ESSAYS ON INTERNATIONAL FRAGMENTATION AND INTERMEDIATE GOODS TRADE

by

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To My Wife

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CHAPTER 1

The Choice of a Firm in Quality Competition: National Firm or Multinational Firm

1.1 Introduction

There have been many studies that shed light on the reasons that multinational firms expand from a developed country (the North) to a developing country (the South). Factor price differences between countries cause multinational firms to emerge (Helpman 1984, Helpman and Krugman 1985). Strong scale economies in headquarters activities relative to scale economies in production cause foreign direct investment (FDI) (Markusen 1984, Brainard 1993, 1997). The desire of firms to internalize their unique intangible assets such as patents and marketing skills encourages FDI (Dunning 1981, 1993). Lower corporate taxes in the host country attract higher FDI (Hines 1997).

Multinational activities in the South cause a leak of technology to Southern firms.[1] This spillover of technology makes Southern firms more efficient and makes Northern firms lose competitiveness against Southern firms. Thus if Northern firms plan to shift their production to the South because of cheap factor costs there, they should consider simultaneously both the factor price difference and the technology spillover. I focus on a tradeoff between the effects of factor price difference and spillover, which other studies have overlooked.

Recently, there has been some research about how such a spillover affects the emergence of multinational firms. Glass and Saggi (2001) look at the spillover of technology as an activity reducing the cost of imitation and thus reducing production cost. However, firms usually compete in the market not only through reducing production cost but also through raising the quality of their product.

[1] I do not handle an issue whether spillovers really exist since this is not the aim of this study. There are many empirical studies to examine existence of spillovers. These studies present different results. Kinoshita (2000), Barba and Castellani (2003), and Keller and Yeaple (2003) show that there are positive spillovers, whereas Aitken and Harrison (1999) show that negative spillovers exist. Braconier, Ekholm and Knarvik (2001), Veugelers and Cassiman (2004) show that there are no spillovers. Also, Smeets and Vaal (2005) look at the issue of spillovers from a slightly different viewpoint. They are interested in reasons why the spillover effect appears so inconsistent in the data. They argue that spillovers are related to the degree of ownership, a variety of spillover transfer channels and absorption capacity. They conclude that earlier empirics do not consider these factors appropriately, so the inconsistency occurs.

In my paper, the North and the South compete with quality. In the North, a single Northern firm produces a high quality product. In the South, many Southern firms produce a low quality product under perfect competition. If the Northern firm shifts its production to the South, the spillover of knowledge improves the quality of the low quality product. Demand for the high quality product decreases since these products are substitutes. The spillover becomes a disincentive for the multinational firm. However, the wage difference between the two countries also provides an incentive to produce in the South. The emergence of the multinational firm is determined by the net effect of these two opposite effects. However, even if the net effect makes the profit of the multinational firm positive, the Northern firm must also compare this profit to what it could earn by remaining a national firm.

I show the importance of factors related to the demand side as well as the cost side of production and R&D in the choice of the Northern firm between remaining national and becoming multinational. In contrast, most research has looked at only the cost side of production. The factors that are analyzed in my paper are (i) the size of world market for the high quality product, (ii) the consumer's preference for low quality, (iii) the consumer's preference for high quality, (iv) the consumer's response to the price of the low quality product, (v) the degree of spillover, (vi) the wage difference between the North and the South, and (vii) the productivity of production technology of the Northern firm and the Southern firms. Using comparative statics, I analyze how these factors affect the choice of the Northern firm.

In particular, I raise a new issue about consumers' preferences and responses, and their interaction with the choice of the firm's production location. Since a change in the consumer's preference for quality and a change in the consumer's response to the price affect the demand for a product and thus the firm's profit, the change in consumption behavior may influence the location choice of the Northern firm.

I find that a strengthening of intellectual property (IP) protection in the South gives rise to a multinational firm to the South and raises the North's product quality. This is different from the result of Glass and Saggi (2001).[2] [3] In fact, developed and developing countries have debated a

[2] A strengthening of IP protection in the South increases Southern firms' cost of imitating products of multinational firms which are produced in the South. This makes the Southern firms hire more labor. And it leads to a tighter South labor market. The tighter labor market reduces labor that will be employed by the multinational firms and crowds them out. Thus foreign direct investment flowing into the South decreases.

[3] There are empirical studies researching effect of intellectual property (IP) protection on foreign direct investment. In Maskus and Konan (1994), there is no significant correlation between the U.S. direct investment position abroad and a level of IP protection. Lee and Mansfield (1996) find a positive relationship between the volume of U.S. foreign direct investment toward developing countries and a level of IP protection in the developing countries.

lot on whether strengthening IP protection is beneficial to the world. My result may provide an answer to this debate since strengthening IP protection induces foreign direct investment to the developing countries and encourages the developed countries to improve the quality of their products.[4]

It is also a politically important issue, how R&D expenditure and quality of product will be affected when the Northern firm shifts its production abroad. I show that the change depends on the extent of difference between world demands for the high quality product, after the substitution effect of the low quality product is removed, facing the national firm and the multinational firm. Another political issue is how a rise in the Northern wage affects the R&D expenditure and the level of quality. These are also examined.

The organization of this paper is as follows. Section 1.2.1 addresses an industry in which there are many Southern firms and a single Northern firm operating as a national firm. Section 1.2.2 addresses the case of the Northern firm as a multinational firm in this industry, including the spillover that arises from the multinational firm to the Southern firms. I set up a model of a multinational firm that faces a tradeoff between the effect of factor price difference and the effect of spillover. In section 1.2.3, the choice of the Northern firm is examined to see whether it remains national or becomes multinational, by comparing the two profits. In section 1.2.4, I investigate how R&D expenditure and quality of product of the Northern firm would change when the firm shifts its production to the South and when the Northern wage rises. I compare the R&D expenditures (the qualities of product) of the national firm and the multinational firm. In section 1.2.5, comparative statics are conducted on the factors influencing the choice of the Northern firm. Subsection 1.2.5.1 examines the effect of stronger IP protection in the South. The effect of the wage in the South is examined in subsection 1.2.5.2, the effect of world market size in subsection 1.2.5.3, the effect of preference for low quality in subsection 1.2.5.4, the effect of preference for high quality in subsection 1.2.5.5, the effect of response to the price of the low quality product in subsection 1.2.5.6, and the effect of productivity of production technology in subsection 1.2.5.7. Conclusions are in section 1.3.

1.2 Model

1.2.1 World Economy with Only National Firms

The world economy is assumed to consist of a developing country (hereafter called the South) and a developed country (hereafter the North). I consider first a world market with many national

[4] Other studies analyze the effect on welfare of stronger IP protection, for example, Chin and Grossman (1990), Deardorff (1992) and Helpman (1993).

Southern firms and a single national Northern firm that all produce a freely traded product only within their respective home countries.[5] Each country has one primary factor, labor. The wage for the South's (the North's) labor is denoted as w_S (w_N). I assume that the Southern wage is lower than the Northern wage: $w_S < w_N$. I use partial equilibrium analysis, focusing on a single industry. Therefore the firms in this industry take as given the factor prices determined by other sectors. In this industry, trade barriers across borders, such as tariffs and transport costs, do not exist.

Let me address the cost function for manufacturing a product of quality q . Higher quality requires more elaborate production methods and facilities. This would necessitate employment of more labor, which raises the marginal cost of production. Thus, cost is a function of quality as well as the wage, $c(q;w_i)$, $i = S$ or N . Assume that λq units of labor are needed to produce one unit of a product of quality q in either country, with $\lambda > 0$. The marginal cost function is

$$c(q;w_i) = \lambda w_i q\,, \quad i = S \text{ or } N\,, \quad \lambda > 0\,. \qquad (1.1)$$

The marginal cost increases if quality and wage rise. It decreases if λ falls. The total cost of producing output x of quality q is

$$C(x,q;w_i) = \lambda w_i q x\,.$$

I explain how quality is determined. First, the case of a low quality product is considered. The low quality represents a basic quality. It is the lowest possible level of quality, and is denoted as $\overline{q}_l$. I assume that no resources are expended on R&D for its creation, since the relevant technological knowledge is known to all firms.

All Southern firms use this technology for producing the low quality product, and have the same cost function of production. The cost function of the Southern firms is

$$C_l(x_l,\overline{q}_l;w_S) = \lambda w_S \overline{q}_l x_l\,.$$

The world market for the low quality product is assumed to be perfectly competitive. All firms face the same price – given by the world market – of the low quality product. This price is denoted as p_l .

The profit function of the Southern firms is

$$\Pi_l = p_l x_l - \lambda w_S \overline{q}_l x_l\,.$$

Each of them chooses quantity to maximize its profit. The first order condition is

[5] The next subsection will handle economies in which the national Southern firms and a multinational firm exist.

$$\frac{\partial \Pi_l}{\partial x_l} = p_l - \lambda w_S \bar{q}_l = 0 .$$

The Southern firms freely exit or enter the market, so their profits become zero. The equilibrium price is

$$\bar{p}_l = \lambda w_S \bar{q}_l . \tag{1.2}$$

Second, I address how the high quality is determined. It is assumed that there is a single Northern firm which produces a product of high quality. The product of high quality, denoted by quality q_h, requires the Northern firm to make an investment in R&D. The R&D is done only by its headquarters in the North. For the Northern firm to develop the quality q_h, it must invest in R&D an amount that depends on the gap in the two qualities $(q_h - \bar{q}_l)$. R&D output is subject to decreasing returns to the amount invested. Since the R&D output translates into the quality of the product, output of quality is concave with respect to R&D expenditure E_h. I assume that $q_h = \sqrt{2E_h} + \bar{q}_l$. E_h is expressed in units of the North's labor. The R&D expenditure is $E_h(q_h) = \frac{(q_h - \bar{q}_l)^2}{2}$. As mentioned earlier, quality in the range $[0, \bar{q}_l]$ does not need R&D expenditure. To create a higher quality, the firm should spend more on R&D. Its R&D expenditure in value terms becomes the Northern wage, w_N, times $E_h(q_h)$:

$$w_N E_h(q_h) = \frac{w_N [Max\{0, (q_h - \bar{q}_l)\}]^2}{2} . \tag{1.3}$$

The R&D expenditure is zero if the quality is in the range $[0, \bar{q}_l]$, and increases if it is higher than $\bar{q}_l$, as in Figure 1.1.

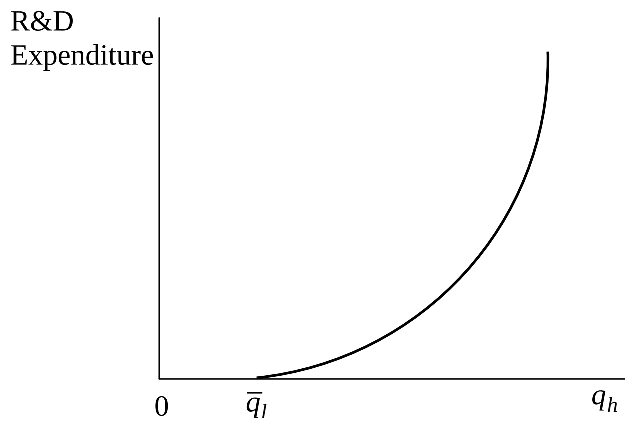

Figure 1.1

When the Northern firm produces x_h of the high quality product in the North, its cost of production is

$$C_h(x_h, q_h; w_N) = \lambda w_N q_h x_h . \qquad (1.4)$$

Note that the cost depends not on the gap $(q_h - \overline{q}_l)$ but on the level of high quality q_h. Since partial equilibrium analysis is used, it is assumed that the Northern wage is not affected by a change in demand for labor in the industry.

Before deriving the Northern firm's profit function, demand for the high quality product is specified. I assume that there is a world demand for this product, which is the aggregate of demand of the South and the North. I specify this demand as follows. The consumer views the low quality product and the high quality product as substitutes and consumes both. Then the demand for the high quality product is decreasing in the price of the high quality product and increasing in the price of the low quality product, when the level of low quality and the level of high quality are unchanged. On the other hand, the demand is increasing in the level of high quality and decreasing in the level of low quality, when the prices of the low quality and the high quality products are unchanged.

The world demand, D_h, for the high quality product is defined as a linear function for simplicity:

$$D_h(p_h, p_l, q_h, q_l) = \alpha + \beta p_l - \theta q_l - p_h + \gamma q_h ,^{6} \quad \text{where } 0 < \beta < 1,\ 0 < \theta < \gamma . \qquad (1.5)$$

The parameter α is a constant indicating market size. p_h (q_h) is the price (quality level) of the high quality product. Let me compare the magnitudes of the coefficients of the demand function:

(i) Comparison of Price Effects: A fall in the price of the high quality product increases the world demand by a larger amount than a fall in the price of the low quality product reduces the world demand: $0 < \beta < 1$.

(ii) Comparison of Quality Effects: A rise in the level of high quality increases the world demand by a larger amount than a rise in the level of low quality decreases the demand: $0 < \theta < \gamma$.

[6] A similar functional form appears in Rhee (1993), who investigates the changes in quality and profit of each firm – competing with price and quality in a duopoly model in the domestic market – in response to government regulated minimum quality standard. I modify his national demand function for the national firm to a world demand function for the multinational firm competing in international market, in which the effect of the international spillover of technological knowledge is reflected.

Since p_l is determined in a perfectly competitive market and $\overline{q}_l$ is given by assumption, the Northern firm treats these as exogenously given. The Southern wage is the numeraire: $w_S = 1$. Substituting $w_S = 1$ into (1.2), $\overline{p}_l = \lambda\overline{q}_l$. Using this, the demand function in (1.5) is rewritten as

$$D_h(p_h, q_h, \overline{q}_l) = \alpha + (\beta\lambda - \theta)\overline{q}_l - p_h + \gamma q_h. \qquad (1.5')$$

This demand is redefined as a function of only the low quality, the price of the high quality product and the high quality. The sign of $(\beta\lambda - \theta)$ may be either positive or negative.[7]

Only one Northern firm supplies the high quality product, so the demand D_h should be equal to its supply x_h: $D_h = x_h$. The inverse world demand function for the high quality product is obtained from (1.5').

$$p_h = (A - \theta\overline{q}_l + \gamma q_h) - x_h, \quad \text{where } A = \alpha + \beta\lambda\overline{q}_l. \qquad (1.6)$$

The term $(A - \theta\overline{q}_l + \gamma q_h)$ indicates an intercept of the inverse demand function. Since it should be defined as positive, I assume $(A - \theta\overline{q}_l)$ is positive.

$$(A - \theta\overline{q}_l) = \alpha + (\beta\lambda - \theta)\overline{q}_l > 0. \qquad (1.7)$$

The Northern firm chooses optimal output and quality of the product. I assume that expenditure for R&D and production occur in the same period of time.[8] I solve the profit maximization problem by choosing output and quality simultaneously. The Northern firm has its profit function:

$$\Pi_h = p_h x_h - C_h(x_h, q_h; w_N) - \frac{w_N[Max\{0, (q_h - \overline{q}_l)\}]^2}{2}.$$

The equation for the inverse demand function (1.6), production cost (1.4) and R&D expenditure (1.3) are used. When I suppose that $q_h > \overline{q}_l$,[9] the R&D expenditure for developing the quality q_h becomes $\frac{w_N(q_h - \overline{q}_l)^2}{2}$. The Northern firm chooses output x_h and quality q_h to maximize its profit.

$$\underset{x_h, q_h}{Max} \quad \Pi_h = (A - \theta\overline{q}_l + \gamma q_h - x_h)x_h - \lambda w_N q_h x_h - \frac{w_N(q_h - \overline{q}_l)^2}{2}. \qquad (1.8)$$

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[7] Which sign should be chosen will be explained in section 1.2.2.

[8] We can also think of a two-period model in which expenditure for R&D is determined in the first period, and production in the second period.

[9] q_h is proved later to be $q_h > \overline{q}_l$.

The first order conditions with respect to x_h and q_h are, respectively:[10]

$$\frac{\partial \Pi_h}{\partial x_h} = -2x_h + (\gamma - \lambda w_N) q_h + (A - \theta \overline{q}_l) = 0 \,, \tag{1.9}$$

$$\frac{\partial \Pi_h}{\partial q_h} = (\gamma - \lambda w_N) x_h - w_N q_h + w_N \overline{q}_l = 0 \,. \tag{1.10}$$

The output and quality are determined by the first order conditions.

For the profit determined at these x_h and q_h to be a maximum, the following three second order conditions should be satisfied. The marginal profit should decrease with respect to output and quality, respectively:

$$\frac{\partial^2 \Pi_h}{\partial {x_h}^2} = -2 < 0 \,, \tag{1.11}$$

$$\frac{\partial^2 \Pi_h}{\partial {q_h}^2} = -w_N < 0 \,. \tag{1.12}$$

Also, the own effects on marginal profit should be bigger than the cross effects.

$$\left(\frac{\partial^2 \Pi_h}{\partial {x_h}^2}\right)\left(\frac{\partial^2 \Pi_h}{\partial {q_h}^2}\right) - \left(\frac{\partial^2 \Pi_h}{\partial q_h \partial x_h}\right)^2 > 0 \,. \tag{1.13}$$

The cross effects are obtained from (1.9) or (1.10).

$$\frac{\partial^2 \Pi_h}{\partial q_h \partial x_h} = \frac{\partial^2 \Pi_h}{\partial x_h \partial q_h} = (\gamma - \lambda w_N) \,. \tag{1.14}$$

Substituting (1.11), (1.12) and (1.14) into (1.13), this is re-expressed as follows, and its sign is assumed to be positive.

$$2w_N - (\gamma - \lambda w_N)^2 > 0 \,. \tag{1.13'}$$

The output and quality are determined from (1.9) and (1.10).[11] The output is

$$x_h = \frac{w_N \{(A - \theta \overline{q}_l) + (\gamma - \lambda w_N) \overline{q}_l\}}{2w_N - (\gamma - \lambda w_N)^2} \,, \quad \text{where } 2w_N - (\gamma - \lambda w_N)^2 > 0 \,,\ (A - \theta \overline{q}_l) > 0 \,.$$

The denominator and the first term in the numerator are positive, respectively. The sign of the second term in the numerator, $(\gamma - \lambda w_N)$, is necessary for x_h. This is the cross effect in (1.14). If the marginal profit of output increases due to a rise in high quality (i.e., $\frac{\partial^2 \Pi_h}{\partial q_h \partial x_h} > 0$), the firm

[10] Appendix 1.1 explains mathematical implications of the first order conditions and second order conditions.
[11] See Appendix 1.2. The calculations for the price and profit are also provided.

would invest in R&D for improving quality. Thus I assume a complementary relationship between the marginal profit of the output and the quality. The cross effect should be positive:

$$(\gamma - \lambda w_N) > 0\,. \tag{1.15}$$

The quality is

$$q_h = \frac{2w_N \bar{q}_l + (\gamma - \lambda w_N)(A - \theta\bar{q}_l)}{2w_N - (\gamma - \lambda w_N)^2}, \quad \text{where } (\gamma - \lambda w_N) > 0\,. \tag{1.16}$$

I now check whether q_h is higher than the basic quality. Compare the magnitudes of the numerator and the denominator in (1.16). The first term in the numerator, $2w_N \bar{q}_l$, is equal to the first term in the denominator, $2w_N$, if $\bar{q}_l$ is normalized to one. While the second term in the numerator, $(\gamma - \lambda w_N)(A - \theta\bar{q}_l)$, is added to the first term, the second term in the denominator, $(\gamma - \lambda w_N)^2$, is subtracted from the first term. Thus the numerator is bigger than the denominator. The high quality q_h is therefore bigger than one. In other words, $q_h > \bar{q}_l$.[12]

The price of the high quality product is determined by substituting x_h and q_h into the equation for the price (1.6):[13]

$$p_h = \frac{w_N[(A - \theta\bar{q}_l)\{1 + \lambda(\gamma - \lambda w_N)\} + (\gamma + \lambda w_N)\bar{q}_l]}{2w_N - (\gamma - \lambda w_N)^2}\,.$$

Substituting the output, price and quality into the profit function (1.8), the profit is obtained:

$$\Pi_h = \frac{w_N\{(A - \theta\bar{q}_l) + (\gamma - \lambda w_N)\bar{q}_l\}^2}{2\{2w_N - (\gamma - \lambda w_N)^2\}}, \quad \frac{\partial \Pi_h}{\partial w_N} < 0\,.\text{[14]} \tag{1.17}$$

It is decreasing in the Northern wage.

1.2.2 World Economy with a Multinational Firm

I set up a model of a multinational firm that faces a tradeoff between the incentive effect provided by the factor price difference and the disincentive effect of a spillover of technological knowledge. I explain how the spillover affects the multinational firm's output, quality, price and profit.

If the Northern firm produces the high quality product in the South instead of producing it in the North, the firm will be able to reduce production cost because the Southern wage is lower than the Northern wage. This creates an incentive to shift its production facilities to the South.

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[12] This is identical to what I assumed earlier.
[13] See Appendix 1.2.
[14] See Appendix 1.3.

However, its headquarters remains in the North since R&D is done only by Northern labor with high skill.

Multinational production causes a spillover of technological knowledge into the South.[15] The Southern firms imitate the technology.[16] Their cost of imitation would be very small compared to the expenditure of the Northern firm on R&D, and it is assumed to be negligible. The spillover causes the low quality to rise. This makes the high quality product less competitive against the low quality product, and reduces the multinational firm's profit. The spillover plays the role of offsetting the incentive to become multinational.

If the incentive effect of the wage difference is larger than the disincentive effect of the spillover, a positive profit is obtained and the multinational firm can arise.[17] However, this is not the final deciding factor for whether the Northern firm becomes multinational. The final decision is made by comparing the profits of the national and multinational firms. If the profit of the multinational firm is higher than that of the national firm, the Northern firm becomes multinational. This issue will be examined in section 1.2.3. In this section, I assume that the Northern firm becomes multinational. I address the cost and profit of the multinational firm which reflect the wage difference and the spillover.

When the multinational firm produces in the South, its cost of production is

$$C_h^*(x_h^*, q_h^*; w_S) = \lambda w_S q_h^* x_h^* . \tag{1.18}$$

The symbol $*$ denotes the case after the transformation from national to multinational firm occurs. It employs only Southern labor whose wage is one. The R&D is conducted in the North. The R&D expenditure for developing the high quality q_h^* is

$$w_N E_h^*(q_h^*) = \frac{w_N (q_h^* - \overline{q}_l)^2}{2} . \tag{1.19}$$

The demand function should be explained again, because the spillover of knowledge appears as a rise in the low quality and thus changes the demand for the product of high quality.[18] First, let me account for the spillover. It arises when the multinational firm operates production in the

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[15] Other channels of spillover would be international trade, international fragmentation, international labor movement, technical and scientific publications, technology licensing, international M&A, and international R&D cooperation. See Branstetter (2001), who finds positive evidence of the spillover of knowledge from Japanese multinational firms that are located in the United States.

[16] Another way is a legal transfer of knowledge by firms owning it, for instance, licensing or sale of the knowledge in return for a reward. I do not consider these cases.

[17] The model of Horstman and Markusen (1987) is similar to my model in that it assumes a product which is differentiated by quality. However, they are interested in the problem of choice between licensing and FDI. When a licensing firm must provide a reward for a licensee to maintain reputation for quality, the firm becomes a multinational firm in order to avoid giving the reward.

[18] Most studies specify the effect of the spillover as a fall in production cost.

South. Production at a location near the Southern firms exposes its knowledge to them. They are able to look closely at its production facilities and scout laborers employed at the multinational firm to obtain the necessary information.

The high quality level is assumed to be positively associated with the level of knowledge of the multinational firm. Higher quality indicates that a higher level of knowledge is embodied in its product. Both Southern firms and multinational firm have access to the knowledge embodied in the basic quality $\bar{q}_l$. To improve the low quality level, the Southern firms therefore need knowledge that is embodied in quality levels above $\bar{q}_l$. The difference between the two qualities, $q_d^* = (q_h^* - \bar{q}_l)$, is used as a proxy for a pool of valuable knowledge which is exposed to Southern firms.

I assume that the increase in low quality due to the spillover is a fraction σ_q, $0 < \sigma_q < 1$, of the exposure q_d^*. The condition $0 < \sigma_q < 1$ means that the spillover does not completely equalize Southern quality to Northern quality. The increase in low quality is

$$\Delta q_l^* = \sigma_q q_d^*, \quad 0 < \sigma_q < 1. \tag{1.20}$$

The low quality reflecting the spillover is defined as the sum of the basic quality $\bar{q}_l$ and the increase in quality:

$$q_l^* = \bar{q}_l + \Delta q_l^* = (1 - \sigma_q)\bar{q}_l + \sigma_q q_h^*. \tag{1.21}$$

The low quality rises as the level of high quality rises. The condition $0 < \sigma_q < 1$ yields $\bar{q}_l < q_l^* < q_h^*$. The Southern firms are technology followers (i.e., technology imitators), so that their quality should be lower than the quality produced by the technology leader (i.e., the multinational firm). This means that the low quality cannot exceed the high quality.

Though the Southern firms' cost of imitation of the knowledge is assumed to be negligible, the marginal cost of production for the low quality product, λq_l^*, increases because the low quality is improved above the basic quality. The price for the low quality product must be its marginal cost, $p_l^* = \lambda q_l^*$. Thus its price also rises.

Substituting $p_l^* = \lambda q_l^*$ into (1.5), the world demand – reflecting the spillover – for the high quality product is rewritten as

$$D_h^*(p_h^*, q_h^*, q_l^*) = \alpha + (\beta\lambda - \theta)q_l^* - p_h^* + \gamma q_h^*. \tag{1.22}$$

The demand D_h^* is equal to its supply x_h^*. The inverse demand function is

$$p_h^* = \{\alpha + (\beta\lambda - \theta)q_l^* + \gamma q_h^*\} - x_h^* . \qquad (1.22')$$

In section 1.2.1, I mentioned that the sign of $(\beta\lambda - \theta)$ may be either positive or negative. To know which sign is meaningful for this model, I check the effect of the low quality on the revenue of the multinational firm, that is $\frac{\partial(p_h^* x_h^*)}{\partial q_l^*} = (\beta\lambda - \theta)x_h^*$. For given output x_h^*, the effect depends on $(\beta\lambda - \theta)$. Suppose that $(\beta\lambda - \theta) < 0$. Then the rise in the low quality reduces the revenue of the multinational firm: $\frac{\partial(p_h^* x_h^*)}{\partial q_l^*} < 0$. This implies that the spillover plays a disincentive role for multinational production because the spillover improves the low quality level and consumers switch consumption from the high quality product to the low quality product. The increased demand for the low quality product would increase the revenue of the Southern firms. The Southern firms have the incentive to imitate. This matches the insights of this paper. Thus the sign of $(\beta\lambda - \theta)$ is chosen to be

$$(\beta\lambda - \theta) < 0 . \qquad (1.23)$$

[19]

Substituting q_l^* (1.21) into p_h^* (1.22'),

$$p_h^* = (G + Hq_h^*) - x_h^* ,$$

$$\text{where } G = \{\alpha + (\beta\lambda - \theta)(1 - \sigma_q)\overline{q}_l\},\ H = \{\gamma + (\beta\lambda - \theta)\sigma_q\} . \qquad (1.24)$$

The first term G of the intercept $(G + Hq_h^*)$ is positive because $G = \{\alpha + (\beta\lambda - \theta)(1 - \sigma_q)\overline{q}_l\}$ > $\{\alpha + (\beta\lambda - \theta)\overline{q}_l\}$. The reasons are: $(\beta\lambda - \theta) < 0$ in (1.23); $0 < (1 - \sigma_q) < 1$ from (1.20); and $\{\alpha + (\beta\lambda - \theta)\overline{q}_l\} > 0$ in (1.7). Also, H is positive. This is because $\theta\sigma_q < \gamma\sigma_q < \gamma$ by the chosen rules of the parameters such that $0 < \theta < \gamma$ in (1.5) and $0 < \sigma_q < 1$, and because $\beta\lambda\sigma_q > 0$. Recall that $0 < \beta < 1$ in (1.5) and $\lambda > 0$ in (1.1). Therefore, the intercept $(G + Hq_h^*)$ is positive.

I address how the multinational firm determines output, quality, price and profit in the presence of the spillover. The revenue of the multinational firm is

$$p_h^* x_h^* = \{(G + Hq_h^*) - x_h^*\}x_h^* , \quad \text{where } G > 0,\ H > 0 . \qquad (1.25)$$

[19] If $(\beta\lambda - \theta) > 0$, the rise in the low quality increases the revenue of the multinational firm. This can mean that the revenue of the Southern firms decreases and the Southern firms lose business to the Northern firm. Thus the Southern firms do not imitate and the knowledge spillover does not occur. This case does not match the issue that this paper intends to analyze.

The profit function of the multinational firm is the revenue (1.25) less both the production cost (1.18) and the R&D expenditure (1.19):

$$\Pi_h^* = \{(G + Hq_h^*) - x_h^*\}x_h^* - \lambda w_S q_h^* x_h^* - \frac{w_N (q_h^* - \overline{q}_l)^2}{2}.$$

Recall that the Southern wage is the numeraire. After Substituting $w_S = 1$ into Π_h^*, I find the first order and second order conditions for profit maximization. The first order conditions with respect to output and quality are

$$\frac{\partial \Pi_h^*}{\partial x_h^*} = -2x_h^* + (H - \lambda)q_h^* + G = 0, \tag{1.26}$$

$$\frac{\partial \Pi_h^*}{\partial q_h^*} = (H - \lambda)x_h^* - w_N q_h^* + w_N \overline{q}_l = 0. \tag{1.27}$$

The second order conditions with respect to output and quality are

$$\frac{\partial^2 \Pi_h^*}{\partial x_h^{*2}} = -2 < 0, \tag{1.28}$$

$$\frac{\partial^2 \Pi_h^*}{\partial q_h^{*2}} = -w_N < 0. \tag{1.29}$$

Also, the own effects on marginal profit should be bigger than the cross effects:

$$(\frac{\partial^2 \Pi_h^*}{\partial x_h^{*2}})(\frac{\partial^2 \Pi_h^*}{\partial q_h^{*2}}) - (\frac{\partial^2 \Pi_h^*}{\partial q_h^* \partial x_h^*})^2 > 0. \tag{1.30}$$

The cross effects are

$$\frac{\partial^2 \Pi_h^*}{\partial q_h^* \partial x_h^*} = H - \lambda. \tag{1.31}$$

As explained in section 1.2.1, the relationship between the marginal profit with respect to output and quality is complementary. The sign of the cross effect is positive:

$$(H - \lambda) > 0. \tag{1.32}$$

Substituting (1.28) (1.29) and (1.31) into (1.30), this is re-expressed as

$$2w_N - (H - \lambda)^2 > 0. \tag{1.30'}$$

Using (1.26) and (1.27), the output of the multinational is[20]

$$x_h^* = \frac{w_N \{G + (H - \lambda)\overline{q}_l\}}{2w_N - (H - \lambda)^2}, \quad \text{where } G > 0,\ (H - \lambda) > 0,\ 2w_N - (H - \lambda)^2 > 0.$$

[20] See Appendix 1.4 for calculations for the output, quality, price and profit.

Its quality is

$$q_h^* = \frac{2w_N \bar{q}_l + G(H-\lambda)}{2w_N - (H-\lambda)^2}. \tag{1.33}$$

Its market price is

$$p_h^* = \frac{w_N\{G + (H+\lambda)\bar{q}_l\} + \lambda G(H-\lambda)}{2w_N - (H-\lambda)^2}.$$

Its profit is

$$\Pi_h^* = \frac{w_N\{G + (H-\lambda)\bar{q}_l\}^2}{2\{2w_N - (H-\lambda)^2\}}. \tag{1.34}$$

I proceed to investigate how an increase in spillover influences the level of output, quality, price and profit. The basic quality is normalized to one: $\bar{q}_l = 1$. The increase in spillover is represented by an increase in σ_q.

$$\frac{\partial x_h^*}{\partial \sigma_q} = \frac{2w_N(\beta\lambda - \theta)(H-\lambda)\{G + (H-\lambda)\}}{\{2w_N - (H-\lambda)^2\}^2} < 0,$$

where $(\beta\lambda - \theta) < 0$, $(H-\lambda) > 0$, $G > 0$, $2w_N - (H-\lambda)^2 > 0$.

$$\frac{\partial q_h^*}{\partial \sigma_q} = \frac{(\beta\lambda - \theta)\{G + (H-\lambda)\}\{2w_N + (H-\lambda)^2\}}{\{2w_N - (H-\lambda)^2\}^2} < 0.$$

$$\frac{\partial p_h^*}{\partial \sigma_q} = \frac{(\beta\lambda - \theta)\{G + (H-\lambda)\}\{2w_N H + \lambda(H-\lambda)^2\}}{\{2w_N - (H-\lambda)^2\}^2} < 0, \quad \text{where } H > 0.$$

$$\frac{\partial \Pi_h^*}{\partial \sigma_q} = \frac{w_N(\beta\lambda - \theta)(H-\lambda)\{G + (H-\lambda)\}^2}{\{2w_N - (H-\lambda)^2\}^2} < 0.$$

The spillover improves the level of the low quality. The demand for the high quality product decreases because consumers switch consumption from the high quality product to the low quality product. The multinational firm's output decreases. The level of high quality and the price of the high quality product fall. Its profit decreases.

1.2.3 Choice of a Northern Firm

The choice of whether the Northern firm becomes a multinational firm or remains a national firm is determined by comparing the profits of the multinational and national firm. If the multinational profit Π_h^* is larger (smaller) than the national profit Π_h, it chooses to become a multinational firm (a national firm).

To compare the two profits, the ratio of the multinational firm's profit to the national firm's profit $\frac{\Pi_h^*}{\Pi_h}$ is used. If the ratio is larger than one, $\frac{\Pi_h^*}{\Pi_h} > 1$, the Northern firm becomes a multinational firm. If it is less than or equal to one, $\frac{\Pi_h^*}{\Pi_h} \le 1$, it remains a national firm. Where the two profits are equal, I assume that it remains a national firm. From (1.17) and (1.34),

$$\frac{\Pi_h^*}{\Pi_h} = \frac{\dfrac{w_N\{G+(H-\lambda)\}^2}{2\{2w_N-(H-\lambda)^2\}}}{\dfrac{w_N\{(A-\theta)+(\gamma-\lambda w_N)\}^2}{2\{2w_N-(\gamma-\lambda w_N)^2\}}}\ , \quad \text{where } \bar{q}_l = 1.$$

For ease of manipulation, this is rewritten as

$$\frac{\Pi_h^*}{\Pi_h} = \frac{\tilde{\Pi}_h^*}{\tilde{\Pi}_h} \gtrless 1,$$

$$\text{where } \tilde{\Pi}_h^* = \frac{\{G+(H-\lambda)\}^2}{2w_N-(H-\lambda)^2} > 0, \quad \tilde{\Pi}_h = \frac{\{(A-\theta)+(\gamma-\lambda w_N)\}^2}{2w_N-(\gamma-\lambda w_N)^2} > 0.$$ [21]

I use log transformation of this criterion. The new criterion becomes

$$\ln\tilde{\Pi}_h^* - \ln\tilde{\Pi}_h \gtrless 0,$$

$$\text{where } \ln\tilde{\Pi}_h^* = 2\ln\{G+(H-\lambda)\} - \ln\{2w_N-(H-\lambda)^2\},$$

$$\ln\tilde{\Pi}_h = 2\ln\{(A-\theta)+(\gamma-\lambda w_N)\} - \ln\{2w_N-(\gamma-\lambda w_N)^2\}. \tag{1.35}$$

Before investigating the sign of $(\ln\tilde{\Pi}_h^* - \ln\tilde{\Pi}_h)$, I explain the features of the graphs of $\ln\tilde{\Pi}_h^*$ and $\ln\tilde{\Pi}_h$ in the space of logarithm of profit and the Northern wage. The graph of $\ln\tilde{\Pi}_h^*$ is decreasing in w_N:

$$\frac{\partial\ln\tilde{\Pi}_h^*}{\partial w_N} = \frac{-2}{2w_N-(H-\lambda)^2} < 0, \quad \text{where } 2w_N-(H-\lambda)^2 > 0, \tag{1.36}$$

$$\frac{\partial^2\ln\tilde{\Pi}_h^*}{\partial w_N{}^2} = \frac{4}{\{2w_N-(H-\lambda)^2\}^2} > 0.$$

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[21] The denominators of $\tilde{\Pi}_h^*$ and $\tilde{\Pi}_h$ are positive since $2w_N-(H-\lambda)^2 > 0$ in (1.30') and $2w_N-(\gamma-\lambda w_N)^2 > 0$ in (1.13'). Thus $\tilde{\Pi}_h^*$ and $\tilde{\Pi}_h$ are positive, respectively.

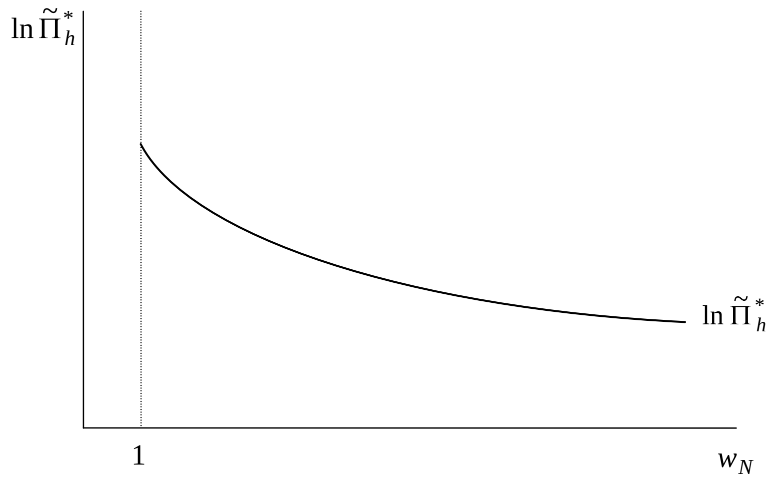

Figure 1.2

Figure 1.2 illustrates the multinational profit, $\ln \tilde{\Pi}_h^*$. Recall that the Northern wage is assumed to be higher than the Southern wage: $w_N > 1$.[22]

The graph of $\ln \tilde{\Pi}_h$ is decreasing in w_N as in Figure 1.3.

$$\frac{\partial \ln \tilde{\Pi}_h}{\partial w_N} = \frac{-2\lambda}{(A-\theta)+(\gamma-\lambda w_N)} - \frac{2\{1+\lambda(\gamma-\lambda w_N)\}}{2w_N-(\gamma-\lambda w_N)^2} < 0, \quad (1.37)$$

where $(A-\theta) > 0$,[23] $(\gamma-\lambda w_N) > 0$, $2w_N-(\gamma-\lambda w_N)^2 > 0$.

$\frac{\partial^2 \ln \tilde{\Pi}_h}{\partial w_N{}^2}$ is positive if the world demand is greater than or equal to two.[24]

$$\frac{\partial^2 \ln \tilde{\Pi}_h}{\partial w_N{}^2} > 0, \quad \text{where } w_N = 1.$$

The graph of $\ln \tilde{\Pi}_h$ is convex toward the origin at $w_N = 1$. However, the slope of $\ln \tilde{\Pi}_h$ is steeper than that of $\ln \tilde{\Pi}_h^*$.[25]

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22 w_N is actually the relative wage, $\frac{w_N}{w_S}$. However, the relative wage is the same as the Northern wage if the Southern wage is normalized to one. For convenience, I call w_N the Northern wage.

23 Since $(A-\theta\bar{q}_l) > 0$ in (1.7), $(A-\theta) > 0$ where $\bar{q}_l = 1$.

24 See Appendix 1.5.

25 It will be explained when I compare the graphs of $\ln \tilde{\Pi}_h^*$ and $\ln \tilde{\Pi}_h$.

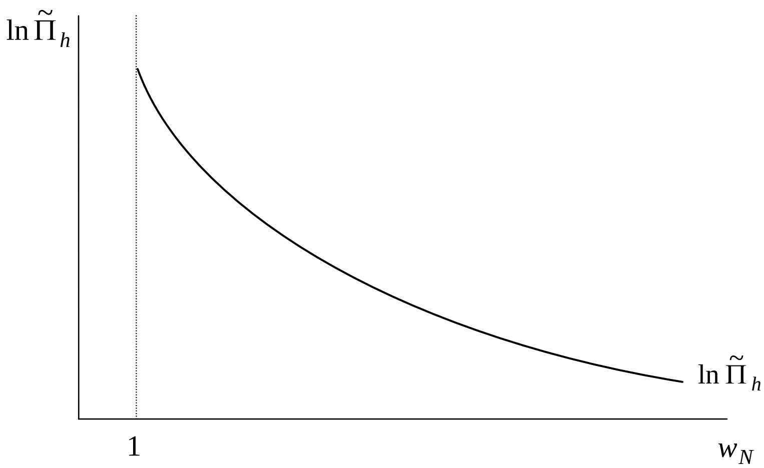

Figure 1.3

To investigate the sign of $(\ln\tilde{\Pi}_h^* - \ln\tilde{\Pi}_h)$, I put the graphs of $\ln\tilde{\Pi}_h^*$ and $\ln\tilde{\Pi}_h$ together in the same space. First, it is necessary to check whether these graphs intersect each other. If there is an intersection, it determines the cutoff wage $\hat{w}_N$.[26] Then $(\ln\tilde{\Pi}_h^* - \ln\tilde{\Pi}_h)$ has the signs, >0, $=0$ or <0 around $\hat{w}_N$. Compare $\ln\tilde{\Pi}_h^*$ and $\ln\tilde{\Pi}_h^*$ in (1.35). The first term in $\ln\tilde{\Pi}_h^*$ is larger than that in $\ln\tilde{\Pi}_h$:

$$2\ln\{G+(H-\lambda)\} > 2\ln\{(A-\theta)+(\gamma-\lambda w_N)\}.$$[27]

Also, if the second term in $\ln\tilde{\Pi}_h^*$ is larger than that in $\ln\tilde{\Pi}_h$,

$$\ln\{2w_N-(H-\lambda)^2\} > \ln\{2w_N-(\gamma-\lambda w_N)^2\},$$

$(\ln\tilde{\Pi}_h^* - \ln\tilde{\Pi}_h)$ has one of three possible cases, >0, $=0$ or <0. The Northern wage satisfying this condition is smaller than $\{1-\frac{(\beta\lambda-\theta)\sigma_q}{\lambda}\}$.[28] I denote $\{1-\frac{(\beta\lambda-\theta)\sigma_q}{\lambda}\}$ to be w_{NUB}. This w_{NUB} is bigger than one since $(\beta\lambda-\theta)<0$ in (1.23). Recall that w_N is also larger than one. Therefore, the Northern wage – ensuring one of the possible cases – will lie between one and w_{NUB}:

[26] I will explain in detail later how the cutoff wage is determined.

[27] Compare $\{G+(H-\lambda)\}$ with $\{(A-\theta)+(\gamma-\lambda w_N)\}$. Using G and H in (1.24), and A from (1.6) and (1.7),
$\{G+(H-\lambda)\}=(\alpha+\gamma-\theta)-\lambda(1-\beta)$ and $\{(A-\theta)+(\gamma-\lambda w_N)\}=(\alpha+\gamma-\theta)-\lambda(w_N-\beta)$.
The first terms in above two equations are the same. The second term in the first equation is smaller than that in the second equation since w_N >1: $\lambda(1-\beta)<\lambda(w_N-\beta)$.
Then $\{G+(H-\lambda)\}>\{(A-\theta)+(\gamma-\lambda w_N)\}$. $2\ln\{G+(H-\lambda)\}>2\ln\{(A-\theta)+(\gamma-\lambda w_N)\}$.

[28] See Appendix 1.6.

$$1 < w_N < w_{NUB} .$$

However, if the second term in $\ln \tilde{\Pi}_h^*$ is smaller than that in $\ln \tilde{\Pi}_h$,

$$\ln\{2w_N - (H-\lambda)^2\} < \ln\{2w_N - (\gamma - \lambda w_N)^2\},$$

$(\ln \tilde{\Pi}_h^* - \ln \tilde{\Pi}_h)$ is always positive. It implies that there is only the multinational firm. This means that a cutoff wage $\hat{w}_N$ does not exist. I do not consider this case.[29]

What I have done above is still not enough to tell us whether the intersection occurs in the range, $1 < w_N < w_{NUB}$. To confirm the existence of an intersection, I have to explain additionally the positions and shapes of the respective graphs for $\ln \tilde{\Pi}_h^*$ and $\ln \tilde{\Pi}_h$ in the range of w_N that is set above. The value of $\ln \tilde{\Pi}_h^*$ at $w_N = 1$ is

$$\ln \tilde{\Pi}_h^* = 2\ln\{G + (H-\lambda)\} - \ln\{2 - (H-\lambda)^2\}. \tag{1.38}$$

The value of $\ln \tilde{\Pi}_h^*$ at $w_N = w_{NUB}$ is

$$\ln \tilde{\Pi}_h^* = 2\ln\{G + (H-\lambda)\} - \ln[2\{1 - \frac{(\beta\lambda - \theta)\sigma_q}{\lambda}\} - (H-\lambda)^2]. \tag{1.39}$$

Second, the value of $\ln \tilde{\Pi}_h$ at $w_N = 1$ is

$$\ln \tilde{\Pi}_h = 2\ln\{(A-\theta) + (\gamma - \lambda)\} - \ln\{2 - (\gamma - \lambda)^2\}. \tag{1.40}$$

The value of $\ln \tilde{\Pi}_h$ at $w_N = w_{NUB}$ is, where $H = \{\gamma + (\beta\lambda - \theta)\sigma_q\}$:

$$\ln \tilde{\Pi}_h = 2\ln\{(A-\theta) + (H-\lambda)\} - \ln[2\{1 - \frac{(\beta\lambda - \theta)\sigma_q}{\lambda}\} - (H-\lambda)^2]. \tag{1.41}$$

Comparing $\ln \tilde{\Pi}_h^*$ and $\ln \tilde{\Pi}_h$, the value of $\ln \tilde{\Pi}_h^*$ at $w_N = 1$ is smaller than that of $\ln \tilde{\Pi}_h$.[30] And the value of $\ln \tilde{\Pi}_h^*$ at $w_N = w_{NUB}$ is larger than that of $\ln \tilde{\Pi}_h$.[31] As shown in Figure 1.2 and Figure 1.3, the slopes of $\ln \tilde{\Pi}_h^*$ and $\ln \tilde{\Pi}_h$ have negative signs, but the slope of $\ln \tilde{\Pi}_h^*$ is flatter than that of $\ln \tilde{\Pi}_h$:[32]

$$\left| \frac{\partial \ln \tilde{\Pi}_h^*}{\partial w_N} \right| < \left| \frac{\partial \ln \tilde{\Pi}_h}{\partial w_N} \right| .$$

[29] Since I want to analyze how the choice of the Northern firm is affected at the cutoff wage by the parameters, this case is meaningless for this purpose.
[30] See Appendix 1.7.
[31] See Appendix 1.8.
[32] See Appendix 1.9.

The graph of $\ln\tilde{\Pi}_h^*$ crosses that of $\ln\tilde{\Pi}_h$ from below in Figure 1.4. A cutoff wage $\hat{w}_N$ is determined where the two curves cross each other. At the cutoff wage, the national firm's profit $\ln\tilde{\Pi}_h$ is equal to the profit of the multinational firm $\ln\tilde{\Pi}_h^*$.

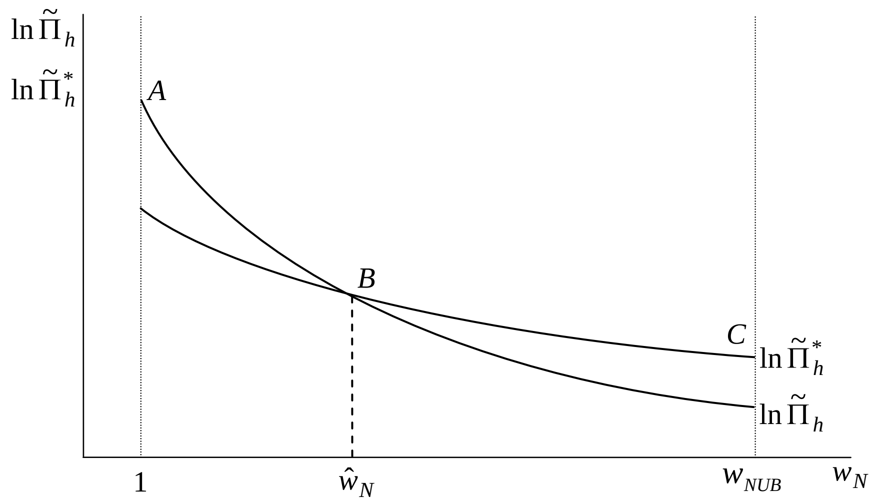

Figure 1.4

When the Northern wage lies in $(1,\ \hat{w}_N)$, the profit of the national firm $\ln\tilde{\Pi}_h$ is higher than that of the multinational firm $\ln\tilde{\Pi}_h^*$. The Northern firm remains national in this case. Its profit is represented by the curve AB. When w_N lies in $(\hat{w}_N,\ w_{NUB})$, the profit of the multinational firm is higher than that of the national firm. Consequently, the Northern firm becomes multinational. Its profit is represented by the curve BC. When w_N is at the point B, the profits of the national and multinational firm are the same. In this case, it is assumed that it remains national.

1.2.4 The Firm's R&D Expenditure and Level of Quality

I address two political issues. The first issue is how R&D expenditure and quality of product of the Northern firm are affected when the Northern wage rises. I first examine the effect on quality of product when the Northern firm is a national firm. Using (1.16),

$$q_h = \frac{2w_N\bar{q}_l + (\gamma - \lambda w_N)(A - \theta\bar{q}_l)}{2w_N - (\gamma - \lambda w_N)^2}, \quad \frac{\partial q_h}{\partial w_N} < 0.\text{[33]} \tag{1.16}$$

A rise in the Northern wage leads to a fall in the quality. Also, using (1.3) and $q_h > \bar{q}_l$,

$$w_N E_h(q_h) = \frac{w_N(q_h - \bar{q}_l)^2}{2}, \quad \frac{\partial\{w_N E_h(q_h)\}}{\partial w_N} < 0.\text{[34]} \tag{1.3}$$

[33] See (A1.10.1) in Appendix 1.10.

The national firm's R&D expenditure is decreased by a rise in the Northern wage.

When the Northern firm becomes multinational, the effect of a change in w_N on q_h^* is from (1.33)

$$q_h^* = \frac{2w_N \bar{q}_l + G(H-\lambda)}{2w_N - (H-\lambda)^2}, \quad \frac{\partial q_h^*}{\partial w_N} < 0 .^{35} \tag{1.33}$$

The effect of a change in w_N on the multinational firm's R&D expenditure is from (1.19)

$$w_N E_h^*(q_h^*) = \frac{w_N (q_h^* - \bar{q}_l)^2}{2}, \quad \frac{\partial \{w_N E_h^*(q_h^*)\}}{\partial w_N} < 0 .^{36} \tag{1.19}$$

A rise in the Northern wage decreases the quality of product and R&D expenditure of the multinational firm.

The second issue is how the Northern firm's R&D expenditure and the quality of its product would change when it shifts the production to the South. To analyze a change in the quality, I compare the qualities in (1.16) and (1.33). The denominator in (1.16) is smaller than the denominator in (1.33) since $(\gamma - \lambda w_N) > (H - \lambda)$ from Appendix 1.6. However, the comparison of magnitudes of the numerators in (1.16) and (1.33) depends on values of $(\gamma - \lambda w_N)(A - \theta \bar{q}_l)$ and $G(H-\lambda)$, respectively. Since $(\gamma - \lambda w_N) > (H - \lambda)$ and $(A - \theta \bar{q}_l) < G$ [37], $(\gamma - \lambda w_N)(A - \theta \bar{q}_l) \gtreqless G(H-\lambda)$. In each case, I compare the quality of the national firm's product to the quality of the multinational firm's product, and then compare the level of R&D expenditure for each of these firms.

(i) case: $(\gamma - \lambda w_N)(A - \theta \bar{q}_l) \geq G(H-\lambda)$

This case says that the numerator in (1.16) is larger than or equal to the numerator in (1.33). However, the denominator in (1.16) is smaller than the denominator in (1.33). This yields $q_h > q_h^*$.

Before addressing intuitively the reason for this result, I explain the implication of each term in case (i). From (1.14) and (1.31), $(\gamma - \lambda w_N)$ and $(H - \lambda)$ are the cross effects that the national firm and the multinational firm encounter, respectively. The cross effects represent how much the marginal profit of output changes when high quality changes. When the level of high quality is

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[34] See (A1.10.8) in Appendix 1.10.
[35] See (A1.11.1) in Appendix 1.11.
[36] See (A1.11.6) in Appendix 1.11.
[37] $A - \theta \bar{q}_l = \{\alpha + (\beta\lambda - \theta)\bar{q}_l\}$ in (1.7). $G = \{\alpha + (\beta\lambda - \theta)(1-\sigma_q)\bar{q}_l\}$ in (1.24). Since $(\beta\lambda - \theta) < 0$ in (1.23) and $0 < (1-\sigma_q) < 1$ from (1.20), $|(\beta\lambda - \theta)\bar{q}_l| > |(\beta\lambda - \theta)(1-\sigma_q)\bar{q}_l|$. Thus $A - \theta \bar{q}_l < G$.

increased, the multinational firm's marginal profit of output increases by less compared with that of the national firm because multinational production causes the spillover of knowledge into the Southern firms: $(H-\lambda)<(\gamma-\lambda w_N)$.

$(A-\theta\overline{q}_l)$ is equal to $\alpha+(\beta\lambda-\theta)\overline{q}_l$. α is a given world demand for the high quality product. As shown in (1.5), the demand for the high quality product is affected positively by the price of the low quality product and negatively by the low quality level. $(\beta\lambda-\theta)\overline{q}_l$ reflects the aggregation of these forces. Since $(\beta\lambda-\theta)$ is negative, $(\beta\lambda-\theta)\overline{q}_l$ represents the low quality product's substitution effect on the demand for the high quality product. Thus $(A-\theta\overline{q}_l)$ is the given net world demand faced by the national firm for the high quality product, after the low quality product's substitution effect is removed from the given world demand α.

G is equal to $\alpha+(\beta\lambda-\theta)(1-\sigma_q)\overline{q}_l$. Since $(1-\sigma_q)$ is positive, $(\beta\lambda-\theta)(1-\sigma_q)$ is negative. $(\beta\lambda-\theta)(1-\sigma_q)\overline{q}_l$ represents the low quality product's substitution effect on the demand for the high quality product when the spillover of knowledge into the Southern firms occurs due to multinational production. However, this substitution effect reflects only part of the actual substitution effect.[38] G is the given net world demand faced by the multinational firm for the high quality product, after the partial substitution effect of the low quality product is removed from the given world demand α.

I compare the magnitudes of G and $(A-\theta\overline{q}_l)$. Since $\left|(\beta\lambda-\theta)(1-\sigma_q)\overline{q}_l\right|<\left|(\beta\lambda-\theta)\overline{q}_l\right|$, the partial substitution effect in the mode of multinational production is smaller than the substitution effect in the mode of national production, such that the spillover does not exist. The given net world demand faced by the multinational firm becomes larger than the given net world demand faced by the national firm: $G>(A-\theta\overline{q}_l)$.

To better understand the implication of the condition in case (i), I re-express it as follows.

$$\left(\frac{\gamma-\lambda w_N}{H-\lambda}\right)\left(\frac{A-\theta\overline{q}_l}{G}\right)\geq 1, \tag{1.42}$$

where $\left(\frac{\gamma-\lambda w_N}{H-\lambda}\right)>1$, $\left(\frac{A-\theta\overline{q}_l}{G}\right)<1$.

Since the ratio of the first term on the LHS in (1.42) is larger than one, the relationship in (1.42) holds if the ratio of the second term approaches one.

---

[38] From (1.22), the actual substitution effect is $(\beta\lambda-\theta)q_l^*$. Using q_l^* in (1.21), the actual substitution effect is re-expressed as $(\beta\lambda-\theta)(1-\sigma_q)\overline{q}_l+(\beta\lambda-\theta)\sigma_q q_h^*$. Thus $(\beta\lambda-\theta)(1-\sigma_q)\overline{q}_l$ reflects the partial substitution effect.

The fact that the ratio of the first term is larger than one says that the cross effect of the multinational firm, $(H-\lambda)$, is small relative to the cross effect of the national firm, $(\gamma-\lambda w_N)$. Thus, the multinational firm has less incentive for developing high quality than the national firm. This causes the quality of the multinational firm's product to be lower than that of the national firm's product.

The interpretation of the ratio in the second term having a value close to one is that the given net world demand faced by the multinational firm, G, and the given net world demand faced by the national firm, $(A-\theta\overline{q}_l)$, are similar. Since the magnitude of the demand affects positively the profit and thus the level of quality, the large ratio (i.e., the ratio is close to one) implies that there is no difference in the two firms' incentives for developing high quality. Consequently, the combined effect of the above two ratios makes the quality of the multinational firm's product lower than that of the national firm's product: $q_h^* < q_h$.[39]

Since the R&D expenditure and the level of quality are positively correlated, $q_h^* < q_h$ implies that R&D expenditure of the multinational firm is smaller than that of the national firm.

(ii) case : $(\gamma-\lambda w_N)(A-\theta\overline{q}_l) < G(H-\lambda)$

The numerator in (1.16) is smaller than the numerator in (1.33). Also the denominator in (1.16) is smaller than the denominator in (1.33). Thus, the comparison of q_h with q_h^* is ambiguous: $q_h \lesseqgtr q_h^*$.

To explain intuitively, the condition in case (ii) is re-expressed as

$$\left(\frac{\gamma-\lambda w_N}{H-\lambda}\right)\left(\frac{A-\theta\overline{q}_l}{G}\right) < 1, \tag{1.43}$$

where $\left(\frac{\gamma-\lambda w_N}{H-\lambda}\right) > 1$, $\left(\frac{A-\theta\overline{q}_l}{G}\right) < 1$.

---

[39] I address this case with the large ratio of the second term in (1.42), in order to explain as clearly as possible why the multinational firm generates lower quality than the national firm. However, the difference in quality can also occur where the ratio in the second term has a less restricted value compared to the case in the main text. The relationship in (1.42) holds if $\left(\frac{A-\theta\overline{q}_l}{G}\right)$ does not have a very small value. In other words, if the given net world demand faced by the multinational firm for the high quality product, G, is not too large relative to the given net world demand faced by the national firm for the high quality product, $(A-\theta\overline{q}_l)$, the multinational firm generates lower quality than the national firm.

The relationship in (1.43) holds if the ratio in the first term has a small enough value that approaches one and if the ratio in the second term has a small enough value that approaches zero.[40]

The interpretation that the ratio of the first term is small enough is that the difference in the cross effects between the multinational firm and the national firm is not sufficiently different, but the cross effect of the multinational firm is still smaller than that of the national firm. This causes the multinational firm to have less incentive to develop high quality than the national firm.

The implication that the ratio of the second term is small enough is that the given net world demand facing the multinational firm is large relative to the given net world demand facing the national firm. This says that the multinational firm's incentive to generate high quality is stronger than the national firm's incentive.

These two incentives affect the quality decision of the firm in opposite directions. Whether the quality of the product of the multinational firm will be higher than that of the national firm depends on which incentive dominates. The comparison of quality of product between these firms becomes ambiguous. Also, the comparison of R&D expenditure of the multinational firm with that of the national firm is ambiguous.

1.2.5 Comparative Statics

The two profits represented by $\ln \tilde{\Pi}_h$ and $\ln \tilde{\Pi}_h^*$ are affected by the parameters $(\alpha, \beta, \gamma, \theta, \lambda, \sigma_q, w_S)$. The cutoff wage is also affected by these parameters. This section examines how the parameters shift the location of the cutoff wage, thus affecting the Northern firm's choice of whether it remains national or becomes multinational.

Before doing comparative statics, I set an economy as a benchmark. The benchmark economy is represented as the point B in Figure 1.4. The actual Northern wage prevailing the economy is w_N. It is also the cutoff wage. $w_N = \hat{w}_N$.

1.2.5.1 Intellectual Property Protection

A strengthening of intellectual property (IP) protection in the South restricts illegal use of the technological knowledge by the Southern firms. This reduces its flow to the Southern competitors. A fall in the diffusion rate σ_q represents stronger IP protection. The graph of $\ln \tilde{\Pi}_h$ does not

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[40] The relationship in (1.43) also holds if the ratio of the first term is small enough but that of the second term is not small enough, or if the ratio of the first term is not small enough but that of the second term is small enough. These cases yield the same result as the case in the main text.

depend on this parameter, so it does not shift in Figure 1.5. However, the fall makes the graph of $\ln\tilde{\Pi}_h^*$ shift upward.

$$\frac{\partial \ln\tilde{\Pi}_h}{\partial \sigma_q} = 0 .$$

$$\frac{\partial \ln\tilde{\Pi}_h^*}{\partial \sigma_q} = \frac{2(\beta\lambda-\theta)(H-\lambda)}{2w_N-(H-\lambda)^2} < 0 , \tag{1.44}$$

where $(\beta\lambda-\theta)<0$, $(H-\lambda)>0$, $2w_N-(H-\lambda)^2>0$.

From (1.44), the extent of the shift of $\ln\tilde{\Pi}_h^*$ is decreasing in w_N, $w_N \in (1, w_{NUB})$. The new graph becomes steeper.

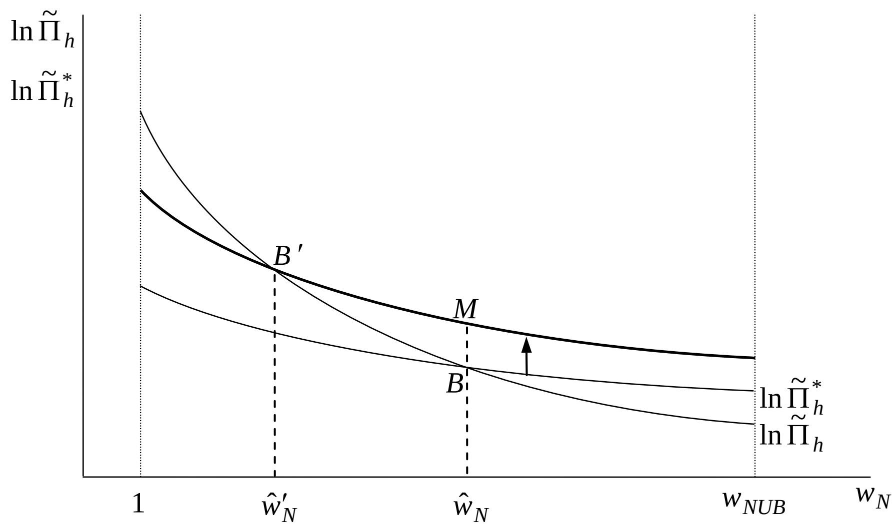

Figure 1.5

The point B in Figure 1.5 represents the benchmark economy and indicates the profit of the national firm. Due to the strengthening of IP protection, it moves along the curve of $\ln\tilde{\Pi}_h$ to B'. The cutoff wage falls from $\hat{w}_N$ to $\hat{w}'_N$. Let me explain the implication of this. The disincentive of the spillover is reduced by stronger IP protection. The profitability of the multinational firm improves. Then the incentive to relocate production to the South occurs at a smaller wage difference between the two countries than before the IP protection is strengthened.

However, the actual wages in the North and the South are not changed, due to the assumption that this industry does not influence the labor markets in both countries. The wage remains at $\hat{w}_N$, which is higher than the new cutoff wage $\hat{w}'_N$. At $\hat{w}_N$, the profit of the national firm is represented by B and the profit of the multinational firm by M. Since M is above B, the profit of the multinational firm is higher than the national firm's profit. Thus the Northern firm has an incentive to locate its production in the South. The multinational firm arises. Also, the strengthening of IP protection encourages the North to improve the quality of its product.

1.2.5.2 Wage in the South

Suppose that the Southern wage falls. For example, some Asian countries' financial crises in the late 1990s reduced their wages. Though w_N is the Northern wage relative to the Southern wage, I have regarded w_N as the Northern wage since the Southern wage is assumed to be one. However, the Southern wage has a value that is not one in this subsection. I express explicitly w_N as a relative wage only in this subsection: $\frac{w_N}{w_S}$. Since the relative wage is at least larger than one, the lower boundary is a value, $\frac{w_{NLB}}{w_S}$, that is larger than one: $\frac{w_{NLB}}{w_S} > 1$. The upper boundary is $\frac{w_{NUB}}{w_S}$ at $w_N = w_{NUB}$.

If the Southern wage falls from w_S to w'_S, the cutoff wage $(\frac{\hat{w}_N}{w_S})$ in Figure 1.6 rises to the new relative wage $(\frac{\hat{w}_N}{w'_S})$. The fall in w_S does not shift the graphs of $\ln\tilde{\Pi}_h$ and $\ln\tilde{\Pi}^*_h$. The values of $\ln\tilde{\Pi}_h$ and $\ln\tilde{\Pi}^*_h$ corresponding to $(\frac{\hat{w}_N}{w'_S})$ are determined along these curves. Since the multinational firm's profit M is above the national firm's profit N, the firm becomes multinational. Intuitively, the fall in the Southern wage decreases the production cost in the South, so this increases the incentive to become multinational.[41] [42]

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[41] When some Asian countries faced financial crises in the late 1990s, the fall in their wage induced foreign direct investments to move there. In other study, Aguiar and Gopinath (2005) contend that the lowered price of assets, due to a liquidity crisis of firm, induced a lot of inflows of foreign capital in the form of M&A there.

[42] I can consider another case; if the Southern wage rises, the Northern firm remains national. For example, when the wage rises due to strong labor unions in the South, the Northern firms tend to avoid production there. In fact, the existence of strong labor unions in the region where production is expected to be established may affect the firms' choice of location since strong labor unions tend to raise wages and benefits, and thus increase the cost of production there.

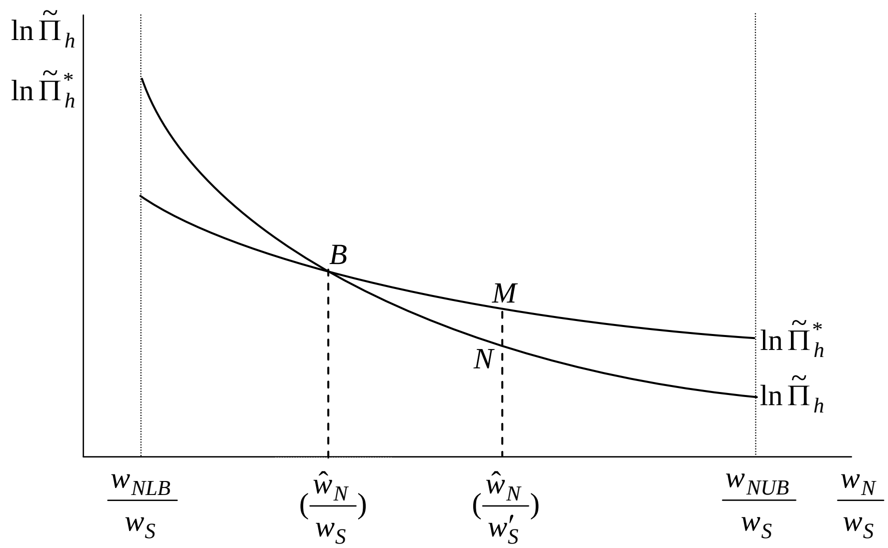

Figure 1.6

1.2.5.3 Growth of the World Market

The parameter α reflects the size of the world market. An increase in α means growth of the world market. The respective demand functions facing the national firm and the multinational firm shift up by the same magnitude when the world market grows. Due to the increase in α, the levels of production, high quality, price and profit that are associated with both the national firm and the multinational firm increase.[43] The growth of the market makes the graphs of $\ln \tilde{\Pi}_h$ and $\ln \tilde{\Pi}_h^*$ shift upward as in Figure 1.7.

$$\frac{\partial \ln \tilde{\Pi}_h}{\partial \alpha} = \frac{2}{(A-\theta)+(\gamma-\lambda w_N)} > 0, \quad \text{where } (A-\theta)>0,\ (\gamma-\lambda w_N)>0. \tag{1.45}$$

$$\frac{\partial \ln \tilde{\Pi}_h^*}{\partial \alpha} = \frac{2}{(A-\theta)+(\gamma-\lambda)} > 0, \quad \text{where } (\gamma-\lambda)>0.^{44} \tag{1.46}$$

Since the extent of the shift of $\ln \tilde{\Pi}_h$ in (1.45) is increasing in w_N, $w_N \in (1,\ w_{NUB})$, the new graph becomes less steep. From (1.46), the new graph of $\ln \tilde{\Pi}_h^*$ has no change in its slope.

The extent of the shift of $\ln \tilde{\Pi}_h$ is bigger than that of $\ln \tilde{\Pi}_h^*$. The reason is that the denominator in (1.45) is smaller than that in (1.46), since $w_N > 1$:

[43] See Appendix 1.12.
[44] $(\gamma-\lambda) > (\gamma-\lambda w_N)$ where $1 < w_N < w_{NUB}$. From (1.15), $(\gamma-\lambda w_N) > 0$. Therefore, $(\gamma-\lambda) > 0$.

$$\frac{\partial \ln \widetilde{\Pi}_h}{\partial \alpha} > \frac{\partial \ln \widetilde{\Pi}_h^*}{\partial \alpha}.$$

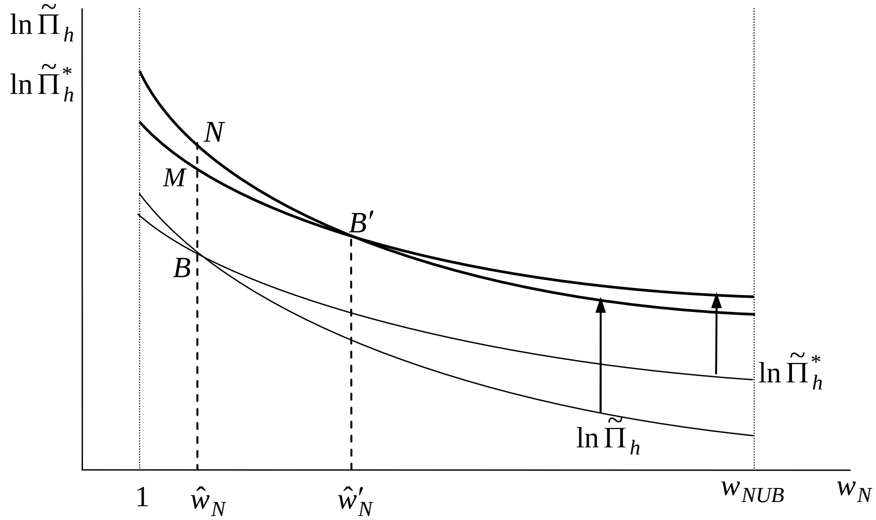

Figure 1.7

The cutoff wage rises from $\hat{w}_N$ to $\hat{w}'_N$. This implies that the Northern firm has the incentive to relocate production to the South only when the wage difference between the two countries is larger than before the market grows. The reason is as follows. The market growth increases the profits of both the national firm and the multinational firm. The Northern firm increases investment in R&D to improve quality. The level of high quality rises. This causes the gap between low quality and high quality to be larger. Thus the Southern firms would imitate more technology. The negative effect of spillover is larger. If the wage difference remains the same as before the market grows, the positive effect of cost reduction from multinational production would be smaller relative to the spillover effect. Therefore, in order for the multinational firm to emerge, the positive effect of cost reduction should be larger than the negative effect of spillover, and at the same time it earns more profit than that of the national firm. This means that the multinational firm should emerge at the larger wage difference.

However, since the wages in the North and the South are assumed to be given, the national firm earns the profit N and the multinational firm earns the profit M. The profit N is higher than M. The firm remains national.

1.2.5.4 Preference for Low Quality

The parameter θ is the degree of preference for low quality. A higher θ means higher preference for low quality.[45] A rise in θ for a given low quality causes consumers to substitute the low quality product for the high quality product. The demand for the high quality product falls.

An increase in the preference for low quality shifts the graphs of $\ln\tilde{\Pi}_h$ and $\ln\tilde{\Pi}_h^*$ downward in Figure 1.8 and Figure 1.9.

$$\frac{\partial \ln\tilde{\Pi}_h}{\partial\theta} = \frac{-2}{(A-\theta)+(\gamma-\lambda w_N)} < 0. \tag{1.47}$$

$$\frac{\partial \ln\tilde{\Pi}_h^*}{\partial\theta} = \frac{-2\sigma_q(H-\lambda)}{2w_N-(H-\lambda)^2} - \frac{2}{(A-\theta)+(\gamma-\lambda)} < 0, \tag{1.48}$$

where $(H-\lambda)>0$, $2w_N-(H-\lambda)^2>0$, $(\gamma-\lambda)>0$.

The extent of the shift of $\ln\tilde{\Pi}_h$ in (1.47) is increasing in w_N, so that the new graph is steeper. From (1.48), the extent of the shift of $\ln\tilde{\Pi}_h^*$ is decreasing in w_N. The new graph of $\ln\tilde{\Pi}_h^*$ becomes less steep.

To figure out which graph shifts more, I compare the magnitudes of $\frac{\partial \ln\tilde{\Pi}_h}{\partial\theta}$ and $\frac{\partial \ln\tilde{\Pi}_h^*}{\partial\theta}$. The term on the RHS in (1.47) is larger in absolute value than the second term on the RHS in (1.48). However, because, in absolute value, the second term is added to the first term on the RHS in (1.48), it is difficult to compare the magnitudes of $\frac{\partial \ln\tilde{\Pi}_h}{\partial\theta}$ and $\frac{\partial \ln\tilde{\Pi}_h^*}{\partial\theta}$.

The choice of the firm is determined by the magnitudes of the shifts of the two graphs. Two cases are considered.

(i) case: $\left|\frac{\partial \ln\tilde{\Pi}_h}{\partial\theta}\right| > \left|\frac{\partial \ln\tilde{\Pi}_h^*}{\partial\theta}\right|$

The extent of the downward shift of $\ln\tilde{\Pi}_h$ is larger than that of $\ln\tilde{\Pi}_h^*$ in Figure 1.8. The cutoff wage falls from $\hat{w}_N$ to $\hat{w}'_N$. The profit of the multinational firm M is higher than the profit of the national firm N. The firm becomes multinational.

[45] The parameter θ can have another possible interpretation. If the low quality rises for a given θ, the parameter θ represents how much the demand for the high quality product decreases. However, I do not consider this implication.

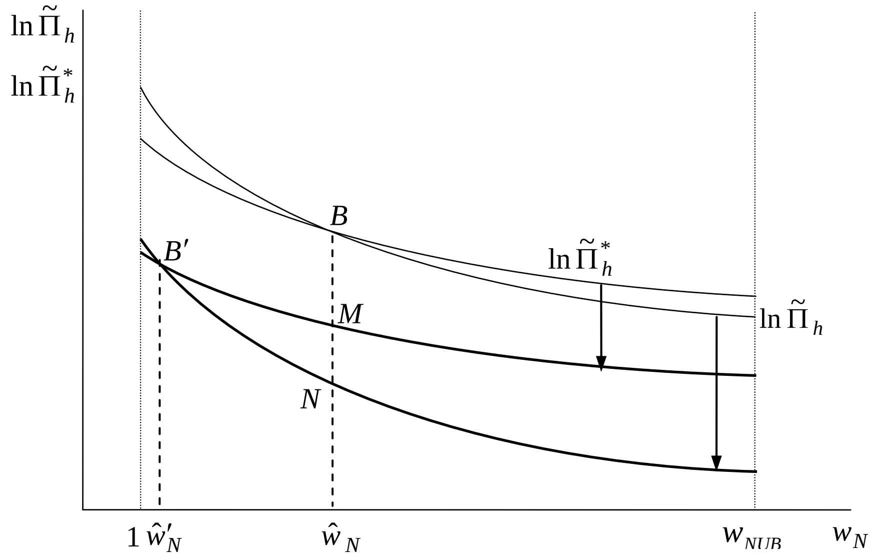

Figure 1.8

(ii) case: $\left|\dfrac{\partial \ln\tilde{\Pi}_h}{\partial\theta}\right| < \left|\dfrac{\partial \ln\tilde{\Pi}_h^*}{\partial\theta}\right|$

The extent of the downward shift of $\ln\tilde{\Pi}_h$ is smaller than that of $\ln\tilde{\Pi}_h^*$ in Figure 1.9. The cutoff wage rises from $\hat{w}_N$ to $\hat{w}'_N$. The profit N is higher than the profit M. The firm remains national.

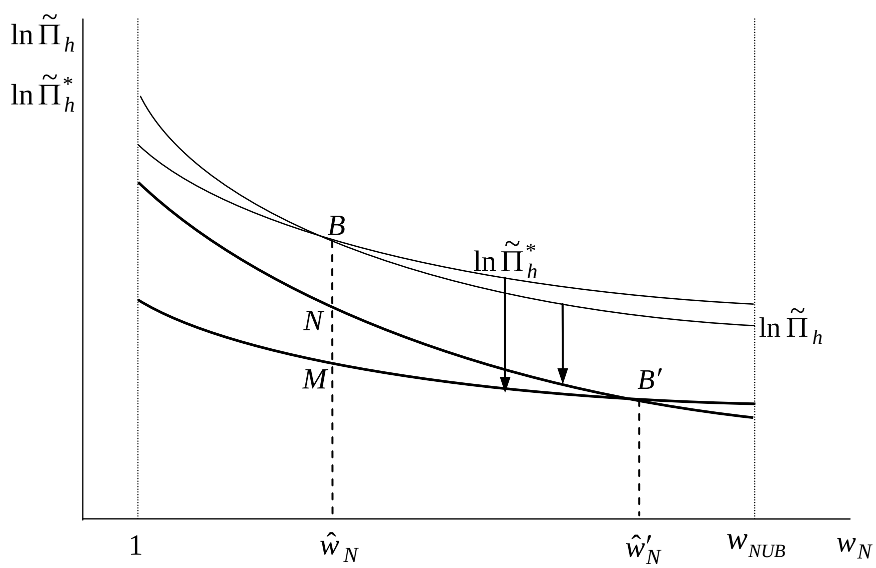

Figure 1.9

As shown in Figure 1.8 and Figure 1.9, the rise in preference for low quality reduces the profits of both the national firm and the multinational firm, since consumers switch consumption from the high quality product to the low quality product. However, these profits vary according to values of the parameters. It is difficult to distinguish which firm's profit would decrease more. Thus it may make the Northern firm become either national or multinational.[46]

1.2.5.5 Preference for High Quality

The parameter γ is the degree of preference for high quality. A higher γ implies higher preference for high quality. A rise in γ makes consumers purchase more of the high quality product. An increase in the preference shifts the graphs of $\ln\tilde{\Pi}_h$ and $\ln\tilde{\Pi}_h^*$ upward in Figure 1.10 and Figure 1.11.

$$\frac{\partial \ln\tilde{\Pi}_h}{\partial\gamma} = \frac{2(\gamma-\lambda w_N)}{2w_N-(\gamma-\lambda w_N)^2} + \frac{2}{(A-\theta)+(\gamma-\lambda w_N)} > 0\,, \tag{1.49}$$

$$\frac{\partial \ln\tilde{\Pi}_h^*}{\partial\gamma} = \frac{2(H-\lambda)}{2w_N-(H-\lambda)^2} + \frac{2}{(A-\theta)+(\gamma-\lambda)} > 0\,, \tag{1.50}$$

where $(\gamma-\lambda w_N) > (H-\lambda)$,[47] $(\gamma-\lambda w_N) < (\gamma-\lambda)$.

The first term on the RHS in (1.49) is decreasing in w_N, but the second term is increasing. The new graph of $\ln\tilde{\Pi}_h$ becomes steeper or less steep according to the net effect of changes in the two terms. From (1.50), the extent of the shift of $\ln\tilde{\Pi}_h^*$ is decreasing in w_N, so that the new graph is steeper.

The extent of the shift of $\ln\tilde{\Pi}_h$ is bigger than that of $\ln\tilde{\Pi}_h^*$, since the first term in (1.49) is bigger than the first term in (1.50) and the second term in (1.49) is bigger than the second term in (1.50):

$$\frac{\partial \ln\tilde{\Pi}_h}{\partial\gamma} > \frac{\partial \ln\tilde{\Pi}_h^*}{\partial\gamma}\,.$$

I investigate the effect of higher preference for high quality when the increase in $\ln\tilde{\Pi}_h$ is increasing or decreasing in w_N.

---

[46] Simulation is needed to know what values of the parameters would lead to a national or multinational firm.

[47] From Appendix 1.6, $(\gamma-\lambda w_N)-(H-\lambda) > 0$.

(i) case: the increase in $\ln \tilde{\Pi}_h$ is increasing in w_N

The new graph of $\ln \tilde{\Pi}_h$ becomes less steep as in Figure 1.10. The cutoff wage rises from $\hat{w}_N$ to $\hat{w}'_N$. The new cutoff wage does not give the Northern firm an incentive to relocate. It remains national and earns the profit N. This profit is higher than the profit of the multinational firm M.

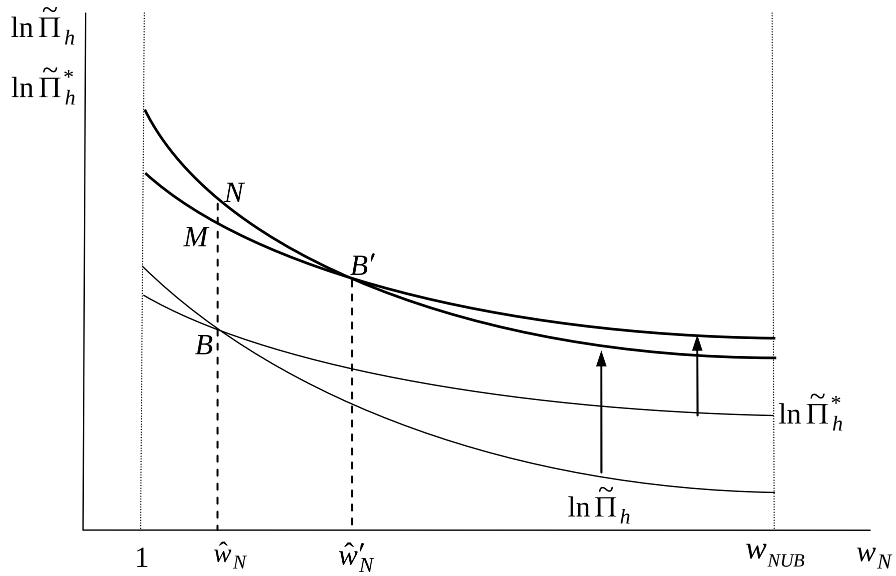

Figure 1.10

(ii) case: the increase in $\ln \tilde{\Pi}_h$ is decreasing in w_N

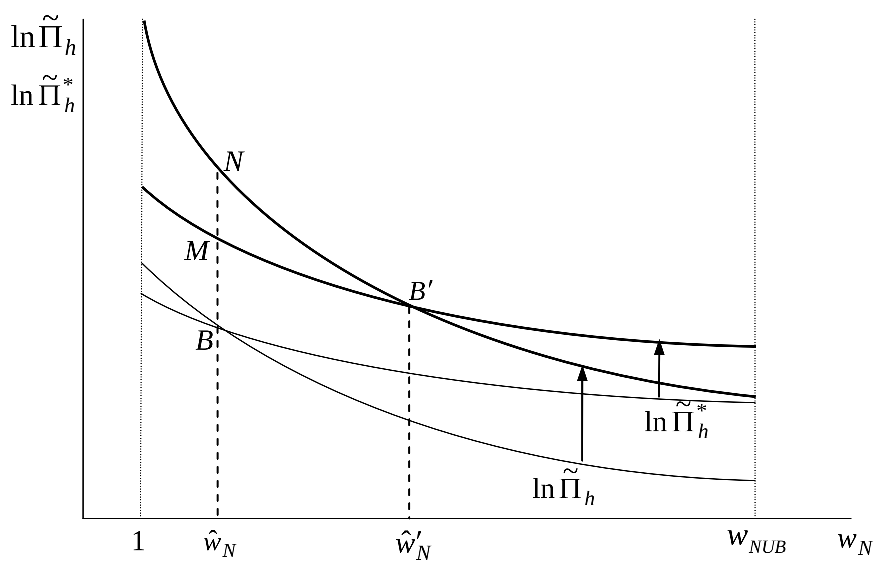

Figure 1.11

The new graph of $\ln\tilde{\Pi}_h$ become steeper as in Figure 1.11. The cutoff wage rises from $\hat{w}_N$ to $\hat{w}'_N$. The Northern firm remains national and earns the profit N.

Regardless of whether the increase in $\ln\tilde{\Pi}_h$ is increasing or decreasing in w_N as shown in Figure 1.10 and Figure 1.11, the rise in preference for high quality makes the Northern firm remain national.[48] The reason is that consumers have higher willingness to pay for high quality and become less sensitive to the price of the high quality product. Thus the firm has less incentive for relocating production to reduce the production cost.

1.2.5.6 Response to the Price of the Low Quality Product

The parameter β is the response to the price of the low quality product. A rise in β for a given price of the low quality product means that consumers feel less satisfied with the given price than before. They think that the price is expensive relative to the level of low quality, and switch consumption from the low quality product to the high quality product. The demand for the high quality product increases.

An increase in the response shifts the graphs of $\ln\tilde{\Pi}_h$ and $\ln\tilde{\Pi}_h^*$ upward in Figure 1.12 and Figure 1.13.

$$\frac{\partial \ln\tilde{\Pi}_h}{\partial\beta} = \frac{2\lambda}{(A-\theta)+(\gamma-\lambda w_N)} > 0 . \tag{1.51}$$

$$\frac{\partial \ln\tilde{\Pi}_h^*}{\partial\beta} = \frac{2\lambda}{(A-\theta)+(\gamma-\lambda)} + \frac{2\lambda\sigma_q(H-\lambda)}{2w_N-(H-\lambda)^2} > 0 . \tag{1.52}$$

The extent of the shift of $\ln\tilde{\Pi}_h$ in (1.51) is increasing in w_N, so that the new graph is less steep. From (1.52), the extent of the shift of $\ln\tilde{\Pi}_h^*$ is decreasing in w_N. The new graph of $\ln\tilde{\Pi}_h^*$ becomes steeper.

To investigate which graph moves more, I compare the magnitudes of $\frac{\partial \ln\tilde{\Pi}_h}{\partial\beta}$ and $\frac{\partial \ln\tilde{\Pi}_h^*}{\partial\beta}$. The term on the RHS in (1.51) is bigger than the first term on the RHS in (1.52). However, since the first term is added to the second term in (1.52), it is difficult to compare the extent of the shift of $\ln\tilde{\Pi}_h$ and that of the shift of $\ln\tilde{\Pi}_h^*$. The following two cases are considered.

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[48] As seen in Figure 1.10 and Figure 1.11, whether the increase in $\ln\tilde{\Pi}_h$ is increasing or decreasing in w_N does not change the direction in which the cutoff wage moves.

(i) case: $\frac{\partial \ln \tilde{\Pi}_h}{\partial \beta} > \frac{\partial \ln \tilde{\Pi}_h^*}{\partial \beta}$

The cutoff wage rises from $\hat{w}_N$ to $\hat{w}'_N$ in Figure 1.12 and the national firm's profit N is higher than the multinational firm's profit M. The firm remains national.

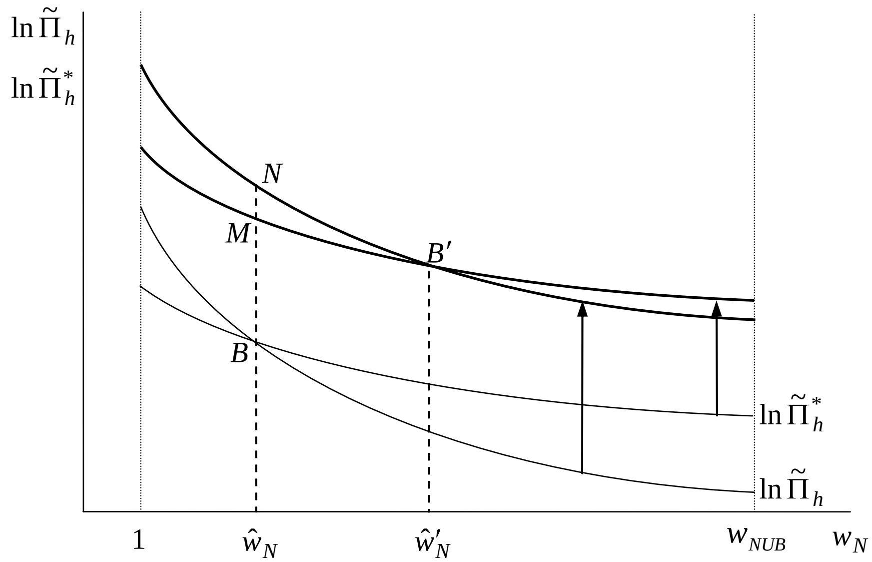

Figure 1.12

(ii) case: $\frac{\partial \ln \tilde{\Pi}_h}{\partial \beta} < \frac{\partial \ln \tilde{\Pi}_h^*}{\partial \beta}$

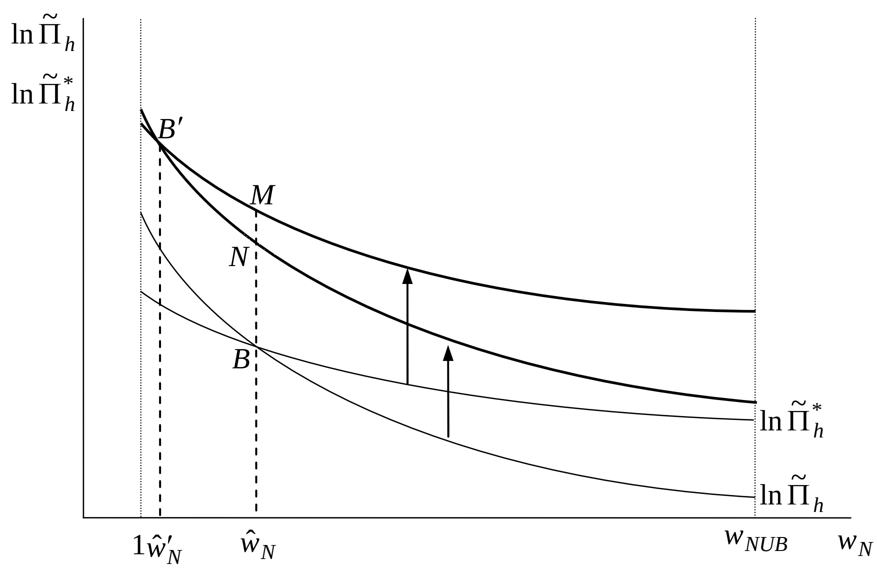

Figure 1.13

The cutoff wage falls from $\hat{w}_N$ to $\hat{w}'_N$ in Figure 1.13. The profit M is higher than the profit N. The firm becomes multinational.

As shown in Figure 1.12 and Figure 1.13, the rise in consumers' response to the price of the low quality product increases the profits of both the national firm and the multinational firm since consumers substitute the high quality good for the low quality product. However, there is ambiguity regarding which firm's profit would increase more, for these profits vary according to values of the parameters. This may make the Northern firm become either national or multinational.[49]

1.2.5.7 Productivity of Production Technology

Suppose that the technology of production – this technology is not for producing the quality attribute – improves in the Northern firm and Southern firms. This reduces the labor input requirement of production. The parameter λ indicates the input requirement for both the Northern firm and the Southern firms. When their productivity of production increases in the same proportion, this means that the value of λ falls.

The increase in productivity shifts the graphs of $\ln\tilde{\Pi}_h$ and $\ln\tilde{\Pi}_h^*$ upward in Figure 1.14.

$$\frac{\partial \ln\tilde{\Pi}_h}{\partial\lambda} = \frac{-2w_N(\gamma-\lambda w_N)}{2w_N-(\gamma-\lambda w_N)^2} - \frac{2(w_N-\beta)}{(A-\theta)+(\gamma-\lambda w_N)} < 0\,, \tag{1.53}$$

$$\frac{\partial \ln\tilde{\Pi}_h^*}{\partial\lambda} = \frac{-2(1-\beta\sigma_q)(H-\lambda)}{2w_N-(H-\lambda)^2} - \frac{2(1-\beta)}{(A-\theta)+(\gamma-\lambda)} < 0\,, \tag{1.54}$$

where $(\gamma-\lambda w_N) > (H-\lambda)$, $(w_N-\beta) > (1-\beta)$, $(\gamma-\lambda w_N) < (\gamma-\lambda)$.

The extent of the shift of $\ln\tilde{\Pi}_h$ in (1.53) can be either increasing or decreasing in w_N. The new graph can be less steep or steeper. Figure 1.14 shows only the decreasing case. In other words, the new graph of $\ln\tilde{\Pi}_h$ becomes steeper. From (1.54), the extent of the shift of $\ln\tilde{\Pi}_h^*$ is decreasing in w_N. The new graph of $\ln\tilde{\Pi}_h^*$ becomes steeper.

The extent of the shift of $\ln\tilde{\Pi}_h$ is bigger than that of $\ln\tilde{\Pi}_h^*$ since the first and second term in (1.53) are bigger in absolute value than the corresponding terms in (1.54):

$$\left|\frac{\partial \ln\tilde{\Pi}_h}{\partial\lambda}\right| > \left|\frac{\partial \ln\tilde{\Pi}_h^*}{\partial\lambda}\right|.$$

[49] Simulation is needed to know what values of the parameters would lead to a national or multinational firm.

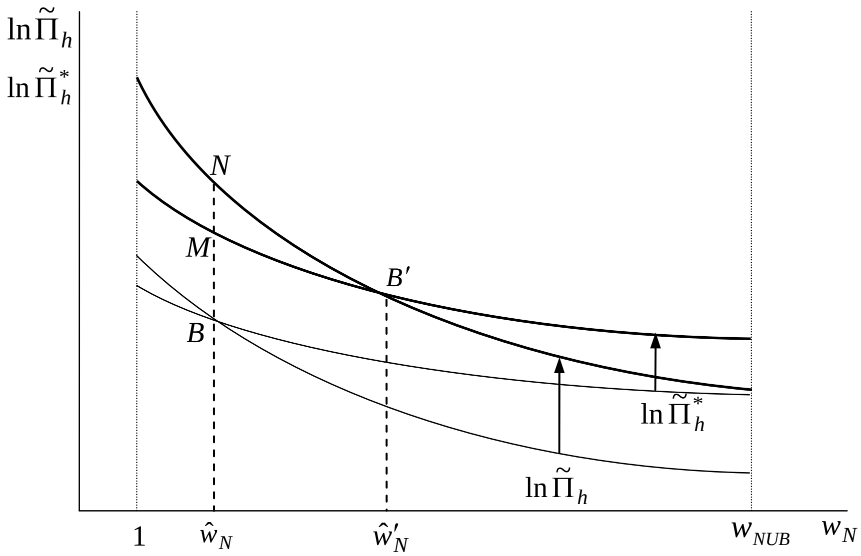

Figure 1.14

The cutoff wage rises from $\hat{w}_N$ to $\hat{w}'_N$. The profit of the national firm N is higher than the profit of the multinational firm M. The firm remains national. This can be understood as follows. An increase in productivity reduces the amount of labor used in production and reduces the production costs in the North and the South for a given output and quality.[50] However, the reduction in production cost is larger in the North than in the South since the Northern wage is higher. Also, the gap in the production cost between the two countries becomes smaller than before the improvement of productivity. This gives less incentive for the Northern firm to shift its production to the South. The Northern firm remains national.

As seen in subsection 1.2.5.5, whether the increase in $\ln\tilde{\Pi}_h$ is increasing or decreasing in w_N does not change the direction in which the cutoff wage moves. I will not address the case that the increase in $\ln\tilde{\Pi}_h$ is increasing in w_N, for the result is the same as the result shown in Figure 1.14.

1.3 Conclusion

A firm in the North considers two main factors when it chooses production location across borders. The first is the factor price difference between the North and the South. A lower wage in the South than in the North gives the Northern firm an incentive to relocate its production to the

[50] Actually, the parameter λ also changes the output and quality, which influence the production cost. These changes, for more exact interpretation, should be taken into account. However, it is hard to separate the effect of λ into the respective effects corresponding to output and quality because all effects are mixed. To explain intuitively, I consider this simple situation.

South to use the cheaper labor. The second is spillover of technological knowledge to Southern firms from the Northern firm, which occurs when the Northern firm's production is in the South. The Northern firm produces the high quality product. The spillover improves the quality of the low quality product produced by the Southern firms. Due to substitution between the low quality product and the high quality product, demand for the high quality product decreases. This provides a disincentive to becoming multinational.

If the net effect of the incentive effect and the disincentive effect is positive, the possibility of the emergence of a multinational firm arises. However, in order for the Northern firm to switch from actually being a national firm to becoming a multinational firm, the profit of the multinational firm must also be larger than that of the national firm. Otherwise, the Northern firm remains national.

When the Northern firm changes from being a national firm to becoming a multinational firm, its R&D expenditure and quality of product decrease if, after the low quality product's substitution effect is removed, a given world demand faced by the multinational firm for the high quality product is not too large relative to a given world demand faced by the national firm for the high quality product. If the given world demand faced by the multinational firm is sufficiently large relative to that faced by the national firm, the Northern firm's R&D expenditure and quality of product can increase, decrease or not change. A rise in Northern wage causes a decrease in R&D expenditure and quality of product of the national firm as well as that of the multinational firm.

Factors influencing the choice of the Northern firm can be categorized as two kinds of factors associated with the cost (supply) side and the demand side. The factors related to the cost side have the following effects. (i) A decrease in the spillover increases the incentive for a Northern firm to become a multinational firm. A policy strengthening intellectual property protection in the South induces a multinational firm to the South. (ii) A fall in the Southern wage decreases the production cost in the South, so that the Northern firm becomes multinational. (iii) If the productivity of production technology of the Northern firm and Southern firms is improved, the amounts of labor used for production are reduced for both. However, the reduction in production cost in the North is larger than in the South, for the Northern wage is higher than the Southern wage. This makes the Northern firm remain national.

The effects of factors associated with the demand side are as follows. (i) Growth of the world market for the high quality product increases the gap between low quality and high quality. This causes more spillover if the Northern firm shifts production to the South. The Northern firm remains national. (*ii*) A rise in consumers' preference for low quality causes consumers to substitute the low quality product for the high quality product. This decreases the profit of the Northern firm regardless of whether this firm remains national or becomes multinational. However, since it is difficult to distinguish which firm's profit decreases more, the Northern firm may remain national or become multinational. (*iii*) A rise in consumers' preference for high quality means that consumers have a higher willingness to pay for high quality. This decreases the incentive for the Northern firm to reduce production costs. It remains national. (*iv*) A rise in consumers' response to the price of the low quality product means that consumers feel less satisfied with a given price, for they perceive the price as high relative to the level of low quality. Thus they switch consumption from the low quality product to the high quality product. However, the Northern firm either remains national or becomes multinational, for there is ambiguity regarding which firm's profit increases more.

Some countries try to institute more effective policies for inducing foreign direct investment. Others tend to be concerned about the shift of activities of firms abroad. My findings provide a useful frame work for evaluating and understanding both.

Appendices

Appendix 1.1

For the profit function Π_h to have a global maximum, the derivatives of Π_h must be zero at some x_h and q_h. $\frac{\partial \Pi_h}{\partial x_h} = 0$ and $\frac{\partial \Pi_h}{\partial q_h} = 0$. Also Π_h must be concave on all x_h and q_h, which will be the case when its Hessian matrix H is negative semidefinite for all x_h and q_h.

$$H = \begin{bmatrix} \frac{\partial^2 \Pi_h}{\partial x_h{}^2} & \frac{\partial^2 \Pi_h}{\partial q_h \partial x_h} \\ \frac{\partial^2 \Pi_h}{\partial x_h \partial q_h} & \frac{\partial^2 \Pi_h}{\partial q_h{}^2} \end{bmatrix}.$$

H is negative semidefinite if and only if every principal minor of the first order is ≤ 0 and principal minor of the second order is ≥ 0. The first order principal minors of H are

$$\left| \frac{\partial^2 \Pi_h}{\partial x_h{}^2} \right| \leq 0\,, \quad \left| \frac{\partial \Pi_h}{\partial q_h{}^2} \right| \leq 0\,. \tag{A1.1.1}$$

The second order principal minor of H is:

$$\begin{vmatrix} \frac{\partial^2 \Pi_h}{\partial x_h{}^2} & \frac{\partial^2 \Pi_h}{\partial q_h \partial x_h} \\ \frac{\partial^2 \Pi_h}{\partial x_h \partial q_h} & \frac{\partial^2 \Pi_h}{\partial q_h{}^2} \end{vmatrix} \geq 0\,. \Rightarrow \left(\frac{\partial \Pi_h}{\partial x_h{}^2}\right)\left(\frac{\partial^2 \Pi_h}{\partial q_h{}^2}\right) - \left(\frac{\partial^2 \Pi_h}{\partial x_h \partial q_h}\right)^2 \geq 0\,. \tag{A1.1.2}$$

Therefore if the conditions (A1.1.1) and (A1.1.2) are satisfied at x_h and q_h, which are obtained from the first order conditions of profit maximization, the maximal profit becomes a local maximum at x_h and q_h.

Appendix1.2

From the two first order conditions (1.9) and (1.10), the output and quality are obtained. The matrix for the two equations is

$$\begin{bmatrix} -2 & (\gamma - \lambda w_N) \\ (\gamma - \lambda w_N) & -w_N \end{bmatrix} \begin{bmatrix} x_h \\ q_h \end{bmatrix} = \begin{bmatrix} -(A - \theta \bar{q}_l) \\ -w_N \bar{q}_l \end{bmatrix}.$$

The determinant is

$$D = \begin{vmatrix} -2 & (\gamma - \lambda w_N) \\ (\gamma - \lambda w_N) & -w_N \end{vmatrix} = 2w_N - (\gamma - \lambda w_N)^2 > 0\,.$$

It is positive since $2w_N - (\gamma - \lambda w_N)^2 > 0$ by (1.13').

x_h is

$$x_h = \frac{\begin{vmatrix} -(A-\theta\bar{q}_l) & (\gamma-\lambda w_N) \\ -w_N\bar{q}_l & -w_N \end{vmatrix}}{D} = \frac{w_N\{(A-\theta\bar{q}_l)+(\gamma-\lambda w_N)\bar{q}_l\}}{2w_N-(\gamma-\lambda w_N)^2}.$$

q_h is

$$q_h = \frac{\begin{vmatrix} -2 & -(A-\theta\bar{q}_l) \\ (\gamma-\lambda w_N) & -w_N\bar{q}_l \end{vmatrix}}{D} = \frac{2w_N\bar{q}_l+(\gamma-\lambda w_N)(A-\theta\bar{q}_l)}{2w_N-(\gamma-\lambda w_N)^2}.$$

p_h is

$$\begin{aligned} p_h &= A-\theta\bar{q}_l+\gamma q_h - x_h \\ &= (A-\theta\bar{q}_l) - \frac{w_N\{(A-\theta\bar{q}_l)+(\gamma-\lambda w_N)\bar{q}_l\}}{2w_N-(\gamma-\lambda w_N)^2} + \frac{\gamma\{2w_N\bar{q}_l+(\gamma-\lambda w_N)(A-\theta\bar{q}_l)\}}{2w_N-(\gamma-\lambda w_N)^2} \\ &= \frac{w_N[(A-\theta\bar{q}_l)\{1+\lambda(\gamma-\lambda w_N)\}+(\gamma+\lambda w_N)\bar{q}_l]}{2w_N-(\gamma-\lambda w_N)^2}. \end{aligned}$$

Π_h is

$$\Pi_h = (p_h-\lambda w_N q_h)x_h - \frac{1}{2}w_N(q_h-\bar{q}_l)^2,$$

where $(p_h-\lambda w_N q_h) = \dfrac{w_N\{(A-\theta\bar{q}_l)+(\gamma-\lambda w_N)\bar{q}_l\}}{2w_N-(\gamma-\lambda w_N)^2}$,

$$w_N(q_h-\bar{q}_l)^2 = w_N\left[\frac{(\gamma-\lambda w_N)\{(A-\theta\bar{q}_l)+(\gamma-\lambda w_N)\bar{q}_l\}}{2w_N-(\gamma-\lambda w_N)^2}\right]^2.$$

Then $\Pi_h = \dfrac{w_N\{(A-\theta\bar{q}_l)+(\gamma-\lambda w_N)\bar{q}_l\}^2}{2\{2w_N-(\gamma-\lambda w_N)^2\}}$.

Appendix 1.3

I take the logarithm of Π_h for its monotonic transformation:

$$\ln\Pi_h = \ln w_N + 2\ln\{(A-\theta\bar{q}_l)+(\gamma-\lambda w_N)\bar{q}_l\} - \ln 2 - \ln\{2w_N-(\gamma-\lambda w_N)^2\}.$$

Taking the derivative with respect to w_N,

$$\frac{\partial\ln\Pi_h}{\partial w_N} = \frac{1}{w_N} - \frac{2\lambda\bar{q}_l}{(A-\theta\bar{q}_l)+(\gamma-\lambda w_N)\bar{q}_l} - \frac{2\{1+\lambda(\gamma-\lambda w_N)\}}{2w_N-(\gamma-\lambda w_N)^2}.$$

The sum of the first term and third term is negative:

$$\frac{1}{w_N} - \frac{2\{1+\lambda(\gamma-\lambda w_N)\}}{2w_N - (\gamma-\lambda w_N)^2} = \frac{-(\gamma+\lambda w_N)(\gamma-\lambda w_N)}{w_N\{2w_N - (\gamma-\lambda w_N)^2\}} < 0,$$

where $(\gamma-\lambda w_N) > 0$ in (1.15) and $2w_N - (\gamma-\lambda w_N)^2 > 0$ in (1.13').

The second term is negative. Thus, $\frac{\partial \ln \Pi_h}{\partial w_N} < 0$. Since $\frac{\partial \Pi_h}{\partial w_N}$ has the same sign as $\frac{\partial \ln \Pi_h}{\partial w_N}$, $\frac{\partial \Pi_h}{\partial w_N} < 0$.

Appendix 1.4

From the first order conditions in (1.26) and (1.27), the output and quality are obtained. The matrix is for the two equations is:

$$\begin{bmatrix} -2 & (H-\lambda) \\ (H-\lambda) & -w_N \end{bmatrix} \begin{bmatrix} x_h^* \\ q_h^* \end{bmatrix} = \begin{bmatrix} -G \\ -w_N\bar{q}_l \end{bmatrix}.$$

The output is

$$x_h^* = \frac{\begin{vmatrix} -G & (H-\lambda) \\ -w_N\bar{q}_l & -w_N \end{vmatrix}}{D^*}.$$

The determinant is

$$D^* = \begin{vmatrix} -2 & (H-\lambda) \\ (H-\lambda) & -w_N \end{vmatrix} = 2w_N - (H-\lambda)^2.$$

The determinant is positive since $2w_N - (H-\lambda)^2 > 0$ from (1.30'). Then

$$x_h^* = \frac{w_N\{G+(H-\lambda)\bar{q}_l\}}{2w_N - (H-\lambda)^2} = \frac{w_N[\{\alpha+(\beta\lambda-\theta)(1-\sigma_q)\bar{q}_l\} + \{\gamma-\lambda+(\beta\lambda-\theta)\sigma_q\}\bar{q}_l]}{2w_N - \{\gamma-\lambda+(\beta\lambda-\theta)\sigma_q\}^2}.$$

Its quality is

$$q_h^* = \frac{\begin{vmatrix} -2 & -G \\ (H-\lambda) & -w_N\bar{q}_l \end{vmatrix}}{D^*} = \frac{2w_N\bar{q}_l + G(H-\lambda)}{2w_N - (H-\lambda)^2}$$

$$= \frac{2w_N\bar{q}_l + \{\alpha+(\beta\lambda-\theta)(1-\sigma_q)\bar{q}_l\}\{\gamma-\lambda+(\beta\lambda-\theta)\sigma_q\}}{2w_N - \{\gamma-\lambda+(\beta\lambda-\theta)\sigma_q\}^2}.$$

Its market price is

$$p_h^* = G - \frac{w_N\{G+(H-\lambda)\bar{q}_l\}}{2w_N - (H-\lambda)^2} + \frac{H\{2w_N\bar{q}_l + G(H-\lambda)\}}{2w_N - (H-\lambda)^2}$$

$$= \frac{w_N\{G + (H+\lambda)\bar{q}_l\} + \lambda G(H-\lambda)}{2w_N - (H-\lambda)^2}$$

$$= \frac{w_N[\{\alpha + (\beta\lambda - \theta)(1-\sigma_q)\bar{q}_l\} + \{\gamma + \lambda + (\beta\lambda - \theta)\sigma_q\}\bar{q}_l]}{2w_N - \{\gamma - \lambda + (\beta\lambda - \theta)\sigma_q\}^2}$$

$$+ \frac{\lambda\{\alpha + (\beta\lambda - \theta)(1-\sigma_q)\bar{q}_l\}\{\gamma - \lambda + (\beta\lambda - \theta)\sigma_q\}}{2w_N - \{\gamma - \lambda + (\beta\lambda - \theta)\sigma_q\}^2}.$$

Its profit is

$$\Pi_h^* = (p_h^* - \lambda q_h^*)x_h^* - \frac{w_N(q_h^* - \bar{q}_l)^2}{2},$$

where $(p_h^* - \lambda q_h^*)x_h^* = [\frac{w_N\{G + (H-\lambda)\bar{q}_l\}}{2w_N - (H-\lambda)^2}]^2$,

$$w_N(q_h^* - \bar{q}_l)^2 = w_N[\frac{(H-\lambda)\{G + (H-\lambda)\bar{q}_l\}}{2w_N - (H-\lambda)^2}]^2.$$

Then $\Pi_h^* = \frac{w_N\{G + (H-\lambda)\bar{q}_l\}^2}{2\{2w_N - (H-\lambda)^2\}} = \frac{w_N[\{\alpha + (\beta\lambda - \theta)(1-\sigma_q)\bar{q}_l\} + \{\gamma - \lambda + (\beta\lambda - \theta)\sigma_q\}\bar{q}_l]^2}{2[2w_N - \{\gamma - \lambda + (\beta\lambda - \theta)\sigma_q\}^2]}$.

Appendix 1.5

I investigate the sign of $\frac{\partial^2 \ln \tilde{\Pi}_h}{\partial w_N{}^2}$.

$$\frac{\partial^2 \ln \tilde{\Pi}_h}{\partial w_N{}^2} = \frac{-2\lambda^2}{\{(A-\theta) + (\gamma - \lambda w_N)\}^2} + \frac{2[\lambda^2\{2w_N - (\gamma - \lambda w_N)^2\} + 2\{1 + \lambda(\gamma - \lambda w_N)\}^2]}{\{2w_N - (\gamma - \lambda w_N)^2\}^2}.$$

The value of $\frac{\partial^2 \ln \tilde{\Pi}_h}{\partial w_N{}^2}$ is, where $w_N = 1$:

$$\frac{\partial^2 \ln \tilde{\Pi}_h}{\partial w_N{}^2} = \frac{-2\lambda^2}{\{(A-\theta) + (\gamma - \lambda)\}^2} + \frac{2\lambda^2}{\{2 - (\gamma - \lambda)^2\}} + \frac{4\{1 + \lambda(\gamma - \lambda)\}^2}{\{2 - (\gamma - \lambda)^2\}^2}, \qquad \text{(A1.5.1)}$$

where $(A-\theta) > 0$, $(\gamma - \lambda) > 0$, $2 - (\gamma - \lambda)^2 > 0$ from the conditions in (1.37).

The first term on the RHS in (A1.5.1) is negative, the second term is positive and the third term is positive. First, compare the numerators of the first and second term on the RHS in (A1.5.1):

$$\left|-2\lambda^2\right| = \left|2\lambda^2\right|.$$

The numerators of the first and second term are equal in absolute value.

Before comparing the denominators of the first and second term on the RHS in (A1.5.1), I need an additional assumption on $(A-\theta)$ in the denominator of the first term. It is assumed to be such as $(A-\theta)\geq 2$. Then the denominator of the first term is bigger than that of the second term:

$$\{(A-\theta)+(\gamma-\lambda)\}^2 > \{2-(\gamma-\lambda)^2\}.$$

The first term in absolute value is smaller than the second term on the RHS in (A1.5.1). Consequently, the sum of the first and second term becomes positive. Since the third term is also positive, $\dfrac{\partial^2 \ln \tilde{\Pi}_h}{\partial w_N{}^2} > 0$.

I check whether the assumption $(A-\theta)\geq 2$ is appropriate. From (1.5'), the world demand function is

$$D_h(p_h, q_h, \bar{q}_l) = (A-\theta) - p_h + \gamma q_h, \quad \text{where } (A-\theta) = \alpha + (\beta\lambda - \theta), \quad \bar{q}_l = 1.$$

If the world demand is greater than or equal to two, the following cases, $(A-\theta)\geq 2$ or $(A-\theta) < 2$, are possible according to the values of p_h and γq_h. However, I consider only the former case since $A\ (= \alpha + \beta\bar{p}_l)$ includes α, that is the market size with a large value. Then $\dfrac{\partial^2 \ln \tilde{\Pi}_h}{\partial w_N{}^2}$ is positive as long as the world demand is greater than or equal to two due to $(A-\theta)\geq 2$.

Appendix 1.6

The condition $\ln\{2w_N - (H-\lambda)^2\} > \ln\{2w_N - (\gamma - \lambda w_N)^2\}$ implies that

$$\{2w_N - (H-\lambda)^2\} > \{2w_N - (\gamma - \lambda w_N)^2\}. \tag{A1.6.1}$$

Rearranging (A1.6.1),

$$\{(\gamma - \lambda w_N) + (H-\lambda)\}\{(\gamma - \lambda w_N) - (H-\lambda)\} > 0.$$

Since $(\gamma - \lambda w_N) > 0$ in (1.15) and $(H-\lambda) > 0$ in (1.32), the first term $\{(\gamma - \lambda w_N) + (H-\lambda)\}$ is always positive. Then the second term should be positive:

$$\{(\gamma - \lambda w_N) - (H-\lambda)\} > 0. \tag{A1.6.2}$$

From (1.24), $H = \{\gamma + (\beta\lambda - \theta)\sigma_q\}$. Substituting H into (A1.6.2), a range of w_N is obtained:

$$w_N < 1 - \frac{(\beta\lambda - \theta)\sigma_q}{\lambda}.$$

In other words, this condition for w_N means that

$$(\gamma - \lambda w_N) > (H - \lambda).$$

Appendix 1.7

The value of $\ln \tilde{\Pi}_h^*$ at $w_N = 1$ is

$$\ln \tilde{\Pi}_h^* = 2\ln\{G + (H - \lambda)\} - \ln\{2 - (H - \lambda)^2\}. \tag{1.38}$$

The value of $\ln \tilde{\Pi}_h$ at $w_N = 1$ is

$$\ln \tilde{\Pi}_h = 2\ln\{(A - \theta) + (\gamma - \lambda)\} - \ln\{2 - (\gamma - \lambda)^2\}. \tag{1.40}$$

Recall that $A = \alpha + \beta\lambda\bar{q}_l$ from (1.6) and (1.7), $G = \{\alpha + (\beta\lambda - \theta)(1 - \sigma_q)\bar{q}_l\}$ and $H = \{\gamma + (\beta\lambda - \theta)\sigma_q\}$ in (1.24), and $\bar{q}_l = 1$.

Substituting for G, H and A, the first terms in (1.38) and (1.40) become equal since $G + (H - \lambda) = (A - \theta) + (\gamma - \lambda) = \alpha + \gamma - \lambda + (\beta\lambda - \theta)$. To compare the second terms in (1.38) and (1.40), I look at the relationship among H, γ and λ. From the definition of H, $H < \gamma$ since $(\beta\lambda - \theta) < 0$ in (1.23) and $H > 0$. From (1.32), $\lambda < H$. Thus $\lambda < H < \gamma$. This means that the second term in (1.38) is larger in absolute value than the second term in (1.40). Therefore,

$$\ln \tilde{\Pi}_h^* < \ln \tilde{\Pi}_h, \text{ where } w_N = 1.$$

Appendix 1.8

The value of $\ln \tilde{\Pi}_h^*$ at $w_N = w_{NUB}$ is

$$\ln \tilde{\Pi}_h^* = 2\ln\{G + (H - \lambda)\} - \ln[2\{1 - \frac{(\beta\lambda - \theta)\sigma_q}{\lambda}\} - (H - \lambda)^2]. \tag{1.39}$$

The value of $\ln \tilde{\Pi}_h$ at $w_N = w_{NUB}$ is

$$\ln \tilde{\Pi}_h = 2\ln\{(A - \theta) + (H - \lambda)\} - \ln[2\{1 - \frac{(\beta\lambda - \theta)\sigma_q}{\lambda}\} - (H - \lambda)^2]. \tag{1.41}$$

The first term in (1.39) and the first term in (1.41) are compared. Substituting $\bar{q}_l = 1$, $G = \{\alpha + (\beta\lambda - \theta)(1 - \sigma_q)\}$ and $(A - \theta) = \alpha + (\beta\lambda - \theta)$. Since $(\beta\lambda - \theta) < 0$ and $(\beta\lambda - \theta)(1 - \sigma_q) > (\beta\lambda - \theta)$, $G > (A - \theta)$. Thus the first term in (1.39) is larger than that in (1.41). The second term in (1.39) is the same as the second term in (1.41). Therefore,

$$\ln \tilde{\Pi}_h^* > \ln \tilde{\Pi}_h, \text{ where } w_N = w_{NUB}.$$

Appendix 1.9

The term on the RHS in (1.36) is negative. The first term and the second term on the RHS in (1.37) are negative, respectively.

$$\frac{\partial \ln \tilde{\Pi}_h^*}{\partial w_N} = \frac{-2}{2w_N - (H-\lambda)^2} < 0\,, \tag{1.36}$$

$$\frac{\partial \ln \tilde{\Pi}_h}{\partial w_N} = \frac{-2\lambda}{(A-\theta)+(\gamma-\lambda w_N)} - \frac{2\{1+\lambda(\gamma-\lambda w_N)\}}{2w_N - (\gamma-\lambda w_N)^2} < 0\,. \tag{1.37}$$

The term on the RHS in (1.36) is compared with the second term on the RHS in (1.37). The numerator of the former is smaller in absolute value than that of the latter, since $\lambda > 0$ in (1.1) and $(\gamma - \lambda w_N) > 0$ in (1.15):

$$|-2| < |-2\{1+\lambda(\gamma-\lambda w_N)\}|\,.$$

The denominator of the former is larger than that of the latter since $(H-\lambda) < (\gamma - \lambda w_N)$ from Appendix 1.6. Therefore, in absolute value, the term on the RHS in (1.36) is smaller than the second term on the RHS in (1.37).

Also, in absolute value, the first term on the RHS in (1.37) is added to the second term on the RHS, so that the slope of $\ln \tilde{\Pi}_h^*$ is flatter than that of $\ln \tilde{\Pi}_h$ in the range of $w_N \in (1,\ w_{NUB})$:

$$\left|\frac{\partial \ln \tilde{\Pi}_h^*}{\partial w_N}\right| < \left|\frac{\partial \ln \tilde{\Pi}_h}{\partial w_N}\right|\,.$$

Appendix 1.10

Using (1.16), the derivative of q_h with respect to w_N is

$$\frac{\partial q_h}{\partial w_N} = \frac{\{2\bar{q}_l - \lambda(A-\theta\bar{q}_l)\}\{2w_N - (\gamma-\lambda w_N)^2\} - 2\{2w_N\bar{q}_l + (\gamma-\lambda w_N)(A-\theta\bar{q}_l)\}\{1+\lambda(\gamma-\lambda w_N)\}}{\{2w_N - (\gamma-\lambda w_N)^2\}^2}$$

$$= \frac{-[(\gamma-\lambda w_N)^2\{2\bar{q}_l + \lambda(A-\theta\bar{q}_l)\} + 2\{\gamma(A-\theta\bar{q}_l) + 2\lambda w_N\bar{q}_l(\gamma-\lambda w_N)\}]}{\{2w_N - (\gamma-\lambda w_N)^2\}^2}\,. \tag{A1.10.1}$$

Since $\lambda > 0$ in (1.1), $(A - \theta\bar{q}_l) > 0$ in (1.7), $\gamma > 0$ in (1.5), and $(\gamma - \lambda w_N) > 0$ in (1.15), (A1.10.1) is negative: $\frac{\partial q_h}{\partial w_N} < 0$.

Using (1.3), the effect of a change in w_N on the national firm's R&D expenditure is

$$\frac{\partial\{w_N E_h(q_h)\}}{\partial w_N} = E_h(q_h)(1+\varepsilon)\,, \tag{A1.10.2}$$

$$\text{where } \varepsilon = \frac{w_N}{E_h(q_h)}\frac{\partial E_h(q_h)}{\partial w_N}\,. \tag{A1.10.3}$$

The derivative of $E_h(q_h)$ with respect to w_N is

$$\frac{\partial E_h(q_h)}{\partial w_N} = (q_h - \bar{q}_l)\frac{\partial q_h}{\partial w_N}. \tag{A1.10.4}$$

Substituting (A1.10.4) and $E_h(q_h) = \frac{(q_h - \bar{q}_l)^2}{2}$ into (A1.10.3), ε is rewritten as

$$\varepsilon = (\frac{2w_N}{q_h - \bar{q}_l})\frac{\partial q_h}{\partial w_N}. \tag{A1.10.3'}$$

Using (1.16),

$$q_h - \bar{q}_l = \frac{2w_N \bar{q}_l + (\gamma - \lambda w_N)(A - \theta\bar{q}_l)}{2w_N - (\gamma - \lambda w_N)^2} - \bar{q}_l$$

$$= \frac{(\gamma - \lambda w_N)\{(A - \theta\bar{q}_l) + \bar{q}_l(\gamma - \lambda w_N)\}}{2w_N - (\gamma - \lambda w_N)^2}. \tag{A1.10.5}$$

Substituting both (A1.10.1) and (A1.10.5) into (A1.10.3'),

$$\varepsilon = \frac{-2w_N[(\gamma - \lambda w_N)^2\{\lambda(A - \theta\bar{q}_l) + 2\bar{q}_l\} + 2\{\gamma(A - \theta\bar{q}_l) + 2\lambda w_N \bar{q}_l(\gamma - \lambda w_N)\}]}{(\gamma - \lambda w_N)\{(A - \theta\bar{q}_l) + \bar{q}_l(\gamma - \lambda w_N)\}\{2w_N - (\gamma - \lambda w_N)^2\}}.$$

To know the sign of (A1.10.2), the sign of $(1 + \varepsilon)$ is needed.

$$1 + \varepsilon = 1 + \frac{-2w_N[(\gamma - \lambda w_N)^2\{\lambda(A - \theta\bar{q}_l) + 2\bar{q}_l\} + 2\{\gamma(A - \theta\bar{q}_l) + 2\lambda w_N \bar{q}_l(\gamma - \lambda w_N)\}}{[(\gamma - \lambda w_N)\{(A - \theta\bar{q}_l) + \bar{q}_l(\gamma - \lambda w_N)\}\{2w_N - (\gamma - \lambda w_N)^2\}}$$

$$= \frac{(B - C - D - E)}{F}, \tag{A1.10.6}$$

where $B = 2w_N(\gamma - \lambda w_N)\{(A - \theta\bar{q}_l) + \bar{q}_l(\gamma - \lambda w_N)\} > 0$,

$C = (\gamma - \lambda w_N)^3\{(A - \theta\bar{q}_l) + \bar{q}_l(\gamma - \lambda w_N)\} > 0$,

$D = 2w_N(\gamma - \lambda w_N)^2\{\lambda(A - \theta\bar{q}_l) + 2\bar{q}_l\} > 0$,

$E = 4w_N\{\gamma(A - \theta\bar{q}_l) + 2\lambda w_N \bar{q}_l(\gamma - \lambda w_N)\} > 0$,

$F = (\gamma - \lambda w_N)\{(A - \theta\bar{q}_l) + \bar{q}_l(\gamma - \lambda w_N)\}\{2w_N - (\gamma - \lambda w_N)^2\} > 0$.

$(B - D - E)$ in (A1.10.6) is equal to

$$-2w_N[(A - \theta\bar{q}_l)\{\gamma + \lambda w_N + \lambda(\gamma - \lambda w_N)^2\} + \bar{q}_l(\gamma - \lambda w_N)\{(\gamma - \lambda w_N) + 4\lambda w_N\}]. \tag{A1.10.7}$$

(A1.10.7) is negative. In other words, $(B - D - E) < 0$. Therefore $(B - C - D - E) < 0$. Also the denominator is positive: $F > 0$. Therefore, $(1 + \varepsilon) < 0$. Recall that $E_h(q_h) = \frac{(q_h - \bar{q}_l)^2}{2} > 0$.

From (A1.10.2),

$$\frac{\partial\{w_N E_h(q_h)\}}{\partial w_N} < 0. \tag{A1.10.8}$$

Appendix 1.11

Using (1.33), the effect of a change in the Northern wage on the quality of the multinational firm's product is

$$\frac{\partial q_h^*}{\partial w_N} = \frac{2\bar{q}_l\{2w_N - (H-\lambda)^2\} - 2\{2w_N\bar{q}_l + G(H-\lambda)\}}{\{2w_N - (H-\lambda)^2\}^2},$$

$$= \frac{-2\bar{q}_l(H-\lambda)\{(H-\lambda)+G\}}{\{2w_N - (H-\lambda)^2\}^2} < 0, \tag{A1.11.1}$$

where $(H-\lambda) > 0$ in (1.32), $G > 0$ in (1.25).

Using (1.19), the change in R&D expenditure by a change in the Northern wage is

$$\frac{\partial\{w_N E_h^*(q_h^*)\}}{\partial w_N^*} = E_h^*(q_h^*)(1+\varepsilon^*), \tag{A1.11.2}$$

$$\text{where } \varepsilon^* = \frac{w_N}{E_h^*(q_h^*)}\frac{\partial E_h^*(q_h^*)}{\partial w_N}. \tag{A1.11.3}$$

The derivative of $E_h^*(q_h^*)$ with respect to w_N is

$$\frac{\partial E_h^*(q_h^*)}{\partial w_N} = (q_h^* - \bar{q}_l)(\frac{\partial q_h^*}{\partial w_N}). \tag{A1.11.4}$$

Substituting (A1.11.4) and $E_h^*(q_h^*) = \frac{(q_h^* - \bar{q}_l)^2}{2}$ into (A1.11.3), ε^* is rewritten as

$$\varepsilon^* = (\frac{2w_N}{q_h^* - \bar{q}_l})(\frac{\partial q_h^*}{\partial w_N}). \tag{A1.11.3'}$$

Using (1.33),

$$q_h^* - \bar{q}_l = \frac{2w_N\bar{q}_l + G(H-\lambda)}{2w_N - (H-\lambda)^2} - \bar{q}_l$$

$$= \frac{(H-\lambda)\{G + \bar{q}_l(H-\lambda)\}}{2w_N - (H-\lambda)^2}. \tag{A1.11.5}$$

Substituting (A1.11.1) and (A1.11.5) into (A1.11.3'),

$$\varepsilon^* = \frac{-4\bar{q}_l w_N}{2w_N - (H-\lambda)^2}.$$

Then $(1+\varepsilon^*)$ on the RHS in (A1.11.2) is equal to $\frac{-\{2w_N + (H-\lambda)^2\}}{2w_N - (H-\lambda)^2}$. This is negative. In other words, $(1+\varepsilon^*) < 0$. Recall that $E_h^*(q_h^*) = \frac{(q_h^* - \overline{q}_l)^2}{2} > 0$. Therefore, from (A1.11.2),

$$\frac{\partial\{w_N E_h^*(q_h^*)\}}{\partial w_N} < 0. \qquad \text{(A1.11.6)}$$

Appendix 1.12

The levels of production, quality and price that are associated with the national firm, increase with respect to growth of the world market. The basic quality is normalized to one. $\overline{q}_l = 1$.

$$\frac{\partial x_h}{\partial \alpha} = \frac{w_N}{2w_N - (\gamma - \lambda w_N)^2} > 0, \quad \text{where } 2w_N - (\gamma - \lambda w_N)^2 > 0.$$

$$\frac{\partial q_h}{\partial \alpha} = \frac{(\gamma - \lambda w_N)}{2w_N - (\gamma - \lambda w_N)^2} > 0, \quad \text{where } (\gamma - \lambda w_N) > 0.$$

$$\frac{\partial p_h}{\partial \alpha} = \frac{w_N\{1 + \lambda(\gamma - \lambda w_N)\}}{2w_N - (\gamma - \lambda w_N)^2} > 0.$$

$$\frac{\partial \Pi_h}{\partial \alpha} = \frac{w_N\{(A-\theta) + (\gamma - \lambda w_N)\}}{2w_N - (\gamma - \lambda w_N)^2} > 0, \quad \text{where } (A-\theta) > 0.$$

The levels of production, quality and price that are determined by the multinational firm, move in the same direction as growth of the world market:

$$\frac{\partial x_h^*}{\partial \alpha} = \frac{w_N}{2w_N - (H-\lambda)^2} > 0, \quad \text{where } 2w_N - (H-\lambda)^2 > 0,$$

$$\frac{\partial q_h^*}{\partial \alpha} = \frac{(H-\lambda)}{2w_N - (H-\lambda)^2} > 0, \quad \text{where } (H-\lambda) > 0,$$

$$\frac{\partial p_h^*}{\partial \alpha} = \frac{w_N + \lambda(H-\lambda)}{2w_N - (H-\lambda)^2} > 0,$$

$$\frac{\partial \Pi_h^*}{\partial \alpha} = \frac{w_N\{G + (H-\lambda)\}}{2w_N - (H-\lambda)^2} > 0.$$

Note that $G + (H-\lambda) = (A-\theta) + (\gamma - \lambda)$ and $(\gamma - \lambda) > 0$.

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CHAPTER 2

Capital Movement and the Proportion of Trade in Intermediate Goods

2.1 Introduction

The economies of the world have become increasingly global. Along with this, international production across multiple countries has become easy and quite common. Such emergence of international production causes each country to specialize in particular production stages – each stage produces a particular part or component. This results in enormous increase in trade in intermediate goods among the countries that are vertically linked in international production, which is one of the features of modern international economies.

Hanson, Mataloni and Slaughter (2005) find empirical support. Where there is intra-firm trade in intermediate goods between parent firms and their vertically linked affiliates, the demand for imported intermediates is negatively correlated with trade costs – tariffs and transport costs – that the affiliates face, and this demand is sensitive to trade costs. These results can be interpreted as follows: fall in trade costs increases intra-firm trade in intermediate goods, and thus trade in intermediate goods has grown fast.

The second feature is that the trade share of GDP has grown fast since World War II. Yi (2003) explains this by the emergence of international vertical specialization in production processes and tariff reduction. When intermediate goods are shipped across borders many times, passing through several production stages, each stage receives the benefit from tariff reduction. Until the production process reaches the final stage, these benefits are obtained repeatedly. Consequently, this has the same effect as that of a big tariff reduction, so that a small tariff reduction leads to steep growth in intermediate goods trade. Since the final good which uses the intermediates is non-tradable and is counted as GDP, the trade share of GDP sharply increases.

The third feature is that trade in intermediate goods has increased rapidly compared to trade in final goods (Hummels, Rapoport and Yi 1998, and Yeats 2001). Thus the share of intermediate goods in total trade, including both final goods and intermediate goods, has increased. Hummels, Ishii and Yi (2001) show that vertical specialization – the use of imported intermediates in producing final goods that are exported – accounts for 21% of total exports of goods as of 1990 and grew almost 30% between 1970 and 1990 in 14 countries.[1] Hummels et al. (1998 and 2001) identify the increase in vertically specialized production across countries as the reason for this growth.

Along with the trend of increasingly free trade, foreign investment has been increasingly liberalized. Capital flows across borders have become important issues in the international economy.[2] One of the interesting issues is what correlation exists between capital movements and trade. Some studies show that the relationship is as substitutes, others, that they are complements, or both. For example, Mundell (1957) explains that final goods flows and factor movements are substitutes in the Heckscher-Ohlin (H-O) model. However, Markusen (1983) shows that they are complements where economies have features such as external economies of scale, imperfect competition and technology differences. According to Antras and Caballero (2007), trade integration and capital inflows are complements from the viewpoint of a developing country if this country has capital markets with higher financial frictions than a developed country, and if final goods have heterogeneous financial dependence. In Ruffin (1984), they tend to be complements in the Kemp-Jones model when technology differences exist. More generally, in Wong (1986), the two can become substitutable or complementary where two countries are in a perfectly competitive situation and have different factor endowments, preferences and technologies with constant returns to scale. However, these studies focus on trade in only final goods.

In this paper, I study the relation between capital movement and the pattern, volume, and proportion, of trade in intermediate goods. I first examine this relation with data on United States-Mexico trade, and then I develop a theoretical model supporting this finding. The data show that capital flows and the share of intermediate goods in total trade have both grown rapidly.

After Mexico allowed the liberalization of foreign investment,[3] capital flows have grown sharply between the United States and Mexico. Figure 2.1 illustrates the inflow of foreign direct investment (FDI) and the stock of inward FDI invested from the U.S. to Mexico every year during the sample period 1982-1997.[4] The solid and dashed lines show growing trends during the sample period. The outflows and outward stock of FDI from Mexico to the U.S. are almost zero.

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[1] The countries are the G-7 nations, plus Australia, Denmark, Ireland, Korea, Mexico, the Netherlands and Taiwan.

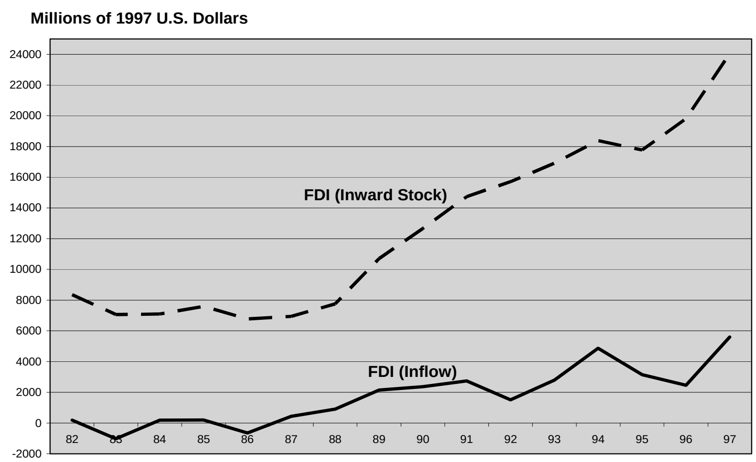
[2] There are many studies investigating how capital flows affect trade. Kemp (1962 and 1966) and Jones (1967) study the interaction among capital movements, optimal tariffs and optimal taxes. Uekawa (1972), Jones and Ruffin (1975), and Ferguson (1978) study how trade patterns and specialization are affected by capital movement.

[3] See Hufbauer and Schott (1992) for the details on the policy changes in Mexico.

[4] This FDI consists of equity capital, invested earnings and other capital (mainly represented by inter-company loans).

Thus the lines in Figure 2.1 represent Mexico's net inflows of FDI and its net inward stocks of FDI from the United States, respectively. These values are expressed in 1997 U.S. dollars.

Mexico's FDIs



Source: OECD. Stat and author's calculation

Figure 2.1

As the solid line shows in Figure 2.1, during the period 1982-83, before the liberalization of foreign investment, Mexico experienced an outflow of foreign capital of U.S. $427 millions on average to the United States. The period 1984-87 recorded a small inflow of capital which was U.S. $41 millions on average. During the period 1988-90, in which the restrictions on foreign investment were greatly eased, the inflow of capital jumped sharply and amounted to U.S. $1804 millions per year. After the beginning of negotiation for the NAFTA, a huge amount of capital flowed into Mexico. From 1991 to the mid-1990s, capital inflows amounted to U.S. $3009 millions. The increasing trend continued during 1996-97, when the inflow of capital sharply increased to U.S. $4028 millions. Also, as the dashed line shows in Figure 2.1, the inward stocks have an upward trend like the inflows.

Next, I examine whether trade in intermediate goods between the United States and Mexico follows a path similar to the trend of Mexico's FDI. Trade in many intermediate goods seems to be characterized by vertical-specialization-based trade. For example, Mexico imports intermediate goods from the United States mainly for processing or assembling these and then exporting back the finished goods to the United States. Also the United States imports intermediates from Mexico for making finished goods and then exporting these back to Mexico. Hummels, Rapport and Yi (1998) measure vertical-specialization-based trade, defined as 2 times the value of imported intermediates embodied in a country's exports.[5] Multiplication by 2 takes into account the fact that the imported intermediates are counted once as imports of the country importing the intermediates, and once as exports of the intermediates embodied in exports of that country. This is identical to the concept of volume of trade in intermediates, so that I interpret the extent of vertical trade as the volume of intermediates trade.

Hummels et al. (1998) measures U.S.-Mexico vertical trade with the maquiladora data. Since the data include the imported intermediates from the United States, and since almost all production is re-exported to the United States, the measure represents vertical trade of the U.S. $\rightarrow$ Mexico $\rightarrow$ the U.S. Their study shows that the share of intermediates trade attributable to Mexico's maquiladoras in total trade between the United States and Mexico has grown over the years. The share averaged about 20% from 1975 to 1979. This share rose to an average of 25% in the 1980s. The share kept rising to an average of 35% in the first half of the 1990s, and reached 39% in 1996.

Also, the United States imports intermediates from Mexico for making finished goods and then exporting these back to Mexico. The data on this trade are not available. However, the value of imported intermediates would probably be small since the level of U.S. wages and the systems of tariff of the U.S. and Mexico were not favorable to this trade flow.[6] Thus I do not consider this trade flow. When only the trade flow in the direction of the U.S. $\rightarrow$ Mexico $\rightarrow$ the U.S. is considered, Figure 2.2 illustrates the share of U.S.-Mexican maquiladora vertical trade in total U.S.-Mexican trade. The share has an upward trend.

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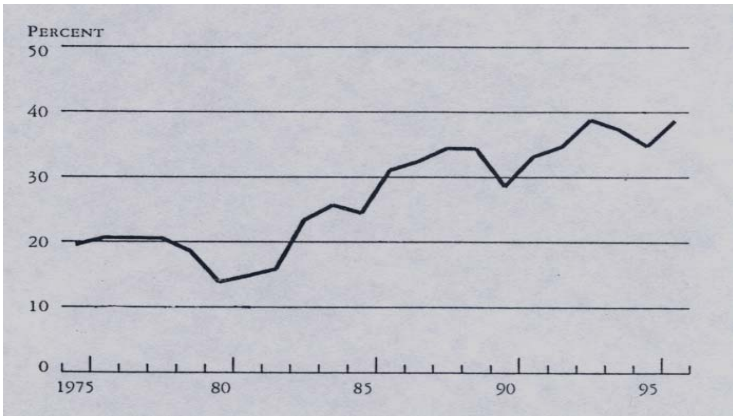
[5] In the case of vertical trade in both directions, this is measured by (*2* $\times$ *the fraction of the Mexican imported intermediates in gross production* $\times$ *the Mexican exports to the U.S.)* $+$ *(2* $\times$ *the fraction of the U.S. imported intermediates in gross production)*.

[6] There is little incentive to assemble or process further intermediates of Mexico-origin in the United States. The reasons are: the cost of wages for further manufacture are higher in the United States than in Mexico; and tariff exemption by the laws of the United States and Mexico was not applied to trade in this direction since the laws applied to the trade only in the opposite direction. For example, until before 1994 when the NAFTA took effect, item 9802 of the U.S. Harmonized Tariff System provides tariff-favor for producers whose overseas production uses parts or materials of U.S. origin. When the products manufactured overseas are imported to the United States, tariffs are not imposed on the value attributable to the parts or materials of U.S. origin. Only the value added that is generated overseas is dutiable. Note that after 1994 when the NAFTA took effect, tariffs and other barriers on all industrial goods have been brought down in a period of less than 15 years. On the other hand, Mexican laws exempt parts and materials from Mexico's tariffs, which Mexican maquiladoras import for making further processed products that are re-exported to the United States.

Intermediate Trade (Vertical Trade) as a Percentage of Total Trade between the U.S. and Mexico



Source: Chart 5 in Hummels, Rapoport and Yi (1998)

Figure 2.2

I find that the solid line in Figure 2.1 representing the trend of the capital inflow has a similar trend to that in Figure 2.2 representing the share of intermediate goods trade in total trade during the period 1982-95. This suggests that capital flows may increase intermediates trade relative to total trade.

The purpose of this study is to explain theoretically how capital flows are able to make trade in intermediate goods grow relative to total trade. I also examine whether capital movement is a substitute for or a complement to trade in intermediate goods, and to trade in final goods.

This model is somewhat similar to the model of Dornbusch, Fischer and Samuelson (D-F-S) (1980) in that the Heckscher-Ohlin theory and the concept of a continuum of final goods are combined. However, these models each deal with different issues. D-F-S (1980) examines the effect of a change in capital endowment on factor price equalization and on trade patterns of final goods.[7] I examine how international capital movement affects factor prices, and the trade patterns

[7] Factor price equalization does not occur in a trading equilibrium if factor endowments differ sufficiently. A capital-abundant country has a higher wage-return ratio than a labor-abundant country. The former specializes completely in production of final goods in the capital-intensive range, while the latter specializes completely in production of final goods in the labor-intensive range. If the capital endowment in the labor-abundant country increases, the equilibrium wage-return ratio falls in the capital-abundant country. The capital-abundant country concentrates in production in a narrower range of goods in which capital is used more intensively. The sorts of goods exported by this country are reduced in number. However, the sorts of goods exported by the labor-abundant country increase. The effect of an increase in capital endowment in the labor-abundant country is ambiguous on the equilibrium wage-return ratio in this country, since this depends on whether elasticities of substitution of capital for labor in production are larger than unity.

of the intermediate good and final goods.[8] Also I analyze the effects of capital movement on trade volumes of the intermediate good and final goods, and the proportion of trade in the intermediate good.

The organization of this paper is as follows. I address technologies and structures of final goods sectors and one intermediate good industry in the autarkic economy in section 2.2.1.1. Consumers' demand is derived in section 2.2.1.2. Section 2.2.1.3 explains factor market equilibria, and derives the relative autarkic prices of the intermediate good and final goods. In section 2.2.2.1, consumers' demand is derived in the open economy. Factor market equilibria, trade patterns and volumes of trade are addressed in sections 2.2.2.2 and 2.2.2.3. Section 2.2.3 derives the effect of capital movements on the amount of intermediate good trade and the share of the intermediate good in total trade. I discuss relaxations of the assumptions, and their implication for the results in section 2.3. Section 2.4 concludes.

2.2 Model

2.2.1 Autarkic Economy

I consider a two-country world economy. The endowments of labor and capital of country i are fixed: $\overline{L}_i$ and $\overline{K}_i$, where $i = 1, 2$. Each country has a sector of a final consumption good, Y, a sector of a continuum of final consumption goods, Z, and an industry of intermediate good, M.

2.2.1.1 Technologies

Sector Y produces a homogeneous final consumption good Y in a perfectly competitive environment. Good Y is produced by locally processing or assembling an intermediate good with local primary production factors before reaching the final state required by consumers. That is, the good Y has the structure of vertical production. In real economies, many firms run operations at a location close to local markets for supplying goods only to the local market rather than to the world market. This case is more often found from firms producing final goods under the mode of vertical production than from firms producing final goods made only from primary production

[8] Unlike factor price equalization in D-F-S (1980) which occurs where capital-labor ratios of two countries are similar, in my model factor price equalization does not occur in a trading equilibrium, though capital-labor ratios are the same across borders. A country with the more efficient technology for the intermediate good has a lower capital return and faces outflow of capital. This increases the capital return in this country. Thus the sorts of the continuum of final goods exported by this country are reduced, and its export of the intermediate good increases. In contrast, a country with less efficient technology for the intermediate good faces inflow of capital and its capital return falls. It comes to export more sorts of final goods, and comes to increase imports of the intermediate good.

factors. Thus, I assume that the non-tradable good Y exists and intermediate good M is used for good Y in order to keep the model close to the real economy.[9]

Technology for good Y requires labor L, capital K and intermediate good M, production of which is considered part of sector Y. It has a Leontief nested Cobb-Douglas form, which is assumed to make my model easily tractable. The technology for Y in country i is

$$y_i = vL_i^{Y\frac{1}{2}}[\min\{K_i^Y, b_i M_i\}]^{\frac{1}{2}}, \quad i = 1, 2. \tag{2.1}$$

Capital and the intermediate good are combined into a component by a Leontief technology. The input coefficient of capital is one and that of the intermediate good is b_i, $0 < b_i < 1$. Then one unit of capital and $\frac{1}{b_i}$ units of M are used for production of one unit of the component. The component is used internally and is not sold to other firms. It is transferred to the next manufacturing process of the Cobb-Douglas technology that assembles it into the final good Y using labor. The component and labor are substitutes. The technology has equal shares $\frac{1}{2}$ of labor and the component, respectively. y_i is output of good Y in country i. v is the efficiency parameter of the technology. For simplicity of the cost function corresponding to the technology for Y, v is assumed to be two: $v = 2$.

For a given y_i, country i's cost function of the final good Y, H_i^Y, is

$$H_i^Y = w_i^{\frac{1}{2}}(r_i + \frac{p_i^M}{b_i})^{\frac{1}{2}} y_i. \tag{2.2}$$ [10]

w_i, r_i and p_i^M are the wage, capital return and price of the intermediate good M in country i, respectively. Note that the term $(r_i + \frac{p_i^M}{b_i})$ on the right hand side (RHS) of (2.2) is the unit production cost of the component. Country i's unit production cost of Y, h_i^Y, is

$$h_i^Y = w_i^{\frac{1}{2}}(r_i + \frac{p_i^M}{b_i})^{\frac{1}{2}}. \tag{2.2'}$$

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[9] Also, there is the case that assembled final good Y is exported back to the country from which the intermediate goods are imported. United States-Mexico trade mentioned in the introduction is the corresponding case. However, in my model, I assume that the assembled final good Y is non-tradable. This is because the model that will be set up is one in which the number of goods is larger than the number of production factors. If Y is tradable, trade patterns in goods Y, M and other final goods in sector Z cannot be determined. To avoid this indeterminacy, the assumption that the good Y should be non-tradable is necessary.

[10] See Appendix 2.1.

Denote the price of good Y as p_i^Y. Perfect competition leads to zero profit. Then

$$p_i^Y = h_i^Y = w_i^{\frac{1}{2}}(r_i + \frac{p_i^M}{b_i})^{\frac{1}{2}}. \tag{2.3}$$

Industry M produces the intermediate good, which is tradable. It is produced by using only labor. One unit of M is produced with b_i units of labor in country i.[11]

$$m_i = \frac{L_i}{b_i}. \tag{2.4}$$

m_i is the output of intermediate good M. Country 1 has a more efficient technology for M compared to country 2. This means $b_1 < b_2$. The unit production cost in country i, h_i^M, is $b_i w_i$. Its production is perfectly competitive. The price must be the unit cost of production by the zero profit condition:

$$p_i^M = h_i^M = b_i w_i. \tag{2.5}$$

p_i^M falls as labor productivity of industry M increases (in other words, b_i falls).

Substituting (2.5) into (2.3), the price of Y is

$$p_i^Y = w_i^{\frac{1}{2}}(r_i + w_i)^{\frac{1}{2}}, \quad \frac{\partial p_i^Y}{\partial w_i} > 0 \text{ and } \frac{\partial p_i^Y}{\partial r_i} > 0.$$

p_i^Y is increasing in the wage and the capital return, respectively.

Sector Z produces a continuum of final consumption goods z that are tradable. These goods are produced by the primary production factors, labor and capital. This assumption represents that goods z are more capital-intensive than M since the production of M is assumed to use only labor. The final goods are indexed by z which is a continuous number. z has a value in the range of $(0, 1]$. These are produced under perfect competition.

The production function of z is of a Leontief form, which is used for simplicity of structure.

$$x_i(z) = \min\{\frac{L_i(z)}{a_i(z)}, K_i(z)\}, \quad z \in (0, 1].$$

[11] I assume the same parameter b_i in the production function for M (2.4) as the parameter used in the production function for Y (2.1). The reason for this assumption is as follows. If the technology of Y and the technology of M are combined, these production functions would collapse into the following Leontief nested Cobb-Douglas function. $y_i = vL_i^{Y\frac{1}{2}}[\min\{K_i^Y, L_i^M\}]^{\frac{1}{2}}$. L_i^M is labor employed in industry M. b_i does not appear in this production function. Thus the production technology of non-tradable good Y is the same in both countries. This helps determine the comparative advantage of the countries. That is, this highlights a pattern of comparative advantage of the tradable goods, M and Z.

$x_i(z)$ is output of z in country i. The technology for z is assumed to be identical between the countries, so that the input-output coefficient on labor $a_i(z) \in (0, 1)$ is identical between countries.

$$a_i(z) = a(z), \quad i = 1, 2.$$

$a(z)$ is assumed to be continuous and increasing in z. Also it is linearly associated with z. This means that labor units $a(z)$ for production of one unit of z increase when the index z rises: $\frac{da(z)}{dz} > 0$. To produce $x_i(z)$, labor is employed in the amount $a(z)x_i(z)$. And capital is used in the amount $x_i(z)$, since one unit of capital is required per unit output of z. However, capital input for production of one unit of z does not depend on the index z.

The unit cost $h_i(z)$ for production of one unit of z in country i takes the following form, since one unit of capital and $a(z)$ units of labor are used.

$$h_i(z) = r_i + a(z)w_i.$$

With $\frac{da(z)}{dz} > 0$, this unit cost represents that the higher the index z, the more labor-intensive is the good z. Since the production of z is perfectly competitive, the price of z, $p_i(z)$ should equal the unit cost:

$$p_i(z) = h_i(z) = r_i + a(z)w_i. \tag{2.6}$$

2.2.1.2 Consumer Demand

A representative consumer demands the final good Y and other final goods z. Preference of the consumer in country i is represented by a utility function of nested Cobb-Douglas form.

$$U_i = c_i^{Y\,\frac{1}{2}} u_i^{\frac{1}{2}}, \quad i = 1, 2.$$

c_i^Y is the quantity of consumption of the final good Y in country i. u_i is a subutility function that represents the utility obtained from consumption of the final goods z. It also has the Cobb-Douglas form.

$$\ln u_i = \int_0^1 \theta(z) \ln c_i(z) dz, \quad i = 1 \text{ or } 2.$$

$c_i(z)$: quantity that country i's consumer demands of final good z.

$\theta(z)$: parameter that represents degree of contribution of consumption of final good z to subutility.

$$\int_0^1 \theta(z)dz = 1 \text{: the sum of the parameters is one.}$$

The utility maximization problem is[12]

$$\underset{c_i^Y,\, c_i(z)}{Max} \quad c_i^{Y\,\frac{1}{2}} u_i^{\frac{1}{2}}$$

$$\text{s.t.} \quad p_i^Y c_i^Y + \int_0^1 p_i(z) c_i(z) dz = I_i \,.$$

The optimal consumptions for the goods Y and z are $c_i^Y = \frac{I_i}{2p_i^Y}$ and $c_i(z) = \frac{\theta(z) I_i}{2p_i(z)}$.

For goods markets equilibrium, the total supply of sector Y equals the total demand for it.

$$y_i = \frac{I_i}{2p_i^Y} \,. \tag{2.7}$$

And the total supply of sector Z equals the total demand for it:

$$\int_0^1 x_i(z) dz = \int_0^1 \frac{\theta(z) I_i}{2p_i(z)} dz \,. \tag{2.8}$$

2.2.1.3 Factor Markets

This paper studies the relation between capital flows and intermediate good trade between countries with different technologies, but identical relative endowments.[13] I assume that the relative factor endowments of both countries are equal: $\bar{k}_1 = \bar{k}_2$ where $\bar{k}_i = \frac{\bar{K}_i}{\bar{L}_i}$, and $\bar{L}_i = \bar{K}_i$.[14]

The supply of z should be equal to the domestic demand. Since one unit of capital is used for production of one unit of z, capital for producing an amount equal to domestic demand for z is

[12] See Appendix 2.2.

[13] A difference in relative factor endowments between countries can cause capital movements. However, my study focuses on the case that both identical relative endowments and different technologies between countries induce capital movement. In the autarky, the identical relative endowments and different technologies do not play the role of causing capital movement. However, in the open economy, these conditions result in a difference in factor prices that is the cause of capital flows.

[14] The assumption $\bar{L}_i = \bar{K}_i$ can be relaxed. If $\bar{L}_i \neq \bar{K}_i$, the condition such that $\bar{k}_1 \lessgtr \bar{k}_2$, exists. I will address these cases in section 2.3.

$\frac{\theta(z)I_i}{2p_i(z)}$. Then total capital for production of goods z that are in the range of $(0, 1]$ is $K_i^Z = \int_0^1 \frac{\theta(z)I_i}{2p_i(z)}dz$. Using $p_i(z) = r_i + a(z)w_i$ in (2.6), the capital demand for Z is replaced by

$$K_i^Z = \frac{I_i}{2}\int_0^1 \frac{\theta(z)}{r_i + a(z)w_i}dz. \quad (2.9)$$

The demand for capital also arises from sector Y. This is obtained by differentiating the cost function for Y, H_i^Y (2.2), with respect to the capital return.[15] Substituting p_i^M (2.5) and y_i (2.7) into K_i^Y and using p_i^Y (2.3), the capital demand for Y is rewritten as

$$K_i^Y = \frac{I_i}{4(r_i + w_i)}. \quad (2.10)$$

The total demand for capital should equal the endowment of capital under full employment. The capital market equilibrium condition is

$$K_i^Z + K_i^Y = \overline{K}_i, \quad i = 1, 2. \quad (2.11)$$

The return to capital in country i is determined by this condition.

Demand for labor in each country arises from their activities of production. First, I consider the labor demand in sector Z. $a(z)$ units of labor are used for production of one unit of z. Since z that are in the range of $(0, 1]$ should be produced in country i in the amount of the domestic demand $\frac{\theta(z)I_i}{2p_i(z)}$, country i employs labor equal to $\frac{a(z)\theta(z)I_i}{2p_i(z)}$. Thus total labor L_i^Z for aggregate production of the goods z is $L_i^Z = \int_0^1 \frac{a(z)\theta(z)I_i}{2p_i(z)}dz$. Using (2.6), the labor demand for Z is replaced by

$$L_i^Z = \frac{I_i}{2}\int_0^1 \frac{a(z)\theta(z)}{r_i + a(z)w_i}dz. \quad (2.12)$$

The labor demand L_i^Y in the stage of assembling the final good in sector Y is obtained by differentiating the cost function H_i^Y (2.2) with respect to the wage.[16] Using p_i^M (2.5), y_i (2.7) and p_i^Y (2.3), the labor demand for Y is replaced by

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[15] $K_i^Y = \frac{\partial H_i^Y}{\partial r_i} = \frac{1}{2} w_i^{\frac{1}{2}} (r_i + \frac{p_i^M}{b_i})^{-\frac{1}{2}} y_i$.

$$L_i^Y = \frac{I_i}{4w_i}. \quad (2.13)$$

To find the labor demand for production of the intermediate good, I need to know the demand for the intermediate good, c_i^M. It is obtained by differentiating H_i^Y (2.2) with respect to the price of M.[17] Using p_i^M (2.5), y_i (2.7) and p_i^Y (2.3), the demand for M is

$$c_i^M = \frac{I_i}{4b_i(r_i + w_i)}. \quad (2.14)$$

The demand for M should equal the supply of it: $c_i^M = m_i$. The production of m_i units of the intermediate good requires $b_i m_i$ units of labor, since one unit of M is produced with b_i units of labor. Then the labor demand L_i^M is

$$L_i^M = b_i m_i = \frac{I_i}{4(r_i + w_i)}. \quad (2.15)$$

Under full employment, labor market equilibrium in country i requires,

$$L_i^Z + L_i^Y + L_i^M = \overline{L}_i, \quad i = 1, 2. \quad (2.16)$$

The equilibrium conditions for the capital market (2.11) and the labor market (2.16) are simplified as follows.

$$K_i^Z = \overline{K}_i - K_i^Y \equiv \overline{K}_i', \quad (2.11')$$

$$L_i^Z + L_i^Y = \overline{L}_i - L_i^M \equiv \overline{L}_i'. \quad (2.16')$$

For comparing the relative factor prices of both countries, I divide (2.11') by (2.16'). That is $\frac{K_i^Z}{L_i^Z + L_i^Y} = \overline{k}_i'$, where $\overline{k}_i' = \frac{\overline{K}_i'}{\overline{L}_i'}$. Since $\overline{K}_i = \overline{L}_i$ by assumption and $K_i^Y = L_i^M$ from (2.10) and (2.15), $\frac{\overline{K}_i - K_i^Y}{\overline{L}_i - L_i^M} = \frac{\overline{K}_i}{\overline{L}_i}$. Note that $\frac{\overline{K}_i}{\overline{L}_i} = \overline{k}_i$. Then $\overline{k}_i' = \overline{k}_i$ since $K_i^Z = \overline{K}_i - K_i^Y$ and $L_i^Z + L_i^Y = \overline{L}_i - L_i^M$. Substituting L_i^Y (2.13) and L_i^Z (2.12) into the labor market equilibrium condition (2.16'),

---

16 $L_i^Y = \frac{\partial H_i^Y}{\partial w_i} = \frac{1}{2} w_i^{-\frac{1}{2}} (r_i + \frac{p_i^M}{b_i})^{\frac{1}{2}} y_i$.

17 $c_i^M = \frac{\partial H_i^Y}{\partial p_i^M} = \frac{1}{2b_i} w_i^{\frac{1}{2}} (r_i + \frac{p_i^M}{b_i})^{-\frac{1}{2}} y_i$.

$$\frac{I_i}{2}[\frac{1}{2w_i}+\int_0^1 \frac{a(z)\theta(z)}{r_i+a(z)w_i}dz]=\overline{L}_i'. \tag{2.17}$$

Substituting K_i^Z (2.9) into the capital market equilibrium condition (2.11'), it is replaced by

$$\frac{I_i}{2}\int_0^1 \frac{\theta(z)}{r_i+a(z)w_i}dz=\overline{K}_i'. \tag{2.18}$$

Then relative autarkic factor price $\frac{r_i}{w_i}$ is obtained by dividing (2.18) by (2.17).

$$\frac{\int_0^1 \frac{\theta(z)}{a(z)+\frac{r_i}{w_i}}dz}{\frac{1}{2}+\int_0^1 \frac{a(z)\theta(z)}{a(z)+\frac{r_i}{w_i}}dz}=\overline{k}_i'=\overline{k}_i,\text{[18]}\quad i=1,2. \tag{2.19}$$

The relative autarkic factor prices of both countries should be equal since $\overline{k}_1=\overline{k}_2$, and the other parameters in (2.19) are the same in both countries.

$$\frac{r_1}{w_1}=\frac{r_2}{w_2}. \tag{2.20}$$

The autarkic prices of M relative to z s are obtained from p_i^M (2.5) and $p_i(z)$ (2.6).

$$\frac{p_i^M}{p_i(z)}=\frac{b_i}{a(z)+\frac{r_i}{w_i}}.$$

Since the relative autarkic factor prices are the same between the countries in (2.20) and $b_1<b_2$ [19],

$$\frac{b_1}{a(z)+\frac{r_1}{w_1}}=\frac{p_1^M}{p_1(z)}<\frac{p_2^M}{p_2(z)}=\frac{b_2}{a(z)+\frac{r_2}{w_2}}\quad \text{for all } z,\ z\in(0,1]. \tag{2.21}$$

2.2.2 Open Economy

Consider the case that the two economies are open. The continuum of final goods z and the intermediate good M are freely traded. Labor is immobile between countries. Suppose that

[18] See Appendix 2.3.

[19] Recall that $b_1<b_2$ since country 1 has a more efficient technology for M compared to country 2.

government policy restricting foreign investment does not allow free capital movements between countries.

To predict patterns of trade, we should consider patterns of production in a trading equilibrium as well as the comparative advantage relation in (2.21). First, I address the patterns of production. In the model, labor and capital are production factors. The technologies for z s use both labor and capital, and the technology for M uses only labor. This can be interpreted as two sectors of different capital / labor intensities: sector Z is capital-intensive relative to industry M. This model is similar to the Heckscher-Ohlin (H-O) model of two countries, two sectors and two factors. Thus each country has a diversified pattern of production at the equilibrium and the nations' transformation curves for producing Z and M would be concave toward the origin as in the standard H-O model. Both countries specialize incompletely if the sizes of both countries are not very different. This means that each country produces some of both goods, but produces them in different proportions. The difference in production is traded.

However, since sector Z consists of a continuum of z s, trade may occur within this sector as well as across the sector Z and industry M. With only the inequality in (2.21), the following possible trade patterns are predicted.

(i) Country 1 with the more efficient technology for the intermediate good will export some of the domestically produced M.

(ii) Country 1 cannot export all sorts of z s within sector Z.

(iii) Country 1 either imports all sorts of z s, or imports some sorts of z s and exports other sorts of z s.

We cannot tell which trade pattern in case (iii) will emerge, because the relative autarkic factor prices are equal in both countries as shown in (2.20). In fact, the comparative advantage chain between goods z is segmented by the equilibrium factor prices that are determined in trading equilibrium, since these equilibrium factor prices are not equal between both countries. To resolve the indeterminacy of trade pattern that appears in case (iii), the equilibrium patterns of production mentioned above should also be considered. The analysis below shows how the trade patterns within sector Z are set.

Factor prices in country i are denoted as w_i^* and r_i^* in trading equilibrium. The symbol $*$ denotes the state in which the countries trade. Country i produces the intermediate good. The domestic price of the intermediate good is determined by country i 's technology for the intermediate good and its wage. The price of M is

$$p_i^{M^*} = b_i w_i^* . \tag{2.22}$$

The intermediate good is freely traded. Thus both countries face the price that is given by the world market. Country i's price is equal to the world price, $p_w^{M^*}$:

$$p_1^{M^*} = p_2^{M^*} = p_w^{M^*} . \tag{2.23}$$

Using (2.22) and (2.23), the wage of country i is

$$w_i^* = \frac{p_w^{M^*}}{b_i} . \tag{2.24}$$

The relation of the two wages is from (2.24)

$$w_1^* = \frac{b_2 w_2^*}{b_1} . \tag{2.25}$$

Since country 1 has a more efficient technology for M compared to country 2 (i.e., $b_1 < b_2$), $\frac{b_2}{b_1} > 1$. This means that w_1^* is higher than w_2^*:

$$w_1^* > w_2^* . \tag{2.26}$$

Country i's cost function of non-tradable final consumption good Y, $H_i^{Y^*}$, is obtained from (2.2). Substituting $p_w^{M^*}$ for $p_i^{M^*}$,

$$H_i^{Y^*} = w_i^{*\frac{1}{2}} (r_i^* + \frac{p_w^{M^*}}{b_i})^{\frac{1}{2}} y_i^* . \tag{2.27}$$

The unit cost of good Y is equal to the price of good Y. The price is

$$p_i^{Y^*} = w_i^{*\frac{1}{2}} (r_i^* + \frac{p_w^{M^*}}{b_i})^{\frac{1}{2}} . \tag{2.28}$$

The price of goods z $(= r_i + a(z) w_i)$ in (2.6) is increasing in z and is affected by both the capital return and the wage. Thus if differences in the factor prices between the two countries exist, this can cause the unit production costs of some sorts of the continuum of goods z to be lower in one country than the other country, and the unit production costs of the remaining sorts of the continuum of goods to be lower in the other country. This says that trade should arise within the sector Z. The unit cost $h_i^*(z)$ for production of one unit of z in country i is from (2.6).

$$h_i^*(z) = r_i^* + a(z) w_i^* , \quad z \in (0, 1] . \tag{2.29}$$

The price of z, $p_i^*(z)$, is equal to the unit cost if it is produced in country i:

$$p_i^*(z) = h_i^*(z) = r_i^* + a(z)w_i^* . \tag{2.30}$$

The fact that $w_1^* > w_2^*$ in (2.26) causes the slope of the unit cost line $h_1^*(z)$ for country 1 to be steeper than that of $h_2^*(z)$ for country 2 as in Figure 2.3. The higher index z that a good has, the larger is the labor cost of the good in country 1 compared to country 2. Also, the capital return affects the capital cost for production of one unit of good z and thus the intercept of the unit cost line $h_i^*(z)$. If the capital return of country 1, r_1^*, were higher than that of country 2, r_2^*, the unit cost line $h_1^*(z)$ would lie above that $h_2^*(z)$ as in Figure 2.3. If $r_1^* = r_2^*$, the lines $h_1^*(z)$ and $h_2^*(z)$ would meet at $z = 0$. This says that the unit production costs of every good z in country 2 would be lower than that of country 1 or would be equal to that of country 1 at $z = 0$. Then all sorts of the goods z would be produced only by country 2 and be exported to country 1.

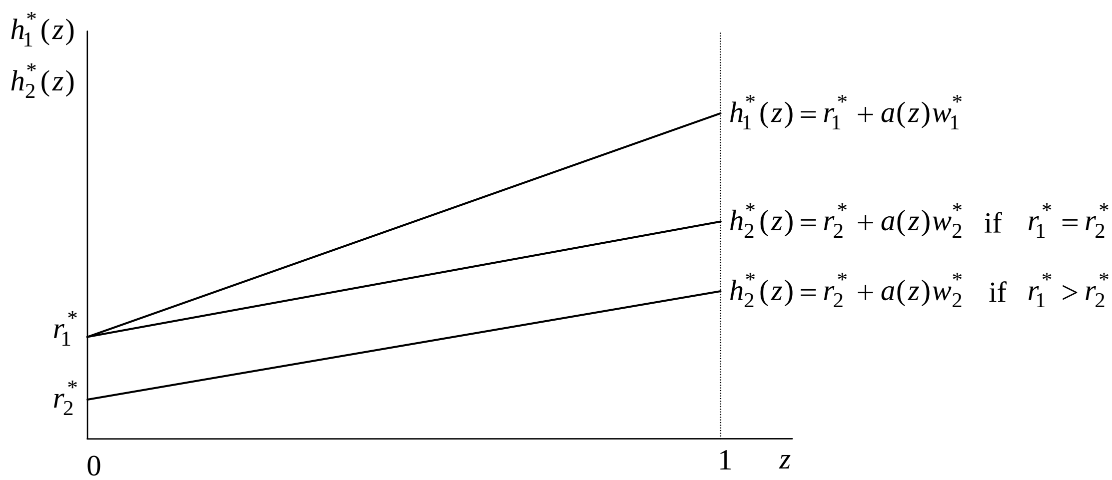

Figure 2.3

I examine whether the case, $\hat{z} = 0$, is a trading equilibrium, where $\hat{z}$ is defined as the value of z such that $h_1^*(\hat{z}) = h_2^*(\hat{z})$. Country 1 should produce a positive amount of Y and M, $y_1^* > 0$ and $m_1^* > 0$, since the final good Y is non-tradable and country 1 has an efficient technology for M compared to country 2. However, country 1 does not produce anything of Z. Then country 1 uses capital only for sector Y: $K_1^{Y^*} > 0$ and $K_1^{Z^*} = 0$. Also, it employs labor for sector Y and industry M, $L_1^{Y^*} > 0$, $L_1^{M^*} > 0$ and $L_1^{Z^*} = 0$. The full employment conditions for capital and labor in country 1 are

$$K_1^{Y^*} = \overline{K}_1,$$

$$L_1^{Y^*} + L_1^{M^*} = \overline{L}_1.$$

Recall that $K_i^{Y^*} = L_i^{M^*}$ from (2.10) and (2.15). Substituting $K_1^{Y^*}$ for $L_1^{M^*}$ into the full employment condition for labor,

$$L_1^{Y^*} + K_1^{Y^*} = \overline{L}_1. \qquad (2.31)$$

Using the full employment condition for capital $K_1^{Y^*} = \overline{K}_1$, equation (2.31) is replaced by

$$L_1^{Y^*} + \overline{K}_1 = \overline{L}_1. \qquad (2.32)$$

From the assumption of identical endowments of capital and labor $\overline{K}_1 = \overline{L}_1$, equation (2.32) requires $L_1^{Y^*} = 0$. This contradicts that $L_1^{Y^*} > 0$. Thus $z = 0$ is not the equilibrium location of $\hat{z}$.

If $r_1^* < r_2^*$, the location of $\hat{z}$ where the two unit cost lines cross is larger than zero, and equal to or less than one: $0 < \hat{z} \leq 1$. As shown in Figure 2.4, if $\hat{z}$ locates at 1, only country 1 produces all sorts of z s and exports all sorts. This contradicts the case, stated in (ii) of section 2.2.1.4, that country 1 cannot export all sorts of z s within sector Z. Thus $\hat{z} = 1$ cannot be the equilibrium. Then the location of $\hat{z}$ should be $0 < \hat{z} < 1$.

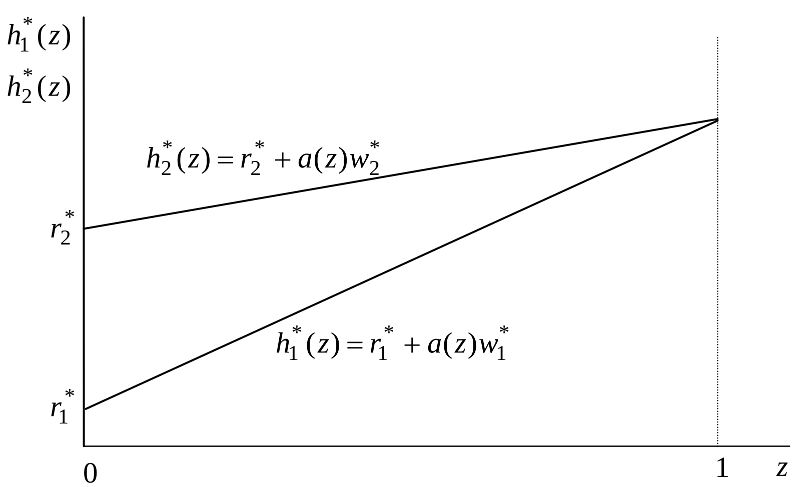

Figure 2.4

The condition $0 < \hat{z} < 1$ says that the line $h_1^*(z)$ crosses the line $h_2^*(z)$ from below as in Figure 2.5. These lines intersect at A where the unit cost lines of the two countries should cross each other somewhere within the range of $z \in (0, 1]$. This means that the production of the goods

z is distributed between the two countries. The minimum unit costs of producing the goods z in the range of $(0, \hat{z}]$ are represented by the line $h_1^*(z)$ since $h_1^*(z) < h_2^*(z)$. Thus country 1 produces a continuum of goods z in the range of $z \in (0, \hat{z}]$.[20] Also the minimum unit production costs of the goods z in the range of $(\hat{z}, 1]$ are represented by the line $h_2^*(z)$ since $h_2^*(z) < h_1^*(z)$. Thus country 2 produces goods z in the range of $z \in (\hat{z}, 1]$.

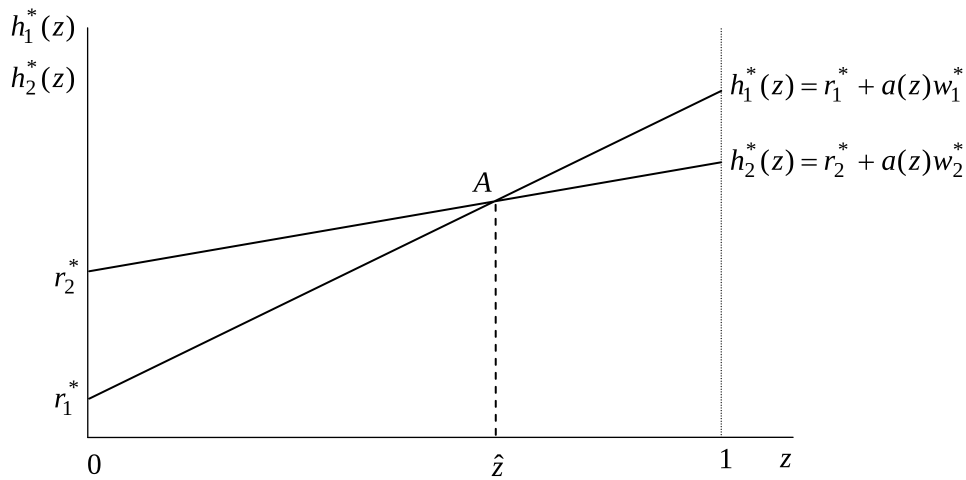

Figure 2.5

As the value of z decreases from $\hat{z}$ to zero in Figure 2.5, labor employed becomes small relative to capital for the production of one unit of z; in other words, z becomes more capital-intensive. This says that country 1 specializes in the production of the capital-intensive z s. As the value of z increases from $\hat{z}$ to one, labor employed becomes large relative to capital; that is, z becomes more labor-intensive. This says that country 2 specializes in the production of the labor-intensive z s.

2.2.2.1 Consumer Demand

The representative consumer in country 1 demands the final goods z in the range $(0, \hat{z}]$ produced at home and the imported final goods z in the range $(\hat{z}, 1]$. The subutility function u_1 is as follows:

$$\ln u_1^* = \int_0^{\hat{z}} \theta(z) \ln c_{11}^*(z) dz + \int_{\hat{z}}^1 \theta(z) \ln c_{12}^*(z) dz$$

[20] The good $\hat{z}$ at the margin is assumed to be produced by country 1.

$c_{11}^{*}(z)$: quantity that country 1's consumer demands of final good z produced in country 1.

$c_{12}^{*}(z)$: quantity that country 1's consumer demands of final good z imported from country 2.

The consumer maximizes utility subject to a budget constraint.

$$\max_{c_1^{Y^*},\, c_{11}^*(z),\, c_{12}^*(z)} \quad U_1^* = c_1^{Y^{*\frac{1}{2}}} u_1^{*\frac{1}{2}}$$

$$\text{s.t.} \quad p_1^{Y^*} c_1^{Y^*} + \{\int_0^{\hat{z}} p_1^*(z) c_{11}^*(z) dz + \int_{\hat{z}}^{1} p_2^*(z) c_{12}^*(z) dz\} = I_1^*$$

$p_1^*(z)$ and $p_2^*(z)$ are the prices of z prevailing in country 1 and country 2, respectively. The import price of country 1 is $p_2^*(z)$. I_1^* is income of country 1 and $I_1^* = r_1^* \overline{K}_1 + w_1^* \overline{L}_1$. From the first order conditions,[21]

$$p_1^{Y^*} c_1^{Y^*} = \frac{I_1^*}{2} \tag{2.33}$$

$$p_1^*(z) c_{11}^*(z) = \frac{\theta(z) I_1^*}{2}, \quad z \in (0,\ \hat{z}] \tag{2.34}$$

$$p_2^*(z) c_{12}^*(z) = \frac{\theta(z) I_1^*}{2}, \quad z \in (\hat{z},\ 1]\ . \tag{2.35}$$

Equation (2.33) says that the consumer spends one half of her income on the final good Y. The integral of equation (2.34) from final good $z = 0$ to $z = \hat{z}$ is expenditure that the consumer pays to the domestic producers of z:

$$\int_0^{\hat{z}} p_1^*(z) c_{11}^*(z) dz = \frac{I_1^*}{2} \int_0^{\hat{z}} \theta(z) dz\ . \tag{2.34'}$$

This expenditure depends positively on the income of country 1, I_1^*, the share of z in the subutility function, $\theta(z)$, and the location of the cutoff z, $\hat{z}$. The integral of equation (2.35) from final good $z = \hat{z}$ to $z = 1$ is expenditure paid to country 2's producers of z:

$$\int_{\hat{z}}^{1} p_2^*(z) c_{12}^*(z) dz = \frac{I_1^*}{2} \int_{\hat{z}}^{1} \theta(z) dz\ . \tag{2.35'}$$

[21] See Appendix 2.4.

This expenditure depends positively on I_1^* and $\theta(z)$. But it is negatively related to $\hat{z}$.

The consumer in country 2 also maximizes utility subject to a budget constraint. The utility function is:

$$U_2^* = c_2^{Y^{*\frac{1}{2}}} u_2^{*\frac{1}{2}}, \quad \text{where} \quad \ln u_2^* = \int_0^{\hat{z}} \theta(z) \ln c_{21}^*(z) dz + \int_{\hat{z}}^{1} \theta(z) \ln c_{22}^*(z) dz .$$

$c_{21}^*(z)$: quantity that country 2's consumer demands of final good z imported from country 1.

$c_{22}^*(z)$: quantity that country 2's consumer demands of final good z produced in country 2.

Country 2 imports the final goods z in $(0, \hat{z}]$; the import price of each is $p_1^*(z)$. The budget constraint is

$$p_2^{Y^*} c_2^{Y^*} + \{\int_0^{\hat{z}} p_1^*(z) c_{21}^*(z) dz + \int_{\hat{z}}^{1} p_2^*(z) c_{22}^*(z) dz\} = I_2^* .$$

From the first order conditions with respect to $c_2^{Y^*}$, $c_{21}^*(z)$ and $c_{22}^*(z)$,[22]

$$p_2^{Y^*} c_2^{Y^*} = \frac{I_2^*}{2} \tag{2.36}$$

$$p_1^*(z) c_{21}^*(z) = \frac{\theta(z) I_2^*}{2}, \quad z \in (0, \ \hat{z}] \tag{2.37}$$

$$p_2^*(z) c_{22}^*(z) = \frac{\theta(z) I_2^*}{2}, \quad z \in (\hat{z}, \ 1] . \tag{2.38}$$

Equation (2.36) says that the consumer in country 2 spends one half of her income on the final good Y. From (2.37), the expenditure on final goods z imported from country 1 is:

$$\int_0^{\hat{z}} p_1^*(z) c_{21}^*(z) dz = \frac{I_2^*}{2} \int_0^{\hat{z}} \theta(z) dz . \tag{2.37'}$$

It is positively related to I_2^*, $\theta(z)$ and $\hat{z}$. From (2.38), the expenditure on final goods z supplied by the domestic producers is:

$$\int_{\hat{z}}^{1} p_2^*(z) c_{22}^*(z) dz = \frac{I_2^*}{2} \int_{\hat{z}}^{1} \theta(z) dz . \tag{2.38'}$$

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[22] These are obtained in the same way as consumptions in country 1 in Appendix 2.4. I will not provide these calculations in the present paper.

It is positively related to I_2^* and $\theta(z)$, and is negatively related to $\hat{z}$.

From (2.33), for country 1, and (2.36), for country 2, the domestic demand for non-tradable good Y in each country is $c_i^{Y^*} = \dfrac{I_i^*}{2p_i^{Y^*}}$. Since the supply of the good Y equals the demand for it,

$$y_i^* = \frac{I_i^*}{2p_i^{Y^*}}, \quad i = 1, 2. \tag{2.39}$$

From (2.34), (2.35), (2.37) and (2.38), the respective demands for a good z are:

$$\begin{aligned} c_{11}^*(z) &= \frac{\theta(z)I_1^*}{2p_1^*(z)}, \quad c_{21}^*(z) = \frac{\theta(z)I_2^*}{2p_1^*(z)}, \quad z \in (0, \hat{z}] \\ c_{12}^*(z) &= \frac{\theta(z)I_1^*}{2p_2^*(z)}, \quad c_{22}^*(z) = \frac{\theta(z)I_2^*}{2p_2^*(z)}, \quad z \in (\hat{z}, 1]. \end{aligned} \tag{2.40}$$

World demand for a good z is obtained by summing each country's demand. I_w^* is the world income.

$$c_{11}^*(z) + c_{21}^*(z) = \frac{\theta(z)I_w^*}{2p_1^*(z)}, \quad z \in (0, \hat{z}], \quad \text{where } I_w^* = I_1^* + I_2^*,$$

$$c_{12}^*(z) + c_{22}^*(z) = \frac{\theta(z)I_w^*}{2p_2^*(z)}, \quad z \in (\hat{z}, 1].$$

2.2.2.2 Factor Markets

The supply of z should be equal to the world demand. Since one unit of capital is used for production of one unit of z, capital for producing an amount equal to world demand for z, $z \in (0, \hat{z}]$, is $\dfrac{\theta(z)I_w^*}{2p_1^*(z)}$ in country 1. Then total capital for production of goods z that are in the range of $(0, \hat{z}]$ is $K_1^{Z^*} = \int_0^{\hat{z}} \dfrac{\theta(z)I_w^*}{2p_1^*(z)} dz$. From (2.30), $p_1^*(z) = r_1^* + a(z)w_1^*$. Using this, $K_1^{Z^*}$ is replaced by

$$K_1^{Z^*} = \frac{I_w^*}{2} \int_0^{\hat{z}} \frac{\theta(z)}{r_1^* + a(z)w_1^*} dz. \tag{2.41}$$

$a(z)$ is defined as a simple linear function of z such that $a(z) = z$. To make the model manageable, the shares of consumption of z s are assumed to be identical. $\theta(z)$ can be replaced by unity.[23] Then equation in (2.41) is re-expressed as

$$K_1^{Z^*} = \frac{I_w^*}{2w_1^*} \ln(1 + \frac{w_1^* \hat{z}}{r_1^*}) .^{24} \qquad (2.41')$$

In country 2, capital for producing an amount equal to the world demand for z that is in $(\hat{z}, 1]$ is $\frac{\theta(z) I_w^*}{2 p_2^*(z)}$. Then total capital for the production of goods z, $z \in (\hat{z}, 1]$ in country 2 is $K_2^{Z^*} = \int_{\hat{z}}^{1} \frac{\theta(z) I_w^*}{2 p_2^*(z)} dz$. From (2.30), $p_2^*(z) = r_2^* + w_2^* z$. Using this, $K_2^{Z^*}$ is rewritten as

$$K_2^{Z^*} = \frac{I_w^*}{2w_2^*} \ln \frac{r_2^* + w_2^*}{r_2^* + w_2^* \hat{z}} .^{25} \qquad (2.42)$$

Capital required for the production of Y in country i is derived by differentiating $H_i^{Y^*}$ (2.27) with respect to the capital return.[26] Using y_i^* (2.39) and $p_i^{Y^*}$ (2.28) into $K_i^{Y^*}$,

$$K_i^{Y^*} = \frac{I_i^*}{4(r_i^* + \frac{p_w^{M^*}}{b_i})} . \qquad (2.43)$$

Since $\overline{K}_i = \overline{L}_i$ by assumption, $I_i^* = \overline{K}_i (r_i^* + w_i^*)$. Substituting w_i^* (2.24) into I_i^*, $K_i^{Y^*}$ is replaced by

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[23] See Appendix 2.5.

[24] $$K_1^{Z^*} = \frac{I_w^*}{2} \int_0^{\hat{z}} \frac{1}{r_1^* + w_1^* z} dz = \frac{I_w^*}{2} \left[\frac{1}{w_1^*} \ln(r_1^* + w_1^* z) \right]_0^{\hat{z}} = \frac{I_w^*}{2w_1^*} \{\ln(r_1^* + w_1^* \hat{z}) - \ln r_1^*\}$$
$$= \frac{I_w^*}{2w_1^*} \ln(1 + \frac{w_1^* \hat{z}}{r_1^*}) .$$

[25] $$K_2^{Z^*} = \frac{I_w^*}{2} \int_{\hat{z}}^{1} \frac{1}{r_2^* + w_2^* z} dz = \frac{I_w^*}{2} \left[\frac{1}{w_2^*} \ln(r_2^* + w_2^* z) \right]_{\hat{z}}^{1}$$
$$= \frac{I_w^*}{2w_2^*} \{\ln(r_2^* + w_2^*) - \ln(r_2^* + w_2^* \hat{z})\} = \frac{I_w^*}{2w_2^*} \ln \frac{r_2^* + w_2^*}{r_2^* + w_2^* \hat{z}} .$$

[26] $$K_i^{Y^*} = \frac{\partial H_i^{Y^*}}{\partial r_i^*} = \frac{1}{2} w_i^{*\frac{1}{2}} (r_i^* + \frac{p_w^{M^*}}{b_i})^{-\frac{1}{2}} y_i^* .$$

$$K_i^{Y^*} = \frac{1}{4}\overline{K}_i \,. \tag{2.43'}$$

In the capital market equilibrium, the capital demand equals the endowment of capital under full employment.

$$K_i^{Z^*} + K_i^{Y^*} = \overline{K}_i \,, \quad i = 1, 2 \,. \tag{2.44}$$

Substituting $K_i^{Y^*}$ (2.43') into (2.44), the capital market equilibrium condition in country i is rewritten as

$$K_i^{Z^*} = \frac{3}{4}\overline{K}_i \,. \tag{2.45}$$

These conditions will be used later, together with labor market equilibrium conditions, for finding the equilibrium capital return in country i and the cutoff good $\hat{z}$.

Demand for labor in each country arises from their activities of production. First, I consider the labor demand in sector Z. $a(z)$ units of labor are used for production of one unit of z. Since z that are in the range of $(0,\ \hat{z}]$ should be produced in country 1 in the amount of the world demand $\frac{\theta(z)I_w^*}{2p_1^*(z)}$, country 1 employs labor equal to $\frac{a(z)\theta(z)I_w^*}{2p_1^*(z)}$. Thus total labor $L_1^{Z^*}$ for production of the goods z, $z \in (0,\ \hat{z}]$, is $L_1^{Z^*} = \int_0^{\hat{z}} \frac{a(z)\theta(z)I_w^*}{2p_1^*(z)}dz$. This is equal to

$$L_1^{Z^*} = \frac{1}{2w_1^*}(\hat{z}I_w^* - \frac{3}{2}r_1^*\overline{K}_1)\,.^{27} \tag{2.46}$$

For z that are in the range $(\hat{z}, 1]$, they should be produced in country 2 in the amount of the world demand $\frac{\theta(z)I_w^*}{2p_2^*(z)}$, and country 2 employs labor equal to $\frac{a(z)\theta(z)I_w^*}{2p_2^*(z)}$. Then total labor $L_2^{Z^*}$ for production of the goods z, $z \in (\hat{z},\ 1]$, is $L_2^{Z^*} = \int_{\hat{z}}^{1} \frac{a(z)\theta(z)I_w^*}{2p_2^*(z)}dz$. This is re-expressed as

$$L_2^{Z^*} = \frac{1}{2w_2^*}\{(1-\hat{z})I_w^* - \frac{3}{2}r_2^*\overline{K}_2\}\,.^{28} \tag{2.47}$$

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[27] See Appendix 2.6.
[28] See Appendix 2.7.

Another sector Y demands labor. Labor that is employed for producing Y in country i is obtained by differentiating the cost function $H_i^{Y^*}$ (2.27) for country i with respect to the wage.[29] Using y_i^* (2.39) and $p_i^{Y^*}$ (2.28),

$$L_i^{Y^*} = \frac{I_i^*}{4w_i^*}. \tag{2.48}$$

To know the demand for labor from industry M, first, the demand for the intermediate good in each country should be derived. The demand for M is obtained by differentiating the cost function $H_i^{Y^*}$ (2.27) for country i with respect to $p_w^{M^*}$.[30] Using y_i^* (2.39) and $p_i^{Y^*}$ (2.28),

$$c_i^{M^*} = \frac{I_i^*}{4b_i\,(r_i^* + \frac{p_w^{M^*}}{b_i})}. \tag{2.49}$$

This is the quantity of the intermediate good that country i's sector Y uses for the production of the non-tradable final consumption good.

The demand for the intermediate good can be explained as follows. Since the intermediate good is used for the production of the component,[31] the demand for the intermediate good is affected by the production of the component. The production of the component is affected by the demand for final good Y. The demand for Y depends positively on income and negatively on the unit cost of the component. Consequently, as shown in (2.49), the demand for the intermediate good is affected positively by income I_i^*, and negatively not only by the efficiency of technology b_i but also by the unit cost of the component $(r_i^* + \frac{p_w^{M^*}}{b_i})$,[32] in which the price of

[29] $L_i^{Y^*} = \frac{\partial H_i^{Y^*}}{\partial w_i^*} = \frac{1}{2} w_i^{*-\frac{1}{2}} (r_i^* + \frac{p_w^{M^*}}{b_i})^{\frac{1}{2}} y_i^*$.

[30] $c_i^{M^*} = \frac{\partial H_i^{Y^*}}{\partial p_w^{M^*}} = \frac{1}{2b_i} w_i^{*\frac{1}{2}} (r_i^* + \frac{p_w^{M^*}}{b_i})^{-\frac{1}{2}} y_i^*$.

[31] Recall that the component is produced by a Leontief technology that uses capital and the intermediate good in the ratio of its productivity coefficient as shown in (2.1).

[32] From (2.2), the unit production cost of the component is $(r_i^* + \frac{p_w^{M^*}}{b_i})$ in the open economy.

the intermediate good is included. Since $\overline{K}_i = \overline{L}_i$, $I_i^* = \overline{L}_i (r_i^* + w_i^*)$. Substituting w_i^* (2.24) into I_i^*, equation (2.49) is replaced by

$$c_i^{M^*} = \frac{\overline{L}_i}{4b_i}. \qquad (2.49')$$

The demand for M depends only on exogenous b_i and $\overline{L}_i$ as shown in (2.49').

Country 1 has a more efficient technology for M compared to country 2, so that country 1 has a comparative advantage in the production of M. Country 1 exports some of the produced M to country 2. This means that country 1's supply of M exceeds its domestic demand. Where m_1^* denotes the quantity of M produced in country 1, country 1's excess supply is $(m_1^* - c_1^{M^*}) > 0$. Also m_1^* is equal to the sum of country 1's demand and its exports. Since one unit of M is produced with b_1 units of labor, country 1's labor demand for the production of m_1^* is

$$L_1^{M^*} = b_1 m_1^*. \qquad (2.50)$$

Country 2 has a comparative disadvantage in the production of M, so that it imports M. This country also produces M. The imports of country 2, which are the difference between its domestic demand and domestic production, are positive: $(c_2^{M^*} - m_2^*) > 0$. Since country 2's import is equal to country 1's export, its domestic production is expressed as

$$m_2^* = c_2^{M^*} - (m_1^* - c_1^{M^*}). \qquad (2.51)$$

Since one unit of M is produced with b_2 units of labor in country 2, the labor $L_2^{M^*}$ employed for the production of m_2^* is

$$L_2^{M^*} = b_2 m_2^* = b_2 \{c_2^{M^*} - (m_1^* - c_1^{M^*})\}. \qquad (2.52)$$

Under full employment, labor market equilibrium in country i requires

$$L_i^{Z^*} + L_i^{Y^*} + L_i^{M^*} = \overline{L}_i, \quad i = 1, 2. \qquad (2.53)$$

As shown in Figure 2.5, the cutoff good $\hat{z}$ is determined where the unit cost of good $\hat{z}$ of country 1 and that of country 2 intersect. Since $a(z) = z$, $h_i^*(z) = r_i^* + w_i^* z$. Then $\hat{z}$ is determined where $h_1^*(\hat{z}) = h_2^*(\hat{z})$.

$$\hat{z} = \frac{r_2^* - r_1^*}{w_1^* - w_2^*}, \quad 0 < \hat{z} < 1. \qquad (2.54)$$

This also says that the capital returns determined in general equilibrium have the following relationship $r_2^* > r_1^*$ since $w_1^* > w_2^*$ in (2.26).

This model contains 36 unknowns and 36 equations. The unknowns are determined by the general equilibrium: the prices of the goods $p_i^{Y^*}$ (2.28) and $p_i^*(z)$ (2.30); I assume that the intermediate good is the numeraire, $p_w^{M^*} = 1$; the consumptions of the goods $c_1^{Y^*}$ (2.33), $c_2^{Y^*}$ (2.36), $c_{11}^*(z)$ (2.40), $c_{21}^*(z)$ (2.40), $c_{12}^*(z)$ (2.40), $c_{22}^*(z)$ (2.40) and $c_i^{M^*}$ (2.49'); the demands for capital and labor $K_i^{Y^*}$ (2.43'), $K_i^{Z^*}$ (2.45), $L_1^{Z^*}$ (2.46), $L_2^{Z^*}$ (2.47), $L_i^{Y^*}$ (2.48), $L_1^{M^*}$ (2.50) and $L_2^{M^*}$ (2.52); the supplies of the goods y_i^* (2.39), m_i^* (2.51) and (2.53); the cutoff good $\hat{z}$ (2.54); the balanced trade; I_i^* is the national income; I_w^* is the world income; the returns r_1^* (2.41') and r_2^* (2.42); the wages w_i^* (2.24). The wages are determined by both the world price of the intermediate good and the efficiency of technology for the intermediate good. However, since the intermediate good is the numeraire (i.e., $p_w^{M^*} = 1$), the wages depend only on the coefficients of technology for the intermediate good, b_i: $w_i^* = \frac{1}{b_i}$ from (2.24).

2.2.2.3 Patterns and Volumes of Trade

The factor prices and $\hat{z}$, which are obtained in the general equilibrium, determine trade patterns. Country 1 exports the intermediate good and the consumption goods z in the range of $(0, \hat{z}]$. Also, country 1 exports z s in value equal to the value that country 2 consumes these goods. Let E_1^Z denote the value of exported z s from country 1. Then, from (2.37'), $E_1^Z = \frac{I_2^*}{2}\int_0^{\hat{z}} \theta(z)dz$. The value of exports of country 1 is the sum of the values of exports of final goods and the intermediate good: $E_1^Z + E_1^M$, where E_1^M denotes the value of country 1's exported intermediate good. The value of exports of country 2 is the value of exports of only the final goods in the range of $(\hat{z}, 1]$. Let E_2^Z denote this value. From (2.35'), $E_2^Z = \frac{I_1^*}{2}\int_{\hat{z}}^{1} \theta(z)dz$.

Under the assumption of balanced trade in the two-country model, the value of exports of one country is equal to the value of exports of the other country. Then balanced trade requires $E_1^M + E_1^Z = E_2^Z$. This is rewritten as

$$E_1^M = E_2^Z - E_1^Z . \tag{2.55}$$

This is also the volume of trade in the intermediate good since only country 1 exports M: $E_1^M = VOT^M$.

The volume of total trade consists of the volume of the intermediate good M and the volume of final goods z: $VOT^M + VOT^Z = E_1^M + E_1^Z + E_2^Z$. The share, S, of trade in the intermediate good in total trade is

$$S = \frac{E_1^M}{E_1^M + E_1^Z + E_2^Z} .$$

Using (2.55), this share is derived as follows:

$$S = \frac{1}{2}(1 - \frac{E_1^Z}{E_2^Z}) . \tag{2.56}$$

Substituting expressions for E_1^Z and E_2^Z obtained earlier into (2.56), S is rewritten as

$$S = \frac{1}{2}(1 - \frac{I_2^* \int_0^{\hat{z}} \theta(z)dz}{I_1^* \int_{\hat{z}}^1 \theta(z)dz}) .$$

$\int_0^{\hat{z}} \theta(z)dz$ is country 2's cumulative expenditure share for final goods z produced by country 1, and $I_2^* \int_0^{\hat{z}} \theta(z)dz$ is country 2's aggregate expenditure on final goods z of country 1, $z \in (0, \hat{z}]$. $\int_{\hat{z}}^1 \theta(z)dz$ is country 1's cumulative expenditure share for final goods z produced by country 2, and $I_1^* \int_{\hat{z}}^1 \theta(z)dz$ is country 1's aggregate expenditure on final goods z of country 2, $z \in (\hat{z}, 1]$. Thus the share S is affected by changes in income and the cumulative expenditure share on the imported continuum of final goods in both countries.

2.2.3 Capital Movement

I define foreign investment as capital movement that occurs across borders to gain from a difference in capital returns. The gain from invested capital belongs to the source country that makes the investment.

Suppose that foreign investment now occurs in the amount $\Delta K_1 > 0$ from country 1 to country 2 because of $r_1^* < r_2^*$, and that ΔK_1 is exogenously determined. The capital flow would allow country 1 to earn the additional return $(r_2^* - r_1^*)\Delta K_1 > 0$ if returns to capital were unchanged at the initial state. The income of country 1 would increase, but the income of country 2 would not change.

The foreign investment also affects capital returns in both countries. To look at changes in the capital returns before and after the capital movement, I will perform comparative statics with the equilibrium condition that represents equilibrium in the capital market before the capital movement. For this, the equilibrium condition needs to be re-expressed as an appropriate formulation instead of (2.45).[33] The capital market equilibrium condition for country i, (2.44), is rearranged as

$$K_i^{Z^*} = \overline{K}_i - K_i^{Y^*}. \tag{2.57}$$

Substituting $K_1^{Z^*}$ (2.41'), $K_1^{Y^*}$ (2.43), $I_1^* = w_1^* \overline{L}_1 + r_1^* \overline{K}_1$ and w_1^* (2.24) into (2.57), the capital market equilibrium condition for country 1 is

$$\ln\{1 + \frac{(\frac{p_w^{M^*}}{b_1})\hat{z}}{r_1^*}\} = \{\frac{(\frac{p_w^{M^*}}{b_1})}{2I_w^*}\}\{3\overline{K}_1 + \frac{(\overline{K}_1 - \overline{L}_1)(\frac{p_w^{M^*}}{b_1})}{r_1^* + (\frac{p_w^{M^*}}{b_1})}\}, \quad \text{where } p_w^{M^*} = 1.\text{[34]} \tag{2.58}$$

I examine the effect of the capital movement on the capital return. A capital outflow decreases the capital stock $\overline{K}_1$ on the RHS of (2.58). In order to equalize the LHS to the lowered RHS, the capital return rises.[35] This means $r_1^{*'} > r_1^*$ where $r_1^{*'}$ denotes the new return.

Similarly, substituting $K_2^{Z^*}$ (2.42), $K_2^{Y^*}$ (2.43), $I_2^* = w_2^* \overline{L}_2 + r_2^* \overline{K}_2$ and w_2^* (2.24) into (2.57), the capital market equilibrium condition for country 2 is

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[33] Since $\overline{K}_i$ is no longer equal to $\overline{L}_i$ due to the capital movement, equation (2.45) cannot be used.

[34] See Appendix 2.8.

[35] See Appendix 2.9.

$$\ln\{\frac{r_2^* + (\frac{p_w^{M^*}}{b_2})}{r_2^* + (\frac{p_w^{M^*}}{b_2})\hat{z}}\} = \{\frac{(\frac{p_w^{M^*}}{b_2})}{2I_w^*}\}\{3\overline{K}_2 + \frac{(\overline{K}_2 - \overline{L}_2)(\frac{p_w^{M^*}}{b_2})}{r_2^* + (\frac{p_w^{M^*}}{b_2})}\}, \quad \text{where } p_w^{M^*} = 1. \qquad (2.59)$$

A capital inflow increases the capital stock $\overline{K}_2$ on the RHS of (2.59). To equalize the LHS to the increased RHS, the capital return falls.[36] This means $r_2^{*'} < r_2^*$ where $r_2^{*'}$ denotes the new capital return. In addition, I assume that the capital movement is too small to make $r_1^{*'} = r_2^{*'}$ or to reverse $r_1^{*'} < r_2^{*'}$.

The changes in capital returns affect incomes of both countries. First, I explain the change in the income of country 1. Country 1 earns the capital return $r_2^{*'}$ per unit of capital invested in country 2. Income corresponding to ΔK_1 is $r_2^{*'}\Delta K_1$. The return per unit of the capital remaining in country 1 is $r_1^{*'}$. The income corresponding to the remaining capital $(\overline{K}_1 - \Delta K_1)$ is $r_1^{*'}(\overline{K}_1 - \Delta K_1)$. Total income from capital in country 1 is the sum of the two incomes from capital.

$$r_1^{*'}\overline{K}_1 + (r_2^{*'} - r_1^{*'})\Delta K_1, \quad \text{where } (r_2^{*'} - r_1^{*'}) > 0.$$

This is larger than income from capital $\overline{K}_1$ before the capital movement, since $r_1^{*'} > r_1^*$ and $r_2^{*'} > r_1^{*'}$:

$$r_1^{*'}\overline{K}_1 + (r_2^{*'} - r_1^{*'})\Delta K_1 > r_1^*\overline{K}_1.$$

I turn now to labor income of country 1. Since the wage of country 1 is determined by the fixed coefficient of technology for M in country 1, w_1^* remains unchanged. Country 1’s labor endowment and its wage are not changed, so that labor income remains unchanged. Thus the foreign investment increases income of country 1 from I_1^* to $I_1^{*'}$ due to only the increase in country 1’s capital income.

[36] This outcome should be the same as the outcome corresponding to country 2 suggested in Appendix 2.9. Thus I will omit proving this directly with the capital market equilibrium condition for country 2.

In contrast, after the capital inflow, income of country 2 from domestic capital $\overline{K}_2$ is $r_2^{*'}\overline{K}_2$. This is smaller than income from capital $\overline{K}_2$ before the capital inflow since $r_2^{*'} < r_2^*$:

$$r_2^{*'}\overline{K}_2 < r_2^*\overline{K}_2 .$$

Also labor income of country 2 does not change since country 2's labor endowment and its wage are unchanged. Recall that w_2^* is determined by the fixed coefficient of technology for M in country 2. Thus income of country 2 falls from I_2^* to $I_2^{*'}$ due to only the decrease in country 2's capital income.

The rise in capital return of country 1 due to the capital outflow makes the unit cost line $h_1^*(z)$ of country 1 shift parallel and upward in Figure 2.6. The decrease in capital return of country 2 makes the unit cost line $h_2^*(z)$ of country 2 shift parallel and downward. The intersection of these lines moves from A to A'.[37] The location of the cutoff good moves from $\hat{z}$ to $\hat{z}'$. The range of goods z that country 1 exports would shrink from $(0, \hat{z}]$ to $(0, \hat{z}']$, and the range of goods z that country 2 exports would expand from $(\hat{z}, 1]$ to $(\hat{z}', 1]$.

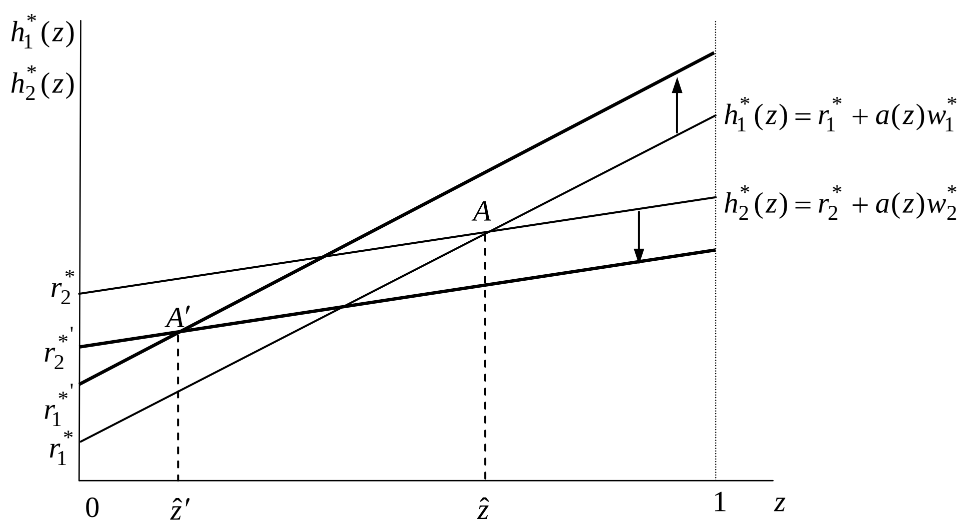

Figure 2.6

This is intuitively explained as follows. A rise of the capital return in country 1 increases the production cost of the continuum of final goods in country 1. Country 1, which specialized in

[37] The magnitudes of the shifts of the two lines, $h_1^*(z)$ and $h_2^*(z)$, are not determined. However, the direction that the cutoff good $\hat{z}$ moves is determined with no information on the magnitudes since the directions of shifts of the two lines are opposite.

production of the capital-intensive z s, loses comparative advantage in some sorts of these goods. Thus country 1 exports fewer sorts of the continuum of final goods and becomes an importer of the final goods that it gives up producing. In contrast, the fall of the capital return in country 2 reduces the production cost of the continuum of final goods in country 2. Country 2, which specialized in production in the labor-intensive z s, gains comparative advantage in some sorts of the capital-intensive z s which were under comparative disadvantage before. Country 2 exports more sorts of the continuum of final goods by displacing the goods z that country 1 exported but now imports.

As shown in Figure 2.6, the shift in cutoff good $\hat{z}$ means that country 2's cumulative expenditure share for final goods z produced by country 1, $\int_0^{\hat{z}'} \theta(z)dz$, falls and country 1's cumulative expenditure share for final goods z produced by country 2, $\int_{\hat{z}'}^1 \theta(z)dz$, rises. Since the changes in capital returns increase country 1's income and decrease country 2's income, country 2's aggregate expenditure, $\frac{I_2^{*'}}{2}\int_0^{\hat{z}'} \theta(z)dz$, falls (in other words, country 1's value of exports of final goods z, E_1^Z, falls) and country 1's aggregate expenditure, $\frac{I_1^{*'}}{2}\int_{\hat{z}'}^1 \theta(z)dz$, rises (in other words, country 2's value of exports of final goods z, E_2^Z, rises).

Also country 2 pays for the services of the imported capital to country 1. The revenue that country 1 earns is $r_2^{*'}\Delta K_1$. Country 2 pays in terms of either a single sort of z or a mix of multiple z s of the continuum of final goods $z \in (\hat{z}', 1]$ that it exports. The remitted payment is $r_2^{*'}\Delta K_1$ that appears as the export of z of country 2. Thus the value of exports of country 2 is the value of exports of the continuum of final goods z, plus the payment for the capital services. After the capital movement, the balanced trade is

$$E_1^M + E_1^Z = E_2^Z + r_2^{*'}\Delta K_1.$$

Country 1's value of exports of the intermediate good, E_1^M, is derived from the balance of trade.

$$E_1^M = E_2^Z - E_1^Z + r_2^{*'}\Delta K_1. \tag{2.60}$$

This is also the trade volume of the intermediate good since only country 1 exports the intermediate good: $E_1^M = VOT^M$.

To look at how capital movement affects the trade volume of the intermediate good, I use (2.60). The flow of capital from country 1 to country 2 increases country 2's value of exports of the continuum of final goods E_2^Z , and decreases country 1's value of exports of the continuum of final goods E_1^Z . Also country 1 earns revenue from the capital services, $r_2^{*'} \Delta K_1 > 0$. This increases the RHS of (2.60), so that the trade volume of the intermediate good increases. This says that the capital movement is a complement to trade in the intermediate good.

To explain intuitively the increased trade volume of the intermediate good, I address first how the demand for the intermediate good in each country responds to the capital movement. The capital outflow from country 1 raises the capital return of country 1 and thus increases the price of the component.[38] The wage is not changed. The price of final good Y rises and thus the demand for it falls for a given income. Sector Y reduces its production and comes to use a smaller quantity of the component and less labor for given price of the component relative to labor. However, since the price of the component relative to labor also rises, sector Y readjusts the use of inputs by substituting labor for the component. This will reduce additionally the use of the component compared to the case when we do not consider the change in the relative factor price. Thus the quantity of the component used for final good Y after the capital outflow is smaller than that before the capital outflow. This causes the demand for the intermediate good to decrease.

On the other hand, the capital outflow from country 1 increases country 1's income. This causes an increase in demand for final good Y – it is a normal good. To match the increased demand, sector Y increases its production and uses more of the component and more labor for the given higher relative factor price. Thus, the demand for the intermediate good increases.

In sum, the actual change in the demand for the intermediate good is the net of the income effect and the relative factor price effect. This change can be shown by the equation for the proportional change in demand for M . Using log transformation of (2.49) and taking the derivative with respect to $\overline{K}_i$,

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[38] After the capital outflow, the price of the component in country 1, $(r_1^{*'} + \frac{p_w^{M*}}{b_1})$, rises because of $r_1^{*'} > r_1^{*}$.

$$\frac{\frac{\partial c_i^{M^*}}{\partial \overline{K}_i}}{c_i^{M^*}} = \frac{\frac{\partial I_i^*}{\partial \overline{K}_i}}{I_i^*} - \frac{\frac{\partial (r_i^* + \frac{p_w^{M^*}}{b_i})}{\partial \overline{K}_i}}{(r_i^* + \frac{p_w^{M^*}}{b_i})} \,, \quad i = 1, 2. \tag{2.61}$$

[39]

$$= \frac{1}{\overline{K}_i (1 + \frac{1}{b_i r_i^*})} \,, \quad \text{where } p_w^{M^*} = 1. \tag{2.61'}$$

The LHS in (2.61) represents the proportion of the change in demand – the net demand – for M induced by the capital movement. The first term on the RHS in (2.61) is the proportion of the change in income induced by the capital movement. The second term is the proportional change in the price of the component induced by the capital movement. These terms on the RHS are simplified as shown in (2.61'). This simplified term is positive. The positive sign means that $\frac{\partial c_i^{M^*}}{\partial \overline{K}_i} > 0$ on the LHS in (2.61). Then, for country 1, a capital outflow decreases the capital stock $\overline{K}_1$ and thus the demand for the intermediate good in country 1, $c_1^{M^*}$, decreases. This is interpreted as follows. The decrease in the demand for M due to the increased price of Y is larger than the increase in the demand for M due to country 1's increased income. This leads to the decrease in the net demand for M in country 1.

In contrast, a capital inflow to country 2 increases the capital stock $\overline{K}_2$. The capital inflow decreases the capital return of country 2 and the component becomes cheaper. Thus the price of final good Y falls. The demand for Y is increased. This causes sector Y to use more intermediate good, compared with before the capital inflow. However, country 2's reduced income decreases the demand for Y and thus the demand for M decreases. Since the increase in the demand for M due to the lower price of Y is larger than the decrease in the demand for M due to country 2's reduced income, the net demand for M in country 2, $c_2^{M^*}$, increases.

I turn now to the supply response of the intermediate good to the capital movement. The fall in the sorts of the continuum of final goods z produced in country 1 makes this country's labor move away from sector Z and enter sector Y and industry M. The production of M increases in country 1. However, since the demand in country 1 for M decreases, the excess supply increases. This appears as an increase in exports of the intermediate good to country 2.

[39] See Appendix 2.10.

In contrast, the rise in the sorts of the continuum of final goods z produced in country 2 makes this country's labor shift from sector Y and industry M to sector Z. The production of M is decreased in country 2. However, since the demand in country 2 for M increases, this causes country 2 to import more of the intermediate good.

The trade volume of final goods z is the sum of country 1's export value of final goods z and country 2's export value of final good z, plus the payment for the capital services,

$$VOT^Z = E_1^Z + E_2^Z + r_2^{*'} \Delta K_1 . \tag{2.62}$$

To explain how capital movement affects the trade volume of final goods, I have to mention the magnitudes of decrement in E_1^Z and increment in E_2^Z. First, suppose that the magnitude of fall in E_1^Z is smaller than that of the rise in E_2^Z. Then $E_1^Z + E_2^Z$ increases. Also, since $r_2^{*'} \Delta K_1$ is positive, VOT^Z increases. This says that the capital movement is a complement to trade in final goods. Second, suppose that the magnitude of the fall in E_1^Z is larger than that of the rise in E_2^Z. Then $E_1^Z + E_2^Z$ decreases. However, since the positive $r_2^{*'} \Delta K_1$ is added, VOT^Z can decrease or increase. This means that the capital movement is a substitute for trade in final goods if VOT^Z decreases, or a complement to trade in final goods if VOT^Z increases.

Since the share of the intermediate good in total trade is the ratio of VOT^M to $VOT^M + VOT^Z$, after the capital movement, the share S' is

$$S' = \frac{E_1^M}{E_1^M + E_1^Z + E_2^Z + r_2^{*'} \Delta K_1} . \tag{2.63}$$

Using the balance of trade in (2.60),

$$S' = \frac{1}{2}\left(1 - \frac{E_1^Z}{E_2^Z + r_2^{*'} \Delta K_1}\right) .$$

Compared to the share S (2.56) before the capital movement, since the capital movement adds $r_2^{*'} \Delta K_1$ to the denominator and raises E_2^Z, the denominator rises, but E_1^Z in the numerator falls. Thus the share increases; that is, $S' > S$.

To explain intuitively the rise in the share, I re-express S' (2.63) as

$$S' = \frac{1}{1 + \dfrac{VOT^Z}{VOT^M}} .$$

First, consider the case that both VOT^M and VOT^Z increase. Country 1, which exported some sorts of goods z before the capital movement, comes to import some of the previously exported goods z after the capital movement. Country 2, which imported these goods before the capital movement, comes to export these after the capital movement. This change in exporter identity increases the trade volume of final goods due to two factors. The first is the difference between the value of export of goods z that country 2 imported from country 1 but now exports and the value of export that occurred when country 1 exported these goods to country 2. This difference occurs due to a change in production cost that results from the switches in exporter identity from country 1 to country 2. The second is the remittance of the capital services payment from country 2. Under balanced trade, country 1 also comes to export more M by the sum of three things: the value of export of goods z that country 2 imported but now exports; the value of export that occurred when country 1 exported these goods; and country 2's payment for the capital services. The trade volume of the intermediate good increases as much as the sum. Thus, the increment in VOT^M is larger than the increment in VOT^Z.[40] This leads to a rise in the share.

Consider the second case that VOT^M increases, but VOT^Z decreases or increases. If VOT^M increases and VOT^Z decreases, the share rises regardless of the absolute magnitudes of change in VOT^M and VOT^Z. If both VOT^M and VOT^Z increase, the share rises since the increment in VOT^M is larger than the increment in VOT^Z as explained in the first case. Thus, these also raise the share.

As seen in the first case and the second case, the capital movement leads to a rise in the share of the intermediate good in total trade regardless of whether the trade volume of final goods increases or decreases. This says that the rise in the share is not affected by whether the capital movement and trade in final goods are complements or substitutes.

[40] From (2.60), the trade volume of the intermediate good is $VOT^M = E_2^Z - E_1^Z + r_2^{*'} \Delta K_1$. From (2.62), the trade volume of final goods z is $VOT^Z = E_1^Z + E_2^Z + r_2^{*'} \Delta K_1$. The difference between the former and the latter is that $VOT^M - VOT^Z = -2E_1^Z$. Taking the derivative with respect to K_1, $\frac{\partial VOT^M}{\partial K_1} - \frac{\partial VOT^Z}{\partial K_1} = -2\frac{\partial E_1^Z}{\partial K_1} > 0$ since $\frac{\partial E_1^Z}{\partial K_1} < 0$. This says that $\frac{\partial VOT^M}{\partial K_1} > \frac{\partial VOT^Z}{\partial K}$ if $\frac{\partial VOT^M}{\partial K_1} > 0$ and $\frac{\partial VOT^Z}{\partial K_1} > 0$.

2.3 Discussion

The assumption of identical relative factor endowments can be relaxed to an environment in which both countries have different relative factor endowments. If country 1, with technological advantage in the intermediate good, is less capital-abundant relative to labor compared to country 2, the relationship of relative endowments is $\frac{\overline{K}_1}{\overline{L}_1} < \frac{\overline{K}_2}{\overline{L}_2}$. When this is compared with the case in the main text $\frac{\overline{K}_1}{\overline{L}_1} = \frac{\overline{K}_2}{\overline{L}_2}$, the capital return relative to wage in country 1 would rise and the capital return relative to wage in country 2 would fall, compared with $\frac{r_1}{w_1} = \frac{r_2}{w_2}$ in (2.20). Then the new autarkic relative factor price relationship is $\frac{r_1}{w_1} > \frac{r_2}{w_2}$. The comparative advantage inequality for M and z s remains unchanged as $\frac{b_1}{a(z) + \frac{r_1}{w_1}} = \frac{p_1^M}{p_1(z)} < \frac{p_2^M}{p_2(z)} = \frac{b_2}{a(z) + \frac{r_2}{w_2}}$ in (2.21). Thus, the trade patterns are the same as the case in the main text. Also, the trade volumes of the intermediate good and final goods, and the share of the intermediate good in total trade move in the same directions as the case in the main text. However, if the relationship of relative endowments is $\frac{\overline{K}_1}{\overline{L}_1} > \frac{\overline{K}_2}{\overline{L}_2}$, this yields the new autarkic relative factor prices relationship $\frac{r_1}{w_1} < \frac{r_2}{w_2}$. This inequality may break the comparative advantage inequality in (2.21), so that the trade patterns, the trade volumes and the share of trade volumes could be ambiguous.

The Leontief technology for the tradable final goods z can be extended to a Cobb-Douglas technology. If this technology is the same in both countries and the corresponding unit cost functions are $h_i(z) = w_i^{1-a(z)} r_i^{a(z)}$, the autarkic prices of M relative to z s are $\frac{p_i^M}{p_i(z)} = \frac{b_i}{(\frac{r_i}{w_i})^{a(z)}}$. Furthermore, the equality of relative autarkic factor prices (2.20) remains unchanged. This yields the following comparative advantage relation: $\frac{b_1}{(\frac{r_1}{w_1})^{a(z)}} = \frac{p_1^M}{p_1(z)} < \frac{p_2^M}{p_2(z)} = \frac{b_2}{(\frac{r_2}{w_2})^{a(z)}}$. This relation has the same inequality as in (2.21). Though Cobb-Douglas technology is used, all results obtained in the main text can hold. However, to show these results, simulation should be used.

I have assumed that the intermediate good uses only labor, and the tradable final goods z use capital and labor. This means that the production of the intermediate good is more labor-intensive than the production of the final goods. However, suppose that the continuum of intermediate goods use capital and labor, and the final good uses only labor. That is, the intermediate goods are capital-intensive relative to the final good. This situation can be represented by the technology for the continuum of intermediate goods $x_i(m)$, $m \in (0, 1]$, where m is the index indicating a sort of intermediate goods, $x_i(m) = \min\{\frac{L_i(m)}{a(m)}, K_i(m)\}$, and by the technology for the tradable final good $z_i = \frac{L_i}{b_i}$. Assuming that the composite good is $d_i = \int_0^1 \theta(m) \ln x_i(m) dm$, where $\theta(m)$ is the share of intermediate goods used, the production function of the non-tradable final good is $y_i = vL_i^{Y\frac{1}{2}} d_i^{\frac{1}{2}}$. The utility function changes to $U_i = c_i^{Y\frac{1}{2}} z_i^{\frac{1}{2}}$.

Since the above changes replace the technology for z by that for M, and the technology for M by that for z, and since there are no changes in functional forms in the model, the same conclusions relating to the final good in the main text can be converted to conclusions about the intermediate good. Then capital movement may decrease or increase the volume of trade in intermediate goods, where the payment for the capital services is paid in terms of the intermediates. The volume of trade in the final good may increase. If VOT^M decreases and VOT^Z increases, the share of the intermediate goods in total trade would fall. If both VOT^M and VOT^Z increase, the share would fall since the increment in VOT^Z is larger than the increment in VOT^M. Thus the share of the intermediate goods in total trade would decrease regardless of whether capital movement and trade in the intermediate goods are complements or substitutes. However, it is necessary to examine whether the intermediates are capital-intensive. This is beyond the scope of the present paper, and is left for future empirical studies.

2.4 Conclusion

This paper examines whether capital movement is a substitute for or a complement to trade in intermediate goods and final goods, and explains why capital movement can increase the share of the intermediate good in total trade.

In a trading equilibrium, the source country, with a lower capital return and a higher wage, exports part of the continuum of final goods using capital relatively intensively compared to labor. Also this country exports the intermediate good that is used for production of a non-tradable final good since it has a more efficient technology for the intermediate good. The host country, with a higher capital return and a lower wage, exports another part of the continuum of final goods using labor relatively intensively compared to capital. Also the host country imports the intermediate good since it has a less efficient technology for the intermediate good.

When capital movement occurs, this raises the capital return in the source country and increases its production cost of the continuum of final goods. Thus the source country exports fewer sorts of the continuum of final goods than before the capital movement. However, the country increases its production of the intermediate good because labor moves away from the sector producing the continuum of final goods, and enters the sector producing the non-tradable final good and the intermediate good.

The rise in capital return in the source country increases the price of the non-tradable final good and thus decreases the demand for the intermediate good. However, the source country's increased income increases the demand for the intermediate good. The net demand for the intermediate good, due to the two opposite effects, is decreased. The increased excess supply of the intermediate good is exported to the host country.

The fall in capital return in the host country decreases the production cost of the continuum of final goods. Thus this country exports more sorts of the continuum of final goods to the source country. This makes the host country's labor shift from the sector producing the non-tradable final good and the intermediate good, to the sector producing the continuum of final goods. The production of the intermediate good is decreased.

Since the fall in capital return in the host country decreases the price of the non-tradable final good, the demand for the intermediate good increases. However, the host country's reduced income decreases the demand for the intermediate good. The former effect is larger than the latter effect, so that the net demand for the intermediate good increases. Thus the host country imports more of the intermediate good.

The volume of trade in final goods can increase or decrease depending on changes in income and cumulative expenditure share on the imported continuum of final goods in both countries, and payment for the capital services. However, the volume of trade in the intermediate good increases in all cases. Thus capital movement and trade in the intermediate good are complements while capital movement and trade in final goods are either complements or substitutes. The share of the intermediate good in total trade rises, regardless of whether capital movement and trade in final goods are complements or substitutes.

Appendices

Appendix 2.1

The production function for Y is

$$y_i = vL_i^{Y\frac{1}{2}}[\min\{K_i^Y, b_i M_i\}]^{\frac{1}{2}}, \quad \text{where } v = 2.$$

Let $d_i = \min\{K_i^Y, b_i M_i\}$ be the production function for a component d. d_i is output of the component. To produce d_i, capital is used in the amount d_i and the intermediate good is used in the amount $\frac{d_i}{b_i}$. When the price of M is p_i^M, the production cost of the component is $(r_i + \frac{p_i^M}{b_i})d_i$. For the production of y_i, the component d_i is inputted into the Cobb-Douglas technology along with L_i^Y units of labor. The production cost for this process is obtained from the following cost minimization:

$$\underset{L_i^Y, d_i}{Min} \quad w_i L_i^Y + (r_i + \frac{p_i^M}{b_i})d_i$$

$$\text{s.t.} \quad vL_i^{Y\frac{1}{2}} d_i^{\frac{1}{2}} = y_i.$$

The cost function is $\frac{2}{v} w_i^{\frac{1}{2}} (r_i + \frac{p_i^M}{b_i})^{\frac{1}{2}} y_i$. Since $v = 2$, the cost function is $w_i^{\frac{1}{2}} (r_i + \frac{p_i^M}{b_i})^{\frac{1}{2}} y_i$.

Appendix 2.2

The utility maximization problem is

$$\underset{c_i^Y, c_i(z)}{Max} \quad c_i^{Y\frac{1}{2}} u_i^{\frac{1}{2}}$$

$$\text{s.t.} \quad p_i^Y c_i^Y + \int_0^1 p_i(z) c_i(z) dz = I_i.$$

Recall that $\ln u_i = \int_0^1 \theta(z) \ln c_i(z) dz$, i =1 or 2.

$$L = c_i^{Y\frac{1}{2}} u_i^{\frac{1}{2}} + \lambda[I_i - p_i^Y c_i^Y - \int_0^1 p_i(z) c_i(z) dz]$$

$$L_{c_i^Y} = \frac{1}{2} c_i^{Y^{-\frac{1}{2}}} u_i^{\frac{1}{2}} - \lambda p_i^Y = 0 \tag{A2.2.1}$$

$$L_{c_i(z)} = \frac{1}{2} c_i^{Y^{\frac{1}{2}}} u_i^{-\frac{1}{2}} \frac{\partial u_i}{\partial c_i(z)} - \lambda p_i(z) = 0, \quad z \in (0, 1] \tag{A2.2.2}$$

$$L_\lambda = I_i - p_i^Y c_i^Y - \int_0^1 p_i(z) c_i(z) dz = 0 \tag{A2.2.3}$$

Multiplying (A2.2.1) by c_i^Y, $c_i^{Y^{-\frac{1}{2}}} u_i^{\frac{1}{2}} = 2\lambda p_i^Y c_i^Y$. (A2.2.4)

Multiplying (A2.2.2) by $c_i(z)$,

$$\theta(z) c_i^{Y^{\frac{1}{2}}} u_i^{\frac{1}{2}} = 2\lambda p_i(z) c_i(z), \quad \text{where } \frac{\partial u_i}{\partial c_i(z)} = \frac{\theta(z) u_i}{c_i(z)}. \tag{A2.2.5}$$

Taking the integral of (A2.2.5) from zero to one,

$$\int_0^1 \theta(z) c_i^{Y^{\frac{1}{2}}} u_i^{\frac{1}{2}} dz = 2\lambda \int_0^1 p_i(z) c_i(z) dz. \tag{A2.2.6}$$

Substituting (A2.2.4) and (A2.2.6) into (A2.2.3), $\lambda = \frac{c_1^{Y^{\frac{1}{2}}} u_i^{\frac{1}{2}}}{I_i}$. (A2.2.7)

Substituting (A2.2.7) into (A2.2.4), $c_i^Y = \frac{I_i}{2 p_i^Y}$.

Substituting (A2.2.7) into (A2.2.5), $c_i(z) = \frac{\theta(z) I_i}{2 p_i(z)}$.

Appendix 2.3

Dividing (2.18) by (2.17),

$$\frac{\int_0^1 \frac{\theta(z)}{r_i + a(z) w_i} dz}{\frac{1}{2w_i} + \int_0^1 \frac{a(z)\theta(z)}{r_i + a(z) w_i} dz} = \frac{\bar{K}_i'}{\bar{L}_i'}. \tag{A2.3.1}$$

The RHS of (A2.3.1) is $\bar{k}_i'$. (A2.3.1) is rearranged and rewritten as

$$\frac{w_i \int_0^1 \frac{\theta(z)}{r_i + a(z) w_i} dz}{\frac{1}{2} + w_i \int_0^1 \frac{a(z)\theta(z)}{r_i + a(z) w_i} dz} = \bar{k}_i'. \tag{A2.3.2}$$

Since w_i is not a function of z, w_i can enter the integral. w_i is cancelled out from both the numerator and the denominator within each integral. Note that $\bar{k}_i' = \bar{k}_i$. Then (A2.3.2) is modified as

$$\frac{\int_0^1 \frac{\theta(z)}{a(z)+\frac{r_i}{w_i}}dz}{\frac{1}{2}+\int_0^1 \frac{a(z)\theta(z)}{a(z)+\frac{r_i}{w_i}}dz} = \bar{k}_i' = \bar{k}_i .$$

Appendix 2.4

The consumer's utility function is

$$U_1^* = c_1^{Y^{*\frac{1}{2}}} u_1^{*\frac{1}{2}}, \quad \text{where } \ln u_1^* = \int_0^{\hat{z}} \theta(z)\ln c_{11}^*(z)dz + \int_{\hat{z}}^1 \theta(z)\ln c_{12}^*(z)dz .$$

The utility maximization problem is:

$$\max_{c_1^{Y^*},\, c_{11}^*(z),\, c_{12}^*(z)} \quad U_1^* = c_1^{Y^{*\frac{1}{2}}} u_1^{*\frac{1}{2}}$$

$$\text{s.t.} \quad p_1^{Y^*} c_1^{Y^*} + \{\int_0^{\hat{z}} p_1^*(z)c_{11}^*(z)dz + \int_{\hat{z}}^1 p_2^*(z)c_{12}^*(z)dz\} = I_1^*$$

$$L = c_1^{Y^{*\frac{1}{2}}} u_1^{*\frac{1}{2}} + \lambda_1[I_1^* - p_1^{Y*}c_1^{Y*} - \{\int_0^{\hat{z}} p_1^*(z)c_{11}^*(z)dz + \int_{\hat{z}}^1 p_2^*(z)c_{12}^*(z)dz\}]$$

$$L_{c_1^{Y^*}} = \frac{1}{2}c_1^{Y^{*-\frac{1}{2}}} u_1^{*\frac{1}{2}} - \lambda_1 p_1^{Y*} = 0$$

$$\frac{1}{2}\frac{c_1^{Y^{*\frac{1}{2}}} u_1^{*\frac{1}{2}}}{c_1^{Y^*}} - \lambda_1 p_1^{Y^*} = 0$$

$$\frac{U_1^*}{2\lambda_1} = p_1^{Y^*} c_1^{Y^*} \qquad \text{(A2.4.1)}$$

$$L_{c_{11}^*(z)} = \frac{1}{2}c_1^{Y^{*\frac{1}{2}}} u_1^{*-\frac{1}{2}} \frac{\partial u_1^*}{\partial c_{11}^*(z)} - \lambda_1 p_1^*(z) = 0, \quad \text{where } \frac{\partial u_1^*}{\partial c_{11}^*(z)} = \theta(z)\frac{u_1^*}{c_{11}^*(z)} .$$

$$\frac{1}{2}c_1^{Y^{*\frac{1}{2}}} \theta(z) u_1^{*\frac{1}{2}} \frac{1}{c_{11}^*(z)} = \lambda_1 p_1^*(z)$$

$$\frac{\theta(z)U_1^*}{2\lambda_1} = p_1^*(z)c_{11}^*(z) \qquad \text{(A2.4.2)}$$

$$L_{c_{12}^*(z)} = \frac{1}{2}c_1^{Y^{*\frac{1}{2}}} u_1^{*-\frac{1}{2}} \frac{\partial u_1^*}{\partial c_{12}^*(z)} - \lambda_1 p_2^*(z) = 0, \text{ where } \frac{\partial u_1^*}{\partial c_{12}^*(z)} = \theta(z)\frac{u_1^*}{c_{12}^*(z)}.$$

$$\frac{1}{2}c_1^{Y^{*\frac{1}{2}}} u_1^{*\frac{1}{2}} \theta(z)\frac{1}{c_{12}^*(z)} = \lambda_1 p_2^*(z)$$

$$\frac{\theta(z)U_1^*}{2\lambda_1} = p_2^*(z)c_{12}^*(z) \qquad \text{(A2.4.3)}$$

$$L_{\lambda_1} = 0$$

$$p_1^{Y^*}c_1^{Y^*} + \int_0^{\hat{z}} p_1^*(z)c_{11}^*(z)dz + \int_{\hat{z}}^1 p_2^*(z)c_{12}^*(z)dz = I_1^* \qquad \text{(A2.4.4)}$$

Substituting (A2.4.1), (A2.4.2) and (A2.4.3) into (A2.4.4),

$$\frac{U_1^*}{2\lambda_1}\{1 + \int_0^{\hat{z}} \theta(z)dz + \int_{\hat{z}}^1 \theta(z)dz\} = I_1^*.$$

Since $\int_0^{\hat{z}} \theta(z)dz + \int_{\hat{z}}^1 \theta(z)dz = 1$, $\lambda_1 = \frac{U_1^*}{I_1^*}$.

Then $p_1^{Y^*}c_1^{Y^*} = \frac{I_1^*}{2}$,

$$p_1^*(z)c_{11}^*(z) = \frac{\theta(z)I_1^*}{2},$$

$$p_2^*(z)c_{12}^*(z) = \frac{\theta(z)I_1^*}{2}.$$

Appendix 2.5

$\theta(z)$ is the expenditure share of z in the utility of Cobb-Douglas form. Recall that the sum of these shares is one: $\int_0^1 \theta(z)dz = 1$. Assume that the shares of every good z are equal. The sum of shares that are equal should be one. I have to determine values of these shares whose sum is equal to one. Since the sum of the shares $\int \theta(z)dz$ is monotonically increasing in z under the assumption made above, and since the value of this sum over the range of z, $(0, 1]$, should be unity, the sum of the shares can be measured by $\int 1\, dz$. However, the integrand in this expression, unity, does not represent the shares. This is just an abstraction used for mathematical manipulation that lets $\int_0^1 1\, dz$ be equal in value to $\int_0^1 \theta(z)dz$. I use the expression $\int 1\, dz$, instead of the original expression $\int \theta(z)dz$, as the sum of the expenditure shares. However, I use the original expression where a description of the concepts is needed.

Appendix 2.6

Total labor demand for production of the goods z, $z \in (0, \hat{z}]$ is $L_1^{Z^*} = \int_0^{\hat{z}} \frac{a(z)\theta(z)I_w^*}{2p_1^*(z)}dz$. From (2.30), $p_1^*(z) = r_1^* + a(z)w_1^*$. Using $a(z) = z$ and $\theta(z) = 1$,

$$L_1^{Z^*} = \frac{I_w^*}{2}\int_0^{\hat{z}} \frac{z}{r_1^* + w_1^* z}dz\,. \tag{A2.6.1}$$

First, I calculate the indefinite integral, $\int \frac{z}{r_i^* + w_i^* z}dz$, where the country index is i. Define G as

$$G = \ln(r_i^* + w_i^* z)\,. \tag{A2.6.2}$$

Taking the derivative with respect to z, $dG = \frac{w_i^*}{r_i^* + w_i^* z}dz$.

Multiplying dG by z, $zdG = \frac{w_i^* z}{r_i^* + w_i^* z}dz$.

Dividing both sides by w_i^*, $\frac{z}{w_i^*}dG = \frac{z}{r_i^* + w_i^* z}dz$.

Switching the LHS and the RHS, and taking the integral on both sides,

$$\int \frac{z}{r_i^* + w_i^* z}dz = \int \frac{z}{w_i^*}dG \text{ + Constant.} \tag{A2.6.3}$$

z is obtained from (A2.6.2). Since (A2.6.2) is identical to the following equation

$$e^G = r_i^* + w_i^* z\,, \tag{A2.6.4}$$

$z = \frac{e^G - r_i^*}{w_i^*}$. Substituting this into the RHS of (A2.6.3), (A2.6.3) is re-expressed as

$$\int \frac{z}{r_i^* + w_i^* z}dz = \int \frac{e^G - r_i^*}{w_i^{*2}}dG \text{ + Constant} = \frac{1}{w_i^{*2}}(e^G - r_i^* G) \text{ + Constant.}$$

Substituting (A2.6.4) for e^G and (A2.6.2) for G, the above equation is rewritten as

$$\int \frac{z}{r_i^* + w_i^* z} dz = \frac{1}{w_i^{*2}}\{(r_i^* + w_i^* z) - r_i^* \ln(r_i^* + w_i^* z)\} + \text{Constant.} \qquad \text{(A2.6.5)}$$

After taking $i = 1$ and substituting (A2.6.5) into $L_1^{Z^*}$ in (A2.6.1),

$$L_1^{Z^*} = \frac{I_w^*}{2}\int_0^{\hat{z}} \frac{z}{(r_1^* + w_1^* z)} dz = \frac{I_w^*}{2}\left[\frac{1}{w_1^{*2}}\{(r_1^* + w_1^* z) - r_1^* \ln(r_1^* + w_1^* z)\}\right]_0^{\hat{z}}$$

$$= \frac{I_w^*}{2w_1^{*2}}\{w_1^* \hat{z} - r_1^* \ln(1 + \frac{w_1^* \hat{z}}{r_1^*})\}. \qquad \text{(A2.6.6)}$$

Substituting $K_1^{Z^*} = \frac{I_w^*}{2w_1^*} \ln(1 + \frac{w_1^* \hat{z}}{r_1^*})$ in (2.41') into (A2.6.6), $L_1^{Z^*} = \frac{1}{w_1^*}(\frac{\hat{z} I_w^*}{2} - r_1^* K_1^{Z^*})$.

From (2.45), $K_1^{Z^*} = \frac{3}{4}\overline{K}_1$. Substituting this into $L_1^{Z^*}$,

$$L_1^{Z^*} = \frac{1}{2w_1^*}(\hat{z} I_w^* - \frac{3}{2} r_1^* \overline{K}_1).$$

Appendix 2.7

Total labor $L_2^{Z^*}$ for production of the goods z, $z \in (\hat{z}, 1]$ is $L_2^{Z^*} = \int_{\hat{z}}^1 \frac{a(z)\theta(z) I_w^*}{2 p_2^*(z)} dz$. I use (A2.6.5) again and take $i = 2$ for the calculation of $L_2^{Z^*}$.

$$L_2^{Z^*} = \frac{I_w^*}{2}\int_{\hat{z}}^1 \frac{z}{(r_2^* + w_2^* z)} dz = \frac{I_w^*}{2}\left[\frac{1}{w_2^{*2}}\{(r_2^* + w_2^* z) - r_2^* \ln(r_2^* + w_2^* z)\}\right]_{\hat{z}}^1$$

$$= \frac{I_w^*}{2w_2^{*2}}[w_2^*(1 - \hat{z}) - r_2^*\{\ln(r_2^* + w_2^*) - \ln(r_2^* + w_2^* \hat{z})\}]$$

$$= \frac{(1 - \hat{z}) I_w^*}{2w_2^*} - \frac{r_2^* I_w^*}{2w_2^{*2}} \ln \frac{r_2^* + w_2^*}{r_2^* + w_2^* \hat{z}}. \qquad \text{(A2.7.1)}$$

Substituting $K_2^{Z^*} = \frac{I_w^*}{2w_2^*} \ln \frac{r_2^* + w_2^*}{r_2^* + w_2^* \hat{z}}$ in (2.42) into (A2.7.1), $L_2^{Z^*} = \frac{(1 - \hat{z}) I_w^*}{2w_2^*} - \frac{r_2^* K_2^{Z^*}}{w_2^*}$.

From (2.45), $K_2^{Z^*} = \frac{3}{4}\overline{K}_2$. Substituting this into $L_2^{Z^*}$,

$$L_2^{Z^*} = \frac{1}{2w_2^*}\{(1-\hat{z})I_w^* - \frac{3}{2}r_2^*\overline{K}_2\}.$$

Appendix 2.8

Using $K_1^{Z^*}$ (2.41'), $K_1^{Y^*}$ (2.43) and $I_1^* = w_1^*\overline{L}_1 + r_1^*\overline{K}_1$, equation (2.57) is re-expressed as

$$\frac{I_w^*}{2w_1^*}\ln(1+\frac{w_1^*\hat{z}}{r_1^*}) = \overline{K}_1 - \frac{w_1^*\overline{L}_1 + r_1^*\overline{K}_1}{4\{r_1^* + (\frac{p_w^{M^*}}{b_1})\}}.$$

Rearranging the equation above,

$$\ln(1+\frac{w_1^*\hat{z}}{r_1^*}) = \frac{w_1^*}{2I_w^*}[\frac{\{3r_1^* + 4(\frac{p_w^{M^*}}{b_1})\}\overline{K}_1 - w_1^*\overline{L}_1}{r_1^* + (\frac{p_w^{M^*}}{b_1})}].$$

Substituting $w_1^* = \frac{p_w^{M^*}}{b_1}$ from (2.24) into this equation,

$$\ln\{1+\frac{(\frac{p_w^{M^*}}{b_1})\hat{z}}{r_1^*}\} = \{\frac{(\frac{p_w^{M^*}}{b_1})}{2I_w^*}\}[\frac{\{3r_1^* + 4(\frac{p_w^{M^*}}{b_1})\}\overline{K}_1 - (\frac{p_w^{M^*}}{b_1})\overline{L}_1}{r_1^* + (\frac{p_w^{M^*}}{b_1})}] = \{\frac{(\frac{p_w^{M^*}}{b_1})}{2I_w^*}\}\{3\overline{K}_1 + \frac{(\overline{K}_1 - \overline{L}_1)(\frac{p_w^{M^*}}{b_1})}{r_1^* + (\frac{p_w^{M^*}}{b_1})}\}.$$

Appendix 2.9

When the capital stock in country i changes, a change in the capital return is denoted as $\frac{\partial r_i^*}{\partial \overline{K}_i}$.

I examine how the sign of $\frac{\partial r_i^*}{\partial \overline{K}_i}$ is determined in the model. First, it is apparent that if the capital stock of each country changes simultaneously in the same direction by the same amount in the absence of an international capital movement, the return in each country moves in the same direction, when other things are unchanged: sign of $\frac{\partial r_1^*}{\partial \overline{K}_1}$ = sign of $\frac{\partial r_2^*}{\partial \overline{K}_2}$, where $\partial\overline{K}_1 = \partial\overline{K}_2$.

These signs are determined by the capital market equilibrium conditions.

Denoting $\frac{p_w^{M^*}}{b_1}$ as η_1, equation (2.58) is re-expressed as

$$\ln(1+\frac{\eta_1\hat{z}}{r_1^*}) = (\frac{\eta_1}{2I_w^*})\{3\overline{K}_1 + \frac{\eta_1(\overline{K}_1 - \overline{L}_1)}{r_1^* + \eta_1}\}. \quad \text{(A2.9.1)}$$

Taking the derivative with respect to $\bar{K}_1$ on both sides in (A2.9.1), the derivative on the LHS is:

$$\frac{\partial}{\partial \bar{K}_1}\{\ln(r_1^* + \eta_1 \hat{z}) - \ln r_1^*\} = \frac{1}{(r_1^* + \eta_1 \hat{z})}(\frac{\partial r_1^*}{\partial \bar{K}_1} + \eta_1 \frac{\partial \hat{z}}{\partial \bar{K}_1}) - (\frac{1}{r_1^*}\frac{\partial r_1^*}{\partial \bar{K}_1}). \qquad \text{(A2.9.2)}$$

From (2.54), $\hat{z} = \frac{r_2^* - r_1^*}{w_1^* - w_2^*}$. A change in $\bar{K}_1$ affects the capital returns, but does not affect the wages because the wages depend on the coefficients of technology for M, b_i: $\frac{\partial w_1^*}{\partial \bar{K}_1} = \frac{\partial w_2^*}{\partial \bar{K}_1} = 0$.

Then taking the derivative with respect to $\bar{K}_1$ of $\hat{z}$,

$$\frac{\partial \hat{z}}{\partial \bar{K}_1} = \frac{1}{(w_1^* - w_2^*)}(\frac{\partial r_2^*}{\partial \bar{K}_2}\frac{d\bar{K}_2}{d\bar{K}_1} - \frac{\partial r_1^*}{\partial \bar{K}_1}).$$

Since the capital outflow from country 1 equals the capital inflow to country 2, $\frac{d\bar{K}_2}{d\bar{K}_1} = -1$. Then $\frac{\partial \hat{z}}{\partial \bar{K}_1}$ is rewritten as

$$\frac{\partial \hat{z}}{\partial \bar{K}_1} = -\frac{1}{(w_1^* - w_2^*)}(\frac{\partial r_1^*}{\partial \bar{K}_1} + \frac{\partial r_2^*}{\partial \bar{K}_2}). \qquad \text{(A2.9.3)}$$

Substitute (A2.9.3) into (A2.9.2). Then (A2.9.2) is replaced by

$$\frac{1}{(r_1^* + \eta_1 \hat{z})}\{\frac{\partial r_1^*}{\partial \bar{K}_1} - (\frac{\eta_1}{w_1^* - w_2^*})(\frac{\partial r_1^*}{\partial \bar{K}_1} + \frac{\partial r_2^*}{\partial \bar{K}_2})\} - (\frac{1}{r_1^*}\frac{\partial r_1^*}{\partial \bar{K}_1})$$

$$= \{\frac{1}{(r_1^* + \eta_1 \hat{z})} - \frac{\eta_1}{(r_1^* + \eta_1 \hat{z})(w_1^* - w_2^*)} - \frac{1}{r_1^*}\}\frac{\partial r_1^*}{\partial \bar{K}_1} - \{\frac{\eta_1}{(r_1^* + \eta_1 \hat{z})(w_1^* - w_2^*)}\}\frac{\partial r_2^*}{\partial \bar{K}_2}$$

$$= -[\frac{\eta_1\{r_1^* + \hat{z}(w_1^* - w_2^*)\}}{r_1^*(r_1^* + \eta_1 \hat{z})(w_1^* - w_2^*)}]\frac{\partial r_1^*}{\partial \bar{K}_1} - \{\frac{\eta_1}{(r_1^* + \eta_1 \hat{z})(w_1^* - w_2^*)}\}\frac{\partial r_2^*}{\partial \bar{K}_2}. \qquad \text{(A2.9.4)}$$

Using (2.54), $\hat{z}(w_1^* - w_2^*) = (r_2^* - r_1^*)$, and substituting this into the numerator of the first term in (A2.9.4), (A2.9.4) is rewritten as

$$-\eta_1[\{\frac{r_2^*}{r_1^*(r_1^* + \eta_1 \hat{z})(w_1^* - w_2^*)}\}\frac{\partial r_1^*}{\partial \bar{K}_1} + \{\frac{1}{(r_1^* + \eta_1 \hat{z})(w_1^* - w_2^*)}\}\frac{\partial r_2^*}{\partial \bar{K}_2}]. \qquad \text{(A2.9.5)}$$

Next, I take the derivative with respect to $\bar{K}_1$ on the RHS in (A2.9.1). For simplicity of the expression, I rewrite the RHS as AB, where $A = \frac{\eta_1}{2I_w^*}$ and $B = 3\bar{K}_1 + \frac{\eta_1(\bar{K}_1 - \bar{L}_1)}{r_1^* + \eta_1}$. The world income is $I_w^* = w_1^* \overline{L}_1 + r_1^* \overline{K}_1 + w_2^* \overline{L}_2 + r_2^* \overline{K}_2$. Then the derivative on the RHS in (A2.9.1) is expressed as

$$\frac{\partial}{\partial \overline{K}_1}(AB) = B\frac{\partial A}{\partial \overline{K}_1} + A\frac{\partial B}{\partial \overline{K}_1}. \tag{A2.9.6}$$

First, consider $\frac{\partial A}{\partial \overline{K}_1}$.

$$\frac{\partial A}{\partial \overline{K}_1} = -\frac{\eta_1}{2}\left(\frac{\frac{\partial I_w^*}{\partial \overline{K}_1}}{I_w^{*\,2}}\right).$$

Since $\frac{\partial w_1^*}{\partial \overline{K}_1} = \frac{\partial w_2^*}{\partial \overline{K}_1} = 0$, the changed world income is

$$\frac{\partial I_w^*}{\partial \overline{K}_1} = (r_1^* + \overline{K}_1 \frac{\partial r_1^*}{\partial \overline{K}_1}) + (r_2^* \frac{d\overline{K}_2}{d\overline{K}_1} + \overline{K}_2 \frac{\partial r_2^*}{\partial \overline{K}_2}\frac{d\overline{K}_2}{d\overline{K}_1}).$$

Using $\frac{d\overline{K}_2}{d\overline{K}_1} = -1$, $\frac{\partial I_w^*}{\partial \overline{K}_1} = (r_1^* + \overline{K}_1 \frac{\partial r_1^*}{\partial \overline{K}_1}) - (r_2^* + \overline{K}_2 \frac{\partial r_2^*}{\partial \overline{K}_2})$.

Substituting $\frac{\partial I_w^*}{\partial \overline{K}_1}$ into $\frac{\partial A}{\partial \overline{K}_1}$,

$$\frac{\partial A}{\partial \overline{K}_1} = -(\frac{\eta_1}{2I_w^{*\,2}})\{(r_1^* + \overline{K}_1 \frac{\partial r_1^*}{\partial \overline{K}_1}) - (r_2^* + \overline{K}_2 \frac{\partial r_2^*}{\partial \overline{K}_2})\}.$$

Consider $\frac{\partial B}{\partial \overline{K}_1}$ on the RHS in (A2.9.6).

$$\frac{\partial B}{\partial \overline{K}_1} = 3 + \{\frac{\eta_1(r_1^* + \eta_1) - \eta_1(\overline{K}_1 - \overline{L}_1)\frac{\partial r_1^*}{\partial \overline{K}_1}}{(r_1^* + \eta_1)^2}\} = 3 + \frac{\eta_1}{(r_1^* + \eta_1)}, \quad \text{where } \overline{L}_1 = \overline{K}_1.$$

Also $B = 3\overline{K}_1$, where $\overline{L}_1 = \overline{K}_1$.

Substitute B, $\frac{\partial A}{\partial \overline{K}_1}$, A and $\frac{\partial B}{\partial \overline{K}_1}$ into (A2.9.6). Then (A2.9.6) is re-expressed as

$$-(\frac{3\eta_1\overline{K}_1}{2I_w^{*\,2}})\{(r_1^* + \overline{K}_1 \frac{\partial r_1^*}{\partial \overline{K}_1}) - (r_2^* + \overline{K}_2 \frac{\partial r_2^*}{\partial \overline{K}_2})\} + (\frac{\eta_1}{2I_w^*})\{3 + \frac{\eta_1}{(r_1^* + \eta_1)}\}$$

$$= -\eta_1[(\frac{3\overline{K}_1}{2I_w^{*\,2}})\{(r_1^* + \overline{K}_1 \frac{\partial r_1^*}{\partial \overline{K}_1}) - (r_2^* + \overline{K}_2 \frac{\partial r_2^*}{\partial \overline{K}_2})\} - (\frac{1}{2I_w^*})\{3 + \frac{\eta_1}{(r_1^* + \eta_1)}\}]. \tag{A2.9.7}$$

The derivative on the LHS, (A2.9.5), should be equal to the derivative on the RHS, (A2.9.7):

$$\{\frac{r_2^*}{r_1^*(r_1^*+\eta_1\hat{z})(w_1^*-w_2^*)}\}\frac{\partial r_1^*}{\partial \overline{K}_1}+\{\frac{1}{(r_1^*+\eta_1\hat{z})(w_1^*-w_2^*)}\}\frac{\partial r_2^*}{\partial \overline{K}_2}$$

$$=(\frac{3\overline{K}_1}{2I_w^{*2}})\{(r_1^*+\overline{K}_1\frac{\partial r_1^*}{\partial \overline{K}_1})-(r_2^*+\overline{K}_2\frac{\partial r_2^*}{\partial \overline{K}_2})\}-(\frac{1}{2I_w^*})\{3+\frac{\eta_1}{(r_1^*+\eta_1)}\}.$$

Rearranging and re-expressing this as follows,

$$C\frac{\partial r_1^*}{\partial \overline{K}_1}+D\frac{\partial r_2^*}{\partial \overline{K}_2}=E, \qquad \text{(A2.9.8)}$$

where $C=\{\frac{r_2^*}{r_1^*(r_1^*+\eta_1\hat{z})(w_1^*-w_2^*)}-(\frac{3\overline{K}_1^2}{2I_w^{*2}})\}$, $D=\{\frac{1}{(r_1^*+\eta_1\hat{z})(w_1^*-w_2^*)}+(\frac{3\overline{K}_1\overline{K}_2}{2I_w^{*2}})\}$,

$$E=(\frac{1}{2I_w^*})[\{\frac{3\overline{K}_1(r_1^*-r_2^*)}{I_w^*}\}-\{3+\frac{\eta_1}{(r_1^*+\eta_1)}\}].$$

To identify the signs for $\frac{\partial r_1^*}{\partial \overline{K}_1}$ and $\frac{\partial r_2^*}{\partial \overline{K}_2}$, I need to check signs of C, D and E. First, for checking the sign of C, dividing both the numerator and denominator of the first term of C by r_2^* and those of the second term of C by $3\overline{K}_1^2$ and using $I_w^*=\overline{K}_1(w_1^*+r_1^*)+\overline{K}_2(w_2^*+r_2^*)$, where $\overline{L}_i=\overline{K}_i$, $i=1,2$, C is rewritten as

$$C=\{\frac{1}{(\frac{r_1^*}{r_2^*})(r_1^*+\eta_1\hat{z})(w_1^*-w_2^*)}\}-[\frac{1}{\frac{2}{3}\{(w_1^*+r_1^*)^2+2(w_1^*+r_1^*)(w_2^*+r_2^*)(\frac{\overline{K}_2}{\overline{K}_1})+(w_2^*+r_2^*)^2(\frac{\overline{K}_2}{\overline{K}_1})^2\}}]$$

$$=\{\frac{1}{(\frac{r_1^*}{r_2^*})(r_1^*+\eta_1\hat{z})(w_1^*-w_2^*)}\}-[\frac{1}{\frac{2}{3}\{(w_1^*+r_1^*)+(w_2^*+r_2^*)\frac{\overline{K}_2}{\overline{K}_1}\}^2}]. \qquad \text{(A2.9.9)}$$

Since $\eta_1\equiv\frac{p_w^{M*}}{b_1}$ and $\frac{p_w^{M*}}{b_1}=w_1^*$ in (2.24), $\eta_1\equiv\frac{p_w^{M*}}{b_1}=w_1^*$. Using $\eta_1=w_1^*$ and $\hat{z}=\frac{r_2^*-r_1^*}{w_1^*-w_2^*}$ in (2.54), the denominator of the first term in (A2.9.9) is

$$(\frac{r_1^*}{r_2^*})(r_1^*+\eta_1\hat{z})(w_1^*-w_2^*)=(\frac{r_1^*}{r_2^*})\{r_1^*(w_1^*-w_2^*)+w_1^*(r_2^*-r_1^*)\}$$

$$=(\frac{r_1^*}{r_2^*})r_1^*(w_1^*-w_2^*)+w_1^*r_1^*(1-\frac{r_1^*}{r_2^*}). \qquad \text{(A2.9.10)}$$

The first term on the RHS in (A2.9.10), $(\frac{r_1^*}{r_2^*})r_1^*(w_1^* - w_2^*)$, is less than $r_1^*(w_1^* - w_2^*)$ since $0 < \frac{r_1^*}{r_2^*} < 1$ and $w_1^* - w_2^* > 0$ in (2.26). The second term on the RHS in (A2.9.10), $w_1^* r_1^*(1 - \frac{r_1^*}{r_2^*})$, is less than $w_1^* r_1^*$ since $0 < (1 - \frac{r_1^*}{r_2^*}) < 1$. Then the RHS has the following relation:

$$(\frac{r_1^*}{r_2^*})r_1^*(w_1^* - w_2^*) + w_1^* r_1^*(1 - \frac{r_1^*}{r_2^*}) < r_1^*(w_1^* - w_2^*) + w_1^* r_1^* .$$

Since the LHS of this inequality is equal to $(\frac{r_1^*}{r_2^*})(r_1^* + \eta_1 \hat{z})(w_1^* - w_2^*)$ from (A2.9.10), this inequality is re-expressed as

$$r_1^*(w_1^* - w_2^*) + w_1^* r_1^* > (\frac{r_1^*}{r_2^*})(r_1^* + \eta_1 \hat{z})(w_1^* - w_2^*) . \quad \text{(A2.9.11)}$$

Now, let us look at the denominator of the second term in (A2.9.9). Rearranging it,

$$\frac{2}{3}\{(w_1^* + r_1^*) + (w_2^* + r_2^*)\frac{\overline{K}_2}{\overline{K}_1}\}^2 = \frac{2}{3}\{(w_1^* + w_2^*\frac{\overline{K}_2}{\overline{K}_1}) + (r_1^* + r_2^*\frac{\overline{K}_2}{\overline{K}_1})\}^2 . \quad \text{(A2.9.12)}$$

The calculation of the RHS is

$$\frac{2}{3}[\{w_1^{*2} + 2w_1^* w_2^*\frac{\overline{K}_2}{\overline{K}_1} + (w_2^*\frac{\overline{K}_2}{\overline{K}_1})^2\} + 2\{(w_1^* + w_2^*\frac{\overline{K}_2}{\overline{K}_1})r_1^* + (w_1^* + w_2^*\frac{\overline{K}_2}{\overline{K}_1})r_2^*\frac{\overline{K}_2}{\overline{K}_1}\} + \{r_1^{*2} + 2r_1^* r_2^*\frac{\overline{K}_2}{\overline{K}_1} + (r_2^*\frac{\overline{K}_2}{\overline{K}_1})^2\}].$$

This is larger than the sum of terms, $\frac{2}{3}w_1^{*2}$, $\frac{2}{3}\{2(w_1^* + w_2^*\frac{\overline{K}_2}{\overline{K}_1})r_1^*\}$ and $\frac{2}{3}r_1^{*2}$, that are included in this expression, $\frac{2}{3}\{(w_1^{*2} + r_1^{*2}) + 2(w_1^* + w_2^*\frac{\overline{K}_2}{\overline{K}_1})r_1^*\}$, since there are the remaining terms that are positive. Using the fact that $(w_1^{*2} + r_1^{*2}) \geq 2w_1^* r_1^*$, the sum has the following relation:

$$\frac{2}{3}\{(w_1^{*2} + r_1^{*2}) + 2(w_1^* + w_2^*\frac{\overline{K}_2}{\overline{K}_1})r_1^*\} \geq \frac{2}{3}\{2w_1^* r_1^* + 2(w_1^* + w_2^*\frac{\overline{K}_2}{\overline{K}_1})r_1^*\} = \frac{4}{3}\{w_1^* r_1^* + (w_1^* + w_2^*\frac{\overline{K}_2}{\overline{K}_1})r_1^*\}.$$

Since the LHS of this inequality is less than the RHS in (A2.9.12) that is equal to the LHS in (A2.9.12), the LHS in (A2.9.12) has the following relation.

$$\frac{2}{3}\{(w_1^* + r_1^*) + (w_2^* + r_2^*)\frac{\overline{K}_2}{\overline{K}_1}\}^2 > \frac{4}{3}\{w_1^* r_1^* + (w_1^* + w_2^*\frac{\overline{K}_2}{\overline{K}_1})r_1^*\} . \quad \text{(A2.9.13)}$$

Also, using the two facts that $\frac{4}{3}$ is larger than 1 and $(w_1^* + w_2^* \frac{\overline{K}_2}{\overline{K}_1})$ is larger than $(w_1^* - w_2^*)$,

$$\frac{4}{3}\{w_1^* r_1^* + (w_1^* + w_2^* \frac{\overline{K}_2}{\overline{K}_1}) r_1^*\} > \{w_1^* r_1^* + (w_1^* - w_2^*) r_1^*\}. \qquad \text{(A2.9.14)}$$

From (A2.9.13), (A2.9.14) and (A2.9.11), the relation between the denominator of the second term in (A2.9.9), $\frac{2}{3}\{(w_1^* + r_1^*) + (w_2^* + r_2^*)\frac{\overline{K}_2}{\overline{K}_1}\}^2$, and the denominator of the first term in (A2.9.9), $(\frac{r_1^*}{r_2^*})(r_1^* + \eta_1 \hat{z})(w_1^* - w_2^*)$, is as follows:

$$\frac{2}{3}\{(w_1^* + r_1^*) + (w_2^* + r_2^*)\frac{\overline{K}_2}{\overline{K}_1}\}^2 > \frac{4}{3}\{w_1^* r_1^* + (w_1^* + w_2^* \frac{\overline{K}_2}{\overline{K}_1}) r_1^*\} > \{w_1^* r_1^* + (w_1^* - w_2^*) r_1^*\} > (\frac{r_1^*}{r_2^*})\{(r_1^* + \eta_1 \hat{z})(w_1^* - w_2^*)\}.$$

Comparing the top ranking one and the bottom ranking one in this chain of ranks, the following relation is obtained:

$$\{\frac{1}{(\frac{r_1^*}{r_2^*})(r_1^* + \eta_1 \hat{z})(w_1^* - w_2^*)}\} > [\frac{1}{\frac{2}{3}\{(w_1^* + r_1^*) + (w_2^* + r_2^*)\frac{\overline{K}_2}{\overline{K}_1}\}^2}].$$

In other words, the first term in (A2.9.9) is larger than the second term in (A2.9.9). This means that $C > 0$. The signs for D and E in (A2.9.8) are straightforward. $D > 0$ since $w_1^* - w_2^* > 0$ by (2.26), and $E < 0$ since $r_1^* - r_2^* < 0$ by (2.54).

Now turn to (A2.9.8) to identify the signs for $\frac{\partial r_1^*}{\partial \overline{K}_1}$ and $\frac{\partial r_2^*}{\partial \overline{K}_2}$. The coefficients in the first and second terms on the LHS are positive (i.e., $C > 0$ and $D > 0$), but the value on the RHS is negative (i.e., $E < 0$). Since $\frac{\partial r_1^*}{\partial \overline{K}_1}$ and $\frac{\partial r_2^*}{\partial \overline{K}_2}$ should have the same sign, and since the RHS is negative, both $\frac{\partial r_1^*}{\partial \overline{K}_1}$ and $\frac{\partial r_2^*}{\partial \overline{K}_2}$ have negative signs. If both $\frac{\partial r_1^*}{\partial \overline{K}_1}$ and $\frac{\partial r_2^*}{\partial \overline{K}_2}$ are positive, the LHS is also positive, so that the equality, (A2.9.8), is not satisfied. Therefore $\frac{\partial r_i^*}{\partial \overline{K}_i} < 0$, $i = 1, 2$. This means that if $\overline{K}_1$ decreases, r_1^* rises and r_2^* falls because the decrease in $\overline{K}_1$ appears as an increase in $\overline{K}_2$ of the same amount.

Appendix 2.10

Using log transformation of (2.49), $\ln c_i^{M^*} = \ln I_i^* - \ln 4 - \ln b_i - \ln(r_i^* + \frac{p_w^{M^*}}{b_i})$.

Taking the derivative with respect to $\overline{K}_i$,

$$\frac{\frac{\partial c_i^{M^*}}{\partial \overline{K}_i}}{c_i^{M^*}} = \frac{\frac{\partial I_i^*}{\partial \overline{K}_i}}{I_i^*} - \frac{\frac{\partial (r_i^* + \frac{p_w^{M^*}}{b_1})}{\partial \overline{K}_i}}{(r_i^* + \frac{p_w^{M^*}}{b_i})}, \quad \text{where } i = 1, 2. \tag{2.61}$$

The numerator of the first term on the RHS is

$$\frac{\partial I_i^*}{\partial \overline{K}_i} = r_i^* + \overline{K}_i \frac{\partial r_i^*}{\partial \overline{K}_i}, \quad \text{where } I_i^* = w_i^* \overline{L}_i + r_i^* \overline{K}_i.$$

Then the first term on the RHS is

$$\frac{\frac{\partial I_i^*}{\partial \overline{K}_i}}{I_i^*} = \frac{r_i^* + \overline{K}_i \frac{\partial r_i^*}{\partial \overline{K}_i}}{w_i^* \overline{L} + r_i^* \overline{K}_i} = \frac{r_i^* + \overline{K}_i \frac{\partial r_i^*}{\partial \overline{K}_i}}{\overline{K}_i (w_i^* + r_i^*)}, \quad \text{where } \overline{K}_i = \overline{L}_i. \tag{A2.10.1}$$

Since $w_i^* = \frac{p_w^{M^*}}{b_i}$ in (2.24), (A2.10.1) is replaced by $\frac{r_i^* + \overline{K}_i \frac{\partial r_i^*}{\partial \overline{K}_i}}{\overline{K}_i (r_i^* + \frac{p_w^{M^*}}{b_i})}$. This is equal to

$$\frac{1}{\overline{K}_i (1 + \frac{p_w^{M^*}}{b_i r_i^*})} + \frac{\frac{\partial r_i^*}{\partial \overline{K}_i}}{(r_i^* + \frac{p_w^{M^*}}{b_i})}.$$

The second term on the RHS in (2.61) is

$$\frac{\frac{\partial (r_i^* + \frac{p_w^{M^*}}{b_i})}{\partial \overline{K}_i}}{(r_i^* + \frac{p_w^{M^*}}{b_i})} = \frac{\frac{\partial r_i^*}{\partial \overline{K}_i}}{(r_i^* + \frac{p_w^{M^*}}{b_i})}, \quad \text{since } \frac{\partial (r_i^* + \frac{p_w^{M^*}}{b_i})}{\partial \overline{K}_i} = \frac{\partial r_i^*}{\partial \overline{K}_i}.$$

Then (2.61) is re-expressed as

$$\frac{\frac{\partial c_i^{M^*}}{\partial \overline{K}_i}}{c_i^{M^*}} = \{\frac{1}{\overline{K}_i(1+\frac{p_w^{M^*}}{b_i r_i^*})} + \frac{\frac{\partial r_i^*}{\partial \overline{K}_i}}{r_i^* + \frac{p_w^{M^*}}{b_i}}\} - \frac{\frac{\partial r_i^*}{\partial \overline{K}_i}}{r_i^* + \frac{p_w^{M^*}}{b_i}} = \frac{1}{\overline{K}_i(1+\frac{p_w^{M^*}}{b_i r_i^*})}\ .$$

Since the intermediate good is the numeraire (i.e., $p_w^{M^*} = 1$), $\frac{\frac{\partial c_i^{M^*}}{\partial \overline{K}_i}}{c_i^{M^*}} = \frac{1}{\overline{K}_i(1+\frac{1}{b_i r_i^*})}$.

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CHAPTER 3

International Fragmentation and Work Effort: Networks, Loyalty and Wages

3.1 Introduction

The world has seen a marked increase in global economic activities among countries. One of these trends is international fragmentation – firms shift some part of production to a foreign country out of the home country to produce the same product more efficiently. Factor endowment differences, factor intensity differences and increasing returns to scale have been identified as the main reasons for the international fragmentation of production.

Harris (2001) emphasizes increasing returns to scale of global trade networks – a set of links connecting a large number of related demanders for and suppliers of intermediate inputs – that the firm runs, in order to explain the phenomenon of international fragmentation among developed countries. The fixed cost of running these networks is an important determinant of the extent of international fragmentation.[1]

The innovative development of physical network technology has given the world's economies new infrastructure for communications, such as the Internet and telecommunication networks. These have made massive flows of information possible, and coordination of business partners scattered worldwide easier. Thus the development of physical networks increases international trade. Freund and Weinhold (2002) find a positive correlation between the Internet in a foreign country, and U.S. service imports and service exports.[2] Also, the development of physical networks makes it possible to control from one location many business divisions scattered geographically. This prompts the multinational firm's activities. I study how physical networks in a developed country (the North) and a developing country (the South) affect international fragmentation of the vertical multinational firm.

[1] Deardorff (2001 a) explains that business and social networks reduce the costs of trade, and their welfare effect is likely to be beneficial for the world as a whole.

[2] A 10% increase in internet use in a foreign country leads to about a 1.7% increase in the growth of U.S. service imports and a 1.1% increase in the growth of U.S. service exports from 1995 to 1997.

Another strand of literature examines how work effort of workers influences factor prices, employment, trade and welfare. [3] I explore how fragmentation of production affects work effort.[4] In an environment where ability of the headquarters to monitor employees is imperfect, employees would tend to shirk, since they feel disutility in making an effort, and since there is the possibility that shirking is not discovered by the headquarters. When the headquarters splits a single-stage production process into multiple-stages, the split plays the role of dividing employees into smaller groups and assigns one group to each stage. This makes it easier to detect the group of employees in which shirking arises. Employees raise their work effort to avoid the penalty of a wage cut. Thus, fragmentation can lead to an increase in work effort in the corresponding country. However, present studies associated with fragmentation seem to have overlooked this characteristic of fragmentation.

As international economies become globalized and free trade zones resulting from an increase in free trade agreements expand, the new competitive business environment makes employment relationships between employees and firms unstable. Firms compete intensely with domestic and foreign firms for markets. The volatility of their business increases and retaining employment becomes more difficult for the worker. Thus the employment contracts of firms are easily broken, or the duration of employment shortens. Hence, loyalty of employees to their firms falls.[5] Since disutility of work effort of the employees with low loyalty is high, their effort level decreases and thus their productivity becomes low. However, the new competitive environment makes the employment relationship more unstable in the South compared to that in the North, since the South, under a more protection that limits free trade, becomes relatively more exposed to international competition than the North. This means loyalty in the South, and thus Southern work effort, is relatively reduced. Therefore, loyalty emerges as a factor affecting the determination of outsourcing to the South. I explore this relationship between loyalty, work effort and international fragmentation.

Wages in developing countries have fast increased. Can this increasing wage cause a Northern firm to reduce outsourcing to the South? Since the increasing wage in the South increases production costs, the Northern firm will tend to reduce outsourcing. However, since a higher

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[3] Copeland (1989) examines the effect of cross-country differences in the disutility of effort on the pattern of trade and welfare. Brecher (1992) analyzes the effect of a tariff on imports on employment and social welfare in an efficiency-wage model with explicit monitoring. Matusz (1996) shows that international trade leads to increased employment in both countries in a model in which intermediate inputs are produced under monopolistic competition and wages are determined according to efficiency wages of Shapiro-Stiglitz type (1984). Leamer (1999) examines the effects of factor intensity on effort level and wage at the sectoral level.

[4] As another issue, Deardorff (2001 b) analyzes how fragmentation affects specialization and trade.

[5] See Reichheld and Teal (1996), and Minkler (2004).

wage gives Southern employees the incentive to increase effort, the level of effort also rises. This reduces production costs in the South, which will lead to the Northern firm increasing outsourcing. These opposite effects offset each other, so that the Southern wage does not influence the degree of outsourcing. If the wage in developed countries rises, the effect on the degree of outsourcing would depend on whether the wage effect on both the headquarters cost and the production cost dominates the effort effect on the production cost.

Finally, when the Northern firm changes its mode of production from national fragmentation to international fragmentation, I investigate how the number of total stages would change. I show that, compared with national fragmentation, international fragmentation can increase, decrease or not change the number of total stages according to differences in the amount of employment between the modes of production.

As for the contribution of this paper to methodology, contrary to the existing method that focuses on finding the level of output for an exogenously given number of stages, this paper focuses on how the firm optimally splits a single-stage production process into multiple-stages, for exogenously given output. Also, under international fragmentation, I explain how the firm optimally determines the number of total stages and how the firm splits the stages between the North and the South.

The organization of this paper is as follows. Section 3.2.1 explains the technology of multiple-stage production and the conditions of industry equilibrium in autarky. Subsection 3.2.1.1 addresses the optimal provision of effort by workers. The number of stages and the effort in the autarkic economy are determined in 3.2.1.2. In section 3.2.2, international fragmentation is considered in a world consisting of the North and the South. Subsection 3.2.2.1 explains how the effort is provided by employees under international fragmentation. Also, I compare average employee tenures between the developed countries and the developing countries for inferring loyalty differences between the two. The Northern firm's cost under international fragmentation is explained in 3.2.2.2. I compare the cost under national fragmentation with the cost under international fragmentation in 3.2.2.3. Subsection 3.2.2.4 addresses how a multinational firm determines the optimal number of total stages and distributes the production stages across borders. In subsection 3.2.2.5, I compare the number of total stages in national fragmentation to that in international fragmentation. In section 3.2.3, I investigate the effects of network, loyalty and wage on outsourcing and the effort levels in the South and the North. Section 3.3 concludes.

3.2 Model

3.2.1 Autarkic Economy

There is a homogeneous final consumption good X which is supplied by a competitive industry. The good X is produced by a production technology that consists of multiple production stages. The primary production factor is labor.

In this model, production is fragmented for two reasons. The first is differences in wages between the North and the South. By splitting the technology for producing the final good into other technologies with different factor intensities for producing components, some of these production stages can be shifted to the South in which labor costs are lower. The second reason is increasing returns to specialization in production. Since output increases more when the number of production stages increases, there is an incentive to split production into many stages, even if the technologies for different stages do not have different factor intensities.

Increasing returns with respect to specialization can be represented by output increasing by more with an increase in the variety of inputs (components). The increasing returns production functions of CES form (Ethier 1979) and Cobb-Douglas form (Edwards and Starr 1987) have been used in the literature. These functions have the feature that the inputs are substitutable for each other. However, I develop a Leontief production function that has the characteristic of increasing returns due to specialization. In products assembled from many components – for example, machinery, electronics, ships, automobiles and airplanes – each component is produced by technology specific to the respective component, and all components are complementary to each other. If a component is removed, the assembled product does not work. This description matches well with the property of the Leontief production function.

First, I consider a case that the production function for the final good X has a single-production stage. The production function is

$$x = \min\{z, \frac{el}{b}\}.$$

x is output of the good X. z is units of the intermediate good Z used. l is units of labor employed. b is units of labor that are required to produce one unit of the good X. e is the level of work effort.

However, when the production process is split into multiple stages, each stage produces a distinct intermediate good, using labor and an intermediate input produced at the previous stage. The final stage produces the finished final good X. Let the production function for an intermediate good of the nth stage be of the Leontief form.

$$z_n = \min\{z_{n-1}, \frac{el_n}{b(\bar{n})}\}, \quad n = 2, \ldots, \bar{n}.$$

z_n is output of an intermediate good Z_n produced by the nth stage in the production with $\bar{n}$ stages for X. z_{n-1} is units of an intermediate input provided by the $(n-1)$th stage. l_n is units of labor employed at the nth stage. The productivity of each stage depends on the degree of labor division; greater division of labor makes labor more productive. If the degree of labor division is identical to the number of total stages, labor productivity $(= \frac{1}{b(\bar{n})})$ rises with the number of total stages $\bar{n}$. The reason is that if $\bar{n}$ increases, the units of labor that each stage uses to produce one unit of the intermediate good, $b(\bar{n})$, become smaller.[6] However, for a given number of total stages, $\bar{n}$, the productivity in each stage is the same, $\frac{1}{b(\bar{n})}$. Note that $\bar{n}$ is not only the number of total stages but also the number indexing the final stage. e is the level of work effort in each stage, which is determined by workers.[7]

I explain how labor is employed by each stage when the firm has $\bar{n}$ stages. The first stage produces its intermediate good Z_1 with only labor, since the intermediate good produced at the zero stage does not exist conceptually. The production function of the first stage is

$$z_1 = \frac{el_1}{b(\bar{n})}.$$

z_1 is output of the intermediate good produced at the first stage and l_1 is labor employed by this stage. The amount of labor of the first stage is $l_1 = \frac{b(\bar{n})z_1}{e}$. $\frac{b(\bar{n})}{e}$ is the labor employed per unit output of the intermediate good Z_1.

The second stage produces an intermediate good Z_2. This stage uses the intermediate good Z_1 produced in the first stage as an intermediate input and labor. Its production technology is

$$z_2 = \min\{z_1, \frac{el_2}{b(\bar{n})}\}.$$

z_2 is output of Z_2. z_1 is units of intermediate good Z_1 used. Suppose that the second stage produces $\bar{z}_2$ units of Z_2. Since the input-output coefficient on Z_1 is 1, the second stage uses Z_1 in the amount $\bar{z}_2$. Thus z_1 becomes $\bar{z}_2$ units: $z_1 = \bar{z}_2$. This means that the first stage will

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[6] The function for $b(\bar{n})$ will be explained later.

[7] I will explain how workers determine the level of work effort in 3.2.1.1.

provide $\bar{z}_2$ units of Z_1. Then the intermediate input used at the second stage embodies $\frac{b(\bar{n})\bar{z}_2}{e}$ units of labor since one unit of Z_1 is produced with $\frac{b(\bar{n})}{e}$ units of labor. The embodied labor $\frac{b(\bar{n})\bar{z}_2}{e}$ becomes the labor demand of the first stage. The second stage also employs labor, $l_2 = \frac{b(\bar{n})\bar{z}_2}{e}$. Thus total labor embodied for the production of Z_2 is the sum of these two labor demands, $\frac{b(\bar{n})\bar{z}_2}{e} + l_2 = \frac{2b(\bar{n})\bar{z}_2}{e}$. The labor employed per unit of Z_2 is $\frac{2b(\bar{n})}{e}$.

The third stage produces an intermediate good Z_3. The production technology is

$$z_3 = \min\{z_2, \frac{el_3}{b(\bar{n})}\}.$$

z_3 is output of Z_3. z_2 is units of intermediate good Z_2 used. Suppose that the third stage produces $\bar{z}_3$ units of Z_3. Since the input-output coefficient on Z_2 is 1, the third stage uses Z_2 in the amount $\bar{z}_3$ and thus $z_2 = \bar{z}_3$. Since one unit of Z_2 is produced with $\frac{2b(\bar{n})}{e}$ units of labor, the intermediate input used at the third stage embodies $\frac{2b(\bar{n})\bar{z}_3}{e}$ units of labor that are accumulated up to the second stage. The third stage also employs labor: $l_3 = \frac{b(\bar{n})\bar{z}_3}{e}$. Thus total labor embodied for the production of Z_3 is $\frac{2b(\bar{n})\bar{z}_3}{e} + l_3 = \frac{3b(\bar{n})\bar{z}_3}{e}$. The labor employed per unit of Z_3 is $\frac{3b(\bar{n})}{e}$.

When this logic is generalized to the nth stage, the labor employed is explained as follows. The production function of the nth stage is

$$z_n = \min\{z_{n-1}, \frac{el_n}{b(\bar{n})}\}, \quad n \geq 2. \tag{3.1}$$

If output of the nth stage is $\bar{z}_n$, this stage demands $\bar{z}_n$ units of Z_{n-1} as an intermediate input: $z_{n-1} = \bar{z}_n$. Since one unit of Z_{n-1} at the $(n-1)$th stage is produced with $\frac{(n-1)b(\bar{n})}{e}$ units of labor,[8] $\bar{z}_n$ units of Z_{n-1} embody $\frac{(n-1)b(\bar{n})\bar{z}_n}{e}$ units of labor that are accumulated up to the $(n-1)$ th stage. Also the n th stage employs labor, $l_n = \frac{b(\bar{n})\bar{z}_n}{e}$. Thus the total labor employed, which is accumulated until the n th stage is $\frac{(n-1)b(\bar{n})\bar{z}_n}{e} + l_n = \frac{nb(\bar{n})\bar{z}_n}{e}$.

At the n th stage, the production cost PC is the product of the total labor employed and the wage:

$$PC_n = \frac{nb(\bar{n})\bar{z}_n w}{e}. \qquad (3.2)$$

The labor employed per unit output of Z_n is $\frac{nb(\bar{n})}{e}$. Then the cumulative unit production cost until the n th stage is $\frac{nb(\bar{n})w}{e}$, which is illustrated in Figure 3.1. The unit cost of producing Z_{n-1} at the $(n-1)$ th stage is CA, which consists of CB (that is cost incurred by labor employment at the $(n-1)$ th stage) and BA (that is cost incurred by intermediate input from the previous $(n-2)$ th stage). The unit cost of producing Z_n at the n th stage is FD, which is the sum of the labor cost FE at the n th stage and the cost ED for intermediate input from the previous $(n-1)$ th stage.

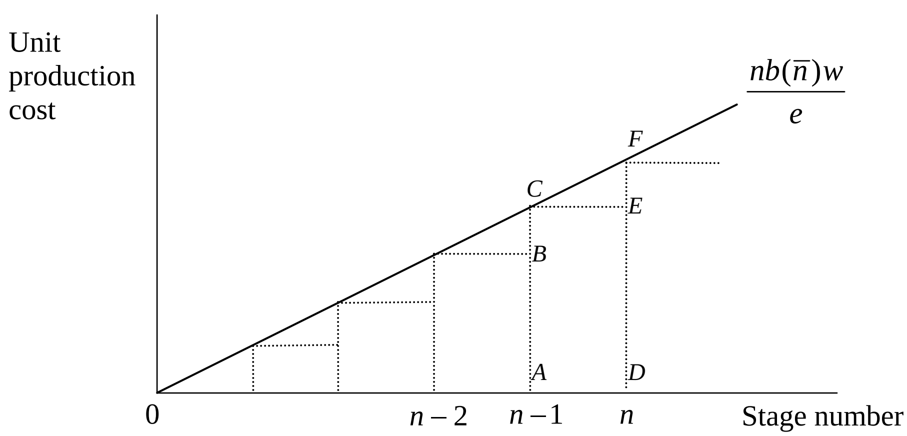

Figure 3.1

[8] Since one unit of Z_1 is produced with $\frac{b(\bar{n})}{e}$ units of labor, one unit of Z_2 is produced with $\frac{2b(\bar{n})}{e}$ units and Z_3 is produced with $\frac{3b(\bar{n})}{e}$ units, one unit of Z_{n-1} should be produced with $\frac{(n-1)b(\bar{n})}{e}$ units of labor.

I now explain labor productivity. As the number of total stages $\bar{n}$ increases, the labor productivity $(=\frac{1}{b(\bar{n})})$ for each stage $n \in \{1,\ldots,\bar{n}\}$ rises. This means that the labor input that is required to produce one unit of the good, $b(\bar{n})$, becomes smaller. Thus $b(\bar{n})$ is decreasing in the number of total stages $\bar{n}$. I define a functional form for $b(\bar{n})$.

$$b(\bar{n}) = (\frac{1}{\bar{n}})^{\delta}, \quad \text{where } \delta \geq 1 \text{ and thus } \frac{\partial b(\bar{n})}{\partial \bar{n}} < 0. \tag{3.3}$$

The condition $\delta \geq 1$ stands for the increasing returns due to labor division – specialization. It is well known that the division of labor raises the productivity of the laborers working at each stage and leads to increasing returns to specialization.[9]

The input-output coefficients on intermediate goods Z_n, $n \in \{1,\ldots,\bar{n}\}$, are one, so that the demand equals the supply for the respective intermediate goods. This makes the relationships among outputs of intermediates Z_n, $n = 1,\ldots,\bar{n}-1$, and final good X at $\bar{n}$ be

$$\bar{z}_1 = \bar{z}_2 = \ldots = \bar{z}_n = \ldots = \bar{z}_{\bar{n}-1} = \bar{z}_{\bar{n}}. \tag{3.4}$$

The output at the final stage $\bar{n}$ equals the output of the final good X: $\bar{z}_{\bar{n}} = \bar{x}$.

However, the firm cannot expand endlessly the number of stages so as to increase the productivity of labor employed. Difficulty in operating all the production stages would rise as the number of production stages increases. Coordinating and monitoring the whole production process are necessary in order to lead to the optimal performance of production. The role of the headquarters is to provide the services of coordination and monitoring. The headquarters cost for providing these services rises as the number of production stages increases. Thus, this would limit the number of stages.[10] Also, I assume that the headquarters cost of providing the services increases if the output of each stage $\bar{z}_n$ increases. As a variable indexing the headquarters service, I use total output

$$\sum_{n=1}^{\bar{n}} \bar{z}_n \tag{3.5}$$

which is increasing both in the number of stages and in the output level of each stage.

For headquarters service, another factor is the ease of information flows that make communications among business divisions possible and make monitoring easy. The development

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[9] See Adam Smith (1776). He explains the division of labor in the example of a pin factory.

[10] Adam Smith (1776) explains that the division of labor (specialization) is limited by market size. Edwards and Starr (1987) explain that it is limited by indivisibilities of labor or setup costs in the transition of labor between production tasks. In Becker and Murphy (1992), specialization depends on the cost of coordinating specialized workers who perform complementary tasks, and on the extent of knowledge available.

of physical networks like the Internet and telecommunications network makes it easier for a firm to access to networks, and to exchange information faster and in greater quantities. Thus traveling time and cost of managers for coordination and monitoring decrease. The home country and foreign country are both assumed to have networks, which are the existing infrastructure that each country has installed, and are considered public goods. Also, the development of network technology has greatly reduced the cost of accessing networks, and the cost is very small relative to other costs. Thus every firm is assumed to use the networks free. The networks improve the efficiency of the headquarters in coordinating and monitoring all production activities. Therefore, networks would reduce the value of the variable indexing headquarters services.

To reflect the effect of the network on the headquarters cost, I need to define accessibility to the network. N is the network size of the economy. This should be a large value such that $N > 1$. At the nth stage, the degree of accessibility to the network for each stage of the firm is assumed to be $\alpha_n(N)$. This has a value between zero and one. The assumption of partial accessibility, $0 < \alpha_n(N) < 1$, implies that the individual firm uses only a part of the network in the economy, which is associated with its business activities. If the economy has a larger network, the accessibility of each stage to the network increases: $\frac{\partial \alpha_n(N)}{\partial N} > 0$. An increase in accessibility causes improvement in efficiency. I assume that the stage n obtains improvement in efficiency by $\alpha_n(N)$ times its output $\bar{z}_n$ which is the index of the headquarters services of the corresponding stage: $\alpha_n(N)\bar{z}_n$. Then the total improvement in efficiency for the firm with $\bar{n}$ stages is the sum of the improvement in efficiency of individual stages: $\sum_{n=1}^{\bar{n}} \alpha_n(N)\bar{z}_n$. For simplicity, I assume that every stage has the same degree of accessibility $\alpha_n(N) = \alpha(N)$ for all $n \in \{1, \ldots, \bar{n}\}$. Then the headquarters gets efficiency of

$$\sum_{n=1}^{\bar{n}} \alpha_n(N)\bar{z}_n = \alpha(N)\sum_{n=1}^{\bar{n}} \bar{z}_n \ . \qquad (3.6)$$

I represent the degree of accessibility $\alpha(N)$ as an index and specify the function constructing the index.

$$\alpha(N) = (1 - \frac{1}{N}), \quad \text{where } N > 1. \qquad (3.7)$$

As I mentioned before, (3.7) satisfies the conditions such that $0 < \alpha(N) < 1$ and $\frac{\partial \alpha(N)}{\partial N} = \frac{1}{N^2} > 0$. When the effect of the network is considered, the index for actual headquarters services is the difference between the headquarters services (3.5) and the improvement in efficiency (3.6): $\{1-\alpha(N)\}\sum_{n=1}^{\bar{n}} \bar{z}_n$. This is re-expressed, using $\{1-\alpha(N)\} = \frac{1}{N}$ from (3.7),

$$\frac{1}{N}\sum_{n=1}^{\bar{n}} \bar{z}_n . \qquad (3.8)$$

The employees at the headquarters play the roles not only of coordinator but also of monitor. These jobs are given many responsibilities for performance similar to those of the owner or principal. This makes employees at headquarters become highly loyal to the firm and work honestly. Thus, I assume that these employees do not shirk.[11] This means that the headquarters employees provide effort $e^{\max}$, which is the upper bound of the set of possible effort levels that they can provide physically: $e^{\max} > e$, for all e. Let θ be the input requirement of effective labor – effective labor is defined as units of labor times an effort level per unit of labor – per unit of headquarters service. Then, the number of units of labor included in θ is $\frac{\theta}{e^{\max}}$ since labor in the headquarters provides effort level $e^{\max}$. Note that θ has a small value relative to the network size of the economy N: $\theta < N$. The amount of labor employed for the headquarters is, from (3.8),

$$\frac{\theta}{e^{\max} N}\sum_{n=1}^{\bar{n}} \bar{z}_n , \quad \text{where } \theta < N . \qquad (3.9)$$

I define national fragmentation as a production pattern locating all production stages within the national border, but all stages are involved in producing a single final good. This section focuses on how national fragmentation is determined in the North.[12] N is the Northern network size and $\bar{n}$ is the number of total stages in the North.

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[11] In reality, the employees who work in the headquarters may shirk. However, I do not consider this case. The reason is that if they are shirkers, their role as monitor of the employees who work in production could be compromised.

[12] Section 3.2.2 will address how international fragmentation is determined in a world of consisting the North and the South.

When the Northern wage is w, the wage cost for coordination and monitoring is from (3.9) $\frac{\theta w}{e^{\max} N} \sum_{n=1}^{\bar{n}} \bar{z}_n$, which is the headquarters cost. Using (3.4), the headquarters cost HC is

$$HC = \frac{\theta w \bar{z}_n \bar{n}}{e^{\max} N}. \tag{3.10}$$

This cost rises if the unit labor input requirement for coordination and monitoring θ, the wage w, the output of each stage $\bar{z}_n$ and the number of total stages $\bar{n}$ increase. However, the cost falls if the network size N is larger. The headquarters cost per unit of output is

$$hc = \frac{\theta w \bar{n}}{e^{\max} N}. \tag{3.10'}$$

I call this the unit headquarters cost.

The production cost for each stage is now explained. Imagine that the firm with a production technology that is made up of $\bar{n}$ stages is now producing the good at the nth stage such that $n \le \bar{n}$. The production cost PC_n for the good at the nth stage is given by (3.2) and (3.3).

$$PC_n = \frac{nb(\bar{n})\bar{z}_n w}{e}, \quad \text{where } b(\bar{n}) = (\frac{1}{\bar{n}})^{\delta}. \tag{3.11}$$

This cost is a cumulative cost from the first stage to the nth stage. As n becomes larger, PC_n increases. As $\bar{n}$ increases, PC_n decreases since labor productivity rises with $\bar{n}$ (in other words, $b(\bar{n})$ is decreasing in $\bar{n}$). As $\bar{z}_n$ becomes larger and w rises, production cost increases. If the work effort e rises, the cost falls since the labor input to produce one unit of good is reduced. Its production cost per unit output of the good is

$$pc_n = \frac{nb(\bar{n})w}{e}, \quad \text{where } b(\bar{n}) = (\frac{1}{\bar{n}})^{\delta} \tag{3.11'}$$

which is the unit production cost.

The effort of the employees who work in production should be explained. I assume that the firm's ability to monitor the employees who work in production is imperfect. For better checking of the employees' performance, the firm splits its production into multiple-stage production. The split plays the role of dividing the employees into groups with a narrower range of activities. This makes it easier to identify the group of employees in which shirking arises. If the number of total stages increases, the firm can monitor the performance of the employees more effectively. This causes the employees to work harder since they know that they will be penalized if idleness is detected.[13] Thus their effort is associated positively with the number of total stages $\bar{n}$:

$$e = e(\bar{n}), \quad \frac{\partial e(\bar{n})}{\partial \bar{n}} > 0. \tag{3.12}$$

The total cost of the firm for production up to the nth stage is the sum of the headquarters cost (3.10) and the production cost (3.11). Substituting $e(\bar{n})$ for e,

$$TC_n = HC + PC_n = w\bar{z}_n\{\frac{\theta\bar{n}}{e^{\max}N} + \frac{nb(\bar{n})}{e(\bar{n})}\}. \tag{3.13}$$

The total cost per unit of output of the good up to the nth stage becomes

$$tc_n = w\{\frac{\theta\bar{n}}{e^{\max}N} + \frac{nb(\bar{n})}{e(\bar{n})}\}. \tag{3.13'}$$

I call this the unit total cost. The first term in the square brackets is the unit headquarters cost. This is affected by the parameters characterizing a country, such as θ and N. This is a country-specific cost, which is neutral across the interim stages. The second term in the square brackets is the unit production cost. Since it is cumulative, the unit total cost tc_n (3.13') is also a cumulative cost.

For clear understanding of the unit total cost, I will graphically explain tc_n with Figure 3.2.[14]

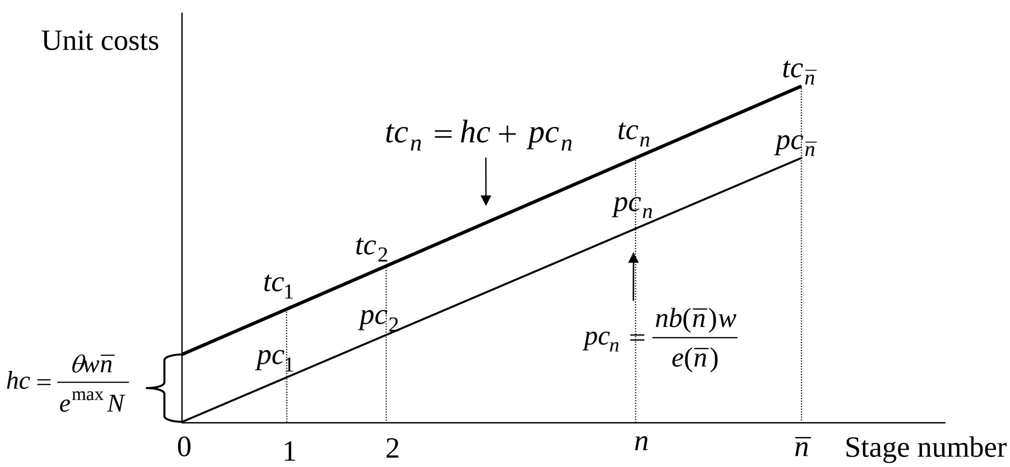

Figure 3.2

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[13] I will address in detail how the employees determine the level of effort and how the firm penalizes them when it identifies shirkers in 3.2.1.1.

[14] The number for the production stage n should be an integer. Thus, the graphs, strictly speaking, cannot be drawn continuously. However, for convenience, I disregard the integer problem and draw continuously the graphs.

The unit total cost at the first stage, tc_1, is the sum of $hc = \frac{\theta w \bar{n}}{e^{\max} N}$ and pc_1 $(= \frac{b(\bar{n})w}{e(\bar{n})})$. The unit total cost accumulated until the second stage, tc_2, is hc plus pc_2 $(= \frac{2b(\bar{n})w}{e(\bar{n})})$. Then the unit total cost accumulated until the nth stage, tc_n, is the sum of hc and pc_n $(= \frac{nb(\bar{n})w}{e(\bar{n})})$. The locus of tc_n is a straight line with the intercept $\frac{\theta w \bar{n}}{e^{\max} N}$, and the slope $\frac{b(\bar{n})w}{e(\bar{n})}$.

I explain the costs of producing the final good. Recall that the industry X is perfectly competitive and the output at the $\bar{n}$th stage is $\bar{x}$. Since the final good X is produced at the final stage $\bar{n}$ of the production process, the total cost and the unit total cost of the final stage are obtained by substituting $\bar{n}$ for n in TC_n (3.13) and tc_n (3.13'):

$$TC_{\bar{n}} = w\bar{x}\{\frac{\theta \bar{n}}{e^{\max} N} + \frac{\bar{n} b(\bar{n})}{e(\bar{n})}\},$$

$$tc_{\bar{n}} = w\{\frac{\theta \bar{n}}{e^{\max} N} + \frac{\bar{n} b(\bar{n})}{e(\bar{n})}\}.$$

Substituting $b(\bar{n}) = (\frac{1}{\bar{n}})^{\delta}$ in (3.3) into $TC_{\bar{n}}$ and $tc_{\bar{n}}$,

$$TC_{\bar{n}} = w\bar{x}\{\frac{\theta \bar{n}}{e^{\max} N} + \frac{\bar{n}^{1-\delta}}{e(\bar{n})}\}, \tag{3.14}$$

$$tc_{\bar{n}} = w\{\frac{\theta \bar{n}}{e^{\max} N} + \frac{\bar{n}^{1-\delta}}{e(\bar{n})}\}. \tag{3.14'}$$

3.2.1.1 Decision of Effort by Employee

An employee provides effort to the firm. The employee is compensated by a wage, which finances consumption. Thus, she obtains higher utility, but feels disutility from effort. She determines a level of effort where the difference between the utility and disutility is maximized. The employee has consumption c and her utility is $u(c)$: $\frac{\partial u(c)}{\partial c} > 0$ and $\frac{\partial^2 u(c)}{\partial c^2} < 0$. She makes an effort e, $0 \le e \le e^{\max}$, and feels disutility $v(e)$: $\frac{\partial v(e)}{\partial e} > 0$ and $\frac{\partial^2 v(e)}{\partial e^2} = 0$. A low level of effort means that the employee is very idle. $e = 0$ stands for complete idleness. If the employee provides full effort without shirking, the level of effort is $e^{\max}$ and she becomes a fully effective worker.

The disutility of effort is also affected by the employee's work ethic. Sennett (1998) and Minkler (2004) contend that work ethic is influenced by the economic environment. With the development of the economy and globalization, the firm faces a more competitive environment in domestic and international markets. The volatility of its business increases, and its survival becomes more difficult. This situation makes for unstable employment relations between the firm and its employees. Employment contracts with the firm are easily broken, for example, the possibility of being unemployed rises, or the duration of employment shortens. This causes the employees' work ethic to be diminished: their loyalty to the firm weakens so that their motivation and willingness to work hard are reduced (Reichheld and Teal 1996, and Minkler 2004).

In this paper, I do not deal with how the firm makes employment contracts to cope with the new competitive business environment.[15] Instead, more importantly, I will focus on how the lower loyalty affects productivity and production cost. First, assume that employees with low loyalty have high disutility of effort. This assumption implies that the disutility depends not only on effort but also on loyalty: $v(e;\eta)$. η is the index for loyalty, $0 < \eta \le 1$; a high value of η represents a high level of loyalty.

The firm expects its employees to work with maximal effort, $e^{\max}$. It monitors their performance and punishes the employees who are caught shirking. However, the headquarters' ability to monitor is imperfect. The employees would shirk on the job since effort involves disutility, and since there is a possibility that shirking is not detected by the headquarters. Their performance would be lower compared with that in the absence of shirking. Let P be the probability of an employee's performance being checked by the headquarters. If the performance of an employee is checked and if her level of effort is revealed to be below the level of full effort such that $0 \le e < e^{\max}$, she is penalized.[16] Let $(1-P)$ be the probability of an employee's performance not being checked. If the performance of an employee is not checked, the firm treats the employee as a non-shirker, and thinks that she works with full effort $e^{\max}$.[17] Under this

[15] The general trend in business today does not seem to involve ways to keep employees longer. Firms often seem to find ways to pay employees less, which leads to lower levels of loyalty (Reichheld and Teal 1996).

[16] Copeland (1989), Brecher (1992), and Matusz (1996) based on the efficiency wage model assume that if shirking of the employee is caught, she is fired. However, I follow the assumption of Calvo and Wellisz (1978), that is, when the shirker is caught, she receives the penalty that her wage is cut.

[17] Though the firm treats the employee as a non-shirker, it acknowledges that she can still shirk. However, without any evidence on her performance, if the firm regards her as a shirker, this incurs complaints from

scheme, if the detection probability is $P = 0$, the employee provides no effort. The equilibrium level of effort is zero. If the probability is $P = 1$, her performance is perfectly revealed and thus she will provide full effort. The equilibrium level of effort is $e^{\max}$.

The employee can face two states. One is that she is not checked, with probability ($1 - P$), and is considered a non-shirker working with full effort $e^{\max}$. She receives a wage w from the firm, which is measured in units of the final good X. Assuming that the wage income is consumed, consumption is $c = w$. The net utility is $u(w) - v(e;\eta)$.

The other state, which occurs with probability P, is that she is checked and is penalized if her effort is less than the full effort, $e < e^{\max}$. She is then paid a wage that is prorated to her effort. Since the wage per unit of effort that the firm considers is $\frac{w}{e^{\max}}$, the paid wage corresponding to her effort e is $\frac{ew}{e^{\max}}$. Her consumption is $c = \frac{ew}{e^{\max}}$. Her net utility is $u(\frac{ew}{e^{\max}}) - v(e;\eta)$.

The employee chooses a level of effort maximizing her expected net utility.

$$\underset{e}{\text{Max}}\ EU = P\{u(\frac{ew}{e^{\max}}) - v(e;\eta)\} + (1-P)\{u(w) - v(e;\eta)\}, \quad \text{s.t. } 0 \le e \le e^{\max}. \qquad (3.15)$$

The maximization problem restricts effort to be $0 \le e \le e^{\max}$. In view of the restriction, three

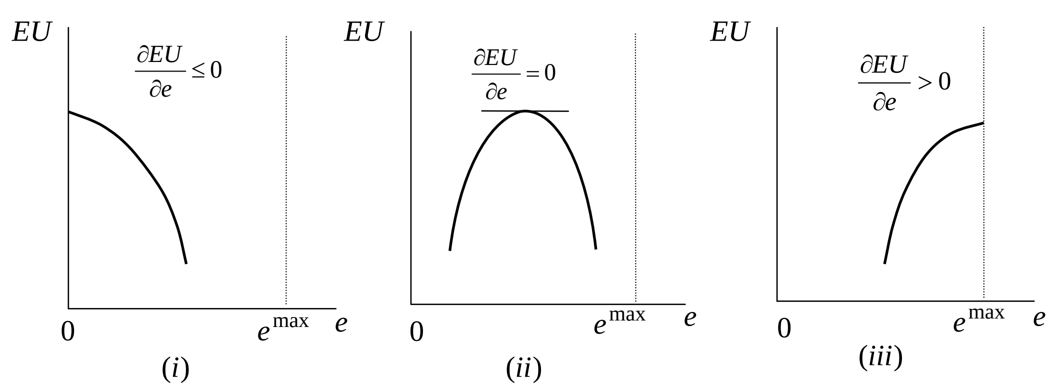

Figure 3.3

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her and lowers her productivity. However, if the firm regards her as a non-shirker (i.e., a fully effective worker), this can (or cannot) increase her motivation to work hard. Anyway, the latter case of regarding a worker as the non-shirker can become beneficial to the firm than the former case of regarding the worker as the shirker. The financial company, A.G. Edwards, suggests a similar idea: “The most important element of the firm’s management approach is to follow the golden rule – treating people the way you would like to be treated.” When the firm treats workers with some form of the golden rule, this improves the level of productivity (Reichheld and Teal 1996).

possible situations may arise: (i) EU is maximized at $e=0$, (ii) EU is maximized at $0<e<e^{\max}$ and (iii) EU is maximized at $e^{\max}$.

To find the optimal effort, specific functional forms for $u(c)$ and $v(e;\eta)$ are defined.

$$u(c)=2\sqrt{c}\,,\quad \text{where } \frac{\partial u}{\partial c}=\frac{1}{\sqrt{c}}>0\,,\ \ \frac{\partial^2 u}{\partial c^2}=\frac{-1}{2\sqrt{c^3}}<0\,.$$

I define the disutility of the employee in the North with a level of loyalty η as

$$v(e;\eta)=e(2-\eta)\,,\quad 0<\eta\le 1\,,$$

$$\text{where } \frac{\partial v(e;\eta)}{\partial e}=2-\eta>0\,,\ \ \frac{\partial v(e;\eta)}{\partial \eta}=-e<0\,.$$

The constant 2 in the disutility function makes the level of disutility positive for a given positive effort level. The sign of the derivative of the first means that if effort increases, the disutility increases. The sign of the derivative of the second says that a fall in loyalty η leads to an increase in the disutility of effort.

Substituting $c=\frac{ew}{e^{\max}}$ for the shirker or $c=w$ for the non-shirker into $u(c)=2\sqrt{c}$, and using $v(e;\eta)=e(2-\eta)$, (3.15) is re-expressed as

$$\underset{e}{\text{Max}}\ EU=P\{2\sqrt{\frac{ew}{e^{\max}}}-e(2-\eta)\}+(1-P)\{2\sqrt{w}-e(2-\eta)\}\,,\quad \text{s.t. } 0\le e\le e^{\max}\,, \qquad (3.15')$$

$$\text{where } \frac{\partial EU}{\partial e}=Pe^{\frac{-1}{2}}(\frac{w}{e^{\max}})^{\frac{1}{2}}-(2-\eta)\,,\quad \frac{\partial^2 EU}{\partial e^2}=-\frac{1}{2}Pe^{\frac{-3}{2}}(\frac{w}{e^{\max}})^{\frac{1}{2}}<0\,. \qquad (3.16)$$

As e approaches zero, $\frac{\partial EU}{\partial e}$ in (3.16), approaches positive infinity. This sign is not what case (i) requires as shown in Figure 3.3. Thus the optimal effort is not zero. The optimal effort would be determined in either of the cases (ii) or (iii). From the constrained maximization problem of expected utility, case (ii) yields a higher expected utility than case (iii).[18] Thus case (ii) is chosen for the optimal effort. The optimal effort is now an interior solution and is smaller than $e^{\max}$:

$$e=\frac{wP^2}{e^{\max}(2-\eta)^2}<e^{\max}\,,\quad \text{where } 0<P\le 1,\ \ 0<\eta\le 1\,. \qquad (3.17)$$

[18] The details are in Appendix 3.1.

A higher wage gives the employee the incentive to be willing to raise her level of effort. Also, a higher probability of being checked raises the possibility that the employee is penalized, thus she raises her work effort. If the employee has high loyalty, she provides high effort.

I now address the detection probability. When the headquarters splits the single-stage production process into multiple-stages, the split gives it more detailed information about the stage in which shirking occurs. This raises the detection probability. A probability function showing a positive relation between the detection probability and the number of total stages can be explained as follows. First, imagine a single stage production process that produces the final good. The firm is able to check the employees' performance only after the production of the good is completed, since the output is observed only at that time. However, all employees produce the same good in one stage, so the firm cannot observe exactly each individual's performance. Instead, it observes the performance of the pool of employees. Thus the possibility that a single employee's performance is checked depends on her portion of the employee pool, that is, the number of employees. Since the firm competes in a perfectly competitive environment, its output is assumed to be exogenous.[19] Thus the number of employees, l, can be considered as given. This number is assumed to be large. The employees are assumed to be distributed uniformly over the line in the range $(0, l]$ as (*i*) in Figure 3.4. Then the size of the employee pool is represented by this line.

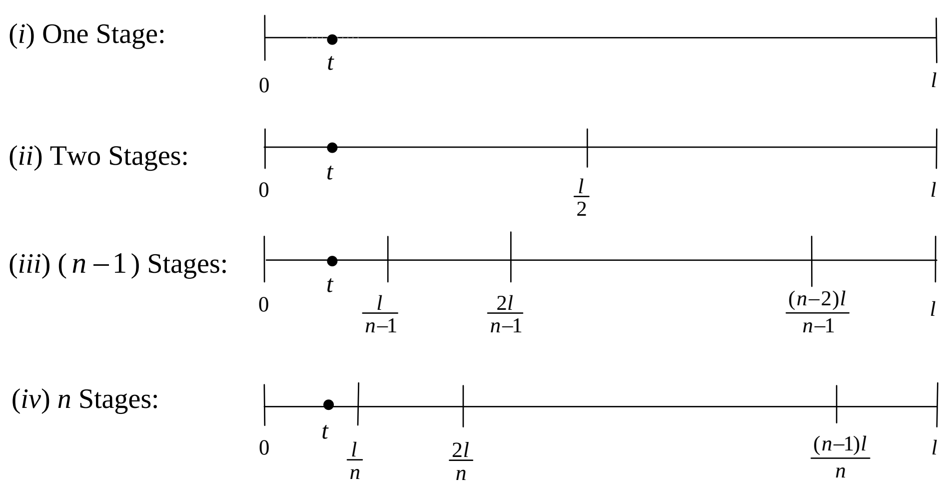

Figure 3.4

[19] In perfect competition, price equals average cost and marginal cost. Since the firm in my model has constant marginal cost, this equilibrium condition does not pin down a level of output. With only the equilibrium condition, a specific level of output cannot be determined. Also, this model is a partial equilibrium model that does not consider the factor market. Thus, the output level should be treated as given exogenously.

Consider a single employee in the pool who is represented as one point, such as an arbitrary point, t, on the line. The length of the range in which she is included (this is identical to the size of the pool for the production mode of one stage) is $\frac{l}{1}$, where one in the denominator is the number of total stages. This length of the range affects the probability that she is checked; I denote the probability simply as a function of the length of the range: $P(\frac{l}{1})$.

If the firm changes the production mode from single-stage production to two-stage production, the probability that she is checked would change. The first stage of the two-stage production produces a distinct intermediate good that is assembled into the final good. The second stage produces the final good. The employees engaged in production would be rearranged in two groups on the assumption that each employee is assigned to only one stage. One group is for the first stage that is represented by the range $(0, \frac{l}{2}]$ as (*ii*) in Figure 3.4. The other is for the second stage which is represented by the range $(\frac{l}{2}, l]$. Since the headquarters observes the output of each stage, if shirking occurs, it knows the group in which shirking occurs. Her location at t belongs to the range of $(0, \frac{l}{2}]$. The range that the firm has to scrutinize, conditional on the shirker belonging to the first stage, would decrease to half of the size of the pool, $\frac{l}{2}$. Then the probability that she is detected becomes higher than the probability in the case of the single-stage production mode: $P(\frac{l}{1}) < P(\frac{l}{2})$.

The same logic works for the general case of n stages. If production is split into n stages, the range is split into n parts as (*iv*) in Figure 3.4. Her location at t is in $(0, \frac{l}{n}]$. The length of the range in which she is included is $\frac{l}{n}$. This is shorter than $\frac{l}{n-1}$ that is the length under the $(n-1)$-stages as (*iii*). Then the probability that she is detected under the n-stage production becomes higher than that under the $(n-1)$-stage production: $P(\frac{l}{n-1}) < P(\frac{l}{n})$. Thus the probability is increasing in the number of stages. $P(\frac{l}{1}) < P(\frac{l}{2}) < ... < P(\frac{l}{n-1}) < P(\frac{l}{n})$. This also means that the detection probability has an inverse relation to the length of the range in which the employee is included; if the length of the range is long (short), the detection probability is low (high).

As the length of the range in which the employee is included becomes shorter, the probability that she is detected becomes higher. However, the degree of the increase in the detection probability decreases as the length of the range shortens. The reason is as follows. The increase in the number of stages makes the range in which she is included smaller, so that the risk of detection becomes higher; the higher risk makes her become more cautious and shirk less; thus the rate of increase in the detection probability decreases if the number of stages increases. To capture all these characteristics, the probability is expressed as

$$P(\frac{l}{n}) = \sqrt{\frac{1}{l/n}}\,, \quad \text{where } \frac{\partial P}{\partial n} = \frac{n^{\frac{-1}{2}}}{2\sqrt{l}} > 0\,, \quad \frac{\partial^2 P}{\partial n^2} = \frac{-n^{\frac{-3}{2}}}{4\sqrt{l}} < 0\,. \tag{3.18}$$

If $n = l$, each stage has only a single employee. This says that monitoring is perfect and the detection probability is one, $P(\frac{l}{n}) = 1$. If $n < l$, the detection probability lies in the range $0 < P(\frac{l}{n}) < 1$. If $n > l$, this means that a single employee works in multiple stages. However, the firm is assumed to allocate more than one of its employees to each of the stages, so that I exclude this case of $n > l$.

Since a larger number of total stages increases the probability that shirking employees are detected, their work effort increases with the number of total stages. Replacing n in (3.18) by $\overline{n}$ and substituting (3.18) for P in (3.17),

$$e(\overline{n}; w, \eta, l) = \frac{w}{e^{\max}(2-\eta)^2}(\frac{\overline{n}}{l})\,, \tag{3.19}$$

$$\text{where } \frac{\partial e(\overline{n}; w, \eta, l)}{\partial \overline{n}} = \frac{w}{e^{\max}(2-\eta)^2 l} > 0\,, \quad \frac{\partial^2 e(\overline{n}; w, \eta, l)}{\partial \overline{n}^2} = 0\,. \tag{3.20}$$

The first equation in (3.20) shows that the effort level increases if the number of total stages increases. And the marginal increase in effort with respect to the number of total stages is constant. Thus the effort function (3.19) is linear with respect to $\overline{n}$. If $\overline{n} = 1$, $e = \frac{w}{e^{\max}(2-\eta)^2 l} > 0$, where $0 < \eta \leq 1$. Also, since an employee cannot provide more than the maximal effort that is physically possible, there would be an upper limit on the effort provided by an employee, $e^{\max}$. Then (3.19) can determine the upper bound of the range of the number of total stages in which effort is provided. This happens where the obtained effort becomes equalized to the maximum effort: $\bar{n} = \frac{\{e^{\max}(2-\eta)\}^2 l}{w}$. Although effort is a function of $\bar{n}$, w, η and l, for convenience, I express it as $e(\bar{n})$. Figure 3.5 illustrates the above relation between the number of total stages and the effort level in the range, $1 \le \bar{n} \le \frac{\{e^{\max}(2-\eta)\}^2 l}{w}$.

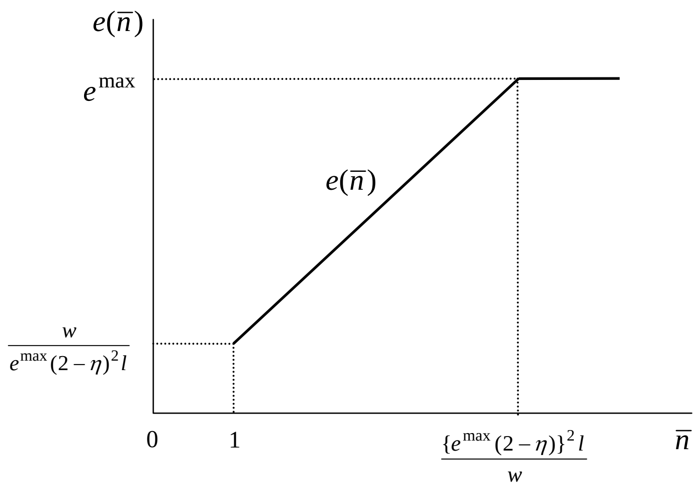

Figure 3.5

3.2.1.2 Equilibrium in National Fragmentation

I address how the firm optimally determines the number of total stages in the North. Since the final good is made at the final stage, the number indexing the final stage becomes equal to the number of total stages.

Where the unit total cost of producing the final good is minimized, the firm sets the number of the final stage, that is, the number of total stages. Recall that the unit total cost function for the final good is expressed with $\bar{n}$ indexing the final stage, $tc_{\bar{n}} = \frac{w\theta\bar{n}}{e^{\max}N} + \frac{w\bar{n}^{1-\delta}}{e(\bar{n})}$ in (3.14'). The first term on the RHS is the unit headquarters cost, $hc = \frac{w\theta\bar{n}}{e^{\max}N}$. This rises with the number of the final stage, and thus hc is linearly increasing in $\bar{n}$ at the rate of $\frac{w\theta}{e^{\max}N}$ as shown in the following Figure 3.6. The second term on the RHS is the unit production cost, $pc_{\bar{n}} = \frac{w\bar{n}^{1-\delta}}{e(\bar{n})}$.

Since effort and labor productivity rise as the number of the final stage increases, the unit production cost is decreasing in $\bar{n}$ as in the following equation (3.21). Here, I define elasticity of effort with respect to the number of the final stage as $\varepsilon = \frac{\partial e(\bar{n})}{\partial \bar{n}} \frac{\bar{n}}{e(\bar{n})}$. Since $\frac{\partial e(\bar{n})}{\partial \bar{n}} > 0$ from (3.20), ε is positive. Then the first derivative of the unit production cost at $\bar{n}$ is expressed as

$$\frac{\partial pc_{\bar{n}}}{\partial \bar{n}} = \frac{(1-\delta-\varepsilon) w \bar{n}^{-\delta}}{e(\bar{n})} < 0, \quad (3.21)$$

[20]

where $\delta \geq 1$, $\varepsilon > 0$, $1 \leq \bar{n} \leq \frac{\{e^{\max}(2-\eta)\}^2 l}{w}$.

And the second derivative is

$$\frac{\partial^2 pc_{\bar{n}}}{\partial \bar{n}^2} = \frac{-(\delta+\varepsilon)(1-\delta-\varepsilon) w \bar{n}^{-(\delta+1)}}{e(\bar{n})} > 0. \quad (3.22)$$

[21]

The conditions, (3.21) and (3.22), say that $pc_{\bar{n}}$ is downward sloping and convex as seen in Figure 3.6.

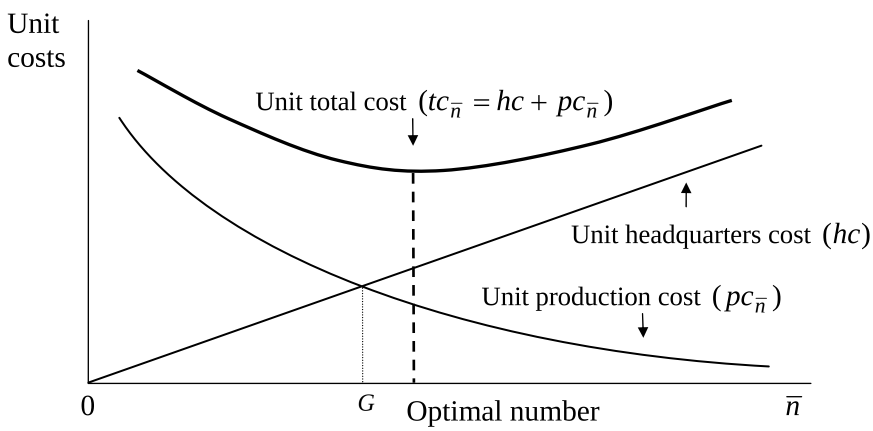

Figure 3.6

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[20] $\frac{\partial pc_{\bar{n}}}{\partial \bar{n}} = \frac{w\bar{n}^{-\delta}\{(1-\delta)e(\bar{n}) - \bar{n}\frac{\partial e(\bar{n})}{\partial \bar{n}}\}}{e(\bar{n})^2} = \frac{w\bar{n}^{-\delta}(1-\delta)}{e(\bar{n})} - \frac{w\bar{n}^{-\delta}}{e(\bar{n})}\{\frac{\bar{n}}{e(\bar{n})}\frac{\partial e(\bar{n})}{\partial \bar{n}}\} = \frac{(1-\delta-\varepsilon)w\bar{n}^{-\delta}}{e(\bar{n})}$.

[21] $\frac{\partial^2 pc_{\bar{n}}}{\partial \bar{n}^2} = \frac{-\{\delta(1-\delta-\varepsilon)w\bar{n}^{-(\delta+1)}e(\bar{n}) + (1-\delta-\varepsilon)w\bar{n}^{-\delta}\frac{\partial e(\bar{n})}{\partial \bar{n}}\}}{e(\bar{n})^2}$

$= -[\frac{\delta(1-\delta-\varepsilon)w\bar{n}^{-(\delta+1)}}{e(\bar{n})} + \frac{(1-\delta-\varepsilon)w\bar{n}^{-(\delta+1)}}{e(\bar{n})}\{\frac{\bar{n}}{e(\bar{n})}\frac{\partial e(\bar{n})}{\partial \bar{n}}\}] = \frac{-(\delta+\varepsilon)(1-\delta-\varepsilon)w\bar{n}^{-(\delta+1)}}{e(\bar{n})}$.

When hc and $pc_{\overline{n}}$ are summed, the graph of $tc_{\overline{n}}$ is convex. The optimal number of stages is determined where $tc_{\overline{n}}$ is minimized with respect to $\overline{n}$. This is graphically shown in Figure 3.6.[22] The first and second order conditions for the cost minimization are

$$\frac{\partial tc_{\overline{n}}}{\partial \overline{n}} = w\{\frac{\theta}{e^{\max} N} + \frac{(1-\delta-\varepsilon)\overline{n}^{-\delta}}{e(\overline{n})}\} = 0 , \quad \frac{\partial^2 tc_{\overline{n}}}{\partial \overline{n}^2} = \frac{\partial^2 pc_{\overline{n}}}{\partial \overline{n}^2} > 0 .$$

The cost-minimizing number of the final stage $\overline{n}$ is

$$\overline{n}(e(\overline{n}); \delta, \varepsilon, N, \theta) = \{\frac{(\delta+\varepsilon-1)e^{\max} N}{e(\overline{n})\theta}\}^{\frac{1}{\delta}} . \tag{3.23}$$

Note that the number of stages is greater than one. $\overline{n} > 1$. The reasons are: $\delta \geq 1$; $\varepsilon = 1$ from (3.19) since e is proportional to $\overline{n}$; $e^{\max} > e(\overline{n})$; and $N > \theta$ in (3.9). Though the number of stages is a function of $e(\overline{n}),\ \delta,\ N$ and $\theta,$ for convenience, I express it as $\overline{n}(e(\overline{n}))$, where $e(\overline{n})$ is the level of effort that the firm expects from its workers. The number of stages is decreasing in $e(\overline{n})$ and is convex toward the origin: $\frac{\partial \overline{n}(e(\overline{n}))}{\partial e(\overline{n})} < 0$ and $\frac{\partial^2 \overline{n}(e(\overline{n}))}{\partial e(\overline{n})^2} > 0$.[23] The curve for $\overline{n}(e(\overline{n}))$ in (3.23) is depicted in Figure 3.7. This means that if the effort of employees increases, the necessity of dividing the production process would be reduced, and thus the firm would decrease the number of total stages.

The equilibrium number of total stages $\tilde{\tilde{n}}$ and the equilibrium effort $\tilde{e}$ are determined by equations (3.19) and (3.23). Substituting (3.19) for $e(\overline{n})$ in (3.23),

$$\tilde{\tilde{n}} = [\frac{(\delta+\varepsilon-1)\{e^{\max}(2-\eta)\}^2 lN}{\theta w}]^{\frac{1}{\delta+1}} , \tag{3.24}$$

$$\tilde{e} = (e^{\max})^{\frac{1-\delta}{\delta+1}} \{\frac{(\delta+\varepsilon-1)N}{\theta}\}^{\frac{1}{\delta+1}} \{\frac{w}{(2-\eta)^2 l}\}^{\frac{\delta}{\delta+1}} .$$

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[22] The optimal number can also lie on the LHS of $G,$ at the point $G,$ or on the RHS of $G.$ The location depends on functional forms of the unit headquarters cost and the unit production cost. For convenience, the optimal number is located on the RHS of G in Figure 3.6.

[23] $\frac{\partial \overline{n}(e(\overline{n}))}{\partial e(\overline{n})} = -\frac{1}{\delta}\{\frac{(\delta+\varepsilon-1)e^{\max} N}{\theta}\}^{\frac{1}{\delta}} e(\overline{n})^{-(1+\frac{1}{\delta})} < 0$,

$$\frac{\partial^2 \overline{n}(e(\overline{n}))}{\partial e(\overline{n})^2} = \frac{1}{\delta}(1+\frac{1}{\delta})\{\frac{(\delta+\varepsilon-1)e^{\max} N}{\theta}\}^{\frac{1}{\delta}} e(\overline{n})^{-(2+\frac{1}{\delta})} > 0 .$$

Also, the equilibrium $\tilde{\bar{n}}$ and $\tilde{e}$ can be shown, by putting the graph for $e(\bar{n})$ in Figure 3.5 and the curve for $\bar{n}(e(\bar{n}))$ together in the same space, as in Figure 3.7. $\tilde{\bar{n}}$ is smaller than the number of total stages where effort is maximal, $\frac{\{e^{\max}(2-\eta)\}^2 l}{w}$. The employee provides $\tilde{e}$ that is less than $e^{\max}$. This says that she is not completely idle, as well as not being a fully effective worker, since she knows that the detection probability is $0 < P < 1$. The situation described above will be applied to a mode of international fragmentation in the next section.

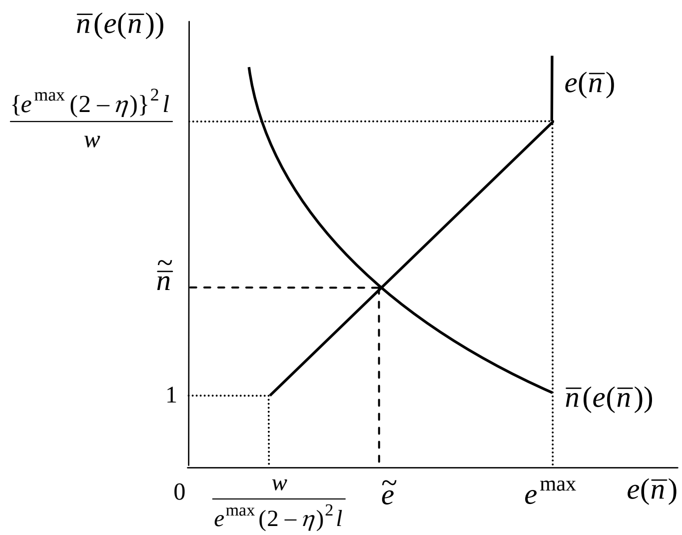

Figure 3.7

3.2.2 Open Fragmented Economies

Revolutionary progress in communication technologies has weakened the link between specialization and geographic concentration.[24] Such progress makes communication across borders possible without physical traveling and loss of time. Thus, regardless of production location, Northern firms can coordinate and monitor all production stages remotely. This causes Northern firms to get the benefit of the low Southern wage, and gives them incentives to relocate their production to the South. That is, international fragmentation becomes more economical than ever before.

[24] See Grossman and Rossi-Hansberg (2007 and 2008). They also say that a revolution in transportation makes it easy to transport partially processed goods quickly, and at lower cost. Thus specialization can be achieved without geographic concentration.

I assume that the firm's headquarters is located in the home country, the North. However, the headquarters faces more difficulty running the Southern stages (i.e., stages outsourced to the South) than the Northern stages (i.e., stages located in the North). The first reason is Southern border barriers, such as different culture, language and legal system. These require more headquarters services. Second, since the network size of the South is assumed to be smaller than that of the North, the small Southern network makes the Northern firm less efficient. Thus the firm faces a higher headquarters cost. However, the low wage of the South reduces the firm's production cost.

If the reduction in the production cost is larger than the increase in the headquarters cost, and thus if the total cost with international fragmentation becomes lower than that with national fragmentation, the firm would change the mode of production from national fragmentation to international fragmentation. The firm will split the production stages across the two regions as a way of incurring the lowest total cost. I define international fragmentation as a production pattern locating some production stages in the North and some production stages in the South, but all stages are involved in producing a single final good. The firm keeps the headquarters in the North and might shift all production stages to the South. However, I exclude this case.[25]

To show that the total cost with international fragmentation is lower than that with national fragmentation, we need to know the cost of the Northern firm as a multinational firm. For this, I first suppose ex-ante that international fragmentation arises as follows: $\hat{n}$ is the cutoff stage number at which the stages are split between the North and the South; m is the number of total stages across both regions.

3.2.2.1 Effort of Employee under International Fragmentation

The effort level of employees affects the multinational firm's cost, therefore, I address how effort is provided by employees under international fragmentation. The firm's total employment in the North and the South is l^f. The symbol f denotes international fragmentation. The employees' performance is monitored in each of the m stages by the same headquarters regardless of which region they reside in. m should be equal to, or greater than two: $m \geq 2$. Then, the size of any group in which an individual employee is included is $\frac{l^f}{m}$. The detection probabilities for the Northern and Southern employees are associated inversely with the size of group, and are the same.

[25] This is explained in subsection 3.2.2.4.

The effort function of the Northern employees under international fragmentation has the same form as that in (3.19) under national fragmentation, except that the detection probability is different. From (3.18), when the mode of production changes from national fragmentation to international fragmentation, the detection probability in the North changes from $\sqrt{\frac{\overline{n}}{l}}$ to $\sqrt{\frac{m}{l^f}}$.

Using (3.17),

$$e(m) = \frac{w}{e^{\max}(2-\eta)^2}\left(\frac{m}{l^f}\right). \tag{3.25}$$

Additionally, we need to know the disutility function of effort in order to pin down the effort level of the employees in the South. Letting an asterisk $*$ denote the South, I define it as $v^* = e^*(2-\eta^*)$, $0 < \eta^* \leq 1$ and $\frac{\partial v^*}{\partial e^*} = 2-\eta^*$. Using (3.25), the effort in the South is

$$e^*(m) = \frac{w^*}{e^{*\max}(2-\eta^*)^2}\left(\frac{m}{l^f}\right). \tag{3.26}$$

When the number of total stages m changes, Southern effort changes by the same proportion, $\frac{1}{m}$, as Northern effort.[26]

I address the degrees of loyalty in the North and the South to compare the effort levels in both regions. More global and more advanced economies (i.e., the North) have experienced more competition than less global and less advanced economies (i.e., the South). The competitive environment in the North makes the employment contracts between the firm and its employees more fragile, and thus duration of employment is shortened. Under more protection that limits free trade, the South has been less exposed to international competition, and has kept relatively stable employment relationships. To show these employment relations in the North and the South, I compare average employment tenure of nine countries: Australia, Germany, the Netherlands, the United Kingdom and the United States as the North, and Malaysia, the Philippines, the Republic of Korea and Taiwan as the South.

Table 3.1 shows the distribution of workers' average tenure from one country to another in 1991.[27] Unlike the developed countries and Taiwan, Malaysia, the Philippines and Korea did not

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[26] From (3.26), $\frac{\partial e^*(m)}{\partial m} = \frac{w^*}{e^{*\max}(2-\eta^*)^2 l^f}$. Using this and (3.26), $\frac{\frac{\partial e^*(m)}{\partial m}}{e^*(m)} = \frac{1}{m}$.

[27] Difference in tenure across nations might be affected by differences in economic cycle, social institutions, employment protection policy, structure of labor market, population, sector coverage and related definitions.

collect official data on tenure, so that I use data from Bai and Cho who conducted their survey in 1991. They selected metropolitan cities such as Kuala Lumpur in Malaysia, Manila in the Philippines and Seoul in Korea, and surveyed average tenure in the manufacturing industry.[28] Since the economic powers of Malaysia, the Philippines and Korea are concentrated in each capital city and its vicinity,[29] their survey data are likely to be representative of the nationwide tenure of these countries.[30]

Table 3.1 **Distribution of Employment by Enterprise Tenure, 1991**

	Australia	Germany *a*	Netherlands *a*	United Kingdom	United States	Malaysia *b*	Philippines *c*	Republic of Korea *d*	Taiwan *e*
Average tenure (years)									
All persons	6.8	10.4	7	7.9	6.7				7.7
Men	7.8	12.1	8.6	9.2	7.5	8.6	10.0	5.2	
Women	5.4	8.0	4.3	6.3	5.9	7.6	9.0	3.5	

(*a*) Data were collected in 1990
(*b*) In Kuala Lumpur, Malaysia, 739 men and 706 women were surveyed from May to August, 1991.
(*c*) In Manila, the Philippines, 784 men and 786 women were surveyed from May to August 1991.
(*d*) In Seoul, Korea, 701 men and 689 women were surveyed in the first survey took place in February and March, 1991, and in the second survey took place in August 1991.
(*e*) Data for men are not available. Data for women are not available.

Source: OECD Employment Outlook (1993) for developed countries, Bai and Cho's survey report (1995) for Malaysia, the Philippines and Korea, and Report on the Manpower Utilization Survey (2005) for Taiwan

As shown in table 3.1, for all persons, Taiwan has an average tenure that is longer than the average tenure in Australia, the Netherlands and the United States. However, Taiwan and the United Kingdom have similar tenure. The tenures for women are longer in Malaysia and the

[28] To compare exactly the tenures between the North and the South, I should use the nationwide average tenure for both developed countries and developing countries. However, such data for the developing countries are not available.
[29] Kuala Lumpur and Selanger made up 33 percent of GDP in 1988 (Lee and Sivananthiran, 1991); Metro Manila provided 30.8 percent of GDP in 1989 (Institute of Labor Studies, 1991). In the case of Korea, the value added of both Seoul and its vicinity Kyongki Province was 45.1 percent of GDP in 1990 (Report on Mining and Manufacturing Census for Korea, 1990). I cite these data from Bai and Cho (1995).
[30] Rural areas may be very different from cities. However, since the year 1965, the importance of agriculture has sharply decreased in Asian-Pacific countries. The shares of agriculture in GDP in 1990 were 9 percent in Korea, 19 percent in Malaysia, and 22 percent in the Philippines (World Bank, World Development Report, 1992 and Asian Development Bank, Key Indicator of Developing Member Countries of ADB, 1992). Also the proportion of employment in agriculture has decreased, during the period 1980-1989, in Asia-Pacific countries. At the same time, the share of the manufacturing industry in GDP has increased. The share of the manufacturing industry in 1991 in Korea was 31 percent, 27 percent in Malaysia, and 25 percent in 1990 in the Philippines (World Bank, World Development Report, 1992 and Asian Development Bank, Key Indicator of Developing Member Countries of ADB, 1992). I cite these data from Bai and Cho (1995). Thus, since the agriculture share of GDP is lower than the manufacturing share of GDP, the tenures in table 1 can be representative of the nationwide tenure.

Philippines than in Australia, the Netherlands, the United Kingdom and the United States. For men, Malaysia and the Philippines have longer tenures than Australia and the United States. Malaysia and the Netherlands have the same tenures. However, Malaysia has shorter tenure than the United Kingdom. Germany has the longest tenure for all persons and men. Korea has the shortest tenure for men and women. Although these comparisons call for some caution, tenure in the developing countries seems higher than tenure in the developed countries. Of course, Germany (as a developed country) and Korea (as a developing country) are opposite cases. I do not consider the trend of these two countries in order to focus on the main trend mentioned above.

To the extent that tenure is positively associated with the level of loyalty, and that tenure in the developed countries tends to be shorter than that in the developing countries, it is possible to assume that the loyalty in the North is lower than that in the South: $\eta < \eta^*$. This assumption says that the marginal disutility of effort in the North, $(2-\eta)$, is higher than that in the South, $(2-\eta^*)$.

Assume that the maximal effort levels in the North and the South are equal, because the upper limit on the effort that employees are able to provide physically is the same: $e^{\max} = e^{*\max}$. Then the effort of the South relative to the North, $\frac{(3.26)}{(3.25)}$, is

$$\frac{e^*(m)}{e(m)} = \frac{w^*}{w}\left(\frac{2-\eta}{2-\eta^*}\right)^2, \quad \text{where } (2-\eta) > (2-\eta^*). \tag{3.27}$$

This equation does not include the number of total stages m. That is, a change in the total number of stages does not influence the relative effort since the Northern effort and the Southern effort are influenced by the same total number of stages. Since $\left(\frac{2-\eta^*}{2-\eta}\right) < 1$, equation (3.27) implies that $\left(\frac{w^*}{w}\right)^{-1}\frac{e^*(m)}{e(m)} > 1$. This inequality is rewritten as

$$\frac{w}{e(m)} > \frac{w^*}{e^*(m)}. \tag{3.28}$$

This says that the wage per unit of effort in the North is higher than the wage per unit of effort in the South. Thus the Northern firm has an incentive to shift production stages to the South.

3.2.2.2 Northern Firm's Cost under International Fragmentation

Before examining the Northern firm's choice of whether to switch from national fragmentation to international fragmentation, its cost under international fragmentation should be first explained. Assume that the firm locates the stages with stage number n, $n \in (0, \hat{n}]$, in the North, and the stages with stage number n^*, $n^* \in (0^*, m-\hat{n}]$, in the South.[31] The firm faces the headquarters costs and production costs occurred in the North and the South, respectively. First, let me explain the headquarters cost incurred by the $\hat{n}$ Northern stages. Since the headquarters remains in the North and employs Northern labor, the unit headquarters cost, hc^f, is obtained by substituting $\hat{n}$ for $\bar{n}$ in (3.10').

$$hc^f = \frac{\Psi w \hat{n}}{e^{\max}}, \quad \text{where } \Psi = \frac{\theta}{N}. \tag{3.29}$$

I explain production in the North. Under international fragmentation, effort and labor productivity are affected by the total number of stages m. The production function for the Northern stages $n \in (0, \hat{n}]$ is derived by substituting $e(m)$ for e in the production function (3.1) in national fragmentation, l_n^f for l_n, and $b(m)$ for $b(\bar{n})$:

$$z_n = \min\{z_{n-1,} \frac{e(m) l_n^f}{b(m)}\}. \tag{3.30}$$

Using (3.11'), the unit production cost at the $\hat{n}$th stage changes to

$$pc_{\hat{n}} = \frac{\hat{n} b(m) w}{e(m)}. \tag{3.31}$$

From (3.29) and (3.31), the unit total cost at the $\hat{n}$th stage is

$$tc_{\hat{n}}^f = hc^f + pc_{\hat{n}} = \frac{\Psi w \hat{n}}{e^{\max}} + \frac{\hat{n} b(m) w}{e(m)}. \tag{3.32}$$

The headquarters in the North also manages the $(m-\hat{n})$ Southern stages. It employs Northern labor and faces the headquarters cost incurred by the Southern stages. The headquarters faces more difficulty operating the Southern stages than the Northern stages for a given number of stages, because of Southern border barriers such as difference in culture, language and legal system. The smaller Southern network also reduces efficiency of the headquarters service. Thus, the costs for the headquarters increase. This causes the unit headquarters cost to be higher for the Southern stages than for the Northern stages.

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[31] I take $\hat{n}$ and m as exogenous variables for the moment. However, in the subsequent analysis, m and $\hat{n}$ will be determined endogenously by the firm's optimal choice.

I evaluate the headquarters cost for the Southern stages for simplicity on the basis of $\Psi(=\dfrac{\theta}{N})$, which consists of the Northern network N and the input requirement of effective Northern labor θ for providing the headquarters services for the Northern stages. This cost is denoted as hc^{f*}. However, the small Southern network and the high Southern border barriers lower the firm's efficiency. The lower efficiency should be embedded in this unit headquarters cost. I formalize this by defining hc^{f*} as a two-part functional form. The first part is linear with respect to Ψ and the number of the Southern stages, respectively: $\dfrac{\Psi(m-\hat{n})w}{e^{\max}}$. The second part is expressed as an increasing marginal cost of the Southern stages, $\dfrac{\beta(m-\hat{n})^{\mu}w}{e^{\max}}$. The parameter β, $\beta>0$, reflects an increase in the headquarters cost due to the small Southern network. Also, the parameter μ, $\mu>1$, reflects an increase in the headquarters cost due to the high Southern border barriers.

$$hc^{f*}=\frac{\{\Psi(m-\hat{n})+\beta(m-\hat{n})^{\mu}\}w}{e^{\max}}, \quad \text{where } \beta>0,\ \mu>1. \tag{3.33}$$

Before addressing the production cost for the Southern stages such that $n^{*}\in(0^{*}, m-\hat{n}]$, I look at the production function for the Southern stages. The first stage in the South is denoted by $(\hat{n}+1^{*})$. This stage uses intermediate good $Z_{\hat{n}}$ which the $\hat{n}$th stage in the North produces and Southern labor $l^{f}_{\hat{n}+1^{*}}$. Since the Southern stage is one part of the production process of m stages, the productivity of labor in the South depends on the total number of stages m, and is the same as in the North. The production function for the $(\hat{n}+1^{*})$th stage is derived by substituting $z_{\hat{n}+1^{*}}$ for z_n in (3.30), $z_{\hat{n}}$ for z_{n-1}, $e^{*}(m)$ for $e(m)$ and $l^{f}_{\hat{n}+1^{*}}$ for l^{f}_{n},

$$z_{\hat{n}+1^{*}}=\min\{z_{\hat{n}}, \frac{e^{*}(m)l^{f}_{\hat{n}+1^{*}}}{b(m)}\}. \tag{3.34}$$

Suppose that output of this stage is $\bar{z}_{\hat{n}+1^{*}}$. Then the $(\hat{n}+1^{*})$th stage uses $\bar{z}_{\hat{n}+1^{*}}$ units of the intermediate $Z_{\hat{n}}$ in the South. The $\hat{n}$th stage produces $\bar{z}_{\hat{n}+1^{*}}$ units of $Z_{\hat{n}}$: $z_{\hat{n}}=\bar{z}_{\hat{n}+1^{*}}$. Since the production of one unit of $Z_{\hat{n}}$ uses $\dfrac{\hat{n}b(m)}{e(m)}$ units of Northern labor, $\bar{z}_{\hat{n}+1^{*}}$ units of $Z_{\hat{n}}$ embodies Northern labor of $\dfrac{\hat{n}b(m)\bar{z}_{\hat{n}+1^{*}}}{e(m)}$ units. The $(\hat{n}+1^{*})$th stage also uses labor for the production of

$\bar{z}_{\hat{n}+1^*}$ units of $Z_{\hat{n}+1^*}$ by $l^f_{\hat{n}+1^*} = \frac{b(m)\bar{z}_{\hat{n}+1^*}}{e^*(m)}$. The total labor input is the sum of embodied labor and the directly used labor: { $\frac{\hat{n}b(m)}{e(m)}$ units of Northern labor + $\frac{b(m)}{e^*(m)}$ units of Southern labor} $\bar{z}_{\hat{n}+1^*}$. The expression in the curly brackets is the total labor input per unit of $Z_{\hat{n}+1^*}$.

Consider the second stage in the South. The production function at the $(\hat{n}+2^*)$ th stage is

$$z_{\hat{n}+2^*} = \min\{z_{\hat{n}+1^*}, \frac{e^*(m)l^f_{\hat{n}+2^*}}{b(m)}\}.$$

I apply here the same logic as applied for the $(\hat{n}+1^*)$ th stage. For the production of $\bar{z}_{\hat{n}+2^*}$ units of $Z_{\hat{n}+2^*}$, the $(\hat{n}+2^*)$ th stage uses $\bar{z}_{\hat{n}+2^*}$ units of $Z_{\hat{n}+1^*}$. And $\bar{z}_{\hat{n}+2^*}$ units of $Z_{\hat{n}+1^*}$ embodies labor by { $\frac{\hat{n}b(m)}{e(m)}$ units of Northern labor + $\frac{b(m)}{e^*(m)}$ units of Southern labor} $\bar{z}_{\hat{n}+2^*}$ because the total labor input per output of $Z_{\hat{n}+1^*}$ is { $\frac{\hat{n}b(m)}{e(m)}$ units of Northern labor + $\frac{b(m)}{e^*(m)}$ units of Southern labor}. The Southern labor used for the $(\hat{n}+2^*)$ th stage is $l^f_{\hat{n}+2^*} = \frac{b(m)\bar{z}_{\hat{n}+2^*}}{e^*(m)}$. The total labor used for the $(\hat{n}+2^*)$ th stage is the sum of the embodied labor and the labor used for this stage: { $\frac{\hat{n}b(m)}{e(m)}$ units of Northern labor + $\frac{2b(m)}{e^*(m)}$ units of Southern labor} $\bar{z}_{\hat{n}+2^*}$. The expression in the curly brackets is the total labor input per unit of $Z_{\hat{n}+2^*}$.

Applying this logic to the $(\hat{n}+n^*)$ th stage, the total labor used for the production of $\bar{z}_{\hat{n}+n^*}$ units of $Z_{\hat{n}+n^*}$ is { $\frac{\hat{n}b(m)}{e(m)}$ units of Northern labor + $\frac{n^*b(m)}{e^*(m)}$ units of Southern labor} $\bar{z}_{\hat{n}+n^*}$. The unit production cost for the $(\hat{n}+n^*)$ th stage, $pc^f_{\hat{n}+n^*}$, is obtained from the product of wage and units of labor per unit of $Z_{\hat{n}+n^*}$.

$$pc^f_{\hat{n}+n^*} = \frac{\hat{n}b(m)w}{e(m)} + \frac{n^*b(m)w^*}{e^*(m)}. \tag{3.35}$$

The unit total cost at the $(\hat{n}+n^*)$ th stage, $tc^f_{\hat{n}+n^*}$, is the sum of the two unit headquarters costs, hc^f (3.29) and hc^{f*} (3.33), plus the unit production cost, $pc^f_{\hat{n}+n^*}$ (3.35). This unit total cost is expressed as (3.36). For simplicity, I assume that $\mu = 2$.

$$tc^f_{\hat{n}+n^*} = \frac{\Psi \hat{n} w}{e^{\max}} + \frac{\hat{n} b(m) w}{e(m)} + \frac{\{\Psi(m-\hat{n}) + \beta(m-\hat{n})^2\} w}{e^{\max}} + \frac{n^* b(m) w^*}{e^*(m)}. \qquad (3.36)$$

The sum of the first term and the second term on the RHS of (3.36) is the cumulative unit total cost from the first stage to the $\hat{n}$ th stage. The sum of the third term and the fourth term on the RHS of (3.36) is the cumulative unit total cost from the $(\hat{n}+1^*)$ th stage to the $(\hat{n}+n^*)$ th stage.

3.2.2.3 National Fragmentation versus International Fragmentation

The Northern firm changes its mode of production from national fragmentation to international fragmentation if the unit total cost under international fragmentation is lower than the unit total cost under national fragmentation.

To compare these unit total costs, I consider two cases. The first case is that the Northern firm locates all $\tilde{\tilde{n}}$ stages in the North. Recall that $\tilde{\tilde{n}}$ is the optimal number of total stages under national fragmentation. In the second case, I consider a deviation from equilibrium of national fragmentation: the firm locates the first $(\tilde{\tilde{n}}-1)$ stages in the North and moves the $\tilde{\tilde{n}}$ th stage to the South.[32]

In the first case, since the $\tilde{\tilde{n}}$ th stage is in the North, the production function for this stage is obtained by (3.1) and (3.12). The production function of $Z_{\tilde{\tilde{n}}}$ is $z_{\tilde{\tilde{n}}} = \min\{z_{\tilde{\tilde{n}}-1}, \frac{e(\tilde{\tilde{n}}) l_{\tilde{\tilde{n}}}}{b(\tilde{\tilde{n}})}\}$. The unit production cost for the $\tilde{\tilde{n}}$ th stage is $\frac{\tilde{\tilde{n}} b(\tilde{\tilde{n}}) w}{e(\tilde{\tilde{n}})}$ from (3.11') and (3.12). The unit headquarters cost for all $\tilde{\tilde{n}}$ stages is $\frac{\Psi w \tilde{\tilde{n}}}{e^{\max}}$ from (3.10').[33] The unit total cost for the $\tilde{\tilde{n}}$ th stage is the sum of these two costs:

[32] This is suboptimal. Exclusively in this section, I consider this suboptimal state to confirm whether there is an incentive to split the stages across the borders. I will address in section 3.2.2.4 how the firm optimally splits the stages into the North and the South.

[33] Recall that $\Psi = \frac{\theta}{N}$.

$$\frac{\Psi w\tilde{\tilde{n}}}{e^{\max}}+\frac{\tilde{\tilde{n}}b(\tilde{\tilde{n}})w}{e(\tilde{\tilde{n}})}. \tag{3.37}$$

Turning to the second case, the firm keeps $(\tilde{\tilde{n}}-1)$ stages in the North and shifts one stage to the South. First, I address the size of the employment pool since this may be affected by the shift of one stage. If the shift takes place, the cost for the final stage $\tilde{\tilde{n}}$ in the North under national fragmentation and the cost for the final stage $\tilde{\tilde{n}}$ in the South under international fragmentation will diverge, while the cost for the first $(\tilde{\tilde{n}}-1)$ stages have remained the same. However, the difference between the two costs is very small compared with the total cost covering all the stages under national fragmentation. The firm would not feel the necessity to adjust the size of the pool in order to internalize the difference between the two costs for the final stage. I assume that the sizes of the pool for national fragmentation and international fragmentation would be the same, l.

The production function for the Southern stage is obtained by replacing $\hat{n}$ in (3.34) with $(\tilde{\tilde{n}}-1)$: $z_{(\tilde{\tilde{n}}-1)+1^*}=\min\{z_{\tilde{\tilde{n}}-1},\frac{e^*(\tilde{\tilde{n}})l^f_{(\tilde{\tilde{n}}-1)+1^*}}{b(\tilde{\tilde{n}})}\}$. One unit of $Z_{(\tilde{\tilde{n}}-1)+1^*}$ is produced with $\{\frac{(\tilde{\tilde{n}}-1)b(\tilde{\tilde{n}})}{e(\tilde{\tilde{n}})}$ units of Northern labor and $\frac{b(\tilde{\tilde{n}})}{e^*(\tilde{\tilde{n}})}$ units of Southern labor}. Using (3.35), the unit production cost for the $(\tilde{\tilde{n}}-1)$th stage in the North is $\frac{(\tilde{\tilde{n}}-1)b(\tilde{\tilde{n}})w}{e(\tilde{\tilde{n}})}$. The unit headquarters cost for the $(\tilde{\tilde{n}}-1)$ stages in the North is $\frac{\Psi(\tilde{\tilde{n}}-1)w}{e^{\max}}$ from (3.29). Using (3.35), the unit production cost for the first stage in the South is $\frac{b(\tilde{\tilde{n}})w^*}{e^*(\tilde{\tilde{n}})}$. Substituting $\{\tilde{\tilde{n}}-(\tilde{\tilde{n}}-1)\}$ for $(m-\hat{n})$ in (3.33),[34] the unit headquarters cost for only the first stage in the South is $\frac{(\Psi+\beta)w}{e^{\max}}$. Then the unit total cost for the $\tilde{\tilde{n}}$th stage is the sum of these four costs:

$$\{\frac{\Psi(\tilde{\tilde{n}}-1)w}{e^{\max}}+\frac{(\Psi+\beta)w}{e^{\max}}\}+\{\frac{(\tilde{\tilde{n}}-1)b(\tilde{\tilde{n}})w}{e(\tilde{\tilde{n}})}+\frac{b(\tilde{\tilde{n}})w^*}{e^*(\tilde{\tilde{n}})}\}. \tag{3.38}$$

In order for the Northern firm to shift the $\tilde{\tilde{n}}$th stage to the South, the cost in (3.38) should be lower than the cost in (3.37): $(3.38)-(3.37)<0$. In both cases, the unit total costs up to the

[34] This means that the number of Southern stages, $\{\tilde{\tilde{n}}-(\tilde{\tilde{n}}-1)\}$, is one.

$(\tilde{\tilde{n}}-1)$ th stage are the same since these stages are located in the North. Thus, the difference between the unit total cost for all the stages $\tilde{\tilde{n}}$ in the first case and that for all the stages $\tilde{\tilde{n}}$ in the second case comes from the difference between the unit total cost for the $\tilde{\tilde{n}}$ th stage in the North and the unit total cost for the $\tilde{\tilde{n}}$ th stage in the South. For simplicity, assuming that $\delta = 1$, the condition for international fragmentation, $(3.38) - (3.37) < 0$, is identical to the following condition:[35]

$$\beta \ < \ \Psi\{1-(\frac{2-\eta^*}{2-\eta})^2\} < 1. \qquad (3.39)$$

I assume that the model satisfies this condition.

3.2.2.4 Optimal Determination of International Fragmentation

The Northern firm determines the optimal number of total stages and split these optimally between the North and the South as a way of incurring the lowest total cost. For the moment, assume that there is a number of total stages across both regions, m, minimizing the unit total cost. For a given m, the firm locates $\hat{n}$ stages in the North and the other $(m-\hat{n})$ stages in the South. The unit total cost function for the final stage m is obtained by replacing n^* in (3.36) with $(m-\hat{n})$.

$$tc^f_{\hat{n}+(m-\hat{n})} = \{\frac{\Psi\hat{n}w}{e^{\max}} + \frac{\hat{n}b(m)w}{e(m)}\} + [\frac{\{\Psi(m-\hat{n})+\beta(m-\hat{n})^2\}w}{e^{\max}} + \frac{(m-\hat{n})b(m)w^*}{e^*(m)}].$$

$$\text{where } \frac{b(m)}{e(m)} = \frac{dm^{-2}}{w}, \ \ d \equiv e^{\max}(2-\eta)^2 l^f , \qquad (3.40)$$

$$\frac{b(m)}{e^*(m)} = \frac{d^* m^{-2}}{w^*}, \ \ d^* \equiv e^{\max}(2-\eta^*)^2 l^f , \qquad (3.41)$$

$$d > d^* \text{ since } \eta < \eta^*. \qquad (3.42)$$

This is simplified as

$$tc^f_m = \frac{w}{e^{\max}}\{\Psi m + \beta(m-\hat{n})^2\} + \{d\hat{n} + d^*(m-\hat{n})\}m^{-2}. \qquad (3.43)$$

For the minimization of this cost with respect to $\hat{n}$ for a given m, the first derivative with respect to $\hat{n}$ is zero and the second derivative is positive.

[35] See Appendix 3.2.

$$\frac{\partial tc_m^f}{\partial \hat{n}} = \frac{-2\beta w(m-\hat{n})}{e^{\max}} + (d-d^*)m^{-2} = 0, \tag{3.44}$$

$$\frac{\partial^2 tc_m^f}{\partial \hat{n}^2} = \frac{2\beta w}{e^{\max}} > 0.$$

The number of Northern stages, $\hat{n}$, is obtained from (3.44). It is a function of m:

$$\hat{n}(m) = m - (d-d^*)(\frac{e^{\max}}{2\beta w})m^{-2}. \tag{3.45}$$

Substituting (3.45) into (3.43),

$$tc_m^f = \frac{w}{e^{\max}}[\Psi m + \beta\{m-\hat{n}(m)\}^2] + [d\hat{n}(m) + d^*\{m-\hat{n}(m)\}]m^{-2}. \tag{3.46}$$

The number of total stages is obtained by minimizing tc_m^f (3.46) with respect to m. For the cost minimization, the first derivative is zero and the second derivative is positive. For a given w, the first derivative is

$$\frac{\partial tc_m^f}{\partial m} = \frac{w\Psi}{e^{\max}} - dm^{-2} + \{\frac{e^{\max}(d-d^*)^2}{\beta w}\}m^{-5} = 0. \tag{3.47}$$ [36]

The second derivative is positive:

$$\frac{\partial^2 tc_m^f}{\partial m^2} = 2dm^{-3} - \{\frac{5e^{\max}(d-d^*)^2}{\beta w}\}m^{-6} > 0, \tag{3.48}$$

where $m > (\frac{5e^{\max}(d-d^*)^2}{2d\beta w})^{\frac{1}{3}}$. [37]

The optimal number of total stages is determined by (3.47). However, for graphical analysis, I rewrite (3.47) as a new expression with A and B that are defined as follows.

$$A - B = 0,$$

$$\text{where } A \equiv \frac{w\Psi}{e^{\max}} + \{\frac{e^{\max}(d-d^*)^2}{\beta w}\}m^{-5}, \quad B \equiv dm^{-2}. \tag{3.49}$$

[36] See Appendix 3.3.

[37] For the second derivative in (3.48) to be positive, $\{2dm^3 - \frac{5e^{\max}(d-d^*)^2}{\beta w}\}\ m^{-6} > 0$. Since the final stage m is a positive number, $m^{-6} > 0$. $\{2dm^3 - \frac{5e^{\max}(d-d^*)^2}{\beta w}\}$ has to be positive. Thus, $m > \{\frac{5e^{\max}(d-d^*)^2}{2d\beta w}\}^{\frac{1}{3}}$. This condition makes the second derivative positive.

The optimal number of total stages $\tilde{m}$ is determined where $A = B$. The shapes of A and B are monotonic in m. The slope of A is flatter than that of B.[38] As m becomes small, the value of A becomes smaller than that of B.[39] As m becomes large, the value of A becomes larger than that of B.[40] Thus A and B are drawn as in Figure 3.8. They intersect at c where $\tilde{m}$ is determined.

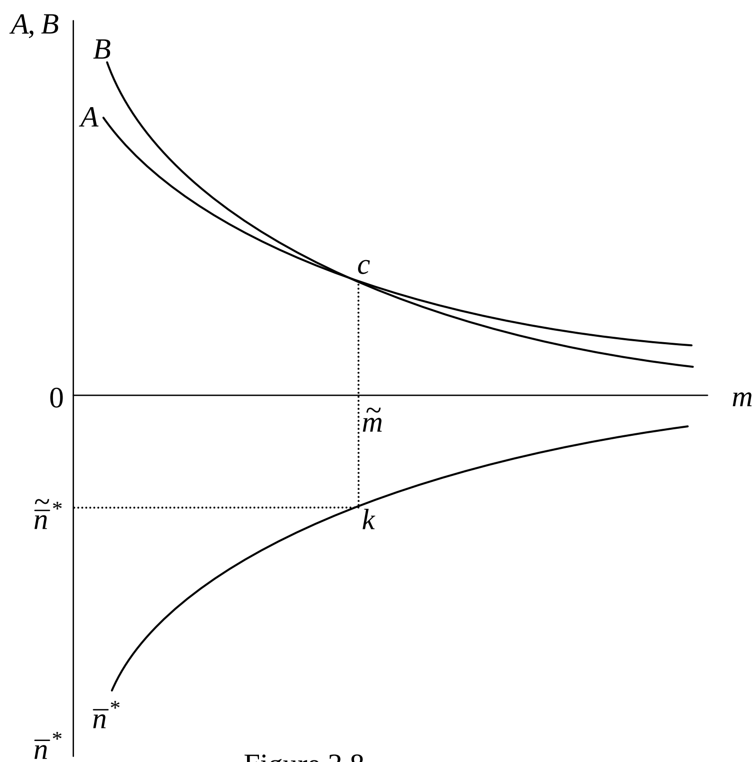

Figure 3.8

The number of the stages located in the South is obtained by using (3.45).

$$\bar{n}^*(m) = m - \hat{n}(m) = (d - d^*)(\frac{e^{\max}}{2\beta w})m^{-2}, \tag{3.50}$$

where $\frac{\partial \bar{n}^*(m)}{\partial m} = \{-\frac{e^{\max}(d - d^*)}{\beta w}\}m^{-3} < 0$, $\frac{\partial^2 \bar{n}^*(m)}{\partial m^2} = \{\frac{3e^{\max}(d - d^*)}{\beta w}\}m^{-4} > 0$.

---

[38] $\frac{\partial A}{\partial m} = \{\frac{-5e^{\max}(d - d^*)^2}{\beta w}\}m^{-6}$. $\frac{\partial B}{\partial m} = -2dm^{-3}$. From $\frac{\partial^2 tc_m^f}{\partial \hat{n}^2} > 0$ in (3.48), $\left|\frac{\partial A}{\partial m}\right| < \left|\frac{\partial B}{\partial m}\right|$.

[39] See Appendix 3.4.

[40] $\lim_{m\to\infty} A = \frac{w\Psi}{e^{\max}}$ and $\lim_{m\to\infty} B = 0$. Then $\lim_{m\to\infty} A > \lim_{m\to\infty} B$.

The curve of $\overline{n}^*(m)$ is drawn in the space $(m, \overline{n}^*)$ below the horizontal axis of Figure 3.8. The vertical axis below the origin represents a positive number of Southern stages $\overline{n}^*$. This number becomes larger moving downward from the origin along this axis. The signs in (3.50) indicate that the curve of $\overline{n}^*(m)$ is convex toward the origin in the space $(m, \overline{n}^*)$. This can be understood as follows. As the total number of stages m increases, the effect of the increasing returns to the labor division increases, and thus the production cost falls; the firm has a smaller incentive for shifting the stages to the South to reduce the production cost; and this means that the number of Southern stages $\overline{n}^*$ decreases with the total number of stages m. The optimal number of Southern stage $\tilde{\overline{n}}^*$ is determined at k where the vertical line extended downward from the point c touches the curve $\overline{n}^*(m)$.

I have to mention the possibility that the headquarters remains in the North and all production stages shift to the South. The possibility of this case is very low in the model, but this case can also be ruled out if η^* satisfies $\eta^* > \frac{\eta}{\sqrt{2}} + (2-\sqrt{2})$.[41]

3.2.2.5 Effect of International Fragmentation on the Total Number of Stages

I address what happens to the total number of stages when the firm changes its mode of production from national fragmentation to international fragmentation. I compare the total number of stages in national fragmentation $\tilde{\overline{n}}$ to that in international fragmentation $\tilde{m}$.

The curves A and B in Figure 3.8 are used for this purpose. First, consider a point on the horizontal axis which is located at $\tilde{m}$. Where m is equal to $\tilde{m}$, the values of A and B are represented by c, and are the same. Thus,

$$\frac{A(m)}{B(m)} = 1, \quad \text{where } m = \tilde{m}. \tag{3.51}$$

Second, consider a point on the left of $\tilde{m}$ on the horizontal axis. Where $0 < m < \tilde{m}$, the value on the curve A corresponding to this point is smaller than the value on the curve B in Figure 3.8. Using (3.51),

$$\frac{A(m)}{B(m)} < \frac{A(\tilde{m})}{B(\tilde{m})} = 1, \quad \text{where } 0 < m < \tilde{m}. \tag{3.52}$$

Third, consider a point on the right of $\tilde{m}$, that is, $m > \tilde{m}$. The value on A corresponding to this point is larger than the value on B in Figure 3.8. By (3.51),

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[41] See Appendix 3.5.

$$\frac{A(m)}{B(m)} > \frac{A(\tilde{m})}{B(\tilde{m})} = 1, \quad \text{where } m > \tilde{m}. \tag{3.53}$$

Using $A(m)$ and $B(m)$ in (3.49), $\frac{A(m)}{B(m)}$ is obtained as follows.

$$\frac{A(m)}{B(m)} = \frac{1}{d}\{\frac{w\Psi m^2}{e^{\max}} + \frac{e^{\max}(d-d^*)^2 m^{-3}}{\beta w}\},^{42} \quad \text{for all } m, \tag{3.54}$$

where $d = e^{\max}(2-\eta)^2 l^f$, $d^* = e^{\max}(2-\eta^*)^2 l^f$.

This ratio is used to compare $\tilde{m}$ and $\tilde{\tilde{n}}$. If m has the same number as the equilibrium number of total stages in national fragmentation $\tilde{\tilde{n}}$, this ratio is expressed as $\frac{A(\tilde{\tilde{n}})}{B(\tilde{\tilde{n}})}$.[43] By applying the criteria in (3.51), (3.52) and (3.53) to $\frac{A(\tilde{\tilde{n}})}{B(\tilde{\tilde{n}})}$, I can compare $\tilde{m}$ and $\tilde{\tilde{n}}$. Recall that $\frac{A(\tilde{m})}{B(\tilde{m})} = 1$ in (3.51). If $\frac{A(\tilde{\tilde{n}})}{B(\tilde{\tilde{n}})}$ is less than or equal to one, this implies that $\tilde{\tilde{n}} \le \tilde{m}$ from (3.51) and (3.52):

$$\frac{A(\tilde{\tilde{n}})}{B(\tilde{\tilde{n}})} \le \frac{A(\tilde{m})}{B(\tilde{m})} = 1 \quad \Rightarrow \quad \tilde{\tilde{n}} \le \tilde{m}. \tag{3.55}$$

However, if $\frac{A(\tilde{\tilde{n}})}{B(\tilde{\tilde{n}})}$ is larger than one, this implies that $\tilde{\tilde{n}} > \tilde{m}$ from (3.53):

$$\frac{A(\tilde{\tilde{n}})}{B(\tilde{\tilde{n}})} > \frac{A(\tilde{m})}{B(\tilde{m})} = 1 \quad \Rightarrow \quad \tilde{\tilde{n}} > \tilde{m}. \tag{3.56}$$

To find the value of $\frac{A(\tilde{\tilde{n}})}{B(\tilde{\tilde{n}})}$, I derive this ratio. After substituting $\tilde{\tilde{n}}$ for m in (3.54), use $\tilde{\tilde{n}}$ in (3.24), $\delta = 1$ and $\varepsilon = 1$.

$$\frac{A(\tilde{\tilde{n}})}{B(\tilde{\tilde{n}})} = \frac{l}{l^f}[1 + \frac{\{(2-\eta)^2 - (2-\eta^*)^2\}^2 \theta^{\frac{3}{2}} w^{\frac{1}{2}}}{e^{\max}\beta(2-\eta)^5 l^{\frac{1}{2}} (\frac{l}{l^f})^2 N^{\frac{3}{2}}}],^{44} \tag{3.57}$$

---

[42] See (A3.6.1) in Appendix 3.6.
[43] The equation in (3.54) is defined for all values of m regardless of whether m represents the number of stages in international fragmentation or that in national fragmentation, though the expressions for $A(m)$ and $B(m)$ are derived from the production mode of international fragmentation. Thus the ratio $\frac{A(m)}{B(m)}$ can be used as a barometer indicating the points where $\tilde{\tilde{n}}$ and $\tilde{m}$ locate on the horizontal axis in Figure 3.8.
[44] See (A3.6.5) in Appendix 3.6.

where $0 < \eta < \eta^* < 1$.

The second term in the square brackets on the RHS in (3.57) is positive. The value in the square brackets is larger than one. Thus, whether the value of $\frac{A(\tilde{\tilde{n}})}{B(\tilde{\tilde{n}})}$ is larger than one or not depends on the size of the employment pool in national fragmentation relative to that in international fragmentation, $\frac{l}{l^f}$. The following cases are considered.

(i) case: $l^f \leq l$

If the size of the employment pool with international fragmentation is less than or equal to that with national fragmentation, the ratio of employment is greater than or equal to one: $\frac{l}{l^f} \geq 1$. The RHS in (3.57) is the product of the ratio $\frac{l}{l^f}$ and the value in the square brackets. The product is larger than one, so that the LHS is larger than one: $\frac{A(\tilde{\tilde{n}})}{B(\tilde{\tilde{n}})} > 1$. This occurs, as explained in (3.56), where the total number of stages in national fragmentation is larger than that in international fragmentation: $\tilde{\tilde{n}} > \tilde{m}$.

This can be explained as follows. The internationally fragmented firm faces a low Southern wage as well as a high Northern wage, but the nationally fragmented firm faces only a high Northern wage. The number of workers hired by the internationally fragmented firm is smaller than or equal to that hired by the nationally fragmented firm. Thus the former firm faces a lower production cost than the latter firm.[45] The internationally fragmented firm feels less pressure to reduce the production cost compared with the nationally fragmented firm. The former firm has a smaller incentive to increase the total number of stages so as to raise their productivity than the latter firm. This says that the total number of stages should be smaller in international fragmentation than in national fragmentation.

[45] The total cost – the sum of the headquarters cost and the production cost – is affected by the network size, loyalty, border barrier, size of employment pool and wage. The degree of segmentation of the production process is determined by the minimization of the total cost. These factors are reflected on the RHS in (3.57). Since the value in the square brackets on the RHS is larger than one, whether or not $\frac{A(\tilde{\tilde{n}})}{B(\tilde{\tilde{n}})}$ is larger than one is determined by the ratio of the numbers of workers hired by the two firms. That is, the second term in the square brackets, which includes the factors mentioned, does not play a key role in determining whether the value of the ratio in (3.57) is larger than one. Therefore, I focus on the wage cost of production.

(ii) case: $l^f > l$

If the size of the employment pool is larger with international fragmentation than with national fragmentation, the ratio of employment is smaller than one: $\frac{l}{l^f} < 1$. The RHS in (3.57) is the product of the ratio $\frac{l}{l^f}$ and the value in the square brackets, where the former is smaller than one and the latter is larger than one. According to which one dominates, the value of the product is greater than, equal to, or less than one. Thus, the value of the LHS in (3.57) is $\frac{A(\tilde{\tilde{n}})}{B(\tilde{\tilde{n}})} \lesseqgtr 1$. This implies that $\tilde{\tilde{n}} \lesseqgtr \tilde{m}$ from (3.55) and (3.56).

Intuitively, the internationally fragmented firm in comparison to the nationally fragmented firm faces a lower wage, but hires a larger number of workers. This could cause the production cost with international fragmentation to be larger than, equal to, or smaller than the production cost with national fragmentation. Thus, it is difficult to say which firm has a greater incentive to reduce its production cost. Therefore, the total number of stages with international fragmentation could be larger than, equal to, or smaller than that with national fragmentation.

3.2.3 Comparative Statics

I focus on the internationally fragmented firm which splits its production stages between the North and the South. I will examine how the network, loyalty of employees and the wage affect the total number of stages, the number of outsourced stages and the effort level.

3.2.3.1 Networks

Suppose the network size of the South becomes larger and the network size of the North does not change. The degree of accessibility of the firm to the Southern network rises. This improves efficiency of the headquarters service. Since β reflects the increment in the headquarters cost due to inefficiency of the small Southern network, an increase in the Southern network size brings on a decrease in β.

From (3.49),

$$\frac{\partial A}{\partial \beta} = \frac{-e^{\max}(d-d^*)^2 m^{-5}}{w\beta^2} < 0\,, \quad \frac{\partial B}{\partial \beta} = 0\,.$$

A decrease in β shifts the curve A upward for a given m in Figure 3.9. However, the extent of the shift of A is decreasing in m, so that the new curve is steeper. The curve B does not change. The point c moves to c' along the curve B. The total number of stages m falls.

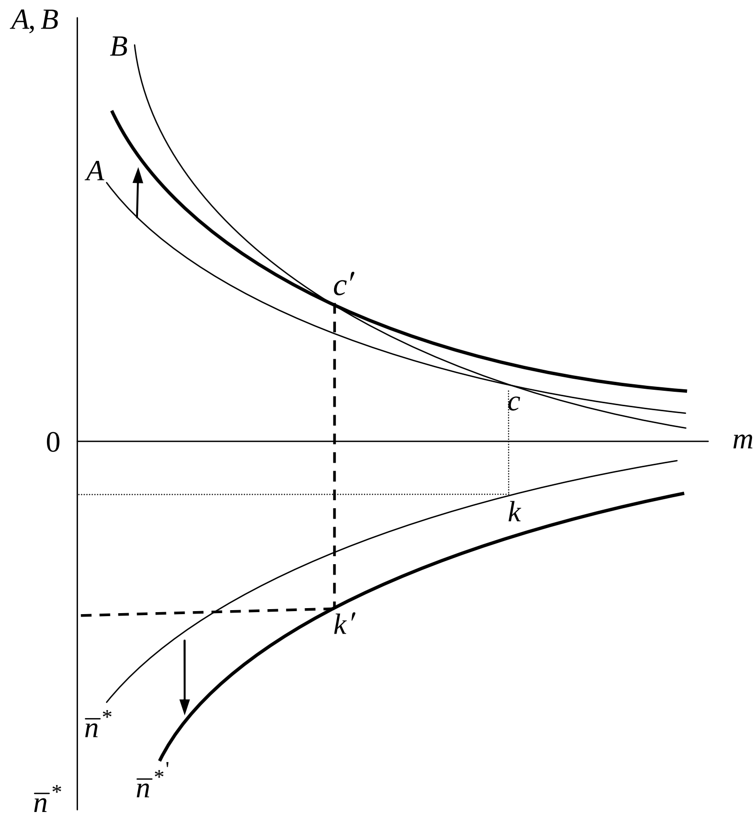

Figure 3.9

Using (3.50),

$$\frac{\partial \bar{n}^*}{\partial \beta} = \frac{-e^{\max}(d - d^*)m^{-2}}{2w\beta^2} < 0 .$$

An increase in the Southern network size (i.e., a decrease in β) leads to an increase in the number of Southern stages. This can be illustrated graphically by a shift of the curve $\bar{n}^*$ in Figure 3.9. Imagine that the curve $\bar{n}^*$ shifts downward (i.e., in a positive direction on the $(m, \bar{n}^*)$ axes) to the curve $\bar{n}^{*'}$. The number of Southern stages increases for a given m. Therefore, the fact that a decrease in β increases the number of Southern stages is represented by the downward shift of the curve $\bar{n}^*$. The extent of the shift of $\bar{n}^*$ is decreasing in m, so that the new curve becomes steeper. The point k, prior to the shift, becomes k' after the shift. The number of Southern stages $\bar{n}^*$ increases.

Intuitively, when the Southern network size increases, the firm connected to this network can communicate information faster and easily with the Southern stages. This makes its headquarters efficiently manage these stages, thus the headquarters cost for these stages decreases. The firm increases the number of Southern stages. On the other hand, the firm decreases the total number of stages. The reason is as follows. The lowered headquarters cost and the increase in the number of Southern stages using low wage labor decrease the firm's total cost; this reduces the necessity for dividing production into more stages for improving the efficiency.

The headquarters supervises all employees in the North and the South with the same monitoring ability, which is measured by the total number of stages in the North and the South. All employees in both countries face the same detection probability. This, along with the decrease in the total number of stages, decreases effort in each country in (3.25) and (3.26) by the same proportion. Thus, the relative effort in (3.27) is not affected.

3.2.3.2 Loyalty of Employees

Suppose that the new competitive environment through globalization makes the employment relationship more unstable in the South compared to that in the North, since the South faces increased international competition. This decreases loyalty in the South compared to the North, thus η^* falls for a given η. To investigate how a decrease in η^* influences the total number of stages and the number of Southern stages, I differentiate A and B in (3.49) with respect to η^*, respectively.

$$\frac{\partial A}{\partial \eta^*} = \{-\frac{2e^{\max}(d-d^*)m^{-5}}{\beta w}\}(\frac{\partial d^*}{\partial \eta^*}) > 0\,, \quad \frac{\partial B}{\partial \eta^*} = m^{-2}(\frac{\partial d}{\partial \eta^*}) = 0\,,$$

$$\text{where } \frac{\partial d^*}{\partial \eta^*} = -2e^{\max}(2-\eta^*)l^f < 0 \text{ from (3.41)}, \qquad (3.58)$$

$$\frac{\partial d}{\partial \eta^*} = 0 \text{ from (3.40).}$$

This says that only the curve A shifts downward as η^* falls for a given m as in Figure 3.10. Since the extent of the shift of A is decreasing in m, the new curve becomes less steep. The curve B does not change. The point c moves to c' along the curve B. The total number of stages m increases.

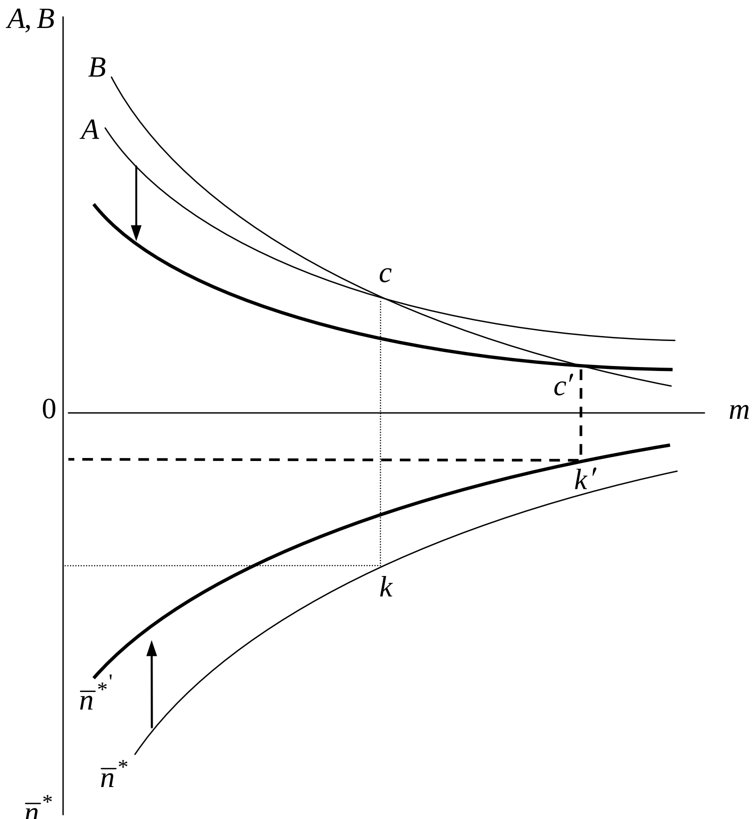

Figure 3.10

From (3.50),

$$\frac{\partial \bar{n}^*}{\partial \eta^*} = (-\frac{e^{\max} m^{-2}}{2\beta w})(\frac{\partial d^*}{\partial \eta^*}) > 0\,, \quad \text{where } \frac{\partial d^*}{\partial \eta^*} < 0 \text{ in (3.58).}$$

A fall in Southern loyalty leads to a decrease in the number of Southern stages. This can be illustrated by a shift of the curve $\bar{n}^*$ in Figure 3.10. If the curve $\bar{n}^*$ shifts upward (i.e., in a negative direction on the $(m, \bar{n}^*)$ axes) to the curve $\bar{n}^{*'}$, the number of Southern stages decreases for a given m. Therefore, the fact that a fall in η^* decreases the number of Southern stages is represented by the upward shift of the curve $\bar{n}^*$. The extent of the shift of $\bar{n}^*$ is decreasing in m, so that the new curve becomes less steep. The point k shifts to k'. The number of Southern stages $\bar{n}^*$ decreases.

If Southern loyalty falls, its effect on Northern effort is from (3.25):

$$\frac{\partial e(m)}{\partial \eta^*} = \frac{w}{e^{\max}(2-\eta)^2 l^f} \frac{\partial m}{\partial \eta^*} < 0\,, \quad \text{where } \frac{\partial m}{\partial \eta^*} < 0\,. \tag{3.59}$$

The negative signs in (3.59) are explained as follows. The fallen Southern loyalty increases the total number of stages (i.e., $\frac{\partial m}{\partial \eta^*} < 0$); this increases the firm's detection probability and thus Northern effort rises.

The effect on Southern effort of lower Southern loyalty is from (3.26):

$$\frac{\partial e^*(m)}{\partial \eta^*} = \{\frac{2w^* m}{e^{\max}(2-\eta^*)^3 l^f} + \frac{w^*}{e^{\max}(2-\eta^*)^2 l^f}\frac{\partial m}{\partial \eta^*}\} \gtrless 0\,, \quad \text{where } e^{*\max} = e^{\max}\,.$$

The first term in the curly brackets is positive. This says that lowered Southern loyalty leads to a fall in Southern effort, given that m does not change. The second term in the curly brackets is negative; the increased total number of stages increases the detection probability and thus Southern effort. These opposite movements make the effect on Southern effort ambiguous.

I now examine the relationship between Southern loyalty and relative effort. From (3.27),

$$\frac{\partial\{\frac{e^*(m)}{e(m)}\}}{\partial \eta^*} = [\frac{\partial\{\frac{e^*(m)}{e(m)}\}}{\partial(\frac{2-\eta}{2-\eta^*})}]\{\frac{\partial(\frac{2-\eta}{2-\eta^*})}{\partial \eta^*}\} > 0\,, \tag{3.60}$$

$$\text{where } [\frac{\partial\{\frac{e^*(m)}{e(m)}\}}{\partial(\frac{2-\eta}{2-\eta^*})}] = 2(\frac{w^*}{w})(\frac{2-\eta}{2-\eta^*}) > 0\,, \quad [\frac{\partial(\frac{2-\eta}{2-\eta^*})}{\partial \eta^*}] = \frac{(2-\eta)}{(2-\eta^*)^2} > 0\,.$$

The positive sign in (3.60) implies that a fall in η^* causes a fall in $\frac{2-\eta}{2-\eta^*}$ and thus a fall in $\frac{e^*(m)}{e(m)}$ from (3.27). Thus, lowered Southern loyalty reduces Southern effort relative to Northern effort.

These results are explained intuitively. If loyalty in the South falls relative to the North, the South's relative disutility of effort rises. The employees in the South provide less effort relative to the employees in the North. This increases the production cost of outsourced stages. The firm decreases outsourcing to the South and brings back some of the Southern stages to the North. The increase in Northern stages using high wage labor, and the lowered Southern effort, increase the total cost. To improve the efficiency of production, the firm increases the total number of stages. This reduces its total cost.

3.2.3.3 Wages

Suppose that the Southern wage rises for a given Northern wage.[46] The change in the relative wage does not affect the position of the curves A and B, since A and B in (3.49) have no Southern wage term.

$$\frac{\partial A}{\partial w^*} = \frac{\partial B}{\partial w^*} = 0 .$$

The total number of stages m is not changed by the Southern wage. The effect on the number of Southern stages $\overline{n}^*$ is from (3.50):

$$\frac{\partial \overline{n}^*}{\partial w^*} = 0 .$$

The number of Southern stages is not affected by the Southern wage.

If the Southern wage rises, Southern effort increases and Northern effort does not change.

$$\frac{\partial e^*(m)}{\partial w^*} = \{\frac{1}{e^{\max}(2-\eta^*)^2}\frac{m}{l^f}\} > 0 , \quad \frac{\partial e(m)}{\partial w^*} = 0 .$$ [47]

Also, a rise in the wage of the South relative to the North increases the effort of the South relative to the North. From (3.27),

$$\frac{\partial\{\frac{e^*(m)}{e(m)}\}}{\partial(\frac{w^*}{w})} = (\frac{2-\eta}{2-\eta^*})^2 > 0 . \tag{3.61}$$

These results can be understood as follows. The rise in the Southern wage increases Southern employees' effort. This relatively decreases the production cost in the South. At the same time,

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[46] I treat the wages as exogenously given. However, it has also become an important issue how wages are affected by international fragmentation. In the single sector H-O model with skilled and unskilled labor of Deardorff (2005), the services outsourcing of skilled-labor from the developed country (the North) causes the wage of unskilled labor in the North to fall below that in the developing country (the South), if the North continues to diversify. However, the high-skilled and low-skilled labor in the North gains if specialization occurs due to largely different factor endowments. Also, Grossman and Rossi-Hansberg (2008) examine how falling costs of offshoring affect high-skill and low-skill wages in the source country in a H-O model with two sectors. They show that offshoring benefits the factors whose tasks are moved more easily overseas and generates shared gains for all domestic factors.

[47] Using (3.26), $\frac{\partial e^*(m)}{\partial w^*} = \{\frac{1}{e^{*\max}(2-\eta^*)^2}\frac{m}{l^f} + \frac{w^*}{e^{*\max}(2-\eta^*)^2}\frac{1}{l^f}\frac{\partial m}{\partial w^*}\}$. Since $e^{*\max} = e^{\max}$ and $\frac{\partial m}{\partial w^*} = 0$, $\frac{\partial e^*(m)}{\partial w^*} = \{\frac{1}{e^{\max}(2-\eta^*)^2}\frac{m}{l^f}\} > 0$. Using (3.25), $\frac{\partial e(m)}{\partial w^*} = \{\frac{w}{e^{\max}(2-\eta)^2 l^f}\}\frac{\partial m}{\partial w^*}$. Since $\frac{\partial m}{\partial w^*} = 0$, $\frac{\partial e(m)}{\partial w^*} = 0$.

the increased wage raises the production cost in the South. These opposite effects are traded off, and the degree of outsourcing is not changed. Also, the total number of stages is not changed.

I consider another case of a rise in the Northern wage for a given Southern wage. The direction of shift of the curve A depends on the values of parameters, while the curve B does not change:

$$\frac{\partial A}{\partial w} = \frac{\Psi}{e^{\max}} - \frac{e^{\max}(d-d^*)^2 m^{-5}}{\beta w^2} \gtrless 0, \quad \frac{\partial B}{\partial w} = 0.$$

The direction of movement of m is ambiguous, so that the change in the total number of stages is ambiguous. The effect on the number of Southern stages $\bar{n}^*$ is

$$\frac{\partial \bar{n}^*}{\partial w} = \frac{-e^{\max}(d-d^*)}{2\beta m^2 w}\left(\frac{1}{w} + \frac{2}{m}\frac{\partial m}{\partial w}\right) \gtrless 0, \quad \text{where } \frac{\partial m}{\partial w} \lessgtr 0.$$

The number of Southern stages can decrease, increase or not change, since $\bar{n}^*$ depends on the change in m with respect to w.

If the Northern wage rises, the changes in effort in both regions are ambiguous due to the ambiguity of the change in the total number of stages. This is shown by using (3.25) and (3.26).

To examine how the rise in the Northern wage affects Northern effort relative to Southern effort, I use (3.61). The sign in (3.61) indicates that a decrease in the ratio of the Southern wage to the Northern wage $\frac{w^*}{w}$ leads to a decrease in the ratio of Southern effort to Northern effort $\frac{e^*(m)}{e(m)}$. Since a decrease in $\frac{w^*}{w}$ means an increase in $\frac{w}{w^*}$, and since a decrease in $\frac{e^*(m)}{e(m)}$ means an increase in $\frac{e(m)}{e^*(m)}$, an increase in $\frac{w}{w^*}$ leads to an increase in $\frac{e(m)}{e^*(m)}$. In other words, a rise in the Northern wage for a given Southern wage causes a rise in Northern effort relative to Southern effort.

Now I explain intuitively why the changes in the total number of stages and outsourcing with respect to a change in the Northern wage are ambiguous. A rise in the Northern wage increases Northern employees' effort relative to Southern employees' effort. This relatively decreases the production cost in the North. At the same time, the increased wage raises the headquarters cost and the production cost in the North. These work in opposite directions so that the total cost can increase, decrease or not change. Thus, the effect of the increased Northern wage on the total number of stages is ambiguous. Since this ambiguity influences production costs of both Northern stages and Southern stages, the comparison of production costs between the North and the South is also difficult. This causes the effect on outsourcing to be ambiguous.

3.3 Conclusion

This paper addresses how physical networks, loyalty of workers and wages affect international fragmentation by a multinational firm and the work effort of its employees in a world consisting of a developed country – the North – and a developing country – the South.

Splitting a single production process into multiple-stages increases a headquarters' monitoring ability to find where shirking occurs. As the number of stages increases, the probability of shirking being caught rises. This increases the employees' work effort so as not to be penalized by a wage cut. This positive effect gives the firm an incentive to fragment production. However, since the headquarters cost for coordinating and monitoring the stages increases with the number of production stages, this limits the number of total stages.

The international differences in the network sizes, border barriers, loyalty and wages influence both the firm's production cost and headquarters cost, so the differences cause the stages to be separated into the Northern stages and the Southern stages. Also, the Northern firm determines the optimal number of total stages as a way of incurring the lowest total cost.

When the mode of production changes from national fragmentation to international fragmentation, if international fragmentation brings on a decrease in employment, the internationally fragmented firm compared with the nationally fragmented firm faces low wage and decreased employment. This weakens the internationally fragmented firm's incentive to improve efficiency by increasing the total number of stages, thus leading to a decrease in the total number of stages. However, if international fragmentation brings on an increase in employment, this raises the incentive to improve efficiency by increasing the total number of stages. On the contrary, the low wage weakens this incentive. These opposite effects make the effect of international fragmentation on the total number of stages ambiguous.

An increase in the Southern network decreases the headquarters cost for the Southern stages compared to the headquarters cost for the Northern stages. This increases outsourcing from the North to the South. Since the increased Southern network reduces the firm's cost, the necessity for the division of production is diminished. The firm decreases the total number of stages. Since all employees in both regions face a lowered and identical detection probability, this decreases the level of effort in the North and the South, and keeps the effort of the South relative to the North unchanged.

The new competitive environment through globalization makes employment relationships more unstable in the South compared to that in the North, since the South is under more protection that limits free trade. As a result, Southern employees' loyalty falls and their effort relative to the Northern employees' effort decreases. This increases the production cost in the South and reduces outsourcing. However, the firm increases the total number of stages to improve productivity.

If the Southern wage rises, Southern effort increases, but Northern effort does not change. The increasing effort in the South decreases the production cost in the South. However, a rise in the Southern wage could also increase the production cost in the South. These opposite effects on the production cost are traded off, so outsourcing does not change. Also, the total number of stages is not changed.

If the Northern wage rises, the headquarters cost and the production cost in the North increase. At the same time, the higher Northern wage increases Northern effort relative to Southern effort. This decreases the production cost in the North. The effect of the risen wage and the effect of the risen effort work in opposite directions, so outsourcing is determined by which of these effects dominates. The effect of the higher Northern wage on outsourcing is ambiguous.

Appendices

Appendix 3.1

The maximal expected utility is set in either of the cases (ii) and (iii). This is obtained by solving the constrained maximization problem, which maximizes EU subject to a single constraint such that $\varphi(e) = e \le e^{\max}$. To solve this problem, it should first be checked whether the constraint qualification is satisfied. The condition of constraint qualification is as follows. If the expected utility is maximized at an effort level $\bar{e}$ such that $\bar{e} \le e^{\max}$, and if the constraint is binding at $\bar{e}$ (i.e., $\varphi(\bar{e}) = e^{\max}$), then $\frac{\partial \varphi(e)}{\partial e}$ should not be zero at $\bar{e}$ on the boundary of the constraint: $\frac{\partial \varphi(\bar{e})}{\partial e} \ne 0$. When I turn to the maximization problem at hand, the constraint has the feature that $\frac{\partial \varphi(e)}{\partial e} = 1$ for all e. This means that $\frac{\partial \varphi(e)}{\partial e} \ne 0$ at a value of e on the boundary, so the constraint, $\varphi(e) = e \le e^{\max}$, satisfies the constraint qualification. Then we are able to form the Lagrangean function and to calculate the optimal effort.

$$L(e,\ \lambda) = EU - \lambda(e - e^{\max}).$$

The first order conditions are

$$\frac{\partial L}{\partial e} = \frac{\partial EU}{\partial e} - \lambda = 0, \tag{A3.1.1}$$

$$\lambda(e - e^{\max}) = 0, \tag{A3.1.2}$$

$$\lambda \ge 0, \tag{A3.1.3}$$

$$e \le e^{\max}. \tag{A3.1.4}$$

If $\lambda > 0$ in (A3.1.3), $\frac{\partial EU}{\partial e} > 0$ from (A3.1.1) and $(e - e^{\max}) = 0$ from (A3.1.2). This means that $e = e^{\max}$. However, if $\lambda = 0$ in (A3.1.3), $\frac{\partial EU}{\partial e} = 0$ from (A3.1.1). Then, $\frac{\partial EU}{\partial e}$ in (3.16) should be $Pe^{\frac{-1}{2}}(\frac{w}{e^{\max}})^{\frac{1}{2}} - (2-\eta) = 0$ and the effort level is determined at $e = \frac{wP^2}{e^{\max}(2-\eta)^2}$.

The two effort levels, $e^{\max}$ and $e = \frac{wP^2}{e^{\max}(2-\eta)^2}$, are candidates that can be considered in the range of $e \le e^{\max}$ in (A3.1.4). The first candidate represents case (iii) in Figure 3.3. The second candidate represents an interior solution, that is, the case (ii) in Figure 3.3 and it should be less than $e^{\max}$ by (A3.1.4): $e = \dfrac{wP^2}{e^{\max}(2-\eta)^2} < e^{\max}$.

Compare the levels of expected utility at these candidates, $e = \dfrac{wP^2}{e^{\max}(2-\eta)^2}$ and $e = e^{\max}$.

First, substituting $e = \dfrac{wP^2}{e^{\max}(2-\eta)^2}$ into the expected utility function in (3.15'),

$$EU = P\{\frac{2wP}{e^{\max}(2-\eta)} - \frac{wP^2}{e^{\max}(2-\eta)}\} + (1-P)\{2\sqrt{w} - \frac{wP^2}{e^{\max}(2-\eta)}\}$$

$$= \{\frac{wP^2}{e^{\max}(2-\eta)} + 2(1-P)\sqrt{w}\} > 0, \quad \text{where } 1 \le (2-\eta) < 2 \text{ and } 0 < (1-P) < 1. \qquad \text{(A3.1.5)}$$

Second, substituting $e = e^{\max}$ into (3.15'),

$$EU = P\{2\sqrt{w} - e^{\max}(2-\eta)\} + (1-P)\{2\sqrt{w} - e^{\max}(2-\eta)\} = 2\sqrt{w} - e^{\max}(2-\eta) \lesseqgtr 0. \qquad \text{(A3.1.6)}$$

If (A3.1.6) is negative or zero, employees would provide effort $e = \dfrac{wP^2}{e^{\max}(2-\eta)^2}$ since (A3.1.5) is positive. However, if (A3.1.6) is positive, the difference between (A3.1.5) and (A3.1.6) is

$$\{\frac{w}{e^{\max}}(\frac{P^2}{2-\eta}) + 2(1-P)\sqrt{w}\} - \{2\sqrt{w} - e^{\max}(2-\eta)\}$$

$$= \{\frac{wP^2}{e^{\max}(2-\eta)}\} - 2P\sqrt{w} + \{e^{\max}(2-\eta)\}$$

$$= \frac{1}{e^{\max}(2-\eta)}[wP^2 - 2P\sqrt{w}\{e^{\max}(2-\eta)\} + \{e^{\max}(2-\eta)\}^2]$$

$$= \frac{1}{e^{\max}(2-\eta)}[P\sqrt{w} - \{e^{\max}(2-\eta)\}]^2 > 0.$$

The difference is positive. This says that (A3.1.5) is larger than (A3.1.6). Thus employees choose (A3.1.5) that yields the higher expected utility. They provide effort $e = \dfrac{wP^2}{e^{\max}(2-\eta)^2}$.

Regardless of whether (A3.1.6) is negative, zero or positive, (A3.1.5) is always larger than (A3.1.6). Thus, case (ii) has higher expected utility than case (iii).

Appendix 3.2

The condition that international fragmentation occurs is that $(3.38) - (3.37) < 0$:

$$[\{\frac{\Psi w(\tilde{\tilde{n}}-1)}{e^{\max}}+\frac{(\Psi+\beta)w}{e^{\max}}\}+\{\frac{(\tilde{\tilde{n}}-1)b(\tilde{\tilde{n}})w}{e(\tilde{\tilde{n}})}+\frac{b(\tilde{\tilde{n}})w^*}{e^*(\tilde{\tilde{n}})}\}]-\{\frac{\Psi w\tilde{\tilde{n}}}{e^{\max}}+\frac{\tilde{\tilde{n}}b(\tilde{\tilde{n}})w}{e(\tilde{\tilde{n}})}\}<0\,.$$

This is rearranged as

$$\frac{\beta w}{e^{\max}}+b(\tilde{\tilde{n}})\{\frac{-w}{e(\tilde{\tilde{n}})}+\frac{w^*}{e^*(\tilde{\tilde{n}})}\}<0\,. \tag{A3.2.1}$$

Using (3.19), where $\bar{n}=\tilde{\tilde{n}}$,

$$\frac{w}{e(\tilde{\tilde{n}})}=\frac{e^{\max}(2-\eta)^2 l}{\tilde{\tilde{n}}}\,. \tag{A3.2.2}$$

Recall that the sizes of the pool for national fragmentation and international fragmentation are assumed to be the same, l. Using (A3.2.2),

$$\frac{w^*}{e^*(\tilde{\tilde{n}})}=\frac{e^{\max}(2-\eta^*)^2 l}{\tilde{\tilde{n}}}\,. \tag{A3.2.3}$$

For simplicity, I assume that $\delta=1$. From (3.3),

$$b(\tilde{\tilde{n}})=\frac{1}{\tilde{\tilde{n}}}\,. \tag{A3.2.4}$$

Substituting (A3.2.2), (A3.2.3) and (A3.2.4) into (A3.2.1),

$$\frac{\beta w}{e^{\max}}+\frac{e^{\max}l}{\tilde{\tilde{n}}^2}\{-(2-\eta)^2+(2-\eta^*)^2\}<0\,. \tag{A3.2.5}$$

From (3.24), where $\delta=1$, $\varepsilon=1$ and $\Psi=\frac{\theta}{N}$, $\tilde{\tilde{n}}=[\frac{\{e^{\max}(2-\eta)\}^2 l}{\Psi w}]^{\frac{1}{2}}$.

Substituting $\tilde{\tilde{n}}$ into (A3.2.5),

$$\frac{w}{e^{\max}}[\beta+\Psi\{-1+(\frac{2-\eta^*}{2-\eta})^2\}]<0\,.$$

Since $\frac{w}{e^{\max}}>0$, $[\beta+\Psi\{-1+(\frac{2-\eta^*}{2-\eta})^2\}]<0$.

This means $\beta<\Psi\{1-(\frac{2-\eta^*}{2-\eta})^2\}$. Since $\Psi=\frac{\theta}{N}<1$ in (3.9) and $0<(\frac{2-\eta^*}{2-\eta})<1$ from (3.27),

$$\beta\ <\ \Psi\{1-(\frac{2-\eta^*}{2-\eta})^2\}\ <\ 1.$$

Appendix 3.3

The first derivative of tc_m^f in (3.46) with respect to m is

$$\frac{\partial tc_m^f}{\partial m} = \frac{w}{e^{\max}}[\Psi + 2\beta\{m - \hat{n}(m)\}\{1 - \frac{\partial \hat{n}(m)}{\partial m}\}] + [d\{\frac{\partial \hat{n}(m)}{\partial m}\} + d^*\{1 - \frac{\partial \hat{n}(m)}{\partial m}\}]m^{-2}$$

$$-2[d\hat{n}(m) + d^*\{m - \hat{n}(m)\}]m^{-3} = 0\,.$$

This is rearranged as

$$\frac{w}{e^{\max}}[\Psi + 2\beta\{m - \hat{n}(m)\}\{1 - \frac{\partial \hat{n}(m)}{\partial m}\}] + \{(d - d^*)\frac{\partial \hat{n}(m)}{\partial m} + d^*\}m^{-2} - 2\{(d - d^*)\hat{n}(m) + d^* m\}m^{-3} = 0.$$

(A3.3.1)

Rearranging (3.45),

$$\{m - \hat{n}(m)\} = (d - d^*)(\frac{e^{\max}}{2\beta w})m^{-2}\,. \qquad \text{(A3.3.2)}$$

The derivative of $\hat{n}(m)$ in (3.45) with respect to m is

$$\frac{\partial \hat{n}(m)}{\partial m} = 1 + (d - d^*)(\frac{e^{\max}}{\beta w})m^{-3}\,. \qquad \text{(A3.3.3)}$$

Substituting (3.45), (A3.3.2) and (A3.3.3) into (A3.3.1),

$$\frac{\partial tc_m^f}{\partial m} = \frac{w}{e^{\max}}[\Psi + 2\beta\{(d - d^*)(\frac{e^{\max}}{2\beta w})m^{-2}\}\{-(d - d^*)(\frac{e^{\max}}{\beta w})m^{-3}\}]$$

$$+[(d - d^*)\{1 + (d - d^*)(\frac{e^{\max}}{\beta w})m^{-3}\} + d^*]m^{-2}$$

$$-2[(d - d^*)\{m - (d - d^*)(\frac{e^{\max}}{2\beta w})m^{-2}\} + d^* m]m^{-3} = 0\,. \qquad \text{(A3.3.4)}$$

(A3.3.4) is rearranged as

$$\frac{w}{e^{\max}}\{\Psi - \beta(d - d^*)^2(\frac{e^{\max}}{\beta w})^2 m^{-5}\} + \{d + (d - d^*)^2(\frac{e^{\max}}{\beta w})m^{-3}\}m^{-2} - 2\{dm - (d - d^*)^2(\frac{e^{\max}}{2\beta w})m^{-2}\}m^{-3} = 0.$$

$$= \{\frac{w\Psi}{e^{\max}} - (d - d^*)^2(\frac{e^{\max}}{\beta w})m^{-5}\} + \{dm^{-2} + (d - d^*)^2(\frac{e^{\max}}{\beta w})m^{-5}\} - 2\{dm^{-2} - (d - d^*)^2(\frac{e^{\max}}{2\beta w})m^{-5}\} = 0.$$

Then

$$\frac{\partial tc_m^f}{\partial m} = \frac{w\Psi}{e^{\max}} - dm^{-2} + \{\frac{e^{\max}(d - d^*)^2}{\beta w}\}m^{-5} = 0\,.$$

Appendix 3.4

I need to prove that $(B-A)>0$ for a small value of m. Rearrange the second derivative in (3.48),

$$\{2dm^{-3}-5(d-d^{*})^{2}(\frac{e^{\max}}{\beta w})m^{-6}\}>0.$$

Factoring out,

$$\frac{m^{-1}}{2}\{dm^{-2}-(2.5)(d-d^{*})^{2}(\frac{e^{\max}}{\beta w})m^{-5}\}>0.$$

Since $\frac{m^{-1}}{2}>0$, $\{dm^{-2}-(2.5)(d-d^{*})^{2}(\frac{e^{\max}}{\beta w})m^{-5}\}>0$.

Rewriting this,

$$\{dm^{-2}-(d-d^{*})^{2}(\frac{e^{\max}}{\beta w})m^{-5}-(1.5)(d-d^{*})^{2}(\frac{e^{\max}}{\beta w})m^{-5}\}>0. \qquad \text{(A3.4.1)}$$

From (3.49), $(B-A)=dm^{-2}-\{\frac{w\Psi}{e^{\max}}+(d-d^{*})^{2}(\frac{e^{\max}}{\beta w})m^{-5}\}$.

Substituting this equation into (A3.4.1),

$$[\{(B-A)+\frac{w\Psi}{e^{\max}}\}-(1.5)(d-d^{*})^{2}(\frac{e^{\max}}{\beta w})m^{-5}]>0.$$

Rearranging this,

$$(B-A)\ >\ -\frac{w\Psi}{e^{\max}}+(1.5)(d-d^{*})^{2}(\frac{e^{\max}}{\beta w})m^{-5}. \qquad \text{(A3.4.2)}$$

We want to know the sign of $(B-A)$ in the range left of $\tilde{m}$ on the horizontal axis in Figure 3.8. This sign is determined when the value on the RHS of (A3.4.2) is obtained. Thus, I calculate this value where m has a value smaller than $\tilde{m}$, such that $m=1$ as an example. That is

$$-\frac{w\Psi}{e^{\max}}+(1.5)(d-d^{*})^{2}(\frac{e^{\max}}{\beta w}). \qquad \text{(A3.4.3)}$$

Using the notations for d and d^{*} in (3.40) and (3.41),

$$(d-d^{*})^{2}=[\{e^{\max}(2-\eta)^{2}l^{f}\}-\{e^{\max}(2-\eta^{*})^{2}l^{f}\}]^{2}=[e^{\max}l^{f}\{(2-\eta)^{2}-(2-\eta^{*})^{2}\}]^{2}$$

$$=[e^{\max}l^{f}\{(\eta^{*}-\eta)(4-\eta-\eta^{*})\}]^{2}. \qquad \text{(A3.4.4)}$$

Substituting (A3.4.4) and $\Psi=\frac{\theta}{N}$ into (A3.4.3), (A3.4.3) is rewritten as

$$\frac{w}{e^{\max}}[-\frac{\theta}{N}+\frac{(1.5)e^{\max 4}\{(\eta^{*}-\eta)(4-\eta^{*}-\eta)\}^{2}l^{f\,2}}{\beta w^{2}}]. \qquad (A3.4.5)$$

I check the magnitudes of the two terms in the square brackets in (A3.4.5). First, consider the numerators. The parameter θ is the input requirement of effective labor per unit of headquarters service. l^f is the total labor input for the production process. Conceptually, θ should be smaller than l^f : $\theta < l^f$. Assume that $e^{\max} \geq 1$. The conditions that $0<\eta\leq 1$, $0<\eta^{*}\leq 1$ and $\eta^{*}>\eta$ make $0<(\eta^{*}-\eta)<1$ and $2<(4-\eta^{*}-\eta)<4$. I assume that $(\eta^{*}-\eta)$ ensures that $(1.5)e^{\max 4}\{(\eta-\eta^{*})(4-\eta^{*}-\eta)\}^{2}l^{f}\geq 1$. Also, since $\theta < l^f$, the relation between the two numerators in the square bracket is

$$\theta < (1.5)e^{\max 4}\{(\eta-\eta^{*})(4-\eta^{*}-\eta)\}^{2}l^{f\,2}.$$

Next, consider the denominators in the square brackets in (A3.4.5). If the Northern network size N is represented by the value of the network infrastructure, N is larger than w^2. Note that w is the wage per worker. The condition in (3.39), $\beta<1$, makes that $\beta w^2 < w^2$. This yields that $N>\beta w^2$. Then the absolute value of the first term in the square brackets in (A3.4.5) is smaller than the value of the second term:

$$\left|-\frac{\theta}{N}\right| < \frac{(1.5)e^{\max 4}\{(\eta^{*}-\eta)(4-\eta^{*}-\eta)\}^{2}l^{f\,2}}{\beta w^{2}}.$$

This makes (A3.4.5) positive and thus (A3.4.3) is positive. This says that $(B-A)>0$ from (A3.4.2). In other words, $B>A$ in the range of $\tilde{m}$.

Appendix 3.5

I have to check whether there is a case that the headquarters remains in the North and all production stages shift to the South at the values of m which are determined by (3.47), in other words, at the values of m that are determined by the intersection of the curves A and B. This case means that there is no production stage in the North: $\hat{n}(m)=0$. Substituting $\hat{n}(m)=0$ into (3.45), m is obtained. When I define it as m',

$$m'=\{\frac{e^{\max}(d-d^{*})}{2\beta w}\}^{\frac{1}{3}}. \qquad (A3.5.1)$$

If this extreme case exists, the condition in (3.47), $\frac{\partial tc_m^f}{\partial m}=0$, should also be satisfied where $m=m'$.

$$\underbrace{\frac{w\Psi}{e^{\max}} - dm'^{-2} + \{\frac{e^{\max}(d-d^*)^2}{\beta w}\}m'^{-5}}_{H} = 0\,.$$

If H is not zero, this extreme case does not exist. If H is zero, this extreme case can exist. To check whether H is zero, rewrite H as follows.

$$\frac{1}{m'^5}[\frac{w\Psi}{e^{\max}}m'^5 - dm'^3 + \{\frac{e^{\max}(d-d^*)^2}{\beta w}\}]\,. \tag{A3.5.2}$$

Since $m'^5 > 0$, I should check whether the value in the square brackets is zero. Using (A3.5.1), the expression in the square brackets of (A3.5.2) is rewritten:

$$[\frac{w\Psi}{e^{\max}}\{\frac{e^{\max}(d-d^*)}{2\beta w}\}^{\frac{2}{3}} + (d-2d^*)]\{\frac{e^{\max}(d-d^*)}{2\beta w}\}\,. \tag{A3.5.3}$$

Since $d > d^*$ in (3.42), $\frac{w\Psi}{e^{\max}}\{\frac{e^{\max}(d-d^*)}{2\beta w}\}^{\frac{2}{3}} > 0$ and $\{\frac{e^{\max}(d-d^*)}{2\beta w}\} > 0$. Then, from (A3.5.3), three cases are considered.

(i) If $d-2d^* \geq 0$, (A3.5.3) is non zero.

(ii) If $d-2d^* < 0$ and if $\frac{w\Psi}{e^{\max}}\{\frac{e^{\max}(d-d^*)}{2\beta w}\}^{\frac{2}{3}} \neq \left|d-2d^*\right|$, (A3.5.3) is non zero.

(iii) If $d-2d^* < 0$ and if $\frac{w\Psi}{e^{\max}}\{\frac{e^{\max}(d-d^*)}{2\beta w}\}^{\frac{2}{3}} = \left|d-2d^*\right|$, (A3.5.3) is zero.

The extreme case exists if case (iii) occurs.

I now explain the likelihood that case (iii) occurs, compared with the cases (i) and (ii). The case (iii) has more restrictive conditions than the case (ii). The reason is that the situations satisfying the second condition in (iii), $\frac{w\Psi}{e^{\max}}\{\frac{e^{\max}(d-d^*)}{2\beta w}\}^{\frac{2}{3}} = \left|d-2d^*\right|$, occur less frequently than the situations satisfying the second condition in (ii). Also, case (iii) has more restrictive conditions than case (i). Then the likelihood that case (iii) can occur is lower than the cases (i) and (ii). In fact, since the second condition in (iii) consists of many parameters, it is very hard to find a combination of the values of the parameters that satisfy the second condition in (iii). Thus, the likelihood of case (iii) is very low.

Another way to avoid the extreme case is to assume that $d-2d^* \geq 0$, like the condition in (i). This condition makes that $\eta^* > \frac{\eta}{\sqrt{2}} + (2-\sqrt{2})$, when $d = e^{\max}(2-\eta)^2 l^f$ in (3.40) and $d^* = e^{\max}(2-\eta^*)^2 l^f$ in (3.41) are substituted into $d-2d^* \geq 0$. Also this inequality ensures that $0<\eta<1$ and $0<\eta^*<1$.

Appendix 3.6

Using $A(m)$ and $B(m)$ in (3.49),

$$\frac{A(m)}{B(m)} = \frac{\frac{w\Psi}{e^{\max}} + \{\frac{e^{\max}(d-d^*)^2}{\beta w}\}m^{-5}}{dm^{-2}}$$

$$= \frac{1}{d}\{\frac{w\Psi m^2}{e^{\max}} + \frac{e^{\max}(d-d^*)^2 m^{-3}}{\beta w}\}, \quad \text{(A3.6.1)}$$

where $d = e^{\max}(2-\eta)^2 l^f$, $d^* = e^{\max}(2-\eta^*)^2 l^f$

The ratio where $m = \tilde{\tilde{n}}$, $\frac{A(\tilde{\tilde{n}})}{B(\tilde{\tilde{n}})}$, is obtained as follows. Substituting $\tilde{\tilde{n}}$ for m in (A3.6.1),

$$\frac{A(\tilde{\tilde{n}})}{B(\tilde{\tilde{n}})} = \frac{1}{d}\{\frac{w\Psi \tilde{\tilde{n}}^2}{e^{\max}} + \frac{e^{\max}(d-d^*)^2 \tilde{\tilde{n}}^{-3}}{\beta w}\}. \quad \text{(A3.6.2)}$$

Using $\tilde{\tilde{n}}$ in (3.24), $\delta = 1$ and $\varepsilon = 1$,

$$\tilde{\tilde{n}} = [\frac{\{e^{\max}(2-\eta)\}^2 lN}{\theta w}]^{\frac{1}{2}}. \quad \text{(A3.6.3)}$$

From the definitions of d and d^*,

$$(d-d^*) = e^{\max} l^f \{(2-\eta)^2 - (2-\eta^*)^2\}. \quad \text{(A3.6.4)}$$

Substituting (A3.6.3) and (A3.6.4) into (A3.6.2),

$$\frac{A(\tilde{\tilde{n}})}{B(\tilde{\tilde{n}})} = \{\frac{1}{e^{\max}(2-\eta)^2 l^f}\}[\frac{\Psi e^{\max}(2-\eta)^2 lN}{\theta} + \frac{{l^f}^2\{(2-\eta)^2-(2-\eta^*)^2\}^2 \theta^{\frac{3}{2}} w^{\frac{1}{2}}}{\beta (lN)^{\frac{3}{2}}(2-\eta)^3}]$$

$$= \frac{l}{l^f}[1 + \frac{\{(2-\eta)^2-(2-\eta^*)^2\}^2 \theta^{\frac{3}{2}} w^{\frac{1}{2}}}{e^{\max}\beta(2-\eta)^5 l^{\frac{1}{2}} (\frac{l}{l^f})^2 N^{\frac{3}{2}}}]. \quad \text{(A3.6.5)}$$

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