WEIGHTED MUTUAL REACTANCES FOR A TRIPLE-CAGE MOTOR CIRCUIT

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INTRODUCTION

At a symposium on double-cage motors held at Toledo in October, 1952, the motor designers had difficulty in calculating the performance of the 75-hp motor which was built and tested by Westinghouse. This information is published in "Power Apparatus and Systems," A.I.E.E. Transactions Papers, for August, 1953.

There are many reasons why the calculated and test results did not check too well, the test conditions being then unknown.

1. Considerable time was taken to get the locked rotor test, thus getting the rotor hot.

2. The rotor currents in the three cages are considerably out of phase, and it is difficult to calculate the mutual effects.

3. Different designers do not use the same method of calculating coil-end and zig-zag reactances, nor of taking account of changes in reactance due to saturation.

The author thought it should be possible, with the information available, to take account of all the variables involved and set up a circuit that would give close results. This led to a different method of handling the mutual reactances, which is the main contribution of the paper. Results obtained are close to test values, without making unreasonable assumptions for the effects of saturation.

ANALYSIS OF ROTOR-BAR TEMPERATURES UNDER LOCKED TEST

The average time to take locked readings was 5.7 seconds. During this time the outer cage bars (hereafter referred to as 2) are loaded 84,000 to 65,000 amperes per square inch. At the latter value, the bars will rise 100°C in about 0.5 second with no heat loss. Curves are shown giving the effect of different loading for copper and 50% conductivity aluminum with no heat loss (see Fig. 1). The surrounding rotor iron will start taking heat from the bars as soon as there is a temperature difference between them. Professor A. D. Moore, of the University of Michigan, suggested a method calculating the conduction of heat from the aluminum to the iron, and calculations were made. Since the aluminum shrinks away from the rotor teeth when it cools, a model was made up and tested. It was found that heat was transmitted from a brass bar to a small stack of 0.75 x 0.75 laminations only 5/8 as fast with about 0.002 (the approximate shrinkage) clearance in the hole as compared to a tight fit. The temperature rises in the outer bar and surrounding iron with no other heat conduction were calculated on this basis and are shown in Fig. 2, which also shows the slot shape.

These computations are not included since they would take considerable space; but the information given in Fig. 2 will indicate that the enormous amount of heat cannot get into the iron fast enough to avoid a considerable rise in the aluminum-bar temperature.

The middle cage bar (3) or bridge between the outer and inner cage bar (4) will also be heavily loaded and will be losing heat to the rotor iron. At the same time it will conduct heat from bar 2 to bar 4 at the rate of 4.6 watt-seconds per second per degree difference in temperature. Accordingly, bar 2 is assumed to reach a temperature of 175 instead of 280 as shown in Fig. 2. Other temperatures used are shown on the circuit diagram, Fig. 3.

THE REACTANCES AND SATURATION

Knowing the current at starting, the ampere turns per slot can be calculated. At 563 amperes, there will be 281.5 amperes per circuit, or 2815 ampere conductors per slot. This will be 2815 ampere turns acting across the 0.150 slot opening, giving a maximum flux density of 84,600. No appreciable ampere turns can be absorbed by the iron at this density; therefore, no saturation will be allowed for the stator slot reactance. The pitch is 1-12; hence, there are only four slots in which both coil sides carry current of the same phase, for each of the three phases. In the other slots the current is less.

Bar 4 carries about 825 amperes, so it can produce a maximum flux density of 106,000 across the 0.035 slot. The iron on each side of the slot can absorb only about 15 ampere turns; hence, no saturation will be used for bar 4. The only saturated path at starting is the bridge over the tunnelled slot 2, which may be 0.002 - 0.004 thick at the center, and it will be carrying a flux density much beyond the range of our magnetization curves. This flux will be thrown in with the zig-zag reactance, which is calculated as 0.114 ohm unsaturated. This value is split 59% for the rotor and 41% for the stator. The reductions for saturation are to 63% for cold start, 70% for hot start, and 85% for pull-out. These are arbitrary values, in fact the only ones, in these calculations. A skew reactance of .0175 ohm is considered constant in each circuit.

MUTUAL REACTANCES AND METHOD OF WEIGHTING

When locked, the rotor bar currents will be of different value and phase. It would be convenient to use a value of self reactance plus two mutual reactances in each bar circuit. In circuit 2, I_2X_2 is the self reactance drop, I_2X_{2m3} is the mutual reactance drop in bar 2 caused by I_3, and I_2X_{2m4} is the same for the effect of I_4. The slot reactances are calculated nearly the same by everyone, and the following permeances should be easily verified. $K_2 = 0.66$, $K_3 = 4.5$, and $K_4 = 11.215$. These multiplied by .044 for this motor give reactances $X_2 = 0.029$, $X_3 = 0.198$, and $X_4 = 0.493$. The motor constants are $K = (100)^2 \times 3 \times 60 \times 10^{-8} = 0.018$. $K_s = (20.05 \times 8)/60 = 2.67$ for the stator. $K_r = (20.05 \times 8)(0.958 \times 0.914)^2/50 = 2.45$ for the rotor. $K_r = 0.018 \times 2.45 = 0.044$.

The mutual effect of bars 3 and 4 on bar 2 will use the permeance of a round slot with the current at a point on the circle or outside it, and this is 1.55. Since all the flux across the slot will not cut all of bar 2, it can be seen that half can be assumed to cut all the bar. Therefore, $.044 \times 1.55/2$ gives 0.0341 ohm as the mutual reactance between bars 3 or 4 and 2, and also between bar 2 and 3 or 4, when the currents are equal and in phase. Bar 4 sets up flux across slot 2, all of which cuts bar 3, and also a flux across slot 3, only half of which may be assumed to cut bar 3. So the mutual reactance between bars 3 and 4 is $.044(1.55 + 0.31/.035 \times 0.5) = 0.263$ ohm.

In order to avoid handling reactance drops that are not in phase, the following procedure is used. Say I_2 and I_3 differ by θ degrees. Mutual reactance to be put in circuit 2 for voltage induced by I_3, is $0.0341 \times I_3 \cos \theta/I_2$, and that to be put in circuit 3 for the effect produced by I_2 is $0.0341 \times I_2 \cos \theta/I_3$.

The circuit is set up and calculated with the best assumed values of total reactance for each circuit, then the currents and phase angles are found, the reactances corrected, and the circuit recalculated. If the first values were too far off, a third calculation may be necessary to get mutual reactances which will not change. This, of course, is not strictly correct, but it greatly simplifies the calculations.

THE CIRCUIT FOR STARTING HOT

In Fig. 3, the circuit is shown with the various final values of resistance and reactance, together with the currents and voltages in the equivalent circuit. Rotor bar currents can be found by multiplying by 5.25, the turns ratio. The actual voltage induced in a bar is $E_o/87.5$, the divisor being the effective conductors in series per phase. The equivalent impedance of the three bar circuits in parallel is $Z' = 0.1554 + j0.094 = 0.182$. The values in the common circuit are added to this giving $Z_r = 0.1759 + j0.2106 = 0.275$. The starting torque, hot, is $3(549)^2(0.1759) = 159000$ synchronous watts, which gives 2.79 per unit torque or 624 pound-feet. I_1 is 563 amperes and I_r is 549 amperes. Test values were 563 amperes and 622 pound-feet.

It is probable that the rotor averages 50°C higher than the stator, in which case the air gap would close down 0.025 on each side; but this was not used.

The circuit was tried starting cold to check the test values extrapolated back to zero time. Resistances were reduced to 25°C values, which gives 1.25 times the calculated value for R_4, to take account of unequal current distribution. When the current shifts towards the air gap, the slot reactance will also be reduced. However, for bar 4, this correction applies to only a small part of its total; hence, it is neglected both here and in the hot-start calculations. The mutual reactances changed slightly for bars 2 and 3 and increased from 0.1656 to 0.248 for bar 4 (see totals of bar reactances in Fig. 3). The zig-zag reactance was reduced to 63% for increased saturation. The calculations then gave 639 amperes at start and 607 pound-feet of torque, which closely checks the test values of 650 and 605.

BREAK-DOWN OR PULL-OUT TORQUE

Under this condition, the current in bar 4 is higher than at rotor locked, being 233 amperes (as compared to 154 amperes) in the equivalent circuit, or 5.25 x 233 or 1225 amperes in the bar. This would give a maximum flux density of 158,000 across the 0.035 gap if all the ampere turns were used on the gap. However, at this density, the adjacent iron will absorb an appreciable amount and the maximum density will be reduced to about 135,000. This will reduce the total flux about 6.3% by flattening the peak of the sine wave. This affects the major item of slot 4 reactance; the middle term in $K_4 = 0.805 + 8.86 + 1.55$, so this is changed to $K_4 = 0.805 + 8.3 + 1.55$. This times 0.044 gives 0.47 instead of the unsaturated value of 0.493 for slot 4 reactance, a rather small reduction.

Using the reduced X_4 and 85% of the X_{22} and the d-c value of R_4 with all resistances at 65°C except R_1 (which was taken at 35 degrees), the pull-out calculations gave $I_1 = 346$ and the torque 530 pound-feet against test values of 362 amperes and 528 pound-feet for 0.15 slip. Seventy-five percent of zig-zag reactance was also tried and gave 352 amperes and 540 pound-feet of torque, but it was assumed that the change from starting should be greater than that. However, it does bring the current closer.

FULL LOAD TORQUE

The full-load calculations gave 94.2 amperes and 105% torque at 0.01667 slip. Since there is no load loss in this circuit, both power factor and efficiency are high. The total reactance is low, which indicates that one or more of the reactances are not correctly calculated for this motor.

Alger and Wray[1] used for this motor 0.171-0.088 or 0.088 for the stator end-turn reactance instead of 0.057. If this value were used, the current would be reduced to 93.5 amperes. The stray load loss by test was 927 watts, which would add 0.036 ohm to the stator circuit. If this were also added, the current would drop to 92.7 amperes and the torque to 102% which is close to test values. However, the full-load calculations do not give so much trouble as the others. It may be that a higher stator coil-end reactance should have been used, that the zig-zag and skew reactances combined, and that both reduced drastically for starting.

SUMMARY

Another method has been presented for calculating the starting and pull-out performance of double- and triple-cage motors, in which the mutual effect of the rotor bars on each other is based on the physical conception of the conditions. Effects of saturation were investigated and found to require practically no correction except in the case of the zig-zag reactance. Since the rotor slot reactances can be figured accurately, the method used, although it entails some extra labor in making computations, leads to a greater assurance concerning the accuracy of the slot reactance values than can be said of the resistances.

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[1] "Power Apparatus and Systems", August, 1953, page 642.

The circuit was tried on a 7.5-hp double-cage motor for which satisfactory data were at hand and the greatest discrepancy was 36.25 pound-feet starting torque against 35 test. It is not often that a motor is tested so carefully and completely as was this 75-hp motor, so the opportunity to try a circuit on it was too inviting to resist.

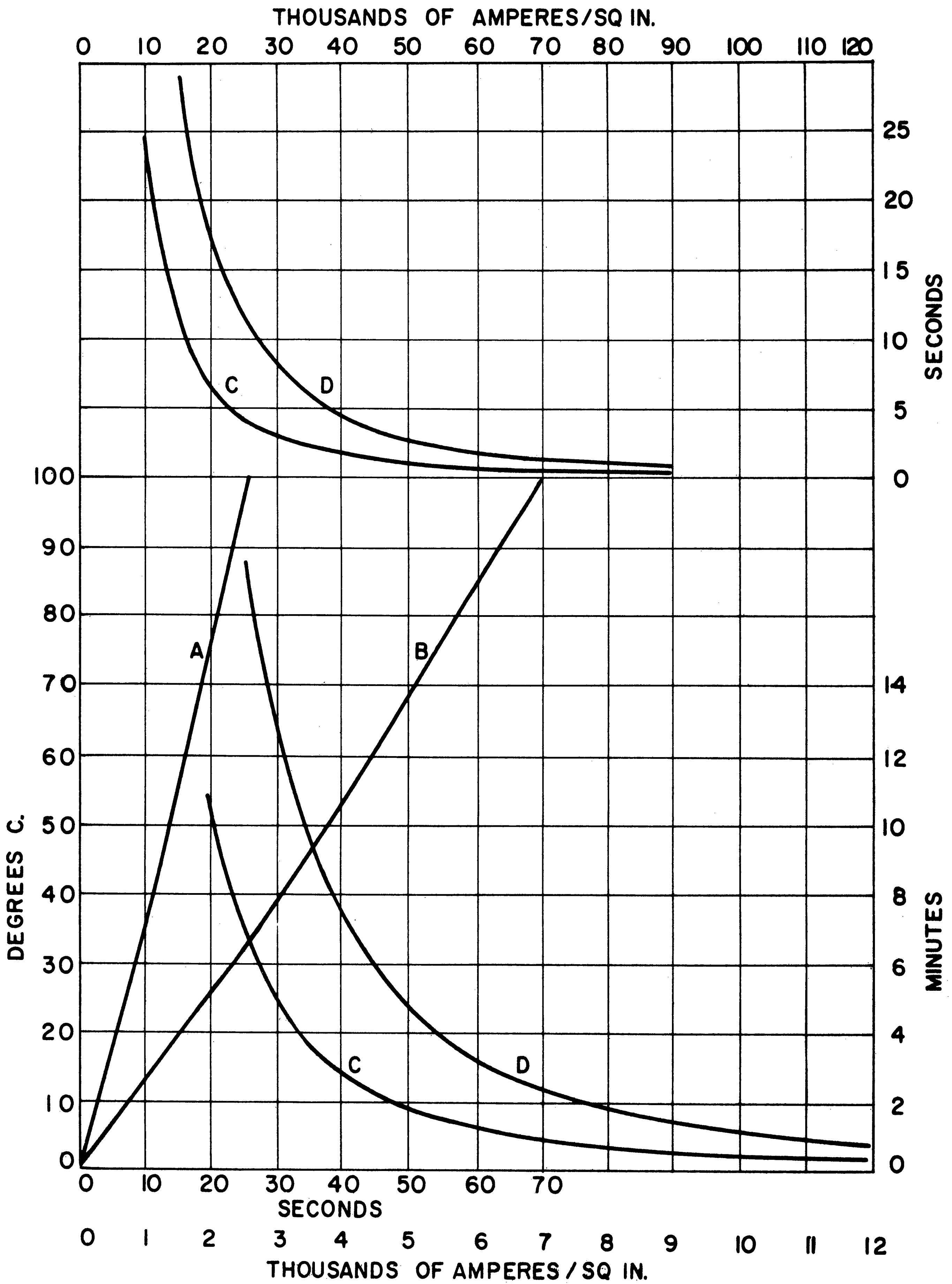
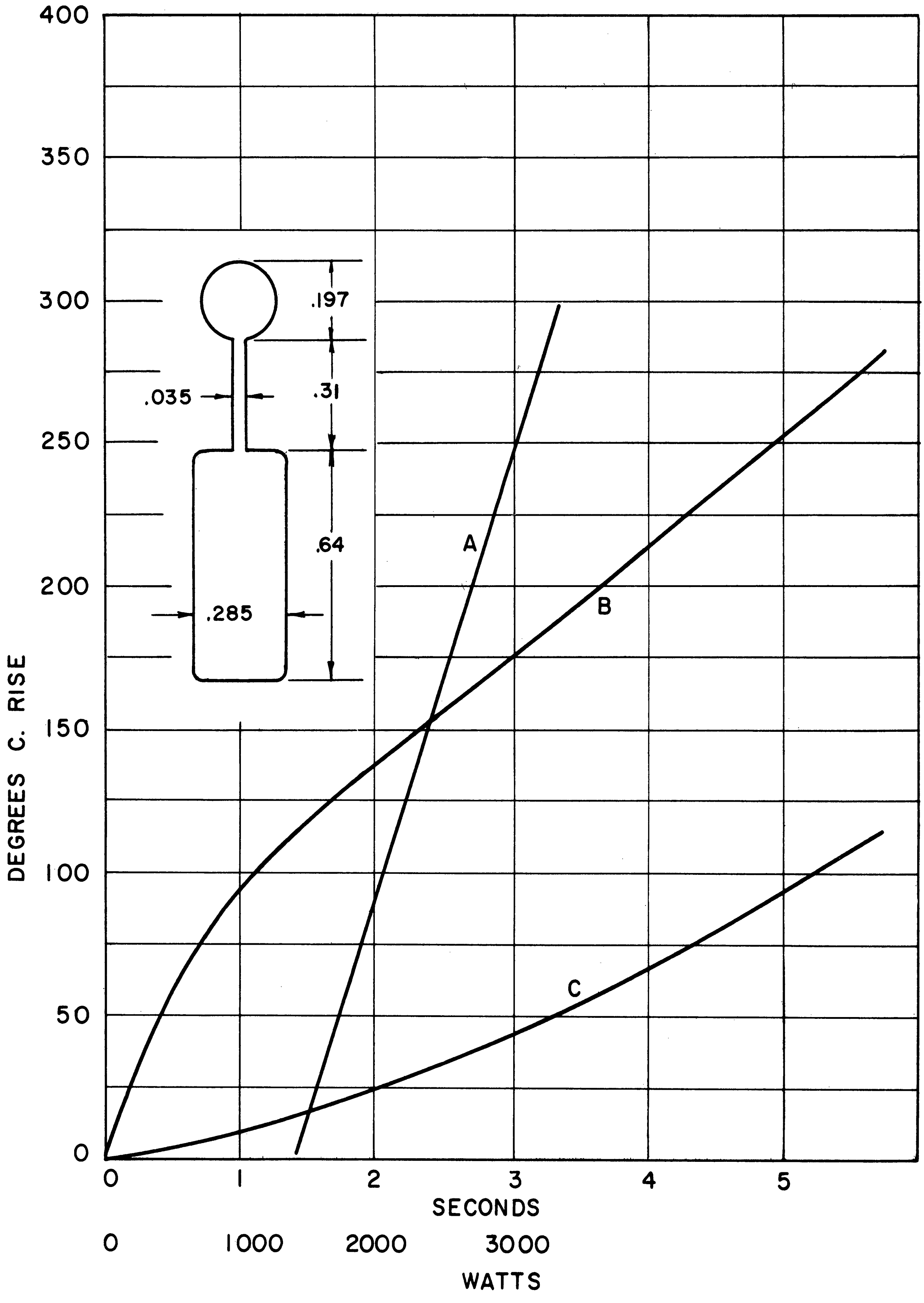
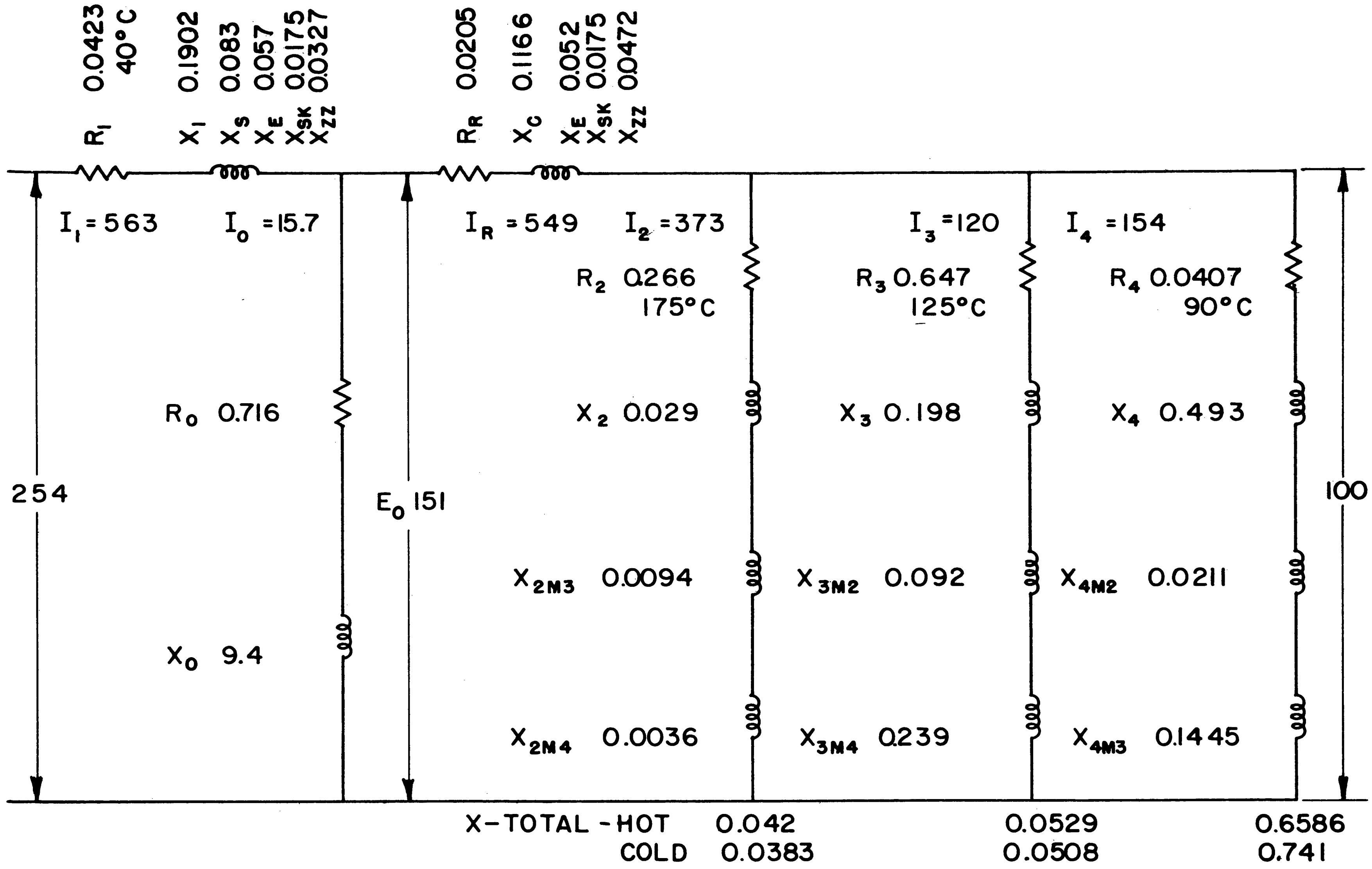

THOUSANDS OF AMPERES/SQ IN.
0 10 20 30 40 50 60 70 80 90 100 110 120
25
20
15
10
5
0
SECONDS
C
D
100
90
80
70
60
50
40
30
20
10
0
DEGREES C.
A
B
14
12
10
8
6
4
2
0
MINUTES
C
D
0 10 20 30 40 50 60 70
SECONDS
0 1 2 3 4 5 6 7 8 9 10 11 12
THOUSANDS OF AMPERES / SQ IN.

FIG. 1



400
350
300
250
200
150
100
50
0
DEGREES C. RISE
.197
.31
.035
.64
.285
A
B
C
0
1
2
3
4
5
SECONDS
0
1000
2000
3000
WATTS

FIG. 2



R1 0.0423 40°C
X1 0.1902
XS 0.083
XE 0.057
XSK 0.0175
XZZ 0.0327
RR 0.0205
XC 0.1166
XE 0.052
XSK 0.0175
XZZ 0.0472
I1 = 563
I0 = 15.7
IR = 549
I2 = 373
I3 = 120
I4 = 154
R2 0.266 175°C
R3 0.647 125°C
R4 0.0407 90°C
R0 0.716
X2 0.029
X3 0.198
X4 0.493
254
E0 151
100
X2M3 0.0094
X3M2 0.092
X4M2 0.0211
X0 9.4
X2M4 0.0036
X3M4 0.239
X4M3 0.1445
X-TOTAL-HOT 0.042 0.0529 0.6586
COLD 0.0383 0.0508 0.741

FIG. 3