[4] L. Dunleavy, Ph.D. thesis, University of Michigan, 1988.

[5] R. Harrington, "Time-Harmonic Electromagnetic Fields," New York: McGraw-Hill, 1961.

[6] A. Papoulis, "The Fourier Integral and Its Applications," New York: McGraw-Hill, 1962.

[7] R. Mittra, ed., "Computer Techniques in Electromagnetics," pp. 305-342, New York: Pergamon Press, 1973.

Appendix A

Derivation of the Green's matrix

Hybrid mode analysis with

$$\bar{E}(x,y,z) = -j\omega\bar{A} + \frac{1}{j\omega\mu\varepsilon}\nabla\nabla \bullet \bar{A} + \frac{1}{\varepsilon}\nabla \times \bar{F} \tag{A-1}$$

$$\bar{H}(x,y,z) = j\omega\bar{F} - \frac{1}{j\omega\mu\varepsilon}\nabla\nabla \bullet \bar{F} + \frac{1}{\mu}\nabla \times \bar{A}$$

Choose $\bar{A} = \hat{z}A_z$ and $\bar{F} = \hat{z}F_z$ where

$$A_z^{(i)}(x,y,z) = \frac{1}{(2\pi)^2}\iint_{-\infty}^{\infty} \tilde{A}_z^{(i)}(\alpha,y,\beta)\, e^{-j\alpha x}\, e^{-j\beta z}\, d\alpha d\beta \tag{A-2}$$

$$\tilde{A}_z^{(i)} = -\left(\frac{\omega\mu\varepsilon}{\beta}\right)\tilde{\psi}_A^{(i)}(\alpha,y,\beta) \quad (i = 1a, 1b, 2a, 2b)$$

$$F_z^{(i)}(x,y,z) = \frac{1}{(2\pi)^2}\iint_{-\infty}^{\infty} \tilde{F}_z^{(i)}(\alpha,y,\beta)\, e^{-j\alpha x}\, e^{-j\beta z}\, d\alpha d\beta \tag{A-3}$$

$$\tilde{F}_z^{(i)} = \left(\frac{\omega\mu\varepsilon}{\beta}\right)\tilde{\psi}_F^{(i)}(\alpha,y,\beta) \qquad (i = 1a, 1b, 2a, 2b)$$

$A_z^{(i)}(x,y,z)$ and $F_z^{(i)}(x,y,z)$ must satisfy

$$\left(\nabla^2 + k_i^2\right)\begin{Bmatrix} A_z^{(i)}(x,y,z) \\ F_z^{(i)}(x,y,z) \end{Bmatrix} = 0 \tag{A-4}$$

Substituting (A-2) and (A-3) into (A-4) implies:

$$\left[\frac{\partial^2}{\partial y^2} - \left(\alpha^2 + \beta^2 - k_i^2\right)\right] \begin{Bmatrix} \tilde{\psi}_A^{(i)}(\alpha, y, \beta) \\ \tilde{\psi}_F^{(i)}(\alpha, y, \beta) \end{Bmatrix} = 0$$

Letting $\gamma_i^2 = \alpha^2 + \beta^2 - k_i^2$ shows

$$\text{(A-5)} \qquad \left(\frac{\partial^2}{\partial y^2} - \gamma_i^2\right) \begin{Bmatrix} \tilde{\psi}_A^{(i)}(\alpha, y, \beta) \\ \tilde{\psi}_F^{(i)}(\alpha, y, \beta) \end{Bmatrix} = 0$$

Appropriate solutions to (A-5) in each region of the structure shown in Figure 1 are:

$$\tilde{\psi}_A^{(1a)} = \tilde{A}^{(1a)} \sinh(\gamma_{1a} y) + \tilde{B}^{(1a)} \cosh(\alpha_{1a} y)$$

$$\tilde{\psi}_A^{(2a)} = \tilde{D}^{(2a)} \sinh\left[\gamma_{2a}\left(y-b\right)\right]$$

$$\text{(A-6)} \qquad \tilde{\psi}_A^{(1b)} = \tilde{A}^{(1b)} \sinh(\gamma_{1b} y) + \tilde{B}^{(1b)} \cosh(\alpha_{1b} y)$$

$$\tilde{\psi}_A^{(2b)} = \tilde{P}^{(2b)} \sinh\left[\gamma_{2b}\left(y+b\right)\right]$$

$$\tilde{\psi}_F^{(1a)} = \tilde{C}^{(1a)} \sinh(\gamma_{1a} y) + \tilde{D}^{(1a)} \cosh(\alpha_{1a} y)$$

$$\tilde{\psi}_F^{(2a)} = \tilde{N}^{(2a)} \cosh\left[\gamma_{2a}\left(y-b\right)\right]$$

$$\text{(A-7)} \qquad \tilde{\psi}_F^{(1b)} = \tilde{C}^{(1b)} \sinh(\gamma_{1b} y) + \tilde{D}^{(1b)} \cosh(\alpha_{1b} y)$$

$$\tilde{\psi}_F^{(2b)} = \tilde{N}^{(2b)} \cosh\left[\gamma_{2b}\left(y+b\right)\right]$$

Note: The ~ notation means Fourier transform domain.

The transformed fields must satisfy

$$\tilde{E}_z^{(1q)}(\alpha,h_q,\beta) = \tilde{E}_z^{(2q)}(\alpha,h_q,\beta)$$

$$\tilde{E}_x^{(1q)}(\alpha,h_q,\beta) = \tilde{E}_x^{(2q)}(\alpha,h_q,\beta)$$

$$\tilde{H}_z^{(1q)}(\alpha,h_q,\beta) = \tilde{H}_z^{(2q)}(\alpha,h_q,\beta)$$

$$\text{(A-8)} \qquad \tilde{H}_x^{(1q)}(\alpha,h_q,\beta) - \tilde{H}_x^{(2q)}(\alpha,h_q,\beta) = \tilde{K}_{z_q}(\alpha,h_q,\beta)$$

$$\tilde{E}_x^{(1q)}(\alpha,0,\beta) = 0$$

$$\tilde{E}_z^{(1q)}(\alpha,0,\beta) = \tilde{M}_x(\alpha,0,\beta)$$

$$(q = a,b)$$

$\tilde{K}_{z_q}(\alpha,h_q,\beta)$ is the Fourier transform of the electric surface current on the qth microstrip. $\tilde{M}_x(\alpha,0,\beta)$ is the Fourier transform of magnetic surface current on the aperture which is common to the two microstrip structures.

At this point in the derivation it should be noted that symmetry about the line x=0, y=0 exists between guides (a) and (b). In order to derive $\tilde{G}_{11}(\alpha,\beta)$ and $\tilde{G}_{12}(\alpha,\beta)$, we apply the boundary conditions (A-8) and express $\tilde{E}_z^{(1a)}(\alpha,h_a,\beta)$ appropriately. $\tilde{G}_{32}(\alpha,\beta)$ and $\tilde{G}_{33}(\alpha,\beta)$ are obtained from, respectively, $\tilde{G}_{12}(\alpha,\beta)$ and $\tilde{G}_{11}(\alpha,\beta)$ as

$$\tilde{G}_{32}(\alpha,\beta) = \tilde{G}_{12}(\alpha,\beta) \Big|_{h_a = -h_b;\ k_{1a} = k_{1b}}$$

(A-9)

$$\tilde{G}_{33}(\alpha,\beta) = \tilde{G}_{11}(\alpha,\beta) \Big|_{h_a = -h_b;\ k_{1a} = k_{1b}}$$

Consequently, solving for $\tilde{G}_{12}(\alpha,\beta)$ and $\tilde{G}_{11}(\alpha,\beta)$ yields $\tilde{G}_{32}(\alpha,\beta)$ and $\tilde{G}_{33}(\alpha,\beta)$ in a direct manner. Imposing the boundary conditions (A-8) and appropriately manipulating terms yields the components of the Green's matrix as:

$$\tilde{G}_{11}(\alpha,\beta) = -j\omega\mu\left(k_{1a}^2 - \beta^2\right)\left(k_{2a}^2 - \beta^2\right) \Big/ \Big\{ \alpha^2\beta^2\left(k_{1a}^2 - k_{2a}^2\right)^2$$

$$- \gamma_{2a}^2 k_{2a}^2\left(k_{1a}^2 - \beta^2\right) - \gamma_{1a}^2 k_{1a}^2\left(k_{2a}^2 - \beta^2\right)$$

$$+ \gamma_{1a}\gamma_{2a}\left(k_{1a}^2 - \beta^2\right)\left(k_{2a}^2 - \beta^2\right) \Big\{ k_{1a}^2 \tanh\left[\gamma_{2a}(h-b)\right] \coth\left(\gamma_{1a}h_a\right)$$

$$+ k_{2a}^2 \tanh\left(\gamma_{1a}h_a\right) \coth\left[\gamma_{2a}\left(h_a-b\right)\right] \Big\} \Big\}$$

$$\tilde{G}_{21}(\alpha,\beta) = \frac{-j\alpha\beta\left(k_{2a}^2 - k_{1a}^2\right)\cosh\left(\gamma_{1a}h_a\right)\tilde{G}_{11}(\alpha,\beta)}{\omega\mu\left[\gamma_{2a}\left(k_{1a}^2 - \beta^2\right)\tanh\left[\gamma_{2a}\left(h_a-b\right)\right] - \gamma_{1a}\left(k_{2a}^2 - \beta^2\right)\tanh\left(\gamma_{1a}h_a\right)\right]}$$

$$\tilde{G}_{31} = 0$$

$$\tilde{G}_{12}(\alpha,\beta) = \cosh\left(\gamma_{1a}h_a\right) - 2\beta\left(k_{1a}^2 - \beta^2\right)\left(k_{2a}^2 - \beta^2\right)\left(k_{1a}^2 - k_{2a}^2\right)\frac{\tilde{\Gamma}_a\left(\alpha,\beta,h_a\right)}{\tilde{\xi}_a\left(\alpha,\beta,h_a\right)}$$

$$\tilde{\Gamma}_a\left(\alpha,\beta,h_a\right) = \sin\left(\gamma_{1a}h_a\right) \Big\{ \tanh\left[\gamma_{2a}\left(h_a-b\right)\right] [\beta^2\gamma_{2a}\left(k_{1a}^2 - \beta^2\right)\left(k_{2a}^2 + \gamma_{2a}^2\right)$$

$$- \alpha^2\beta^2\gamma_{2a}\left(k_{1a}^2 - k_{2a}^2\right) + \alpha^2\gamma_{2a}\left(k_{1a}^2 + \gamma_{2a}^2\right)\left(k_{1a}^2 - \beta^2\right) - k_{1a}^2\gamma_{2a}^2\left(k_{1a}^2 - \beta^2\right)\Big]$$

$$+ \tanh\left(\gamma_{1a}h_a\right)\Big[k_{1a}^2\gamma_{2a}^2\gamma_{1a}\left(k_{2a}^2 - \beta^2\right) - \beta^2\gamma_{1a}\left(k_{2a}^2 + \gamma_{2a}^2\right)\left(k_{2a}^2 - \beta^2\right)$$

$$- \alpha^2\gamma_{1a}\left(k_{1a}^2 + \gamma_{2a}^2\right)\left(k_{2a}^2 - \beta^2\right)\Big] - k_{2a}^2\gamma_{1a}\gamma_{2a}\left(k_{2a}^2 - \beta^2\right)\coth\left[\gamma_{2a}\left(h_a - b\right)\right]\Big\}$$

$$+ \cosh\left(\gamma_{1a}h_a\right)\left[\alpha^2\beta^2\gamma_{1a}\left(k_{1a}^2 - k_{2a}^2\right) - \gamma_{1a}\gamma_{2a}^2k_{2a}^2\left(k_{1a}^2 - \beta^2\right)\right]$$

$$\tilde{\xi}_a\left(\alpha,\beta,h_a\right) = \alpha^2\beta^2\left(k_{1a}^2 - k_{2a}^2\right)^2 + k_{2a}^2\gamma_{2a}^2\left(k_{1a}^2 - \beta^2\right)^2 + k_{1a}^2\gamma_{1a}^2\left(k_{2a}^2 - \beta^2\right)^2$$

$$+ \gamma_{1a}\gamma_{2a}k_{2a}^2\left(k_{2a}^2 - \beta^2\right)\left(k_{1a}^2 - \beta^2\right)\tanh\left(\gamma_{1a}h_a\right)\coth\left[\gamma_{2a}\left(h_a - b\right)\right]$$

$$- \gamma_{1a}\gamma_{2a}k_{1a}^2\left(k_{2a}^2 - \beta^2\right)\left(k_{1a}^2 - \beta^2\right)\coth\left(\gamma_{1a}h_a\right)\tanh\left[\gamma_{2a}\left(h_a - b\right)\right]$$

$$\tilde{G}_{22}(\alpha,\beta) = \tilde{\eta}(\alpha,\beta) \Big/ \tilde{\tau}(\alpha,\beta)$$

where

$$\tilde{\eta}(\alpha,\beta) = -j\left\{\frac{\sinh\left(\gamma_{1a}h_a\right)}{\omega^2\mu\gamma_{1a}\left(k_{2a}^2 - \beta^2\right)}\left[\beta^2\left(k_{1a}^2 - k_{2a}^2\right) - \left(k_{1a}^2 + \gamma_{2a}^2\right)\left(\alpha^2 + \beta^2\right) + k_{1a}^2\gamma_{2a}^2\right]\right.$$

move over.

$$\left. + \frac{\omega\varepsilon_{1a}\gamma_{1a}}{\left(k_{1a}^2 - \beta^2\right)}\cosh\left(\gamma_{1a}h_a\right)\coth\left(\gamma_{1a}h_a\right)\right\} \times \left\{\alpha\beta\left(k_{1a}^2 - k_{2a}^2\right) \Big/\right.$$

$$\left.\left[\varepsilon_{2a}\gamma_{2a}\left(k_{1a}^2 - \beta^2\right)\coth\left[\gamma_{2a}\left(h_a - b\right)\right] - \varepsilon_{1a}\gamma_{1\alpha}\left(k_{2\alpha}^2 - \beta^2\right)\coth\left(\gamma_{1a}h_a\right)\right]\right\}$$

$$+ j\alpha\beta\left[\frac{2}{\left(k_{1a}^2 - \beta^2\right)}\cosh\left(\gamma_{1a}h_a\right) - \frac{\gamma_{2a}\sinh\left(\gamma_{1a}h_a\right)}{\gamma_{1a}\left(k_{2a}^2 - \beta^2\right)}\tanh\left[\gamma_{2a}\left(h_a - b\right)\right]\right]$$

$$\tilde{\tau}(\alpha,\beta) = \frac{j\cosh\left(\gamma_{1a}h_a\right)}{\left(k_{2a}^2 - \beta^2\right)} \left\{ \frac{\omega\mu}{\beta} \left[\gamma_{2a}\left(k_{1a}^2 - \beta^2\right) \tanh\left[\gamma_{2a}\left(h_a - b\right)\right]\right.\right.$$

$$\left. - \gamma_{1a}\left(k_{2a}^2 - \beta^2\right) \tanh\left(\gamma_{1a}h_a\right) \right] - \frac{\alpha^2\beta}{\omega} \left(k_{1a}^2 - k_{2a}^2\right)^2 \Big/ \left[\varepsilon_{2a}\gamma_{2a}\left(k_{1a}^2 - \beta^2\right) \cdot \right.$$

$$\left.\left. \coth\left[\gamma_{2a}\left(h_a - b\right)\right] - \varepsilon_{1a}\gamma_{1a}\left(k_{2a}^2 - \beta^2\right) \coth\left(\gamma_{1a}h_a\right) \right] \right\}$$

$\tilde{G}_{32}(\alpha,\beta)$ is obtained through equation A-9.

$$\tilde{G}_{13}(\alpha,\beta) = 0$$

$$\tilde{G}_{23}(\alpha,\beta) = 0$$

$\tilde{G}_{33}(\alpha,\beta)$ is obtained through equation A-9.