Report No. UMICH 014571-17-I

The University of Michigan

Department of Mechanical Engineering

Cavitation and Multiphase Flow Laboratory

# Investigation of Secondary Liquid Phase Droplet Accelerations

written by:

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Supported by: National Science Foundation Grant No. ENG 75-2315
and U-M internal (SEP) funds.

April 1978

## Introduction

Due to less scheduled maintenance and the increased cost of downtime, turbine blade errosion (Fig. 1) [1]* has become an important consideration in the design of low pressure stages of today's steam turbines. Krzyzanowski [7] has estimated turbine blade errosion for a small system (500 kw) can be as costly as $1000/kw over the 40 year life of the system. Such economic considerations have motivated the study of turbine blade errosion.

This investigation deals with the motion of secondary droplets in a steam wake behind a stationary turbine blade. Such secondary droplets result from a thin liquid film which forms on the stationary blade due to the accumulation of primary droplets. The main stream steam flow causes this liquid film to break up at the trailing edge of the blade resulting in the formation of secondary droplets. As these droplets leave the trailing edge they are accelerated into the main stream steam wake. This acceleration was observed experimentally and the results are presented. Analytical prediction of droplet acceleration based on drag considerations is also attempted.

Figure 2 depicts the experimental apparatus while Figure 3 [8] shows its operation schematically.

Statistical approach is also attempted using a least squares regression analysis.

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*Numbers in brackets designate References at end of paper.

## Discussion of Results

Experimental secondary droplet accelerations were obtained from the observation of high speed motion pictures of the trailing edge of the stationary blade. Droplet size and position from the trailing edge were recorded for the steam velocities and liquid film flows shown in tables 1 and 2. Once knowing the droplet position and the motion picture film speed of 5000 frames per second, the velocity ( $U_d = \Delta x_d / \Delta t$ ) and the acceleration ( $a_d = \Delta U_d / \Delta t$ ) were easily calculated. This reduction of data (droplet position as a function of time) was accomplished through the use of a digital computer program [6]. Average accelerations for the flow conditions outlined above are also listed in tables 1 and 2 and shown graphically in Figure 4. These average accelerations were based on the arithmetic average of all those droplets observed for each liquid film flow rate.

Analytical prediction of droplet acceleration was based upon the following considerations:

1) the droplet after leaving the trailing edge assumed a spherical shape.
2) the main stream steam was one dimensional steady flow.
3) the main stream steam velocity is much larger than the droplet velocity, $U_d$ .

   Hence, $U_r = U_s - U_d$ (with $U_d \approx 0$)

   $U_r = U_s$
4) the main stream steam density, $\rho_s$ , and the droplet density, $\rho_d$ , remain constant.
5) Drag force has an overwhelming influence on the droplet acceleration. See Figure 5.

From (5) above it follows

$$m_d \frac{dU_d}{dt} = C_D A_d \rho_s \frac{U_r^2}{2} \qquad \sim (1)$$

noting,

$$A_d = \pi d^2/4 \quad \text{and} \quad V_d = \pi d^3/6$$

also, $m_d = \rho_d V_d$

Rearranging and substituting into equation (1) yields the following working equation,

(Droplet acceleration) $$a_d = \frac{du_d}{dt} = \left\{ C_D \frac{\rho_s}{\rho_d} \frac{3}{4d} \right\} U_s^2 \quad \sim (2)$$

where $C_D$ is taken from [4] as

$$C_D = \frac{24}{R_e} \left(1 + 0.17\ R_e^{2/3}\right)$$

This resulted in $C_D$ values that agreed with [5] within 2-20% for the range of Reynolds' numbers encountered. Density of the water droplet was taken to be constant, $\rho_d = 1.0\ gm/cm^3$ ($62.4\ lbm/ft^3$). The density of the steam was also taken to be constant and equal to that of saturated steam at 2.5 psia and $134.4^oF$, namely, $\rho_s = 1.135 \times 10^{-4}\ gm/cm^3$ ($7.086 \times 10^{-3}\ lbm/ft^3$). Analytically calculated droplet accelerations have been calculated for each droplet observed; however, only the average analytical accelerations for each liquid film flow are shown in tables 1 and 2 and Figure 4. Observing the ratio of experimental acceleration to analytical acceleration, as shown in tables 1 and 2, one finds this ratio to be less than 1.0 for the 305, 525 and 1100 steam velocities. This would be expected since the analytical model considers only drag effects and should tend to predict accelerations larger than those observed.

For the 825 and 975 steam velocities one notices this ratio to fluctuate about 1.0. This suggests some phenomenon in addition to simple drag considerations. Perhaps the steam velocity profile just past the trailing edge of the fixed blade is such that vortices are created. These vortices may tend to increase or retard the droplet accelerations depending on the droplets' initial position relative to the center of the vortex.

In any case, caution should be exercised when reviewing the results of the higher steam velocities. For example, one notices the observed accelerations for the 825 ft/sec run to be double those of the 520 ft/sec run. This means that while studying every fifth frame of the high speed motion pictures, a droplet would traverse the width of the frame in one half the time or in one half the number of frames observed. Hence one would obtain nearly one half the number of raw data points. Consequently, the experimentally observed acceleration for these cases is less likely to be as precise a measure of true droplet acceleration. Perhaps a study of every second frame may result in a more accurate droplet acceleration at large steam velocities.

Figure 4 does show the droplet accelerations to be essentially independent of liquid film flow for the lower steam velocities. However, it is apparent that droplet accelerations do increase with increasing steam velocity.

## Statistical Considerations

Due to the nature of the physical phenomenon under consideration and due to the large amount of experimental data collected, certain statistical tools can be used to better understand the experimental results. The first approach was to perform a multivariate regression analysis using the MIDAS [3] package of statistical programs. This multivariate approach is shown schematically in figure 6. Specific results are contained in Appendix A. These results lead to the more meaningful first order regression analyses of experimental droplet acceleration, E.A., on flow rate, Q, and experimental droplet acceleration, E.A., on steam velocity, $U_S$.

Table 3 summarizes the results of a first order regression analysis of E.A. on Q. Here we see an equation of the form

$$E.A. = \mathcal{M}(Q) + b$$

where,

E.A. = experimental droplet acceleration

$\mathcal{M}$ = slope

b = "y" intercept

Table 3 shows the slope $\mathcal{M}$ not to be significantly different than zero slope in ~~the~~ THREE cases out of five. This indicates "flat" or level lines on plots of E.A. vs. Q (see Figure 4). R-SQR or "goodness of fit" parameter bears this out in the same three cases, in that, it says only a very small percentage of the variation in E.A. is explained by a variation in Q. Hence one can conclude, in a statistical sense, that E.A. is not affected by Q. Since the slopes can be considered statistically equal to zero, the intercepts, b, then become some indication of an "average" acceleration for a given steam velocity irrespective of flow rate. The statistically average values shown in table 3 compare favorably with those shown in table 1. S.E. values shown in table 3 are an estimate of standard error or standard deviation of E.A. about the regression line shown in Figure 4. Hence if 2*(S.E.) lines ($2\sigma$ bands) are drawn above and below the respective regression lines, 95.4% of the E.A.'s would lie within the bands.

When considering a first order regression analysis of E.A. on $U_S$ one must consider the relationship shown in equation (2), namely

$$a_d = \left\{ C_D \frac{\rho_s}{\rho_d} \frac{3}{4d} \right\} U_s^2 \quad \sim (2)$$

taking the square root of both sides,

$$\sqrt{a_d} = \left\{ C_D \frac{\rho_s}{\rho_d} \frac{3}{4d} \right\}^{1/2} U_s \quad \sim (3)$$

Equation (3) shows $\sqrt{E.A.}$ to be a linear function of $U_S$, hence a transformation of the experimental acceleration data is required in order to perform the most meaningful first order regression of $\sqrt{E.A.}$ on $U_S$. Table 4 shows resultant E.A. data while table 5 and Figure 7 show results of the regression analysis.

Again, one notices an equation of the form

$\sqrt{E.A.} = M(U_S) + b$

where:

$\sqrt{E.A.}$ = square root of experimental droplet acceleration

$M$ = slope

b = "y" intercept = <u>zero</u>

In this case all the slopes we found to be statistically different than zero. Goodness of fit, R-SQR shows, for the Q = 17.7 case, 76.1% of variation in $\sqrt{E.A.}$ can be explained by the variation in $U_S$, which indicates E.A. to be a strong function of $U_S$ (as is indicated by equation (2)). Intercept, b, equal zero, i.e., E.A. = 0 at $U_S = 0$. Makes sense, no droplets are being accelerated when there is no steam flow. The slope $M$ corresponds to the term in brackets in equation (3), hence substituting the following known quantities at $Q = 35.3 \times 10^{-5}\ ft^3/min$ and $\mho_S = 305\ ft/sec^2$:

$$R_e = 1036$$

$$C_D = \frac{24}{R_e}\left[1 + 0.17\ (R_e)^{.667}\right] = 0.4265$$

$$d = 0.003281\ ft$$

$$\rho_d = 62.4\ lbm/ft^3$$

$$\rho_S = .007086\ lbm/ft^3$$

$$\left\{C_D \frac{\rho_S}{\rho_d} \frac{3}{4d}\right\}^{1/2} = \frac{(0.4265)\ (.007086)\ (3)}{(62.4)\ (4)\ (.00328)} = 0.1052$$

Note 0.1052 compares favorably with the statistically obtained slopes shown in table 5. Also S.E. or standard error is an estimate of the standard deviation about the regression line shown in Figure 7. The two conditions (Q = 10 and 20) are representative of the remaining data.

Conclusions

1. Droplet accelerations are unaffected by liquid film flow for steam velocities of 500 ft/sec or less.

2. The absolute magnitude of droplet acceleration does increase with increasing steam velocity.

3. Equation (2) may be used to predict the magnitude of droplet accelerations for $305 \lesssim U_s \lesssim 1100$ feet per second.

4. First order statistical regression analysis supports all of the above conclusions.

List of Symbols

$U$ - velocity

$\rho$ - density

$M$ - segression slope

$m$ - mass

b - regression intercept

$C_D$ - drag coefficient

A - crossectional area

$V$ - volume

a - acceleration

E.A. - average experimentally observed droplet acceleration

A.A. - average analytical droplet acceleration

d - droplet

$R_e$ - Reynolds number

$\Delta$ - change in, as, $\Delta U$ - change in velocity

Q - liquid film flow rate

t - time

Subscripts

d - droplet

s - steam

r - relative, as in relative velocity $U_r$

Acknowledgements

Special thanks go to:

1) Mr. R. Niedzielski for his diligent work in obtaining raw data by reviewing all the high speed motion pictures.

2) Mr. M. Wegenka for his excellent work in developing a digital computer program to reduce the raw data.

3) Professor F. G. Hammitt and Mr. W. Kim for their guidence and many helpful suggestions.

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D. G. CHRISTIE, G. W. HAYWARD, H. L. LOWE, A. N. McDONALD, AND P. SCULPHER

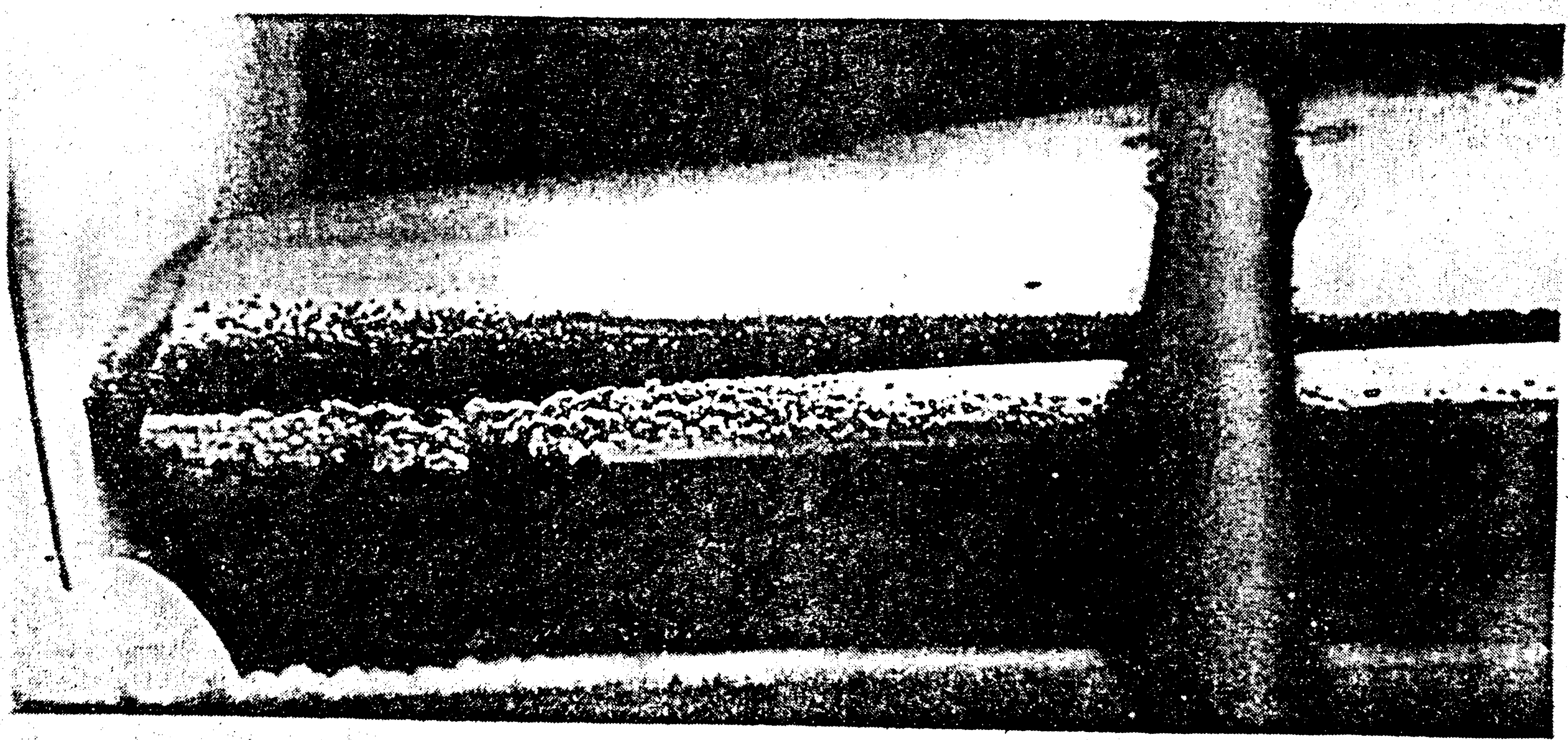

Fig. 4.2. Water erosion damage to blade and shield at inlet edge

FIGURE 1 — EXAMPLE OF WATER EROSION DAMAGE.

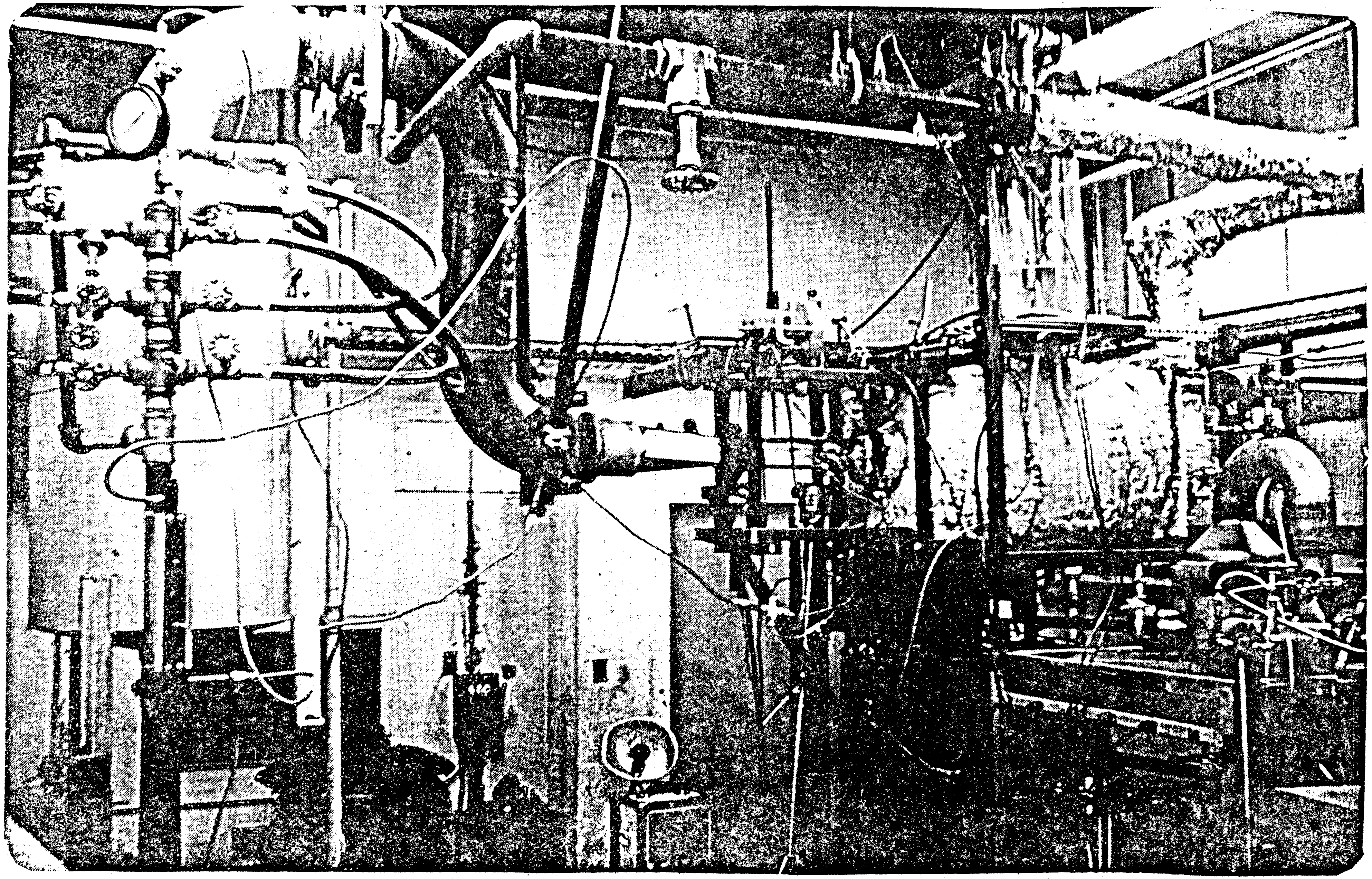

FIGURE 2 – TEST EQUIPMENT, UNIV. OF MICH. STEAM TUNNEL

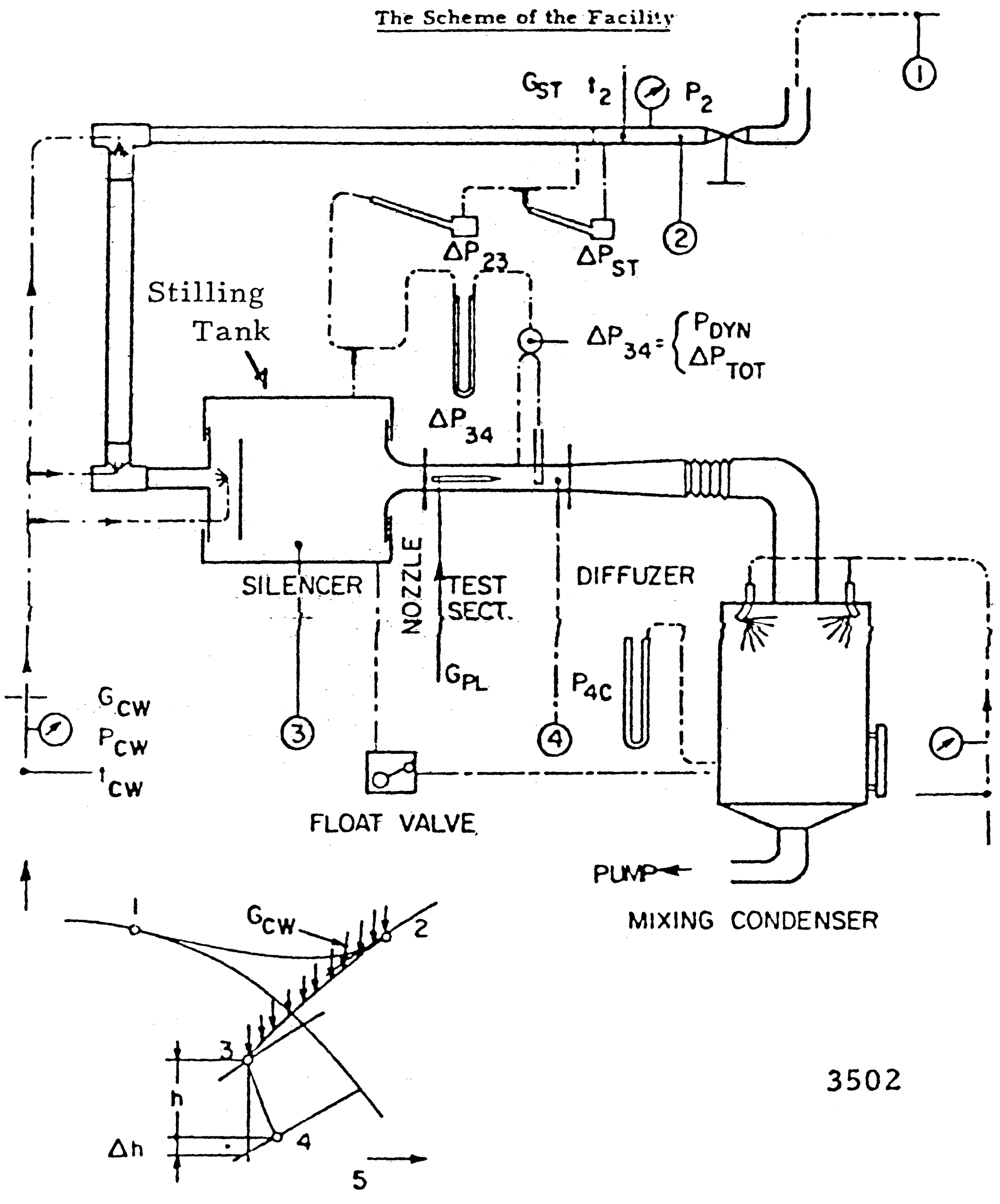


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Figure 3 - Schematic Diagram of the University of Michi
Steam Tunnel

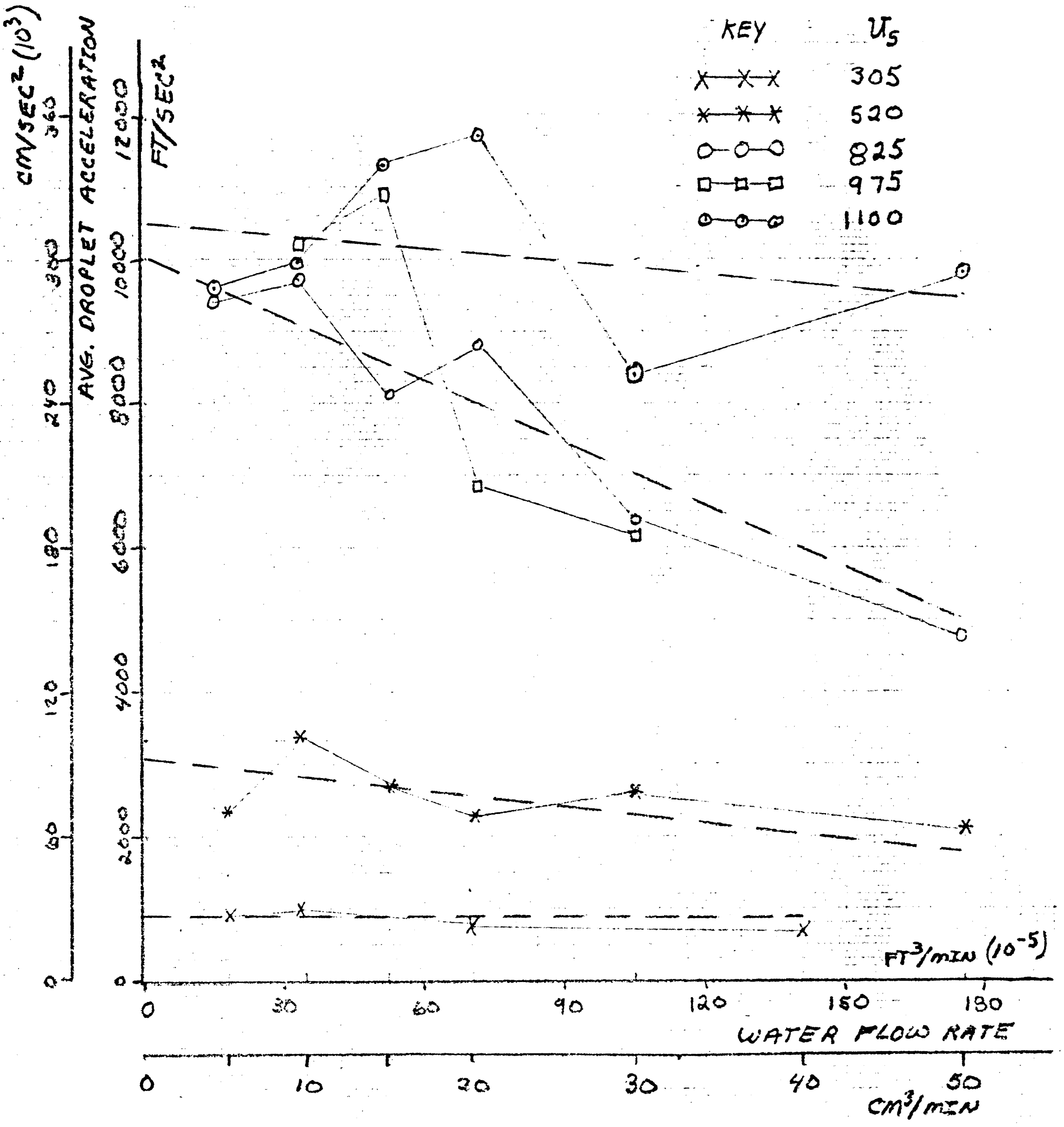
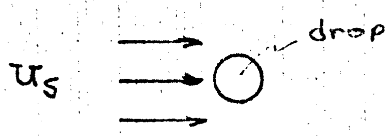



FIG. 4 — "AVERAGE OBSERVED DROPLET ACCELERATION VERSUS WATER FLOW RATE AT VARIOUS STEAM VELOCITIES". DOTTED LINES SHOW REGRESSION ANALYSIS RESULTS. NOTE: SLOPES OF REGRESSION LINES WERE NOT FOUND TO BE SIGNIFICANTLY DIFFERENT THAN ZERO. SEE STATISTICAL ANALYSIS.

FIRST ORDER ANALYTICAL MODEL

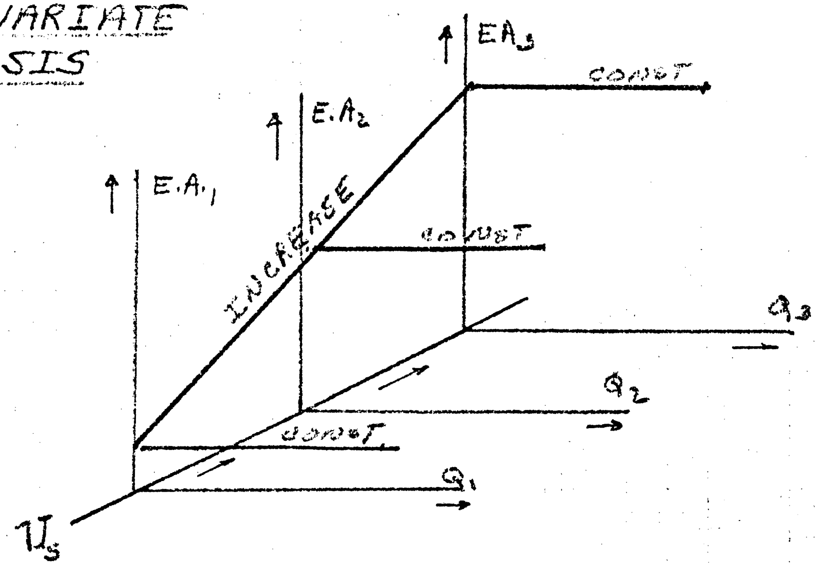



$$F = ma$$

$$a_d = \left\{ C_D \frac{\rho_s}{\rho_d} \frac{3}{4d} \right\} U_s^2$$

FIGURE 5 — SCHEMATIC OF ANALYTICAL MODEL

MULTIVARIATE ANALYSIS




REGRESSION EQA. FORM

$$E.A. = \alpha_1 (Q) + \beta_1 (U_s) + e_i$$

$\alpha_1 (Q)$ → WEAK; $\beta_1 (U_s)$ → STRONG; $e_i$ → STD. ERROR

FIGURE 6 — SCHEMATIC OF MULTIVARIATE REGRESSION ANALYSIS

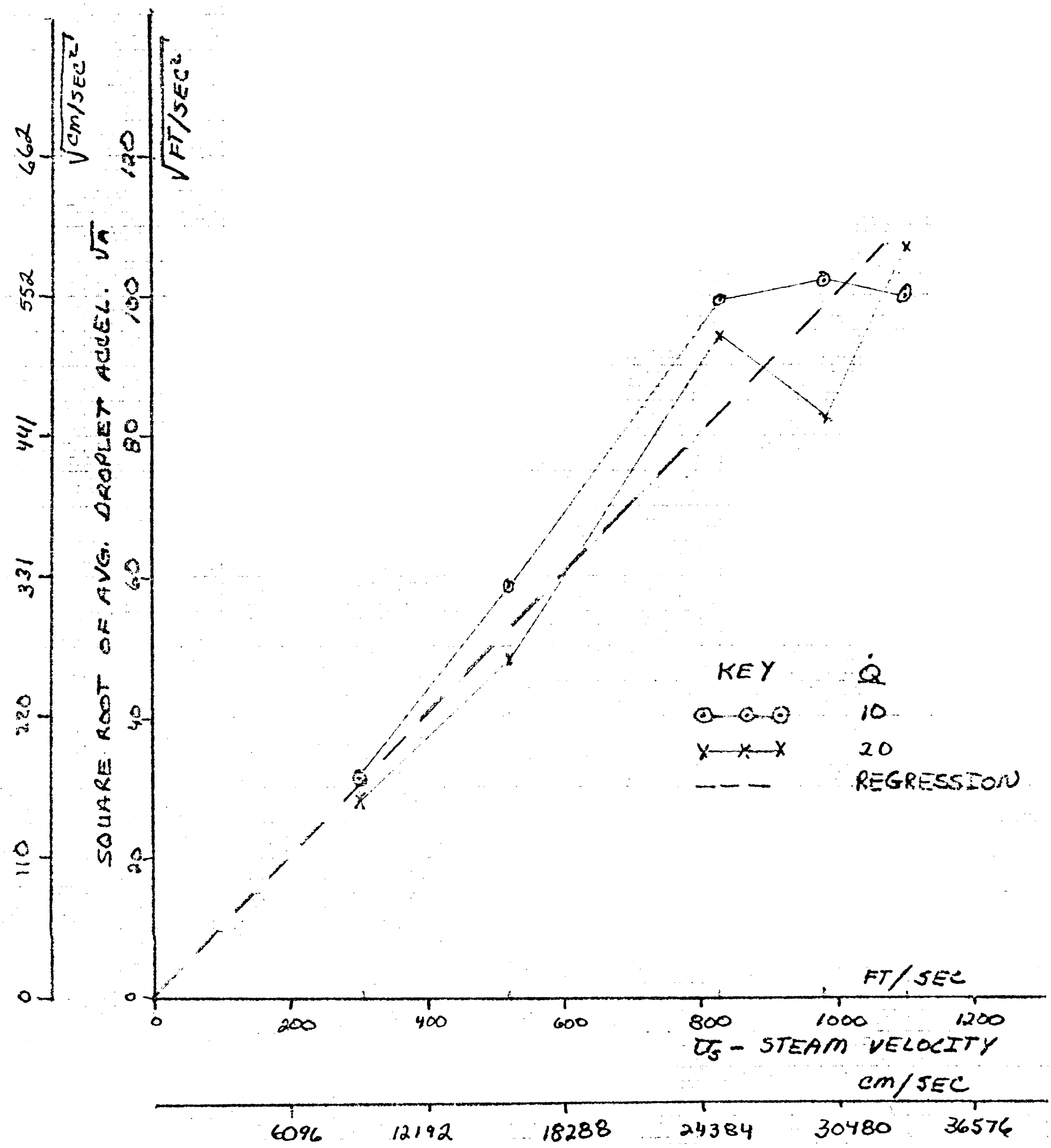



FIG. 7 - SQUARE ROOT OF AVERAGE DROPLET ACCELERATION VERSUS STEAM VELOCITY. DOTTED LINE SHOWS REGRESSION ANALYSIS RESULTS.

| $U_s$ FT/SEC / Q $10^5$ FT³/RAD | 305 | | | 520 | | | 825 | | | 975 | | | 1100 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | E.A. FT/SEC² | A.A. FT/SEC² | R. | E.A. FT/SEC² | A.A FT/SEC² | R | E.A. FT/SEC² | A.A. FT/SEC² | R | EA. FT/SEC² | A.A. FT/SEC² | R | EA. FT/SEC² | A.A. FT/SEC² | R. |
| 17.7 | 838 | 1242 | 0.782 | 2465 | 3666 | 0.699 | 9432 | 8018 | 1.418 | — | — | — | 9527 | 11916 | 0.910 |
| 35.3 | 944 | 1014 | 1.010 | 3442 | 3867 | 0.983 | 9670 | 5245 | 2.059 | 10242 | 10173 | 1.152 | 9943 | 15272 | 0.700 |
| 53.0 | — | — | — | 2721 | 2914 | 0.966 | 8071 | 7044 | 1.262 | 10950 | 16211 | 1.270 | 11319 | 17346 | 0.696 |
| 70.6 | 770 | 1396 | 0.775 | 2282 | 2885 | 0.791 | 8858 | 5307 | 1.820 | 6861 | 10270 | 0.770 | 11700 | 16829 | 0.699 |
| 105.9 | — | — | — | 2601 | 3374 | | 6352 | 6517 | 0.953 | 6205 | 10903 | 0.625 | 8378 | 16730 | 0.606 |
| 141.2 | 735 | 1004 | 0.727 | — | — | — | — | — | — | — | — | — | — | — | — |
| 176.6 | — | — | — | 2139 | 2966 | 0.746 | 4830 | 6089 | 0.964 | — | — | — | 9706 | — | — |

TABLE 1 - COMPARSION OF EXPERIMENTAL "EA" AND ANALYTICAL "AA" DROPLET ACCELERATIONS.
<u>ENGLISH UNITS</u>. R - ACCELERATION RATIO.
ALL VALUES ARE AVERAGE VALUES FOR THE CONDITIONS SHOWN.

| $U_s$ cm/sec / Q cm³/min | 9296.4 | | | 15849.6 | | | 25146 | | | 29718 | | | 33528 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | E.A. cm/sec² | A.A. cm/sec² | R. | EA. cm/sec² | A.A. cm/sec² | R. | E.A. cm/sec² | A.A. cm/sec² | R. | E.A. cm/sec² | A.A. cm/sec² | R. | EA. cm/sec² | A.A. cm/sec² | R |
| 5 | 25550 | 37855 | 0.782 | 75144 | 109893 | 0.699 | 287500 | 244372 | 1.418 | — | — | — | 290390 | 363203 | 0.910 |
| 10 | 28780 | 30893 | 1.010 | 104906 | 117845 | 0.983 | 294750 | 159862 | 2.059 | 312187 | 310066 | 1.152 | 303406 | 465454 | 0.700 |
| 15 | — | — | — | 82940 | 88814 | 0.966 | 246000 | 214481 | 1.262 | 333750 | 311230 | 1.270 | 345000 | 528665 | 0.690 |
| 20 | 23454 | 42556 | 0.775 | 67542 | 87936 | 0.791 | 270000 | 161763 | 1.820 | 209117 | 312999 | 0.770 | 356625 | 512930 | 0.699 |
| 30 | — | — | — | 80275 | 102841 | 0.828 | 193593 | 198627 | 0.953 | 189140 | 332318 | 0.625 | 255374 | 509915 | 0.606 |
| 40 | 22117 | 30593 | 0.727 | — | — | — | — | — | — | — | — | — | — | — | — |
| 50 | — | — | — | 65203 | 90400 | 0.746 | 147187 | 185582 | 0.964 | — | — | — | 295833 | 478515 | 0.680 |

TABLE 2 - COMPARISON OF EXPERIMENTAL "EA" AND ANALYTICAL "AA" DROPLET ACCELERATIONS. CGS UNITS. R-ACCELERATION RATIO. ALL VALUES ARE AVERAGE VALUES FOR THE CONDITIONS SHOWN.

$$E.A. = m(Q) + b$$

| $U_s$ FT/SEC² | $m$ SLOPE | b INTERCEPT | R-SQR[1] GOODNESS OF FIT, % | S.E.[3] STANDARD ERROR |
|---|---|---|---|---|
| 305 | (-6.0)[2] | 945 | 1.1 | 740 |
| 520 | (-29.5) | 3403 | 2.5 | 2374 |
| 825 | -104.6 | 9999 | 22.0 | 2849 |
| 975 | -265.3 | 13526 | 32.5 | 3033 |
| 1100 | (-17.3) | 10512 | 0.4 | 3836 |

TABLE 3 - RESULTS OF STATISTICAL REGRESSION OF AVERAGE EXPERIMENTAL ACCELERATION "EA" ON FLOW RATE "Q".

(1) R-SQR, STATISTICAL PARAMETER - PER CENT VARIATION IN E.A. THAT IS ATTRIBUTABLE TO VARIATION IN Q, FLOW RATE. [5]

(2) SLOPES APPEARING IN PARENTHESIS ARE NOT STATISTICAL DIFFERENT THAN ZERO SLOPE.

(3) ESTIMATE OF STD. DEVIATION OF E.A. ALSO KNOWN AS STD. ERROR. [5]

| $Q$ ($10^{-5}$ FT$^3$/MIN) \ $U_s$ (FT/SEC) | 305 | 520 | 825 | 975 | 1100 |
|---|---|---|---|---|---|
| | E.A. $\sqrt{FT/SEC^2}$ | E.A. $\sqrt{FT/SEC^2}$ | E.A. $\sqrt{FT/SEC^2}$ | E.A. $\sqrt{FT/SEC^2}$ | E.A. $\sqrt{FT/SEC^2}$ |
| 17.7 | 28.9 | 49.7 | 97.1 | — | 97.6 |
| 35.3 | 31 | 58.7 | 98.3 | 101.2 | 99.7 |
| 53.0 | — | 54 | 89.8 | 104.6 | 106.4 |
| 70.6 | 28 | 47.8 | 94.1 | 82.8 | 108.2 |
| 105.9 | — | 51 | 79.7 | 78.8 | 91.5 |
| 141.2 | 27 | — | — | — | — |
| 176.6 | — | 46.3 | 69.5 | — | 98.5 |

TABLE 4 — SQUARE ROOT OF EXPERIMENTAL DROPLET ACCELERATIONS. ENGLIS UNITS. AVERAGE VALUES FOR TH CONDITIONS SHOWN.

$$\sqrt{E.A.} = M(U_s) + b^{\to 0}$$

| Q FT³/MIN | M SLOPE | b INTECEPT | R-SQR ① GOODNESS OF FIT % | SE ② STANDARD ERROR |
|---|---|---|---|---|
| 17.7 | 0.098 | 0.0 | 76.1 | 16.4 |
| 35.3 | 0.100 | 0.0 | 75.1 | 15.5 |
| 53.0 | 0.101 | 0.0 | 60.9 | 17.5 |
| 70.6 | 0.092 | 0.0 | 74.4 | 16.0 |
| 105.9 | 0.084 | 0.6 | 42.8 | 16.5 |
| 141.2 | 0.081 | 0.0 | — | 11.7 |
| 176.6 | 0.085 | 0.0 | 53.2 | 19.4 |

TABLE 5 – RESULTS OF STATISTICAL REGRESSION OF SQUARE ROOT OF AVERAGE EXPERIMENTAL DROPLET ACCELERATION, $\sqrt{EA}$, ON STEAM VELOCITY, $U_S$,

① R-SQR STATISTICAL PARAMETER - PERCENT VARIATION IN $\sqrt{EA}$ ATTRIBUTABLE TO VARIATION IN $U_S$.

② ESTIMATE OF STD. DEVIATION OF $\sqrt{EA}$ ALSO KNOWN AS STD. ERROR.

Appendix - A "MIDAS Results"

Due to the nature of the physical phenomenon under consideration and due to the large amount of experimental data collected certain statistical tools were used to better understand the experimental results. Reduction of the initial raw data [6] resulted in 855 droplet accelerations for the flow condition outlined in tables 1 and 2. These 855 cases were broken into 327 subsets for which average droplet accelerations were obtained. (Do not confuse the average accelerations mentioned here with those shown in tables 1 and 2. Tables 1 and 2 data are average accelerations from the "855" dataset for each specific flow condition, i.e., for

$U_S$ = 305 ft/sec and Q = 17.7 x $10^{-5}$ $ft^3$/min

average E.A. = 838 ft/$sec^2$ is an average of the "855" dataset).

When performing a first order regression of E.A. on Q the 327 dataset was arranged into the following cases corresponding to constant steam velocity, $U_S$ , and stored on data file "AVGACCEL":

| Cases | $U_S$ (ft/sec) |
|---|---|
| 1-45 | 305 |
| 46-129 | 520 |
| 130-185 | 825 |
| 186-246 | 975 |
| 247-327 | 1100 |

In a similar manner the "327" dataset was rearranged in order to perform a first order regression of $\sqrt{E.A.}$ on $U_S$ with liquid water film flow, Q, being held constant.

| Cases | Q ($cm^3$/min) |
|---|---|
| 1-3 | 0 |
| 4-58 | 5 |
| 59-130 | 10 |
| 131-198 | 15 |
| 199-250 | 20 |
| 251-294 | 30 |
| 295-302 | 40 |
| 303-327 | 50 |

Note, these data were stored on data file "QACCEL."

MIDAS "REGRESSION" command was used to perform the regression analyses refered to throughout the text. These results can be found between the heavy dark lines on the attached output. Under "Regression" of E.A. on Q at constant $U_S$, for $U_S$ = 520 ft/sec the various important output parameters are noted. Consult MIDAS user's manual [3] concerning details of other output parameters or other MIDAS commands.

```
>   249        [illegible]            [illegible]          [illegible]            [illegible]
>   250        12549.21094,          0.49587,         1100.00000,          5.00
>END ØF FILE
#
```

Notation Used
AVGACCEL ⇒ AA, R, U, Q

ΩGABAR ⇒ AAC R, U, Q

$\log_{10}$ of AA = LAA

" of AAC = LAAC

**VARIABLE LABELS**

DATA FILE : AVGACCEL

AA - AVE. "327" DATA SET ACCELERATION, FT/SEC

R - ACCEL RATIO

U - STEAM VELOCITY, FT/SEC

Q - FILM FLOW, $cm^3/min$

RES

REGRESSION ANALYSES DENOTED BY HEAVY LINES SEE FOLLOWING PAGES

```
 CØMMAND
?HIST V=AA CASES=1-45 ØP=HISTZ
 INTERVAL EXPRESSIØN -- #INT:(MIN,MAX)   (MIN,MAX)/WIDTH   #PER/(MIN,MAX)
=
```

REGRESSION OF E.A ON Q @ CONSTANT $U_s$

```
 HISTØGRAM   CASES=CASE#:1-45
```

TUR=805 Hz

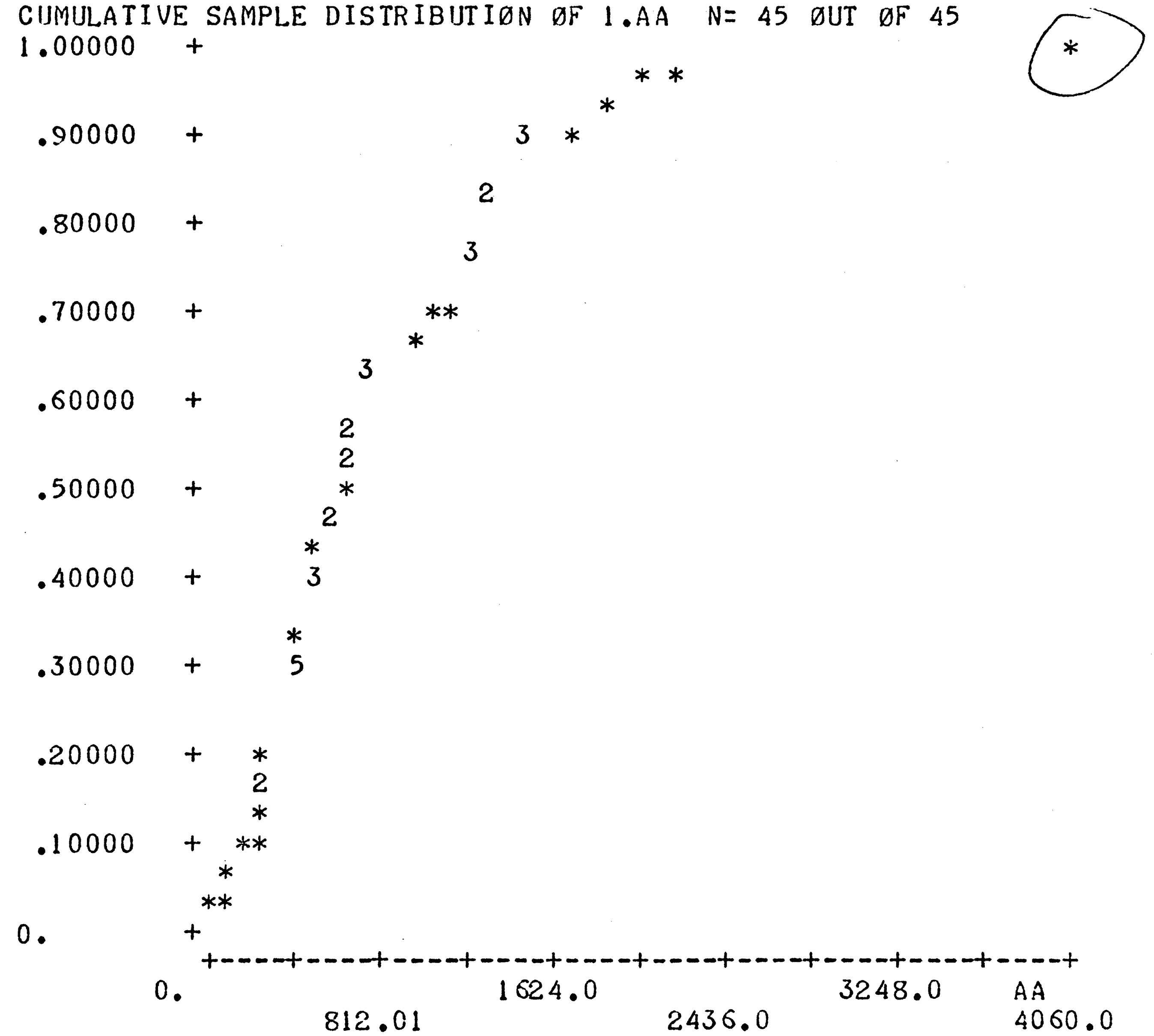
```
 MIDPØINT    HISTZ  CØUNT FØR 1.AA  (EACH X= 1)

  0.          20.0     9 +XXXXXXXXX
  676.67      46.7    21 +XXXXXXXXXXXXXXXXXXXXX
  1353.3      22.2    10 +XXXXXXXXXX
  2030.0       8.9     4 +XXXX
  2706.7      0.       0 +
  3383.4      0.       0 +
  4060.0       2.2     1 +X

  TØTAL               45  (INTERVAL WIDTH= 676.67)


 CØMMAND
?HIST V=AA CASES=1-45 ØP=HISTZ
 INTERVAL EXPRESSIØN -- #INT:(MIN,MAX)   (MIN,MAX)/WIDTH   #PER/(MIN,MAX)
=*


 HISTØGRAM   CASES=CASE#:1-45

 MIDPØINT    HISTZ  CØUNT FØR 1.AA  (EACH X= 1)

  0.          20.0     9 +XXXXXXXXX
  676.67      46.7    21 +XXXXXXXXXXXXXXXXXXXXX
  1353.3      22.2    10 +XXXXXXXXXX
  2030.0       8.9     4 +XXXX
  2706.7      0.       0 +
  3383.4      0.       0 +
  4060.0       2.2     1 +X

  TØTAL               45  (INTERVAL WIDTH= 676.67)


 CØMMAND
?DESCRIBE V=AA CASES=1-45


 DESCRIPTIVE MEASURES   CASES=CASE#:1-45

      VARIABLE             N   MINIMUM      MAXIMUM        MEAN      STD DEV
```

```
1.AA                    45  0.          4060.0     851.28      735.72
```

```
CØMMAND
?DIST V=AA CASES=1-45 PRØB=.5
```

DISTRIBUTIØNAL ANALYSIS  CASES=CASE#:1-45

CUMULATIVE SAMPLE DISTRIBUTIØN ØF 1.AA  N= 45 ØUT ØF 45




| PRØB | QUANTILE |
|---|---|
| .5000 | 632.73 |

```
CØMMAND
?REGRESS V=AA;Q CASES=1-45
```

LEAST SQUARES REGRESSIØN  CASES=CASE#:1-45    u = 305 ft/sec

ANALYSIS ØF VARIANCE ØF 1.AA  N= 45 ØUT ØF 45

| SØURCE | DF | SUM SQRS | MEAN SQR | F-STAT | SIGNIF |
|---|---|---|---|---|---|
| REGRESSIØN | 1 | .25527 +6 | .25527 +6 | .46589 | .4985 |
| ERRØR | 43 | .23561 +8 | .54793 +6 | | |
| TØTAL | 44 | .23816 +8 | | | |

MULT R= .10353  R-SQR= .01072  SE= 740.22

consistent with notation that no relationship

| VARIABLE | PARTIAL | CØEFF | STD ERRØR | T-STAT | SIGNIF |
|---|---|---|---|---|---|
| CØNSTANT | | 944.69 | 175.80 b | 5.3736 | .0000 |
| 4.Q | -.10353 | -5.9627 | 8.7357 m | -.68256 | .4985 |

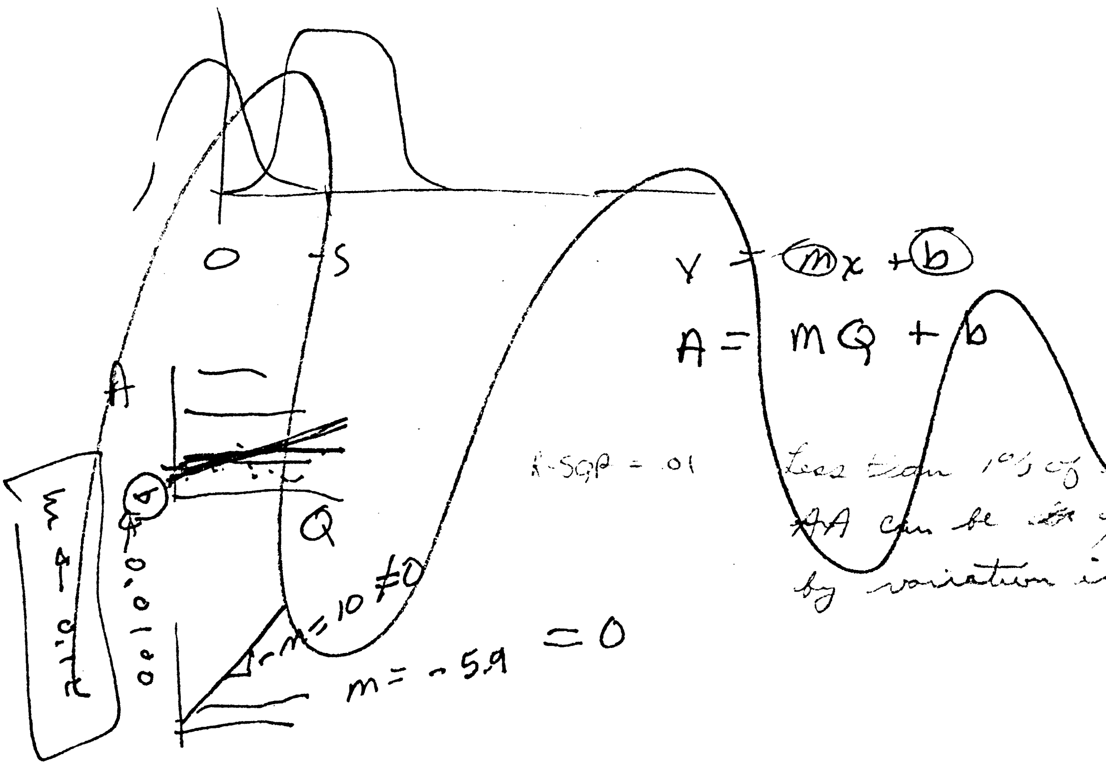
compare to zero

COMMAND
?

$y = mx + b +$

$$AA = 944 - 5.9(Q)$$

Std Erro (125) (8.7)




R-SQR = .01  Less than 1% of variation of AA can be explained by variation in Q

U ≈ 520

```
?HIST V=AA CASES=46-129 ØP=HIST%
 INTERVAL EXPRESSIØN -- #INT:(MIN,MAX)  (MIN,MAX)/WIDTH  #PER/(MIN,MAX)
=


 HISTØGRAM   CASES=CASE#:46-129

 MIDPØINT    HIST%  CØUNT FØR 1.AA   (EACH X= 2)

 -7381.8       1.2      1 +X
 -4921.2       0.       0 +
 -2460.6       1.2      1 +X
  .62497 -1    3.6      3 +XX
  2460.7      75.0     63 +XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
  4921.3      14.3     12 +XXXXXX
  7381.9       1.2      1 +X
  9842.5       2.4      2 +X
  12303.       0.       0 +
  14764.       1.2      1 +X

  TØTAL                84  (INTERVAL WIDTH= 2460.6)


 CØMMAND
?DESCRIBE V=AA CASES=46-129)←


 DESCRIPTIVE MEASURES   CASES=CASE#:46-129

     VARIABLE          N  MINIMUM    MAXIMUM      MEAN     STD DEV

   1.AA               84 -7381.8     14764.     2879.7     2389.0
```

U = 520 ft/sec

```
 CØMMAND
?DIST V=AA CASES-←=46-129 PRØB=.5


 DISTRIBUTIØNAL ANALYSIS  CASES=CASE#:46-129


 CUMULATIVE SAMPLE DISTRIBUTIØN ØF 1.AA   N= 84 ØUT ØF 84
1.00000   +                                          2          *
                                           *   *
                                          4*
 .90000   +
                                         4
                                        2
 .80000   +                            5

                                      3*
 .70000   +                           2
```

```
.60000   +                        3
                                  5

.50000   +                       6

                                 5
.40000   +
                                 7

.30000   +                      3
                                *
                                4
.20000   +                     2
                               9

.10000   +
                              **
                    *     *   *
0.       +*
          +----+----+----+----+----+----+----+----+----+----+
    -7381.8          1476.4              10335.     AA
              -2952.7             5905.5            14764.

  PROB  QUANTILE

  .5000  2509.8
```

*SAMPLE REGRESSION RESULTS*

---

```
COMMAND
?REGRESS V=AA;Q CASES46←←=46-129

LEAST SQUARES REGRESSION  CASES=CASE#:46-129
```

*⇒ $U_S$ = 520 ft/sec*

```
ANALYSIS OF VARIANCE OF 1.AA   N= 84 OUT OF 84
```

| SOURCE | DF | SUM SQRS | MEAN SQR | F-STAT | SIGNIF |
|---|---|---|---|---|---|
| REGRESSION | 1 | .11675 +8 | .11675 +8 | 2.0719 | .1538 |
| ERROR | 82 | .46205 +9 | .56347 +7 | | |
| TOTAL | 83 | .47372 +9 | | | |

MULT R= .15699  R-SQR= .02464  SE= 2373.8

*← Goodness of fit (R-SQR); ← Std error (SE)*

| VARIABLE | PARTIAL | COEFF | STD ERROR | T-STAT | SIGNIF |
|---|---|---|---|---|---|
| CONSTANT | | 3402.7 | 446.23 | 7.6255 | .0000 |
| 4.Q | -.15699 | -29.388 | 20.417 | -1.4394 | .1538 |

*$y = b + m(x)$*

*$AA = 3402 - 29.4(q)$*

*(446)  (20.4)*

*Compares to zero slope: T-stat test, anything less than .0010 bad.*

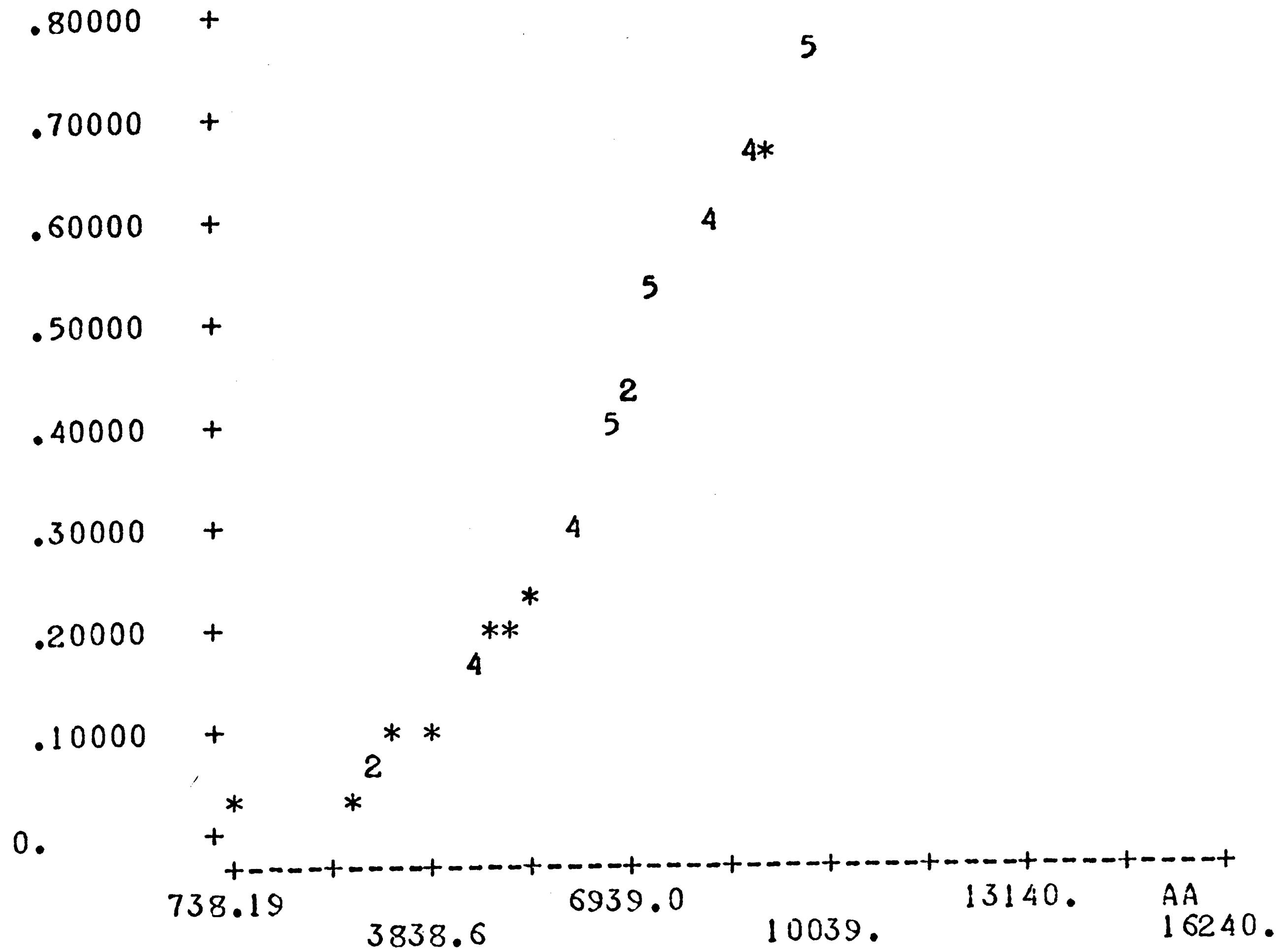
```
COMMAND
?
```

u = 825

```
?HIST V=AA CASES=130-185 ØP=HIST%
 INTERVAL EXPRESSIØN -- #INT:(MIN,MAX)  (MIN,MAX)/WIDTH  #PER/(MIN,MAX)
=
```

u = 825 ft/sec

```
 HISTØGRAM  CASES=CASE#:130-185

 MIDPØINT   HIST%  CØUNT FØR 1.AA  (EACH X= 1)

  738.19     1.8     1 +X
  2952.8     8.9     5 +XXXXX
  5167.3    19.6    11 +XXXXXXXXXXX
  7381.9    28.6    16 +XXXXXXXXXXXXXXXX
  9596.5    23.2    13 +XXXXXXXXXXXXX
  11811.    12.5     7 +XXXXXXX
  14026.     3.6     2 +XX
  16240.     1.8     1 +X

  TØTAL             56  (INTERVAL WIDTH= 2214.6)

 CØMMAND
?DESV←SCRIBE V=AA CASES=130-185

 ERRØR -- INVALID CØMMAND:  "DESSCRIBE"
 CØMMAND CANCELLED


 CØMMAND
?DESCRIBE V=AA CASES=130-185


 DESCRIPTIVE MEASURES  CASES=CASE#:130-185

      VARIABLE        N  MINIMUM   MAXIMUM     MEAN     STD DEV

    1.AA             56  738.19    16240.     7871.8    3195.2


 CØMMAND
?DIST V=AA CASES=130-185 PRØB=.5


 DISTRIBUTIØNAL ANALYSIS  CASES=CASE#:130-185


 CUMULATIVE SAMPLE DISTRIBUTIØN ØF 1.AA  N= 56 ØUT ØF 56
 1.00000  +                                                  *
                                                     *  *
                                                   3
 .90000   +                                      3
```




PRØB QUANTILE

.5000 7381.9

---

CØMMAND
?REGRESS V=AA;Q CASES=130-185

LEAST SQUARES REGRESSIØN CASES=CASE#:130-185 ⇒ $\bar{U}_s$ = 825 ft/sec.

ANALYSIS ØF VARIANCE ØF 1.AA N= 56 ØUT ØF 56

| SØURCE | DF | SUM SQRS | MEAN SQR | F-STAT | SIGNIF |
|---|---|---|---|---|---|
| REGRESSIØN | 1 | .12325 +9 | .12325 +9 | 15.185 | .0003 |
| ERRØR | 54 | .43828 +9 | .81163 +7 | | |
| TØTAL | 55 | .56153 +9 | | | |

MULT R= .46849 R-SQR= .21949 SE= 2848.9

| VARIABLE | PARTIAL | CØEFF | STD ERRØR | T-STAT | SIGNIF |
|---|---|---|---|---|---|
| CØNSTANT | | 9991.2 | 663.89 | 15.050 | .0000 |
| 4.Q | -.46849 | -104.57 | 26.835 | -3.8968 | .0003 |

CØMMAND
?

AA = 9991 − 104(Q)

Std Error [redacted] [redacted]

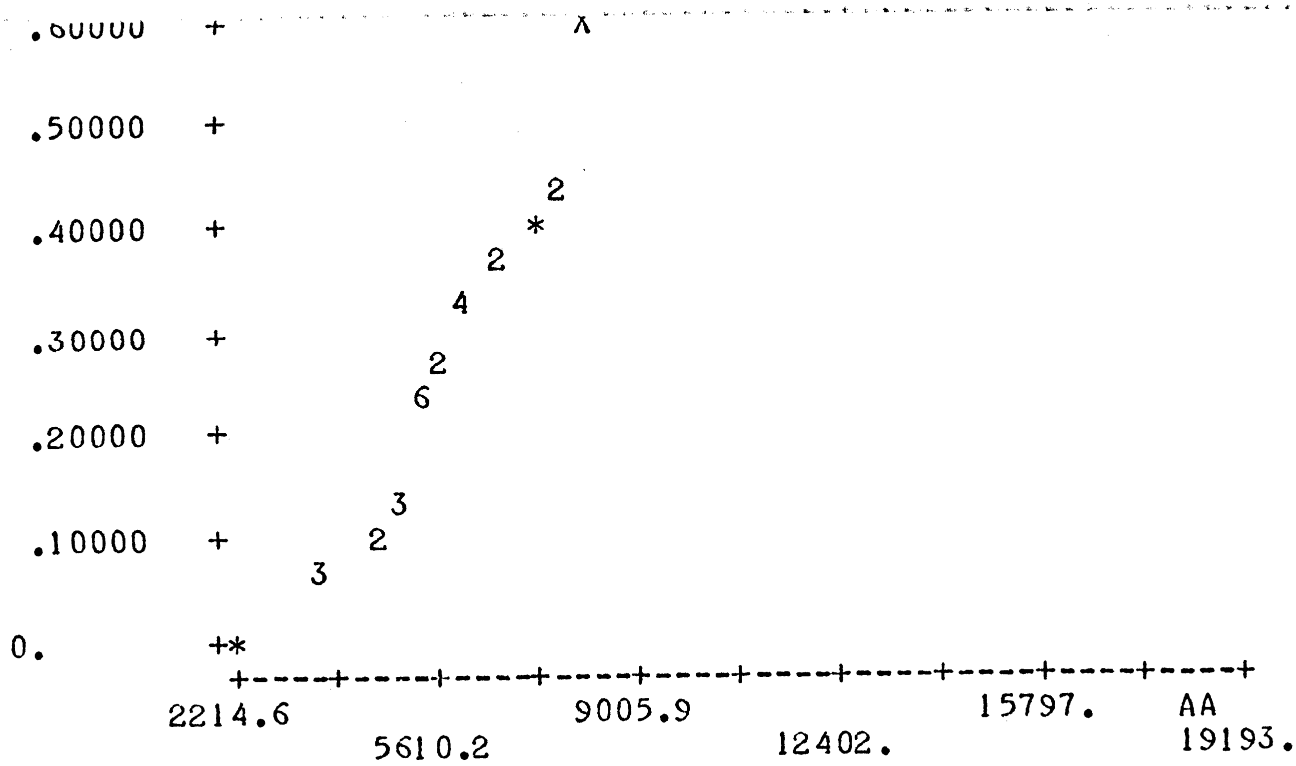
---

u=975

```
HIST V=AA CASES=186-246 I←ØP=HIST%
INTERV@L EXPRESSIØN -- #INT:(MIN,MAX)  (MIN,MAX)/WIDTH  #PER/(MIN,MAX)
=
```

u=975 ft/sec

```
HISTØGRAM  CASES=CASE#:186-246

MIDPØINT   HIST%  CØUNT FØR 1.AA  (EACH X= 1)

 2214.6     1.6     1 +X
 4640.0    26.2    16 +XXXXXXXXXXXXXXXX
 7065.5    31.1    19 +XXXXXXXXXXXXXXXXXXX
 9491.0    13.1     8 +XXXXXXXX
 11916.    14.8     9 +XXXXXXXXX
 14342.     8.2     5 +XXXXX
 16767.     3.3     2 +XX
 19193.     1.6     1 +X

 TØTAL             61  (INTERVAL WIDTH= 2425.5)


CØMMAND
?DESCRIBE V=186-246#
?DESCRIBE V=AAB← CASES=186-246


DESCRIPTIVE MEASURES  CASES=CASE#:186-246

    VARIABLE          N  MINIMUM    MAXIMUM      MEAN     STD DEV

   1.AA              61  2214.6     19193.      8525.5    3661.7


CØMMAND
?DIST V=AA CASES=186-246 PRØB=.5


DISTRIBUTIØNAL ANALYSIS  CASES=CASE#:186-246


CUMULATIVE SAMPLE DISTRIBUTIØN ØF 1.AA  N= 61  ØUT ØF 61
1.00000   +                                                   2      *
                                                           *
                                                     3 *

 .90000   +
                                                4 *

 .80000   +                                   4

                                            *
 .70000   +                               2
                                       5
```




| PRØB | QUANTILE |
|---|---|
| .5000 | 8120.1 |

---

CØMMAND
?REGRESS V=AA;Q CASES=186-246 ⇒ $v_s$ = 975 ft/sec

LEAST SQUARES REGRESSIØN CASES=CASE#:186-246

ANALYSIS ØF VARIANCE ØF 1.AA N= 61 ØUT ØF 61

| SØURCE | DF | SUM SQRS | MEAN SQR | F-STAT | SIGNIF |
|---|---|---|---|---|---|
| REGRESSIØN | 1 | .26170 +9 | .26170 +9 | 28.446 | .0000 |
| ERRØR | 59 | .54280 +9 | .92000 +7 | | |
| TØTAL | 60 | .80450 +9 | | | |

MULT R= .57035 R-SQR= .32530 SE= 3033.1
32%

| VARIABLE | PARTIAL | CØEFF | STD ERRØR | T-STAT | SIGNIF |
|---|---|---|---|---|---|
| CØNSTANT | | 13526. | 1014.8 | 13.328 | .0000 |
| 4.Q | -.57035 | -265.25 | 49.733 | -5.3335 | .0000 |

CØMMAND
?

AA = 13526 − 265(Q)
1014 49.7

```
 HIST#
?HIST V=AA CASES=247-327 ØP=HIST%
 INTERVAL EXPRESSIØN -- #INT:(MIN,MAX)  (MIN,MAX)/WIDTH  #PER/(MIN,MAX)
=
```

$U_5$ = 1100 ft/sec

```
 HISTØGRAM  CASES=CASE#:247-327

 MIDPØINT   HIST%  CØUNT FØR 1.AA  (EACH X= 1)

  2952.8     1.2     1 +X
  5085.3    12.3    10 +XXXXXXXXXX
  7217.8    22.2    18 +XXXXXXXXXXXXXXXXXX
  9350.4    25.9    21 +XXXXXXXXXXXXXXXXXXXXX
  11483.    19.8    16 +XXXXXXXXXXXXXXXX
  13615.     2.5     2 +XX
  15748.     9.9     8 +XXXXXXXX
  17881.     3.7     3 +XXX
  20013.     1.2     1 +X
  22146.     1.2     1 +X

  TØTAL             81  (INTERVAL WIDTH= 2132.5)


 CØMMAND
?DESCRIBE V=AA CASES=247-327


 DESCRIPTIVE MEASURES  CASES=CASE#:247-327

     VARIABLE          N  MINIMUM    MAXIMUM     MEAN     STD DEV

   1.AA               81  2952.8     22146.     10201.    3819.4
```

---

```
 CØMMAND
?REGRESS V=AA;Q CASES=247-2←327
```

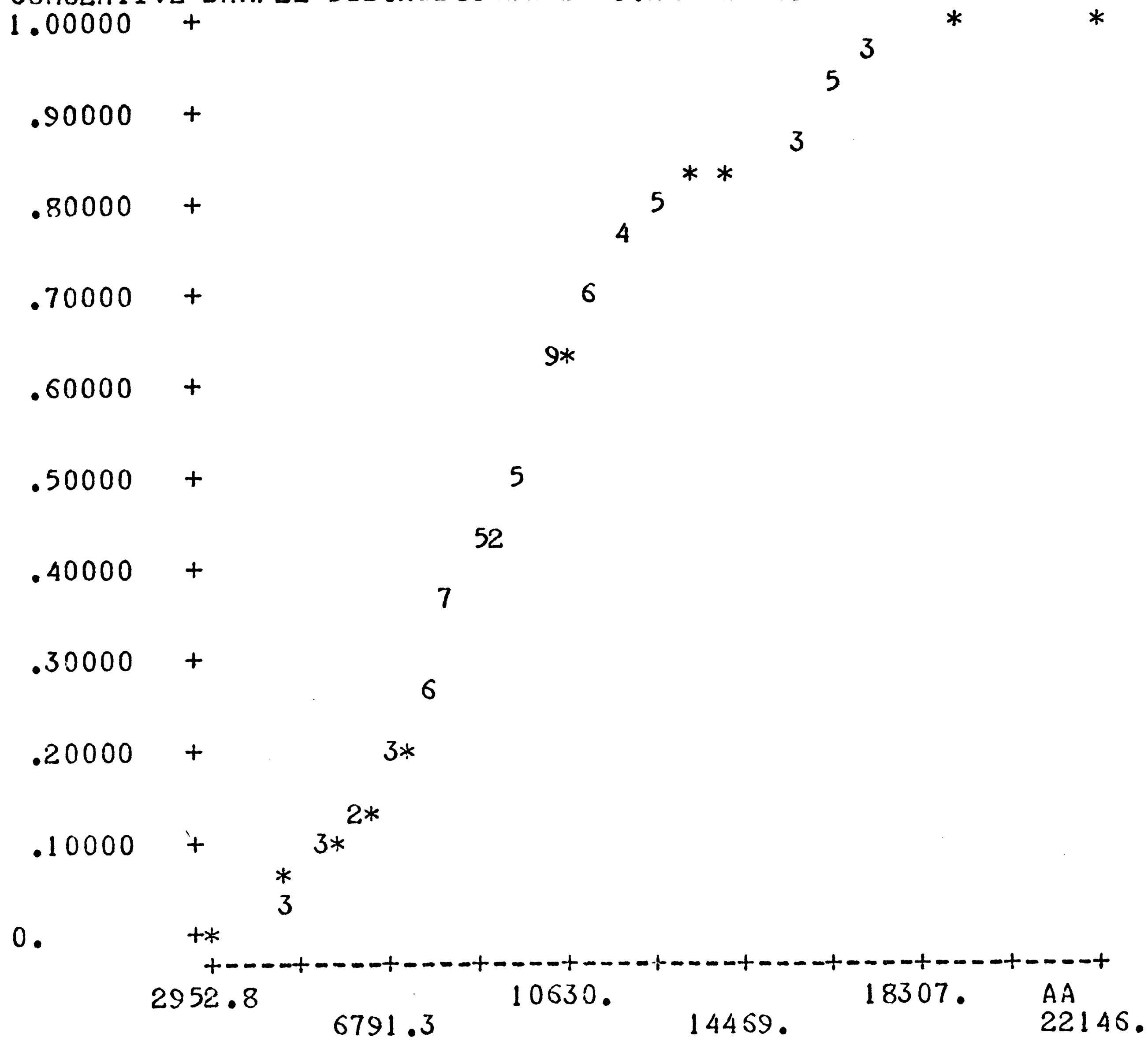
⇒ $U_5$ = 1100 ft/sec

```
 LEAST SQUARES REGRESSIØN  CASES=CASE#:247-327


 ANALYSIS ØF VARIANCE ØF 1.AA  N= 81 ØUT ØF 81

     SØURCE              DF  SUM SQRS    MEAN SQR    F-STAT     SIGNIF

     REGRESSIØN           1  .45454 +7   .45454 +7   .30890     .5799
     ERRØR               79  .11625+10   .14715 +8
     TØTAL               80  .11670+10

     MULT R= .06241  R-SQR= .00389 SE= 3836.0

     VARIABLE     PARTIAL     CØEFF     STD ERRØR    T-STAT     SIGNIF

     CØNSTANT                10512.      703.91     14.934      .0000
   4.Q           -.06241    -17.273      31.079     -.55579     .5799
```

```
CØMMAND
?DIST V=AA CASES=247-327 PØ←RØB=.5
```

DISTRIBUTIØNAL ANALYSIS  CASES=CASE#:247-327

CUMULATIVE SAMPLE DISTRIBUTIØN ØF 1.AA  N= 81  ØUT ØF 81




| PRØB | QUANTILE |
|---|---|
| .5000 | 9596.5 |

```
CØMMAND
?
```

AA = 10512 - 17.2 (Q)

703    31

↑ Bad

Neglect Remaining Output ↓

```
 REGRESS V=AA;Q
 CASES TØ SELECT
=ALL


 LEAST SQUARES REGRESSIØN


 ANALYSIS ØF VARIANCE ØF 1.AA   N= 327  ØUT ØF 327

     SØURCE                DF  SUM SQRS    MEAN SQR    F-STAT     SIGNIF

     REGRESSIØN             1  .71320 +8   .71320 +8   3.3347      .0688
     ERRØR                325  .69509+10   .21387 +8
     TØTAL                326  .70222+10

     MULT R= .10078  R-SQR= .01016 SE= 4624.6


     VARIABLE      PARTIAL      CØEFF     STD ERRØR    T-STAT     SIGNIF

     CØNSTANT                   6998.8     450.21      15.546      .0000
   4.Q             -.10078   -37.216      20.380      -1.8261      .0688


 CØMMAND
?TRANS
 TRANSFØRMATIØN EXPRESSIØN  (E.G., V9=V2+V5  ØR  V10=SQRT(V8)  )
=V10=LØG10(AA) L=LA

 ERRØR -- SYNTAX:  "V10=LØG10(AA) L=LA"
 ENTER NEW VALUE FØR TRANSFØRMATIØN EXPRESSIØN  (E.G., V9=V2+V5  ØR
 V10=SQRT(V8)  )
=

 ERRØR -- ILLEGAL VALUE

 THE EXPRESSIØN STARTS WITH "VK=", WHERE 0 < K < 10000; THE RIGHT-SIDE
 MAY BE  "A", "A ØP B", "FCN", "FCN(A)", "FCN(A,B)", WHERE A AND B ARE
 VARIABLES AND/ØR CØNSTANTS WITH DECIMAL PØINTS, FCN IS FUNCTIØN NAME.

 ENTER NEW VALUE FØR TRANSFØRMATIØN EXPRESSIØN  (E.G., V9=V2+V5  ØR
 V10=SQRT(V8)  )
=V20=LØG10(AA)
 LABEL FØR THE RESULT VARIABLE(S)
=LAA


 LØG10 TRANSFØRMATIØN

     VARIABLE  TØTAL  VALID   MISS

   20.LAA        327    323      4


 CØMMAND
?REGRESS V=LAA;Q CASESE←=ALL
```

```
 LEAST SQUARES REGRESSIØN


 ANALYSIS ØF VARIANCE ØF 20.LAA   N= 323  ØUT ØF 327

      SØURCE                DF   SUM SQRS     MEAN SQR     F-STAT      SIGNIF

      REGRESSIØN             1   .39460       .39460       1.9284       .1659
      ERRØR                321   65.685       .20463
      TØTAL                322   66.080

      MULT R= .07728  R-SQR= .00597 SE= .45236

      VARIABLE      PARTIAL      CØEFF      STD ERRØR     T-STAT      SIGNIF

      CØNSTANT                   3.6946      .44400 -1    83.211      0.
    4.Q              -.07728  -.27783 -2   .20007 -2  -1.3887       .1659


 CØMMAND
?DIST V=LAA CASES=ALL ØP==←HIST%

 ERRØR -- INVALID KEYWØRD:  "ØP=HIST%"


 DISTRIBUTIØNAL ANALYSIS


 CUMULATIVE SAMPLE DISTRIBUTIØN ØF 20.LAA   N= 323  ØUT ØF 327
 1.00000    +                                                             52*
                                                                        X
                                                                      9

  .90!
 $.07 USED.  CØNTINUE(YES/NØ)%0200 DATA CHECK.
?%0200 DATA CHECK.
??#
#
RES
 "Y"  ØR  "N"?N


 CØMMAND
?HIST=V#
?HIST V=LAA CASES=ALL ØP=HIST%
 INTERVAL EXPRESSIØN -- #INT:(MIN,MAX)   (MIN,MAX)/WIDTH   #PER/(MIN,MAX)
=


 HISTØGRAM

 MIDPØINT    HIST%   CØUNT FØR 20.LAA   (EACH X= 2)

  1.8682        .6      2 +X
  2.0139      0.        0 +
  2.1596      0.        0 +
  2.3053        .6      2 +X
  2.4510      1.2       4 +XX
  2.5967      1.9       6 +XXX
  2.7424      3.1      10 +XXXXX
  2.8882      1.5       5 +XXX
  3.0339      2.5       8 +XXXX
  3.1796      6.8      22 +XXXXXXXXXXX
  3.3253      7.7      25 +XXXXXXXXXXXXX
  3.4710      7.7      25 +XXXXXXXXXXXXX
```

```
 3.6167      9.6     51 +XXXXXXXXXXXXXXXX
 3.7624     13.0     42 +XXXXXXXXXXXXXXXXXXXXX
 3.9081     17.6     57 +XXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 4.0539     19.2     62 +XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 4.1996      5.9     19 +XXXXXXXXXX
 4.3453       .9      3 +XX

 MISSING              4
 TOTAL              327  (INTERVAL WIDTH= .14571)


 COMMAND
?READ

 NOTE.  THIS DATASET CURRENTLY HAS 5 VARIABLES AND 327 CASES
 FILE CONTAINING DATA OR *  TO ENTER DATA HERE
=WGABAR
 FORMAT SPECIFICATION OR *  TO ENTER DATA SEPARATED BY COMMAS
=:←*
 VARIABLES TO READ (E.G., 1-10)
=1-4
 LABELS CORRESPONDING TO THESE VARIABLES OR *  FOR STANDARD LABELS
=AAC,RC,U,Q
 CASES TO ASSIGN TO THE DATA BEING READ (E.G., 1-98)
=1-327
```

WG BAR

```
 READ OBSERVATIONS 1-327
 VARIABLES BY CASE

 327 CASES READ FOR 4 VARIABLES

 CASES CHANGED FOR 4 EXISTING VARIABLES


 COMMAND
?REGRESS V=AAC;Q CASES=ALL


 LEAST SQUARES REGRESSION


 ANALYSIS OF VARIANCE OF 1.AAC  N= 327 OUT OF 327

      SOURCE              DF  SUM SQRS    MEAN SQR    F-STAT     SIGNIF

      REGRESSION           1  .66258+11   .66258+11   3.3347      .0688
      ERROR              325  .64576+13   .19869+11
      TOTAL              326  .65238+13

      MULT R= .10078  R-SQR= .01016 SE= .14096 +6


      VARIABLE     PARTIAL     COEFF     STD ERROR    T-STAT     SIGNIF

      CONSTANT               .21332 +6   13722.       15.546      .0000
    4.Q            -.10078  -1134.4      621.19      -1.8261      .0688


 COMMAND
?TRANS
 TRANSFORMATION EXPRESSION  (E.G., V9=V2+V5  OR  V10=SQRT(V8)  )
=V30=LOG10(AAC)
 LABEL FOR THE RESULT VARIABLE(S)
=LAAC
```

```
 LØG10 TRANSFØRMATIØN

      VARIABLE  TØTAL  VALID   MISS

   30.LAAC        327    323      4


 CØMMAND
?REGRESS V=LAAC;Q CASES=ALL


 LEAST SQUARES REGRESSIØN


 ANALYSIS ØF VARIANCE ØF 30.LAAC  N= 323 ØUT ØF 327

      SØURCE              DF  SUM SQRS   MEAN SQR   F-STAT    SIGNIF

      REGRESSIØN           1  .39460     .39460     1.9284     .1659
      ERRØR              321  65.685     .20463
      TØTAL              322  66.080

      MULT R= .07728  R-SQR= .00597 SE= .45236


      VARIABLE     PARTIAL     CØEFF     STD ERRØR   T-STAT    SIGNIF

      CØNSTANT                5.1786     .44400 -1  116.64    0.
    4.Q            -.07728  -.27783 -2  .20007 -2 -1.3887     .1659


 CØMMAND
?SIGNØFF

 ERRØR -- INVALID CØMMAND:  "SIGNØFF"
 CØMMAND CANCELLED


 CØMMAND
?STØP
 USE $RES TØ RE-ENTER MIDAS
#SIGNØFF
#ODLU 15:39:40-17:22:21 FRI MAR 31/78
#TERM,NØRMAL,UNIV
#ELAPSED TIME       102.685 MIN.         $2.40
#CPU TIME USED        3.021 SEC.          $.49
#CPU STØR VMI         4.116 PAGE-MIN.     $.24
#WAIT STØR VMI      141.476 PAGE-HR.
#DRUM READS        5806
#APPRØX. CØST ØF THIS RUN IS     $3.23
#DISK STØRAGE       128      PAGE-HR.     $.01
#APPRØX. REMAINING BALANCE:    $73.29
(@
```

QACCEL - ft/sec
arranged by Q

SR-AA - Sq root ave accel

REGRESSION OF $\sqrt{EA}$ ON $U_s$ @ CONSTANT Q

REGRESSION ANALYSIS FOR EACH Q DENOTED BY HEAVY LINES ON FOLLOWING PAGES

VARIABLE LABELS

DATA FILE: QACCEL

V1. AA - AVG ACCEL, FT/SEC²

V2. R - ACCEL RATIO

V3. U - STEAM VELOCITY, FT/SEC

V4. Q - FILM FLOW RATE, cm³/MIN

V20. SRAA - SQUARE ROOT AVG. ACCEL, $\sqrt{FT/SEC^2}$

04/05/78

```
 SIGNON ODLU
#ENTER USER PASSWORD.
?STEAM
#TERM,NORMAL,UNIV
#**LAST SIGNON WAS: 00:19:25   TUE APR 04/78
# USER "ODLU" SIGNED ON AT  17:27:51 ON WED APR 05/78
#F
 AVGACCEL   CUMACCEL   LF         LINEFIT    MDPRDATA  PFILE      PLOT1
 PLOT2      QACCEL     TDATA      TEMP       WATERDROP WD         WDATA
 WDGRAF     WG         WGABAR     WGAVG
#LIST QACCEL
>      1          19192.91406          0.75839      1100.00000          0.0
>      2          15501.96875          0.61254      1100.00000          0.0
>      3           7381.89063          0.29169      1100.00000          0.0
>      4           2030.01978          2.24796       305.00000          5.0
>      5           1328.74023          1.10043       305.00000          5.0
>      6           1722.44116          1.42648       305.00000          5.0
>     !
>ATTN!
#RUN STAT;_:MIDAS
 EXECUTION BEGINS


 M I D A S
 STATISTICAL RESEARCH LABORATORY
 UNIVERSITY OF MICHIGAN
 17:30:46
 APR  5, 1978



 COMMAND
?READ
 FILE CONTAINING DATA OR  *  TO ENTER DATA HERE
:*_QACCEL
 FORMAT SPECIFICATION OR  *  TO ENTER DATA SEPARATED BY COMMAS
:4(F15.5,1X)
 VARIABLES TO READ (E.G., 1-10)
:1-4
 LABELS CORRESPONDING TO THESE VARIABLES OR  *  FOR STANDARD LABELS
:AA,R,U,Q
 CASES TO ASSIGN TO THE DATA BEING READ (E.G., 1-98)
:1-327


 READ OBSERVATIONS 1-327
 VARIABLES BY CASE

 327 CASES READ FOR 4 VARIABLES


 COMMAND
?WRITE V=ALL CASES=1-6
```

```
FILE TO RECEIVE DATA OR  *  TO WRITE DATA HERE
*
FORMAT SPECIFICATION OR  *  TO LIST DATA WITH HEADINGS
*

WRITE OBSERVATIONS  CASES=CASE#:1-6
VARIABLES BY CASE

1.           2.          3.          4.
AA           R           U           Q


 19193.      .75839      1100.0      0.

 15502.      .61254      1100.0      0.

 7381.9      .29169      1100.0      0.

 2030.0      2.2480      305.00      5.0000

 1328.7      1.1004      305.00      5.0000

 1722.4      1.4265      305.00      5.0000

6 CASES WRITTEN FOR 4 VARIABLES


COMMAND
DESCRIBE V=AA CASES=ALL


DESCRIPTIVE MEASURES

     VARIABLE           N  MINIMUM    MAXIMUM      MEAN      STD DEV

   1.AA               327 -7381.8     22146.      6322.2     4641.2


COMMAND
DIST V=AA CASES=ALL PROB=NOGRAPH:.25,.5,.75


DISTRIBUTIONAL ANALYSIS


CUMULATIVE DISTRIBUTION OF 1.AA  N= 327 OUT OF 327

  PROB  QUANTILE

 .2500  2399.1

 .5000  5659.4

 .7500  9596.5


COMMAND
```

```
HIST V=AA CASES=ALL OP=HIST%
INTERVAL EXPRESSION -- #INT:(MIN,MAX)  (MIN,MAX)/WIDTH  #PER/(MIN,MAX)
=


HISTOGRAM

MIDPOINT    HIST%  COUNT FOR 1.AA  (EACH X= 2)

-7381.8       .3      1 +X
-5741.4     0.        0 +
-4101.0     0.        0 +
-2460.6       .3      1 +X
-820.14       .6      2 +X
 820.28     15.9     52 +XXXXXXXXXXXXXXXXXXXXXXXXXX
 2460.7     16.2     53 +XXXXXXXXXXXXXXXXXXXXXXXXXXX
 4101.1     10.7     35 +XXXXXXXXXXXXXXXXXX
 5741.5      9.5     31 +XXXXXXXXXXXXXXXX
 7381.9     15.0     49 +XXXXXXXXXXXXXXXXXXXXXXXXX
 9022.4      9.5     31 +XXXXXXXXXXXXXXXX
 10663.      7.6     25 +XXXXXXXXXXXXX
 12303.      6.1     20 +XXXXXXXXXX
 13944.      2.8      9 +XXXXX
 15584.      3.1     10 +XXXXX
 17224.      1.5      5 +XXX
 18865.       .6      2 +X
 20505.     0.        0 +
 22146.       .3      1 +X

 TOTAL              327  (INTERVAL WIDTH= 1640.4)


COMMAND
?REGRESS V=AA;U CASES=ALL OPTION=MEANZERO


LEAST SQUARES REGRESSION


ANALYSIS OF VARIANCE OF 1.AA  N= 327 OUT OF 327

     SOURCE                DF  SUM SQRS    MEAN SQR    F-STAT     SIGNIF

     REGRESSION             1  .16705+11   .16705+11   1607.8     0.
     ERROR                326  .33871+10   .10390 +8
     TOTAL                327  .20092+11

     OPT:  MEANZERO  R-SQR= .57491 SE= 3223.4


     VARIABLE      PARTIAL    COEFF     STD ERROR   T-STAT     SIGNIF

   3.U              .91182   8.6755     .21636      40.098     0.

COMMAND
```

REGRESS V=AA;U CASES=ALL

LEAST SQUARES REGRESSION

ANALYSIS OF VARIANCE OF 1.AA  N= 327 OUT OF 327

| SOURCE | DF | SUM SQRS | MEAN SQR | F-STAT | SIGNIF |
|---|---|---|---|---|---|
| REGRESSION | 1 | .40371+10 | .40371+10 | 439.55 | .0000 |
| ERROR | 325 | .29851+10 | .91848 +7 | | |
| TOTAL | 326 | .70222+10 | | | |

MULT R= .75823  R-SQR= .57491 SE= 3030.6

| VARIABLE | PARTIAL | COEFF | STD ERROR | T-STAT | SIGNIF |
|---|---|---|---|---|---|
| CONSTANT | | -3204.5 | 484.32 | -6.6164 | .0000 |
| 3.U | .75823 | 12.325 | .58787 | 20.965 | .0000 |

COMMAND

```
?SCATTER V=AA;U CASES=ALL


 SCATTER PLOT
            N= 327 OUT OF 327  1.AA VS. 3.U
 AA
 22146.    +                                                  *


           +                                                  2

                                                      2       3
 16240.    +                                 *                5
                                                      *       3
                                             2        *       *
           +                                 *        3       *
                                             3        *       5
                                             4        8       X
 10335.    +                                 2        2       X
                                             6        2       7
                          2                  8        X       X
           +                                 7        3       7
                          *                  5        2       4
                          6                  6        X       6
 4429.2    +*             5                  5        5       4
                          X                  2        3
            2             X                  3        *       *
           +X             X
            X             *                  *
            *             *
-1476.3    +

                          *
           +


-7381.8    +              *
            +----+----+----+----+----+----+----+----+----+----+
         305.00              623.00             941.00     U
                   464.00             782.00              1100.0

 COMMAND
?
```

```
TRANS
TRANSFORMATION EXPRESSION  (E.G., V9=V2+V5  OR  V10=SQRT(V8)  )
V20=SQRT(AA) LABELS=SRAA

ERROR -- SYNTAX:  "V20=SQRT(AA) LABELS=SRAA"
ENTER NEW VALUE FOR TRANSFORMATION EXPRESSION  (E.G., V9=V2+V5  OR
V10=SQRT(V8)  )
V20=SQRT(AA)
LABEL FOR THE RESULT VARIABLE(S)
SRAA


SQRT TRANSFORMATION

    VARIABLE  TOTAL  VALID   MISS

  20.SRAA        327    324      3


COMMAND
WRITE V=S_ALL CASES=1-6
FILE TO RECEIVE DATA OR  *  TO WRITE DATA HERE
*
FORMAT SPECIFICATION OR  *  TO LIST DATA WITH HEADINGS
*


WRITE OBSERVATIONS  CASES=CASE#:1-6
VARIABLES BY CASE

1.          2.          3.          4.          20.
AA          R           U           Q           SRAA


 19193.      .75839      1100.0      0.          138.54

 15502.      .61254      1100.0      0.          124.51

 7381.9      .29169      1100.0      0.          85.918

 2030.0      2.2480      305.00      5.0000      45.056

 1328.7      1.1004      305.00      5.0000      36.452

 1722.4      1.4265      305.00      5.0000      41.502

6 CASES WRITTEN FOR 5 VARIABLES


COMMAND
DESCRIBE V=SRAA CASES=ALL


DESCRIPTIVE MEASURES

    VARIABLE            N  MINIMUM     MAXIMUM       MEAN      STD DEV

  20.SRAA             324  0.           148.81      73.991     30.692


COMMAND
```

```
HIST V=SRAA CASES=ALL OP=HIST%
INTERVAL EXPRESSION -- #INT:(MIN,MAX)  (MIN,MAX)/WIDTH  #PER/(MIN,MAX)
=


HISTOGRAM

MIDPOINT   HIST%  COUNT FOR 20.SRAA  (EACH X= 1)

 0.          .3      1 +X
 8.2675      .6      2 +XX
 16.535     3.4     11 +XXXXXXXXXXX
 24.802     4.9     16 +XXXXXXXXXXXXXXXX
 33.070     3.4     11 +XXXXXXXXXXX
 41.337     7.7     25 +XXXXXXXXXXXXXXXXXXXXXXXXX
 49.605     9.0     29 +XXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 57.872     7.7     25 +XXXXXXXXXXXXXXXXXXXXXXXXX
 66.140     6.5     21 +XXXXXXXXXXXXXXXXXXXXX
 74.407     9.3     30 +XXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 82.675     8.3     27 +XXXXXXXXXXXXXXXXXXXXXXXXXXX
 90.942    12.0     39 +XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
 99.209     8.6     28 +XXXXXXXXXXXXXXXXXXXXXXXXXXXX
 107.48     7.1     23 +XXXXXXXXXXXXXXXXXXXXXXX
 115.74     5.2     17 +XXXXXXXXXXXXXXXXX
 124.01     3.4     11 +XXXXXXXXXXX
 132.28     1.5      5 +XXXXX
 140.55      .6      2 +XX
 148.81      .3      1 +X

 MISSING             3
 TOTAL             327  (INTERVAL WIDTH= 8.2675)


COMMAND
?DIST V=R_SRAA CASES=ALL PROB=NOGRAPH;.25,.5,.75


DISTRIBUTIONAL ANALYSIS


CUMULATIVE DISTRIBUTION OF 20.SRAA  N= 324 OUT OF 327

  PROB  QUANTILE

 .2500  49.605

 .5000  76.847

 .7500  97.962


COMMAND
?
```

```
REGRESS V=SRAA;U CASES=ALL OP=MEANZERO

LEAST SQUARES REGRESSION

ANALYSIS OF VARIANCE OF 20.SRAA   N= 324 OUT OF 327

     SOURCE               DF   SUM SQRS    MEAN SQR    F-STAT     SIGNIF

     REGRESSION            1   .19801 +7   .19801 +7   6527.6     0.
     ERROR               323   97979.      303.34
     TOTAL               324   .20781 +7

     OPT:  MEANZERO  R-SQR= .68150 SE= 17.417


     VARIABLE      PARTIAL     COEFF      STD ERROR    T-STAT     SIGNIF

   3.U             .97614    .94625 -1   .11712 -2   80.794     0.

COMMAND
```

```
?SCATTER V=SRAA;U


 SCATTER PLOT
            N= 324 OUT OF 327  20.SRAA VS. 3.U
 SRAA
 148.81    +                                                  *

                                                              2
           +
                                             *        2       8
                                                      *       3
 119.05    +                                 2        *       *
                                             4        4       6
                                             3        4       4
           +                                 *        4       7
                                             7        4       X
                          2                  5        5       7
 89.289    +                                 4        X       7
                                             7        *       7
                          *                  5        2       4
           +              2                  5        6       3
                          5                  2        9       4
            *             5                  4        2       3
 59.526    +              9                  2        3
                          X                  2                *
                          X                  *
           +3             X                           *
            4             X
            7             2
 29.763    +*             *
            X                                *
            9
           +6
            2

 0.        +*
            +----+----+----+----+----+----+----+----+----+----+
         305.00              623.00              941.00    U
                   464.00              782.00              1100.0


 COMMAND
?
```

```
HIST V=R_SRAA CASES=1-3 I=*

HISTOGRAM  CASES=CASE#:1-3

MIDPOINT    COUNT FOR 20.SRAA  (EACH X= 1)

 85.918        1 +X
 138.54        2 +XX

 TOTAL         3  (INTERVAL WIDTH= 52.621)

COMMAND
DIST V=SRAA CASES=1-3 PROB=NOGRAPH;.25,.5,.75

DISTRIBUTIONAL ANALYSIS  CASES=CASE#:1-3

CUMULATIVE DISTRIBUTION OF 20.SRAA  N= 3 OUT OF 3

 PROB  QUANTILE

.2500  85.918

.5000  124.51

.7500  138.54

OMMAND
ESCRIBE V=SRAA CASES=1-3

ESCRIPTIVE MEASURES  CASES=CASE#:1-3

    VARIABLE          N  MINIMUM    MAXIMUM      MEAN      STD DEV

 20.SRAA              3  85.918     138.54      116.32     27.249

OMMAND
```

```
REGRESS V=SRAA;U CASES=1-3 OP=MEANZERO
```

$\varphi = 0$ cm³/min

```
LEAST SQUARES REGRESSION   CASES=CASE#:1-3

ANALYSIS OF VARIANCE OF 20.SRAA   N= 3 OUT OF 3

     SOURCE              DF   SUM SQRS    MEAN SQR    F-STAT     SIGNIF

     REGRESSION           1   40592.      40592.      54.670      .0178
     ERROR                2   1485.0      742.49
     TOTAL                3   42077.

     OPT:  MEANZERO  R-SQR= .00000 SE= 27.249

     VARIABLE       PARTIAL     COEFF     STD ERROR    T-STAT     SIGNIF

   3.U              .98220    .10575     .14302 -1   7.3939      .0178
```

```
COMMAND
?SCATTER V=SRAA;U CASES=1-3

 SCATTER PLOT   CASES=CASE#:1-3

 INVALID INTERVAL FOR 3.U:  1100.0, 1100.0

 N= 0 OUT OF 3  20.SRAA VS. 3.U

 COMMAND
?
```

Q = 5

SCATTER V=SRAA;U CASES=4-58

```
SCATTER PLOT   CASES=CASE#:4-58
          N= 54 OUT OF 55  20.SRAA VS. 3.U
SRAA
 130.30   +                                                  *
                                                             *
                                            *
          +                                                  *
                                                             *

 104.24   +                                 *                *
                                            *                2
                                            2                2
          +                                 3                *
                                            *                2
                                                             *
 78.181   +
                         *                                   2

          +              2                                   *
                         3
                         *
 52.120   +              2
                         *
           2             *
          +*             *
           4
           *
 26.060   +4
           *
           2
          +*


 0.       +*
           +----+----+----+----+----+----+----+----+----+----+
        305.00              623.00              941.00    U
                  464.00              782.00              1100.0
```

COMMAND

```
HIST V=SRAA CASES=4-58 OP=HIST% I=*


HISTOGRAM  CASES=CASE#:4-58

MIDPOINT    HIST%  COUNT FOR 20.SRAA  (EACH X= 1)

 0.           1.9      1 +X
 18.614      14.8      8 +XXXXXXXX
 37.229      18.5     10 +XXXXXXXXXX
 55.843      14.8      8 +XXXXXXXX
 74.458      11.1      6 +XXXXXX
 93.072      25.9     14 +XXXXXXXXXXXXXX
 111.69       7.4      4 +XXXX
 130.30       5.6      3 +XXX

 MISSING               1
 TOTAL                55  (INTERVAL WIDTH= 18.614)


COMMAND
?DIST V=SRAA CASES=4-58 PROB=NOGRAPH;.25,.5,.75


DISTRIBUTIONAL ANALYSIS  CASES=CASE#:4-58


CUMULATIVE DISTRIBUTION OF 20.SRAA  N= 54 OUT OF 55

  PROB  QUANTILE

 .2500  36.452

 .5000  64.677

 .7500  94.118


COMMAND
?
```

DESCRIBE V-SRAA_____=SRAAA_ CASES=4-58

DESCRIPTIVE MEASURES CASES=CASE#:4-58

| VARIABLE | N | MINIMUM | MAXIMUM | MEAN | STD DEV |
|---|---|---|---|---|---|
| 20.SRAA | 54 | 0. | 130.30 | 65.850 | 33.073 |

---

COMMAND
REGRESS V=SRAA;U CASES=AL__4-58 OP=MEANZERO    Q = 5 cm³/min

LEAST SQUARES REGRESSION CASES=CASE#:4-58

ANALYSIS OF VARIANCE OF 20.SRAA N= 54 OUT OF 55

| SOURCE | DF | SUM SQRS | MEAN SQR | F-STAT | SIGNIF |
|---|---|---|---|---|---|
| REGRESSION | 1 | .27783 +6 | .27783 +6 | 1029.8 | .0000 |
| ERROR | 53 | 14299. | 269.79 | | |
| TOTAL | 54 | .29213 +6 | | | |

OPT: MEANZERO R-SQR= .76080 SE= 16.425 ~ Std Deviation of Errors about regression (fit not of residuals)

| VARIABLE | PARTIAL | COEFF | STD ERROR | T-STAT | SIGNIF |
|---|---|---|---|---|---|
| 3.U | .97522 | .95737 -1 | .29834 -2 | 32.090 | .0000 |

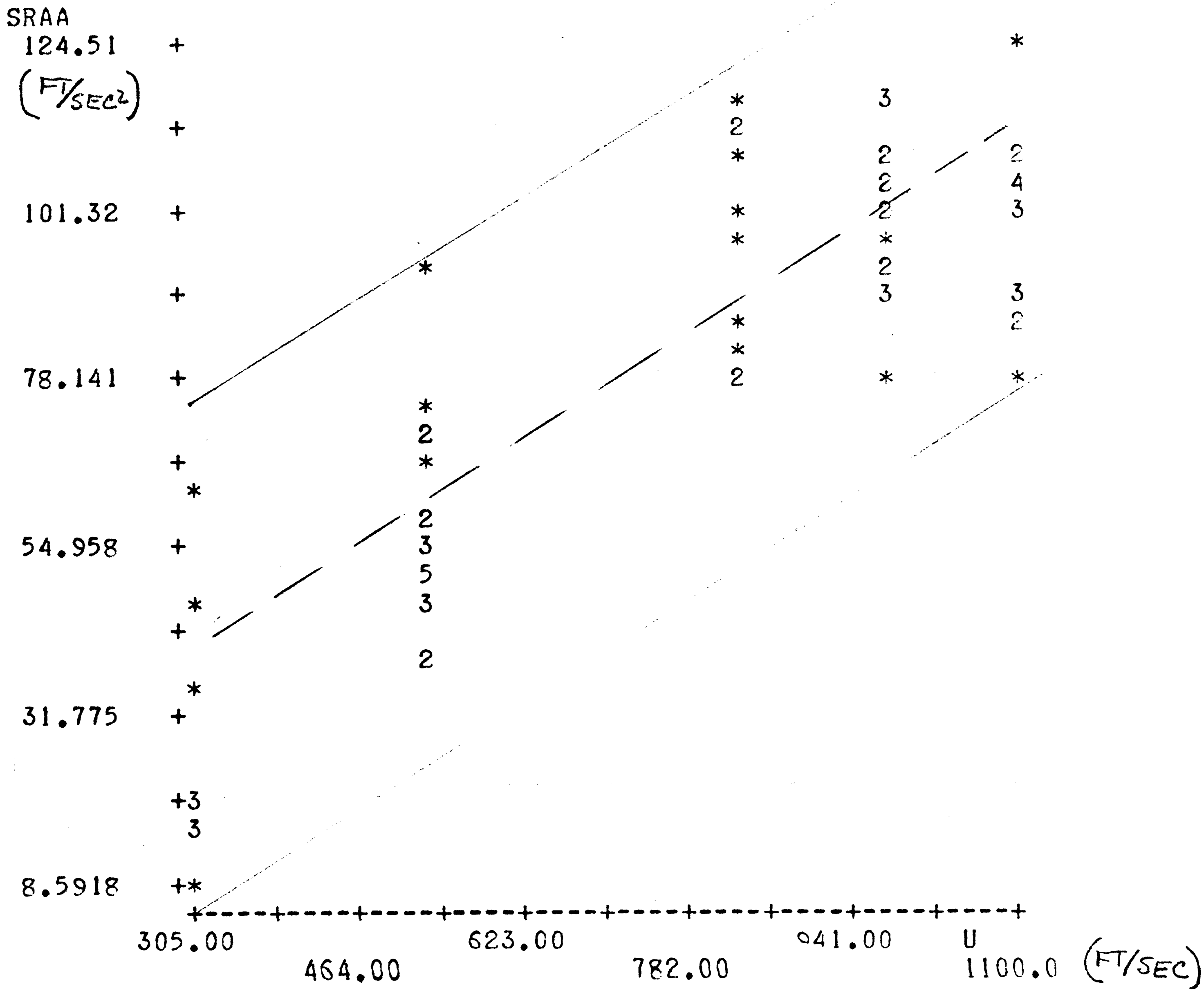
---

COMMAND

SCATTER V=SRAA;U CASES=59-130

→ Q = 10 (cc/min)

SCATTER PLOT CASES=CASE#:59-130
N= 72 OUT OF 72 20.SRAA VS. 3.U




COMMAND
?DESCRIBE V=SRAA CASES=59-130

DESCRIPTIVE MEASURES CASES=CASE#:59-130

| VARIABLE | N | MINIMUM | MAXIMUM | MEAN | STD DEV |
|---|---|---|---|---|---|
| 20.SRAA | 72 | 8.5918 | 124.51 | 77.396 | 30.637 |

COMMAND
?

```
HIST V=SRAA CASES=59-130 OP=HIST% I=*


HISTOGRAM  CASES=CASE#:59-130

MIDPOINT   HIST%  COUNT FOR 20.SRAA  (EACH X= 1)

 8.5918      2.8      2 +XX
 23.081      6.9      5 +XXXXX
 37.571      4.2      3 +XXX
 52.060     18.1     13 +XXXXXXXXXXXXX
 66.549      6.9      5 +XXXXX
 81.039     12.5      9 +XXXXXXXXX
 95.528     23.6     17 +XXXXXXXXXXXXXXXXX
 110.02     22.2     16 +XXXXXXXXXXXXXXXX
 124.51      2.8      2 +XX

 TOTAL               72  (INTERVAL WIDTH= 14.489)


COMMAND
DIST V=SRAA CASES=59-130 PROB=NOGRAPH;.25,.5,.75


DISTRIBUTIONAL ANALYSIS  CASES=CASE#:59-130


 CUMULATIVE DISTRIBUTION OF 20.SRAA  N= 72 OUT OF 72

   PROB  QUANTILE

  .2500  52.026

  .5000  85.918

  .7500  101.66

COMMAND
```

REGRESS V=SRAA;U CASES=59-130 OP=MEANZERO

Q = 10 cm³/min

LEAST SQUARES REGRESSION CASES=CASE#:59-130

\NALYSIS OF VARIANCE OF 20.SRAA N= 72 OUT OF 72

| SOURCE | DF | SUM SQRS | MEAN SQR | F-STAT | SIGNIF |
|---|---|---|---|---|---|
| REGRESSION | 1 | .48093 +6 | .48093 +6 | 2007.4 | .0000 |
| ERROR | 71 | 17010. | 239.58 | | |
| TOTAL | 72 | .49794 +6 | | | |

OPT: MEANZERO R-SQR= .75104 SE= 15.478

| VARIABLE | PARTIAL | COEFF | STD ERROR | T-STAT | SIGNIF |
|---|---|---|---|---|---|
| 3.U | .98277 | .10040 | .22408 -2 | 44.804 | .0000 |

COMMAND
?

SCATTER V=SSR__S_RAA;U CASES=131-198

Q = 10

```
SCATTER PLOT   CASES=CASE#:131-198
          N= 66 OUT OF 68  20.SRAA VS. 3.U
SRAA
 138.54   +                                                  *
                                                  2
          +                          *                       *
                                                             *
 118.19   +                                       *          *
                                     *
                                                  *          3
          +                          *            *          *
                                                  2          *
                                                             2
 97.838   +                          *            *          3
           *                         2            *          *
                                                  *          *
          +
                                     3                       *
           *                         *
 77.488   +                                       *
                                     *
           2
          +*                         4            *          *
           *
 57.138   +*
           3
           2
          +*
           2
           2
 36.788   +4
           +----+----+----+----+----+----+----+----+----+----+
        520.00              752.00              984.00     U
                  636.00              868.00              1100.0
```

COMMAND

DESCRIBE V=SRAA CASES=131-198

DESCRIPTIVE MEASURES   CASES=CASE#:131-198

| VARIABLE | N | MINIMUM | MAXIMUM | MEAN | STD DEV |
|---|---|---|---|---|---|
| 20.SRAA | 66 | 36.788 | 138.54 | 84.547 | 27.825 |

COMMAND

```
 RE#
?HIST V=SRAA CASES=131-198 OP=HIST% I=*
```

φ = 15

```
 HISTOGRAM  CASES=CASE#:131-198

 MIDPOINT    HIST%   COUNT FOR 20.SRAA  (EACH X= 1)

  36.788     10.6      7 +XXXXXXX
  49.507     10.6      7 +XXXXXXX
  62.226     13.6      9 +XXXXXXXXX
  74.944      6.1      4 +XXXX
  87.663     12.1      8 +XXXXXXXX
  100.38     22.7     15 +XXXXXXXXXXXXXXX
  113.10     15.2     10 +XXXXXXXXXX
  125.82      7.6      5 +XXXXX
  138.54      1.5      1 +X

  MISSING              2
  TOTAL               68  (INTERVAL WIDTH= 12.719)


 COMMAND
?DIST V=SRAA CASES=131-198 PROB=NOGRAPH;.25,.5,.75


 DISTRIBUTIONAL ANALYSIS  CASES=CASE#:131-198


 CUMULATIVE DISTRIBUTION OF 20.SRAA  N= 66 OUT OF 68

   PROB  QUANTILE

  .2500  66.552

  .5000  85.918

  .7500  105.23


 COMMAND
?REGRESS V=SRAA;U CASES=131-198 OP=MEANZERO
```

---

```
 LEAST SQUARES REGRESSION  CASES=CASE#:131-198
```

φ = 15 cm³/min

```
 ANALYSIS OF VARIANCE OF 20.SRAA  N= 66 OUT OF 68

     SOURCE               DF  SUM SQRS    MEAN SQR    F-STAT    SIGNIF

     REGRESSION            1  .50215 +6   .50215 +6   1635.1     .0000
     ERROR                65  19962.      307.10
     TOTAL                66  .52211 +6

     OPT:  MEANZERO  R-SQR= .60911 SE= 17.524


     VARIABLE        PARTIAL    COEFF     STD ERROR    T-STAT    SIGNIF

   3.U               .98070    .10115     .25016 -2   40.437     .0000
```

---

```
 COMMAND
```

```
SCATTER V=SRAA CASES=199-250    9=20

ERROR -- WRONG # OF VARS:  "SRAA"
ENTER NEW VALUE FOR VARIABLES -- VERTICAL; HORIZONTAL
SRAA;U


SCATTER PLOT  CASES=CASE#:199-250
          N= 52 OUT OF 52  20.SRAA VS. 3.U
SRAA
 130.30   +                                                  *
                                                             2
                                                     *
          +

                                                             *
 108.45   +                                 *        *
                                                             *
                                            *                *
          +                                 *                2
                                            *        2
                                                     3       *
 86.599   +
                                            *        *       *

          +                                 *        2
                                                     2
                                                     *
 64.748   +                                          *
                                                     2

          +              2
                         2
                         3                           *
 42.897   +              *
           *             *

          +2
           *
           4
 21.046   +2
           +----+----+----+----+----+----+----+----+----+----+
       305.00              623.00              941.00    U
                 464.00              782.00              1100.0

COMMAND
#
#
DESCRIBE V=SRAA CASES=199-250


DESCRIPTIVE MEASURES  CASES=CASE#:199-250
```

| VARIABLE | N | MINIMUM | MAXIMUM | MEAN | STD DEV |
|---|---|---|---|---|---|
| 20.SRAA | 52 | 21.046 | 130.30 | 71.219 | 31.705 |

```
COMMAND
```

```
?#
?HIST V=SRAA CASES=199-250 OP=HIST% I=*


 HISTOGRAM  CASES=CASE#:199-250

 MIDPOINT   HIST%  COUNT FOR 20.SRAA  (EACH X= 1)

  21.046     13.5      7 +XXXXXXX
  36.653      9.6      5 +XXXXX
  52.261     15.4      8 +XXXXXXXX
  67.869     13.5      7 +XXXXXXX
  83.477     17.3      9 +XXXXXXXXX
  99.085     17.3      9 +XXXXXXXXX
  114.69      5.8      3 +XXX
  130.30      7.7      4 +XXXX

  TOTAL               52  (INTERVAL WIDTH= 15.608)


 COMMAND
?DIST V=SRAA CASES=199-250 PROB=NOGRAPH;.25,.5,.75


 DISTRIBUTIONAL ANALYSIS  CASES=CASE#:199-250


 CUMULATIVE DISTRIBUTION OF 20.SRAA  N= 52 OUT OF 52

   PROB  QUANTILE

  .2500  47.059

  .5000  71.884

  .7500  94.118


 COMMAND
?
```

#
#

```
REGRESS V=SRAA;U CASES=199-250 OP=MEANZERO
```

$\phi = 20\ cm^3/min$

```
LEAST SQUARES REGRESSION  CASES=CASE#:199-250

ANALYSIS OF VARIANCE OF 20.SRAA  N= 52 OUT OF 52

    SOURCE                DF   SUM SQRS    MEAN SQR    F-STAT     SIGNIF

    REGRESSION             1   .30189 +6   .30189 +6   1173.1      .0000
    ERROR                 51   13124.      257.34
    TOTAL                 52   .31502 +6

    OPT:  MEANZERO  R-SQR= .74400 SE= 16.042

    VARIABLE       PARTIAL      COEFF      STD ERROR    T-STAT     SIGNIF

  3.U               .97895    .91948 -1   .26845 -2   34.251      .0000
```

COMMAND

```
?#
?SCATTER V=SRAA CAS____;U CASES=251-294  9-30


 SCATTER PLOT  CASES=CASE#:251-294
            N= 44 OUT OF 44  20.SRAA VS. 3.U
  SRAA
   127.44  +                                                  *


           +

                                      *
   107.38  +                                                  *

                                                              *
           +
                                                              2
                                      *            3          *
   87.330  +                                       3
                                      *
                                      *            *          *
           +                          *            *
                                                   *
            *                         *            6          2
   67.277  +                          *

            *                                      *
           +*
            *                                                 *
            2
   47.223  +
            *

           +2

            *
   27.170  +                          *
            +----+----+----+----+----+----+----+----+----+----+
         520.00              752.00             984.00     U
                   636.00              868.00              1100.0
```

COMMAND
?DESCRIBE V=SRAA CASES=251-294

DESCRIPTIVE MEASURES  CASES=CASE#:251-294

| VARIABLE | N | MINIMUM | MAXIMUM | MEAN | STD DEV |
|---|---|---|---|---|---|
| 20.SRAA | 44 | 27.170 | 127.44 | 73.952 | 21.156 |

COMMAND
?

```
HIST V=SRAA CASES=251-294 OP=HIST% I=*


HISTOGRAM  CASES=CASE#:251-294

MIDPOINT   HIST%  COUNT FOR 20.SRAA  (EACH X= 1)

 27.170     4.5     2 +XX
 43.881    11.4     5 +XXXXX
 60.592    13.6     6 +XXXXXX
 77.303    38.6    17 +XXXXXXXXXXXXXXXXX
 94.014    25.0    11 +XXXXXXXXXXX
 110.73     4.5     2 +XX
 127.44     2.3     1 +X

 TOTAL             44  (INTERVAL WIDTH= 16.711)


COMMAND
DIST V=SRAA CASES=* PROB=NOGRAPH;:_*

ERROR -- INVALID CONSTANT:  "NOGRAPH;*"
ENTER NEW VALUE FOR PROBABILITY POINTS
NOGRAPH;.5,.25,.75


DISTRIBUTIONAL ANALYSIS  CASES=CASE#:251-294


CUMULATIVE DISTRIBUTION OF 20.SRAA  N= 44 OUT OF 44

  PROB  QUANTILE

 .5000  71.884

 .2500  60.753

 .7500  88.040


COMMAND
```

```
#
?REGRESS V=SRAA;U CASES=251-294 OP=MEANZERO

 LEAST SQUARES REGRESSION  CASES=CASE#:251-294

 ANALYSIS OF VARIANCE OF 20.SRAA   N= 44 OUT OF 44

      SOURCE                 DF   SUM SQRS     MEAN SQR     F-STAT     SIGNIF

      REGRESSION              1   .24816 +6   .24816 +6    909.49      .0000
      ERROR                  43   11733.       272.86
      TOTAL                  44   .25989 +6

      OPT:   MEANZERO  R-SQR= .42784 SE= 16.518

      VARIABLE       PARTIAL      COEFF      STD ERROR    T-STAT     SIGNIF

    3.U              .97717     .83657 -1   .27740 -2    30.158      .0000
```

$Q = 30\ cm^3/min$

```
 COMMAND
?
```

```
SCATTER V=SRAA CASES=295-302

ERROR -- WRONG # OF VARS:  "SRAA"
ENTER NEW VALUE FOR VARIABLES -- VERTICAL; HORIZONTAL
SRAA,_;U


SCATTER PLOT  CASES=CASE#:295-302

INVALID INTERVAL FOR 3.U:  305.00, 305.00

N= 0 OUT OF 8  20.SRAA VS. 3.U


COMMAND
DESCRIBE V=SRAA CASES=*


DESCRIPTIVE MEASURES  CASES=CASE#:295-302

    VARIABLE         N  MINIMUM    MAXIMUM      MEAN     STD DEV

 20.SRAA             8  9.0565      42.959     24.830     11.166


COMMAND
DIST V=SRAA CASES=* PROB=NOGRAPH;.25,.5,.75


DISTRIBUTIONAL ANALYSIS  CASES=CASE#:295-302


CUMULATIVE DISTRIBUTION OF 20.SRAA  N= 8 OUT OF 8

 PROB  QUANTILE

.2500  16.384

.5000  22.184

.7500  25.616


COMMAND
```

```
 #
?#
?HIST V=SRAA CASES=* OP=* I=*


 HISTOGRAM  CASES=CASE#:295-302

 MIDPOINT    COUNT FOR 20.SRAA  (EACH X= 1)

  9.0565       2 +XX
  26.008       4 +XXXX
  42.959       2 +XX

  TOTAL        8  (INTERVAL WIDTH= 16.951)
```

---

```
 COMMAND
?REGRESS V=SRAA;U CASES=* OP=MEANZERO
```

$Q = 40\ cm^3/min$

```
 LEAST SQUARES REGRESSION  CASES=CASE#:295-302


 ANALYSIS OF VARIANCE OF 20.SRAA  N= 8 OUT OF 8

     SOURCE              DF  SUM SQRS   MEAN SQR   F-STAT    SIGNIF

     REGRESSION           1  4932.1     4932.1     39.557     .0004
     ERROR                7  872.77     124.68
     TOTAL                8  5804.8

     OPT:  MEANZERO  R-SQR= .00000 SE= 11.166


     VARIABLE      PARTIAL    COEFF     STD ERROR   T-STAT    SIGNIF

   3.U              .92176   .81409 -1  .12944 -1  6.2895     .0004
```

---

```
 COMMAND
?
```

```
SCATTER V=SRAA CASES=303-327

ERROR -- WRONG # OF VARS:  "SRAA"
ENTER NEW VALUE FOR VARIABLES -- VERTICAL; HORIZONTAL
SRAA;U


SCATTER PLOT  CASES=CASE#:303-327
          N= 25 OUT OF 25  20.SRAA VS. 3.U
SRAA
 148.81  +                                                  *


         +

                                                            *
 126.07  +


         +


 103.32  +                                                  *
                                    *
                                                            *
         +

                                    *                       *
80.571   +                          *                       *
                                                            *
         
         +                                                  *
                                                            *
                                    *
57.824   +*                         *
                                    2
          2                         *
         +*
          2
          *
35.076   +*
          +----+----+----+----+----+----+----+----+----+----+
       520.00              752.00              984.00    U
                 636.00              868.00              1100.0


COMMAND
DESCRIBE V=SRAA CASES=*


DESCRIPTIVE MEASURES  CASES=CASE#:303-327
```

| VARIABLE | N | MINIMUM | MAXIMUM | MEAN | STD DEV |
|---|---|---|---|---|---|
| 20.SRAA | 25 | 35.076 | 148.81 | 70.386 | 28.315 |

```
COMMAND
```

```
DI__HIST V=SRAA CASES=* OP=HIST% I=*___ I=*


HISTOGRAM  CASES=CASE#:303-327

MIDPOINT   HIST%  COUNT FOR 20.SRAA  (EACH X= 1)

 35.076     16.0     4 +XXXX
 57.824     44.0    11 +XXXXXXXXXXX
 80.571     20.0     5 +XXXXX
 103.32     12.0     3 +XXX
 126.07      4.0     1 +X
 148.81      4.0     1 +X

 TOTAL              25  (INTERVAL WIDTH= 22.748)


COMMAND
?DIST V=SRAA CASES=* PROB=NOGRAPH;.25,.5,.75


DISTRIBUTIONAL ANALYSIS  CASES=CASE#:303-327


CUMULATIVE DISTRIBUTION OF 20.SRAA  N= 25 OUT OF 25

  PROB  QUANTILE

 .2500  48.981

 .5000  60.753

 .7500  83.742

COMMAND
?
```

REGRESS V=SRAA;U CASES=* OP=MEANZERO

$\varphi = 50\ cm^3/min$

LEAST SQUARES REGRESSION   CASES=CASE#:303-327

ANALYSIS OF VARIANCE OF 20.SRAA   N= 25 OUT OF 25

| SOURCE | DF | SUM SQRS | MEAN SQR | F-STAT | SIGNIF |
|---|---|---|---|---|---|
| REGRESSION | 1 | .13409 +6 | .13409 +6 | 357.18 | .0000 |
| ERROR | 24 | 9009.8 | 375.41 | | |
| TOTAL | 25 | .14310 +6 | | | |

OPT:  MEANZERO  R-SQR= .53178 SE= 19.375

| VARIABLE | PARTIAL | COEFF | STD ERROR | T-STAT | SIGNIF |
|---|---|---|---|---|---|
| 3.U | .96801 | .85139 -1 | .45049 -2 | 18.899 | .0000 |

COMMAND