MARKET SYSTEMS MODELING FOR PUBLIC VERSUS PRIVATE TRADEOFF ANALYSIS IN OPTIMAL VEHICLE DESIGN

by

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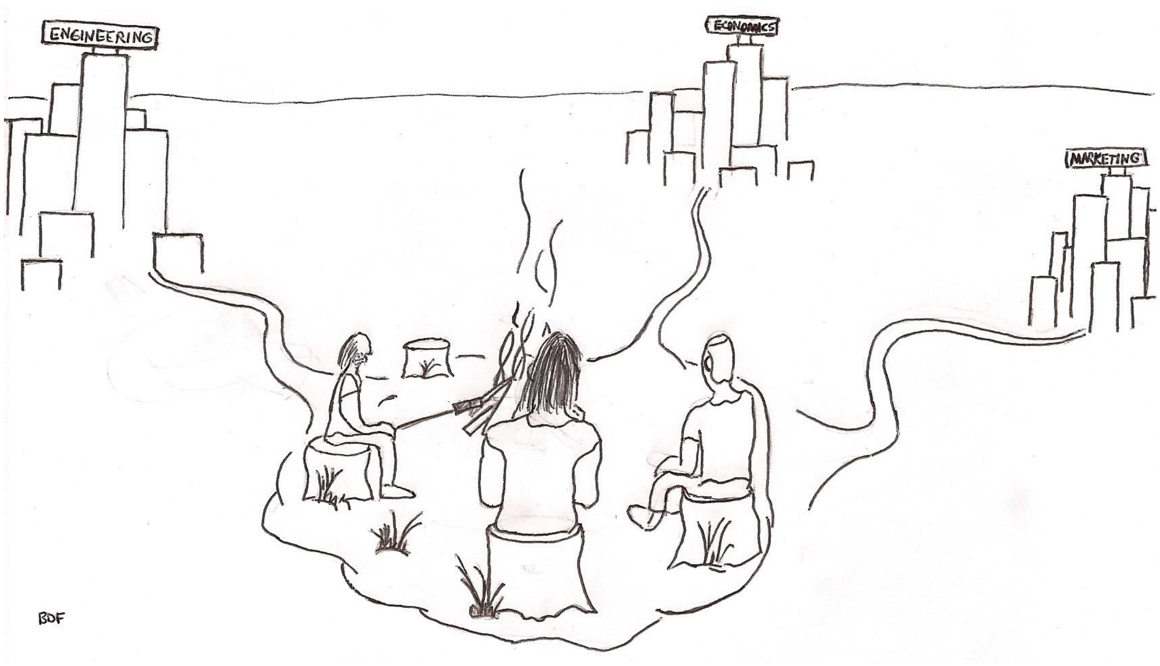
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PREFACE

At first blush a dissertation with the words "market systems" in the title appears out of place in the mechanical engineering department. The dissertation is motivated by two observations. The first is that establishing the product development problem as I have posed it requires engineering insight. The goal here is to make product design decisions endogenous to the overall product planning decision. Achieving this goal requires the ability to model the technology capability of the firm. This is precisely what engineers do: describe the physical interactions within a product that lead to the complex attribute colinearities lamented by marketers and economists.

The second observation is that engineers are the primary customers of this work. The benefits from the insights gained from a market system approach that includes the ability to change vehicle design are most valuable in the early stages of product planning. At the early stage planners have the most design freedom. A methodology for exploring the potential economic desirability of future technologies provides insight into R&D investment. Additionally, integrating engineering design models in a market system context begins to provide tools to engineers to enable communication with the broader product development organization. As communication across the organization improves, the expected outcome is improved profitability, product quality, and reduced time to market.

BF

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Δ_{X_s}	The sensitivity metric for the Pareto set defining the change in the value of one objective with respect to the change from the nadir to the ideal value of the other objective
Γ	Portion of the choice share derivative with respect to price as defined in Equation (5.16)
$\mathbf{\Gamma_v}$	The coupling vector derived from the KKT optimality conditions for a weighted-sum objective for a system design problem
Λ	Portion of the choice share derivative with respect to price as defined in Equation (5.16)
Φ	The area metric for a Pareto set defined as the area of the rectangle that inscribes the Pareto Set
ω_r	Cost model coefficient for cost model parameter r
α	Product attributes observed by consumers
β	Fixed coefficients for product attribute and demographic interactions for individual i and product j that enter the utility expression with fixed effects
δ	Fixed coefficiencts for product attributes that enter the utility expression with fixed effects
ϵ_{ij}	Independent and identically distributed random variable that represents a portion of the random component of utility for individual i for product j
ε_{ij}	Random component of utility
γ	Combined OEM and dealer profit margin
κ	Effective curvature of the Pareto frontier that defines the relative convexity or concavity of a particular tradeoff
λ	The products of the choice-share derivatives
λ_n	The scaling parameter used for the area and sensitivity Pareto set metrics

μ Random coefficiencts for product attributes and product and demographic interactions for individual i and product j that enter the utility expression with random effects

ν_{ij} Systematic component of utility for consumer i for product j

ϕ The constraint decoupling ratio indicating the ratio of the angle where constraint decoupling exists and the angle between the objective gradients

π Single period profit

ρ^2 Likelihood ratio index

$\theta^<$ The polar cone angle between two objective gradients

ω_{ABS} Empirical cost model parameter coefficient for anti-lock brakes

ω_{AWD} Empirical cost model parameter coefficient for all wheel drive or 4 wheel drive vehicle

ω_{CDI} Empirical cost model parameter coefficient for common-rail direct injection fuel injection system

$\omega_{C_{Lvan}}$ Empirical cost model parameter coefficient for large cargo van

$\omega_{C_{mvan}}$ Empirical cost model parameter coefficient for small cargo van

$\omega_{EDisp_{CI}}$ Empirical cost model parameter coefficient for compression-ignition engine displacement

$\omega_{EDisp_{SI}}$ Empirical cost model parameter coefficient for spark-ignition engine displacement

ω_{Eng1} First cost parameter coefficient for spark ignition engine cost formula

ω_{Eng2} Second cost parameter coefficient for spark ignition engine cost formula

ω_{Eng3} First cost parameter coefficient for compression ignition engine cost formula

ω_{Eng4} Second cost parameter coefficient for compression ignition engine cost formula

ω_{HEVcon} HEV powertrain cost calibration constant

ω_{HEVm} Multiplicative constant on all HEV powertrain costs

ω_{Man} Empirical cost model parameter coefficient for manual transmission

ω_{SC} Empirical cost model parameter coefficient for vehicle stability control feature

$\omega_{T_C I}$ Empirical cost model parameter coefficient for turbo-charged compression-ignition (diesel) engine

$\omega_{T_S I}$ Empirical cost model parameter coefficient for for turbo-charged or twin turbo-charged spark-ignition (gasoline) engine

ω_{VCUV} Empirical cost model parameter coefficient for crossover vehicle size

ω_{VLSUV} Empirical cost model parameter coefficient for large sport utility vehicle size

$\omega_{VLpupLvan}$ Empirical cost model parameter coefficient for large pickup or large van vehicle size

ω_{VLxSUV} Empirical cost model parameter coefficient for Luxury SUV or crossover vehicle size

ω_{VMSUV} Empirical cost model parameter coefficient for midsize sport utility vehicle size

ω_{VSpup} Empirical cost model parameter coefficient for small pickup truck vehicle size

ω_{Vmvan} Empirical cost model parameter coefficient for minivan vehicle size

ω_{con} Empirical cost model parameter coefficient for constant parameter

ξ The angle between two objective gradients

$4WD$ Four wheel drive

$\mathcal{A}$ The attainable set of objective values in a multiobjective optimization problem

AWD All wheel drive

$\mathbf{b}_{ij}$ Product attribute and demographic interactions for individual i and product j that enter the utility expression with fixed effects

B_{ik}^m The partial derivative of utility for individual i with respect to X_k^m of product k

$b_{m\cdot c}$ Minivan and children in household interaction dummy variable

$BMEP$ Brake mean effective pressure

$b_{p\cdot r}$ Pickup truck and rural living interaction dummy variable

$b_{s\cdot c}$ SUV and children in household interaction dummy variable

c Product unit cost

c_{avg} Per vehicle unit variable cost for average vehicle

c_{bod} The variable cost per unit of the body

c_{chas} The variable cost per unit of the chassis

c_{CIEng} Cost component from the compression ignition engine for empirical cost model

C_f Total fixed cost for manufacturer

c_{fun} The hypothetical variable cost to produce a single unit of the average vehicle given the specified learning curve effect

$c_{fun,bod}$ The hypothetical variable cost to produce a single unit of the body for an average vehicle given the specified learning curve effect

$c_{fun,chas}$ The hypothetical variable cost to produce a single unit of the chassis for an average vehicle given the specified learning curve effect

$c_{fun,pwtrn}$ The hypothetical variable cost to produce a single unit of the powertrain for an average vehicle given the specified learning curve effect

$c_{fun,whe}$ The hypothetical variable cost to produce a single unit of the wheels for an average vehicle given the specified learning curve effect

c_{HEV} Total unit variable cost for the HEV powertrain components

c_{HEVB} Cost for battery for HEV powertrain

c_{HEVBC} Cost for brackets and cables for HEV powertrain

c_{HEVcon} Cost for controller for HEV powertrain

CI Subscript used to indicate a compression-ignition or diesel engine

C_j Product total production cost

c_j Per vehicle unit variable cost

$c_{j,a}$ The assumed unit vehicle cost based on a given markup relationship between the OEM and the dealer

c_{MG1} Cost for electric machine 1 for HEV powertrain

c_{MG2} Cost for electric machine 2 for HEV powertrain

c_{pwtrn} The variable cost per unit of the powertrain

c_{SIEng} Cost component from the spark ignition engine for empirical cost model

c_{whe} The variable cost per unit of the wheels

D	The total number of draws for each individual i from the distributions of the random attribute partworths $\boldsymbol{\mu}$
d_{Chr}	European brand dummy variable
d_{Eur}	European brand dummy variable
d_{GM}	European brand dummy variable
$\mathbf{d}_j$	Product attributes for product j that enter the utility expression with fixed effects
d_{Jap}	European brand dummy variable
d_{Kor}	European brand dummy variable
$E_{ijX_k^m}$	The elasticity of demand for individual i for vehicle j is the percentage change in demand for j given a percentage change in attribute m of vehicle k
$E_{jX_k^m}$	The elasticity of demand for vehicle j is the percentage change in demand for j given a percentage change in attribute m of vehicle k
F	The total number of firms competing in the market
f	Subscript representing an individual firm $1, \ldots, F$
$\mathbf{f}^{\circ}$	The ideal or utopia point; the vector of ideal values for all criteria
f°_{MPG}	Ideal value for fuel economy for a given design optimization problem
f^{N}_{MPG}	Nadir value for fuel economy for a given design optimization problem
f°_n	The ideal value for objective f_n. In other words the optimum for objective f_n solved as a single objective problem
f^{N}_n	The nadir value for objective f_n. In other words the worst value for objective f_n that is also some Pareto point $\mathbf{f} = [f_n^N, f_l]$
FWD	Front wheel drive
f_{yj}	A dummy variable equal to 1 when for choice observation y product j is selected and $f_{yj} = 0$ otherwise
G	The combined gradient norm for the price equilibrium problem defined to be the L_∞-norm of the vector composed of the products of the choice-share derivatives $\boldsymbol{\lambda}$ and the difference between the initial prices and computed fixed-point prices $\mathbf{p} - \mathbf{q}$
g	Inequality constraint

GTDI	Gas turbo direct-injection engine
G_{tol}	The convergence tolerance for the combined gradient norm as defined in Equation (6.6)
H	Learning curve effect; value of 1 represents no learning
h	Equality constraint
HEV	Hybrid electric vehicle
I	The total number of individuals in a sample population
i	Subscript representing an individual consumer $1, \ldots, I$
J	The total number of individuals in a sample population
j	Subscript representing an individual product $1, \ldots, J$
J_f	The total number of products j designed by firm f
L	Likelihood function
L_{ij}	The conditional likelihood individual i chooses vehicle j for a particular μ_i
L_∞	The L-infinity norm taken to be the maximum value from a vector of values
LL	Log-likelihood function
M	Total market size
m_{CUV}	Crossover vehicle class dummy variable
$\mathbf{m}_{ij}$	Product attribute and product and demographic interactions for individual i and product j that enter the utility expression with random effects
$m_{LpupLvan}$	Large pickup truck or large van vehicle class dummy variable
m_{LSUV}	Large sport utility vehicle class dummy variable
m_{LxSUV}	Luxury sport utility or crossover vehicle (all sizes) class dummy variable
m_{MSUV}	Midsize sport utility vehicle class dummy variable
m_{mvan}	Minivan vehicle class dummy variable
m_{pup}	Pickup truck vehicle class dummy variable
m_{Spup}	Small pickup truck vehicle class dummy variable
$MSRP$	Manufacturer suggested retail price. This value is used to represent the price paid by the consumer

m_{SUV}	Sport utility vehicle class dummy variable
m_{tsmc}	Two seater or minicompact vehicle class dummy variable
m_{van}	Full size van vehicle class dummy variable
n	Subscript representing a specific objective criterion $1, \ldots, N$
OEM	Average operating leverage, which is the relative distribution of fixed versus variable costs
OEM	Original equipment manufacturer. In the context of this dissertation, OEM signifies a automobile producer such as Ford or Toyota.
p_{avg}	The average vehicle price for vehicles in the considered class used in the scaling cost model
P_{ij}	The unconditional likelihood individual i chooses vehicle j
P_j	Choice share for product j
p_j	Product unit price
$\mathbf{Pl}$	Matrix of choice shares where each row represents an individual from a population and each column represents a vehicle alternative
$\mathbf{p}'$	The computed fixed-point prices
P_{peak}	Peak engine power output
P_{tol}	A tolerance value indicating the minimum magnitude of the choice probability for a given vehicle in order for it to get updated by the fixed-point iteration
p_{tol}	The convergence tolerance for changes in price during the single-stage design and price equilibrium game computation
q	The total number of designing firms
Q_{avg}	The sales volume of the average vehicle
$\mathbf{Q}^{\geq}(\mathbf{x})$	Points in the decision space that are inferior to $\mathbf{x}$
Q_j	Quantity demanded for product j
$\mathbf{Q}^{<}(\mathbf{x})$	Points in the decision space that are superior to $\mathbf{x}$
$\mathbf{Q}^{\sim}(\mathbf{x})$	Points in the decision space that are not comparable to $\mathbf{x}$
R	Product total revenue
r	Product quality

r	Subscript representing a cost parameter $1, \ldots, R$
r_{stol}	The tolerance for the minimum improving stepsize in the fixed-point algorithm for the price equilibrium calculations
RWD	Rear wheel drive
$\mathbf{s}$	Individual demographics
$\mathcal{S}$	The set constraint for design variables
SI	Subscript used to indicate a spark-ignition or gasoline engine
s_{inc}	Individual annual income
SI	Spark-ignition gasoline engine
S_j	The observed choice share for product j from the data sample
U_{ij}	Utility of consumer i for product j
$\mathbf{v}$	model parameters assumed fixed during design optimization
v_{4WD}	Dummy variable for four wheel drive
v_{AWD}	Dummy variable for all wheel drive
v_{CD}	Vehicle coefficient of drag
v_{cI}	Nominal peak current for HEV powertrain
v_{cm}	Mass of a single battery cell for HEV powertrain
v_{cQ}	Nominal battery cell charge for HEV powertrain
v_{cV}	Nominal battery cell voltage for HEV powertrain
v_{Dies}	Dummy variable for diesel engine
$v_{EDTBc_3^*}$	Baseline $EDTBc_3$ value for HEV powertrain
v_{FWD}	Dummy variable for front wheel drive
v_{G1}	Transmission gear ration for first gear
v_{G2}	Transmission gear ration for second gear
v_{G3}	Transmission gear ration for third gear
v_{G4}	Transmission gear ration for fourth gear
v_{H156}	Vehicle minimum ground clearance

v_{HEV} Dummy variable for hybrid electric vehicle powertrain

$v_{K_{BM}}$ Scaling coefficient affecting spread in battery costs based on changes in the specific energy of the battery with respect to the reference battery

v_{MCC^*} The reference manufacturing cost in dollars per kg, for batteries given the reference specific energy

v_{MMPG} Parameter value indicating the minimum allowable fuel economy for a given design optimization problem

v_{MRW} Engine mounting midrail width

v_{MSH} Minimum required driver sitting height

v_{pay} Vehicle minimum payload capacity

v_{RWD} Dummy variable for rear wheel drive

v_{TC} Dummy variable for turbo-charged engine

v_{tw} Tire width

$v_{tw,avg}$ The average tire width for vehicles in the considered class used in the scaling cost model

v_w Wheel diameter

$v_{w,avg}$ The average wheel diameter for vehicles in the considered class used in the scaling cost model

W_r Cost model parameter r

W_{ABS} Empirical cost model parameter for anti-lock brakes

W_{AWD} Empirical cost model parameter for all wheel drive or 4 wheel drive vehicle

$W_{C_{Lvan}}$ Empirical cost model parameter for large cargo van

$W_{C_{mvan}}$ Empirical cost model parameter for small cargo van

W_{CDI} Empirical cost model parameter for common-rail direct injection fuel injection system

W_{con} Empirical cost model parameter for constant parameter

$W_{EDisp_{CI}}$ Empirical cost model parameter for compression-ignition engine displacement

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W_{SC}	Empirical cost model parameter for vehicle stability control feature
W_{T_CI}	Empirical cost model parameter for turbo-charged compression-ignition (diesel) engine
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W_{VLSUV}	Empirical cost model parameter for large sport utility vehicle size
W_{VLxSUV}	Empirical cost model parameter for Luxury SUV or crossover vehicle size
W_{VMSUV}	Empirical cost model parameter for midsize sport utility vehicle size
W_{Vmvan}	Empirical cost model parameter for minivan vehicle size
W_{VSpup}	Empirical cost model parameter for small pickup truck vehicle size
$x_{W105,avg}$	The average vehicle width for vehicles in the considered class used in the scaling cost model
W_{HEVBpk}	Battery per kilogram cost for HEV powertrain
w_n	Weighting value for objective f_n
$\mathbf{x}$	Design variables
$\mathcal{X}$	Feasible design domain
x_B	Engine bore diameter
x_{BPow}	Peak power output for hybrid electric vehicle battery pack
x_{BtS}	Engine bore to stroke ratio
x_{FD}	Final drive ratio
x_{H101}	Vehicle total height as a design variable
$x_{H101,avg}$	The average vehicle height for vehicles in the considered class used in the scaling cost model
x_{L101}	Vehicle wheelbase as a design variable
$x_{L101,avg}$	The average vehicle wheelbase for vehicles in the considered class used in the scaling cost model
x_{L103}	Vehicle total length as a design variable

$x_{L103,avg}$ The average vehicle length for vehicles in the considered class used in the scaling cost model

X_k^m The value of attribute m for vehicle k

x_{PGR} Planetary gear ratio

X_s The scaled difference between the nadir and ideal values for the first objective function

x_{W105} Vehicle total width as a design variable

y Subscript representing a choice observation $1, \ldots, Y$

Y_s The scaled difference between the nadir and ideal values for the second objective function

$\mathbf{z}$ Product performance criteria

z_{060} Time for vehicle to accelerate from 0 to 60 miles per hour time

z_{3050} Time for vehicle to accelerate from 30 to 50 miles per while towing

z_{65T} Maximum climbing grade at 65 miles per hour while towing

$z_{\$/mi}$ Dollars per mile fuel cost for a specific vehicle and price of gas

z_{A107} Angle of departure from rear wheel to rear bumber

z_{A147} Ramp breakover angle between front and rear wheels

z_{CAh} The stored charge per battery cell in Amp-hours for HEV powertrain

z_{CE} The energy storage capacity of a battery cell for HEV powertrain

z_{CGlong} Vehicle center of gravity in longitudinal direction

z_{CGvert} Vehicle center of gravity in vertical direction

z_{CS} Underhood crush space between bumper and driver heel point

z_{CVI} Cargo volume index measured behind 2nd row

z_{EDisp} Engine displacement

$z_{EDisp,avg}$ The average engine displacement for vehicles in the considered class used in the scaling cost model

z_{EDTBc_3} Energy density of a battery cell in Watt-hours per kilogram for HEV powertrain

z_{EL} Engine length

z_{GVWR} Gross vehicle weight rating

z_H Vehicle total height

z_{hp} Peak engine horsepower

z_L Vehicle total length

z_{MCS} Minimum calculated required underhood crush space

z_{MD} Deceleration in 35 miles per hour frontal crash

z_{MG1} Peak power output for motor-generator one

z_{MG2} Peak power output for motor-generator two

z_{MPG} Combined city and highway fuel economy

z_{NC} Number of battery cells for HEV powertrain

z_{Roll} Vehicle rollover score

z_{TF} Tire flop, distance in the width direction to allow tire movement

z_{tol} The convergence tolerance for changes in vehicle attributes during the two-stage design and price equilibrium game computation

z_{TS} Vehicle top speed

z_{VM} Vehicle mass, also known as curbweight

z_W Vehicle total width

ABSTRACT

Market Systems Modeling for Public Versus Private Tradeoff Analysis in Optimal Vehicle Design

by

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Chair: Panos Y. Papalambros

The motivating application for this work is the tension between the public versus private tradeoff in the automotive industry between firm profit and the public negative externalities of automotive transportation, particularly fossil fuel energy use and greenhouse gas emissions.

This dissertation establishes a methodology for evaluating automotive vehicle design according to private (firm profit) and public (fuel consumption) criteria. The methodology set forth relies on developments from engineering, economics, and marketing. The primary contribution of this dissertation is that these disparate developments have been brought together in a single mathematical problem formulation for a large-scale problem. The integrated problem formulation will allow study of interdisciplinary issues related to product development in a new way. Other work has begun to develop similar comprehensive problem formulations. This work points to some of the challenges that must be addressed in such formulations. Specifically, the functional form of the cost models, and the utility specification of the demand models can have a large impact on the market outcomes even when the differing models ap-

pear to fit the underlying data similarly well. A second contribution is the application of the notion that we can explore the tradeoff between private interests and public interests by simulating market response under different hypothetical scenarios. We can then gain deeper insights by examining the tradeoff relationships between the different scenarios.

The strength of the approach, at its current state of development, lies not in a claim to predict future automotive market behavior but in establishing a quantitative approach for evaluating the implications of future scenarios were they to become reality. Individual firms and policy makers can learn from this approach by comparing the differences between the scenario outcomes.

The problem formulation integrates models of demand, cost, and product performance in order to implement a game-theoritic formulation of producer behavior where producers choose the attributes of the products they produce and the prices they will charge in order to maximize profit. Two variations of a newly estimated mixed-logit discrete choice model of new car buyer purchase behavior are developed for incorporation as demand models. Three cost model formulations are developed and compared in the context of the problem formulation. An explicit representation of an automotive manufacturer's technology capability in the form of a comprehensive yet stylized engineering performance model is developed. Novel metrics are established for comparing the Pareto set of solutions from one hypothetical scenario to the next. Hypothetical scenarios are evaluated involving the design of a single vehicle within a price-equilibrium market context and the design of multiple same-segment vehicles within a price-equilibrium market context. The differences in scenario outcomes based on differences in the demand and cost models are explored. The results show that improving the fuel economy of a specific vehicle does not always lead to a reduction in US fleet fuel consumption.

Several areas for modeling improvement are identified.

CHAPTER I

Introduction

Increasing environmental sustainability related to the automotive vehicle industry can be seen as a public good. Negative externalities from the existing automotive-centric transportation system vary from harmful airborne emissions, traffic congestion, noise, and crashes, to greenhouse gas emissions. Fuel taxes, emissions standards, and fuel economy standards transfer the costs of these externalities to private actors, namely automotive manufacturers and individual drivers. There is much debate about the effectiveness of the current set of regulatory policies in mitigating the negative externalities of automotive transportation especially as climate change has captured public attention. A diversity of technologies is expected to proliferate over the next decades including hybrid, plug-in hybrid, clean diesel, bio-fuels, and fuel cells [*J.D. Power and Associates* (2006)] that could have a substantive impact on greenhouse gas emissions from automotive trasnportation.

This dissertation establishes a methodology for evaluating automotive vehicle design according to private (firm profit) and public (fuel consumption) criteria. The methodology set forth relies on developments from engineering, economics, and marketing. The primary contribution of this dissertation is that these disparate developments have been brought together in a single mathematical problem formulation for a large-scale problem. The integrated problem formulation will allow study of interdisciplinary issues related to product development in a new way. Other work has begun to develop similar comprehensive problem formulations. This work points to some of the challenges that must be addressed in such formulations. Specifically, the functional form of the cost models, and the utility specification of the demand models can have a large impact on the market outcomes even when the differing models appear to fit the underlying data similarly well. A second contribution is the application of the notion that we can explore the tradeoff between private interests and public interests by simulating market response under different hypothetical scenarios. We can then gain deeper insights by examining the tradeoff relationships between the different scenarios.

The strength of the approach, at its current state of development, lies not in a claim to predict future automotive market behavior but in establishing a quantitative approach for evaluating the implications of future scenarios were they to become reality. Several areas for modeling improvement are identified. Individual firms and policy makers can learn from this approach by comparing the differences between the scenario outcomes.

This dissertation develops model components that represent the technical feasibility, the cost, and the consumer demand of new automotive vehicles. The approach is not tied to the automotive industry. It can be applied to a range of products and markets as well as a range of public interests.

1.1 Product Development as a Quantitative Decision-making Process

The description of a product depends on the disciplinary perspective adopted. For example, a product may be a complex assembly of interacting components to an engineering designer. It may be a sequence of development and production process steps to a supply-chain manager. The product may be a bundle of attributes to a marketer, or it may be viewed by an executive as the outcome of an organizational process [*Krishnan and Ulrich* (2001)]. Each description is useful in achieving the goals of a product development organization. None of the descriptions are complete. Analogous to the communication in actual organizations, we can develop mathematical abstractions of each description. Using the mathematical descriptions of a product we can observe the interactions between stakeholders, and we can exercise the mathematical framework under different scenarios to gain insights into the actual product development problem. The decision-making framework to be implemented is adapted from that introduced by Georgiopoulos [*Georgiopoulos* (2003)] and Michalek [*Michalek* (2005)].

1.1.1 Design for market systems framework

We adopt quantitative measures for each product development perspective. Engineering design will be represented as a vector of performance criteria $\mathbf{z} = \mathbf{f}(\mathbf{x})$ that are a function of product design variables $\mathbf{x}$. The marketing interest is assumed to be a representation of consumer demand $Q_j(\mathbf{U}_{ij}); i = 1, \ldots, I; j = 1, \ldots, J$ where $U_{ij} = u(\boldsymbol{\alpha}(\mathbf{z_j}), p_j)$ is the utility of consumer i as a function of product attributes Product attributes $\boldsymbol{\alpha}(\mathbf{z_j})$ including price p_j for product j. Product cost $C_j = c(\mathbf{x_j}, Q_j(\mathbf{U_{ij}}))$, a function of product design variables $\mathbf{x_j}$ and quantity demanded $Q_j(\mathbf{U_{ij}})$, represents operations. Single-period profit $\pi = Qp - C$ is taken as the organizational objective. In general, each firm will produce multiple differentiated products ($Q_f = \sum_{j=1}^{J_f} Q_j$). Finally, each firm in the market faces competition from other firms. By adopting a consumer utility measure based on product attributes and price we assume that a firm's products compete in the market on these attributes. The demand function for firm f is modified to account for competitors $Q_f = \sum_{j=1}^{J} \sum_{i=1}^{I} Q_j(\mathbf{U}_{ij}, \mathbf{U}_{ik}),\ \forall k = 1, \ldots, J \neq j$.

The interactions of the various product descriptions can be formalized into a

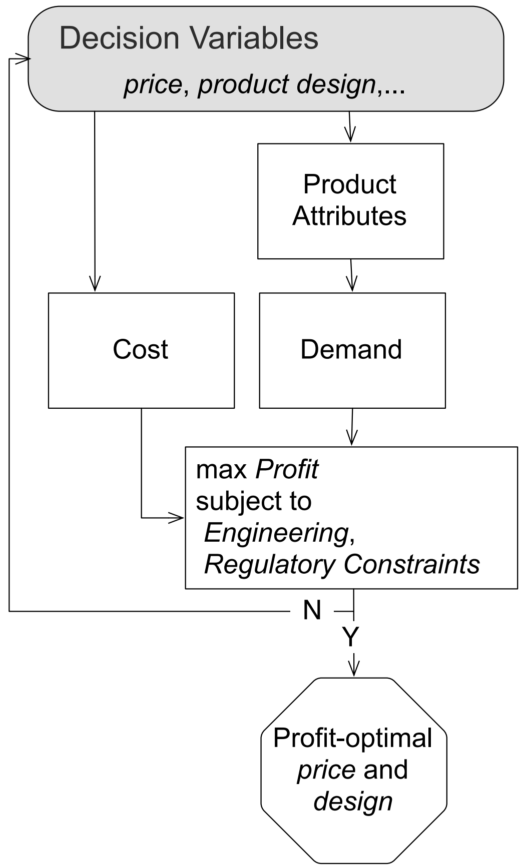


Figure 1.1: Single-firm design-decision-model schematic

mathematical optimization problem representing the firm's profit interest.

$$\begin{aligned} \max_{p,\mathbf{x}} \quad & \pi(p, \mathbf{x}; \mathbf{v}) \\ \text{subject to:} \quad & \textit{engineering constraints} \\ & \textit{regulatory constraints} \end{aligned} \tag{1.1}$$

where the objective is to maximize profit with respect to design variables $\mathbf{x}$ and price p, given a set of fixed parameters $\mathbf{v}$, subject to engineering and regulatory constraints. A graphical description of the interacting functions is shown in Figure 1.1.

The design decision model can be expanded to include multiple firms. The firms' decisions are coordinated using a game-theoretic structure describing the competition between firms. The interaction between firms is illustrated in Figure 1.2.

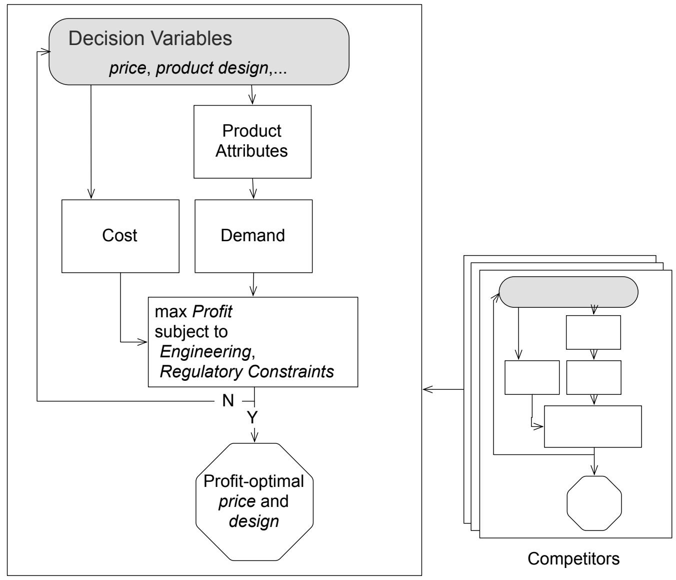


Figure 1.2: Multi-firm design-decision-model schematic

A mathematically formalized decision model as described in Equation (1.1) is a suitable subject for counterfactual experiments. Counterfactual experiment is a term derived from economics to describe the exercise of modifying model parameters in order to study the model response to the new parameter values. The word counterfactual is used to textslasize that the model is exercised using parameters other than those originally found to estimate best the observed market activity without implying that the results reflect a prediction of future market activity. For example, using data on vehicle miles traveled in the US, it may be possible to fit a model that estimates the vehicle miles traveled in a given year as a function of gasoline price, gross domestic product, etc. A counterfactual experiment would be to change the price of gas for a given year and observe the difference in vehicle miles traveled between the original model and the model with the change in gas price.

The work in design for market systems has evolved from the notion that counterfactual experiments can indeed inform future design decisions either as scenario generators or as full-fledged prescriptive models. This is a similar view to the marketing community whose work seeks to be normative rather than positive [*Chintagunta* *et al.* (2006)]. A question in design for market systems is how to identify sensible optimization outcomes within a counterfactual experiment. If the original market is in price and design equilibrium and the demand and cost models capture this, then a design optimization market simulation should replicate the existing vehicle prices and designs. The researcher could then experiment with counterfactual demand- or supply-side shocks.

If the market is not in design (or price) equilibrium, then a design optimization market simulation will suggest changes to firms' designs (prices) in order to maximize profits. The confounding question becomes how to differentiate a situation where the market is in design and price equilibrium but the cost and demand models are inadequate from a situation where the market is not in equilibrium and the cost and demand models are reflective of the market. We conjecture that one way forward is to develop models with sufficient flexibility to allow testing of either scenario. Such models would necessarily accommodate the effects of attribute and not only price changes. This dissertation is a first step in establishing such models.

Although the majority of the design examples in the literature have adopted published choice models, we hypothesize that this practice may contribute to poor market simulation results regardless of whether the market is in equilibrium or not. This can occur when the estimated choice model describes well the aggregate consumer behavior given a fixed vehicle fleet, but misses preference nonlinearities and correlations between attributes, which would mislead the design optimization. In this dissertation we seek to expand the understanding of demand modeling in a design context, leaving the comprehensive evaluation of integrated engineering, product cost, and demand models to future work.

1.1.2 Economic Assumptions

The development of the previous sections implies three economic assumptions. Namely, producers are profit-maximizers; consumers are utility-maximizers; and competitive behavior is well-defined. There is a tension between these assumptions and the aims of the analyst. The assumptions are intuitively appealing, and they make the problem computationally tractable. However, we desire the results to be useful for normative rather than positive analysis. If the basic assumptions held in reality, then producers would have already determined methods for maximizing profits, and the market would be in price and design equilibrium. This dissertation assumes that the automotive market demonstrates static price equilibrium during a single period (year). We also assume vehicle producers seek to produce vehicles that maximize profit. However, we make no assumption about the state of market equilibrium with respect to vehicle designs. Calculating price equilibrium of the US automotive market under policy changes by a firm or firms (i.e., design changes in one or a small number of vehicles) is then used as one method for evaluating market response to the introduction of new technology or regulation.

We make several assumptions about the structure of competition. Much of the economics literature evaluating the automotive market assumes that the vehicle producer sets vehicle price directly. Recent work in the design for market systems literature confirmed that modeling the vehicle producer and the vehicle dealer as independent decision-makers changes the nature of the design decision problem for the vehicle producer [*Shiau and Michalek* (2009)]. To simplify the discussion of the work in this dissertation we evaluate the case where the dealer markups are fixed and assumed known a priori.

Policy analysts have used a single-stage equilibrium where a producer makes all decisions simultaneously. Industrial organization economists view this approach as simplistic. They suggest a subgame perfect equilibrium where all producers make product design decisions before making a subsequent decision to update prices. This idea is supported by the notion that producers have more freedom to control prices than vehicle designs [*Tirole* (1988)]. We adopt a subgame perfect equilibrium approach where the pricing game is the subgame within the product design game. We adopt a fixed-point iteration algorithm for computing price equilibrium of the automotive fleet as developed in [*Morrow* (2008)].

1.1.3 Automotive Vehicle Application

The mathematical model described by Equation (1.1) and Figure 1.2 is illustrated through an example representing the US automotive market. Models have been developed to represent product attributes, demand, and cost.

Product attributes

Product attributes $\boldsymbol{\alpha}$ are assumed to be functions of product design variables $\mathbf{x}$. Product attributes may be a one-to-one or some other transformation of product performance criteria $\mathbf{z}(\mathbf{x})$. The fidelity of product attribute representation is determined by the quality of the models translating the physical design variables to product performance criteria. For the purposes of this research we assume that the engineering models of vehicle performance adequately describe the relationships between product design decisions and product attributes. Sensitivity analysis can be employed to look at the effects of uncertainty.

Engineering design models were built to represent a five-passenger mid-size crossover vehicle to evaluate proposed vehicle concepts in terms of vehicle attributes such as fuel economy, acceleration, range, crashworthiness, and cargo capacity. Three powertrain options are modeled for the vehicle: gasoline spark-ignition internal combustion engine; gasoline turbocharged direct-injection internal combustion engine; and a split-mode hybrid vehicle design with a battery for energy storage, a gasoline internal combustion engine, and two electric machines for battery charging and locomotion. New engineering models were developed for this dissertation with the exception of the hybrid vehicle model which was developed by Kukhyun Ahn from other work [*Ahn* (2008)]. The engineering design model is described in Chapter 3.

Consumer demand

Consumer demand for new automotive vehicles is modeled based on random utility theory [*Keeny and Raiffa* (1976)]. A choice model indicates the probability that individual i selects a product j given a utility expression U_{ij}. A specific choice model may represent consumers as having homogeneous or heterogenous preferences. We adopt utility expressions that are a function of individual demographics $\mathbf{s}$ such as income, household size, location of residence, etc.; and product attributes $\boldsymbol{\alpha}$. We assume that the aggregate choice shares P_j for each product j can be represented by summing the individual choice shares P_{ij} for each product.

$$P_j = \frac{1}{I}\sum_{i=1}^{I} P_{ij} \tag{1.2}$$

Market demand is the product of aggregate choice share and total marketsize M.

$$Q_j = MP_j \tag{1.3}$$

In this dissertation we briefly discuss existing choice models from the literature in Chapters 2 and 4, and we estimate new choice models for the US automotive industry as described in Section 4.2. The data for the new models comes from a survey of new car buyers and detailed vehicle data.

Cost model

A cost model represents all of the relevant economic information about a firm's technology for maximum profit considerations [*Varian* (1992)]. The cost model relates company cost to specified levels of outputs or, in our case, to a product with specified attribute levels. A cost model can be expressed as a function of product attributes or directly as a function of design and process variables. The domain of the cost model describes the feasible output levels of the firm. We present and compare three cost modeling approaches for the US automotive market. One approach seen in the marketing and economics literature is to assume pricing decisions represent market equilibrium outcomes. Then, cost and utility model forms are postulated and unknown coefficients are estimated for both models. This approach was taken in [*Berry et al.* (1995)], for example. We implement this approach as described in Section 5.1.4.

A second approach we propose here is to identify cost drivers from the physical components of the product and regress a cost relationship based on price (see Section 5.1.3). We have implemented such an approach using data from model year 2005 in Ward's Automotive Yearbook [*Wards Communications* (2006)].

The third approach modifies a cost model from De Weck [*De Weck et al.* (2006)] and Cook [*Cook* (1997)] and is described in Section 5.1.2. It is based on assigning a cost to a hypothetical average vehicle and then computing the cost for a specific vehicle based on design deviations from the average. Approaching cost modeling in this way enables design-specific cost differences to be considered without requiring a complete bottom up cost structure. This approach requires individual cost models for each vehicle class of interest, as opposed to the other approaches that suggest cost relationships for the entire vehicle fleet.

Competition

The mathematical modeling of the competitive interaction among firms is described in Chapter 6.

Optimal Vehicle Design Problem

The implementation of the optimal vehicle design problem is described in Chapter 8.

1.2 Public versus Private Tradeoffs

This dissertation follows previous work on public and private interest in vehicle design [*Michalek et al.* (2004); *Wissmann and Yassine* (2005)]. We look at this problem in more detail by developing an engineering model with greater delity for a specic vehicle class, by developing new demand models from disaggregate consumer data, by considering the impact of the entire market through rm pricing behavior, and by formalizing metrics for comparing bi-objective optimization problems that can be applied to public versus private tradeoff scenarios.

Increasing the energy efficiency of personal transportation, reducing vehicle miles traveled, or switching to more efficient modes of transportation are three high-level strategies for decreasing the environmental impact of personal transportation. Given a fixed number of vehicle miles traveled, emissions outcomes are directly related to the fuel source and fuel economy of transportation vehicles. Fuel consumption is adopted as the public good objective in these studies.

1.2.1 Externalities

Economic externalities affect decision-making. An externality exists when consumers and producers do not explicitly consider all costs and benefits associated with

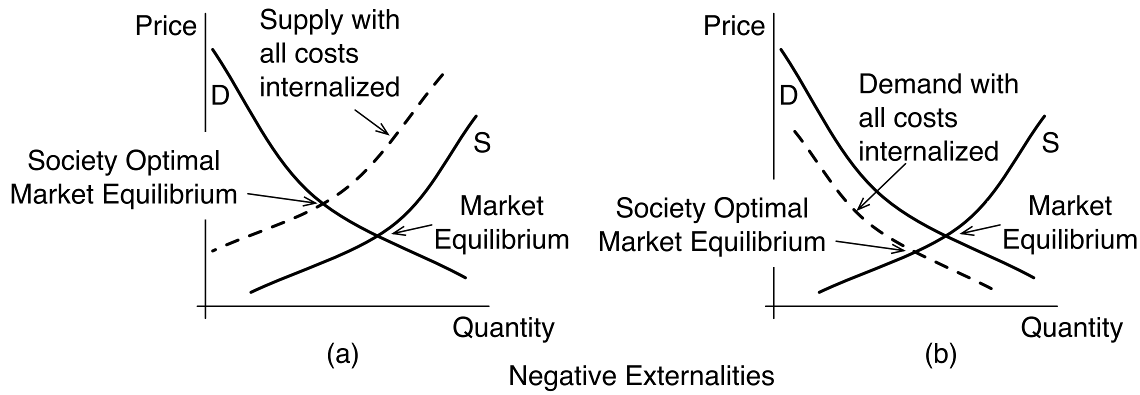


Figure 1.3: (a) Market equilibrium compared to societal ideal with externality internalized by supplier. (b) Market equilibrium compared to societal ideal with externality internalized by consumer

their choices, as observed by society. A negative externality occurs when a decision maker does not bear the full cost of a decision and over-produces or over-consumes. Figure 1.3 shows two cases where the market equilibrium point is different compared with the societal ideal. Figure 1.3a shows the case where all external costs have been internalized by the supplier. For example, this situation could result if automotive vehicle producers were taxed directly for each vehicle they produced. Figure 1.3b shows the case where all external costs have been internalized by the consumer. For example, this situation could result if consumers were taxed directly for each vehicle they purchase. In fact, both of these examples already occur in the US automotive market. The size of the tax on consumers and producers determines how different the observed market equilibrium is from the hypothetical unregulated market equilibrium.

Externalities occur frequently with public goods such as clean air because a public good is not excludable and is nonrival [*Varian* (1992)]. This means that an individual can not be excluded from consuming the good and one person's consumption does not limit another persons ability to consume the good. Negative externalities related to automobiles include traffic congestion, harmful pollutant emissions, road degradation, accidents, and greenhouse gas (GHG) emissions. For example, increased vehicle fuel economy has a personal benefit through lower cost of transportation. However, the societal benefit of lower greenhouse gas and other harmful emissions and greater energy security are not captured in the vehicle or fuel purchase transaction. The result is lower production of higher fuel economy vehicles, or fuel and vehicle prices that are too low.

The basic strategy for eliminating externalities is to create a new market (convert the public good into a private good), or to incorporate the costs and benefits that were once external into an existing market. The debate surrounding externalities is not usually about their existence, but the price society should be willing to pay to encourage positive externalities and discourage negative externaities.

Mechanisms such as fuel taxes, emissions standards, and fuel economy standards are all measures that place additional cost on consumers or vehicle producers. However, one may argue that these measures do not internalize the total cost to society, and that an externality still exists, particularly with respect to GHG emissions. This dissertation will not discuss public valuation [*Arrow et al.* (1993)] or finding the "right" price for GHG emissions. Instead, we present analysis of the trade-off between profit and fuel consumption to support the private decision of a producer to act in its best interest. This construction has the side-benefit (i.e., positive externality) that it can be used to aid policy makers in quantifying one aspect of the public versus private debate with respect to fuel consumption and the automotive vehicle industry.

1.2.2 Firm Perspective

We adopt an enterprise-wide trade-off model [*Hazelrigg* (1998); *Michalek et al.* (2004); *Wassenaar and Chen* (2003)] with two objectives: a private one (a firm's stated business objective to maximize profit) and a public one (a firm's stated social objective to minimize environmental impact). The enterprise balances these competing objectives with price and product design as decision variables. Firm motivation to pursue a social objective may come from a desire to pursue corporate social responsibility for brand building purposes, from anticipation of future government regulations, from anticipation of changing consumer preferences, from a desire to proactively shift consumer preferences, or from a desire to mitigate risk under uncertain exogenous effects such as fuel prices.

The firm may choose to see its influence on the public objective on multiple levels. Firms whose revenue stream relies on the sale of new vehicles may influence the public outcome primarily by reducing the on-road GHG intensity of one or multiple vehicle offerings. They may assess their impact on multiple levels: the firm's fleet GHG intensity, the firm's sales-weighted fleet GHG intensity, or the entire US automotive new vehicle sales-weighted fleet GHG intensity compared to a baseline scenario.

1.2.3 Greenhouse Gas Emissions as a Negative Externality in the Automotive Industry

Transportation in the US delivers a significant environmental burden, causing about 28% of the greenhouse gas emissions in the US in 2004. Light-duty trucks alone accounted for over 27% of the total greenhouse gas emissions from the transportation sector [*USEPA* (2006a)]. In addition, light duty ground vehicles (passenger cars and trucks) contribute about 28% of CO, 11% NO_X, and 15% VOC emissions in the U.S. and are also sources of PM, SO_2, and NH_3 [*USEPA* (2000)].

Numerous methods have been proposed for analyzing and labeling environmental impact of products and services [*EUCAR et al.* (2004)]. Areas of concern for the automobile include contribution to global warming, harmful emissions, and nonrenewable resource and energy use. The vehicle-use phase constitutes as much as 85% of GHG and other emissions [*Keoleian et al.* (1997); *Graedel and Allenby* (1998)]. Research has found significant correlation between cumulative fossil fuel energy use and numerous other environmental indicators [*Huijbregts et al.* (2006)], so fossil fuel use attributed to the use phase of a vehicle life cycle (fuel economy) appears to be a suitable metric for assessing environmental performance. For advanced technologies, gasoline equivalent fuel economy measures are being proposed and developed [*Gonder and Simpson* (2006); *Markel et al.* (2006)].

A miles per gallon gasoline equivalent metric for advanced technology vehicles may be difficult to interpret. What could once be communicated in a single number now requires multiple metrics to communicate (e.g., $/mile, frequency of trips to the pump, barrels of foreign oil consumption, greenhouse gas (GHG) emissions). Should a miles per gallon gasoline equivalent (MPGGE) that is calculated on a mile/joule or mile/$ basis become prevalent in the future, some other metric for recognizing the GHG contribution of driving a vehicle may be relevant. A straightforward approach for doing this is to follow a well-to-wheels approach for energy required in the use phase similar to what will be done for any MPGGE calculation. Several well-to-wheels studies have been performed that track such factors as total energy use, fossil fuel use, GHG emissions, criteria pollutant emissions, etc. in the US [*Wang* (2001); *Brinkman et al.* (2005); *ANL* (2007); *Wang* (2002)] and in Europe [*EUCAR et al.* (2004, 2006)]. These studies typically consider numerous energy paths including production and distribution of conventional gasoline, hydrogen, biofuels, and electricity. Data from these models could support future work for well-to-wheels GHG emissions calculations for proposed vehicle designs.

1.2.4 Previous Studies

The National Energy Modeling System developed by the Energy Information Administration [*Gabriel et al.* (2001)] uses an equilibrium framework to project energy consumption and prices for the US. It has been used to analyze various scenarios regarding penetration of advanced technology vehicles among others. Argonne National

Lab's VISION model [*ANL* (2006)] is a spreadsheet model specifically for projecting impacts of advanced technology transportation vehicles on energy and greenhouse gas emissions outcomes. The base case for this model is updated from the Energy Information Administration's annual report.

General Motors, Argonne National Lab, and Shell partnered to develop well-to-wheels analysis for a full-sized pickup truck [*Brinkman et al.* (2005)]. Their intent was not to look at market dynamics, but instead to capture the vehicle characteristics and the associated environmental impact that would result from a fully-functional pickup truck using various powertrains and fuel pathways. They could then use this data to assess the economic viability of each option.

A 2004 NESCCAF report [*Cooper et al.* (2004)] studied the emissions impacts of various advanced vehicle technology packages across five different vehicle classes. They considered a cost-effectiveness metric to make recommendations about which technologies should be adopted by the market.

These previous studies are primarily analysis tools. They do not project automotive vehicle demand; rather, they require this as an input to then analyze the environmental outcomes.

As cited previously, some preliminary work has proposed using a design for market systems approach to study the public vs. private goods automotive design problem. Michalek et al. [*Michalek et al.* (2004)] focused on the impact of producer design decisions (engine selection) in the face of various hypothetical regulatory regimes. Wissmann and Yassine [*Wissmann and Yassine* (2005)] developed a design for market systems framework that demonstrated how producers may adapt designs over time given changes in fuel price, price of steel, and regulation.

The goal of this dissertation is to bring greater fidelity to the engineering application and greater sophistication and modeling insights to the demand and cost sides to enable more realistic application of the design for market systems idea.

1.3 Expected Contributions

The methodology set forth relies on developments from engineering, economics, and marketing. The primary contribution of this dissertation is that these previously-made disparate developments have been brought together in a single mathematical problem formulation for a large-scale problem. The integrated problem formulation will allow a new way to study interdisciplinary issues related to product development. Other work has also begun to develop similar comprehensive problem formulations. This work points to some of the challenges that must be addressed in such formulations. Specifically, the functional form of the cost models, and the utility specification of the demand models can have a large impact on the market outcomes even when the differing models appear to fit the underlying data similarly well. We show this by comparing outcomes for the vehicle design problem formulation using two different cost models and two different demand models.

A second contribution is the application of the notion that we can explore the tradeoff between private interests and public interests by simulating market response under different hypothetical scenarios. We can then gain deeper insights by examining the tradeoff relationships between the different scenarios. Novel metrics are established for comparing the Pareto set of solutions from one hypothetical scenario to the next that can be public and private tradeoffs derived from hypothetical market simulations.

Practical contributions include the development of several models that can be applied to other similar problems. Specifically, the problem formulation integrates models of demand, cost, and product performance in order to implement a game-theoritic formulation of producer behavior where producers choose the attributes of the products they produce and the prices they will charge in order to maximize profit. Two variations of a newly estimated mixed-logit discrete choice model of new car buyer purchase behavior are developed for incorporation as demand models. Three cost model formulations are developed and compared in the context of the problem formulation. An explicit representation of an automotive manufacturer's technology capability in the form of a comprehensive yet stylized engineering performance model of a midsize crossover vehicle is developed. A methodology for developing and estimating mixed-logit choice models accessible to the design for market systems researcher is documented.

Hypothetical scenarios are evaluated in order to test the vehicle design problem formulation involving the design of a single vehicle within a price-equilibrium market context and the design of multiple same-segment vehicles within a price-equilibrium market context. The differences in scenario outcomes based on differences in the demand and cost models are explored. The results show that improving the fuel economy of a specific vehicle does not always lead to a reduction in US fleet fuel consumption.

Caution should be taken in interpreting the results of this study. The optimal vehicle design problem solutions presented in Chapters 7 and 8 represent a stylized equilibrium rather than a formal, full-market design and price equilibrium. Significant obstacles remain in studying maximum profit formulations in vehicle design including questions about underlying demand model validity and realistic cost models among others. Furthermore, regulatory scenarios can be considered, such as the CAFE standard, a fuel tax, or a CO_2 tax. Therefore, the numerical results presented here are useful in illustrating the proposed concept of public and private alignment rather than suggesting specific decisions or policy outcomes.

1.4 Dissertation Overview

The dissertation proceeds as follows. Chapter 2 reviews the literature from discrete-choice modeling, emperical cost modeling, multi-objective programming, game theory, market simulations in the automotive industry, and design for market systems research. Chapter 3 develops the engineering model for a midsize crossover vehicle. Chapter 4 describes the newly-estimated demand models of the US automotive industry and presents methods for evaluating the suitability of such demand models. Chapter 5 presents the three cost modeling approaches. Chapter 6 briefly reviews the mechanics of game-theoretic market simulations. Chapter 7 expands the public versus private tradeoff discussion by presenting measures for comparing tradeoffs between hypothetical future scenarios. Chapter 8 details several hypothetical scenarios. The first set of studies focuses on a single producer modifying a single vehicle in the fleet. The second set of studies focuses on the midsize crossover segment where three firms can modify their midsize crossover vehicle. Chapter 9 concludes with a summary, contributions, and future work. One appendix supplements the dissertation. Appendix A provides detailed background on the development of the vehicle engineering model.

CHAPTER II

Literature Review

This chapter reviews literature and background related to the many disciplines touched by this dissertation. Section 1 reviews discrete-choice modeling. Section 2 reviews empirical cost modeling. Section 3 reviews game theory. Section 4 reviews multiobjective programming. Section 5 reviews previous work in market simulations of the automotive vehicle industry. Section 6 reviews design for market systems research.

2.1 Discrete-choice Modeling

Choice models can be useful in estimating demand for products. Many econometric demand models have been estimated for the automotive market, for example Berry, Levinsohn, and Pakes; Beresteanu and Li; and Train and Winston [*Berry et al.* (1995); *Beresteanu and Li* (2008); *Train and Winston* (2007)]. These models predict choice share as a function of product attributes, individual demographics, product-demographic interactions, and other factors.

Table 2.1 lists the model variables or, covariates, used in three mixed-logit choice models developed for the automotive industry. (BLP 95: [*Berry et al.* (1995)]; B&L: [*Beresteanu and Li* (2008)]; T&W: [*Train and Winston* (2007)]). The first column labels the choice model. The second column lists the years over which the choice model was estimated. For example, BLP95 used vehicle and sales data from 1971-1990 to estimate their choice model. The third, fourth, and fifth columns list the covariates for each model. The covariates are divided by vehicle attributes, household attributes, and other factors. Some covariates can take on continuous values such as vehicle price or size. Other covariates are discrete such as household size, and others are binary taking a value either 0 or 1 such as Automatic transmission or Air conditioning. The last column lists the cost model attributes for cases where a cost model was jointly estimated with the choice model. The respective references contain the full definition of each covariate and the functional form of consumer utility.

2.1.1 Choice Paradigm

Random utility theory is the most common choice paradigm in marketing and economics work. It assumes that consumers seek to maximize satisfaction or utility from their purchases. An individual's utility U_{ij} is assumed to be composed of two parts, a systematic component ν_{ij} and a random component ε_{ij}. The role of the utility specification is to describe the preference structure for an individual by providing a mapping from a preference ordering of each choice alternative to a numerical ordering of each choice alternative. The analyst assigns the systematic component of utility ν_{ij} to each individual i for each choice alternative j.

In one typical arrangement, the utility is expressed as a linear combination of product attributes $\mathbf{d}_j$ with fixed effects δ, demographic and attribute interactions $\mathbf{b}_{ij}$ with fixed effects β, attributes or interactions $\mathbf{m}_{ij}$ with individual-specific random effects μ_i [*Train and Winston* (2007)], and in independent and identically distributed random variable ϵ_{ij}. The random effects allow for individual taste differences independent of demographics or other observed factors, Equation (2.1).

$$U_{ij} = \delta'\mathbf{d}_j + \beta'\mathbf{b}_{ij} + \mu_i'\mathbf{m}_{ij} + \epsilon_{ij} \tag{2.1}$$

Table 2.1: Model covariates in three revealed-preference mixed-logit vehicle demand models

Model	Data years	Vehicle attributes	Household attributes	Other factors	Cost model attributes
BLP 95	1971-1990	Ln(Income-MSRP) MP$ [mi/$] HP/WT Size [L*W] Air Conditioning	Income	Product Specific	MPG HP/Wt Size Air Tech Trend Sales
B&L	1999-2006	MSRP/Income DPM [$/mi] HP Size [L*W] Segment Hybrid	Income Location Children Household Size Rented house Head age Travel time Education	Local support for hybrids Product Specific (unknown)	MPG HP Size Weight Ln(Sales) Segment (unknown) Year (unknown)
T&W	2000	MSRP, MSRP/Income Fuel consumption [gal/mi] HP/WT Wheelbase Length - Wheelbase Automatic transmission Reliability Segment Manufacturer	Income Children Leased vehicle Women 30+	Dealers within 50 mi Previous manufacturer purchases Product Specific	None

The probability P_{ij} that an individual i chooses product j is the probability that the difference in the systematic component of utility between product j and any other product k is greater than the difference in the random component of utility of the other product k and product j,

$$P_{ij} = \Pr(\nu_{ij} - \nu_{ik} > \epsilon_{ik} - \epsilon_{ij} | \forall k). \tag{2.2}$$

For the special case where we assume that the portion of the random component ϵ_{ij} is independent and identically distributed across individuals according to the extreme value type 1 distribution and the product and product and demographic interaction random components are assumed known for each individual, a closed-form expression exists for the choice probabilities [*Train* (2003)].

$$P_{ij} = \frac{e^{\delta' \mathbf{d}_j + \beta' \mathbf{b}_{ij} + \mu_i' \mathbf{m}_{ij}}}{\sum_{k=1}^{J} e^{\delta' \mathbf{d}_k + \beta' \mathbf{b}_{ik} + \mu_i' \mathbf{m}_{ik}}} \tag{2.3}$$

When a sample population is assumed representative, the aggregate choice shares are taken to be the weighted average of the individual choice shares.

$$P_j = \frac{1}{I} \sum^{I} P_{ij} \tag{2.4}$$

The simple logit model results when we assume that all individuals have homogeneous taste preferences (Equation (2.5)). This model still allows heterogeneity to enter through observed demographics.

$$P_j = \frac{1}{I} \sum^{I} \frac{e^{\delta' \mathbf{d}_j + \beta' \mathbf{b}_{ij}}}{\sum^{K} e^{\delta' \mathbf{d}_k + \beta' \mathbf{b}_{ik}}} \tag{2.5}$$

2.1.2 Consumer preference and product differentiation

Economic theory describes two forms of product differentiation: Vertical differentiation results when consumers agree on the relative value ordering (or quality) of competing products or attributes of a product but differ in their willingness to pay for increased quality. Horizontal differentiation results when consumers disagree about the relative ordering of goods or attribute levels of a good. In the new vehicle automotive market we observe products that appear both vertically (e.g., various grades of luxury for a full-size sedan) and horizontally (e.g., various vehicle classes) differentiated. We also observe that manufacturers produce a portfolio of products rather than a single product offering. Consumer preferences can be one factor motivating firms to produce multiple products that are both vertically differentiated and horizontally differentiated. For example, industrial organization theory suggests that when consumers have nonidentical preferences product differentiation tends to weaken price competition [*Tirole* (1988)].

The previous discussion motivates the challenge of developing choice models that capture consumers' vertical as well as horizontal taste differences. It is expected that a choice model that only represents vertical preference differences (willingness to pay) would suggest policies different from a model that represents horizontal preference differences ("distance from ideal"). In the former, the dominant tradeoff for the firm is between cost and improvement of the attribute but in the latter, the important tradeoff is choosing which group of consumers to address. We may conjecture that for the new car buyer vehicle acceleration, fuel economy, safety features, and luxury appointments are examples of attributes for which preference may be characterized best by vertical differentiation. On the other hand, preference for brand, vehicle class, overall size, drivetrain (i.e., FWD, RWD, AWD, 4WD), ride, handling, and transmission (i.e., automatic, manual) may be best characterized by horizontal differentiation.

The consumer preference structures across individuals in the population can be categorized by their tendency to promote vertical differentiation between product offerings or horizontal differentiation. Following the development of Tirole [*Tirole* (1988)], one example of a preference structure leading to vertical differentiation is described by the utility function in terms of price p and quality r for consumer i and product j in Equation (2.6).

$$U_{ij} = \delta_i r_j - p_j \tag{2.6}$$

The utility of the outside good is assumed to be 0.

Two market cases can result for a monopolistic firm considering two products with different prices (p_1, p_2) and qualities (r_1, r_2). We consider a consumer population that is represented by a distribution of preference parameters μ_j that are assumed distributed according to some known distribution ($\mu_j \sim f(\mu)$). In the case where $r_2/p_2 \geq r_1/p_1$ then Product 2 dominates Product 1. Demand for Product 2 will be

$$Q_2 = M\left(1 - F\left(\frac{p_2}{r_2}\right)\right), \tag{2.7}$$

where $F()$ denotes a functional relationship. In other words, consumers with $\mu_j \geq p_2/r_2$ purchase Product 2; otherwise, they do not purchase either product. When $r_2/p_2 \leq r_1/p_1$, consumers with $\mu_j > (p_2 - p_1)/(r_2 - r_1)$ purchase the higher quality good (i.e., Product 2). Consumers with $p_1/r_1 < \mu_j \leq (p_2 - p_1)/(r_2 - r_1)$ purchase the lower quality good (i.e., Product 1). Demand for Product 2 will be

$$Q_2 = M\left(1 - F\left(\frac{p_2 - p_1}{r_2 - r_1}\right)\right), \tag{2.8}$$

and demand for Product 1 will be

$$Q_1 = M\left(F\left(\frac{p_2 - p_1}{r_2 - r_1}\right) - F\left(\frac{p_1}{r_1}\right)\right). \tag{2.9}$$

One example of a preference structure leading to horizontal differentiation is described by the utility function for consumer i and product j in Equation (2.10).

$$U_{ij} = \beta_i(\gamma_i - \alpha_j)^2 - p_j \tag{2.10}$$

Two parameters identify each consumer; μ_i represents willingness to pay similar to the vertical differentiation case but should have a negative sign; γ_i represents consumer i's preferred level of attribute α. The specific functional form (in this case a quadratic) controls the change in utility as an attribute's value is further away from the ideal one (γ_i). Following the linear city analogy [*Hotelling* (1929)] consumers' preferred attribute levels are assumed distributed on a continuum. If we assume two competing firms will offer one product each, most consumers will experience some disutility (or "transportation cost") by purchasing a good away from their ideal. In the first case, Product 1 is priced below Product 2 such that the reduced price compensates for the transportation cost for all consumers. In the second case, there is a tradeoff between price and transportation cost such that some consumers prefer Product 1 and some consumers prefer Product 2. A variant on this case occurs when both products are priced so high that the consumers in the middle purchase neither product due to the additional transportation cost.

With the exception of dummy coding [*Beresteanu and Li* (2008)], the typical utility function of mixed-logit specifications found in the literature is formulated as in Equation (2.1). Studying this equation reveals that linear-in-attribute specifications primarily imply vertical differentiation through the monotonicity of utility with respect to each attribute.

Linear-in-attribute specifications do allow for two commonly seen cases and a third less utilized case implying horizontal differentiation. The first is when a random coefficient straddles 0 so that increase in the given attribute provides utility to some individuals and disutility to other individuals—implying that consumers either like or dislike the attribute monotonically.

The second opportunity for horizontal differentiation from the conventional utility forms comes from the dummy variables associated with brand, vehicle class, or other attributes. When the coefficients on these dummies are treated as random, it is possible for the preference ordering between vehicle class, for example, to vary across the population. This will occur if the estimated variances are large enough to dominate the estimated mean effects. Interacting vehicle class dummies with observed demographics is the systematic analog to this idea. When the estimated parameters are constant across individuals then the utility function represents a homogeneous ranked-ordering of brands or vehicle classes (i.e., vertical differentiation). This approach is quite common; however, it requires specifying the structure of differentiation a priori (e.g., the vehicle classes).

A third, less common approach is to transform a product attribute such that it does not enter into the utility function monotonically. In Chapter 4 we propose an ideal-point utility formulation for vehicle size that gives an example of this approach. We choose vehicle footprint for the ideal-point because we believe it is the continuous attribute that is most likely to demonstrate horizontally differentiated preference.

2.1.3 Stated vs. Revealed Choices

Demand models can be estimated from a variety of data sources including revealed choice (actual product purchase) and stated choice (respondent's preferred alternative in a thought experiment). Individual-level revealed-choice data can be collected at the point of sale (e.g., scanner data), or through a survey mechanism after the sale. Aggregate sales data may also be collected through various reporting mechanisms. Individual-level purchase observations represent the gold standard for data in much applied economics work. These data have high face validity because they represent an actual choice (involving an exchange of money or allocation of some other resource). However, revealed-choice data only inform models of consumer preference based on the alternatives that each consumer actually saw.

Stated-choice data allow researchers to extend investigations beyond the scope of current product availability and investigate product attributes or other factors of interest at finer detail than provided by the market [*Louviere et al.* (2000)]. One form of stated-choice data useful for demand modeling is collected by gathering responses to hypothetical-purchase-choice experiments. Because this data comes from planned experiments, it is almost always collected at the individual level. The stated-choice approach is particularly appropriate for considering future demand for alternative fuel vehicles given their limited market penetration and rapidly evolving technology [*Potoglou and Kanaroglou* (2007); *Bunch et al.* (1993)]. However, there is no guarantee that a consumer will behave the same way in the market as in the stated-choice experiments.

It is possible to combine revealed and stated preference data. Hybrid models combining stated and revealed choice are an active area of research [*Kumar et al.* (2007); *Feit et al.* (In Review)]. Louviere et al. include a discussion on this topic [*Louviere et al.* (2000)], and Brownstone et al. show a combined data model for alternative fuel vehicles [*Brownstone et al.* (2000)]. The model we present in Chapter 4 follows the development of Train and Winston [*Train and Winston* (2007)] where we supplement observed vehicle purchases and demographics with stated information about other considered vehicles.

2.1.4 Choice Model Evaluation

Chintagunta et al. [*Chintagunta et al.* (2006)] propose four criteria for evaluating choice models in their review of the economic and marketing literature regarding structural choice models. They are fit, interpretability, predictive validity, and plausibility. The econometrics literature has developed and applied many statistical tests to address these criteria in the context of choice share predictions [*Train* (2003)]. However, other properties of choice models are important to investigate for engineering design, particularly how the design optimization model will behave based on the given demand model. One approach to begin to address this question is to consider how the demand model covariates (particularly those associated with product attributes) and their functional forms affect choice shares.

Interpretability and Fit

Traditional measures of interpretability are tests of significance of the estimated parameters, and checking parameter sign against intuition.

A standard measure of fit for logit models is the likelihood ratio index:

$$\rho^2 = 1 - LL(\hat{\delta}, \hat{\beta}, \hat{\mu})/LL(\mathbf{0}), \tag{2.11}$$

which measures how well the estimated model performs compared to a model where all of the parameters are zero (i.e., no model).

The likelihood function L is the product of choice probabilities P_{yj} for each individual for all choice observations for the choice the individual actually made given a set of estimates for the model parameter values $\{\hat{\delta}, \hat{\beta}, \hat{\mu}\}$, where f_{yj} is a dummy variable equal to 1 when for choice observation y product j is selected and $f_{yj} = 0$ otherwise.

$$L = \prod_{y=1}^{Y} \prod_{j=1}^{J} P_{yj}^{f_{yj}} \tag{2.12}$$

The log likelihood function is the natural logarithm transformation of the likelihood function.

$$LL = \sum_{y=1}^{Y} \sum_{j=1}^{J} f_{yj} \ln P_{yj} \tag{2.13}$$

We take $LL(\mathbf{0})$ to be the log likelihood function where all parameter values are set equal to 0. This is equivalent to predicting that each choice alternative has the same choice probability, i.e., $P_{yj} = 1/J$. An alternative definition for $LL(\mathbf{0})$ is to substitute S_j for all P_{yj} in Equation (2.13), where S_j is the percentage of observed choices for product j from the data sample.

Values between 0.2-0.4 represent very good model fits, and have been equivalenced to 0.7-0.9 R^2 values for linear ordinary least squares regression [*Louviere et al.* (2000)]. This statistical measure can only be used to compare the goodness-of-fit of two or more models if they are estimated from identical data sets and choice alternatives [*Train* (2003)].

Predictive Validity and Plausibility

The goal of estimating the demand model in design for market systems is to predict demand for products under counterfactual scenarios. Properties of particular interest to design optimization relate to how the model predicts consumers trade off product attributes and how their willingness to pay for improving an attribute compares to the cost of improvement.

Whereas fit measures the ability of the model to describe the in-sample data, predictive validity evaluates the ability to describe out-of-sample data. This may include a hold-out sample from the same time period for which the model was estimated, or it could be a sample from another time period or population. The likelihood ratio index can be used to evaluate a model's ability to predict choice shares from this out-of-sample set. This evaluation is one measure of a model's ability to capture consumer tradeoffs among attributes because one way to interpret superior model performance (i.e., higher ρ^2) on out-of-sample data is that the model better captures consumer tradeoffs better than simply describing the data (i.e., a good fit).

Similarly, another test that can be performed to evaluate indirectly the validity of the attribute valuation described by a particular model is predicting prices for products in an equilibrium framework as performed by Morrow [*Morrow* (2008)]. If accurate cost data are available, then the demand model can be used to generate equilibrium prices for all products under assumptions of competitive behavior. This evaluation is stronger than the likelihood ratio index because it can be used to identify systematic errors in predicting attribute tradeoffs such as if prices for higher quality products were underpredicted compared to observed market behavior.

We define plausibility as the ability of the estimated model to produce outcomes that represent market behavior based on theory or observations. One way to assess plausibility in the absence of cost and constraint models is to examine substitution patterns between competing goods given changes in price and other attributes, as measured by own- and cross-elasticities [*Train* (2003); *Nevo* (2000)]. The substitution patterns can then be compared to observed market behavior where possible.

2.2 Cost Modeling

Product planning and investment decisions rely on estimates of design and production costs, and yet cost estimates are a key element of uncertainty in the formulation of design and product development problems. The cost of a product is an outcome of various supply markets and labor inputs. Simplifying the discussion to focus on how product attribute changes affect cost is no less problematic. Costs change for identical components based on volume and from year to year. Cost to produce a vehicle includes not only the direct production costs, but also a firm's overhead. The cost models developed in this study do not attempt a bottom-up estimation for an entire vehicle. Rather the cost formulations focus on differences in cost from a baseline vehicle due to differences in powertrain and vehicle geometry. More sophisticated models could be included in the design optimization framework as they become available.

Examples of cost models for automotive vehicles include an ACEEE model that estimates incremental cost increases for advanced powertrains [*Kliesch and Langer* (2006)], and studies by the national labs [*Markel et al.* (2006)]. The AVCEM model provides estimates for alternative and traditional powertrains [*Delucchi* (2005)].

Three approaches to cost modeling are illustrated in Chapter 5: (1) an analytical model based on attribute differences, (2) a parametric model based on attribute regression of presumed costs (based on market prices and dealer markups), and (3) a parametric model derived from the price-equilibrium conditions given a demand model. The first two models are primarily motivated from an engineering perspective. The third model is motivated from an economics perspective. The following sections briefly review cost modeling from an engineering and economics perspective.

2.2.1 Cost Models from the Engineering Literature

Cost modeling as described in the engineering literature usually falls into one of three categories, or the approach is a combination of two or three of the categories. These catagories are parametric, analogous, and analytical. Parametric modeling seeks to define relationships between attributes of interest (design variables) and cost. This can be accomplished by using regression to match historical data. Analogous models use historical cost data from similar products in order to estimate the cost of a new product. Analytical models are explicit functions based on known (or presumed) relationships used to predict cost. One type of analytical modeling is based on detailed models of manufacturing processes. Another is based on abstract relationships between product characteristics and cost. Fixson reviews several levels of cost analysis and hypothesizes that as the scope of the analysis broadens from estimating the cost of operating a single machine towards estimating the costs of the entire enterprise, the fraction of indirect cost increases and the cost becomes increasingly nonlinear [*Fixson* (2004)].

Direct and abstract modeling are the two primary means whereby the three cost modeling approaches are applied. Direct modeling links cost directly to manufacturing and production operations through analytical, parametric, or analogous models. This approach has the potential to give an accurate cost estimate. However, these models are generally more difficult to associate directly with design variables and they are resource intensive to create and maintain.

Abstract modeling uses a proposed analytical or parametric model as a surrogate model for a detailed cost model. The abstract model is intended to provide some of the trade-off features of a detailed cost model without being tied directly to specific cost drivers such as machining time or energy use. Such models may use information such as sales prices, or they may develop cost of variety functions based on the number and variety of components. Design variables appear in these models, but they make assumptions about the impact the design variables will have on final cost of the product rather than rely on historical data or detailed manufacturing models. Although they predict design and production costs, they cannot usually assure manufacturing feasibility. This type of model is meant to guide product planning decisions and not to be used as a rigorous cost analysis tool. For a more detailed review of cost modeling in the engineering literature see [*Frischknecht* (2006)].

2.2.2 Cost Models and Economic Theory

Market equilibrium simulations have become common practice in the marketing and economics literature, see for example, the various examples cited in [*Chintagunta et al.* (2006)]. Two approaches in these studies for handling costs are (1) obtain actual cost data [*Chintagunta et al.* (2003)]; or (2) estimate a cost model from price equilibrium conditions assuming a specific model of competition [*Berry et al.* (1995); *Beresteanu and Li* (2008)]. Obtaining manufacturing costs for the automotive market appears highly unlikely. This would require multiple manufacturers to share closely-held cost information. It may be possible for someone close to the industry to develop working approximations of vehicle costs for various firms. However, this approach is not practical for academic studies. We therefore look to the work based on equilibrium assumptions described briefly below.

Economists expect producers to behave as profit maximizers. This implies that for any two production plans with the same quantity and quality of outputs, the producer will choose the production plan with the lowest cost. We can then say that producers are cost minimizers. Therefore the cost model that is relevant to the economist is not the cost of every possible production plan (i.e., the type and number of inputs required to produce a certain quality and quantity of outputs), but the minimum cost for each set of outputs. Given a description of a technology (i.e., the relationship between costs and quantities of inputs to quantities and qualities of outputs), the relevant cost function can be derived by solving the cost minimization problem for each output level [*Varian* (1992)].

Technology descriptions are complex, proprietary, and often not fully articulated even within a given firm. One economics approach is then to work backwards by making some assumptions about the competitive behavior in a market. For example, if we assume a given market is in Nash-Bertrand price equilibrium we can assume that the market prices reflect the profit maximizing behavior of the firms in the market. We can then write out the equilibrium conditions implied for each firm by setting the derivative of each firm's profit function equal to 0. In the simplest case profit π is a function of revenue R and cost C, where revenue and costs depend on demand Q and price p, $\pi = Q(p)(p - c)$. The costs can then be computed by rearranging the equilibrium conditions. A functional form for cost can be postulated (typically a function of product attributes believed to impact demand), and the cost values can be regressed on cost factors. A variation on this approach is to co-estimate the demand and cost parameters simultaneously by enforcing the equilibrium conditions at each iteration of the estimation procedure [*Berry et al.* (1995)].

The cost models developed using the equilibrium approach are typically described as functions of product attributes. It is an open question whether cost models that were a function of more elementary vehicle characteristics could yield better results. For example, intuition indicates that a cost model that was a function of engine power and size (measured as the product of vehicle length and width) may better capture cost tradeoffs than one that is rather a function of acceleration (related to the ratio of engine power to vehicle mass), fuel consumption, and size. Although acceleration and fuel consumption are important to the consumer, there is no physical reason why these attributes should contribute to vehicle cost in an additively separable way as is frequently modeled.

2.3 Multi-objective Optimization

Mathematical multi-objective problem formulations are appropriate when there are important criteria, believed to be competing, that are noncommensurable (i.e., they cannot be compared directly with the same units of measurement and it is difficult to convert them into a common unit of comparison such as dollars). A mult-objective optimization problem can be mathematically stated as:

$$\min_{\mathbf{x}} \quad \mathbf{f}(\mathbf{x};\mathbf{v})|\ \mathbf{h}(\mathbf{x};\mathbf{v}) = \mathbf{0},\ \mathbf{g}(\mathbf{x};\mathbf{v}) \leq \mathbf{0},\ \mathbf{x} \in \mathcal{X} \tag{2.14}$$

Here $\mathbf{f}(\mathbf{x};\mathbf{v})$ is a vector of criteria of interest f_n, $n = 1, \ldots, N$. The set of variable values $\mathbf{x}$ that satisfy all constraints is the feasible (design) domain, $\mathcal{X}$. The set of parameters $\mathbf{v}$ take on fixed values. The set of all vectors $\mathbf{f}$ mapped from the feasible domain is the attainable set $\mathcal{A} = \{\mathbf{f}(\mathbf{x};\mathbf{v})|\mathbf{x} \in \mathcal{X}\}$. A point in $\mathcal{A}$, $\mathbf{f}(\mathbf{x}^*)$, is said to be non-dominated or Pareto optimal, if there exist no $\mathbf{f}(\mathbf{x};\mathbf{v})$ such that $\mathbf{f}(\mathbf{x};\mathbf{v}) \leq \mathbf{f}(\mathbf{x}^*;\mathbf{v})$ and $f_n(\mathbf{x};\mathbf{v}) < f_n(\mathbf{x}^*;\mathbf{v})$ for at least one n. Ideal values f_n° are the optimal criterion values obtained optimizing one criterion at a time. The ideal or utopia point is the vector of ideal values for all criteria, $\mathbf{f}^\circ = [f_1^\circ, f_2^\circ]'$.

Special cases of multicriteria optimization occur when the criteria are dependent. Following from the Karush-Kuhn-Tucker conditions for noninferiority of vector optimization problems [*Karush* (1939); *Kuhn and Tucker* (1951); *Kuhn* (1976)][1], when the gradient of one criterion function can be expressed as a linear combination of the other criteria functions ($\nabla f_r(\mathbf{x}) = \sum_{n \neq r}^{N} w_n \nabla f_n(\mathbf{x})$ then one of the following conditions exists: (1) If $w_n \geq 0$ for all n, then objective f_r can be eliminated from further consideration. (2) If the summation is negative and $w_n \neq 0$ for all $n \neq r$, then all feasible solutions are noninferior [*Cohon* (1978)].

The Pareto set of a multicriterion optimization problem may have regions where a very small improvement in one objective leads to a large decline in another objective. Kuhn and Tucker [*Kuhn and Tucker* (1951)], and Geoffrion [*Geoffrion* (1968)] labeled these regions as improper Pareto points.

2.3.1 Multi-objective decision making

The concept of Pareto optimality was introduced by Edgeworth in 1881 [*Edgeworth* (1881)] and advanced by Pareto in 1906 [*Pareto* (1906)]. During the first half of the 20th century numerical techniques for multi-objective optimization were developed by mathematicians and economists. Engineers began to apply these techniques widely in the 1960s [*Stadler* (1979)]. Within engineering most early applications were large-scale civil projects. Most of civil applications were water resource management applications [*Stadler* (1981)]. Often these decisions were concerned with public policy. The utilization of multi-objective optimization can serve in the public debate by making trade-offs explicit and analysis transparent [*Cohon* (1978)]. Early engineering academic work concentrated on linear programming. Closed-form expressions can be developed for the Pareto set for small problems [*Lin et al.* (1975)].

[1] A summary of the unpublished 1939 Master's thesis of W. Karush was published as an appendix to [*Kuhn* (1976)].

Multicriterion decision making can be broken into three different approaches [*Evans* (1984)]: (1) Preferences are determined a priori and expressed as a value function; (2) Preferences evolve iteratively by solving a problem and adjusting preferences multiple times; 3) The range of solutions is presented and preferences are established a posteriori. In all cases, making a final decision about a single solution requires decisions to be made about the decision maker's preferences. Preferences can be described mathematically by value functions. A value function is often called a utility function, and it can include consideration of risk or uncertainty [*Keeny and Raiffa* (1976)].

If preferences for all criteria are monotonic, then the decisions under consideration can be reduced from the attainable set to the Pareto set [*Athan* (1994)]. Pareto optimality is the most common vector (multiobjective) optimality concept; however, there are others, especially from game theory, such as some equilibrium concepts and min-max solutions [*Stadler* (1988)].

This dissertation adopts utility functions from random-utility theory as the value function for consumer preferences. No preference structure is defined for the firm-level public private tradeoff between firm profit and vehicle fuel consumption. Instead we present the first part for approach three above by presenting the range of solutions before establishing preferences.

2.3.2 Solution techniques

Several solution techniques exist for generating Pareto optimal points. A common approach is to convert the vector objective function into a scalar objective and solve the problem once, or a number of times, using the techniques of scalar optimization. Multiobjective genetic algorithm methods generate an approximation to the Pareto set at once as an envelop of the attainable set [*Fonseca and Fleming* (1993)].

Popular scalarization methods include the weighted criteria, global criterion, and ε-constraint method (or constraint trade-off method) [*Osyczka* (1984)] including the lexicographic method [*Waltz* (1967)], and others [*Rao and Papalambros* (1989); *Athan and Papalambros* (1996); *Das* (1999)]. A limitation of the linear weighted criteria method is that it cannot find Pareto points in a non-convex region of the Pareto frontier. Generalized weighted criteria methods consider functions in the place of constant weighting parameters [*Athan and Papalambros* (1996)]. We adopt the ε-constraint method because of its straight-forward application in the case of two criteria and the ability to identify non-convex Pareto solutions.

A brief review is presented of common solution techniques. These techniques can be divided into those that provide Pareto optimal solutions and those that provide solutions based on some other solution strategy.

Pareto-optimal techniques

ε-constraint

The approach is to minimize one criterion with constraints on all other criteria. It can find Pareto optimal solutions in regions of nonconvexity. However, it may require a large number of constraint level combinations when the number of criteria is large. A special case of this method is the lexicographic method. The lexicographic method is not a Pareto optimal solution strategy and is described below.

Weighted Criteria

The weighted criteria method solves a single objective problem that consists of the weighted sum of the individual objectives. This method can be incorporated with no change to existing single objective optimization routines. However, because a weighted sum is a convex combination of the two objectives, the solution algorithm will not identify nonconvex regions of the Pareto frontier. Also, an even distribution of weights will not lead to even distribution of points on the Pareto curve. The distribution of the solutions generated therefore is highly dependent on the scale of each criterion and the weighting factors chosen.

Compromise Programming

Compromise programming sets a target point $\mathbf{f}^t$. Then, a regret function, defined to be a norm of the target point and the criterion vector, is minimized [*Yu and Leitmann* (1974)]. The target point can be set to the ideal point to ensure that solutions are Pareto optimal. Determining the ideal point requires separate scalar optimization runs for each criterion. The form of the regret function is typically $R(\mathbf{f}) = (\sum_{i=1}^{n}(f_i - f_i^t)^p)^{(1/p)}$. A $p = 1$ value treats each criterion as equally important and commensurable (i.e., the lowest sum of values in the criterion vector is sought). $p = 1$ is similar to goal programming and majority rule. For a bicriterion minimization problem the slope of a line tangent to the Pareto curve at the $p = 1$ solution is equal to -1. A $p = 2$ value corresponds to the Euclidean distance from the target point to the Pareto surface. It treats each criterion as equally important but noncommensurable (i.e., the value that is geometrically closest to the ideal point is preferred). A $p = \infty$ value corresponds to the L_∞ norm. It treats each criterion as equally important and seeks to minimize the maximum deviation from the target value for each criterion. For a bicriterion problem the solution will be found at the intersection of the Pareto curve and a line with slope equal to one that also intersects the target point. Setting $p = \infty$ is also called the min-max, or Tchebycheff method.

Given that the range of each criterion may be vastly different and compromise programming with regret functions typically treats each criterion as equally important, designers typically scale each criterion over its range in order to normalize the criteria comparison. Yu and Leitman document some characteristics of compromise programming including how the methodology implicitly imposes an intercomparison among criteria because the solution is not independent of a positive linear transformation of criteria (i.e., scaling of objectives matters). Also, for problems where the individual criterion represent the objectives of multiple independent stakeholders, as p increases, group utility decreases, but individual regret reduces as well. As p decreases group utility increases, but individual regret increases as well [*Yu and Leitmann* (1974)].

Curve Tracing using Homotopy Techniques

This method is specifically for bi-objective problems. The principle is to move along the Pareto frontier from one Pareto-optimal point to the next using local information at the current solution. For further description of this method see [*Rao and Papalambros* (1989)].

Normal-Boundary Intersection

This method begins with the ideal values for each objective (or an approximation). A line search is then conducted in the direction normal to the hyperplane intersecting the ideal values. The search continues until no further improvement in the objective is achieved. The solution is guaranteed to be Pareto optimal for most cases. There are notable exceptions when a Pareto set contains "folds" that the algorithm can terminate prematurely at a dominated point. This method is very useful for developing an even distribution of solutions across the Pareto frontier [*Das* (1999)].

Non Pareto optimal solution techniques

Goal Programming

Goal programming requires setting a priority for all criteria and establishing constraints for each criteria. The objective is to minimize deviations from each criterion goal [*Ignizio* (1976)]. Pareto optimality is not relevant in this framework because the preferences for criteria are not monotonic but based on the criteria prioritizing and the goal values. The goal values or targets may or may not be attainable. This is a primary difference from compromise programming where the targets are unattainable to ensure that the solution will be Pareto optimal.

Game Theory

In game theory the multiple criteria are typically broken down and adopted by multiple agents, or players [*Fudenberg and Tirole* (1991)]. One objective of game theory is to observe outcomes based on the actions of multiple agents given specific decision strategies. The strategies may or may not be based on monotonic preference for a given set of criteria. Some game theory solutions will generate Pareto optimal points and some will not depending on the particular strategy employed. For example, some models of competition may put more textslasis on reducing the outcome for a competitor rather than maximizing own benefit. Game theory is discussed in more detail below as it relates to market simulations.

Multilevel programming (lexicographic or hierarchical)

A special case of the ε-constraint method is multilevel programming, sometime called the lexicographic or hierarchical method. The lexicographic method is not a Pareto optimal solution strategy because the problem is divided into subproblems that are solved sequentially rather than all at once. The decision maker incorporates preferences into the solution strategy by ranking criteria in order of importance. Then, a series of sequential scalar optimization problems are solved in the order of the ranked criteria subject to problem constraints. The attained value of each succeeding criterion is maintained as a constraint for the remaining problems [*Waltz* (1967)]. The technique can be useful when there are many degrees of freedom. If there are low degrees of freedom the solution will be fixed after the first optimization runs.

Parameter Space Investigation and Genetic Algorithms

Parameter Space Investigation was developed for low-dimensional problems that are highly nonlinear and nonsmooth. Adaptation of the method beyond ten decision variables has proven difficult computationally. Furthermore there is no guarantee of Pareto optimality. The method involves sampling design points randomly or by some prescribed method. Infeasible points are discarded, and the remaining points are ordered. Sampling and selection occur in an iterative fashion [*Steuer and Sun* (1995)].

Genetic algorithms have been applied to multi-objective problems in much the same way as for parameter space investigation. However the genetic algorithm handles the sampling, ordering, and selection procedure. See for example [*Horn et al.* (1994)].

2.4 Game Theory

Game theory is a large and active area of research. Many of the ideas were first formalized by Von Neumann [*Morgenstern and Von Neumann* (1944)]. Several textbooks have been dedicated to the topic including [*Fudenberg and Tirole* (1991); *Friedman* (1986)]. Several key terms are provided here as background for the development of the market simulations described in Chapters 6 and 8.

2.4.1 Structure of a Game

A game is defined by identifying the players (i.e., decision makers), the strategies (i.e., the choices faced by each decision maker), the payoffs (i.e., the consequence for a player of each choice), and the sequence of decisions to be made. The simplest games assume players simultaneously make one decision each.

The decision-making strategy of each player will determine the choices the player makes. Individual utility (i.e., payoff) maximization is a frequently adopted strategy. In this case each decision maker will seek to maximize individual utility while consider the actions of the other players. A mixed strategy is one where the decision maker does not always make the same choice in a repeated game. A pure strategy is where a particular choice will be made every time.

A Nash equilibrium is defined as a solution strategy for each player such that one individual cannot unilaterally change strategies and increase payoff. A Nash equilibrium solution is expected when all players have correct information about the likelihood of each individual choosing each strategy, and each individual is acting to maximize payoff. Nash showed that given these assumptions with a finite number of agents and a finite number of pure strategies an equilibrium will always exist [*Varian* (1992)].

2.4.2 Game variants

Single vs. Multi-stage Game

A single-stage game is a game where players make choices simultaneously. Other games can be developed where players make a series of choices either simultaneously or in sequence. One example of such a game is where one player is the leader and the others are the followers. That is, one player makes his or her choice first. The other players then simultaneously make their choices. When players face a set of decisions given that some decision has already been made, the reduced decision context is called a subgame. A subgame perfect equilibrium is a strategy for each player that results in an equilibrium to the overall problem, which is also an equilibrium of the subgames [*Varian* (1992)].

Static vs. Dynamic Games

A single-stage game can be repeated. When the players know that the single-stage game will be repeated we have a simple dynamic game. A player's preferred strategy may change from the single-stage game strategy knowing that the game will be repeated. More complicated dynamic games can be developed that incorporate multi-stage games.

2.4.3 Application to Market Simulations

Published market simulations for the automotive industry have focused on static games. This is clearly a simplification because we expect manufacturers to have expectations about the future actions of competitors that would influence decisions today. Dynamic games are computationally much less tractable than static games.

We follow previous research and choose to focus on a static-game representation of the automotive industry. The players (vehicle manufacturers) are assumed to be profit maximizers. Price is chosen as the strategic variable controlled by firms implying that the resulting equilibrium solution to the market pricing game will be a Nash-Bertrand equilibrium [*Tirole* (1988)]. This means that we assume that producers are capable of supplying arbitrary quantities demanded exactly. This is a simplification adopted by other researchers for the automotive industry. Another simplifying assumption we make is that each producer will adopt only pure strategies. This means that each producer will charge a specific price for each vehicle, rather then, for example, charging one price 40% of the time and another price 60% of the time.

A different situation (typical of many commodities) would arise if we assumed that each producer was production constrained. In this case, firms would allocate resources to produce a specific quantity of a good. Price would then be determined by the overall market demand and the total quantity supplied. The result would be a Cournot equilbrium [*Tirole* (1988)].

The textslasis in design for market systems is to study the implications of the market on the decisions that firms make about product design decisions, not only price. We therefore adopt a two-stage (or subgame) approach where firms first design products and then set prices. The subgame is the pricing problem given all vehicle designs. We limit the number of players to one player or a small number of players to clarify the discussion of the results for the market simulations. In the case of one player, the manufacturer will design a given vehicle (the first stage) considering that the price for all own and competing vehicles can be adjusted based on the given designs (the second stage). In the multi-player case, a limited number of firms design one vehicle each (the first stage) considering that the price for all own and competing vehicles can be adjusted based on the given designs (the second stage). Conducting two-stage games where each player designs multiple vehicles is beyond the scope of this dissertation.

2.5 Market Simulations in Automotive Industry

Many econometric demand models have been estimated for the automotive market [*Beresteanu and Li* (2008); *Petrin* (2002); *Brownstone et al.* (2000); *Train and Winston* (2007); *Goldberg* (1995); *Sudhir* (2001)]. These models are often tailored for specific analyses such as interpreting firm pricing behavior rather than pricing and design decisions. For example, Berry, Levinsohn, and Pakes [*Berry et al.* (1995)], abbreviated BLP95, estimated a mixed-logit model of the automotive market to introduce a method for estimating demand in differentiated-products markets accounting for price endogeneity, and using only aggregate level demand and known population demographic distributions. They simultaneously recovered demand and cost parameters assuming observed prices satisfied Nash equilibrium. Many succeeding papers describing the automotive market have followed the BLP methodology.

Several drawbacks to the BLP approach can be identified. It is not clear that the BLP-style model supports counterfactual analyses based on changes in product attributes because the estimation procedure assumes design decisions are exogenous [*Nevo* (2000)]. Also, the exogenous specification of distributions for demographics, such as income, risk affecting parameter identification through erroneous correlations between demographics and product choices. Additionally, the utility specification enforces counterintuitive notions about consumer preferences. For instance, the utility specification contains the attribute “miles per dollar” (fuel economy divided by the price of gas), which lowers the relative importance of fuel economy as gas price increases. Although all three of the more recent demand models corrected the “miles-per-dollar” error—in favor of either a “dollars per mile” or “gallons per mile" attribute—both [*Petrin* (2002)] and [*Beresteanu and Li* (2008)] formulate utility as monotonic with vehicle size when it is possible that an individual could prefer a specific size vehicle.

2.6 Design for Market Systems Research

Engineers have begun to adopt, modify, and develop econometric models to serve in design for market systems applications, especially applied to automotive vehicle design [*Michalek et al.* (2004); *Shiau and Michalek* (2007); *Frischknecht and Papalambros* (2008); *Wassenaar et al.* (2005); *Donndelinger et al.* (2008)]. Michalek [*Michalek et al.* (2004)] and Shiau and Michalek [*Shiau and Michalek* (2007)] extracted the price, fuel economy, and acceleration pieces of utility from an existing automotive demand model, assuming all other product attributes (e.g., dimensions) are fixed. Frischknecht and Papalambros [*Frischknecht and Papalambros* (2008)] used these same assumptions while including vehicle size decisions but also adjusted the parameters for fuel economy and acceleration, recognizing the fleet average (i.e., consumers' expectations) of these attributes have changed since the time the model was estimated. These heuristic methods allow demand models to be used in design optimization for illustrative purposes but the interpretation of the results is uncertain.

Kumar [*Kumar et al.* (2007)] and Shiau [*Shiau and Michalek* (2009)] constructed their own demand models for the purpose of design optimization. However, little textslasis has been placed on the choice of the functional form of utility used in these models. Wassenaar et al. [*Wassenaar et al.* (2005)] presented a first attempt at addressing this issue, applying the Kano method to select the functional form for each product attribute, but offered no methods for evaluating the suitability for design optimization of the resulting demand model beyond measures of fit.

Other work in design for market systems research includes topics such as how to allocate resources between projects and what should be the product of each project [*Georgiopoulos et al.* (2005); *Michalek et al.* (2006); *De Weck et al.* (2006); *Kumar et al.* (2009)]; how to translate product attribute targets into engineering design and compare the value of different designs [*Cooper and Papalambros* (2003)]; how to integrate experimental techniques from marketing with engineering design [*Michalek et al.* (2005); *MacDonald et al.* (2007a); *Hoyle et al.* (2008); *Kumar et al.* (2007)]; how distribution channels affect product design [*Williams et al.* (2008)]; and many others [*Lewis et al.* (2006)]. Survey pieces include [*Michalek* (2008); *Frischknecht et al.* (2009a)].

CHAPTER III

Product Performance

An engineering model representing a midsize crossover vehicle is developed by combining powertrain performance simulations and other empirically or analytically derived models representing other vehicle characteristics. Three separate powertrain configurations are developed: conventional spark-ignition (SI) gasoline engine, gas turbo direct-injection gasoline engine (GTDI), and a split-mode hybrid electric vehicle with a conventional spark ignition gasoline engine (HEV). The hybrid electric powertrain model is a backwards-looking simulation built by Kukhyun Ahn based on his dissertation work [*Ahn* (2008)]. Section 3.1 defines the modeled vehicle characteristics. Section 3.2 defines the design variables, parameter values, and the engineering constraint functions. Section 3.3 presents model validation data, and Section 3.4 summarizes the chapter. Additional details describing the engineering performance model are found in Appendix A.

3.1 Vehicle characteristics

Vehicle characteristics are presented in three categories: powertrain performance, packaging (which includes curbweight), and safety.

3.1.1 Powertrain

Vehicle simulations for SI and GTDI were developed using the AVL *Cruise* software package [*AVL* (2008)]. The vehicle powertrain was configured to represent a standard automatic transmission front-wheel-drive vehicle with a gasoline engine. In addition to powertrain specifications (i.e., gear ratios, gear shifting schedule, engine number of cylinders, vee or inline configuration, bore, and stroke, valvetrain configuration, and final drive ratio) *Cruise* also receives other vehicle parameters as inputs, including curbweight, frontal area, drag coefficient, tire radius, and center of gravity location under various loads. These parameters were developed from the other characteristic models as described below or taken from data describing one 2007 model (i.e., Ford Edge). In all, over 30 parameters were tuned for midsize crossover vehicles. All other parameters were left at the default passenger vehicle levels.

Five performance tests were simulated in *Cruise*: The US city driving cycle (FTP75), the US highway driving cycle (HFET), used to calculate combined city and highway fuel economy, z_{mpg}; an acceleration test starting from rest, used to calculate 0-60 mph time, z_{060}, and vehicle top speed, z_{TS}; an elasticity test, used to calculate 30-50 mph acceleration time while towing, z_{3050}; and a gradeability test, used to calculate maximum grade at 65 mph while towing, z_{65T}. The 30-50 mph acceleration test simulated towing by adding the maximum towing capacity to the mass of the vehicle. The gradeability test simulated towing by using the virtual trailer option, which allows specification of trailer mass and an estimate of losses.

Cruise characterizes engine performance by reference to engine maps derived from experimental results of a baseline engine. The fuel consumption map for SI was taken from a 2.5 l, V-6 engine with $BMEP_{P_{peak}} = 1068$ kPa. The full load characteristic was scaled from the Duratec35 engine ($BMEP_{P_{peak}} = 1085$ kPa) used in the Ford Edge. The GTDI baseline fuel consumption map was adapted from an SAE paper [*Kleeberg et al.* (2006)]. The full load characteristic was adopted from a "best guess" transcription from the EcoBoost YouTube video hosted by Derek Kuzak of Ford [*Ford Environmental* (2009)]. Additionally, *Cruise* provides modules for modeling turbo-charger behavior (see Appendix A for more details). The fuel consumption map and full-load performance characteristic for HEV were taken from a data based on the engine in the Toyota Prius.

Engine maps were scaled for a given evaluation as functions of x_B and x_{BtS} following established scaling relationships [*Chon and Heywood* (2000)]. We assume the peak power brake mean effective pressure of the engine is 1085 kPa and mean piston speed at peak power out is 18.1 m/s for all SI designs. We assume the peak power brake mean effective pressure of the engine is 2047 kPa and mean piston speed at peak power out is 16.7 m/s for all GTDI designs. The advanced friction module found in *Cruise*, which incorporates engine and valvetrain architecture, based on [*Patton et al.* (1989)] was used to integrate frictional engine losses into the simulations for SI and GTDI.

Surrogate models were obtained from *Cruise* simulations to reduce computational expense during design optimization. A Latin hypercube experimental design with 1000 numerical experiments was executed for the SI. A similar experiment with 1600 experiments was executed for GTDI and 500 experiments for HEV. The HEV simulation was developed in Matlab rather than *Cruise*, and the HEV simulations were executed by Kukhyun Ahn. Table 3.1 shows the experimental factors and the upper and lower bounds for each factor for each powertrain.

3.1.2 Packaging

Vehicle characteristics derived from simplified assumptions of vehicle geometry include an estimated engine length, z_{EL}; cargo volume index behind 2nd row, z_{CVI}; ramp breakover angle, z_{A147}; angle of departure, z_{A107} [*SAE International* (2005)]; and distance in the width direction to allow tire movement (i.e., tireflop), z_{TF}.

Table 3.1: Upper and lower bounds on experimental factors for powertrain simulations

	x_B	x_{BtS}	x_{FD}	v_{G1}	v_{G2}	v_{G3}	v_{G4}	x_{L101}	v_{H156}	x_{L101}	x_{L103}	x_{W105}	v_{CD}	x_{PGR}	x_{BPow}
Units	mm	-	-	-	-	-	-	mm	mm	mm	mm	mm	-	-	kW
SI, V-6	86	0.95	1.1					1600		2286	3556	1600	0.33		
	100	1.18	4.0					1930		3048	5080	2000	0.42		
GTDI, I-4	80	0.9	2.5	2.85	1.85	1.40	1.0	1651	175		4318	1727	0.33		
	92	1.18	5.0	5.0	2.85	1.85	1.4	1803	215		5080	1982	0.42		
HEV, I-4	88	0.9	3.5					1651		2286	4319	1727	0.33	0.3333	45
	92	1.18	5.0					1803		3048	5080	1982	0.42	0.6666	65

Table 3.2: Coefficient values for crossover vehicle mass calculation

c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9
5141	-60.82	1.992	2001	2180	2106	1936	358.5	2233

Vehicle characteristics related to the mass properties of the vehicle include vehicle curbweight and gross vehicle weight rating, z_{VM}, z_{GVWR}; and the minimum required payload capacity, v_{pay}. A regression was fit (R^2:0.92) to estimate curb weight (reported in lbm) using data for 2005 light-duty trucks from Ward's automotive yearbook [*Wards Communications* (2006)]. Here z_{EDisp} is the engine displacement volume in cubic centimeters, length and width in inches. v_{FWD}, v_{AWD}, v_{4WD}, v_{RWD} are dummy variables {0,1} for driveline configuration, and v_{HEV} is a dummy variable {0,1} for a hybrid electric powertrain. Table A.3 lists the parameter values.

$$
\begin{aligned}
z_{VM} &= c_1((x_{L103} \times x_{W105})/100)^2 + c_2(x_{L103} \times x_{W105}) + c_3 z_{EDisp} \\
&\quad + c_4 v_{FWD} + c_5 v_{AWD} + c_6 v_{4WD} + c_7 v_{RWD} + c_8 v_{HEV} + c_9
\end{aligned}
\tag{3.1}
$$

3.1.3 Safety

Vehicle characteristics modeled that influence vehicle safety include the static rollover score based on the static stability factor [*NHTSA* (2009)], z_{Roll}; vehicle center of gravity position in longitudinal and vertical direction, z_{CGlong}, z_{CGvert}; bumper to driver heel crush space, z_{CS}; Minimum required crush space for a given average force, v_{MCS}; and estimated peak deceleration in front crash test, v_{MD};

The static stability factor is a function of vehicle geometry and vertical center of gravity position. A correlation has been made between the static stability factor and the risk of rollover [*NHTSA* (2009)]. Vehicle center of gravity is determined by vehicle geometry and a prescribed subsystem mass distribution [*Malen* (2005)]. Bumper to heel crush space, minimum required crush space, and estimated peak deceleration are derived from vehicle geometry, underhood component geometry, and assumptions of crush efficiency and 50% of crush load born by midrails.

Table 3.3: Upper and lower bounds on design variables

	x_B	x_{BtS}	x_{FD}	x_{H101}	x_{L101}	x_{L103}	x_{W105}	x_{PGR}	x_{BPow}
Units	mm	-	-	mm	mm	mm	mm	-	kW
SI, V-6	86	0.95	1.1	1600	2286	3556	1600		
	100	1.18	4.0	1930	3048	5080	2000		
GTDI, I-4	80	0.9	2.5	1651	2286	4318	1727		
	92	1.18	5.0	1803	2286	5080	1982		
HEV, I-4	88	0.9	3.5	1651	2286	4319	1727	0.3333	45
	92	1.18	5.0	1803	3048	5080	1982	0.6666	65

3.2 Design Variables and Constraints

The design variables are: engine bore, x_B; engine bore to stroke ratio, x_{BtS}; final drive ratio, x_{FD}; vehicle length, width, and height, x_{L103}, x_{W105}, x_{H101}; vehicle wheelbase, x_{L101}; and for HEV planetary gear ratio, x_{PGR}; and peak power for the battery, x_{BPow}. Other parameters include the minimum height between seat and roof, v_{MSH}, and the underhood midrail width, v_{MRW} among others. Table 3.3 gives the upper and lower bounds of the design variables organized by powertrain.

The constraint set is as follows:

$$
\begin{aligned}
g_1 &= 5\% - z_{65T} \leq 0 \\
g_2 &= 13^\circ - z_{A107} \leq 0 \\
g_3 &= 12^\circ - z_{A147} \leq 0 \\
g_4 &= 29\text{ft}^3 - z_{CVI} \leq 0 \\
g_5 &= z_{CVI} - 60\text{ft}^3 \leq 0 \\
g_6 &= z_{Roll} - .21 \leq 0 \\
g_7 &= 50\% - 100(1 - z_{CGlong} - x_{L104}/x_{L101} \leq 0 \\
g_8 &= v_{pay} + z_{VM} - z_{GVWR} \leq 0 \\
g_9 &= z_{MCS} - z_{CS} \leq 0 \\
g_{10} &= z_{MD} - 20(9.81\text{m/s}^2) \leq 0 \\
g_{11} &= (2z_{TF} + 2v_{MRW} + z_{EL} + 50.8) - (x_{W105} - 254) \leq 0 \\
g_{12} &= x_{L101} + v_{L104} - x_{L103} \leq 0 \\
g_{13} &= 115\text{mph} - z_{TS} \leq 0 \\
g_{14} &= v_{MSH} - x_{H101} \leq 0
\end{aligned}
\tag{3.2}
$$

Table 3.4 lists an explanation for each constraint. Cargo volume and rollover constraints [*NHTSA* (2009)] were relaxed: g_4 (min cargo volume) from 32 ft^3 to 29 ft^3; g_6 (max rollover score) from 0.1999, a 4-star rating, to 0.21, to account for differences between the model and real-world data.

3.3 Model Performance

This section presents data to illustrate the performance of the engineering models. Results for the three powertrain configurations are given. Each of the powertrain sections presents the curve-fits for the surrogate models. The SI and GTDI sections also compare the AVL *Cruise* simulations with data from specific vehicle actual vehicles. The curbweight section compares the model predictions to actual data for the

Table 3.4: Explanation of engineering constraints

Constraint	Description
g_1	Minimum grade towing 3500 lbm at 65 miles per hour greater than 5%
g_2	Rear wheel to bumper angle of departure greater than 13 degrees
g_3	Ramp breakover angle between wheels greater than 12 degrees
g_4	Minimum cargo volume index greater than 29 cubic feet
g_5	Maximum cargo volume index less than 60 cubic feet
g_6	Maximum rollover score less than 0.21
g_7	Minimum of 50% of vehicle mass distributed on front axle
g_8	Vehicle gross vehicle weight rating greater than the vehicle curbweight plus a minimum payload $v_{pay} = 100$ kg
g_9	Minimum hood compartment crush space greater than calculated required crush space
g_{10}	Maximum deceleration less than 20 g's in a 35 mile per hour front crash
g_{11}	Vehicle width great than tire flop, midrail width ($v_{MRW} = 74.5$ mm), and engine length
g_{12}	Vehicle length greater than front overhang and wheelbase
g_{13}	Vehicle top speed greater than 115 miles per hour
g_{14}	Minimum vehicle height high enough to allow a minimum sitting height $v_{MSH} = 840$ mm

curbweight.

3.3.1 Powertrain performance

Conventional spark-ignition

Simulation fits

Satisfactory polynomials were found for both driving cycles, and the gradeability simulation (R^2: 0.998 City, 0.994 Hwy, 0.997 Grade). Three neural nets were generated in Matlab, one for z_{060}, one for z_{TS}, and one for, and z_{3050}. z_{060} and z_{3050} had R^2 values for the training and test points of 0.999. z_{TS} had an R^2 of 0.988. The R^2 values reported for the neural net fits are for the test points. Figure 3.1 plots the predicted vs. actual *Cruise* responses.

Single model comparisons

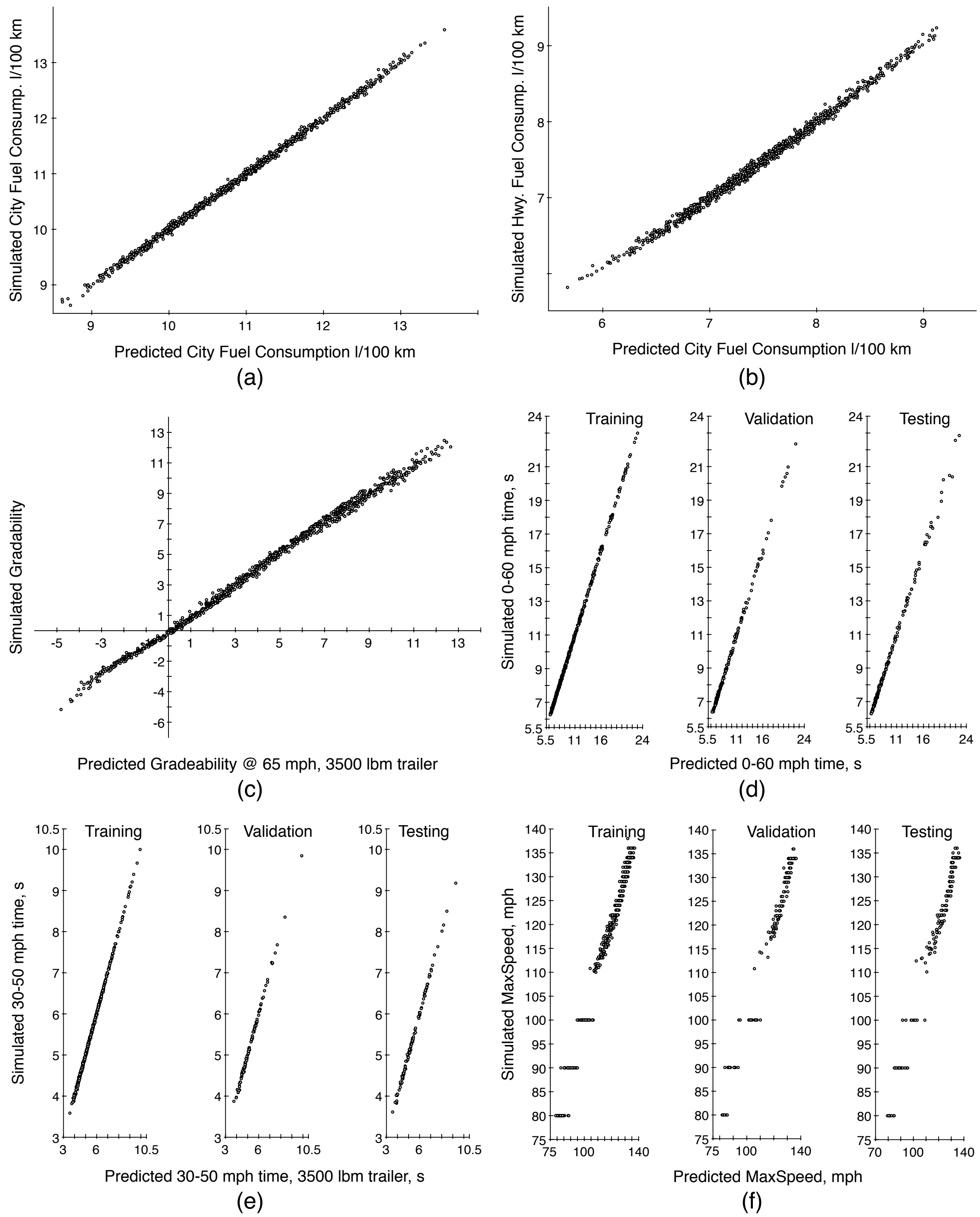


Figure 3.1: (a) City driving cycle fuel economy; (b) Highway driving cycle fuel economy; (c) Maximum gradeability at 65 mph while towing; (d) 0-60 mph acceleration time; (e) 30-50 mph acceleration time; (f) Maximum vehicle velocity

Attributes for midsize crossover vehicles were gathered from the Internet including values for all design variables, transmission ratios, and other model parameters [*Conceptcarz.com* (2008); *Edmunds Inc.* (2008); *LLC Classified Ventures* (2008)]. In addition to scaling the fuel consumption map and full-load performance characteristic by engine geometry, the fuel consumption map was further scaled by the ratio of the baseline and modeled engine peak power brake mean effective pressure. Single *Cruise* simulation runs were performed for each vehicle and the values of the attributes were recorded. The vehicle simulation was rerun using the reported curb weight for z_{VM} in cases where the vehicle curb weight prediction deviated by more than 50 kg from the reported curb weight. This was to make a fair assessment of the simulation tool rather than bias the results based on weakness in the curbweight model. Table 3.5 lists the comparisons between the modeled vehicles and the actual quoted performance values. Fuel economy values for the actual vehicles are those reported as combined fuel economy ratings according to the pre-2008 window sticker reporting method. The 0-60 times for the actual vehicles are approximate and were primarily gathered from popular press. The Nissan Murano was not simulated in *Cruise* due to its continuously variable transmission. Figure 3.2 shows the difference in reported versus simulated fuel economy. Most vehicles are simulated within 6% of the reported value. The Santa Fe simulation deviates 9% and the Suzuki XL7 deviates 19%.

Gas Turbo-charged direct Injection

Simulation fits

Satisfactory polynomials were found for both driving cycles, the gradeability simulation, and 30-50 mph acceleration time while towing (R^2: 0.998 City, 0.995 Hwy, 0.999 z_{65T}, 0.998 z_{3050}). Two neural nets were generated in Matlab, one for z_{060} (R^2: 0.99) and one for z_{TS} (R^2: 0.987), where the R^2 values reported are for the test points.

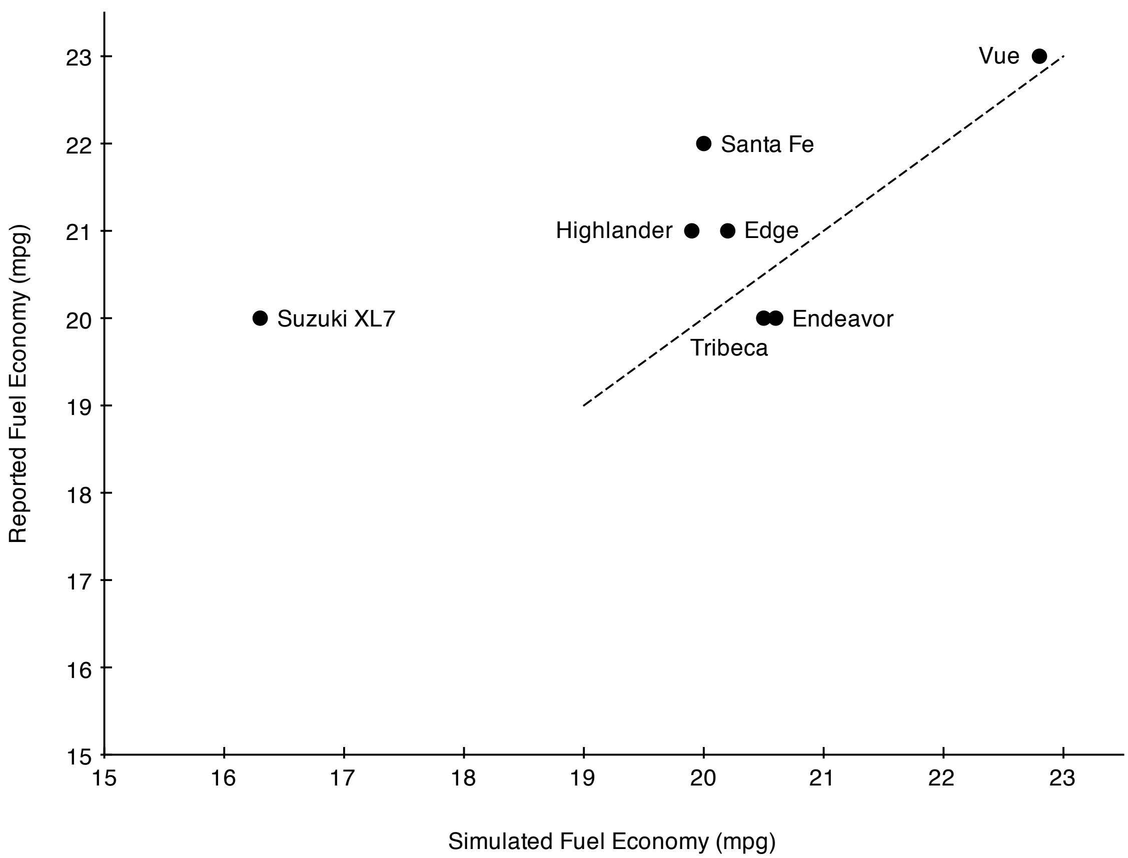


Figure 3.2: Comparison of SI reported fuel economy and simulated fuel economy

Table 3.5: Comparison between actual vehicles and SI simulated vehicles

	z_{MPG}	z_{060}	z_{3050}	z_{65T}	z_{TS}	z_{VM}	z_{EDisp}	x_B	x_{BtS}	x_{FD}
Units	mpg	s	s	%	mph	kg	cc	mm	-	-
2007 Edge	21	7.4-8.4	unavail.	unavail.	111 gov.	1859	3496	92.5	1.07	2.77
Edge Calc. z_{VM}	20.5	7.5	5.2	6.8	134	1768	3496	92.5	1.07	2.77
Edge Act. z_{VM}	20.2	7.7	5.3	6.6	134	1858	3496	92.5	1.07	2.77
2006 Murano	22	8.0	unavail.	unavail.	unavail.	1747	3494	95.5	1.18	5.17
Murano Calc. z_{VM}	unavail.	unavail.	unavail.	unavail.	unavail.	1744	3494	95.5	1.18	5.17
2006 Highlander	21	7.8-9.4	unavail.	unavail.	unavail.	1655	3309	91.9	1.11	3.48
Highlander Calc. z_{VM}	19.9	7.9	6.0	5.1	131	1658	3309	91.9	1.11	3.48
2006 Santa Fe	22	8.7	unavail.	unavail.	unavail.	1691	3339	91.9	1.10	3.68
Sant Fe Calc. z_{VM}	20.0	7.7	6.0	6.1	125	1708	3339	91.9	1.10	3.68
2006 Tribeca	20	7.4-9.5	unavail.	unavail.	unavail.	1885	2997	89.2	1.11	3.58
Tribeca Calc. z_{VM}	20.9	7.8	5.6	6.1	128	1717	2997	89.2	1.11	3.58
Tribeca Act. z_{VM}	20.5	8.3	5.9	5.8	128	1885	2997	89.2	1.11	3.58
2006 Vue	23	8-9	unavail.	unavail.	unavail.	1578	3462	88.9	0.96	4.06
Vue Calc. z_{VM}	22.6	8.7	6.9	3.5	112	1640	3462	88.9	0.96	4.06
Vue Act. z_{VM}	22.8	8.4	6.8	3.6	112	1578	3462	88.9	0.96	4.06
2006 Suzuki XL7	20	8.2	unavail.	unavail.	unavail.	1763	3563	94.0	1.10	2.48
Suzuki XL7 Calc. z_{VM}	16.3	8.0	4.2	0.0	132	1794	3563	94.0	1.10	2.48
2006 Endeavor	20	8.4	unavail.	unavail.	unavail.	1810	3824	95.0	1.06	4.32
Endeavor Calc. z_{VM}	20.6	9.9	7.0	3.4	124	1793	3824	95.0	1.06	4.32

Figure 3.3 plots the predicted vs. actual *Cruise* responses.

Single model comparisons

The GTDI concept was introduced by Ford in 2007-08. For a comparison with actual vehicle data we use the 20-30% fuel economy improvement quoted for the 2008 Explorer America concept car from the 2008 North American Auto Show [*Ford Motor Company* (2009)] compared to the V-6 conventional Explorer. The simulated values are reported in Table 3.6 for the Explorer America concept car, the conventional Explorer, and a conventional Explorer with the V-6 engine replaced by the 2 liter, 4 cylinder GTDI engine. The simulated fuel economy difference between the conventional and the concept car was 29%. We also present the results for a GTDI equipped Ford Edge with similar 0-60 time as the baseline Ford edge and compare the fuel economy improvement in the GTDI Ford Edge versus the baseline (approximately 13%).

Split-mode hybrid electric

Simulation fits

For the HEV model, satisfactory polynomials were found for all powertrain performance characteristics (R^2: 0.998 City; 0.999 z_{060}; 0.99 z_{65T}; 0.999 z_{3050}; 1.0 Peak power output from motor-generator one, z_{MG1}; 1.0 Peak power output from motor-generator two, z_{MG2};) except the fuel economy for the highway driving cycle, and the vehicle top speed. These two outputs were fit with neural nets (R^2: 0.99 Hwy, 0.97 z_{TS} for the test points). Figures 3.4 and 3.5 plot the predicted vs. actual Matlab simulation responses. The HEV fuel economy and 0-60 time were adjusted to account for the overly optimistic behavior of the Matlab simulation.

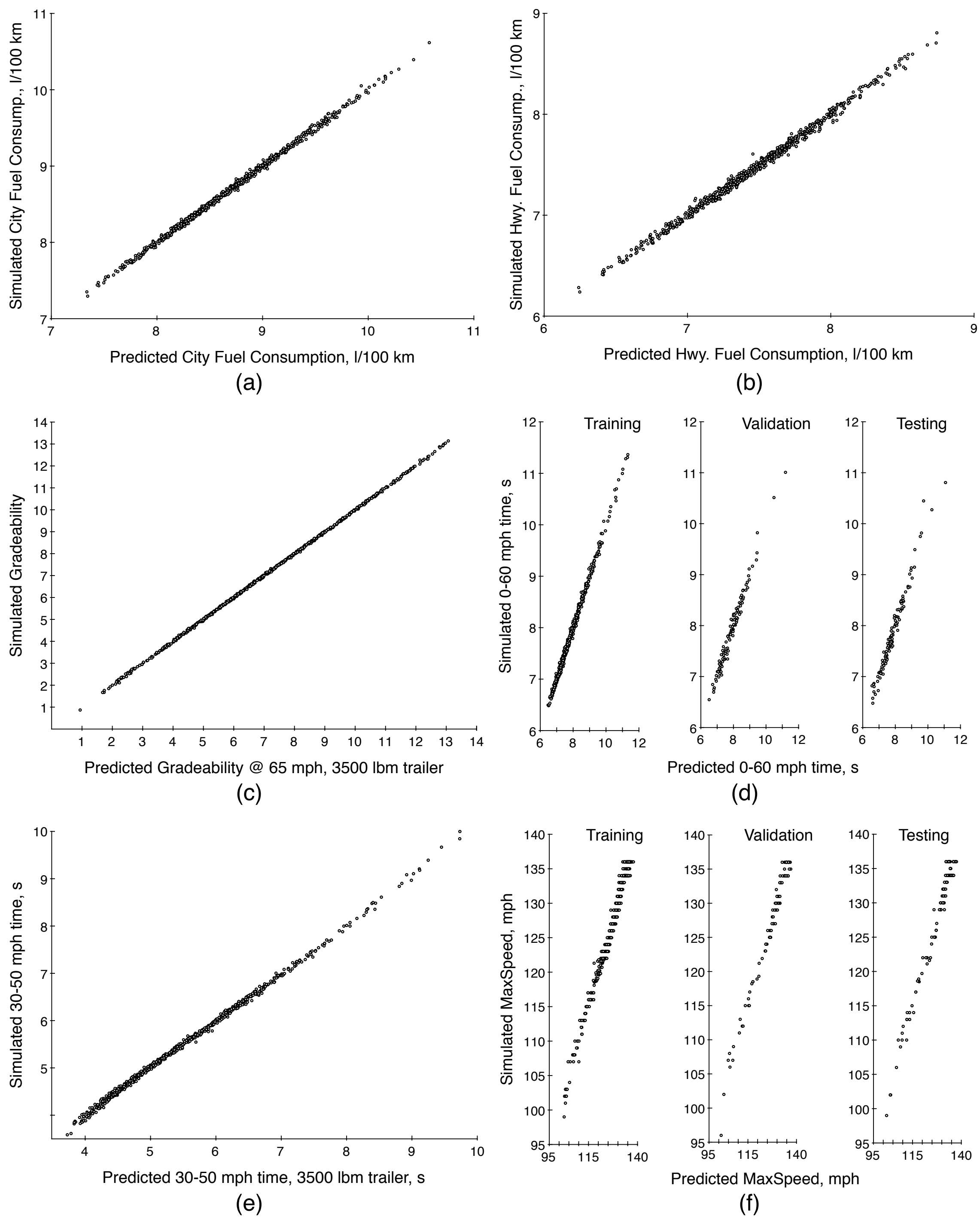


Figure 3.3: (a) City driving cycle fuel economy; (b) Highway driving cycle fuel economy; (c) Maximum gradeability at 65 mph while towing; (d) 0-60 mph acceleration time; (e) 30-50 mph acceleration time; (f) Maximum vehicle velocity

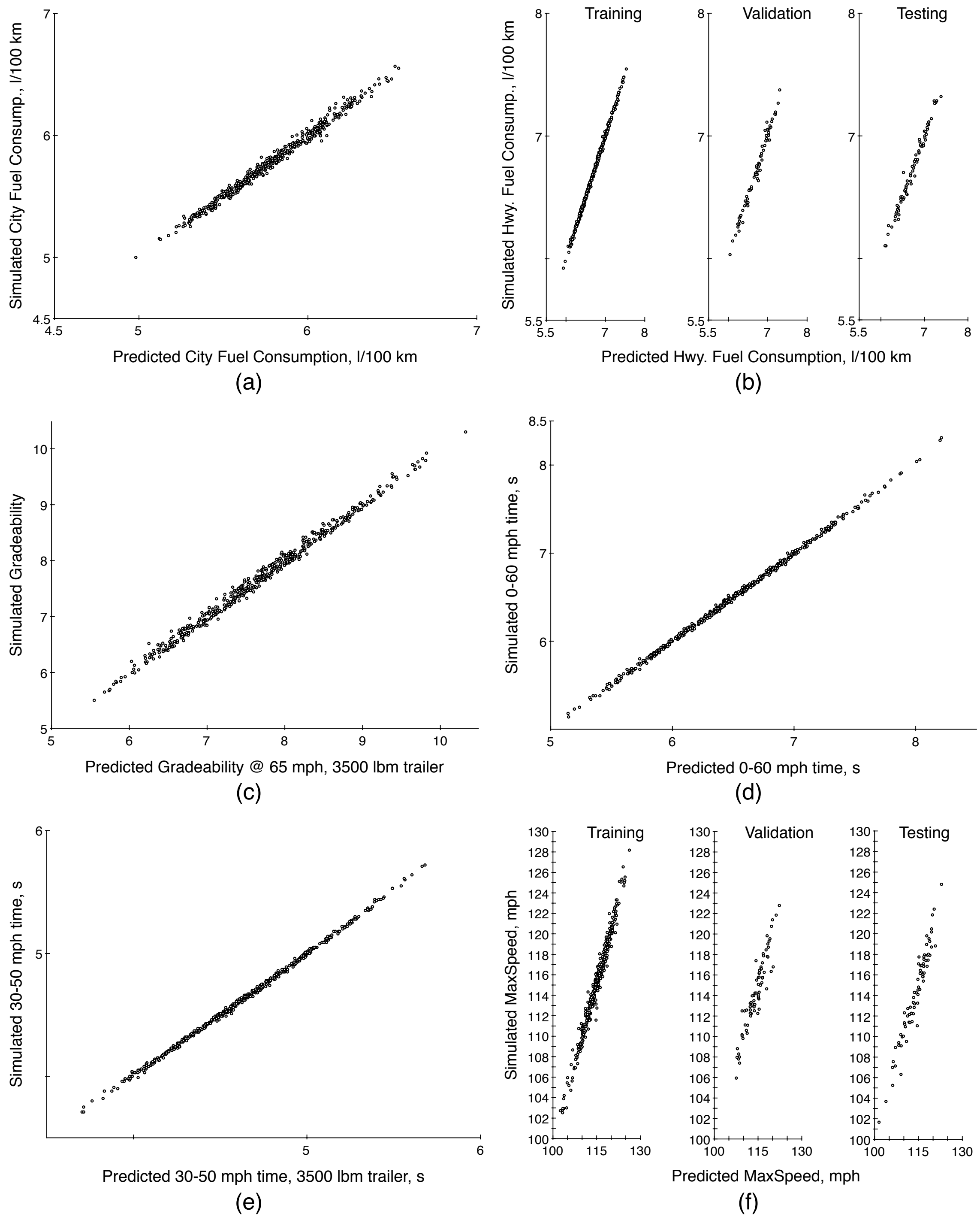


Figure 3.4: (a) City driving cycle fuel economy; (b) Highway driving cycle fuel economy; (c) Maximum gradeability at 65 mph while towing; (d) 0-60 mph acceleration time; (e) 30-50 mph acceleration time; (f) Maximum vehicle velocity

Table 3.6: Comparison between 2007 Ford edge and 2008 Explorer America concept car with SI and GTDI simulated vehicles

	z_{MPG}	z_{060}	z_{3050}	z_{65T}	z_{TS}	z_{VM}	z_{EDisp}	x_B	x_{BtS}	x_{FD}
Units	mpg	s	s	%	mph	lbm	cc	mm	-	-
Simulated 2008 Explorer America concept	21.3					4300	2000	87.4	1.07	
Simulated SI Explorer	16.5					4460	4000	100.3	1.19	3.55
Simulated GTDI Explorer	20.3					4460	2000	87.4	1.07	
2007 Ford Edge	21	7.4-8.4	unavail.	unavail.	111 gov.	4096	3496	92.5	1.07	2.77
Simulated SI Edge	20.2	8.29	5.64	5.86	134	4096	3496	92.5	1.07	2.77
Simulated GTDI Edge	22.9	8.25	5.53	5.94	134	4095	2000	87.4	1.05	2.5

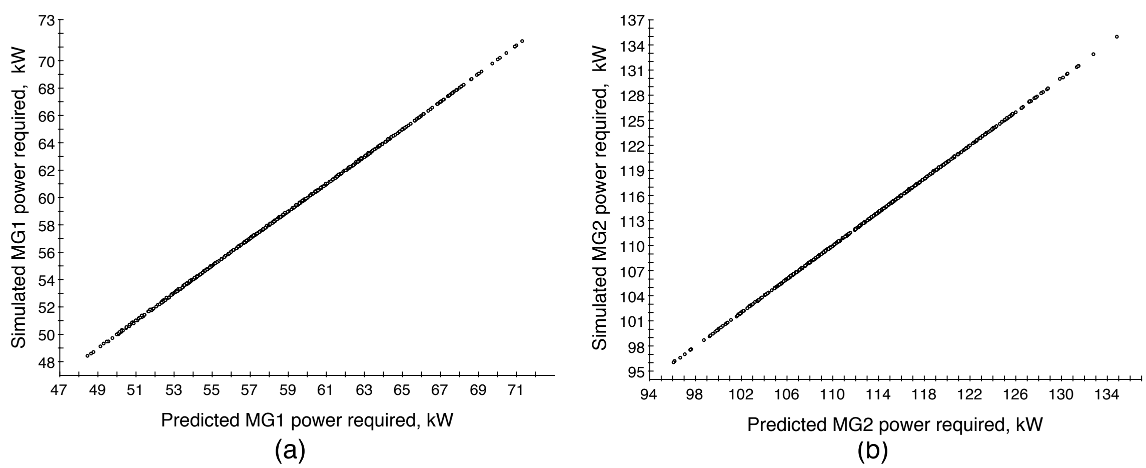


Figure 3.5: (a) Power required for electric machine z_{MG1} (b) Power required for electric machine z_{MG2}

An additional 0.9 seconds was added to the 0-60 mph acceleration time for the HEV model. A scaling factor of 5.778/4.5772 was applied to the test cycle city fuel consumption for the HEV model. These changes were recommended by the author of the HEV model to account for the overly optimistic performance of the HEV model.

The Ford Escape Hybrid/Mercury Mariner Hybrid, Toyota Highlander Hybrid, and Lexus RX400h were all SUV hybrids in the 2006 model year. The Saturn VUE Hybrid was added in 2007.

3.3.2 Vehicle Mass Prediction

curbweight

Figure 3.6 plots the predicted versus reported curbweight (z_{VM}) for the crossover segment based on Equation (3.1).

3.4 Summary

Engineering models representing the midsize crossover vehicle segment were developed representing three powertrain options: conventional spark-ignition engine (SI), gas turbocharged direct-injection engine (GTDI), split-mode electric-gasoline hybrid (HEV). Powertrain attributes were modeled using the AVL *Cruise* simulation software. Surrogate models were generated for each powertrain simulation. The simulated versus predicted powertrain attribute results were presented. Packaging and safety attributes were modeled using a combination of empirical and analytical equations. Comparisons of the modeled attributes to the reported attributes were given.

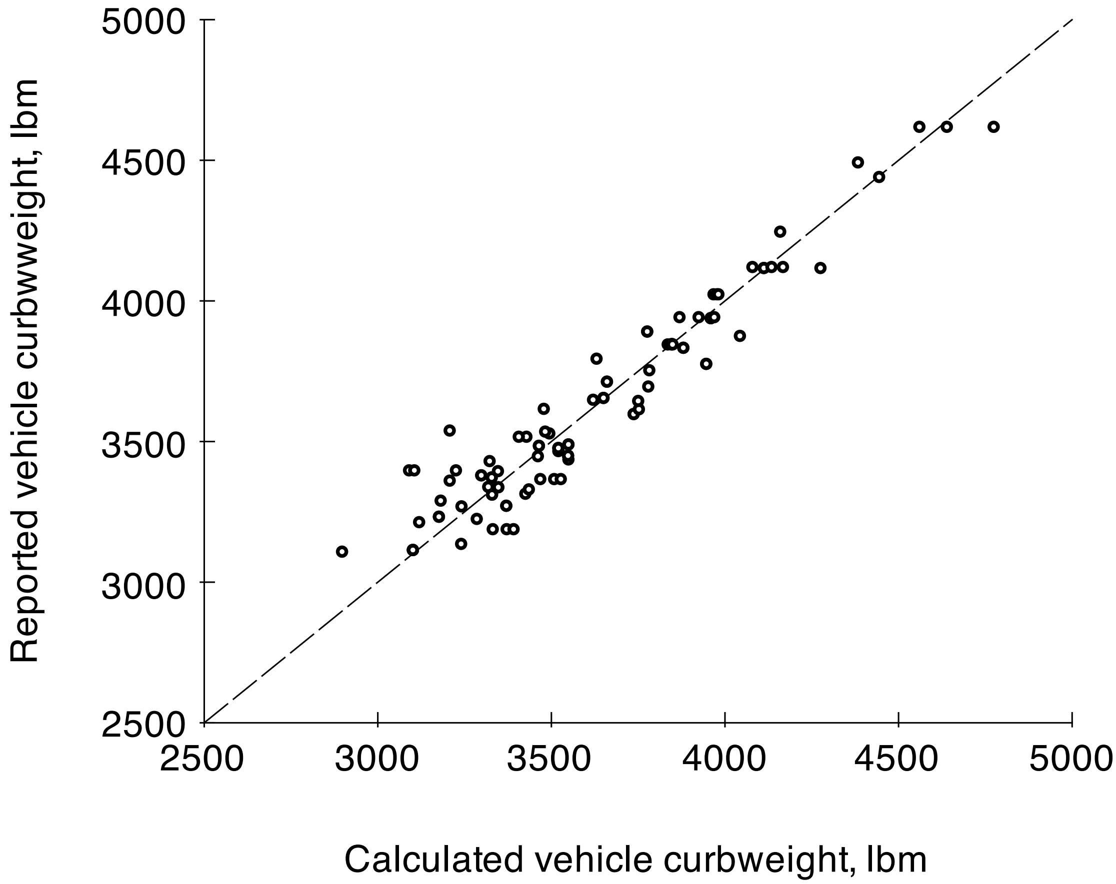


Figure 3.6: Calculated curbweight based on vehicle attributes versus reported curbweight

CHAPTER IV

Demand

This chapter describes various methods adapted from the literature for developing and then assessing discrete choice models for the purpose of design optimization. A suitable demand model is one piece of the required framework for conducting design studies with market simulations. Two newly developed discrete-choice models are presented that were developed from individual-level survey responses of new car buyers. Particular emphasis is made on comparing the substitution patterns implied by the attribute elasticities of demand between the two models. Both newly developed models appear to have avoided extreme price sensitivity as observed with the BLP95 model. The US automotive market was selected to illustrate the choice modeling for design optimization approach. However, the same approach can be applied to a wide range of products and industries. Key features of a product or market that could benefit by the approach are an established market with observable consumer purchase data and a product with technical attributes and constraints that require engineering modeling to compare feasibility of the product design.

Some challenges with estimating choice models are selecting a suitable utility specification where the utility specification is the mathematical formulation that maps product and consumer attributes to consumer preference for product choices; lack of disaggregate consumer data where disaggregate consumer data mean recorded purchase observations and demographic information by individual; correlation of price with unobserved attributes of products; and modeling of the outside good, where the outside good represents the value to the consumer of not purchasing any product in the market.

We test two variations of a utility specification that contains similar elements to previous choice models of the automotive industry. We obtained individual consumer data from a new car buyer survey conducted by Maritz Research. To account for the correlation between price and unobserved product attributes (i.e., price endogeneity), we instrumented for price endogeneity using a previously demonstrated approach. We ignore the possibility of product attribute endogeneity, or the correlation of unobserved vehicle attributes with vehicle attributes that are included in the systematic component of utility. While we think this is an important issue to be explored, it was beyond the scope of this work. Additionally, we did not model an outside good in our choice model. This is common practice in many automotive studies because the choice model is estimated on observed data (i.e., only buyers are observed not individuals who "walked away").

There are two elements that are new with our approach that have not been highlighted in the literature. The first element is that we take a larger than normal set of vehicle alternatives. Most automotive studies have used on the order of 200 vehicles per model year. Our study includes 473 vehicle alternatives. Increasing the number of alternatives allows finer product differentiation by permitting attribute differences associated with power and fuel economy to be captured between same models that come with different engine options. We approximate observed market shares at this level by referencing the EPA collected sales figures, which are organized at this level of detail. The intention is that the large vehicle set allows for improved parameter identification including individual taste differences (i.e., unobserved heterogeneity).

The second novel element is a model specification that incorporates an explicit ideal-point for consumer preferences for vehicle size (i.e., $length \times width$). This model assumes that individuals have an ideal vehicle size rather than a monotonically increasing or decreasing preference for increased vehicle size.

Section 4.1 reviews traditional methods for evaluating demand models . Section 4.2 presents the choice modeling approach and derives the expressions for the relevant vehicle attribute point-elasticities. Section 4.3 presents the horizontal differentiation hypothesis and the model specifications to test the hypothesis, the estimation results, and the model comparisons. Section 4.4 discussions the findings from the model in a design for market systems context, and Section 4.5 summarizes the chapter.

4.1 Methods for Evaluating Demand Models

We define interpretability as a qualitative assessment of how well the functional form of utility is supported by theory or beliefs of market behavior. Questions that a modeler should ask when checking for interpretability before estimation include: Do all components of utility have behavioral or physical significance? Does each behavioral or physical factor influence choice probabilities in a manner that is consistent with theory or belief? After estimation, the modeler should check interpretability by conducting various tests: (1) The significance of the estimated parameters, with particular attention to those deemed to support theory or beliefs; (2) the signs of the estimated parameters; (3) over-fitting; (4) colinearities among attributes. We will report on the new model performance on items 1-2. Items 3-4 require further development and are left for future work.

The form of the choice model can influence decision outcomes more or less in line with economic observations. We postulate that an appropriate choice model should account for price endogeneity, at a minimum. Endogeneity of other vehicle attributes may also be important. The choice model should yield to several intuitive checks: (i) the sum of the non-price component of utility and the price component due to product cost for an individual's utility maximizing vehicle should be interior to the vehicle attribute space for the majority of individuals; (ii) price equilibrium calculations employing the choice model should yield increasing markups with price; (iii) a design optimization should yield firms that are incentivized to produce a portfolio of differentiated products. For example, the simple logit does not meet the second check as it is known to exhibit constant markups for all products of a firm irrespective of price [*Shiau and Michalek* (2009)]. The choice models estimated in this dissertation account for price endogeneity. We leave non-price attribute endogeneity for future work. The performance of the choice models on the three checks is discussed in the market simulation results in Chapter 8.

4.1.1 Measures of Fit

Metrics to evaluate the fit of a choice model can be directly applied from econometrics. Measures of fit emphasize the descriptive power of a model with respect to the same data set used to estimate the model. However, they do not indicate if the model is correctly describing the most important factors or how well the model will predict outcomes based on changes in behavior, both of which are important for design optimization. One primary measure of fit is the likelihood ratio test as described in Chapter 2.

Another common measure of fit is to count the choice shares correctly predicted by the model. However, choice models are probabilistic by nature, dealing in *expected* choice shares. Correct prediction counts are therefore of limited value [*Train* (2003)]. In addition, recent practice in economics is to employ an alternative-specific constant for each alternative during the estimation procedure. This means that each alternative (except one baseline alternative) is identified in the utility specification by a dummy variable. Augmenting the utility specification in this way allows the predicted choice shares to be matched to the observed market shares. We do not include alternative-specific constants in the model specification. However, we emphasize the importance of considering model performance without the aid of the alternative-specific constants because (1) a designer comparing different models should consider how much variance can be described by design attributes before the aid of the constants, (2) the values for alternative-specific constants are not available for many published models, and (3) including alternative-specific constants can add computational burden to the model estimation process. One way to visually assess how the model is doing without alternative-specific constants is to plot observed market shares and predicted market shares on the same graph as a function of a given attribute (e.g., price).

Alternative-specific constants are essential for improving choice share prediction. However, their importance in a design optimization context is less straightforward. It is true that increasing or decreasing the utility of a particular vehicle through an alternative-specific constant could change its own or another vehicle's optimal design decisions. This is expected especially for the mixed-logit case where the cross-elasticities of demand are affected by all vehicle choice alternatives, not only the two vehicles being compared. Future work should study the difference in optimization outcomes between a model that was estimated without alternative-specific constants and one with alternative-specific constants. In the end, this work is computationally limited (i.e., in computer memory) in its ability to explore models with the additional 472 parameters (or 200+ if consolidated to the model level) required to represent the alternative-specific constants. The vehicle brand parameters in the present study act as a coarser version of true alternative-specific constants.

4.1.2 Attribute Elasticity

The elasticity of demand $E_{jX_k^m}$ for vehicle j is the percentage change in demand for j given a percentage change in attribute m of product k. Computing own- and cross-elasticities is then an indication of the substitution patterns expected given changes to vehicle attributes. Given two choice models with identical fit characteristics, the model with the substitution patterns more reflective of the market should yield more useful design optimization results.

The formula for cross- and own-elasticities for individual i given a mixed-logit choice model are as follows [*Nevo* (2000); *Train* (2009)].

$$E_{ijX_k^m} = \frac{\partial P_{ij} X_k^m}{\partial X_j^m P_{ij}} = \begin{cases} \frac{X_k^m}{P_{ij}} \int B_{ik}^m L_{ik}(\mu)(1 - L_{ij}(\mu)) f(\mu) d\mu & \text{if } j = k \\ \\ -\frac{X_k^m}{P_{ij}} \int B_{ik}^m L_{ik}(\mu) L_{ij}(\mu) f(\mu) d\mu & \text{if } j \neq k \end{cases} \tag{4.1}$$

where $P_{ij} = \int (e^{\mu' X_{ij}} / \sum_k e^{\mu' X_{ik}}) f(\mu) d\mu$ is the unconditional likelihood individual i chooses vehicle j, and $L_{ij}(\mu_i) = e^{\mu_i' X_{ij}} / \sum_k e^{\mu_i' X_{ik}}$ is the conditional likelihood individual i chooses vehicle j for a particular μ_i with similar interpretation for L_{ik} for vehicle k, X_k^m is the value of attribute m for vehicle k, and B_{ik}^m is the partial derivative of utility with respect to X_k^m.

The integrals in these equations can be simulated by computing the inner terms for a number of draws from the parameter distributions and then dividing by the number of draws D, where the market-level elasticity $E_{jX_k^m}$ can be approximated by summing the individual elasticities of a representative population and dividing by the number of individuals I.

$$E_{jX_k^m} = \begin{cases} \frac{X_k^m}{ID} \sum^I \frac{1}{P_{ij}} \sum^D B_{ikd}^m L_{ikd}(\mu_d)(1 - L_{ijd}(\mu_d)) & \text{if } j = k \\ \\ \frac{X_k^m}{ID} \sum^I -\frac{1}{P_{ij}} \sum^D B_{ikd}^m L_{ikd}(\mu_d) L_{ijd}(\mu_d) & \text{if } j \neq k \end{cases} \tag{4.2}$$

4.2 Estimating Mixed Logit Choice Models from Disaggregate Data

Two mixed-logit discrete-choice models of the US new vehicle industry were estimated. The goal for the estimation was to develop models that could be used in the design optimization market simulation. A schematic of the choice model development is shown in Figure 4.1. The following subsections describe each of these steps in more detail.

4.2.1 Choice Data

Data for the estimation came from the Maritz Research 2006 New Vehicle Customer Satisfaction survey [*Maritz Holdings Inc.* (2007)] and additional vehicle specification data came from Chrome System Inc.'s New Vehicle Database and VINMatch tool [*Chrome Systems Inc.* (2008)].

The choice set for each individual was selected from 473 vehicles (a subset of 2006 model year vehicle styles corresponding to available make, model, and engine options). We eliminated vehicles priced over $100,000 as well as seven alternatives that were not observed in the survey data, and further reduced the vehicle choice set by consolidating pickup truck and full-size van models with gross-vehicle-weight ratings over 8,500 lb to 2 models each. Summary vehicle data are provided in Table 4.1. The table lists manufacturer suggested retail price $MSRP$; vehicle attributes for fuel economy, engine peak horsepower, vehicle curbweight; and vehicle dimensions length, width, and height.

4.2.2 Instruments

We instrumented for price endogeneity (R^2=0.78) using attribute distance metrics patterned after those reported by [*Train and Winston* (2007)], where instrumental

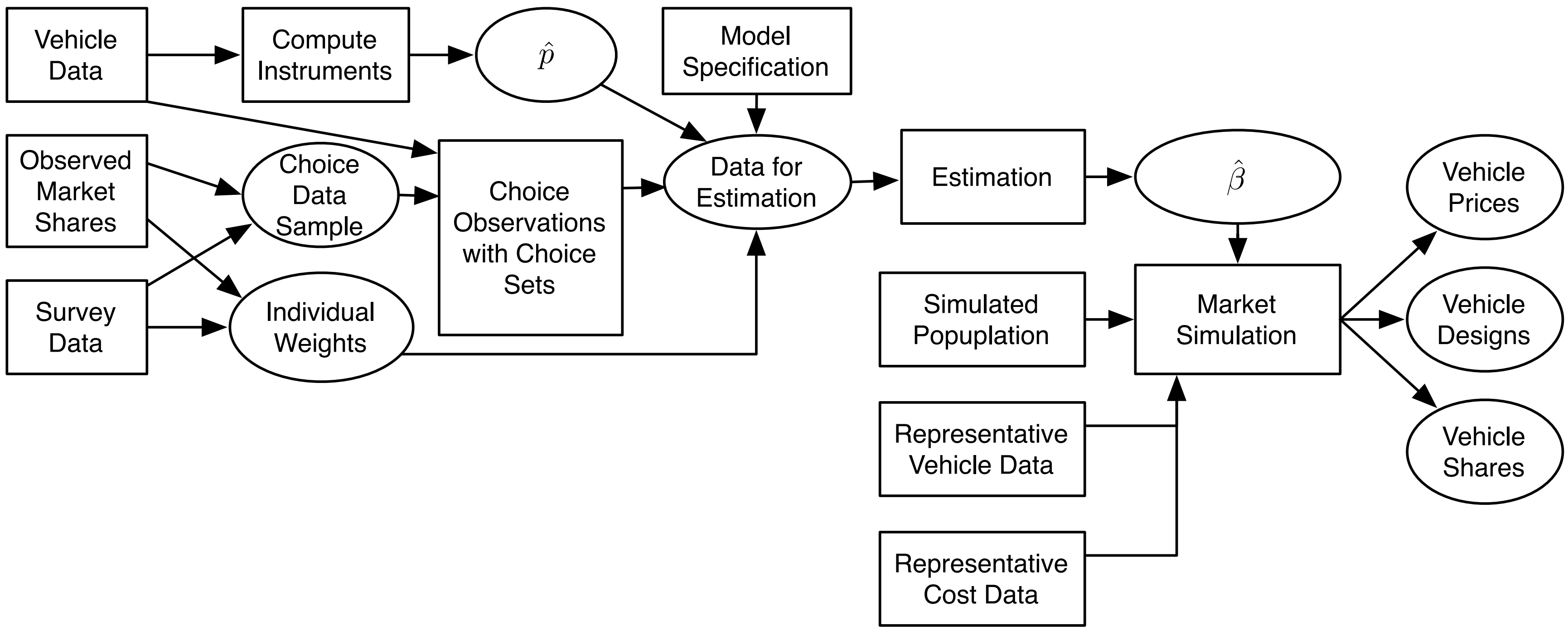


Figure 4.1: Schematic diagram of steps to develop choice models from survey and vehicle data

variables consist of differences between vehicle attributes (z_{hp} and $z_L \times z_W \times z_H$) among a firm's vehicle fleet and among competitors' vehicles as well as the systematic product attribute components of utility. We ignore product attribute endogeneity assuming that the observed non-price attributes are uncorrelated with the unobserved utility component.

4.2.3 Estimation Approach

We estimate new mixed-logit choice models according to the simulated maximum log likelihood approach using Matlab code modified from Kenneth Train's publicly available estimation code [*Train* (2009)].

An individual's choice set was taken to be 100 vehicles including the purchased vehicle, the vehicles strongly considered (up to 3 vehicles as reported by the survey respondent), and uniformly-conditioned randomly-selected vehicles up to the 100 vehicles. We take the reported order of considered vehicles as a preference ranking and treat the overall estimation as an exploded (or rank-ordered) logit. Assuming the error term ϵ_i is independent and identically distributed following the extreme value type 1 distribution in the mixed logit model allows ranked observations to be treated as seperate choice observations [*Train* (2003)]. An individual's choice set for the pseudo-observations is the same as for the purchased vehicle observation choice set with the purchased and higher-ranked considered vehicles removed.

A set of 6,563 individuals were sampled from 81,705 survey respondents[1] using

[1]Nineteen percent of the survey respondents did not report income. These individuals were set

Table 4.1: Summary vehicle attribute data

	$MSRP$	z_{MPG}	z_{hp}	z_{VM}	z_L	z_W	z_H
units	\$	mpg	hp	lbm	in	in	in
min	11925	11.0	65	1975	143.1	65.7	47.6
max	97485	56.6	520	6400	249	86	93.1
median	27330	21.0	230	3682	189.7	72.2	58.7
mean	32675	21.5	241	3887	190.9	72.8	63.4

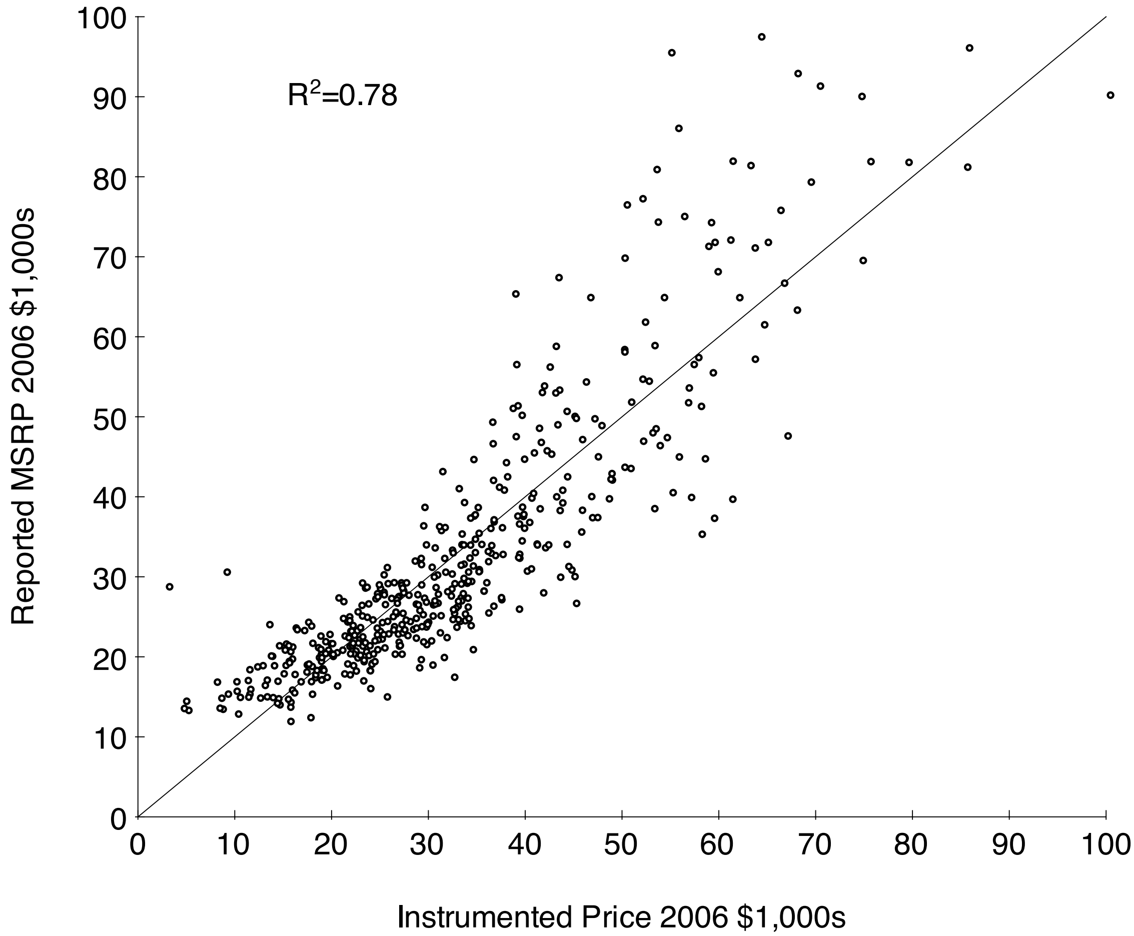


Figure 4.2: Regression showing reported vehicle MSRP as a function of the instrumented price $\hat{p}$

choice-based sampling to approximate 2006 market shares. In some cases either too few vehicle choices were available in the survey to match the sales or too few respondents would be sampled to represent the demographics of purchasers of a vehicle.[2] To account for this, a set of weights was generated for each individual in the sample to adjust the log-likelihood calculation to correctly match 2006 market shares.[3]

aside for sampling purposes as well as individuals who purchased model year 2007 vehicles in 2006.

[2]A minimum of five observations for each vehicle alternative was set (if at least five were available) to increase the sample of demographics for consumers of low market-share vehicles.

[3]The weighting procedure used was available in Train's code. This procedure multiplies the log of each individual's logit probability (including all choice observations) by the weighting value for that individual.

The assumption made with this choice-based sampling approach is that the sampled individuals who purchased a particular vehicle alternative are representative of all individuals who purchased that vehicle.

4.2.4 Attribute Elasticity

We examine the substitution patterns produced by both models by looking at own- and cross-elasticities. Table 4.2 gives the values of B_{ik}^{m} required for Equation (4.2) for the several attributes of interest.

4.3 Testing Newly-estimated Demand Models

4.3.1 Horizontal Differentiation Hypothesis

We conjecture that optimization studies of the automotive market using demand models with little horizontal differentiation will be oversensitive to price and underpredict differentiation in product attributes compared with the observed market. Our hypothesis is that functional forms of econometric demand models that have more freedom to capture horizontal differentiation, if it exists, will perform better in terms of fit than those that imply consumer preferences are predominantly based on vertical differentiation. The improved fit could be taken as one measure that the conventional models underpredict the level of horizontal preference heterogeneity in the market. Additionally, we hypothesize that substitution patterns between vehicles will be noticeably different between the conventional and the increased horizontal differentiation

Table 4.2: Partial derivatives B_{ik}^{m} of attributes evaluated for elasticities

Attribute	Model 1	Model 2
$price$	$\mu_{price}/Income_i$	$\mu_{price}/Income_i$
hp/wt	$\mu_{hp/wt}$	$\mu_{hp/wt}$
$gal/100mi$	$\mu_{gal/100mi}$	$\mu_{gal/100mi}$
$z_L \times z_W$	$\mu_{z_L \times z_W}$	$2\mu_{(z_L \times z_W)^2}(z_{L_k} z_{W_k}) - 2\mu_{(-2z_L \times z_W)}$

models.

We choose vehicle footprint to test an ideal-point preference model because we believe it is the continuous attribute that is most likely to demonstrate horizontally differentiated preference.[4]

4.3.2 Model Specification

The utility specification can be broken into three parts following Equation (2.1): Terms that rely on the product alone $\delta'\mathbf{d}_j$ (brand dummies[5] European d_{Eur}, Japanese d_{Jap}, Chrysler d_{Chr}, General Motors d_{GM}, and Korean d_{Kor}); interactions between product attributes and demographics $\beta'\mathbf{b}_{ij}$ (minivan and children $b_{m\cdot c}$, SUV and children $b_{s\cdot c}$, pickup truck and rural living $b_{p\cdot r}$); product-attribute or attribute-demographic interaction terms with individual-specific random coefficients (assumed normally distributed) $\mu_i'\mathbf{m}_{ij}$ (vehicle price divided by individual income $p_j/s_{inc,i}$, power to weight ratio (a proxy for acceleration) z_{hp}/z_{VM}, combined city and highway fuel consumption $100/z_{MPG}$, vehicle footprint $z_L \times z_W$; class dummies based on EPA vehicle classes: Two seater or minicompact m_{tsmc}, minvan m_{mvan}, Sport utility vehicle m_{SUV}, full size van m_{van}, pickup truck m_{pup}; and a hybrid powertrain dummy v_{HEV}) [6].

Model 1 assumes utility is monotonic in vehicle footprint: $\mu_1(z_L \times z_W)$. Model 2 assumes an ideal-point model of footprint: $\mu_1(\mu_2 - z_L \times z_W)^2$, which implies an interior maximum when μ_1 is negative. Variation across individuals in μ_2 represent individual-specific ideal footprints for a vehicle. In order to use estimation techniques built around linear-in-parameters utilities we simplify the expression by expanding the quadratic: $\mu_1\mu_2^2 - 2\mu_1\mu_2 z_L \times z_W + \mu_1(z_L \times z_W)^2$ and drop $\mu_1\mu_2^2$, leaving $\mu_1(z_L \times z_W)^2 - \hat{\mu}_2 2 z_L \times z_W$ (Note: $\hat{\mu}_2$=$\mu_1\mu_2$), which we use in the estimation. We can drop

[4]We expect preference for acceleration and fuel economy to be monotonic although not necessarily linear.

[5]Ford was considered the baseline brand, so no dummy variable was used for Ford vehicles. This means that all brand coefficient values are with respect to Ford.

[6]z_{hp}/z_{VM} is $z_{hp} \times 10/z_{VM}$ where z_{VM} is measured in lbm, and footprint is $z_L \times z_W$ measured in in.^{2}/10000.

$\mu_1\mu_2^2$ from the expression because it is constant across vehicles and only relative utility affects choice probabilities.

4.3.3 Model Identification

Parameter estimates and standard errors are reported in Table 4.3.

4.3.4 Model Comparisons

Fit

The log-likelihood ratio, or psuedo-R^2 value, for the models were 0.115 and 0.119 respectively. While these values are substantially lower than the target range of 0.2-0.4, the likelihood ratio cannot be compared across other models in the literature estimated on different data. Overall, both models are comparable in terms of fit.

Interpretability

Attributes and demographics were chosen with physical interpretations related to design optimization and product planning. Price divided by income allows sensitivity to price to change nonlinearly with income. This follows the intuition that price influences choices more when it represents a higher percentage of annual income. Horsepower over curbweight is a proxy for acceleration, which we believe is an important factor in purchases. Other performance metrics may be better suited for certain vehicle classes concerned with towing, for example, but this extension is beyond the scope of the present study. Vehicle size should also be relevant to the car-buying decision. However, the monotonic formulation of Model 1 seems nonsensical when carried to extremes. The formulation in Model 2 maps more naturally to the observation that different size vehicles (of the same price) succeed in the market.

The vehicle segment dummies match observed vehicle classes in the market. Sedans (ranging from subcompacts to full-size) represent the baseline vehicle, so values of the segment dummies are relative to those vehicles. The majority of two-seater and mini-compact vehicles are sports cars. We included this parameter to see if this class would command a premium above the average tradeoff for acceleration and other attributes. A hybrid dummy was also included to account for any influences on purchase decisions

Table 4.3: Vehicle demand model parameter estimates

Mean values				
	Model 1		Model 2	
Parameters	Estimate	Std. Error	Estimate	Std. Error
p/s_{inc}	3.29	0.13	4.11	0.16
$10z_{hp}/z_{VM}$	0.4	0.13	0.62	0.13
$100/z_{MPG}$	-0.82	0.03	-0.86	0.03
$z_L \times z_W$	5.33	0.13	-	-
$(z_L \times z_W)^2$	-	-	-15.6	0.94
$-2z_L \times z_W$	-	-	-25.5	1.37
m_{twmc}	-1.7	0.46	0.25*	0.23
m_{mvan}	-6.03	0.69	-4.74	0.53
m_{SUV}	-0.56	0.11	-0.56	0.11
m_{van}	-13.2	3.55	-7.72	0.97
m_{pup}	-4.79	0.46	-1.77	0.14
v_{HEV}	-3.89	0.48	-4.97	0.63
d_{Eur}	-0.38	0.055	-0.18	0.059
d_{Jap}	0.15	0.036	0.24	0.038
d_{Chr}	0.16	0.043	0.13	0.044
d_{GM}	-0.43	0.036	-0.34	0.038
d_{Kor}	-0.63	0.059	-0.55	0.06
$b_{m \cdot c}$	2.87	0.31	2.53	0.26
$b_{s \cdot c}$	0.74	0.13	0.76	0.13
$b_{p \cdot r}$	4.66	0.53	2.07	0.22
Standard Deviations				
p/s_{inc}	0.88	0.17	0.79	0.19
$10z_{hp}/z_{VM}$	0.19*	0.3	0.04*	0.32
$100/z_{MPG}$	0.73	0.03	1.01	0.03
$z_L \times z_W$	0.36*	0.33	-	-
$(z_L \times z_W)^2$	-	-	0.17*	0.17
$-2z_L \times z_W$	-	-	4.29	0.23
m_{twmc}	2.47	0.3	1.08	0.27
m_{mvan}	5.12	0.47	4.16	0.37
m_{SUV}	3.19	0.15	2.93	0.13
m_{van}	8.17	2.15	4.91	0.63
m_{pup}	6.21	0.5	2.42	0.17
v_{HEV}	2.16	0.38	2.94	0.44

* not significant at 95% confidence interval

of hybrids independent of fuel economy.

Brand differences were consolidated into six groups (European, Japanese, Chrysler, GM, Korean, and others–i.e., Ford). Other combinations could also be tested but are outside the scope of this study. The demographic interactions of minivan and SUV purchases in households with children and pickup truck purchases in rural areas are intuitive.

The signs of both sets of parameters are generally as expected including the signs for the footprint terms in Model 2, which implies local maximum values. It is non-intuitive that individuals would prefer lower fuel economy, but the large standard deviations on the fuel consumption parameter implies this behavior. This is a case where either the model lacks appropriate instruments to tease apart the power to weight ratio versus fuel consumption interaction, the model should be respecified to get at a more intuitive preference structure, or the data set should be improved. For example, an alternative would be to use a cost to drive attribute such as $\$/mile$ and estimate a model where gasoline price is determined using date of purchase.

Another observation is that the mean value for the two seater or minicompact vehicle class is positive but not significant in Model 2 and negative and significant in Model 1. Both standard deviations are significant. This is likely related to the differences in the footprint specifications. This case illustrates the challenge working with these models because a change in specification propagates to other elements of the specification that may have very different interpretations in a design optimization context.

Model 2 includes two more terms than Model 1 and has one more term that is significant in a two-tailed t-test at a 95% confidence interval as noted in Table 4.3. Notably, the standard deviations for footprint and power to weight ratio in Model 1 are not significant. Model 2 appears to support our initial hypothesis with both mean and one standard deviation footprint term significant.

Predictive Validity

The use of choice-based sampling did not allow true hold-out sample testing from the 2006 survey data. Comparing model behavior on data from another year is a next step for future work. A psuedo-hold-out-sample test was generated by drawing a new choice-based sample from the 2006 survey data. The log likelihood ratio for the pseudo-hold-out-samples for Model 1 was 0.106 and for Model 2 was 0.113. This was an overlapping sample because the low observation choices in the survey data where identical in both samples.

Plausibility

We investigate the plausibility of the models through visualization schemes. First, we compare existing market shares in Figure 4.3. The purpose for the comparison with the BLP model is not to compare performance directly per se given that BLP was estimated on 1971-1990 vehicle data, but to illustrate a potential pitfall in adopting an "off-the-shelf" choice model for a design for market systems study[7].

We simulated own- and cross-elasticities for each vehicle alternative using the estimation population of individuals ($I = 6,563$) and 100 standard normal random draws for each individual. Figure 4.4 shows the own-elasticities for $price$, hp/wt, $gal/100\ mi$, and $z_L \times z_W$ for both Model 1 and Model 2 for all 473 vehicles. The cross-elasticity for $z_L \times z_W$ is shown for both models in Figure 4.5. In both figures vehicles were ordered to aid interpretation: From left to right the vehicles were grouped by class; for Figure 4.4 vehicles were ordered within class from smallest to largest value of the corresponding attribute; for Figure 4.5 vehicles were ordered within class from

[7]To make the comparison as fair as possible, the following steps were taken. All attributes involving dollar values were scaled from 2006$ to 1983$ for BLP utility evaluation. Additionally, a model of the outside good was reported in BLP but not in model 1 and model 2. We normalize utility comparisons between all models by differencing the maximum utility for each individual from the other utilities for that individual. For BLP we further eliminate individuals whose maximum utility did not exceed the value of the outside good and then rescale the market shares based on the remaining individuals.

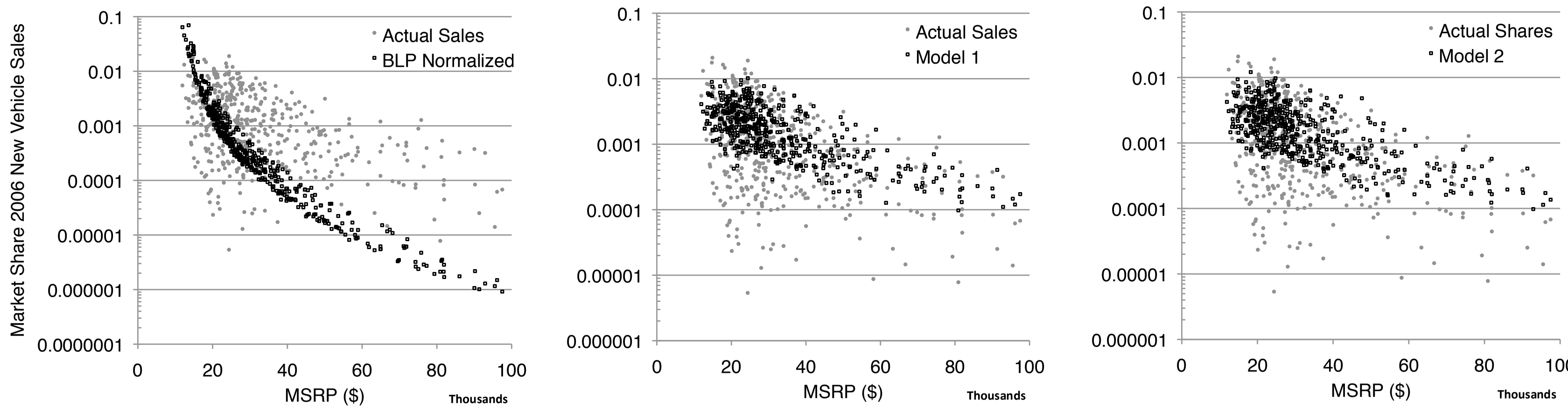


Figure 4.3: Predicted shares vs. actual shares. Left: BLP; Center: Model 1; Right: Model 2

least to most expensive.

Comparing own-elasticities in Figure 4.4 reveals that Model 2 has greater heterogeneity in the $z_L \times z_W$ and $gal/100\ mi$ attributes. The own-elasticities for $z_L \times z_W$ in Model 2 are large positive values indicating that, in the overall market, increasing size is preferred. Another important difference is that while Model 1 indicates that, on average, size is more important as the vehicle size increases (shown by increasing elasticities from smaller to larger classes), Model 2 shows that increased size is more important for the large sedans and much less important for full size vans and pickups.

Figure 4.5 is a gray-scale plot showing cross-elasticities for the $z_L \times z_W$ parameter for Model 1 (a) and Model 2 (b). More negative elasticities are shown in darker shades. Many effects can be observed by studying the plot in detail. For example, changes in the sedan class affect shares in the two seater or minicompact class, but changes in the two seater or minicompact class have less effect on the sedan class. Panel (b) shows that some vehicles have positive cross-elasticities for $z_L \times z_W$ in Model 2 (primarily for changes to the large pickup trucks and vans). This means that increasing size of vehicle k will increase the market share of vehicle j. Both models show much stronger substitution within class than between classes. Panel (b) shows that vehicles within the same class and closer in price have greater magnitude cross-elasticities (an intuitive property). This is seen by the lighter shading on the upper diagonal for each block compared to the lower diagonal. This effect is less pronounced for Model 1, but it is difficult to conclude from the plot if this is a real difference between models or due to the difference in cross-elasticity magnitude.

4.4 Implications for Design for Market Systems

The results from the previous section point to the differences in choice model behavior given different choice model specifications. Two questions relating to the application of the choice models in a design optimization context are addressed in this

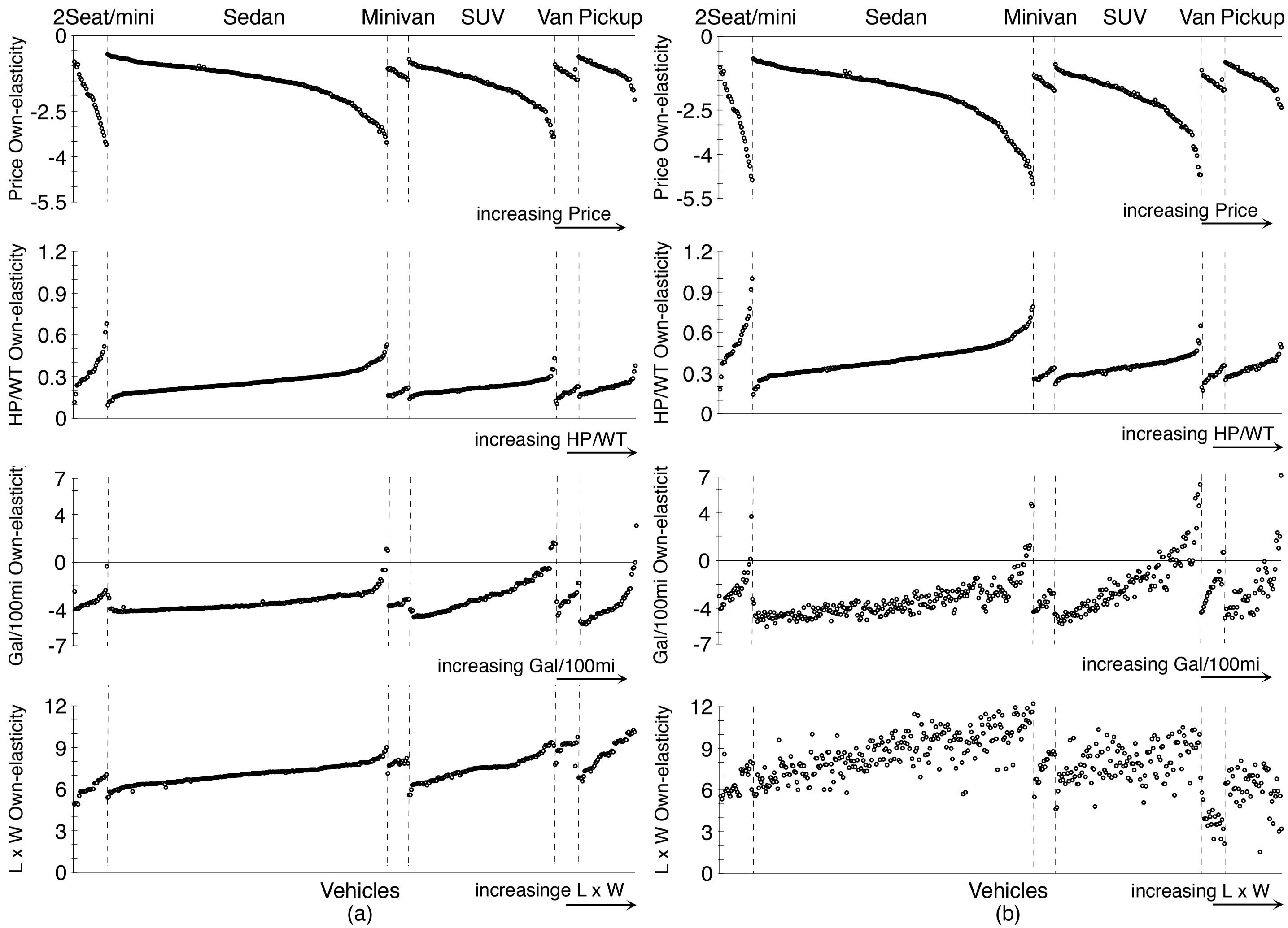


Figure 4.4: Own-Elasticity for all vehicles by attribute (a) Model 1, (b) Model 2

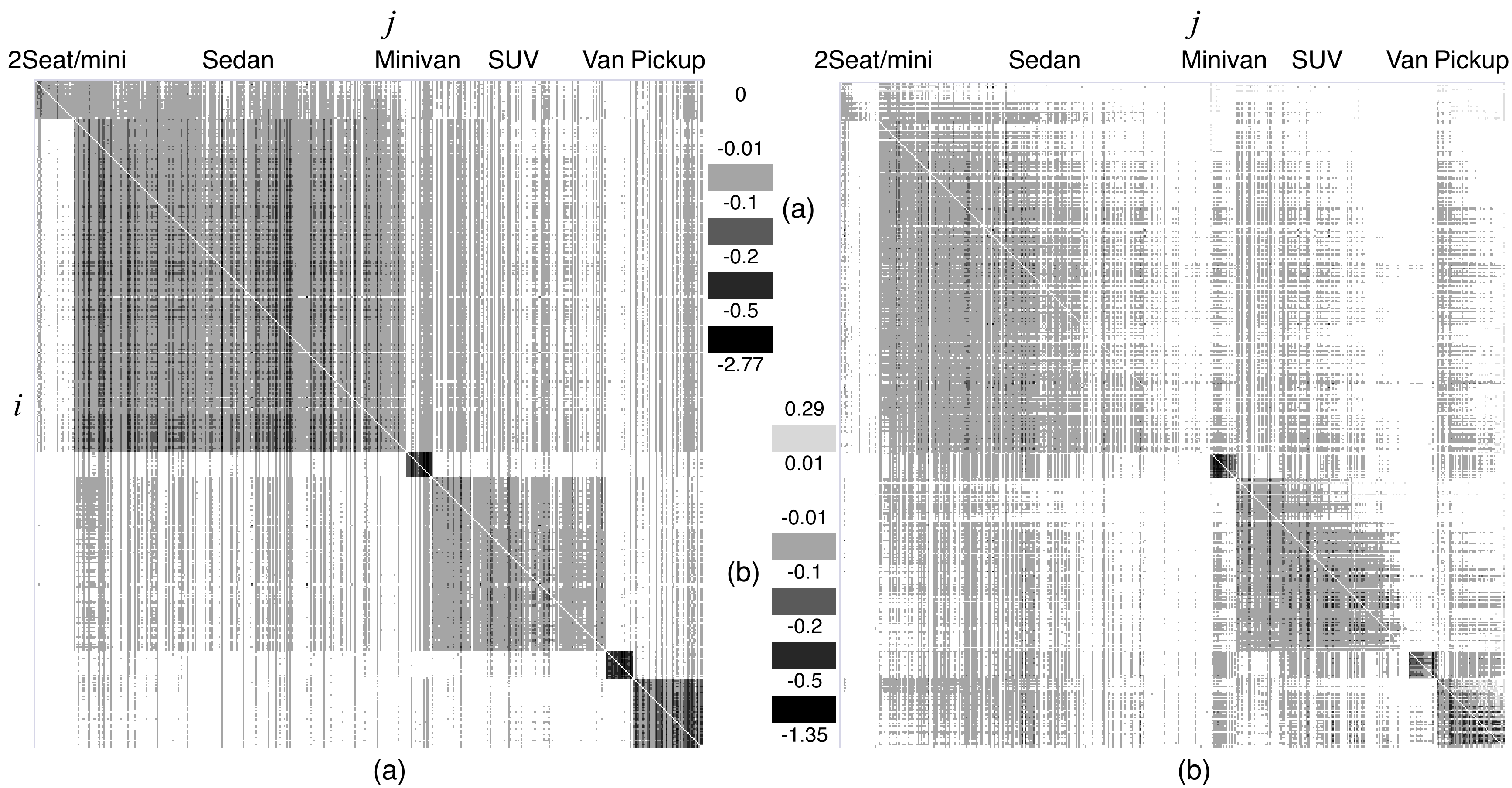


Figure 4.5: Cross-Elasticity for all vehicles for $z_L \times z_W$ attribute (a) Model 1, (b) Model 2

section. The first section looks at the influence of using different numbers of draws to simulate the taste distribution for each individual in the simulated population. The second section briefly highlights the fact that the effect of the specification differences may be most pronounced when the entire vehicle fleet is free to change vehicle design. This step is beyond the scope of this work.

4.4.1 Simulating population heterogeneity

With demand model in hand, the design for market systems goal is to incorporate the demand model into a market simulation. Such a simulation evaluates product demand given product attributes and prices. It may include a game that represents competitive behavior. Product demand is evaluated by simulating a population of car buyers with representative demographic characteristics. Random taste coefficients are drawn for each individual. A set of coefficients μ for individual i from a single draw can be used to compute the conditional probability of individual i choosing each product j given μ. However, the individual's "true" μ's are not known, rather the distribution of μ. The unconditional choice share over the distribution of μ for an individual can be simulated by taking multiple draws of μ and averaging the conditional choice shares. Each draw per individual adds additional computational expense to the market simulation. What is the right number of draws to capture the population heterogeneity? Figures 4.6-4.13 investigate this question by computing the own-elasticity for each attribute for Model 1 and Model 2. The elasticity of demand with respect to some attributes (those with little or no taste variation) changes little between 1 draw/individual and 128 draws/individual. However, the elasticity for attributes such as the footprint attribute in Model 2 change dramatically.

The simulations were performed on the identical population (I=6,563) used to estimate the models. Draws were standard normal random draws generated by Matlab. Figure captions for Figures 4.6-4.13 indicate the number of draws in each illustrated case. The gray data are the elasticities from the previous case (i.e., the figure panel for 2 draws shows the 1 draw results "grayed-out"). Figures 4.6-4.9 correspond to Model 1 (linear in footprint). Figures 4.10-4.13 correspond to Model 2 (quadratic in footprint).

Price elasticities change primarily for the most expensive vehicles, and here the change is small. Therefore, the expectation is that price equilibrium results would change little from the case with 1 draw to the case with many draws (i.e., 128). However, elasticities for the the fuel consumption and size attributes change to a large degree. Therefore, we expect the results of design simulation results that allow vehicle attributes to change to change between the case with 1 draw and the case with 128 draws per individual.

4.4.2 Model specification

We hypothesize that the differences in model parameter values and the differences observed in the attribute elasticities will lead to different design optimization market simulation outcomes given the same inputs. Testing this hypothesis is one of the outcomes of the case studies detailed in Chapter 8. Future work remains to expand the engineering modeling framework to allow modeling of multiple product offerings across the vehicle fleet for each firm. This is an important piece to fully test the above hypothesis because the most dramatic differences in elasticities occur for the luxury models and for specialty market segments such as the two-seaters/minicompacts, and the pickup trucks. For example, comparing Model 1 and Model 2 in Figure 4.4 shows that the own-elasticities for footprint in Model 1a primarily monotonic relationship across a vehicle class for the entire vehicle fleet. Own-elasticities for footprint in Model 2 are scattered. Notably, Figure 4.4 also shows that own-elasticities for footprint for full size vans and pickup trucks classes are among the highest magnitudes across classes in Model 1 and among the lowest magnitudes across all classes in Model 2.

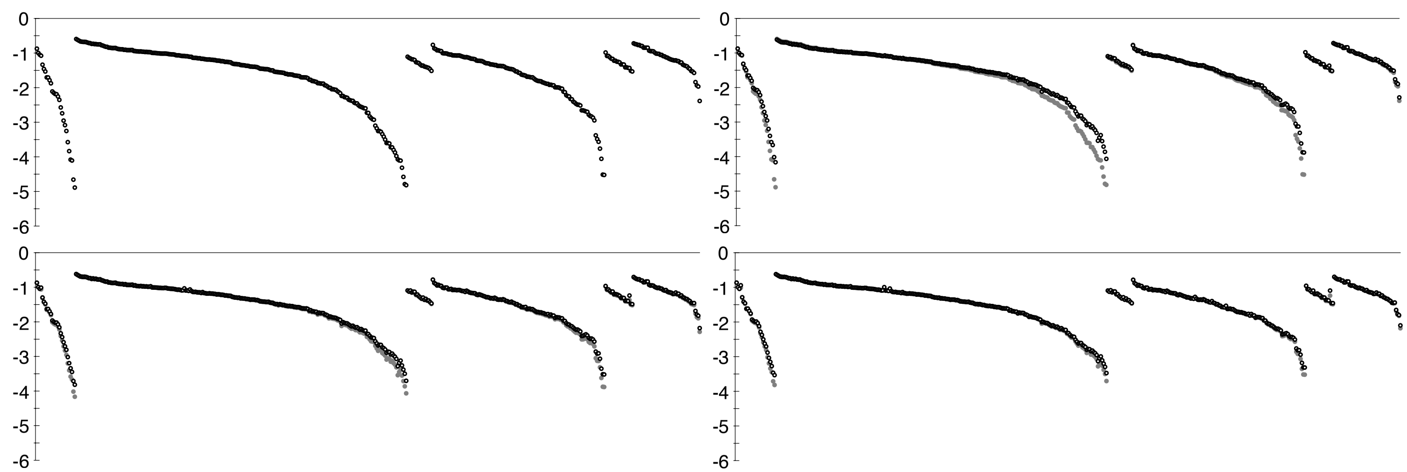


Figure 4.6: Simulated own-elasticity for price attribute for various numbers of draws per individual. From top to bottom, left to right, Number of Draws: 1, 8, 32, 128

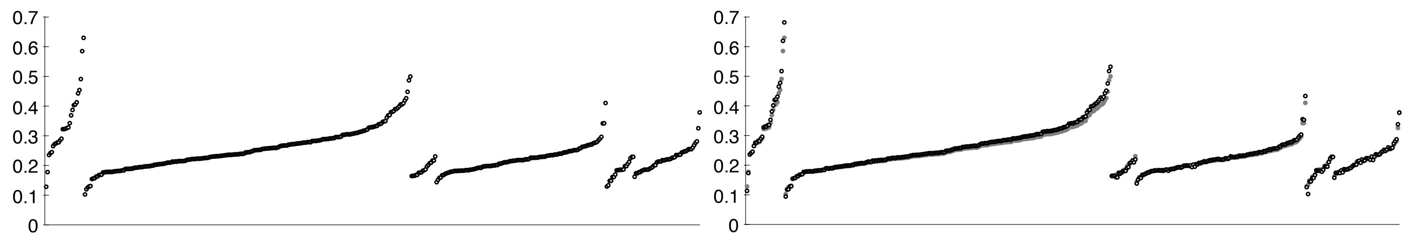


Figure 4.7: Simulated own-elasticity for hp/wt attribute for various numbers of draws per individual. From top to bottom, left to right, Number of Draws: 1, 128

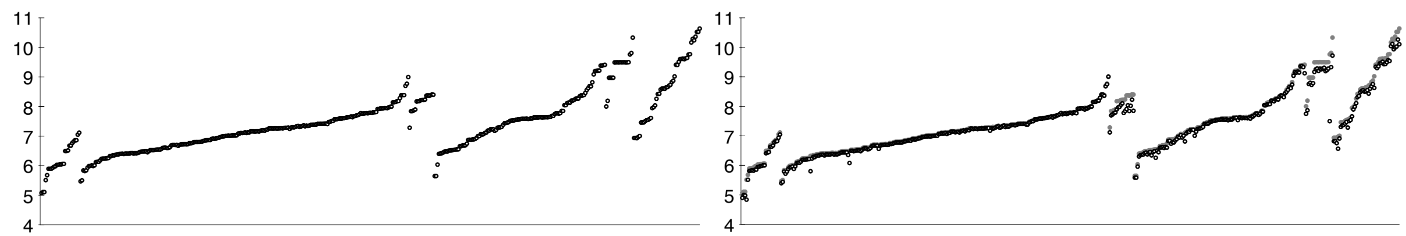


Figure 4.8: Simulated own-elasticity for $z_L \times z_W$ attribute for various numbers of draws per individual. From top to bottom, left to right, Number of Draws: 1, 128

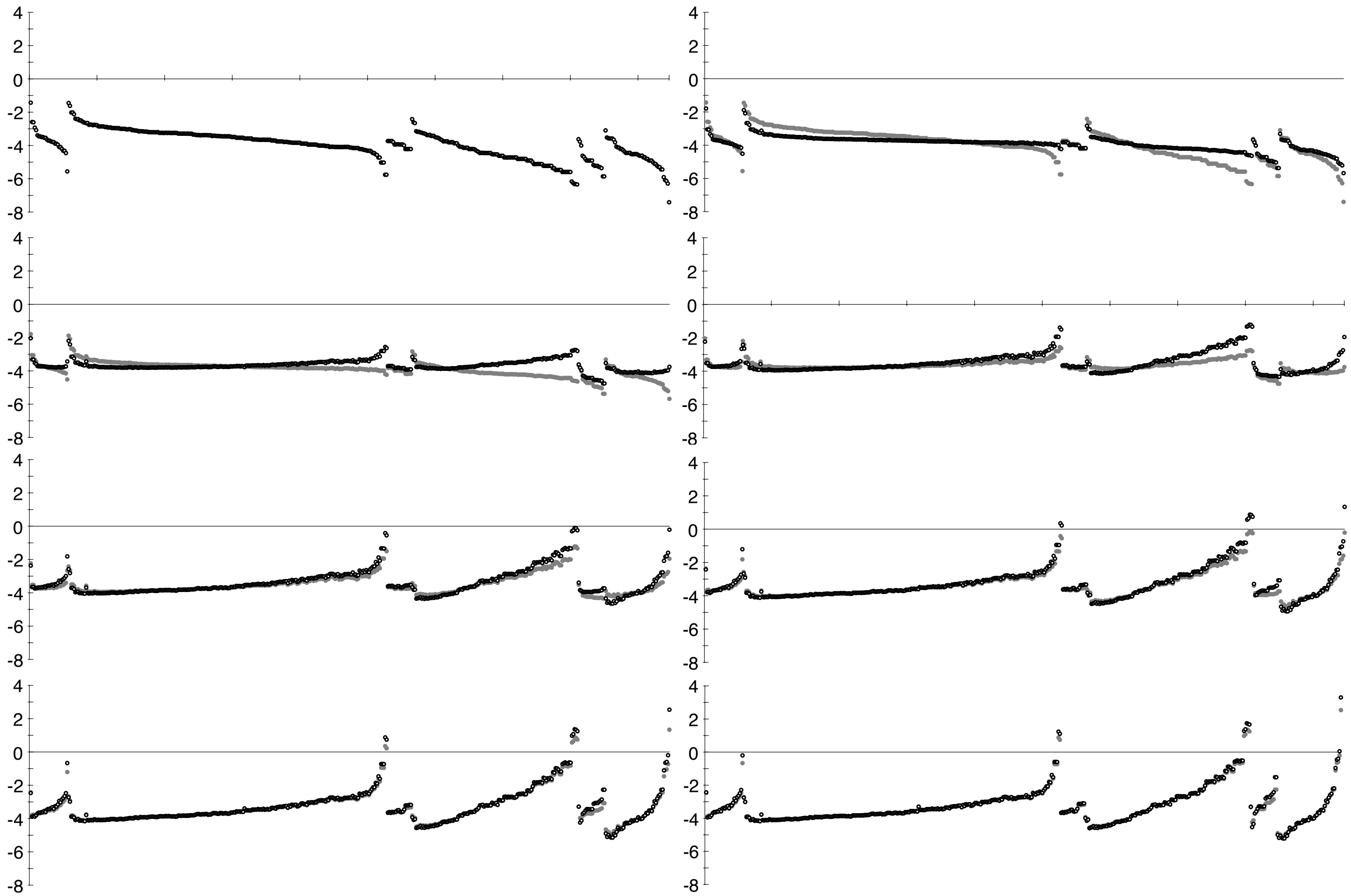


Figure 4.9: Simulated own-elasticity for fuel consumption attribute for various numbers of draws per individual. From top to bottom, left to right, Number of Draws: 1, 2, 4, 8, 16, 32, 64, 128

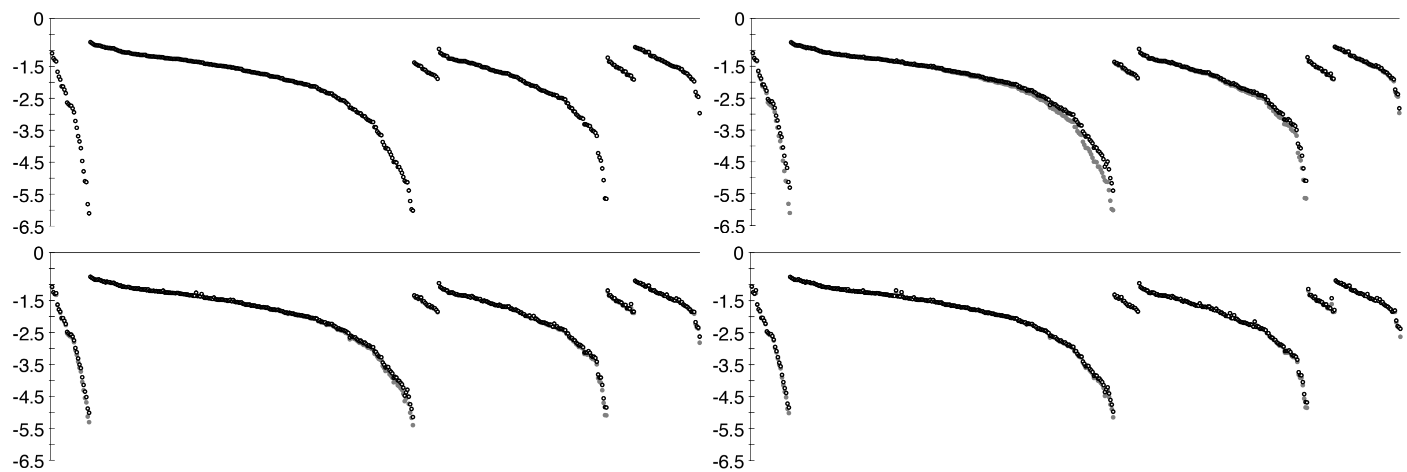


Figure 4.10: Simulated own-elasticity for price attribute for various numbers of draws per individual. From top to bottom, left to right, Number of Draws: 1, 8, 32, 128

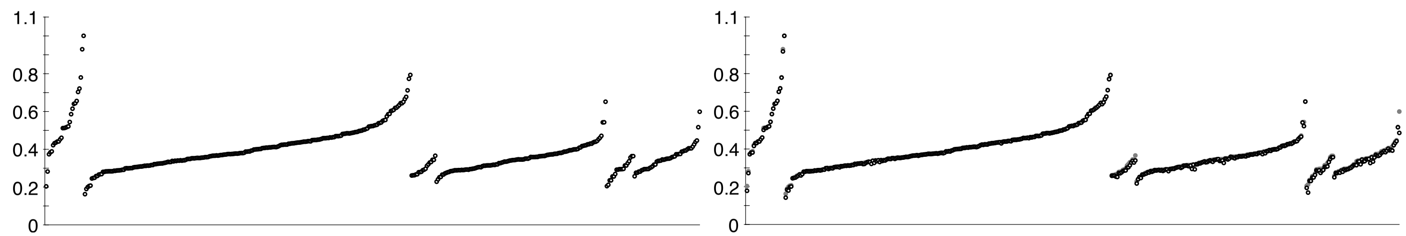


Figure 4.11: Simulated own-elasticity for hp/wt attribute for various numbers of draws per individual. From top to bottom, left to right, Number of Draws: 1, 128

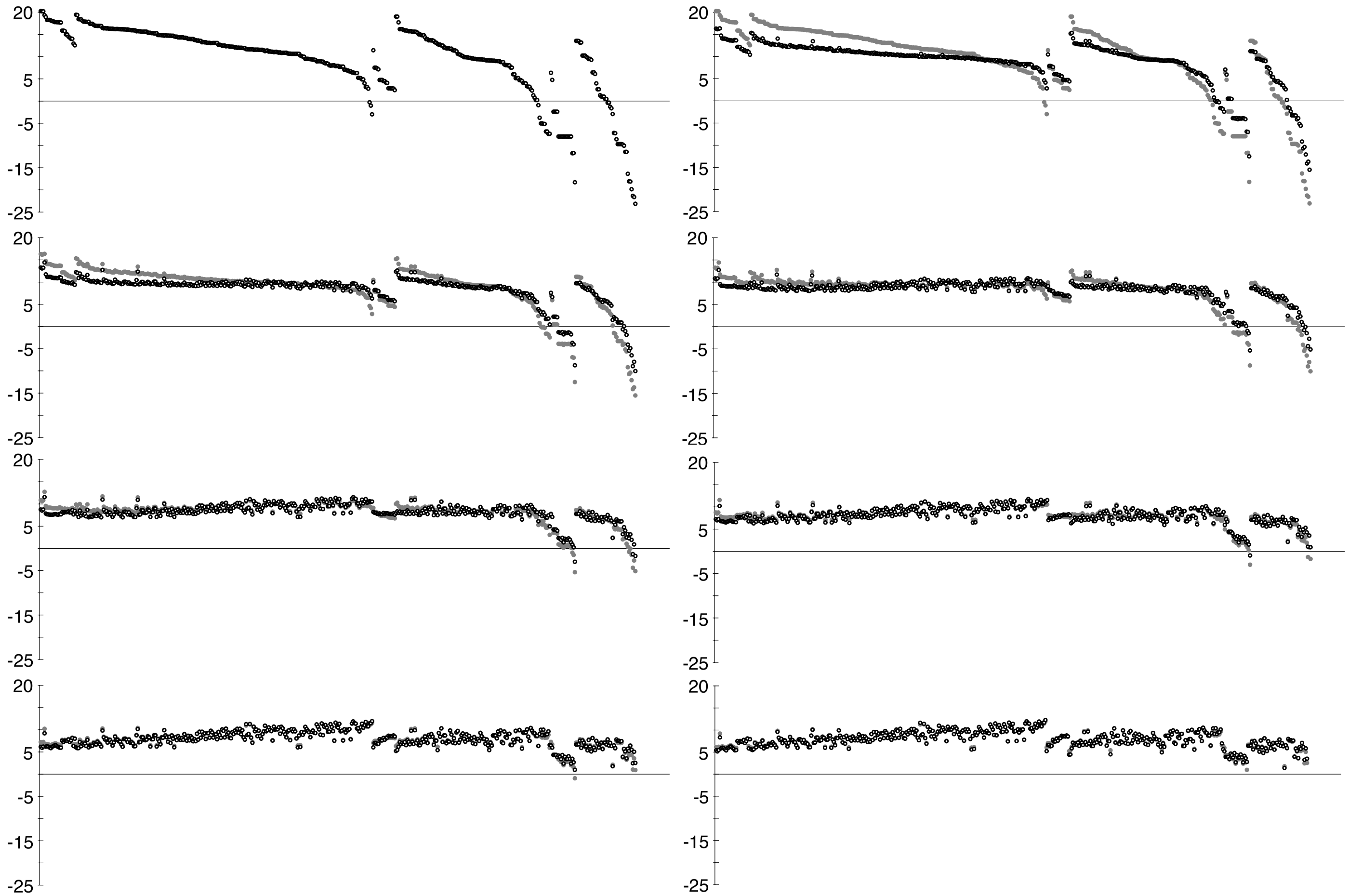


Figure 4.12: Simulated own-elasticity for $z_L \times z_W$ attribute for various numbers of draws per individual. From top to bottom, left to right, Number of Draws: 1, 2, 4, 8, 16, 32, 64, 128

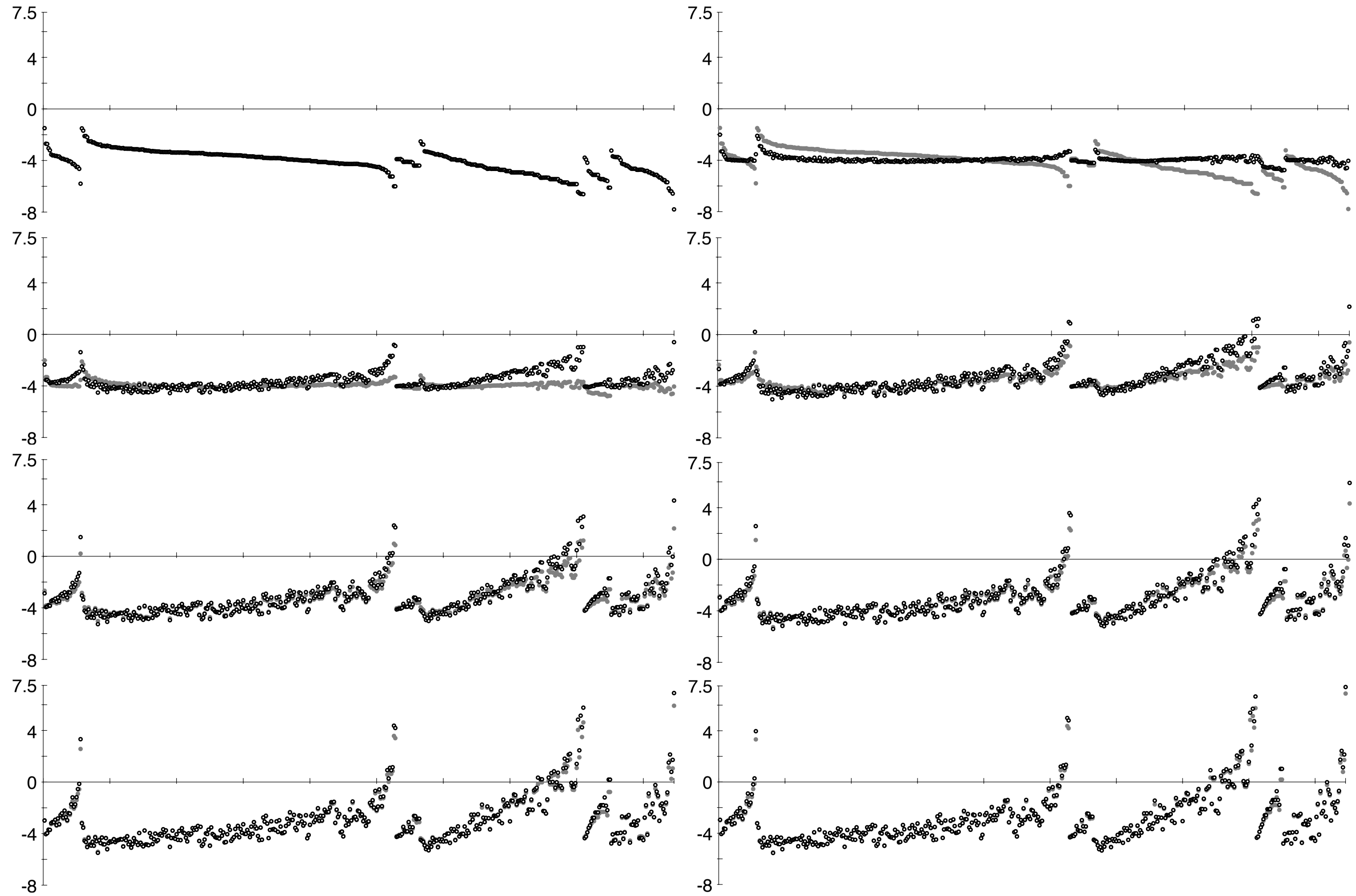


Figure 4.13: Simulated own-elasticity for fuel consumption attribute for various numbers of draws per individual. From top to bottom, left to right, Number of Draws: 1, 2, 4, 8, 16, 32, 64, 128

4.5 Summary

Methods were presented for developing and evaluating consumer choice models that are suitable in a design optimization context. We used these methods to test the hypothesis that allowing horizontal taste preferences to be expressed explicitly and separately from distributions of random coefficients improves the performance of the model with respect to these metrics. Results suggest that the inclusion of horizontal preferences slightly improved fit and predictive validity, but this effect was not strong enough to clearly support the hypothesis. With respect to plausibility, we show that the inclusion of horizontal-preference terms significantly modifies substitution effects. This behavior is expected to change optimization results as shown in Chapter 8.

Applying similar evaluation methods to those presented should be foundational to design for market systems research in order to evaluate hypotheses about consumer-preference relationships. Similar evaluation methods are needed for producer cost models and for game-theoretic competitive behavior. The right combination of appropriate demand and cost models, and competitive assumptions should lead to a set of intuitive checks on plausibility applied to the entire market system.

CHAPTER V

Cost

A key component in testing hypotheses for market and environmental outcomes assuming profit-maximizing firm behavior is an adequate representation of production costs as a function of producer decisions. Here we explore the ability to capture such a model relying on publicly available data rather than proprietary cost estimates or joint demand and cost model estimations. Section 5.1.1 examines some publicly available data for vehicle prices and projected dealer markups.

The objective of the cost model exploration is to identify promising approaches for generating parametric cost representation of vehicle production costs. Parametric cost models typically relate variable costs to a firm's product design and production decisions through a mathematical expression. Cost models of this form are particularly well suited for integration within a design optimization context.

Three cost modeling approaches are examined in this chapter. The first two approaches are motivated by work in the engineering community. The third approach is motivated by the economics community. The first approach (Section 5.1.2) develops a cost representation specifically for the midsize crossover segment. It is based on establishing a baseline vehicle configuration and then linearly scaling costs based on deviations from the baseline attributes. The second approach (Section 5.1.3) develops a regression equation as a function of product characteristics for predicting pre-specified costs based on an assumed relationship between market prices, dealer markups, and original equipment manufacturer (OEM) markups. The third approach (Section 5.1.4) begins with the newly-estimated demand models and backs out the costs for each vehicle by enforcing the equilibrium conditions for the given vehicle designs and prices. The costs are then regressed on vehicle characteristics.

No approach has as yet yielded completely satisfactory results. One of the simplifying assumptions for all three models is that the cost factors are additively separable. This means that the total per vehicle cost is a linear combination of product attributes. It is conceivable that the true cost relationship may be described by a nonlinear interaction between product characteristics. Exploring cost models and their implications for design optimization is an important area for future work.

Additionally modeling is required to supplement each modeling approach in the case of new technologies. Section 5.1.5 presents models to account for incremental costs associated with gas turbo-charged direct-injection engine technology and hybrid electric vehicle technology. Section 5.2 summarizes the chapter.

5.1 Cost Model Approaches

5.1.1 Exploratory Data

The scaling and the empirical markup cost model representations rely on an estimation of dealer and OEM markup, at least for an average vehicle. These estimates were made by examining publicly available price data (i.e., quoted dealer invoice and MSRP values). Wards automotive yearbook provides data on several vehicle characteristics as well as MSRP for new vehicles sold in the US each year [*Wards Communications* (2006)]. In addition, many online sources such as Edmunds.com [*Edmunds Inc.* (2008)] or Cars.com [*LLC Classified Ventures* (2008)] provide an estimate of the dealer invoice price, destination charge, and MSRP. In the absence of transaction price

Table 5.1: Average dealer markups and estimated full markups by class

Class	Sample Size	Min	Max	Avg	Std Dev	OEM+Dealer Markup
Crossover	35	5.9%	12.4%	8.1%	1.9%	17.7%
Large Pickup	10	10.5%	15.5%	13.4%	1.8%	41.3%
Large SUV	11	9.3%	15.6%	13.2%	2.2%	40.2%
Large Van	13	9.9%	15.6%	13.9%	1.8%	43.9%
Mid-size SUV	18	5.5%	12.0%	8.9%	2.2%	20.7%
Small Pickkup	12	5.7%	11.9%	9.9%	1.6%	24.4%
Small Van	14	6.2%	12.3%	9.1%	1.8%	21.5%
Lux. SUV/Cross.	30	6.3%	14.3%	9.9%	2.0%	24.5%

data for vehicle sales, these data sources provide a comprehensive information source that offers insight into the relationship between vehicle characteristics, price, and cost. Examining 143 vehicle makes for model year 2005, as reported by Cars.com, indicates there are differences by vehicle class for the percentage markup between dealer invoice and manufacturer suggested retail price. Table 5.1 and Figure 5.1 show the average dealer markup for various classes of light-duty trucks. The error bars in Figure 5.1 show $\pm$ 1 standard deviation assuming the markups are normally distributed. The trends match the historical belief that large trucks and large SUVs are more profitable for dealers and OEMs than smaller vehicles.

5.1.2 Scaling Models

The cost model, modified from De Weck [*De Weck et al.* (2006)] and Cook [*Cook* (1997)], is based on assigning a cost to a hypothetical average vehicle and then computing the cost for a specific vehicle based on deviations from the average. Approaching cost modeling in this way enables design-specific cost differences to be considered without requiring a complete bottom up cost structure. The initial cost value is generated from assumptions about average profit margin $\gamma = 20\%$ (OEM plus dealer), as well as the average operating leverage $\phi = 0.35$ (relative distribution of fixed vs. variable costs). No learning curve effect was assumed.

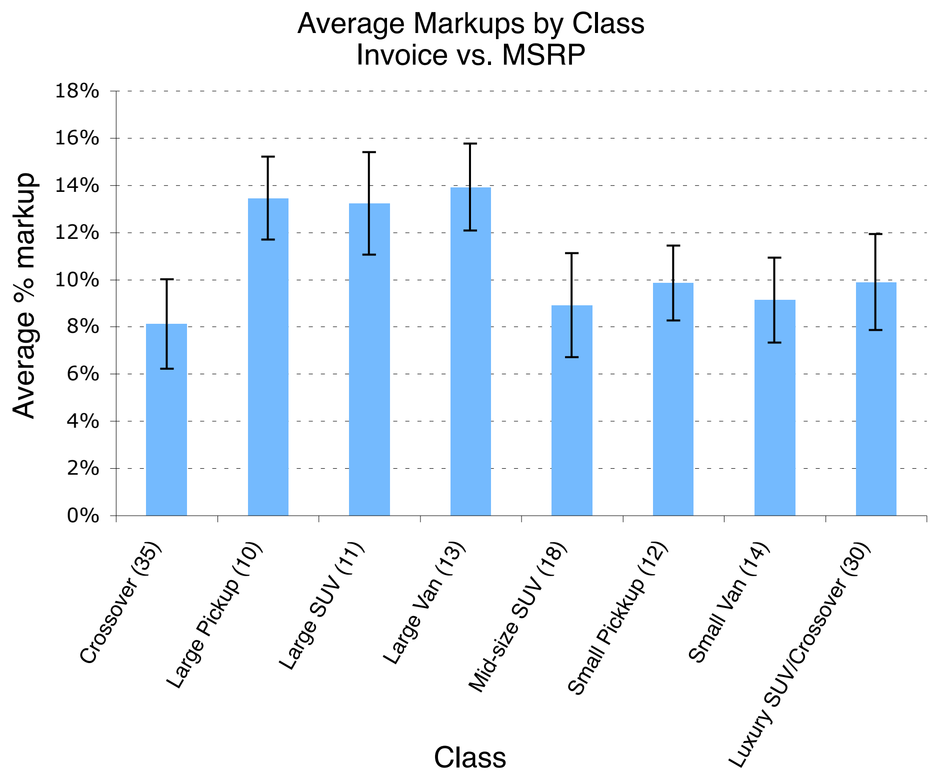


Figure 5.1: Average percentage markups by class: Invoice to MSRP

Per vehicle variable cost $UnitC_{var}$ is broken down into four subsystems, where the relative cost of each subsystem (indicated as a percentage in parentheses) is a function of design variables and parameters. Design variables corresponding to each subsystem are listed after the subsystem percentage: Powertrain (30%): x_B, x_{BtS}; Chassis (35%): x_{W105}, x_{L101}; Body (30%): x_{L103}, x_{H101}, x_{W105}; Wheels (5%): Wheel diameter v_w, Tire width v_tw. Total cost C is calculated according to the following, where C_f corresponds to the total fixed cost for the manufacturer, c_{avg} corresponds to the per vehicle unit variable cost for the average vehicle, Q_{avg} corresponds to the sales volume of the average vehicle, c_{fun} corresponds to the hypothetical variable cost to produce a single unit given the specified learning curve effect with the additional subscripts indicating the hypothetical variable cost to produce a single unit of a given subsytem, and similarly for c_j the per vehicle unit variable cost of vehicle j, G

corresponds to the learning curve effect, the average variable and parameter values (e.g., $x_{L101,avg}$) are also indicated.

$$\begin{aligned}
C_j &= Q_j \times c_j + C_f && (5.1)\\
C_f &= \frac{\phi \times c_{avg} \times Q_{avg}}{1-\phi}\\
c_{avg} &= c_{fun} \times Q_{avg}^G\\
c_{fun} &= c_{fun,pwtrn} + c_{fun,chas} + c_{fun,bod} + c_{fun,whe} && (5.2)\\
c_j &= c_{pwtrn} + c_{chas} + c_{bod} + c_{whe}\\
c_{pwtrn} &= c_{fun,pwtrn}\frac{z_{EDisp}}{z_{EDisp,avg}}Q_j^G\\
c_{chassis} &= c_{fun,chas}\frac{x_{W105} \times x_{L101}}{x_{W105,avg} \times x_{L101,avg}}Q_j^G\\
c_{body} &= c_{fun,bod}\frac{x_{W105} \times x_{L103} \times x_{H101}}{x_{W105,avg} \times x_{L103,avg} \times x_{H101,avg}}Q_j^G\\
c_{wheels} &= c_{fun,whe}\frac{v_{tw} \times v_w}{v_{tw,avg} \times v_{w,avg}}SalesVol^G\\
c_{fun,pwtrn} &= 0.3(1-\phi)(1-\gamma)p_{avg} \times Q_{avg}^{-G}\\
c_{fun,chas} &= 0.35(1-\phi)(1-\gamma)p_{avg} \times Q_{avg}^{-G}\\
c_{fun,bod} &= 0.3(1-\phi)(1-\gamma)p_{avg} \times Q_{avg}^{-G}\\
c_{fun,whe} &= 0.05(1-\phi)(1-\gamma)p_{avg} \times Q_{avg}^{-G} && (5.3)
\end{aligned}$$

Fixed cost C_f is the same for all firms and relates the operating leverage ϕ, the average unit variable cost c_{avg}, and the average sales volume for a model in the midsize crossover segment Q_{avg}. Vehicle model sales volume Q_j is the product of the vehicle choice share P_j and the total market size M. This cost model could be further customized for individual firms by specifying a firm's specific ϕ, γ, Q_{avg}, and average vehicle characteristics.

5.1.3 Empirical Markup Models

A second approach we propose is to identify cost drivers from the physical components of the product and regress a cost relationship based on price. We have attempted such an approach using data from model year 2005 Ward's Automotive Yearbook.

The 2005 Wards light-duty truck database contains approximately 1650 unique nameplates (e.g., Ford Escape XLS). There may be any number of nameplates produced under 1 of the approximately 160 models (e.g., Ford Escape). All models belong to one of 34 makes (e.g., Ford). Vehicles with MSRP above $60,000 were filtered out as well as a small number of specialty vehicles (e.g., Ford Escape Hybrid, Chevy SSR, Dodge Ram Quad Cab SRT 10). An effort was then made to filter all vehicles that had identical characteristics in the Wards data, but were differentiated significantly in price. The higher priced vehicle was filtered from the data set. A large number of pickup trucks were removed in this way. In total 800 nameplates remained in the data set.

Vehicle data include gross vehicle dimensions, engine characteristics (e.g., bore, stroke, compression ratio,), engine performance (e.g., peak horsepower, peak torque), transmission type, drive type, safety features (e.g., antilock brakes), curbweight, fuel economy, and MSRP.

We make the common economic assumption that, for a given firm, the price they charge in the market increases with product quality and that firms practice cost-minimizing behavior, i.e., they seek the minimum cost of inputs to produce an output of a given quality. Therefore, vehicle price and cost should both be monotonic with respect to product quality (quality, here taken to be increasing levels of measurable product characteristics).

We assume that there is a consistent relationship (or apportioning) between the dealer invoice and MSRP markup and the amortized per vehicle OEM cost and the dealer invoice price markup. We assume further that higher percentage dealer markups for certain classes of vehicles indicate vehicles with a superior value proposition affording the supplier more pricing power; and that as value increases, the OEM has more pricing power than the dealer and thereby captures a greater percentage of the available profit. For lower value products we observe lower percentage dealer markups, and we assume the dealer has more pricing power than the OEM. This affect has historically been compounded, at least for the U.S. automakers, by a downward pricing pressure on fuel-efficient vehicles in order to sell those vehicles to satisfy CAFE requirements.

The dealer markup data from Table 5.1 was used to compute average total markups γ_T according to the following relationship. The formula is based on the assumption that OEM markups γ_{OEM} increase more rapidly than dealer markups γ_D.

$$\begin{aligned} \gamma_T &= \gamma_D \left(\frac{1}{6}(100\gamma_D - 7) + 2 \right) \\ \gamma_{OEM} &= \gamma_T - \gamma_D \end{aligned} \tag{5.4}$$

Figure 5.2 plots the relationship from Equation (5.4). The equation, as it is currently tuned, implies that nominally $\gamma_D = \gamma_{OEM}$ when $\gamma_D = 7\%$, and nominally γ_{OEM} will be twice γ_D when $\gamma_D = 13\%$. In practice the OEM markup will be slightly less than the nominal value because the dealer markup acts on the price charged by the OEM, not the cost to the OEM to produce the vehicle.

Relying on such a proposed relationship between price, dealer markups, and OEM markups, we can compute assumed unit vehicle cost $c_{j,a}$ for each vehicle $j = 1, \ldots, J$. We then fit assumed vehicle costs as an output of a regression where components are vehicle characteristics using data from model year 2005 light-duty trucks. A general

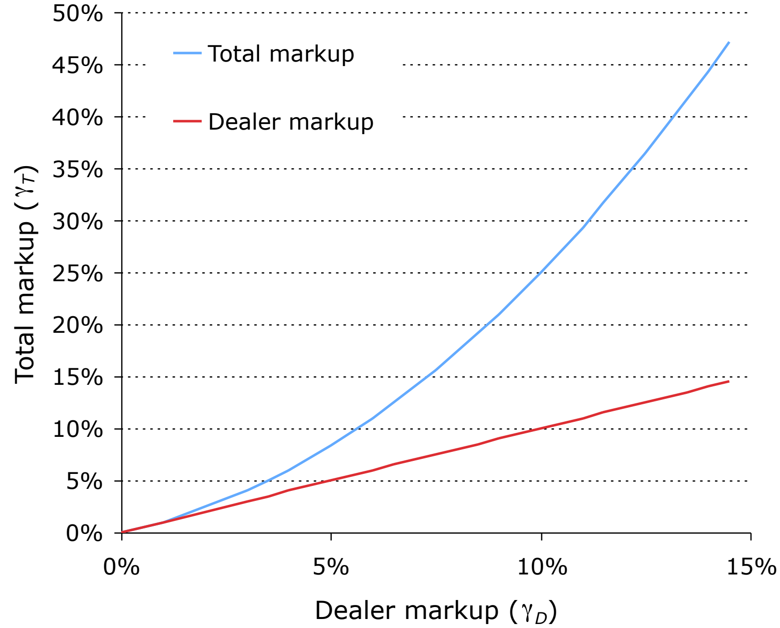


Figure 5.2: Plot showing relationship between dealer markup and assumed total markup

linear least-squares regression problem was solved for a cost function of the form

$$c_j = \exp\left(\sum_{r=1}^{R} \omega_r W_r\right) \tag{5.5}$$

$$C_f = 0 \tag{5.6}$$

by taking the natural log of both sides of the equation and then solving for the unknown coefficients by minimizing the sum of the squared error between the assumed cost based on markup assumptions $c_{j,a}$ and the predicted per vehicle unit cost c_j based on Equation (5.5). Fits with numerous vehicle parameters were made with non-significant parameters being discarded. The set of vehicle cost parameters W is as in Equation (5.8). Table 5.2 describes each parameter, and Table 5.3 gives the values for the cost parameter coefficients ω.

$$
\begin{aligned}
\mathbf{W} = \; & W_{con} \\
& W_{EDisp_{SI}} \\
& W_{EDisp_{CI}} \\
& W_{AWD} \\
& W_{VCUV} \\
& W_{VLpupLvan} \\
& W_{VLSUV} \\
& W_{VMSUV} \\
& W_{VSpup} \\
& W_{Vmvan} \qquad (5.7) \\
& W_{VLxSUV} \\
& W_{ABS} \\
& W_{SC} \\
& W_{Man} \\
& W_{C_{Lvan}} \\
& W_{C_{mvan}} \\
& W_{T_CI} \\
& W_{T_SI} \\
& W_{CDI}
\end{aligned}
$$

(5.8)

Allowing cost parameters to be fit in this way can result in nonintuitive relationships between cost and vehicle parameters. Ideally we could specify a known

Table 5.2: Preliminary empirical cost parameter descriptions

Cost Parameter	Units	Description
W_{con}		Constant parameter value, $W_{con} = 1$
$W_{EDisp_{SI}}$	liters	$z_{EDisp}(1 - v_{Dies})$, spark-ignition engine displacement in liters
$W_{EDisp_{CI}}$	liters	$z_{EDisp}(v_{Dies})$, compression-ignition engine displacement in liters
W_{AWD}		{0,1} 0 = FWD/RWD or 1 = AWD/4WD
W_{VCUV}	inches^3/125000	$z_L z_W z_H m_{CUV}$, the product of vehicle length, width, height, and the crossover dummy variable
$W_{VLpupLvan}$	inches^3/125000	$z_L z_W z_H m_{LpupLvan}$, the product of vehicle length, width, height, and the large pickup or large van dummy variable
W_{VLSUV}	inches^3/125000	$z_L z_W z_H m_{LSUV}$, the product of vehicle length, width, height, and the large SUV dummy variable
W_{VMSUV}	inches^3/125000	$z_L z_W z_H m_{MSUV}$, the product of vehicle length, width, height, and the midsize SUV dummy variable
W_{VSpup}	inches^3/125000	$z_L z_W z_H m_{Spup}$, the product of vehicle length, width, height, and the small pickup dummy variable
W_{Vmvan}	inches^3/125000	$z_L z_W z_H m_{mvan}$, the product of vehicle length, width, height, and the minivan dummy variable
W_{VLxSUV}	inches^3/125000	$z_L z_W z_H m_{LxSUV}$, the product of vehicle length, width, height, and the luxury SUV or crossover dummy variable
W_{ABS}		{0,1} 1 = Anti-lock brakes standard
W_{SC}		{0,1} 1 = Stability control standard
W_{Man}		{0,1} 0 = Automatic, 1 = Manual transmission
$W_{C_{Lvan}}$		{0,1} 1 = Large cargo van
$W_{C_{mvan}}$		{0,1} 1 = Minivan cargo van
$W_{T_{CI}}$		{0,1} 1 = Turbo-charged compression ignition (diesel) engine
$W_{T_{SI}}$		{0,1} 1 = Turbo-charged spark ignition (gasoline) engine
W_{CDI}		{0,1} 1 = Common rail direct injection fuel injection system for diesel engine

Table 5.3: Preliminary empirical cost parameter coefficient values

ω_{con}	$\omega_{EDisp_{SI}}$	$\omega_{EDisp_{CI}}$	ω_{AWD}	ω_{VCUV}	$\omega_{VLpupLvan}$	ω_{VLSUV}	ω_{VMSUV}	ω_{VSpup}	ω_{Vmvan}
9.178	0.0684	0.0658	0.1021	0.0654	0.0286	0.0568	0.0683	0.0453	0.0600

ω_{VLxSUV}	ω_{ABS}	ω_{SC}	ω_{Man}	$\omega_{C_{Lvan}}$	$\omega_{C_{mvan}}$	ω_{T_CI}	ω_{T_SI}	ω_{CDI}
0.0994	0.0313	0.0705	-0.1184	-0.1038	-0.1604	0.1909	0.0891	0.0753

Table 5.4: Cost formula coefficients for conventional powertrains

ω_{Eng1}	ω_{Eng2}	ω_{Eng3}	ω_{Eng4}
400	0.63	1625	0.1758

relationship between cost and a cost parameter before fitting the data. An alternative cost model was created that postulated a specific form for the relationship between engine power, given in units of horsepower, and cost from [*Michalek et al.* (2004)]. The light-duty truck price data was then fit by regressing coefficients for the other vehicle parameters. The regression model had an $R^2 = 0.83$. The form of the cost equation is as follows:

$$\begin{aligned} c_v &= \exp\left(\sum_{r=1}^{R} \omega_r W_r\right) + c_{SIEng} + c_{CIEng} \\ c_{SIEng} &= (1 - v_{Dies})\omega_{Eng1} \exp\left(\omega_{Eng2} z_{hp}/100\right) \\ c_{CIEng} &= (v_{Dies})\omega_{Eng3} \exp\left(\omega_{Eng4} z_{hp}/100\right). \end{aligned} \tag{5.9}$$

The final set of vehicle cost parameters W is as in Equation (5.11). Table 5.5 describes each parameter, and Table 5.6 gives the values for the cost parameter coefficients ω.

$$
\begin{aligned}
\mathbf{W} = \; & W_{con} \\
& W_{AWD} \\
& W_{VCUV} \\
& W_{VLpupLvan} \\
& W_{VLSUV} \\
& W_{VMSUV} \\
& W_{VSpup} \\
& W_{Vmvan} \\
& W_{VLxSUV} \\
& W_{ABS} \\
& W_{SC} \\
& W_{Man} \\
& W_{C_{Lvan}} \\
& W_{C_{mvan}} \\
& W_{T_C I} \\
& W_{T_S I} \\
& W_{CDI}
\end{aligned}
\tag{5.10}
$$

5.1.4 Equilibrium-derived Models

One approach seen in the marketing and economics literature is to assume pricing decisions represent market equilibrium outcomes. Then, cost and utility model forms are postulated and unknown coefficients are estimated for both models. At least two

Table 5.5: Final empirical cost parameter descriptions

Cost Parameter	Units	Description
W_{con}		Constant parameter value, $W_{con} = 1$
W_{AWD}		{0,1} 0 = FWD/RWD or 1 = AWD/4WD
W_{VCUV}	inches^3/125000	$z_L z_W z_H m_{CUV}$, the product of vehicle length, width, height, and the crossover dummy variable
$W_{VLpupLvan}$	inches^3/125000	$z_L z_W z_H m_{LpupLvan}$, the product of vehicle length, width, height, and the large pickup or large van dummy variable
W_{VLSUV}	inches^3/125000	$z_L z_W z_H m_{LSUV}$, the product of vehicle length, width, height, and the large SUV dummy variable
W_{VMSUV}	inches^3/125000	$z_L z_W z_H m_{MSUV}$, the product of vehicle length, width, height, and the midsize SUV dummy variable
W_{VSpup}	inches^3/125000	$z_L z_W z_H m_{Spup}$, the product of vehicle length, width, height, and the small pickup dummy variable
W_{Vmvan}	inches^3/125000	$z_L z_W z_H m_{mvan}$, the product of vehicle length, width, height, and the minivan dummy variable
W_{VLxSUV}	inches^3/125000	$z_L z_W z_H m_{LxSUV}$, the product of vehicle length, width, height, and the luxury SUV or crossover dummy variable
W_{ABS}		{0,1} 1 = Anti-lock brakes standard
W_{SC}		{0,1} 1 = Stability control standard
W_{Man}		{0,1} 0 = Automatic, 1 = Manual transmission
$W_{C_{Lvan}}$		{0,1} 1 = Large cargo van
$W_{C_{mvan}}$		{0,1} 1 = Minivan cargo van
$W_{T_{CI}}$		{0,1} 1 = Turbo-charged compression ignition (diesel) engine
$W_{T_{SI}}$		{0,1} 1 = Turbo-charged spark ignition (gasoline) engine
W_{CDI}		{0,1} 1 = Common rail direct injection fuel injection system for diesel engine

Table 5.6: Final empirical cost parameter coefficient values

ω_{con}	ω_{AWD}	ω_{VCUV}	$\omega_{VLpupLvan}$	ω_{VLSUV}	ω_{VMSUV}	ω_{VSpup}	ω_{Vmvan}
8.261	0.1148	0.7730	0.6110	0.7612	0.7919	0.6896	0.7443

ω_{VLxSUV}	ω_{ABS}	ω_{SC}	ω_{Man}	$\omega_{C_{Lvan}}$	$\omega_{C_{mvan}}$	ω_{T_CI}	ω_{T_SI}	ω_{CDI}
0.9277	0.0423	0.0606	-0.1368	-0.1347	-0.1421	0.2146	0.0301	-0.1527

issues may detract from this approach. The first is that by assuming the market is in price equilibrium it is no longer possible to test if the market is in price equilibrium. Second, the analyst has generated the functional form of the cost model based on experience, insight, or convenience. The static price and cost values may represent an equilibrium. However, if the functional form of the cost model does not describe the underlying relationship between cost and the cost factors, a design optimization that suggests changes in design will result in different design solutions than would be expected in the market

For the purposes of this dissertation, the first issue of assuming price equilibrium is accepted. Prices are easily changed through changes to MSRP and OEM and dealer incentives. Most models are sold over several years, so OEMs have considerable experience pricing their portfolio. The focus of the dissertation is on the implementation of the design game on top of the pricing game, and so assuming price equilibrium simplifies this discussion.

The second issue of unknown functional form of the cost model is truly problematic. This is one of the reasons for including cost models with different functional forms and developed using different approaches. Multiple cost models allow us to compare and contrast the study results in Chapter 8. Improving cost modeling should be the focus of future work.

The equilibrium-derived cost modeling approach is straightforward and intuitively appealing even with its drawbacks. Here we present the derivation of the equilibrium costs.

Firm profit π_f is defined as a function of the choice share of all vehicles in a firm's fleet $\mathbf{P}_f$, vehicle prices $\mathbf{p}_f$ and vehicle costs $\mathbf{c}_f$,

$$\pi_f = \mathbf{P}_f(\mathbf{p}_f - \mathbf{c}_f). \tag{5.11}$$

Assuming prices are in equilibrium means that we assume that a firm's profit can not be increased by unilaterally changing its vehicle's prices. Therefore, we take the derivative of firm profit with respect to each vehicle price and set it equal to **0**:

$$\frac{d\pi_f}{d\mathbf{p}_f} = \frac{d\mathbf{P}_f}{d\mathbf{p}_f}(\mathbf{p}_f - \mathbf{c}_f) + \mathbf{P}_f = \mathbf{0}. \tag{5.12}$$

We now solve for the vector of costs.

$$\mathbf{c}_f = \mathbf{p}_f + \left(\frac{d\mathbf{P}_f}{d\mathbf{p}_f}\right)^{-1} \mathbf{P}_f, \tag{5.13}$$

where, for a simulated population (borrowing notation from [*Morrow* (2008)],

$$\frac{d\mathbf{P}_f}{d\mathbf{p}_f} = \mathrm{diag}(\Lambda) - (\mathrm{diag}(\Lambda))^{-1}\Gamma' \tag{5.14}$$

$$\Lambda = \frac{1}{I}\sum^{I} \frac{d\mathbf{U}_i}{\mathbf{p}} \tag{5.15}$$

$$\Gamma = \frac{1}{I}\mathbf{Pl}\frac{d\mathbf{U}_i}{\mathbf{p}} \tag{5.16}$$

Equilibrium costs were computed for the 2006 model year data with the BLP95, Model 1, and Model 2 demand models using a log normal distribution on income for the BLP95 population and using the 6563 individual population from the demand model estimation for Model 1 and Model 2. Forty draws were taken from the distribution of random coefficients for each individual. Figure 5.3 plots the calculated equilibrium costs versus vehicle prices in 2006 dollars. The BLP95 model costs are clustered closely together, and all costs appear in a range slightly below prices.

The Model 1 and Model 2 costs are more scattered, and a significant portion of the lower-priced vehicles exhibit negative costs. Model 1 has the greater portion of negative costs. The reason that the predicted equilibrium costs are negative relates to the demand model estimation and the market equilibrium assumptions. Considering

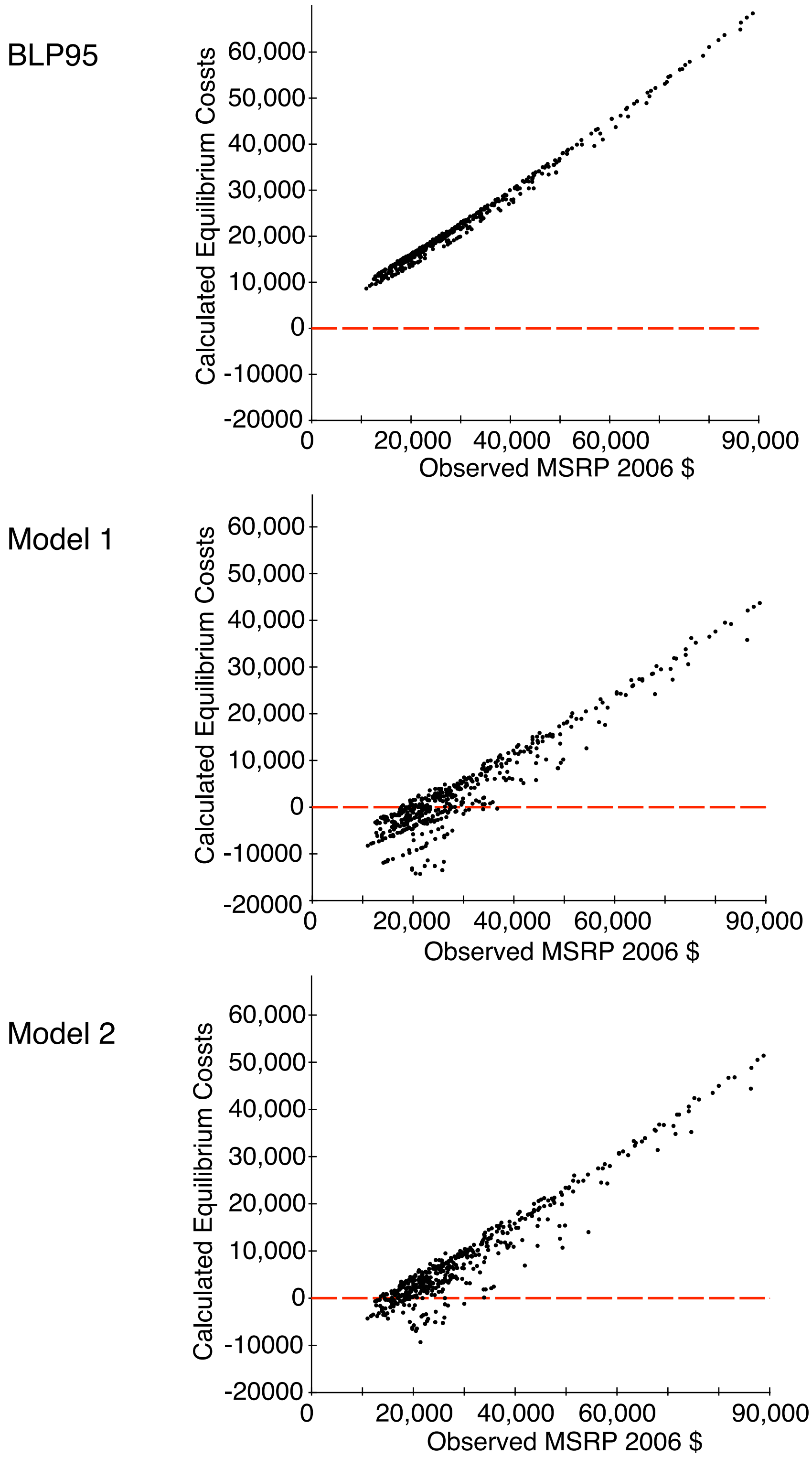


Figure 5.3: Calculated equilibrium costs versus market prices for 2006 model year vehicles for demand model (a) BLP95, (b) Model 1, (c) Model 2

Figure 4.3 indicates that Model 1 and Model 2 underpredict the shares of the most popular vehicles in the market, which are also among the lower-priced vehicles. Model 1 and Model 2 also overpredict the shares of the least popular vehicles, which are also among the higher-priced vehicles. We expect that this over and underprediction would be compensated were we to employ alternative-specific constants in the demand model estimation. However, as it now stands the true market prices appear quite generous to a potential car buyer according to Model 1 or Model 2 who are more willing to buy an expensive car compared to the market data. Therefore, in the equilibrium costs formulation the rational explanation for the low prices must be that costs are very low–so low that they are negative for some vehicles. The corollary of this observation is seen in Section 6.2, where for fixed costs the resulting equilibrium prices are much higher than those observed in the market.

The BLP95 model, on the other hand, is very price sensitive as shown by the choice share distribution in Figure 4.3. Costs are then predicted to be close to prices because if costs were lower, the firm would lower prices in order to capture the price sensitive market.

With calculated costs in hand, a functional form for cost can be postulated and fit. We assume a linear in coefficients model where the cost factors are a constant; horsepower; footprint; diesel dummy v_{Dies}; 4WD or AWD dummy v_{AWD}, v_{4WD}; footprint interacted with two seater or minicompact class, minivan class, SUV class, Van class, Pickup truck class; brand dummies Chrysler, GM, European, Japanese, Korean; Turbo-charged dummy v_{TC}; Hybrid dummy. Table 5.7 lists the coefficient values corresponding to each demand model.

Figure 5.4 shows the vehicle costs with the calculated equilibrium costs versus the regressed cost values in 2006 dollars.

The postulated cost model form fits the data with an R^2 of 0.74 for BLP95, R^2 of 0.77 for Model 1, and with an R^2 of 0.74 for Model 2. The market simulations

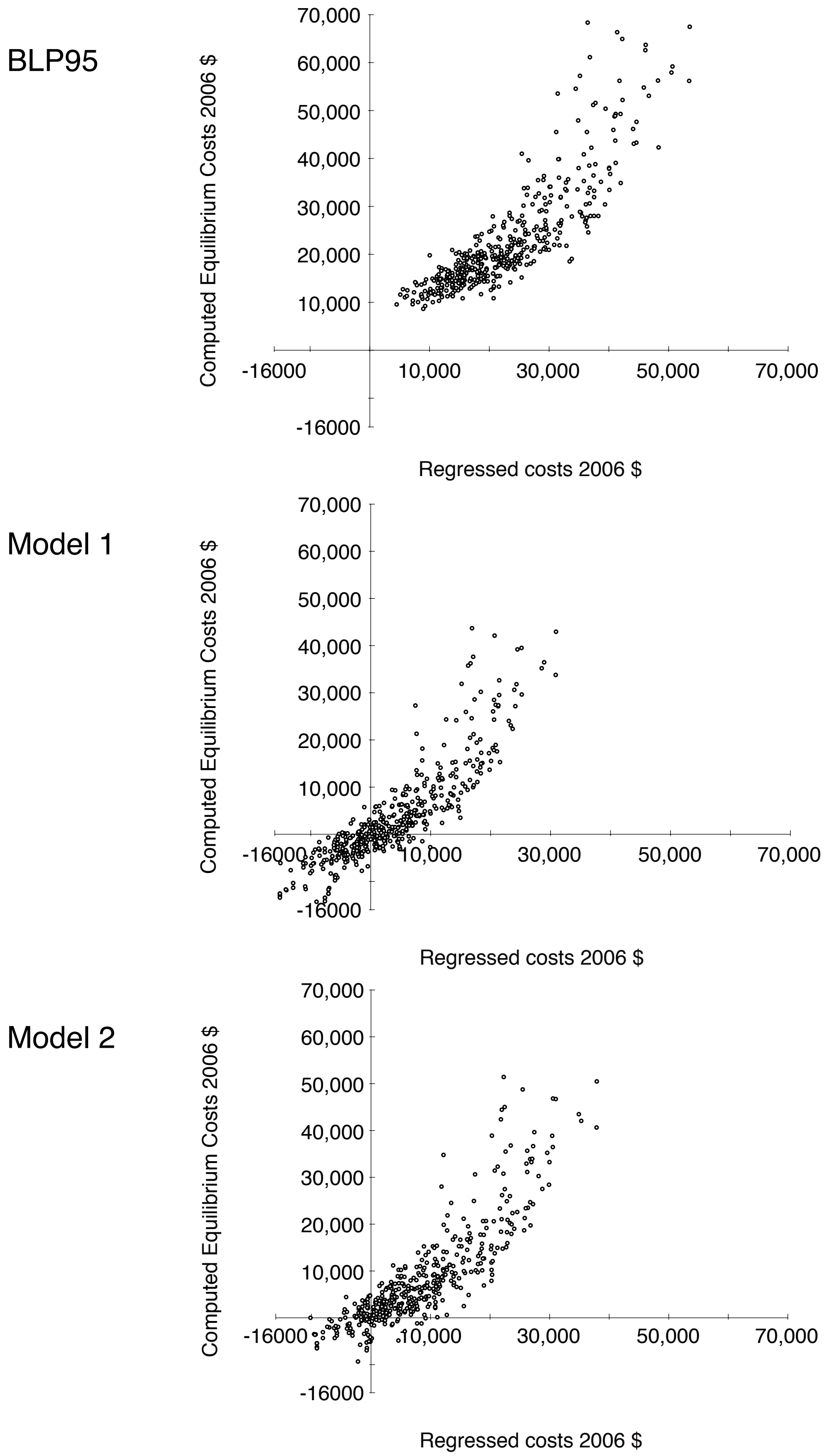


Figure 5.4: Calculated equilibrium costs versus regressed costs for 2006 model year vehicles for demand models BLP95, Model 1, and Model 2

Table 5.7: Coefficients for equilibrium cost models for BLP95, Model 1, and Model 2

Cost Factor	BLP95	Model 1	Model 2
Constant	-11640	-20200	-16790
z_{hp}	8783	7153	7960
$z_L z_W$	8301	4230	3549
v_{Dies}	6249	4286	4844
v_{AWD}, v_{4WD}	990.5	941.5	1008
$m_{tsmc} z_L z_W$	4920	3479	2944
$m_{mvan} z_L z_W$	-378.1	-930.6	-536.4
$m_{SUV} z_L z_W$	-341.9	9.290	-211.4
$m_{van} z_L z_W$	-3531	-6338	-5217
$m_{pup} z_L z_W$	-5115	-5182	-4765
d_{Chr}	-1501	-1790	-1993
d_{GM}	-2004	-5138	-4852
d_{Eur}	9641	9421	9937
d_{Jap}	1222	1946	1829
d_{Kor}	1470	2950	2452
v_{TC}	-5380	-4486	-5419
v_{HEV}	5936	1236	1243

assume that most of the vehicles remain unchanged, and the designed vehicles are modifications from existing vehicles. Therefore, the residuals are kept for each vehicle in the market. The cost model is used to compute changes to the base vehicle cost based on changes in design variables.

5.1.5 Advanced technology costs

Models based on historical data will not give any information about the costs of new technologies. Incremental cost models were developed for two advanced technologies: Gas-turbo-direct-injection gasoline engines (GTDI), and hybrid-electric powertrains (HEV).

For GTDI we simply added a constant to the predicted cost from the cost models listed above. For the empirically fit cost models, the T2TSI parameter captures some of the additional cost of the advanced technology. Additional comparisons are required to form an estimate for the additional cost. In the case of the empirically fit cost model that includes separate engine cost calculations, besides the T2TSI component, engine cost is a function of peak engine power rather than engine displacement, thus accounting for the higher power output of the smaller GTDI engine and fully accounting for the additional cost.

The additional cost of a hybrid electric vehicle powertrain is assumed to comprise electric machine power, battery peak power, energy density, cell capacity, and controller & inverter, and bracket & cable costs[1] [*Lipman and Delucchi* (2006, 2003)]. Equation (5.17). Table 5.8 defines the parameters and lists the values used in the HEV cost equation. The total per unit cost of the hybrid powertrain c_{HEV} is made up of electric machine 1 and 2 costs c_{MG1}, c_{MG2}, the cost of the controller c_{HEVcon}, the cost of brackets and cable c_{HEVBC}, and the cost of the battery c_{HEVB}. All component costs are multiplied by a constant markup factor ω_{HEVm}. Several attributes of the hybrid powertrain system are calculated and used to compute the hybrid powertrain system component costs. The battery cell charge in Amp-hours z_{CAh}, the energy storage capacity of a battery cell z_{CE}, the energy density of a battery cell in Watt-hours per kilogram of battery z_{EDTBc_3}, the per kilogram cost of a battery cell W_{HEVBpk},

[1]This model is adopted from a paper by Lipman and Delucchi from UC Davis. "Retail and LCC of HEVs," Lipman, Delucchi, 2006, *Transportation Research, Part D*; see also, "Hybrid Electric Vehicle Design Retail and Lifecycle Cost Analysis," 2003-UCD-ITS-RR-03-01.

and the total number of cells in the battery pack z_{NC}.

$$\begin{aligned}
c_{HEV} &= (c_{MG1} + c_{MG2} + c_{HEVcon} + c_{HEVBC} + c_{HEVB})\omega_{HEVm} \\
c_{MG1} &= -111.3 + 127.7 \ln(z_{MG1}) \\
c_{MG2} &= -111.3 + 127.7 \ln(z_{MG2}) \\
c_{HEVcon} &= 480 + (2.95(z_{MG1} + z_{MG2})); \\
c_{HEVBC} &= 1.5(z_{MG1} + z_{MG2}); \\
c_{HEVB} &= z_{NC} \times v_{cm} \times W_{HEVBpk} \\
z_{NC} &= 1000 x_{BPow} / (v_{cV} \times v_{cI}) \\
W_{HEVBpk} &= v_{MCC^*} - ((z_{EDTBc_3} - v_{EDTBc_3^*}) / v_{K_{BM}}) \ln(z_{EDTBc_3}) \\
z_{EDTBc_3} &= z_{CE} / v_{cm} \\
z_{CE} &= z_{CAh} \times v_{cV} \\
z_{CAh} &= v_{cQ} / 3600
\end{aligned} \tag{5.17}$$

Table 5.8: HEV cost model parameters

HEV Cost Parameter	Value	Units	Description
v_{cm}	0.17	kg	mass of single battery cell
v_{cI}	175	A	nominal peak current
v_{cV}	1.55	V	nominal cell voltage
v_{cQ}	23400	A-s	nominal cell charge
v_{MCC^*}	17.69	\$/kg	reference battery cost in \$/kg
$v_{EDTBc_3^*}$	75	Wh/kg	baseline $EDTBc_3$ value
$v_{K_{BM}}$	15		scaling coefficient for battery cost
ω_{HEVm}	1		multiplicative constant on all costs

The value computed by the HEV cost equation is added to the unit variable cost for a new c_v.

$$c_v = HEVUnitCost + c_v \tag{5.18}$$

The empirically fit cost model with the explicit engine cost has been further adjusted for the HEV case by including additional details. The cost of the engine in this cost model is based on peak engine power. We assume that HEVs employ spark ignition engines running the Atkinson cycle rather than the Otto cycle, so these engines have lower peak power than would be expected for engines of similar displacement. In these cases engine displacement may be a better predictor of engine cost than engine power. We compensate by adding a constant of 120 HP to z_{hp} when computing the c_{SIEng}. A calibration constant ω_{HEVcon} is also added to c_v resulting in the following equation for unit variable cost.

$$\begin{aligned} c_v &= c_{HEV} + \omega_{HEVCon} + \exp\left(\sum_{r=1}^{R} \omega_r W_r\right) + \\ &\quad \omega_{Eng1} \exp\left(\omega_{Eng2} \frac{z_{hp} + 120}{100}\right) \end{aligned} \tag{5.19}$$

5.2 Summary

A major component in a market equilibrium study is a representation of producer costs. Three approaches were taken: (1) scaling costs around an average expected cost, (2) employing empirical data on prices, (3) using market equilibrium assumptions, given a demand model, to estimate costs. The three approaches differ on the data required and the assumptions made to generate the cost models. The scaling cost model requires only average values for vehicle parameters and sales figures for the vehicle segment of interest. The empirical model requires a set of individual vehicle data. The equilibrium cost model requires a set of individual vehicle data and a consumer demand model.

Some of the estimated costs for the equilibrium model are negative, and the costs in general are much lower than expected. We conjecture that the low cost projections are based on the fact that the cost model was derived from a demand model that did not match the sales for the market from which it was estimated. In other words, in the real market a set of vehicles and vehicle prices produced a certain mix of sales. When we simulate predicted sales volume using the newly-estimated demand model with the same mix of vehicles and vehicle prices we predict a different sales volume mix. If we take the modeled sales volume mix to represent market equilibrium with its unrealistic sales, it makes sense that we will get unrealistic vehicle costs based on the equilibrium costs method. The implications for the market simulation based on the unrealistic cost behavior are discussed in Chapter 8.

The empirical cost model was fit to the data based on vehicle attributes alone. The equilibrium cost model relied on vehicle attributes, but also on vehicle brand because the demand model was estimated with brand coefficients. This is another way that unexpected cost results may be generated by the equilibrium cost model.

Assumptions were stated about incremental costs for GTDI technology. A cost model that was adapted from the literature for hybrid electric vehicles was also presented.

This chapter is a first step to identify the best methods for constructing empirical cost models using publicly available (or at least not proprietary) data. A second step would be to refine the formulations of the cost models to capture better the relationships between vehicle characteristics and costs.

CHAPTER VI

Market Simulation

The development in the previous chapters implies two economic assumptions, namely, producers are profit-maximizers and consumers are utility-maximizers. This chapter will describe a third assumption that is competitive behavior is well-defined. There is a tension between these assumptions and the aims of the analyst. The assumptions are intuitively appealing, and they make the problem computationally tractable. However, we desire the results to be useful for normative rather than positive analysis. If the basic assumptions held in reality, then producers would have already determined methods for maximizing profits, and the market would be in price and design equilibrium. This dissertation assumes that the automotive market demonstrates static price equilibrium during a single period (year). We also assume vehicle producers seek to produce vehicles that maximize profit. However, we make no assumption about the state of market equilibrium with respect to vehicle designs. Calculating price equilibrium of the US automotive market under policy changes by a firm or firms, i.e., design changes in one or a small number of vehicles, is then used as one method for evaluating market response to the introduction of new technology or regulation.

Section 6.1 outlines the computational steps that are used to simulate a firm's profit maximizing behavior with respect to vehicle design decisions in the context of a market in price equilibrium. Section 6.1.1 discusses the case where we would assume a combined design and price equilibrium. This is the approach adopted in the automotive vehicle design example in Chapter 7. Section 6.1.2 discusses the case where price equilibrium is treated as a subgame of the design game. The case of a single designing firm, and the case of multiple designing firms are described. The price equilibrium subgame is the approach adopted in Chapter 8. Section 6.2 shows computational results for the price equilibrium problem alone when vehicle costs and prices are given in order to demonstrate that the price equilibrium assumption appears reasonable for the US automotive vehicle market.

We make several assumptions about the structure of competition. Much of the economics literature evaluating the automotive market assumes that the vehicle producer sets vehicle price directly. Recent work in the design for market systems literature confirms that modeling the vehicle producer and the vehicle dealer as independent decision-makers changes the nature of the decision problem for the vehicle producer [*Shiau and Michalek* (2009)]. To simplify the discussion in this dissertation, we evaluate the case where the dealer markups are fixed and assumed known a priori.

6.1 Game Structures

Policy analysts have often used a single stage equilibrium where a producer makes all decisions simultaneously. Industrial organization economists view this approach as simplistic. They suggest a subgame perfect equilibrium where all producers make product design decisions before making a subsequent decision to update prices. This idea is supported by the notion that producers have more freedom to control prices than vehicle designs [*Tirole* (1988)].

For simple cases it should be possible to derive equilibrium conditions that can be solved directly with respect to price and design decisions. For complex cases such a derivation is difficult. This dissertation employs a convergence criterion that references changes in vehicle attributes from one iteration to the next. When the maximum vehicle attribute change from one iteration to the next is below the convergence tolerance we take the solution as an approximation to an equilibrium solution. In future work, we can verify that our solutions have behavior of equilibrium solutions by checking 1st and 2nd order optimality conditions for the design game.

A complete product equilibrium process would allow each competitor to optimize designs as well as prices with respect to all others' designs and prices. The market simulation scenarios could also be enhanced to allow for product entry and exit. In the case of a homogenous multinomial logit model and identical underlying engineering and cost models, each firm would choose identical designs and prices, as seen in [*Michalek et al.* (2004)]. Instead, assuming some fixed competitor designs coincides with vehicle planning where a firm makes some educated assumptions about the products competitors will produce. During product launch and subsequent sales, all competitors are at liberty to adjust prices freely while the designs remain fixed. Competitive behavior [*Chintagunta et al.* (2006)] among automotive manufacturers considering the full market has been modeled, see for example [*Sudhir* (2001); *Goldberg* (1995)]. The simplified approach shown here includes competitive effects sufficient to illustrate trends without greatly increasing the computational complexity of the model.

6.1.1 Design Problem Formulation for Single-stage Design and Price Equilibrium

The first $F-1$ competitors $f = 1, \ldots, F-1$ maximize profit with respect to vehicle price $p_{j,f}$ given the product designs $\mathbf{z}$ and prices $\mathbf{p}_{k \neq j}$ of all competitors. A single firm F is designated as the designing firm. The designing firm optimizes product design variables $\mathbf{x}_{j,F}$ and the price variable $p_{j,F}$, concluding one iteration. Iterations continue until price changes fall below a convergence tolerance p_{tol}. The vehicle prices (and firm F design variables) are now set such that no firm can make a different decision that would improve its own profits while the choices of the other firms remain fixed, thus approximating a Nash equilibrium [*Fudenberg and Tirole* (1991)].

The following expression in Equation (6.2) is used to simulate computationally the solution to a single-stage game where one firm (firm F) controls design and price variables and other firms (firms $1, \ldots, F-1$) control price variables only. This is the approach that was used in the automotive design example in Section 7.4.1.

$$
\begin{aligned}
\text{Do while} \quad & \max\{\Delta p_j, j = 1, \ldots, J\} > p_{tol} \\
& \text{for} \quad f = 1, \ldots, F-1 \\
& \quad \max_{\mathbf{p}_{j,f}} \quad \pi_f(p_{j,f}; \mathbf{z}, \mathbf{p}_{k \neq j}) \\
& \text{end} \\
& \max_{\mathbf{x}_{j,F}, p_{j,F}} \quad \pi_F(\mathbf{x}_{j,F}, p_{j,F}; \mathbf{z}_{k \neq j}, \mathbf{p}_{k \neq j}) \\
& \quad \text{s.t.} \quad \mathbf{g}(\mathbf{x}_{j,F}) \leq 0 \\
\text{end} \quad &
\end{aligned}
\tag{6.1}
$$

where,

$$\pi_f = (\mathbf{p}_f - \mathbf{c}_f)' \cdot \mathbf{q}_f \tag{6.2}$$

In these expressions, the index j represents the individual vehicles being designed or priced across all of the firms. The index f represents the individual firms that are competing in the market. The combined subscript j, f, for example in the case of $p_{j,f}$ signifies the price of vehicle j, where vehicle j is controlled by firm f. The index k is used to differentiate the remaining vehicle choices in the market from the current vehicle j.

6.1.2 Design Problem Formulation for Two-stage Design and Price Equilibrium

The difference between the two-stage game described in this section and the single-stage game described in Section 6.1.1 is that the pricing problem for all of the vehicles in the market is solved as a subgame to the vehicle design problem. Three market scenarios of varying complexity are presented here. The first two scenarios are later implemented in Sections 8.2-8.5. The third scenario is presented for illustrative purposes only.

Single Firm with Two-stage Game

This market simulation scenario is the one employed in the vehicle design examples in Sections 8.2-8.4.

A single firm is designated as the designing firm F. This firm controls all vehicle design variables for its vehicle $\mathbf{x}_{j,F}$ except for price $p_{j,f}$. The price of the designed vehicle and the prices of all of the other vehicles $\{p_j,\ j = 1, \ldots, J\}$ in the market are determined in a subgame that is computed at each function call of the designing firm's profit objective. This process iterates until the largest change in any of the designed vehicle's attributes $\mathbf{z}_{j,F}$ is below a convergence tolerance z_{tol}.

$$
\begin{aligned}
\text{Do while} \quad & \max\{\Delta \mathbf{z}_{j,F}\} > z_{tol} \\
& \max_{\mathbf{x}_{j,F}, \mathbf{p}} \quad \pi_F(\mathbf{x}_{j,F}, \mathbf{p}; \mathbf{z}_{k \neq j}) \\
& \quad \text{s.t.} \quad \mathbf{g}(\mathbf{x}_{j,F}) \leq 0 \\
& \qquad\qquad \mathbf{p} = \arg\left(\max_{\mathbf{p}} \{\pi_f(\mathbf{p}_f; \mathbf{z}_f),\ f = 1, \ldots, F\} \right) \\
\text{end,} \quad &
\end{aligned} \tag{6.3}
$$

where, π_f is the same as Equation (6.2).

Single Market Segment with Two-stage Game

This market simulation scenario is the one employed in the vehicle design example in Section 8.5.

Multiple firms $F-q+1,\ldots,F$ with competing vehicles in the same segment are designated as the designing firms. The first designing firm $(F-q+1)$ maximizes profit π_{F-q+1} with respect to the vehicle design variables $\mathbf{x}_{j,F-q+1}$ where all vehicle prices in the entire market $\mathbf{p}$ are determined by a pricing subgame as in the single firm two-stage game case. The second designing firm then maximizes profit with respect to the vehicle design variables $\mathbf{x}_{j,F-q+2}$, and so on. This process iterates until the largest change in one of the designed vehicles' attributes is below the convergence tolerance z_{tol}.

$$
\begin{aligned}
\text{Do while} \quad & \max\{\Delta\mathbf{z}_{j,f}; f=F-q+1,\ldots,F\} > z_{tol} \\
& \text{for} \quad f=F-q+1,\ldots,F \\
& \quad \max_{\mathbf{x}_{j,f},\mathbf{p}} \quad \pi_f(\mathbf{x}_{j,f},\mathbf{p};\mathbf{z}_{k\neq j}) \\
& \qquad \text{s.t.} \quad \mathbf{g}(\mathbf{x}_{j,f}) \le 0 \\
& \qquad\qquad \mathbf{p} = \arg\left(\max_{\mathbf{p}}\{\pi_f(\mathbf{p}_f;\mathbf{z}_f);\; f=1,\ldots,F\}\right) \\
& \text{end} \\
\text{end,} &
\end{aligned}
\tag{6.4}
$$

where, π_f is the same as Equation (6.2).

Multi-product Firms with Two-stage Game

This market simulation scenario is not demonstrated in this dissertation and is listed here for illustrative purposes.

Multiple firms $F-q+1,\ldots,F$ are designated as the designing firms. Multiple products $\{j_{F-q+1}; j_{F-q+1} = 1, \ldots, J_{F-q+1}\}$ from each firm are designated as the designed vehicles. The first designing firm maximizes profit with respect to the vehicle design variables for all vehicles it will design $\{\mathbf{x}_{j_{F-q+1}}; j_{F-q+1} = 1, \ldots, J_{F-q+1}\}$ where all vehicle prices in the entire market $\mathbf{p}$ are determined by a pricing subgame as in the single firm two-stage game case. The second designing firm then maximizes profit with respect to the vehicle design variables of the vehicles it will design $\{\mathbf{x}_{j_{F-q+2}}; j_{F-q+2} = 1, \ldots, J_{F-q+2}\}$, and so on. This process iterates until the largest change in one of the designed vehicles' attributes is below the convergence tolerance z_{tol}.

$$
\begin{aligned}
\text{Do while} \quad & \max\{\Delta\mathbf{z}_{j_f}; j_f = 1, \ldots, J_f;\ f = F-q+1, \ldots, F\} > z_{tol} \\
& \text{for} \quad f = F-q+1, \ldots, F \\
& \quad \max_{\mathbf{x}_{j_f}, \mathbf{p}} \quad \pi_f(\mathbf{x}_{j_f}, \mathbf{p}; \mathbf{z}_{k \neq j_f}); j_f = 1, \ldots, J_f \\
& \quad\quad \text{s.t.} \quad \mathbf{g}(\mathbf{x}_{j_f}) \leq 0 \\
& \quad\quad\quad\quad \mathbf{p} = \arg\left(\max_{\mathbf{p}}\{\pi_f(\mathbf{p}_f; \mathbf{z}_f);\ f = 1, \ldots, F\}\right) \\
& \text{end} \\
\text{end,} &
\end{aligned}
\tag{6.5}
$$

where, π_f is the same as Equation (6.2).

6.2 Automotive Market Price Equilibrium Comparisons

The price equilibrium represents the subgame in the vehicle design market simulation. We compare the simulated market behavior for the case of price equilibrium alone without the vehicle design problem for the BLP95 model and the two new models presented in Chapter 4. The pricing problem is posed in the form of a dealer pricing problem. In other words, assuming that each dealer sells only vehicles from a single manufacturer, what prices should dealers set collectively to maximize profits for their vehicle sales. The fixed-point algorithm developed by Morrow [*Morrow* (2008)] is used to compute optimal prices for a portfolio of vehicles for each firm in the market by iteratively solving a fixed-point equation for price. Convergence is evaluated at each fixed-point iteration by comparing a user-specified tolerance value to the combined gradient norm. The combined gradient norm G is defined to be the L_∞-norm of the vector composed of the products of the choice-share derivatives λ and the difference between the initial prices and computed fixed-point prices $\mathbf{p} - \mathbf{p}'$.

$$G = ||\lambda' \times \text{diag}(\mathbf{p} - \mathbf{q})||_\infty \tag{6.6}$$

When the combined gradient norm is close to 0, it means that no firm can improve its profits by changing prices unilaterally. For this reason we take the combined gradient norm convergence below a given tolerance value as an approximation to a Nash equilibrium solution.

The vehicle data came from a JD Power data set of 5,298 vehicle purchases during the 2005 calendar year. This set of observations was reduced to a total of 993 vehicle offerings corresponding to different model years and model names. Characteristics for vehicles with the same model name and model year were averaged. The dealer costs were taken to be the vehicle costs.

A simulated population was generated for each demand model evaluation. For BLP95 the only demographic is income, so 1000 standard normal random draws were taken and converted into annual incomes assuming a log normal distribution of income in 1983 dollars with mean 10.33 and standard deviation of 1. Random coefficient values for z_{hp}/z_{VM}, $z_{\$/mi}$, and $z_L z_W$ were generated in the same way assuming normal distributions on these parameters. All vehicles were assumed to have air-conditioning standard. The utility of each vehicle for each simulated individual was computed. Choice shares were computed according to the standard mixed-logit formulation. Price equilibrium prices were computed using the fixed-point iteration technique.

Prices and shares for the two newly estimated demand models from Chapter 4, Model 1 that is linear in footprint and Model 2 that is quadratic in footprint, were computed following the same methodology as described in the preceding paragraph. However, the simulated population was taken to be the same population used to estimate the models. Fifteen random draws were taken for each random coefficient for each individual. The price equilibrium simulation was repeated several times for each model all with similar results to those reported here.

Equilibrium prices were compared with the (average) transaction prices as shown in Figure 6.1, and the estimated market shares were plotted in Figure 6.1. The equilibrium prices for the BLP95 model match the reported transaction prices very well. The equilibrium prices for Model 1 and Model 2 are offset higher than the reported transaction prices and appear to have a percentage increase, i.e., slope greater than 1, compared to the transaction prices. Model 2 has a slight but noticeably smaller slope than Model 1.

Additionally, the BLP95 simulation achieved a very small combined gradient norm, $G < 1 \times 10^{-17}$. The fixed-point iteration code typically achieved a combined gradient norm around values of $G < 5 \times 10^{-9}$ for both Model 1 and Model 2. The difference can be seen by the increased "thickness" of the band of computed versus reported prices in Figure 6.1. All three demand models preserve the price ordering to a recognizable degree, i.e., the trend for computed vs. given prices lies along a straight line.

We believe that difference in the convergence criterion values between the BLP95 model and the newly estimated model comes because of increased nonlinearities in the choice share calculations for Model 1 and Model 2. Consumers, according to the

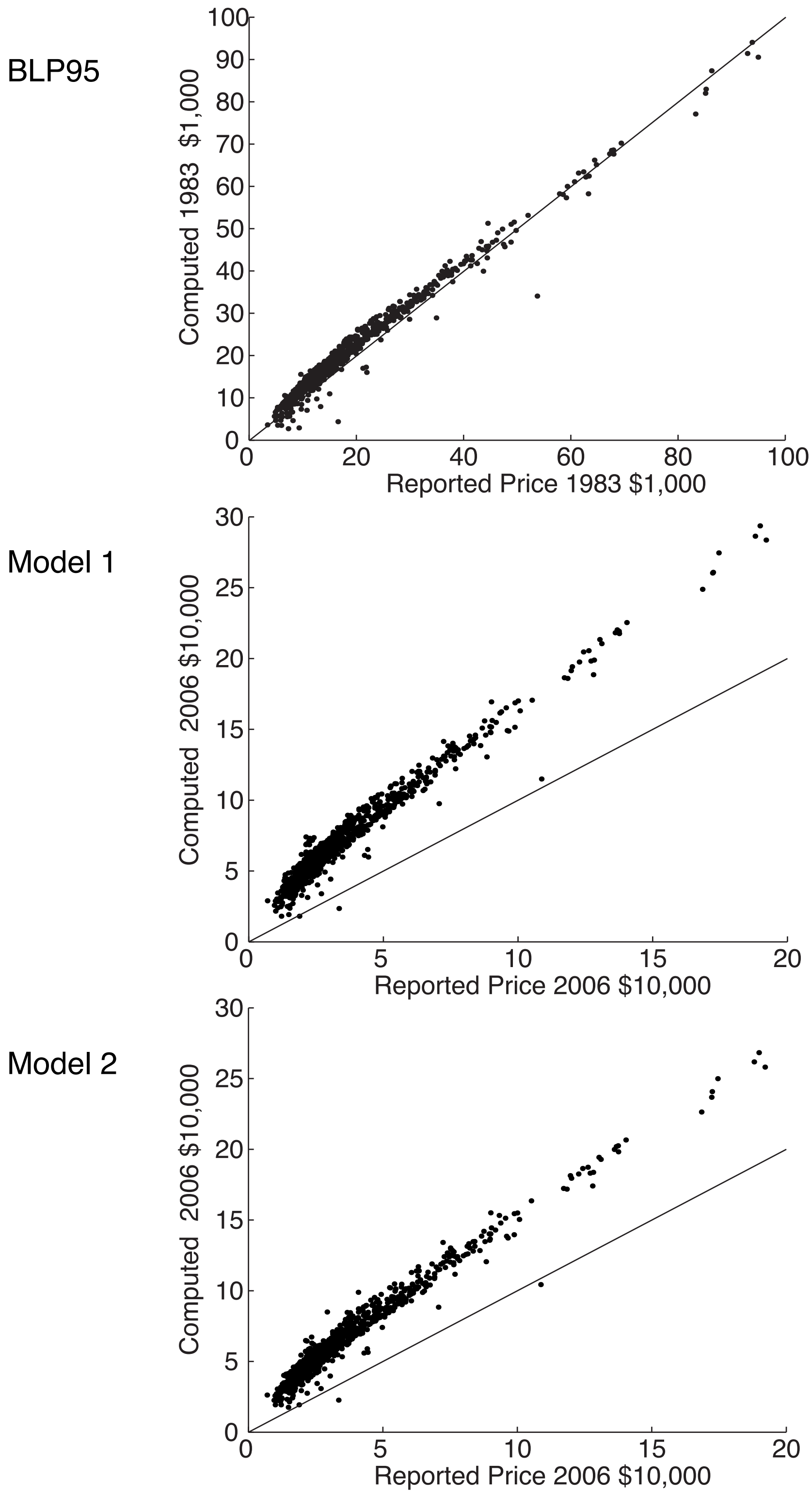


Figure 6.1: Computed prices vs. reported prices. Left: BLP95; Center: Model 1; Right: Model 2

BLP95 model, are quite price sensitive, and so there is a clear ordering of sales volume that corresponds well with vehicle price. The Model 1 and Model 2 demand models show that some high-priced models achieve significant market share. This makes the profit calculation for each firm much more nonlinear because vehicles across the firm's fleet (not only the inexpensive vehicles) contribute to firm profit.

The choice shares plots in Figure 6.2 show a contrast between the BLP95 performance and the new model performance. The BLP95 model predicts very high choice shares for the lowest priced models and almost 0 choice share for the remaining models. This behavior is more accentuated than in the 2006 model comparison shown in Chapter 4. On the other hand, Model 1 and Model 2 show similar choice share behavior to the 2006 comparison in Chapter 4. All models show a tendency toward lower price vehicles as compared to the 2006 model comparison. The suggested cause for this tendency is that the choice model, without alternative-specific constants, does not capture the underlying preference for the most popular less expensive vehicles. However, it appears that the models do capture this underlying preference because when let loose in a market equilibrium simulation, the resulting shares look much more similar to the observed market shares. This hypothesis is supported by the equilibrium cost results in Chapter 5 that show that in order to produce the equilibrium outcome suggested by the demand models at the observed market prices, the cost to manufacture the least expensive vehicles would be negative. It is natural, then that when prices are free to vary, manufactures increase prices in order to rebalance the market shares more similarly to what was observed in the actual market.

It should be noted that the choice set used in the market simulation in this section spans multiple model years and is not known to be representative of the vehicles available in 2005. The robustness of the choice share results for Models 1 and 2 compared to the 2006 data comparison provide a preliminary indication that they may exhibit structural properties well suited for market simulation with design optimization. The

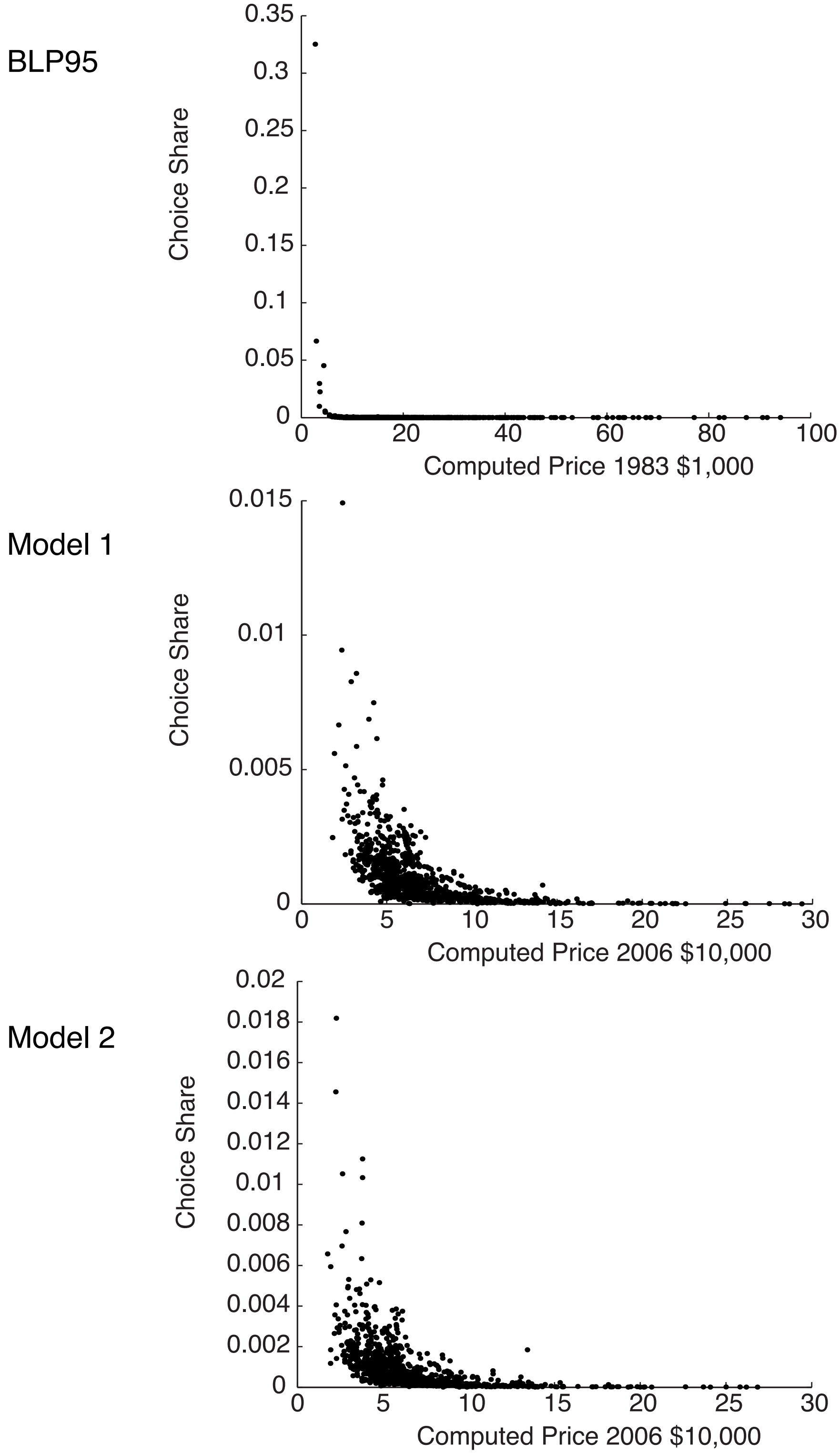


Figure 6.2: Computed shares vs. computed prices. Left: BLP95; Center: Model 1; Right: Model 2

BLP95 model, on the other hand, demonstrates accentuated price sensitivity beyond the already exaggerated behavior shown in Chapter 4.

One hypothesis about the poor performance of the BLP95 model is that consumer preferences have shifted since the model was developed. This idea is worth investigating since the BLP95 model was estimated on vehicle data from 1971-1990, and the test data was from 2005. Figure 6.3 shows market equilibrium simulations for various cost scenarios. Again, this is a dealer pricing problem, since we are maximizing profit based on the retail price values. Panel (a) assumes that dealer markup is 10% of MSRP. Panel (b) assumes that dealer markup is a fixed $512 for each vehicle. Panel (c) assumes that percentage dealer markup increases linearly with MSRP and takes values between 0%-30%.

For all the cases we observe that the choice share of the most popular vehicle has been reduced by an order of magnitude. This observation supports the idea that the BLP95 model is better suited for the older data. Without data on the actual sales of the models in the choice sample it is difficult to draw further conclusions about the fit of the 1985 data. The visual effect of the choice share spread in Figure 6.3 compared to the 2006 market shown in Figure 4.3 suggests that the 1985 market was much more skewed toward inexpensive cars or that the BLP95 model exhibits oversensitivity to price even in the model years for which it was estimated. The oversensitivity to price may be accounted for completely by the alternative specific constants. However, those constants are not reported in the BLP95 paper. The oversensitivity to price of the model without the constants highlights one of the dangers of adopting models from the literature for design for market systems studies.

More specifically in Figure 6.3, we observe that for the constant 10% margin case (a), the shares are more dispersed and the prices are overpredicted compared with Figures 6.1-6.2. For the fixed margin case (b), the prices for high-priced vehicles are overpredicted even more than in case (a) and the share dispersion is similar. For the

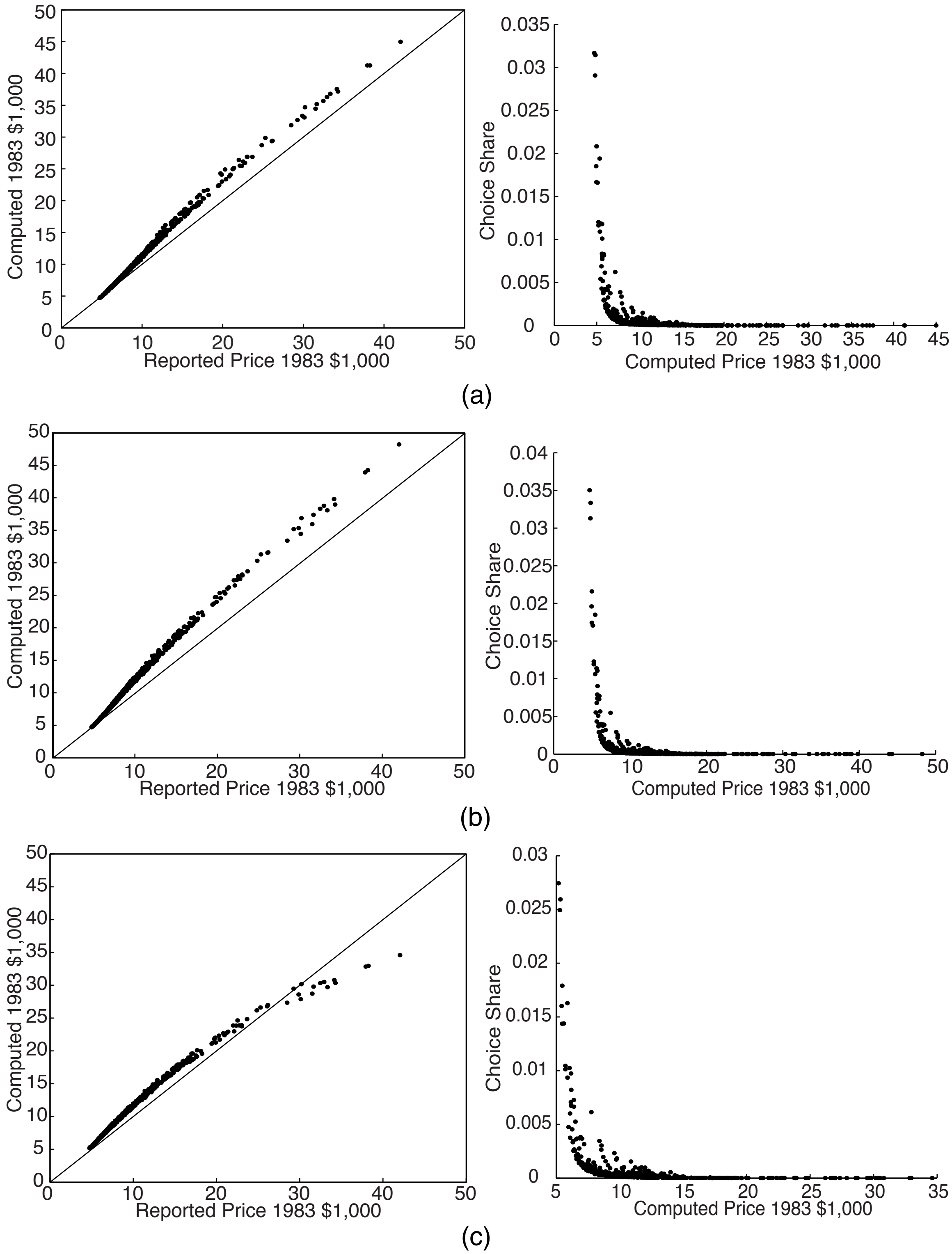


Figure 6.3: Computed shares vs. computed prices for BLP95 model using 1985 vehicle data from Ward's Automotive Yearbook. (a) 10% dealer margin; (b) 512$ dealer margin; (c) linearly-increasing-with-price dealer margin between 0%-30%

increasing percentage markup (c), the prices are overpredicted for low-priced models and underpredicted for high-priced models. We interpret this as an underprediction of margin for the low-priced vehicles and an over prediction of margin for the high-priced vehicles given the demand model. The share dispersion begins to approach the dispersion seen with Model 1 and Model 2 in the year 2006 case. However, shares remain more dispersed.

6.3 Summary

This chapter defined the computational structure that is used for the automotive vehicle design examples in Chapters 7 and 8. We pattern the computations after singe and two-stage market equilibrium games. However, we do not prove convergence to a market equilibrium solution. Instead we iterate among competitors until no further design changes are observed. We also simplify the problem by designating a small number of designing firms and designed vehicles. The designing firms control the vehicle design variables of the designing firms. All other firms and vehicles in the market are considered in the pricing problem, but the vehicle designs of these vehicles remain fixed.

The second element of this chapter is an analysis of the price equilibrium computational results to be employed as a subgame in all of the two-stage design and price formulations. The purpose of these tests were to test the pricing game behavior in isolation of the design game in order to gain confidence about the suitability and feasibility of employing the pricing subgame in the over all design game model. Using data from dealers on vehicle cost and selling price we presented graphically the results of the price equilibrium problem. The choice share results for Model 1 and Model 2 for the dealer data test using 2005 data appear quite similar to the real market choice shares from the 2006 data used in Chapter 4. The pricing game for all models appears to converge although the BLP95 model is capable of a much higher convergence tolerance and well matched predicted to given prices. Both Model 1 and Model 2 have a price offset where the predicted prices are higher than the given prices. This behavior coincides with and is the inverse of the equilibrium pricing behavior seen in Chapter 5 where given the reported prices, the equilibrium predicted costs were much lower than expected. All of these findings build confidence in the computational approach. We conjecture that the offset observed in Model 1 and Model 2 would be accounted for by incorporating alternative specific constants in the demand model estimation.

We observed that the choice share prediction of the BLP95 model was very poor. We presented three different equilibrium price and choice share results for 1985 data using the BLP95 demand model and three different cost assumptions in order to examine the change in the price and choice share behavior when the BLP95 model was used for a model year over which it was estimated. In all three cases the choice share predictions appear more reasonable although perhaps remaining overly price-sensitive. The price predictions were not as accurate as in the 2005 dealer data case, which could be expected based on the simplistic cost assumptions made rather than using dealer cost data.

CHAPTER VII

Local and Global Measures for Bi-objective Tradeoffs

Quantitative studies of tradeoffs between competing objectives are ubiquitous. They typically focus on finding Pareto points [*Steuer* (1986)] and the preference structure for selecting one point among many or vise versa. Preferences or constraints that lead to the tradeoff relationship are assumed fixed.

However, changes to the mathematical structure and input parameter values of the optimization model can lead to changes in the shape of the attainable set and its Pareto boundary. These changes can be captured by the objective function gradients and constraint activity shifts. Furthermore, psychologists have shown, and recent work in the design community has begun to explore, that decision maker preferences do not necessarily exist a priori. This finding implies decision maker preferences may be influenced by evolving tradeoffs—hence the value of studying them systematically [*Slovic* (1995); *Kulok and Lewis* (2007); *Besharati et al.* (2006); *MacDonald et al.* (2007b)].

This chapter encompasses portions of the papers [*Frischknecht and Papalambros* (2008); *Frischknecht et al.* (2009b)]. The goal in this chapter is to establish a methodology and metrics for comparing the type of public versus private goods tradeoffs presented in Chapter 8. We can think of of each set of parameter values used as inputs to a vehicle design and market simulation problem as corresponding to a product development or policy scenario. For a given scenario, the public versus private good tradeoff can be captured as a tradeoff between vehicle fuel consumption and the firm's profit. The optimization results can then be represented as a Pareto set. The methods presented in this chapter provide an approach for measuring differences in the Pareto set from one scenario to another in order to facilitate the discussion of what represents a desirable scenario. Essentially, using the optimization framework, we can study how changes in parameter values (or possibly the analysis models) change the tradeoff, or alignment, of the competing objectives.

The remainder of the chapter is organized as follows. Section 1 describes a bi-objective problem formulation and categorizes changes to this problem formulation that lead to changes in the Pareto set. Section 2 introduces global metrics for comparing Pareto sets, and Section 3 introduces local metrics. Section 4 gives examples of how the metrics can be used, and Section 5 summarizes the chapter.

7.1 Pareto set analysis

A wide body of literature in the decision sciences [*Keeny and Raiffa* (1976)] and the engineering community [*Das* (1999); *Kasprzak and Lewis* (2001)] studies how to formulate preferences given a particular tradeoff. The Pareto set, in particular, has received much attention, including how the Pareto frontier relates to sensitivity in the objective functions [*Lootsma* (1999)]. Additionally, analogies to postoptimal analysis in single objective problems have been proposed, particularly for vector objective linear programming [*Kornbluth* (1974); *Gal and Leberling* (1977)].

This chapter formalizes metrics for comparing tradeoff scenarios. We hypothesize that we can measure how much two objectives compete in a Pareto problem. The less they compete the more aligned they are. A multicriterion or Pareto optimization problem is stated as:

$$\begin{aligned} \min \quad & \mathbf{f}(\mathbf{x}) \\ \text{subject to:} \quad & \mathbf{h}(\mathbf{x}) = \mathbf{0};\ \mathbf{g}(\mathbf{x}) \leq \mathbf{0};\ \mathbf{x} \in \mathcal{S}; \end{aligned} \tag{7.1}$$

Here $\mathbf{f}(\mathbf{x})$ is a vector of criteria of interest f_n, $n = 1, \ldots, N$. The set of variable values $\mathbf{x}$ that satisfy all equality $\mathbf{h}$, inequality $\mathbf{g}$, and set constraints $\mathcal{S}$ is the feasible (design) domain, $\mathcal{X}$. The range set of all vectors $\mathbf{f}$ mapped from the feasible domain is the attainable set $\mathcal{A} = \{\mathbf{f}(\mathbf{x})|\mathbf{x} \in \mathcal{X}\}$. A point in $\mathcal{A}$, $\mathbf{f}(\mathbf{x}^*)$, is said to be non-dominated or Pareto optimal, if there exist no $\mathbf{f}(\mathbf{x})$ such that $\mathbf{f}(\mathbf{x}) \leq \mathbf{f}(\mathbf{x}^*)$ and $f_n(\mathbf{x}) < f_n(\mathbf{x}^*)$ for at least one n.

Ideal values f_n° are the optimal criterion values obtained optimizing one criterion at a time:

$$f_n^\circ = \min(f_n(\mathbf{x})|\mathbf{h}(\mathbf{x}) = \mathbf{0},\, \mathbf{g}(\mathbf{x}) \leq \mathbf{0},\, \mathbf{x} \in \mathcal{X}),\ n = 1, \ldots, N \tag{7.2}$$

Nadir values f_n^N are the worst values for each criterion found in the set of Pareto optimal points. For a bi-criterion problem the nadir value for one criterion can be found when the other criterion reaches its ideal value [*Ehrgott and Tenfelde-Podehl* (2003)]:

$$f_n^N = \{(f_n(\mathbf{x})|f_l(\mathbf{x}) = f_l^\circ\} \tag{7.3}$$

The ideal or utopia point is the vector of ideal values for all criteria, $\mathbf{f}^\circ = [f_1^\circ, f_2^\circ]'$.

We consider now Pareto set analysis, or how we can compare different Pareto sets. A design scenario is defined here as the Pareto set generated by a given problem statement and its associated parameter values. A design scenario can be classified as superior to another using the concept of a meta-Pareto set, which includes all non-dominated criteria vectors selected from the union of all the individual Pareto sets under consideration [*Athan and Papalambros* (1996); *Mattson and Messac* (2003)]. Cohon [*Cohon* (1978)] discusses special cases of mathematical conditions for non-inferiority introduced by Kuhn & Tucker [*Kuhn and Tucker* (1951)] where objectives are redundant or completely conflicting. Lootsma [*Lootsma* (1999)] suggests examining a cross-effect matrix, whose rows are the values for all criteria at the individual optimum for each criterion, to observe competition between objectives. Several degree of conflict and similarity measures have been proposed for pairwise comparison of alternatives or for linear programs. Purshouse & Fleming [*Purshouse and Fleming* (2003)] characterize regions of objective space as independent, conflicting, or harmonious. Carlsson & Fuller [*Carlsson and Fuller* (1995)] define interdependence of objectives, degree of objective conflict, and problem complexity for the linear case. Deng [*Deng* (2007)] presents a conflict index between two alternatives and defines a metric for similarity to the ideal point.

The design of the solution set (rather than a single-point design) is important in many design scenarios. These scenarios share a characteristic that design decisions are not all made simultaneously, but some may be made before others (configuration design), some decisions may be more flexible then others, or be repeated at a higher frequency (dynamic control, product platforming, design for adjustability), and some decisions (or exogeneities) may be uncertain (robust design, product development investment planning, regulatory policy). In the general case, systems characterized by multiple objectives will exhibit a tradeoff relationship between improvements for both objectives. Considering how the Pareto set changes with changes in the problem formulation can facilatate design of the attainable set in addition to illustrating the tradeoffs between specific solutions.

Changes to the mathematical structure and input parameter values of a bi-objective programming problem can lead to changes in the shape of the attainable set and its

Pareto boundary. We illustrate the link between the terms described in Sections 7.2 and 7.3 and outcomes of the Pareto set using a simplified automotive vehicle design problem and a two-dimensional nonlinear programming examples.

The task of the designer abstracted to a mathematical decision-making problem is to specify the functional forms of the objective and constraint functions (referred to as system specification in the dynamic systems terminology), then partition model elements between parameters and variables, specify parameter values, and find efficient values for design variables (referred to as system identification in the dynamic systems terminology). We classify changes to a system design problem formulation (summarized in Table 7.1) according to this definition of system specification and identification. Each of these decisions may affect the Pareto set. For example, changing parameter values is equivalent to a traditional parametric study and fits in system identification. The examples listed in Table 7.1 reflect changes to the example problem specified in Equation (7.18).

Table 7.1: Classification of System Design Model Changes

System Modeling Stage	Change	Example
Specification	Objective functional form	-
	Constraint functional form	-
	Add constraint	$x_2 - 5 + 2x_1 \leq 0$
	Remove constraint	-
Identification	Repartition parameters	-
	Parameter values	$p = 5$

Section 7.2 discusses global metrics that are so named because they require information about the extreme points of the Pareto set and require evaluation of more than one Pareto point. The metrics described in Section 7.3 are referred to as local metrics because they are computed at a specific Pareto point.

7.2 Global multicriterion tradeoff metrics

We introduce here three global metrics to compare the nature of multiobjective tradeoffs that have similarities to previous work with the advantage that the new metrics are relatively inexpensive to compute and can be applied to a wide range of problem types. The new metrics can be applied to compare Pareto sets when the system design model has changed (i.e., the functional form of objectives and constraints) in addition to comparing tradeoffs for a single problem definition.

The concept of criterion alignment is introduced to compare Pareto sets in terms of how much their objectives compete with each other. Two objectives are said to be aligned when both attain their ideal values simultaneously. A Pareto curve is more aligned than another when (i) the effective curvature of the normalized Pareto curve is greater; (ii) it spans a smaller area in the criterion space; (iii) it is less sensitive or "flat". Three metrics, each emphasizing a different aspect of criteria alignment, are proposed in order to facilitate comparisons between design scenerios: Effective curvature, area, and sensitivity. With the exception of curvature, the metrics are relative and can be used to compare tradeoffs only for problems with the same objectives $\mathbf{f}$. Computing the effective curvature metric requires examining the Pareto frontier, while the other two metrics can be computed from knowing the ideal points.

7.2.1 Effective curvature

The effective curvature κ indicates the relative convexity or concavity of a particular tradeoff. To calculate κ we normalize the Pareto set between 0 and 1 for each criterion: $f'_n(\mathbf{x}) = (f_n(\mathbf{x}) - f^\circ_n) / \left(f^N_n - f^\circ_n\right)$, and define the minmax solution $L^A_\infty = \min L_\infty = \min \|\mathbf{f}'\|_\infty = \max\{|f'_1|, |f'_2|\}$, that minimizes the maximum deviation from either ideal value, $0 < L^A_\infty < 1$. This solution is at the intersection of the curve $f'_2 = f'_1$ and the Pareto set, and is used to calculate κ by finding the curvature

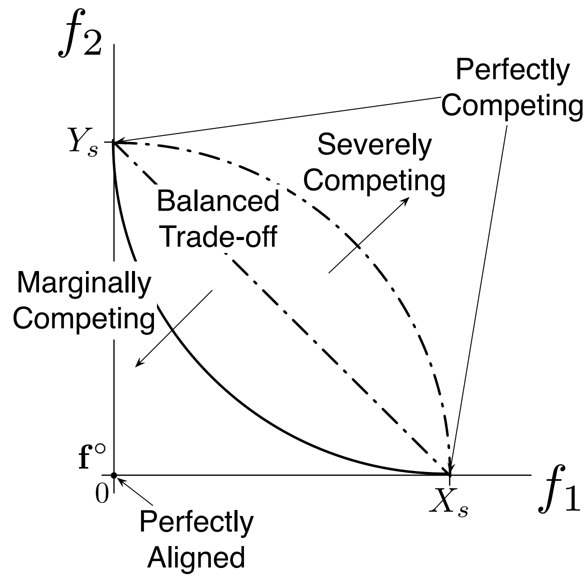


Figure 7.1: Normalized criterion space showing competition severity as a function of Pareto set curvature

of the hyperbola $y = 1/\left((Ax+B)-C\right)$, intersecting the coordinate axes at 1. Then

$$\begin{aligned}
\kappa &= 3\sqrt{2}(2L_\infty^A - 1)^2 / \left(2{L_\infty^A}^2 (L_\infty^A - 1)^2\right), \quad 0 < L_\infty^A \le \tfrac{1}{2}, \quad \text{convex} \\
\kappa &= -3\sqrt{2}(2L_\infty^A - 1)^2 / \left(2{L_\infty^A}^2 (L_\infty^A - 1)^2\right), \quad \tfrac{1}{2} < L_\infty^A < 1, \quad \text{concave}
\end{aligned} \tag{7.4}$$

Thus κ is a monotonically decreasing, smooth, piecewise function with respect to L_∞^A, with vertical asymptotes at 0 and 1 and an inflection point at 0.5. Criteria compete less as κ increases, indicating increasing convexity and L_∞^A closer to 0. Figure 7.1 shows differences in curvature for a normalized Pareto set. In general,

$$-\infty < \kappa < \infty \begin{cases}
\lim_{L_\infty^A \to 1} \kappa \to -\infty, & \text{perfectly competing} \\
-\infty < \kappa < 0, & \text{severely competing} \\
0 < \kappa < \infty, & \text{marginally competing} \\
\lim_{L_\infty^A \to 0} \kappa \to \infty, & \text{perfectly aligned} \\
\kappa = 0, & \text{balanced tradeoff}
\end{cases} \tag{7.5}$$

7.2.2 Area

The area metric is the area of the rectangle that inscribes the Pareto set, defined as $\Phi = |X_s Y_s|$ where $X_s = (f_1^N - f_1^\circ)/\lambda_1$, $Y_s = (f_2^N - f_2^\circ)/\lambda_2$ for the bi-objective problem, with $\lambda_n, n = 1, 2$, chosen by the designer, to compute a scaled range for criterion n. Criteria compete less or are more aligned, as the area is reduced.

A useful heuristic for selecting λ_n is to consider the smallest change in f_n that would be meaningful, i.e., one that gives a unique solution in a practical application. For example, for a miles/gallon criterion, a difference of 0.1 mpg may be the smallest significant unit. The scaled values of f_n would then be multiples of the significant unit. Setting $\lambda = 1$ maintains the original scale. Scaling based on a significant unit is useful for comparing Pareto sets because the scale of the relative changes in each criterion is preserved. This would not be the case if the Pareto set was normalized or if a value unique to each design scenario was used, such as f_n°. Selecting scaling factors implies some judgment on the relative value of each criterion, just as normalizing or leaving criteria unscaled implies a relative weighting.

7.2.3 Sensitivity

The sensitivity metric is defined relative to each criterion $\Delta_{X_s} = Y_s/X_s$, $\Delta_{Y_s} = X_s/Y_s$. A lower value of Δ_{X_s} means criterion Y is less sensitive to changes in criterion X. The sensitivity metric reflects the change in one criterion given a change in the other criterion over the entire Pareto set; it indicates the shape of the rectangle that inscribes the Pareto set. A criterion is more or less sensitive as the rectangle becomes more eccentric. Balanced sensitivity occurs when $Y_s/X_s = 1$.

7.2.4 Pareto set comparisons

Criterion alignment is just one aspect of Pareto set comparison. It should be noted that increased or decreased alignment according to the metrics described above

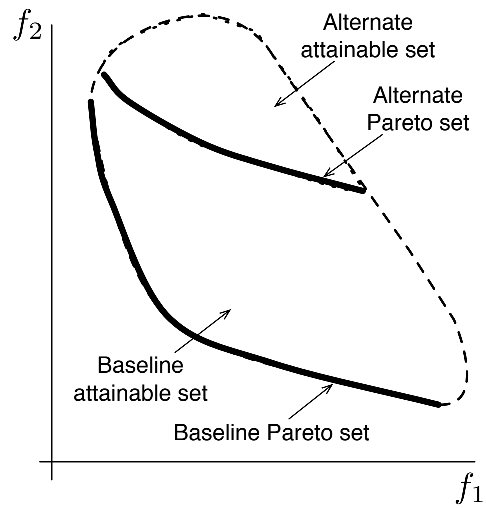


Figure 7.2: Alternative Pareto set exhibiting reduced area and sensitivity at the expense of objective values

does not indicate preference for a given design scenario over another in terms of the numerical objective values of the Pareto set. For example, introduction of additional constraints may reduce the value of the area or sensitivity metrics compared to a baseline case by reducing the attainable set, see for example Figure 7.2). The Pareto set would then exhibit increased alignment at the expense of the objective values.

7.3 Local multicriterion tradeoff metrics

To clarify the motivation of the discussion that follows it is helpful to think about the vehicle design problem as a design and control problem. The design variables are physical characteristics of the vehicle. The control variable is the vehicle price. In the simplest case where a producer is only considering the public impact of its own isolated actions, the public objective (minimize fuel consumption per vehicle sold) is a function of design variables alone. The private objective (maximize profit) is a function of design variables and vehicle price. The public objective is analogous to a design objective and the private objective is analogous to a control objective in a co-design and control problem.

One method for solving a design and control problem is to first solve the design problem to optimality and then solve the control problem to optimality given the results from the first stage. This method works well when the design problem has highest priority and the design problem is not influenced by the control variables. More priority can be given to the control problem by solving the problem as a weighted-sum single objective problem with more weight added to the control problem. Even in this case, the computations can proceed in the same sequential fashion–first design, then control. The analogy to public and private tradeoffs in a market systems context comes when we realize that the two-stage design and price equilibrium game we established in Chapter 6 is analogous to the sequential solution strategy just described for design and control problems. Each vehicle producer solves the vehicle design problem (the top-level game) assuming that the price (control variables) will be solved to optimality given a set of design variables (the subgame).

The design of modern smart products requires concurrent optimization of the artifact design and its controller. This so-called co-design problem [*Fathy et al.* (2001); *Reyer et al.* (2001)] is often performed in a sequential manner for reasons of convenience and tradition: design the artifact first, and then design its controller. In general, such a strategy will yield non-optimal solutions, compared with a simultaneous or all-in-one optimization of the combined system [*Fathy et al.* (2001); *Reyer et al.* (2001)], particularly when bidirectional coupling exists between the two subproblems, for example, when each of the two objectives depends on some variables and parameters of the other subproblem [*Reyer et al.* (2001)]. However, there exists a large class of problems where coupling is unidirectional, for example, the artifact criterion $f_1(\mathbf{x}_1)$ depends only on the artifact design variables $\mathbf{x}_1$ while the control criterion $f_2(\mathbf{x}_1, \mathbf{x}_2)$ depends on both the artifact variables and the controller design variables $\mathbf{x}_2$, so that the system objective becomes: $F = w_1 f_1(\mathbf{x}_1) + w_2 f_2(\mathbf{x}_1, \mathbf{x}_2)$, where w_1, w_2 are weights. An example of such a formulation is a linear positioning device where the artifact objective is steady-state displacement and the controller objective is settling time. Such a partitioning is inherent when the artifact criterion is independent of the controller variables as measured by the partial gradients of the objective and constraint functions with respect to the controller variables. Partitioning artifact and controller variables may be desirable for practical purposes in cases where the effect of the controller variables on the artifact criterion is deemed small enough, or where the analytical or computational means are not available to treat artifact and control variables simultaneously for the controller objective. One strategy for the latter case above is to solve the system-level problem as a nested optimization one [*Reyer et al.* (2001); *Fathy* (2003)], where the system solution is found with respect to $\mathbf{x}_1$, with the optimal $\mathbf{x}_2$ computed as a function of $\mathbf{x}_1$ by solving the "inner" optimal controller problem first [*Fathy et al.* (2001); *Fathy* (2003)]. This nested problem formulation is distinguished from the simultaneous one using the notation $F^n = w_1 f_1(\mathbf{x}_1) + w_2 f_2^n(\mathbf{x}_2^*(\mathbf{x}_1))$.

Note that the linear scalarization used here reflects typical practice in the control literature. Since this is known to fail in cases of nonconvex Pareto sets, the actual computation can be done using nonlinear scalarization [*Athan and Papalambros* (1996)] or some other method, several of which were described in Section 2.3.

Viewing the co-design problem as a bi-objective Pareto formulation without scalarization and weights, we can examine how much the two objectives compete or are aligned [*Cohon* (1978)]. Intuitively, it would appear that objective alignment must relate to objective coupling. Quantifying this relationship will provide deeper insights in the nature of both the alignment and coupling concepts, and their implications for understanding coupled multi-objective problems. In what follows, we show how a measure of objective alignment (the polar cone of objective gradients) is related to the coupling vector derived for a problem with unidirectional coupling, and how the measure of constraint decoupling can be normalized when the system design problem is considered as a bi-objective problem. These measures help to understand how the Pareto set is affected by changes to the system design problem formulation.

7.3.1 Objective Coupling and Objective Alignment

Multi-objective programming typically focuses on finding Pareto points and defining the preference structure for selecting one point among many [*Steuer* (1986)]. Several researchers have applied the concept of objective function gradient differences in order to compare solutions [*Purshouse and Fleming* (2003); *Carlsson and Fuller* (1995); *Deng* (2007)]. Lootsma examined how the Pareto frontier relates to sensitivity in the objective functions [*Lootsma* (1999)]. Additionally, analogies to postoptimal analysis in single objective problems have been proposed, particularly for vector objective linear programming [*Kornbluth* (1974); *Gal and Leberling* (1977)]. Others have also discussed the idea of comparing different Pareto sets using the concept of a meta-Pareto set, which includes all non-dominated criteria vectors selected from the union of all the individual Pareto sets under consideration [*Athan and Papalambros* (1996); *Mattson and Messac* (2003)]. We adopt the polar cone of the negative gradients as our measure of objective alignment, and we will consider how this measure changes, and the attendant implications for the Pareto set, with changes in the problem formulation.

The decision space can be partitioned into three disjoint sets with respect to a feasible point $\mathbf{x}$: Points $[\mathbf{x}_1, \ldots, \mathbf{x}_n]^\top \in \mathbf{R}^n$ that are superior, $\mathbf{Q}^<(\mathbf{x})$; points that are equal or inferior, $\mathbf{Q}^\geq(\mathbf{x})$; and points that cannot be compared, $\mathbf{Q}^\sim(\mathbf{x})$. The set $\mathbf{Q}^<(\mathbf{x})$ is equivalent to the interior of the polar cone of the negative objective gradients

$$\mathbf{Q}^<(\mathbf{x}) = \{\mathbf{k} | -\mathbf{k}^\top \nabla f^i > \mathbf{0}; i = 1, 2\} \tag{7.6}$$

where $\mathbf{k}$ is an n-dimensional vector with origin at $\mathbf{x}$ [*Zadeh* (1963); *Cohon* (1978)]. The angle between the boundaries of the polar cone can then be taken as a measure of objective function alignment at a particular $\mathbf{x}$. A polar cone angle of π corresponds to the case where the gradients of both objectives at $\mathbf{x}$ are parallel. The polar cone angle collapses to 0 when objective gradients are parallel with reversed signs.

The interdependence of the multiple objectives for a given system is critical to its design [*Balling and Sobieszczanski-Sobieski* (1996); *Bloebaum et al.* (1992); *Hajela et al.* (1990)]. The complete co-design problem with unidirectional coupling is formulated as [*Fathy* (2003)]

$$\begin{aligned} \min_{\mathbf{x}_1,\mathbf{x}_2} \quad & w_1 f_1(\mathbf{x}_1) + w_2 f_2(\mathbf{x}_1,\mathbf{x}_2) \\ \text{subject to:} \quad & \mathbf{h}_1(\mathbf{x}_1) = \mathbf{0};\ \mathbf{h}_2(\mathbf{x}_1,\mathbf{x}_2) = \mathbf{0} \\ & \mathbf{g}_1(\mathbf{x}_1) \le \mathbf{0};\ \mathbf{g}_2(\mathbf{x}_1,\mathbf{x}_2) \le \mathbf{0} \end{aligned} \tag{7.7}$$

where $f_1(\mathbf{x}_1)$ is the artifact objective function, $f_2(\mathbf{x}_1,\mathbf{x}_2)$ is the controller objective function, $\mathbf{x}_1$ is the vector of artifact design variables, $\mathbf{x}_2$ is the vector of controller design variables, $\mathbf{h}$ are the system equality constraints, and $\mathbf{g}$ are the system inequality constraints, and w_1 and w_2 are the weights associated with the objective functions f_1 and f_2, respectively.

7.3.2 Definitions

We adopt several terms to aid in explaining changes in the Pareto set. First, we define terms related to the unidirectional coupled system problem. Next, we define terms related to the bi-objective problem. Then, we define terms related to both problems.

Consider the nested system design problem

$$F^n = w_1 f_1(\mathbf{x}_1) + w_2 f_2^n(\mathbf{x}_2^*(\mathbf{x}_1)), \tag{7.8}$$

where the asterisk denotes that the optimal values for $\mathbf{x}_2$ have been found with respect to $\mathbf{x}_1$. The coupling vector $\mathbf{\Gamma_v}$ [*Fathy* (2003)] defined by Equation (7.9) is derived from the Karush-Kuhn-Tucker (KKT) optimality conditions for the weighted-sum objective describing the system design problem.

$$\mathbf{\Gamma_v} = W\left(\frac{\partial f_2}{\partial \mathbf{x}_1} + \frac{\partial f_2}{\partial \mathbf{x}_2^*}\frac{\partial \mathbf{x}_2^*}{\partial \mathbf{x}_1}\right) = W\nabla f_2^n(\mathbf{x}_1) \tag{7.9}$$

To simplify the notation in Equation (7.9), we set w_1 from Equation (7.8) equal to 1 and substitute W for w_2. The inner term is the gradient $\nabla f_2^N(\mathbf{x}_1)$. $\mathbf{\Gamma_v}$ is assumed to be a row vector.

Objective decoupling

Objective decoupling occurs when the inner term of $\mathbf{\Gamma_v}$ vanishes. In this case the solution to the single-objective problem, $\min f_1$, will also be the solution to the weighted-sum system objective problem [*Fathy* (2003)].

Constraint decoupling

Constraint decoupling occurs when there is a range of values for W for which a given $\mathbf{x}^*$ is the system optimal solution. This behavior occurs when the gradients of the active constraints at the system optimal solution can form convex combinations equal to the system objective gradient for a range of objective gradient directions, controlled by W.

Objective alignment

For a bi-objective problem, two objectives are said to be aligned at a particular design point $\mathbf{x}$ if the angle between the objective gradients is 0, or equivalently if the angle described by the polar cone $\theta^{<}$ of the two negative objective gradients is π. The polar cone has an appealing geometric interpretation in that the larger $\theta^{<}$ the greater the region of simultaneously improving directions, or the greater the objective alignment. From the definition of polar cone in Equation (7.6), the polar cone angle is

$$\theta^{<} = \arccos\left(\frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{|\mathbf{k}_1||\mathbf{k}_2|}\right), \ \{\mathbf{k}_1| - \mathbf{k}_1 \nabla f_1(\mathbf{x}) = 0;\ \mathbf{k}_2| - \mathbf{k}_2 \nabla f_2(\mathbf{x}) = 0\}. \quad (7.10)$$

A convenient way to identify the appropriate $\mathbf{k}_1$ and $\mathbf{k}_2$ for a problem with two design variables is to recognize that $\mathbf{k}_1$ should be orthogonal to $-\nabla f_1$ and in the plane defined by f_1 and f_2. We preserve the polar cone measure for the n-dimensional case but calculate it directly from the objective gradients: $\theta^{<} = \pi - \xi$, where $\xi = \arccos((\nabla f_1 \cdot \nabla f_2)/(|\nabla f_1||\nabla f_2|))$.

Coincidence

Two objective criteria are said to be coincident when the single-objective minimizers of the design variables shared between the objectives are equal. Namely, there exists some vector $\mathbf{x}^*|\ f_1(\mathbf{x}^*) = f_1^{\circ}, f_2(\mathbf{x}^*) = f_2^{\circ}$. A relative measure of coincidence to compare two Pareto sets is the L_2 norm of the design variables between objective ideal points $||(\mathbf{x}^{f_1^{\circ}} - \mathbf{x}^{f_2^{\circ}})||_2$.

Dominance

One Pareto set is said to dominate another Pareto set when each member of the dominated Pareto set belongs to $\mathbf{Q}^{\geq}(\mathbf{x})$ for at least one member of the dominating

Pareto set.

Independence

Two objectives are said to be independent at a particular design point $\mathbf{x}$ when $\boldsymbol{\Gamma}_{\mathbf{v}} = \mathbf{0}$ or when $\theta^{<}$ is undefined.

Pareto slope

The coupling vector $\boldsymbol{\Gamma}_{\mathbf{v}}$ is related to the slope of the Pareto frontier of the bi-objective problem [*Peters et al.* (2009)]. At a given Pareto-efficient point $\mathbf{x}^*$ using Equation (7.9) we have,

$$\frac{df_2^*}{df_1^*} = \frac{1}{W}\boldsymbol{\Gamma}_{\mathbf{v}}\frac{d\mathbf{x}_1}{df_1^*} = \nabla f_2^{n\top} \cdot (1/\nabla f_1). \tag{7.11}$$

7.3.3 Quantification of Alignment and Objective Coupling

We begin with the necessary conditions for an efficient point to a bi-objective minimization problem [*Kuhn and Tucker* (1951)] as in Equation (7.1):

$$\begin{aligned} \eta_1 \nabla f_1(\mathbf{x}^*) + \eta_2 \nabla f_2(\mathbf{x}^*) + \lambda^\top \nabla \mathbf{h} + \mu^\top \nabla \mathbf{g}(\mathbf{x}^*) &= \mathbf{0} \\ \mu &\geq \mathbf{0} \\ \lambda &\neq \mathbf{0} \\ \mu^\top \mathbf{g}(\mathbf{x}^*) &= \mathbf{0} \\ \mathbf{h} &= \mathbf{0} \end{aligned} \tag{7.12}$$

Comparing Equation (7.12) to the first-order optimality conditions for Equation (7.7) we see that the co-design problem is a special case of the bi-objective problem where the weighting factors were chosen a priori. In previous work on co-design coupling, emphasis has been placed on comparing f_1 to the weighted system objective $w_1 f_1 + w_2 f_2$

rather than comparing f_1 and f_2 directly. However, the coupling vector $\mathbf{\Gamma_v}$ is difficult to interpret because it is directly proportional to the subjective weighting value W and the units of measurement for the objective function. Comparing the two objectives directly frees the designer from implying a scale W, or "exhange rate", between objectives before studying the attainable set. Objective alignment is a function of gradient direction only (not magnitude). It is still possible that the gradient direction is affected by the scale of the controller variables $\mathbf{x}_2$ since they do not appear in the artifact objective function.

Objective alignment can be calculated for a unidirectional coupled problem following the definition of alignment above. This is a straightforward process if the whole gradients are available: $\nabla f_1(\mathbf{x}_1), \nabla f_2(\mathbf{x}_1, \mathbf{x}_2)$. However, it can be challenging to formulate or compute the whole gradients when the controller objective is formulated as a nested problem with gradient $\nabla f_2^n = \frac{\partial f_2}{\partial \mathbf{x}_1} + \frac{\partial f_2}{\partial \mathbf{x}_2^*}\frac{\partial \mathbf{x}_2^*}{\partial \mathbf{x}_1}$. For example, assume $f_2^n(\mathbf{x}_1)$ is a black-box simulation. We can then compute $\nabla f_2^n(\mathbf{x}_1)$ and observe $\partial \mathbf{x}_2^*/\partial \mathbf{x}_1$. However, we require $\nabla f_2(\mathbf{x}_1, \mathbf{x}_2)$. If we assume we can compute $\nabla f_2(\mathbf{x}_2)$ analytically or by evaluating the conventional control problem $f_2(\mathbf{x_2})$, then we can back out the missing component: $\partial f_2/\partial \mathbf{x}_1 = \nabla f_2^n - \frac{\partial f_2}{\partial \mathbf{x}_2}\frac{\partial \mathbf{x}_2}{\partial \mathbf{x}_1}$.

7.3.4 Normalized Constraint Decoupling

Returning to the geometric interpretation of the weighted-sum objective in the design variable space, the sum of any two vectors with positive weighting factors $((1-w)f_1 + wf_2 |\ w \geq 0)$ will be a new vector that lies between the two original vectors assuming the same origin. The necessary conditions for the optimal system design problem imply that the weighted-sum-objective vector can be formed by a convex combination of the gradients of the active constraints. Constraint decoupling requires that, at a given Pareto point for a system design problem with weight w, the span of the convex combination of satisfied constraints (including degenerate constraints) will similarly satisfy the necessary conditions for optimality for a system design problem with some other weighting factor $t \neq w$.

The constraint decoupling ratio ϕ can be calculated at an ideal point (f_1°) by first calculating the angle between the two single-objective gradients $\nabla f_1, \nabla f_2$. We can then find the limiting weighting value w^*, and compute the angle between the weighted-sum system-objective gradient and ∇f_1. Assuming the system design objective is a convex combination of the single objectives, the limiting weighting value can be found by solving the following problem where $\nabla f_1, \nabla f_2, \nabla \mathbf{g}, \nabla \mathbf{h}$ have been evaluated at $(\mathbf{x}_1, \mathbf{x}_2)_{f_1^\circ}$:

$$\begin{aligned} \min_{w,\boldsymbol{\beta}} \quad & -w \\ \text{subject to:} \quad & (1-w)\nabla f_1 + w\nabla f_2 + \boldsymbol{\beta}^\top \nabla \mathbf{g} + \boldsymbol{\lambda}^\top \nabla \mathbf{h} = \mathbf{0} \\ & -\boldsymbol{\beta} \leq \mathbf{0}; \boldsymbol{\lambda} \neq \mathbf{0}. \end{aligned} \tag{7.13}$$

The ratio ϕ evaluated at an ideal point is then the ratio of the angle between the maximum weighted-sum objective gradient with the same optimal solution as the ideal point and the single-objective gradient, and the angle between the two single-objective gradient vectors:

$$\phi = \arccos\left(\frac{\nabla f_1 \cdot ((1-w^*)\nabla f_1 + w^*\nabla f_2)}{|\nabla f_1| \left| ((1-w^*)\nabla f_1 + w^*\nabla f_2) \right|}\right) / \arccos\left(\frac{\nabla f_1 \cdot \nabla f_2}{|\nabla f_1||\nabla f_2|}\right). \tag{7.14}$$

The amount of constraint decoupling, or the range of weighting values for which the constraint decoupling conditions hold, will change with the objective scaling. However, ϕ, based on the gradient directions will take a value between 0 and 1 and will not change with objective scaling. Figure 7.5(c) illustrates this case where the dashed line shows the limiting gradient direction for which the system design problem will have the same solution as the single-objective problem f_1. A normalized measure can be defined for any Pareto point $\mathbf{x}^*$ by replacing ∇f_1 in Equation (7.13) with

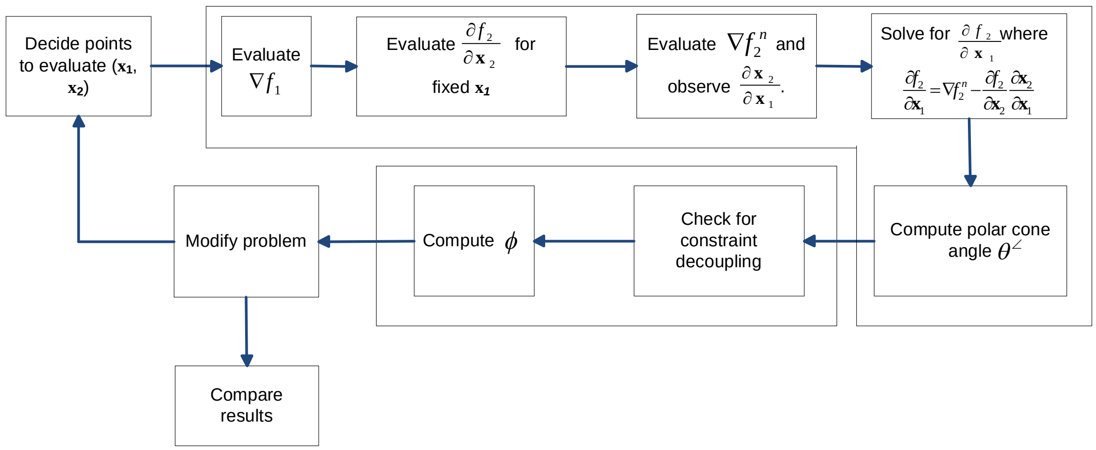


Figure 7.3: Algorithm for evaluating objective alignment and coupling measures for multiple problem formulations figure produced by D.L. Peters

$(1 - \upsilon)\nabla f_1 + \upsilon \nabla f_2$, where υ is the minimum weighting value for which $\mathbf{x}^*$ is the system design problem solution.

Given a fixed set of constraints, increasing objective alignment will result in a higher ϕ. Given a fixed set of objectives, decreasing satisfied-constraint alignment will result in higher ϕ.

Figure 7.3 summarizes the steps described above into an algorithm for systematically analyzing the Pareto set for objective alignment and coupling. The "compare results" step of the algorithm is illustrated in the next sections.

7.4 Pareto set analysis examples

Section 7.4.1 presents an example of Pareto set analysis applying the global measures. Section 7.4.2 presents an example of Pareto set analysis applying the local measures.

Table 7.2: Explanation of fuel economy constraint

Constraint	Description
g_{15}	Minimum combined fuel economy rating must be greater than the minimum fuel economy parameter

7.4.1 Pareto set analysis with global measures

This section illustrates a simplified vehicle design optimization problem that employs the scaling-based cost model from Chapter 5 and a simple logit model from the literature [*Boyd and Mellman* (1980)].

Problem formulation

The engineering models are the same as described in Chapter 3. The constraint set is the same as given as Equation (3.2) with the addition of a constraint:

$$g_{15} = z_{MPG} - v_{MMPG} \leq 0 \tag{7.15}$$

on fuel conusmption used to calculate Pareto points to the following problem using the ϵ-constraint method [*Osyczka* (1984)]:

$$\min \mathbf{f} | \mathbf{f} = [\pi, z_{MPG}] \tag{7.16}$$

by varying the minimum fuel economy constraint parameter v_{MMPG} between the nadir and ideal values of fuel economy f^N_{MPG} and f°_{MPG}. Table 7.2 gives a description of the fuel economy constraint.

Vehicle simulations were configured to represent a standard automatic transmission front wheel drive vehicle with a gasoline engine. In addition to powertrain specifications (i.e., gear ratios, gear shifting schedule, engine number of cylinders, vee or inline configuration, bore, and stroke, valvetrain configuration, and final drive ratio) *Cruise* also receives other vehicle parameters as inputs, including curb weight, frontal area, drag coefficient, tire radius, and center of gravity location under various loads. Over 30 parameters were tuned for midsize crossover vehicles based on data from one 2007 model. All other parameters were left at the default passenger vehicle levels.

Vehicle demand model

A logit model was chosen for representing demand, due to its ease of interpretation congruent with random utility theory [*Train* (2003)], and widespread use. We considered only vehicles from a very narrow class and thus reduced the risk of violating the independence of irrelevant alternatives assumption with the introduction or exclusion of a particular vehicle. All other purchase possibilities are represented in the utility of the outside good ν_{og}.

The choice share of a given product $\Pr(i)$ is defined for the logit model as the probability of choosing product i given products $1, \ldots, n$ as follows,

$$\Pr(i) = \exp(\nu_i) / \left(\nu_{og} + \sum_{j=1}^{n} \exp(\nu_j) \right), \ \nu_i \sum_{k} \delta_k \alpha(z_k) \text{ for } k \text{ attributes}, \tag{7.17}$$

where ν_i is the systematic component of utility for product i, ν_{og} is the utility of the outside good, δ_k is the systematic utility coefficient or part-worth corresponding to attribute k, α is a function that relates vehicle characteristic z_k to the systematic utility expression.

From the literature of demand models for the automotive industry [*Berry et al.* (1995); *Petrin* (2002); *Berry et al.* (2004)] we adopted a model similar to Boyd & Mellman [*Boyd and Mellman* (1980)]. It assumes aggregate (homogeneous) preferences, a logit form of the choice model, and a utility model that is linear in the coefficients with vehicle attributes: price, fuel consumption, the inverse of 0-60 acceleration time, and the styling factor *Styl1* based on external vehicle dimensions.

The Boyd & Mellman model was estimated using model year 1977 vehicle data, and as such, caution should be taken when interpreting results. Vehicle price was converted to 1977 dollars using the consumer price index; 0-60 times improved dramatically from 13.8s in 1977 to 9.6s in 2007 [*USEPA* (2007)]. A 1s shift (rather than the full 4.2s) to acceleration times generated vehicle designs similar to the existing market for the baseline scenario implying that preference for acceleration increased.

Conveniently, the assumed price of gas in 1977 dollars was \$0.70 (roughly \$2.40 in 2007 dollars—at the low end of the observed range of gas price in 2007). The average decrease in fuel consumption ($\approx 30\%$), as tested by the EPA, is accounted for by using the updated adjusted values [*USEPA* (2006b)] for fuel economy, rather than the EPA test values, and then shifting the mpg value by the 0.7mpg remaining difference, implying people value improvements in fuel economy at roughly the same level but expect higher average fuel economies.

The model was further calibrated by setting market size to the total vehicles sold in the US in 2007 (14.87 M), and ν_{og} was set to produce a total demand for the 9 hypothetical vehicles roughly equivalent to the 2007 sales of the real vehicles ($\approx 600,000$). Adjusting market size and ν_{og} in this way rather than using the segment market size and a modest ν_{og} did not shift the design decisions of Firm X, but did provide downward pressure on prices to bring them inline with observed values.

Market demand for each vehicle is estimated to be the product of the market size cap and the choice share. Other choice model formulations such as the mixed logit [*Train* (2003)] allow for individual taste heterogeneity. Studying the impact of choice model selection on the Pareto set outcomes could be the subject of future work.

Calibrating models between years is difficult because the purchase power of the dollar has decreased, the average vehicle attributes have changed, and the average price of vehicles in real dollars has increased [*Berry et al.* (1995)].

Attributes of the competing vehicles are listed in Table 7.3. Vehicle model names are prefaced by 'x-', indicating that the z_{Acc060} and the z_{MPG} attributes given are

Table 7.3: Hypothetical midsize crossover vehicle market excluding Firm X vehicle

Model	z_{Acc060}	z_{MPG}	z_{L103}	z_{W105}	z_{H101}	z_{Sty1}	B&M Util.
xEdge	7.7	19	185.7	75.8	67.0	3.9	5.75
xEndeavor	8.4	19	190.8	73.6	69.6	3.80	5.42
xHighlander	7.9	19	184.6	71.9	67.9	3.78	5.52
xMurano	8.0	20	187.6	74.0	66.5	3.93	5.82
xSanta Fe	8.2	19	184.1	74.4	67.9	3.81	5.48
xXL7	7.9	18	190.8	73.6	69.6	3.80	5.43
xTribeca	8.3	19	189.8	73.9	66.4	3.97	5.68
xVue	8.4	21	181.3	71.5	66.5	3.8	5.64

based on our simulation—not on the corresponding vehicle model's real world reported performance. Time is given in seconds, fuel economy in miles/gallon, and length in inches. The z_{Styl1} attribute and utility are dimensionless. We assume all vehicles use regular gasoline.

Price Equilibrium Solution Strategy

Each competitor optimizes profit with respect to vehicle price given the product designs and prices of all competitors. Firm X then optimizes product design and price variables, concluding one iteration. Iterations continue until price changes fall below a threshold constraint ($\approx$ \$80). The vehicle prices (and Firm X design variables) are now set such that no firm can make a different decision that would improve its own profits while the choices of the other firms remain fixed, thus approximating a Nash equilibrium [*Fudenberg and Tirole* (1991)].

A complete product equilibrium process would allow each competitor to optimize designs as well as prices with respect to all others' designs and prices. However, given the use of a homogenous multinomial logit model and identical underlying engineering and cost models, each firm would choose identical designs and prices, as seen in [*Michalek et al.* (2004)]. Instead, assuming fixed competitor designs coincides with vehicle planning where a firm makes some educated assumptions about the products

Table 7.4: Design scenario parameters for baseline

Market Assumptions	Market Size	Utility of outside good	Dealer Markup	Number of Firms
	14,870,000	9.1	8%	9
Relative Cost Breakdown	Powertrain	Chassis	Body	Wheels
	0.3	0.35	0.3	0.05
Choice Model Coefficients	δ_{MPG}	δ_{Acc060}	δ_{Styl1}	δ_{Price}
	-0.339	0.375	1.37	-0.000286

competitors will produce. During product launch and subsequent sales, all competitors are at liberty to adjust prices freely while the designs remain fixed. Competitive behavior [*Chintagunta et al.* (2006)] among automotive manufacturers considering the full market has been modeled, see for example [*Sudhir* (2001); *Goldberg* (1995)]. The simplified approach shown here includes competitive effects sufficient to illustrate trends without greatly increasing the computational complexity of the model.

Design Scenarios

We examine design scenarios that translate into model changes and new Pareto sets. The criterion alignment metrics and the overall objective values of the Pareto sets are compared. We divide design scenarios into four 'mechanisms': technology, preference, competition, and regulation. Table 7.4 gives the initial parameter values for the baseline case. Table 7.5 lists each design scenario and the corresponding model changes. Each model change is implemented individually, and all other parameter values are kept at the baseline levels. The unscaled results are shown in Figure 7.4. All design scenarios consider 9 producers including Firm X.

Baseline Case

The vehicle price ($MSRP$), market share (within segment), and expected profit for each firm are listed in Table 7.6 for the$f^{\circ}_{-profit}$ solution. Table 7.7 shows the

Table 7.5: Design scenarios listed by mechanism and model parameter level

Mechanism	Levels 1	2	3
Technology			
C_D	0.34	0.40	
Powertrain cost	20%	40%	
Preference			
Accel. Indifference	$\delta_{Acc060} = 0$		
Fuel economy	-0.239	-.439	-.639
Styling preference	$\frac{z_{W105} \times 2 + z_{L103}/2}{z_{H101}}$	$\frac{-(z_{W105} + z_{L103} + z_{H101})}{2000}$	$\frac{10 \times z_{H101}}{z_{W105} + z_{L103}}$
Competition			
Price-cutting	\$21,000 (xVue)	\$23,000 (xVue)	
Market size volatility	12,870,000	16,870,000	
Regulation			
Price-ceiling	\$20,000	\$22,000	\$24,000

Table 7.6: Prices, market shares, and expected profits for all firms for the baseline case

	xEdge	xMurano	xHighlander	xSanta Fe	xTribeca
MSRP	\$26,900	\$26,700	\$26,000	\$26,300	\$25,900
Market Share	12%	13%	10%	9%	12%
Profit	\$314M	\$373M	\$197M	\$151M	\$319M

	xVue	xXL7	xEndeavor	Firm X
MSRP	\$25,900	\$26,900	\$27,100	\$27,400
Market Share	11%	10%	8%	15%
Profit	\$257M	\$193M	\$68M	\$543M

Table 7.7: Firm X vehicle attributes for ideal $-profit$ and $fuelconsump$ values for the baseline case

Model	z_{Acc060}	z_{MPG}	z_{L103}	z_{W105}	z_{H101}	z_{Styl1}	Util.
Firm X $f^{\circ}_{-profit}$	7.2	18.4	196.6	74.4	67.0	4.05	6.03
Firm X $f^{\circ}_{fuelconsump}$	9.8	21.1	187.3	74.2	67.0	3.90	5.48

attribute values for $f^{\circ}_{-profit}$ and $f^{\circ}_{fuelconsump}$ for the baseline case.

Technology

Two technology scenarios are considered. The drag coefficient is changed to 0.4 and 0.34 from 0.37, and the powertrain cost parameter is changed from 30% to 20% and 40%. The difference in cost is absorbed evenly by the chassis and body subsystems.

Preference

Individual preferences for products change over time based on externalities (e.g., rising fuel prices or increased public concern for global warming) and based on changes to the performance levels and salience of the observable characteristics of a product. Advances in technology and government regulation are two influences that may change the observable product characteristics of a product such as the automobile. For example, increased consumer interest may be placed on fuel economy and derivative characteristics of advanced powertrains (e.g., range, access to refueling). Advertising is another mechanism that influences preferences, but it is ignored here.

The importance of acceleration in the baseline case is contrasted to the case where consumers are completely indifferent to 0-60 acceleration given that the vehicle meets towing, top speed, and 30-50 mph acceleration requirements.

We postulate new fuel consumption coefficients in the demand model assuming preference is proportional to cost of transportation in real dollars. This analysis assumes that preference for the other attributes with respect to price remain unchanged, which has the unavoidable side effect of changing the elasticities between fuel consumption and the other attributes. The coefficients are listed with the corresponding fuel price in 2007 dollars: $\delta_{MPG} = \{-.239, \$1.69; -.339, \$2.40; -.439, \$3.10; -.639, \$4.51\}$

Several alternative vehicle styling forms were considered. Three forms are reported, i) A longer, lower, wider form, where increases in width are more important than increases in length; ii) minimalist styling that emphasizes reduction in all three exterior dimensions; iii) an "inverted" form that is taller, shorter, and thinner.

Changing parameter values or functional form of product attributes in the utility model changes the computed values of utility. The utility of the outside good was updated to preserve the relative difference between the average utility value of the 8 original vehicles and the utility of the outside good for all preference and regulatory scenarios. Finally, all firms will react to changes in consumer preference. However, only Firm X changes vehicle design in this example. The resulting profits should be considered inflated relative to market expectations, but the trend in vehicle design (i.e., the relative change of fuel economy) should be preserved.

Competition

Two scenarios are considered that deal with the competitive landscape facing Firm X. First, the effect of a price-cutting strategy by another firm is examined at approximately \$3,000 and \$5,000 below the baseline equilibrium price. Second, market volatility is considered by varying annual US vehicle market size ± 2 million vehicles.

Regulation

Numerous regulatory scenarios can be explored using the proposed framework. Only one policy, a mandatory price ceiling at \$24,000, \$22,000, and \$20,000, is reported here. A natural consequence of a price ceiling policy is that demand exceeds supply. Losses by many firms in these scenarios indicate that the worst performing firms would exit the market and fewer vehicles would be produced overall.

Discussion

The results of each design scenario, including the baseline case, are shown in Figure 7.4 grouped by mechanism. The criterion alignment metrics are listed in Table 7.8. The Scenario Dominance section below compares the relative objective values of the Pareto sets across the scenarios, and the Tradeoff Metric section discusses the criterion alignment metrics.

Vehicle length (z_{L103}) is large at and shrinks along the Pareto frontier. Vehicle cost, fuel consumption, and acceleration are all negatively correlated with vehicle size. Vehicle height (z_{H101}) is constrained by the minimum sitting height constraint. Vehicle width (z_{W105}) is constrained by the rollover constraint and vehicle length and wheelbase are constrained by the minimum angle of departure and cargo volume constraints. In the absence of preference valuation for cargo volume, legroom, or other spatial features of the vehicle, Firm X will seek to build the smallest vehicle possible. The style attribute, which rewards increases in length and width, moves the design away from the constraint boundaries at the most profitable solutions. Vehicle length, wheelbase, and width are above average for the hypothetical marketplace and vehicle height is slightly below average. The minimum towing grade constraint becomes active at $f^{\circ}_{fuelconsump}$.

The attainable set for all scenarios is limited by the same set of constraints on vehicle characteristics. The scenarios have the effect of shifting output levels of the objective functions (e.g., the technology scenarios) and shifting the boundary of the attainable set that is Pareto optimal (e.g., the preference scenarios). New scenarios that modified, introduced, or excluded constraints could change the boundary of the

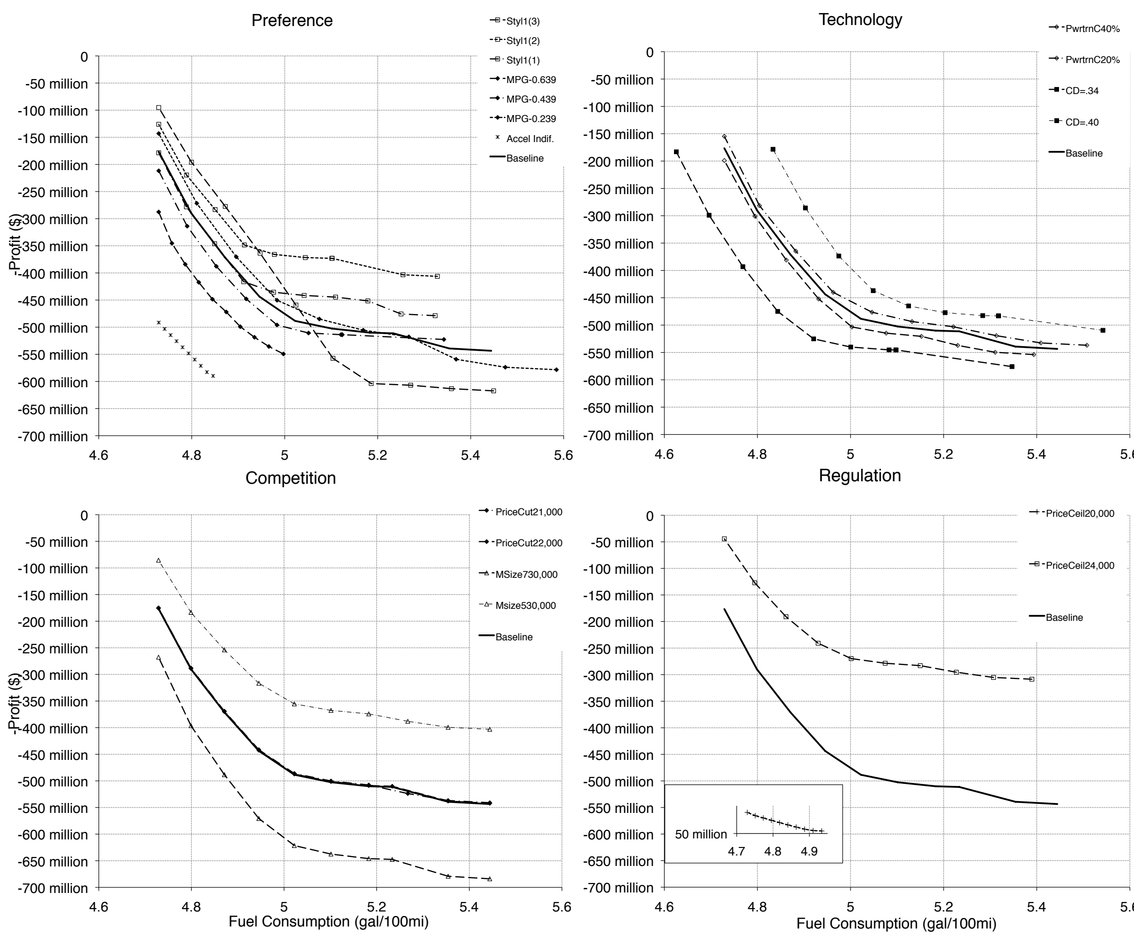
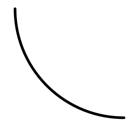
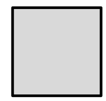
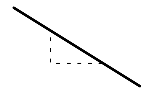


Figure 7.4: Results for all design scenarios showing −Profit vs. Fuel Consumption

Table 7.8: Criteria alignment metrics for each design scenario

Design Scenario	 Curvature	 Area	 Sensitivity Δ_{X_s}
Baseline	5.7	131,000	103
Technology			
$C_D = 0.34$	6.1	142,000	109
$C_D = 0.40$	7.4	117,000	93
Powertrain cost 20%	6.3	149,000	98
Powertrain cost 40%	5.3	118,000	107
Preference			
Accel. Indifference	0.02	5,750	168
Fuel economy -0.239	5.3	186,000	102
Fuel economy -0.439	6.4	95,000	101
Fuel economy -0.639	0.8	35,000	195
$Styl1(1)$: $\frac{W105\times2+L103/2}{H101}$	7.0	89,000	101
$Styl1(2)$: $\frac{-(W105+L103+H101)}{2000}$	7.1	84,000	94
$Styl1(3)$: $\frac{10\times H101}{W105+L103}$	1.8	188,000	145
Competition			
Price-cutting $21,000	5.7	131,000	102
Price-cutting $23,000	5.7	131,000	102
Market size volatility (12,870,000)	5.7	114,000	89
Market size volatility (16,870,000)	5.7	149,000	116
Regulation			
Price-ceiling $24,000	6.1	87,000	80
Price-ceiling $22,000	7.2	57,000	60
Price-ceiling $20,000	0.3	10,000	98

attainable set. On the other hand, the Pareto frontier in an unconstrained problem would depend only on the gradients of the objective functions. Reviewing Figure 7.4 most of the Pareto sets follow a similar form characterized by a distinct “elbow” separating a region of gradual decrease in profits for about 1/2 the fuel consumption change from a steeper region of profit loss for the remaining fuel consumption change. The similar form across most scenarios is a result of the similar constraint activity across those scenarios. The top speed constraint is active up to the elbow in the direction of decreasing fuel consumption and is no longer active beyond that point. Also near the elbow, *EngBore* meets the lower bound imposed by the model. However, *EngBoretoStroke* does not reach its lower bound so the engine displacement continues to decrease modestly as fuel consumption decreases.

Scenario Dominance

The globally dominant meta-Pareto set tracks along the market size of 16.87 million, then follows the indifference to acceleration, and then the $C_D = 0.34$ scenario. In the low fuel consumption region, expensive powertrains produce results in the market similar to higher preference for fuel economy. When powertrains are expensive (40% vs. 30%), fuel economy improves for the max profit solution; however, it is still more profitable to balance acceleration and fuel economy rather than reduce engine size and cost, and focus on fuel economy. Such a result may explain one reason why many hybrids have been tuned towards performance and not solely to maximize fuel economy.

Intuitively, increased market size increases profits. Non-intuitively, artificially lowering the price of a single vehicle had a negligible effect on Firm X decisions. This result demonstrates one of the weaknesses of the simple logit model: there is no way to account for the substitution patterns we would expect, i.e., more sales for the reduced price model coming from shifting demand within the segment rather than the entire vehicle fleet (the outside good). Preliminary studies using the segment size as the market size and a modest value of outside good utility showed results for a price cut by one firm very similar to a price ceiling on the entire segment.

Tradesoff Metric Comparison

Values for the three metrics are listed in Table 7.8. Each metric shows a range of values across the analyzed scenarios. Area and sensitivity were computed using scale factors of $\lambda_{-profit} = \$100,000$ and $\lambda_{fuelconsump} = 0.02$ gal/100 mi.

The acceleration indifference scenario comes closest to showing how the bi-objective problem can collapse to a single solution when the gradients of the two objectives are very similar. The importance of the *Styl1* attribute preserves a tradeoff. Another type of collapse could occur when one objective becomes completely indifferent to the other objective. This can occur when the gradients of each objective are uncoupled or only weakly coupled. In other words, the objectives depend on few, if any, of the same design variables.

Comparing the metric values and Figure 7.4 shows that localized effects of the Pareto curves are not accounted for by the metrics. The tradeoff region of interest to a vehicle producer is the region immediately around the max profit point. For most scenarios, the tradeoff in this region is much more shallow than the overall tradeoff as indicated by the sensitivity metric, and the curvature is much closer to 0 than indicated by the curvature metric. The metrics could be reapplied to a designated region of the Pareto curve to define metrics for local curvature, area, and sensitivity.

There are clear changes to the nature of the tradeoff as the MPG attribute increases in importance. The area decreases as $f^N_{fuelconsump}$ decreases. The curvature decreases dramatically between the baseline and the $\delta_{MPG} = -0.639$ case because the shallow tradeoff region is no longer Pareto-optimal, and the sensitivity increases because the range of fuel consumption decreases dramatically while the range of

Table 7.9: Trends in tradeoff metric values with respect to changes in Pareto set objective values

	Value	Curvature	Area	Sensitivity Δ_{X_s}
Technology	+	−	−	+
Preference	+	mixed	mixed	+
Competition	−	none	−	−
Regulation	−	mixed	−	mixed

profit changes less. The styling importance impact on profit and fuel consumption is mixed. Particularly interesting is the *Styl1*(1) scenario where width is valued more than length. Increased profits can be achieved and a firm is severely penalized for decreasing fuel consumption.

Reduced drag coefficient leads to increase in the area metric as the Pareto set improves compared to the baseline case, which is inconsistent with the trend in the preference scenarios. One interpretation of these results is that improvements in vehicle design (e.g., improved drag characteristics) have great potential for decreasing environmental impact, but these changes alone will lead only to marginal changes at $f^{\circ}_{-profit}$ when they occur in isolation of other model changes.

Price ceiling scenarios decreased fuel consumption at the cost of reduced profit levels. Trends for the metrics are summarized in Table 7.9. For example, the first line of the table should be read, "As preferences change such that the objective values of the Pareto set improve, curvature and area decrease while sensitivity increases." Some trends are not monotonic. Results for effective curvature were mixed over all the scenarios. As a whole, the tradeoff metrics show increasing alignment for Pareto sets with improved objective values when changes are made with respect to technology or preference, and they show increasing alignment for Pareto sets with inferior objective values when changes are made with respect to competition or regulation.

Discussion

The area metric is of practical significance to decision makers in that larger tradeoff area means more to gain (lose) in the tradeoff decision. The area metric also appears to give the best overall assessment of criterion competition considering the working definition of criteria alignment, i.e., both objectives achieve single objective optimality simultaneously. The other two metrics show increased competition for most cases when the area metric decreases. The smallest area values coincide with the smallest curvature values, likely indicating that, as ideal values move closer together for a given problem formulation, smaller regions of the attainable set boundary are Pareto optimal, and are thus more and more closely approximated by a straight line. This effect should be explored in other multi-objective problems with different Pareto frontier shapes (e.g., a concave frontier), and future work should investigate directly how the problem structure (i.e., objective gradients and constraint activity) relates to the metrics. The curvature metric, which can be compared across problems, may serve to classify multiobjective problems according to typical Pareto frontier shapes.

The enterprise vehicle design problem is an example of a class of problems where decision maker preferences are heavily weighted to one objective (e.g., profit). The sensitivity metric can be useful in such problems, especially if it is applied locally around the solution. A firm considering producing a vehicle away from $f^{\circ}_{-profit}$ for strategic reasons could then formulate a risk assessment for each scenario given the sensitivity of profit with respect to its choice. Scenarios with lower Δ_{X_s} values will be less sensitive to design choices away from $f^{\circ}_{-profit}$.

Another insight drawn from the metrics is that decreased objective competition does not predicate superior solutions compared to a more competing scenario. For example, one intuitive yet important trend in the results is that, when the area metric decreases due to changes in the objective gradients (e.g., consumer indifference to acceleration), the design scenario has potential to improve objective values for both stakeholders. However, when area decreases are due to changes in constraints (e.g., a price ceiling), the objective values of the scenario decreases for at least one stakeholder.

Caution should be taken in interpreting the results of this study. The intent is not to represent a true market equilibrium, but to represent a design scenario as it may appear to a vehicle manufacturer making assumptions about the vehicle designs of competitors. Significant obstacles remain in studying maximum profit formulations in vehicle design including questions about underlying demand model validity, econometric interpretations of changes to the demand model parameters, and realistic cost models, among others. Furthermore, other regulatory scenarios can be considered, such as a CAFE standard, fuel tax, or CO_2 tax. Therefore, the numerical results presented here are useful in illustrating the proposed concept of public and private alignment rather than suggesting specific decisions.

7.4.2 Pareto set analysis with local measures

We now demonstrate examples of problem formulation changes and observe the corresponding changes to the Pareto set for a two-variable nonlinear programming problem modified from Problem 10 in [*Hock and Schittkowski* (1981)]:

$$\begin{aligned} &\min\ [f_1 = 0.5x_1^2 - 7x_1,\ f_2 = x_2^2 - x_1x_2 - px_2]^\top \\ &\text{subject to: } x_2^2 + 4x_1^2 - 25 \leq 0,\ p = 7 \end{aligned} \tag{7.18}$$

The problem is illustrated graphically in Figure 7.5. Case (a) shows the unmodified problem, case (b) shows the case where $p = 5$, and case (c) includes the additional constraint $x_2 - 5 + 2x_1 \leq 0$. The dashed line in case (c) indicates the degree of constraint decoupling for the problem.

We compute the measures defined in Section 7.3 and report the values in Ta-

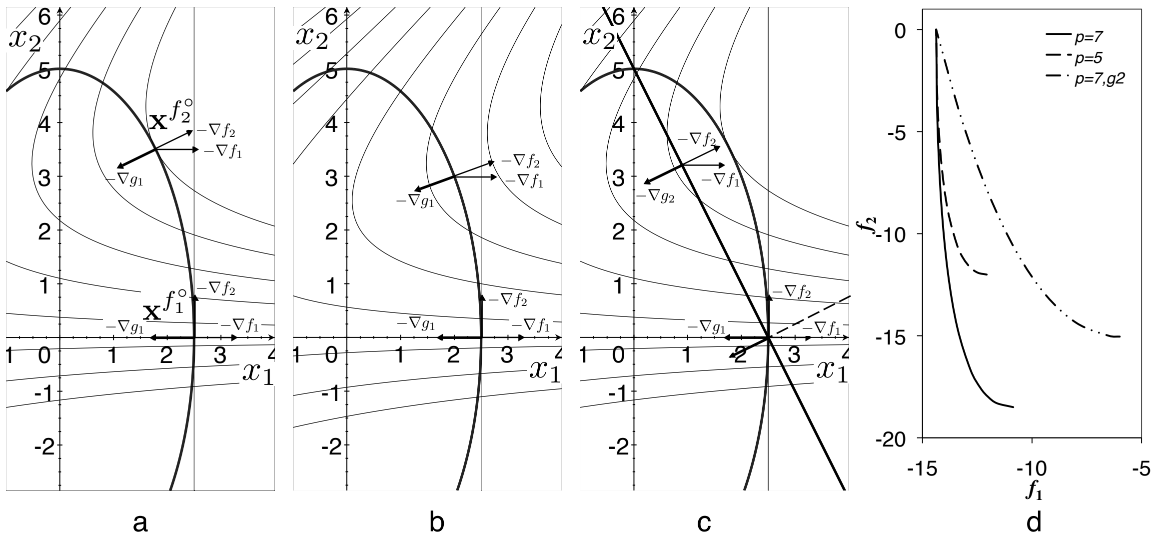


Figure 7.5: Graphical representation of problem given in Equation (7.18) and modifications from Table 7.1. Panel (a) shows the unmodified problem. Panel (b) shows the case where $p = 5$. Panel (c) shows the case where the constraint $x_2 - 5 + 2x_1 \leq 0$ has been added

ble 7.10. The results have been categorized by their reflection on the Pareto set. Performance refers to the placement of the Pareto set in the objective space. When one Pareto set dominates another Pareto set it is said to have improved performance. Sensitivity refers to the the slope of the Pareto frontier. Evaluated locally, sensitivity is defined as df_2^*/df_1^*. Evaluated over the entire Pareto frontier it represents the cost in one objective to achieve the ideal value for the other objective. Parity refers to the similarity in the decision to be made in order to minimize each objective f_1 and f_2 singly. One measure of parity is the measure of coincidence between ideal values $(||(\mathbf{x}^{f_1^\circ} - \mathbf{x}^{f_2^\circ})||_2)$. Complete parity requires that the decision maker chooses the identical decision in order to minimize both objectives, in other words, the objectives are coincident. We use the polar cone angle of the negative objective gradients $\theta^<$ evaluated at an ideal point $\mathbf{x}^{f_1^\circ}$ as an alternative measure of parity given that objectives with a polar cone angle $= \pi$ will be coincident. A degenerate case of complete parity is when the objectives are independent.

Examining Table 7.10, case (a) dominates the other cases and so is superior in the performance criterion. However, if parity is important, then case (b) would be superior. For reducing sensitivity of f_2 with respect to f_1 case (c) would be superior.

Table 7.10: Pareto Set Analysis Results

Criterion		Case	a	b	c
Performance	Dominance		$a \geqslant b$, $a \geqslant c$	-	-
Sensitivity	Local	f_1°	und.	und.	und.
		f_2°	0	0	0
	Global (f_2/f_1)		-5.26	-5.21	-1.80
	$\mathbf{\Gamma_v}$	f_1°	und.	und.	und.
		f_2°	0	0	0
	ϕ		0	0	0.29
Parity	Coincidence		3.59	3.00	3.54
	Polar cone angle	f_1°	$\pi/2$	$\pi/2$	$\pi/2$
		f_2°	2.68	2.79	2.68

It should be noted that objective alignment on the Pareto set as measured by the polar cone of the objective gradients is different from the general notion of objective alignment that is perhaps best characterized by the measure of coincidence. In fact high polar cone angles along the Pareto frontier could be an indication of an undesirable problem formulation in some design scenarios. For example, in an unconstrained bi-objective problem, by definition the polar cone angle will be 0 at all points along the Pareto set. High polar cone angles along the Pareto set may imply that the feasible design space is far away from both unconstrained objective optima.

Increased parity may similarly result from constraint tightening (either through parameter changes or adding constraints). Both polar cone angle increases and lower coincidence distance may also be achieved through reformulation of the objective functions.

The notion of sensitivity is particularly useful for problems characterized by a primary objective and a secondary objective. For example in a product design problem with a market system objective such as maximize profit, a producer will primarily be concerned with satisfying the profit objective although the producer may also be concerned with other objectives such as environmental impact for strategic or other reasons. In this case it would be valuable to assess the local sensitivity, or incremental cost to the profit objective for decreases in environmental impact. Decreased sensitivity may result from a decrease in the coupling term due to constraint reformulation, objective reformulation, parameter/variable repartitioning, or modification to parameter values.

7.5 Discussion

Systematically studying changes to the Pareto set due to changes to the multiobjective formulation can yield deeper insights into the system-level design problem. We motivate Pareto Set analysis, or comparing Pareto sets from alternative design scenarios by abstracting the design problem to a decision making problem that involves system specification and system identification. Changes to the design problem at the specification or identification stages give rise to unique design scenerios with an accompanying unique Pareto set in the case of multiobjective problems. The general Pareto analysis approach involves selecting several Pareto efficient points for analysis. At a minimum, the two ideal points should be selected. These points can be evaluated for each change in problem formulation such as a change in the functional form of the objective function or a repartitioning of variables and parameters. Numerical measures were then presented that describe each Pareto set in terms of its performance or dominance over other problem formulations, sensitivity of one objective to changes in the other objective, and the parity faced by the decision maker in the objective space and in the decision space when considering the difference between the single-objective solutions.

The desirability of a given Pareto set, or problem formulation, over another should be dictated by the design context. There are numerous design contexts where the designer is concerned with the attainable solution set and not only a point design solution. Such problems include the nested co-design problem described here and a wide variety of problems where the “control,” or partitioned variable design decisions, may be made asynchronously to the other design decisions. The performance, sensitivity, and parity attributes of a particular design problem represent a multi-objective problem of their own.

Plots of Pareto sets such as Figure 7.5 are typical and very useful analysis tools for bi-objective problems. However, as computational expense increases, it is not always feasible to generate a suitable graphical representation of the Pareto set. The methods presented in this chapter provide a series of concrete steps to make the most out of a small number of analysis runs by connecting the local behavior of a bi-objective problem to the characteristics of the Pareto set. Future work may seek to extend the numerical measures described here to higher-dimension problems where graphical representation is similarly problematic.

7.6 Summary

We presented results using the three global metrics to measure bi-objective tradeoffs in the vehicle design problem. We illustrated how the metrics could be used to discuss differences across design problem and market simulation problem scenarios of the type formulated in Chapter 6.

We presented additional local measures that can be evaluated at individual Pareto points and illustrated their application with a nonlinear programming example. Borrowing from traditional system design from the design and control literature, we have shown how the particular case of combined optimal design and optimal control, or co-design, can be represented as a system design problem or alternatively as a bi-objective programming problem with an artifact objective and a controller objective.

The coupling vector derived for a system problem with unidirectional coupling was shown to be related to the alignment of competing objectives, as measured by the polar cone of objective gradients, in the bi-objective programming formulation. We also showed how the measure of constraint decoupling can be normalized when the system problem is considered as a bi-objective one.

Chapter 8 illustrates the optimization results for numerous scenarios. The Pareto sets are plotted for two different sets of scenarios. Following the approach illustrated in Section 7.4.1 for the simplified automotive vehicle design example, the metrics described in this chapter could be applied to the results in Chapter 8 to gain further insight into the public versus private good tradeoff.

CHAPTER VIII

Public Versus Private Tradeoff Studies in Vehicle Design

The first goal of this chapter is to examine in practice the modeling framework that has been proposed in the previous chapters including comparing differences between different demand models and different cost models. The second goal of the chapter is to examine the public versus private automotive vehicle design tradeoff problem in the context of the results.

Vehicle manufacturers typically update and redesign the vehicles in their portfolio over several years. Improving the fuel economy of the next vehicle scheduled for introduction or redesign is one way a firm could consider reducing the environmental impact of its fleet. The public versus private tradeoff problem is therefore posed as the fuel consumption rating of the designed vehicle versus the expected firm profit.

Chapter 7 presented a simplified example based on studying a single vehicle class in isolation. That study relied on the scaling cost model from Chapter 5 and a simple logit model of demand from the literature [*Boyd and Mellman* (1980)]. This chapter goes beyond that example by implementing the GTDI and HEV technology models from Chapter 3, the two newly estimated demand models from Chapter 4 as well as a mixed logit model from the literature [*Berry et al.* (1995)], the empirical cost model and equilibrium cost model from Chapter 5, and the full US fleet price equilibrium as a subgame to the design problem as described in Chapter 6. The succeeding sections present results from a range of design scenarios. Each design scenario is defined by the number and name of the firms that are designing vehicles, the powertrain technology of each vehicle, the demand model employed, the number of random coefficient draws for each individual, and the cost model employed. For simplicity and consistency the same 6563 individuals used to estimate the demand models from Chapter 4 were used to execute the market simulations in this chapter except where noted. All scenarios generate the optimal vehicle design of 1 to 3 midsize crossover vehicles. Each scenario is solved for the two ideal points and three intermediate Pareto points of the firm profit versus fuel consumption rating tradeoff.

The outcomes of interest for each scenario are the values of the design variables, the vehicle attributes including price, the designed vehicle sales volume, the designing firm's profit, and measures of the environmental impact of the scenario. The measures of environmental impact include fuel consumption rating of the designed vehicle, expected annual fuel consumption from sales of the designed vehicle, the designing firm's projected fuel consumption from all vehicle sales, the expected midsize crossover vehicle segment fuel consumption based on all midsize crossover vehicles sold, and the expected US fleet fuel consumption based on all vehicles sold. Notably, the annualized fuel consumption measures for the US fleet do not always fall with reduced fuel consumption ratings of the designed vehicle illustrating that the public versus private decision problem facing firms is more complex than simply improving the fuel consumption of a single vehicle in their fleet.

The total market size M was fixed for all scenarios at 16,109,855 vehicles. This is approximately the number of vehicle sales in 2006. While actual firm names are referenced (Hyundai, Toyota, GM), it is noted that the design scenario results should not be interpreted in the context of the 2006 market. As discussed in Chapters 5 and 6, the lack of alternative specific constants in the demand models means that vehicle share predictions do not match the 2006 market. Therefore, policy interpretations including optimal design decisions should not be made with respect to the real market.

Section 1 develops the problem formulation used in each scenario. Section 2 presents the results for scenarios where a single firm designed a vehicle with a conventional powertrain. Section 3 presents the results for scenarios where a single firm designed a vehicle with a GTDI powertrain. Section 4 presents the results for scenarios where a single firm designed a vehicle with a hybrid powertrain. Section 5 presents results for scenarios where three firms designed vehicles of varying powertrains. Section 6 compares the results across demand, cost, and powertrain models. Section 7 discusses the public versus private tradeoff implications from the results. Section 8 summarizes the chapter including points for future work.

8.1 Problem Formulation

The engineering models including the design variables are the same as described in Chapter 3. The constraint set is the same as given as Equation (3.2) with the addition of a constraint:

$$g_{15} = z_{MPG} - v_{MMPG} \leq 0 \tag{8.1}$$

on fuel conusmption used to calculate Pareto points to the following problem using the ϵ-constraint method [*Osyczka* (1984)]:

$$\min \mathbf{f} \mid \mathbf{f} = [\pi_f, z_{MPG}] \tag{8.2}$$

by varying the minimum fuel economy constraint parameter v_{MMPG} between the nadir and ideal values of fuel economy f_{MPG}^{N} and f_{MPG}°. Table 7.2 gives a description of the fuel economy constraint.

The profit objective π_f is the firm profit. This means that the firm takes into account the price and profitability of its entire fleet when setting design variable values for the designed vehicle. Additionally, profits are calculated based on an assumed markup by the dealer. In other words, firm profits are determined by taking the product of a vehicle's sales and the margin between the firm cost and the price paid by the dealer (i.e., not the consumer purchase price $MSRP$). The price paid by the dealer is assumed to be the $MSRP$ less a percentage of the $MSRP$ assigned to be the percentage difference between the dealer invoice price and the $MSRP$ as given in the Chrome Systems database [*Chrome Systems Inc.* (2008)]. Consumer demand is determined based on the $MSRP$. However, the price equilibrium subgame is executed for firm profit and is therefore based on dealer price rather than $MSRP$.

Vehicle simulations were configured to represent a standard automatic transmission front wheel drive vehicle. In addition to powertrain specifications (i.e., gear ratios, gear shifting schedule, engine number of cylinders, vee or inline configuration, bore, and stroke, valvetrain configuration, and final drive ratio) the engineering design model as described in Chapter 3 also receives other vehicle parameters as inputs, including curb weight, frontal area, drag coefficient, tire radius, and center of gravity location under various loads. Over 30 parameters were tuned for midsize crossover vehicles based on data from one 2007 model.

As described in Chapter 6, computations proceeded with designing firms maximizing profit until based on the specific market simulation, the convergence tolerance on changes in design attributes was met. All scenarios were executed as two-stage game formulations where the vehicle prices for the entire market were solved as a subgame to the overall vehicle design problem. These formulations are described in Section 6.1.2.

Unless otherwise noted the population for the BLP95 model was generated from 1000 random draws from a lognormal distribution representing income. The population for Model 1 and Model 2 was generated from the identical 6563 individual survey sample used in the model estimations. One draw for the random coefficients was taken for each individual. The lower bounds on the scaled engine bore variable were lowered from -1 to -1.5 for the conventional model, from -1 to -2 for the GTDI and HEV model, and the upper bound on engine bore was raised from 1 to 2 for the HEV model. This was done because it was noticed that the engine bore variables were consistently hitting their bounds. Relaxing the constraints allowed the final drive ratio and engine bore to stroke ratios to adjust to more common values. The resulting optimal engine displacements and horsepower ratings with unconstrained engine bore values were in the range of midsize crossovers in the market in 2006 although lower than the 2007 Ford Edge after which the conventional vehicle model was originally patterned.

Tolerances for the price equilibrium game for Model 1 and Model 2 were 1e-11 for minimum choice share to be included in calculation P_{tol}, 1e-8 for the maximum combined gradient norm G_{tol}, and 1e-8 for the minimum improving step size r_{stol}. Tolerances for the price equilibrium game for the BLP95 model were the same for the minimum choice share and 1e-12 for G_{tol}, and 1e-15 for r_{stol}. The price equilibrium calculation consistently converged according to the combined gradient norm criterion as defined in Equation (6.6).

The convergence tolerance for the design game z_{tol} was 1e-3. Attribute differences were normalized over the attribute value from the preceding iteration. This means that a z_{tol} =1e-3 required that no vehicle attribute deviated by more than 1/10th of 1 percent from the previous iteration values.

The optimization algorithm implemented for the design problem was a version of sequential quadratic programming implemented in Matlab under the optimization function 'fmincon'. Gradients were computed using finite differencing with the forward difference method. The maximum difference was set to 1e-1. The minimum difference was set to 1e-2. The tolerance on the design variables was set to 1e-6. The tolerance on the objective function was set to 1e+2. The tolerance on the constraints was set to 1e-5. The objective function values represented dollar values and were typically on the order of 1e+8–1e+9. The larger than typical tolerance on the objective function is therefore an artifact of objective scale. The optimization algorithm would terminate when changes in the objective function were on the order of 1e+1 to 1e+3 dollars. This behavior was deemed suitable given the noise from the price equilibrium game and the discontinuities introduced by computing the weighted sum of the choice shares from each individual for each vehicle alternative for changes in the vehicle design.

8.2 Single Designed Vehicle with Conventional Powertrain

Results are presented in this section for various design scenarios where a single firm has been selected as the designing firm and a single midsize crossover vehicle with a conventional gasoline engine has been selected as the designed vehicle. The subsections are organized in order to facilitate comparison between problem formulation differences in the design scenarios. Design scenarios are compared across demand models and cost models (Section 8.2.1), designing firms, the number of random coefficient draws per individual in the simulated population, and changes in consumer preference for fuel consumption (Section 8.2.2). We report the values of the design variables, the vehicle attributes including price, the designed vehicle sales volume, and the designing firm's profit for each scenario. We present the Pareto sets for the profit versus fuel consumption tradeoff for an representative scenario in the discussion in Section 8.2.3 . Comparisons of the other environmental impact metrics are presented in Section 8.7.

8.2.1 Comparing Results across Demand and Cost Models

Table 8.1 presents the design variable values for design scenarios with a single designed vehicle and a single designing firm (Hyundai). Table 8.2 presents the vehicle attributes for design scenarios with a single designed vehicle and a single designing firm (Hyundai), and Table 8.3 presents the simulated market outcomes. Each of the scenarios reported is a maximum profit scenario. In other words, there was no constraint set on vehicle fuel consumption. The table rows are labeled by demand model (BLP95, Model 1, Model 2) and by cost model (Emp, Eq). Examining the design variables in Table 8.1 shows that there are large differences between the optimal vehicle designs under the BLP95 model versus the new models. There are no significant differences in the optimal vehicle designs between Model 1 and Model 2, and there are small differences in the designs between the empirical and the equilibrium cost model.

The BLP95 vehicle has a large engine and small body compared to the Model 1 and Model 2 vehicles leading to a lower fuel economy rating and higher power to weight ratio as shown in Table 8.2. The design differences between the empirical and equilibrium cost models for Model 1 and Model 2 led to small differences in the engine displacement, the power to weight ratio, and other performance metrics.

Referencing the market outcomes found in Table 8.3 shows that the BLP95 model suggests a low price and achieves very low sales volume compared with the Model 1 and Model 2 results with the same empirical cost model. The empirical cost model for Model 1 and Model 2 projects higher prices, sales volumes, and profits compared to the equilibrium cost model. Model 2 consistently prices lower than Model 1 and achieves higher sales volume and higher profits on the designed model. However, Model 1 projects higher overall firm profits than Model 2. The BLP95 demand model favors Hyundai in terms of total sales, and the Model 1 demand model with the empirical cost model projects the highest firm profits.

Table 8.1: Maximum profit design variable values for BLP95, Model 1, and Model 2 with conventional engine technology and empirical cost model (Emp) or equilibrium cost model (Eq)

Model	x_B	x_{FD}	x_{L103}	x_{H101}	x_{W105}	x_{BtS}	x_{L101}
BLP95, Emp	100.0	3.65	4714	1704	1883	1.18	2870
Model 1, Emp	84.6	3.55	5008	1704	2000	1.02	3048
Model 2, Emp	84.6	3.56	5008	1704	2000	1.02	3048
Model 1, Eq	84.5	3.49	5008	1704	2000	1.00	3048
Model 2, Eq	84.5	3.49	5008	1704	2000	1.00	3048

Table 8.2: Maximum profit vehicle attributes for BLP95, Model 1, and Model 2 with conventional engine technology and empirical cost model (Emp) or equilibrium cost model (Eq)

Model	z_{MPG}	$10z_{hp}/z_{VM}$	z_Lz_W	z_{hp}	z_{VM}	z_{Edisp}	z_{060}	z_{3050}	z_{65T}	$z_{MPG_{city}}$	$z_{MPG_{hwy}}$	z_{CVI}	z_{TS}
BLP95, Emp	18.4	0.79	1.38	310	3912	3.99	6.67	4.15	10.8	16.6	21.1	29	115
Model 1, Emp	21.7	0.53	1.55	222	4210	2.80	9.30	6.61	5.0	20.4	23.5	46	115
Model 2, Emp	21.7	0.53	1.55	222	4210	2.80	9.31	6.61	5.0	20.4	23.5	46	115
Model 1, Eq	21.7	0.52	1.55	221	4218	2.84	9.22	6.65	5.0	20.4	23.5	46	115
Model 2, Eq	21.7	0.52	1.55	221	4218	2.84	9.22	6.65	5.0	20.4	23.5	46	115

Table 8.3: Maximum profit market performance for BLP95, Model 1, and Model 2 with conventional engine technology and empirical cost model (Emp) or equilibrium cost model (Eq)

Model	p_j	$P_j \times M$	$\pi_{j,f}$	$M\sum_{j=1}^{J_f} P_{j,f}$	π_f
BLP95, Emp	31,034	14,240	115,241,000	492,846	5,193,009,000
Model 1, Emp	48,042	84,760	2,062,255,000	438,927	7,664,210,000
Model 2, Emp	44,515	125,352	2,607,755,000	448,411	6,641,492,000
Model 1, Eq	29,055	65,729	1,587,701,000	306,614	5,840,329,000
Model 2, Eq	27,117	105,440	2,000,417,000	332,894	5,189,909,000

8.2.2 Comparing Results across Firms, Fuel Economy Preference, and Draws

Table 8.4 presents the design variable values for design scenarios with a single designed vehicle and a single designing firm. All scenarios employ the Model 2 demand model and the equilibrium cost model. However, the first three rows represent three different designing firms (Hyundai, Toyota, or General Motors). The fourth row represents a case where the mean of the random coefficient for fuel consumption in the demand model has been doubled from -0.8553 to -1.7106. The fifth row presents results for the case that the number of draws per individual from the distribution of random coefficients was 40 rather than 1. Table 8.5 presents the vehicle attributes for the same design scenarios as Table 8.4 and Table 8.6 presents the simulated market outcomes for the same scenarios. Each of the scenarios reported is a maximum profit scenario. In other words, there was no constraint set on vehicle fuel consumption. The table rows are labeled by designing firm (Hyundai,Toyota, General Motors), the number of draws (1, 40), and row 4 of each table indicates that fuel consumption preference has been doubled (2xMPG).

Tables 8.7-8.9 present additional results for design scenarios with differing number of draws using Model 1 demand and the equilibrium cost model. In this case a subset of only 500 individuals is used from the full 6563 individuals from the estimation population. The smaller population allowed a higher number of draws to be tested, in this case 128, without running into computer memory constraints. The top row presents results from a scenario with 1 draw for each individual from the distribution of random coefficients, and the bottom row presents the results from a scenario with 128 draws. Examining the design variables in Table 8.4 shows that there are no differences in design variables across designing firms, fuel economy preference, and number of draws except for very small differences in the final drive ratio for different designing firms and very small differences in final drive ratio and engine bore to stroke

Table 8.4: Maximum profit design variable values for equilibrium cost models with conventional engine technology for Model 2 demand model comparing different designing firms, fuel economy preference, and number of random-coefficient draws

Model	x_B	x_{FD}	x_{L103}	x_{H101}	x_{W105}	x_{BtS}	x_{L101}
Hyundai, 1 D	84.5	3.49	5008	1704	2000	1.00	3048
Toyota, 1 D	84.5	3.48	5008	1704	2000	1.00	3048
GM, 1 D	84.5	3.47	5008	1704	2000	1.00	3048
Hyundai, 1 D, 2xMPG	84.5	3.51	5008	1704	2000	1.01	3048
Hyundai, 40 D	84.5	3.49	5008	1704	2000	1.00	3048

Table 8.5: Maximum profit vehicle attributes for equilibrium cost models with conventional engine technology for Model 2 demand model comparing different designing firms, fuel economy preference, and number of random-coefficient draws

Model	z_{MPG}	$10z_{hp}/z_{VM}$	$z_L z_W$	z_{hp}	z_{VM}	z_{Edisp}	z_{060}	z_{3050}	z_{65T}	$z_{MPG_{city}}$	$z_{MPG_{hwy}}$	z_{CVI}	z_{TS}
Hyundai, 1 D	21.7	0.52	1.55	221	4218	2.84	9.22	6.65	5.0	20.4	23.5	46	115
Toyota, 1 D	21.7	0.52	1.55	221	4219	2.84	9.21	6.66	5.0	20.4	23.5	46	115
GM, 1 D	21.7	0.52	1.55	221	4221	2.85	9.19	6.67	5.0	20.4	23.5	46	115
Hyundai, 1 D, 2xMPG	21.7	0.52	1.55	221	4216	2.83	9.24	6.64	5.0	20.4	23.5	46	115
Hyundai, 40 D	21.7	0.52	1.55	221	4218	2.84	9.22	6.65	5.0	20.4	23.5	46	115

Table 8.6: Maximum profit market performance for equilibrium cost models with conventional engine technology for Model 2 demand model comparing different designing firms, fuel economy preference, and number of random-coefficient draws

Model	p_j	$P_j \times M$	$\pi_{j,f}$	$M \sum_{j=1}^{J_f} P_{j,f}$	π_f
Hyundai, 1 D	27,117	105,440	2,000,417,000	332,894	5,189,909,000
Toyota, 1 D	27,797	205,513	4,756,808,000	242,953	3,550,121,000
GM, 1 D	23,431	153,538	3,792,056,000	246,127	3,636,304,000
Hyundai, 1 D, 2xMPG	30,208	113,766	2,508,341,000	386,185	6,366,299,000
Hyundai, 40 D	27,264	101,497	1,940,547,000	330,458	5,171,044,000

ratio in the increased preference for fuel economy scenarios. These differences were not large enough to change the fuel economy and horsepower to weight attributes at the reported precision in Table 8.5.

The Hyundai and Toyota vehicles report similar prices in Table 8.6. However, the Toyota vehicle has double the sales volume. The GM vehicle has the lowest price and a sales volume in between the Hyundai and Toyota vehicle. Although the case with the increased fuel economy preference did not lead to design changes, the market outcome resulted in simultaneously higher price and higher sales volume for the Hyundai vehicle. There are small differences in price, sales, and firm profit between the 1 Draw and the 40 Draw case. Sales volume and profit at the designed vehicle and firm level all declined in the 40 Draw case.

Examining the design variables in Table 8.7 shows that there are no differences in design variables across the number of random coefficient draws. There are noticeable difference in the projected market outcomes just as there were market differences between the 1 Draw case and the 40 Draw case with the 6563 population in Table 8.6. In this case increasing the number of draws led to a lower designed vehicle price and lower sales volume.

8.2.3 Discussion

The primary finding from the results in this section is the similarity in design solutions across all scenarios with the exception of the BLP95 scenarios. The design solutions are all highly constrained. The constraints come from the specific requirements to stay within the midsize crossover vehicle class in terms of size and towing capacity. The range of size for a midsize crossover vehicle in terms of vehicle footprint is very similar to a midsize sedan. The average consumer according to Model 1 prefers larger vehicles, and the average consumer's ideal footprint according to Model 2 is larger than a midsize sedan. The result is that both demand models reward increases

Table 8.7: Maximum profit market performance for 1 Draw per individual and 128 Draws per individual with conventional engine technology, equilibrium cost model, and Model 2 demand model; Population 500

Model	x_B	x_{FD}	x_{L103}	x_{H101}	x_{W105}	x_{BtS}	x_{L101}
1 Draw	84.5	3.49	5008	1704	2000	1.00	3048
128 Draws	84.5	3.49	5008	1704	2000	1.00	3048

Table 8.8: Maximum profit vehicle attributes for 1 Draw per individual and 128 Draws per individual with conventional engine technology, equilibrium cost model, and Model 2 demand model; Population 500

Model	z_{MPG}	$10z_{hp}/z_{VM}$	$z_L z_W$	z_{hp}	z_{VM}	z_{Edisp}	z_{060}	z_{3050}	z_{65T}	$z_{MPG_{city}}$	$z_{MPG_{hwy}}$	z_{CVI}	z_{TS}
1 Draw	21.7	0.52	1.55	221	4218	2.84	9.22	6.65	5.0	20.4	23.5	46	115
128 Draws	21.7	0.52	1.55	221	4219	2.84	9.21	6.66	5.0	20.4	23.5	46	115

Table 8.9: Maximum profit market performance for 1 Draw per individual and 128 Draws per individual with conventional engine technology, equilibrium cost model, and Model 2 demand model; Population 500

Model	p_j	$P_j \times M$	$\pi_{j,f}$	$M \sum_{j=1}^{J_f} P_{j,f}$	π_f
1 Draw	27,079	102,974	1,949,723,000	329,470	5,146,405,000
128 Draws	25,876	74,022	1,312,750,000	304,214	4,583,679,000

in size over the allowable range of size for a midsize crossover vehicle. The market simulations which then take into account the change in demand for the change in cost to produce a larger vehicle all suggested that firms should make the largest vehicle possible. In this case the vehicle size was limited by a practical constraint on the wheelbase design variable, a practical constraint on the vehicle width, and an SAE specification on departure angle that prohibits the tail end of the vehicle protruding too far from the rear wheels.

The other result that was common across the scenarios was that each scenario exhibited the same tradeoff solution between power to weight ratio and fuel consumption. This may have resulted because the degrees of freedom of the powertrain variables were taken away by the gradeability constraint and the top speed constraint. Again it appears that for Model 1 and Model 2 high power to weight ratios were not valued as highly as a larger, more fuel efficient, and less expensive vehicles. Anecdotally, Ford introduced a midsize crossover vehicle to compete in the segment focused on in this study in 2007. Then, they introduced a larger slower crossover in 2009. The trend across the crossover segment has been to lengthen the vehicle with even some small crossovers such as the Toyota RAV4 achieving third row seating.

The designed vehicle according to the BLP95 model in Section 8.2.1 projects lower sales than the other demand models because consumers according to the BLP95 model are more price sensitive. Sales are then weighted to the very least expensive vehicles. The design constraints on size and towing capability inhibit the designed vehicle from becoming smaller and less costly to build, and so it is not competitive with the least expensive vehicles in the market. Additionally, the BLP95 model scenario predicts higher total sales for Hyundai than in the other scenarios because Hyundai has several small inexpensive models on the market.

The scenarios that combine the empirical or equilibrium cost models with the Model 1 or Model 2 demand model result in grossly inflated profits compared to the real market. The combination of demand and cost model result in per vehicle profit margins that are very large. We conjecture that introducing alternative specific constants would resolve this issue by increasing sales volume of less expensive cars and reducing sales volume of luxury models. The overall effect would be downward pressure on prices in the case of the empirical cost model and higher equilibrium costs in the case of the equilibrium cost model.

Model 2 marginally benefits smaller less expensive cars compared to Model 1 as seen in the predicted market shares plot in Figure 4.3. Shares and designed vehicle profit are therefore higher for Model 2. However, total firm profits are higher for Model 1. The higher firm profits for Model 1 versus Model 2 could be some evidence that a demand model that resolved the mismatch between actual and predicted sales would resolve the inflated profit margins.

The small design differences that were observed between designing firms in Section 8.2.2 may mean that different firms are motivated to design different vehicles based on their specific context. Whether these differences are driven by differences in brand value or by a firm's existing portfolio or both should be a topic of future work. For the given problem formulation these details may be difficult to determine because the design decisions are in a highly constrained space. These points of exploration are in addition to exploring the differences that emerge when firms have different technology and cost capabilities.

For the case of Hyundai as the designing firm it appears that increasing the number of draws decreases sales volume and profit. More runs can be taken in future work to examine wether this result is stable and if it holds for other firms or designed vehicle segments.

Figure 8.1 shows the public versus private tradeoff with various measures for the public tradeoff for the design scenario from Tables 8.1-8.3. Figure 8.1(a) shows the designed vehicle fuel consumption rating in gallons per 100 miles plotted versus the firm's profit[1]. As the designed vehicle is constrained to improve fuel economy beyond the maximum profit design firm profit decreases. While the initial thought was to use the fuel consumption rating of the designed vehicle to represent the public interest (Figure 8.1(a)), the market simulation results indicate that the aggregate fuel consumption of the US fleet can actually increase rather than decrease when the designed vehicle fuel consumption is reduced. This is the case for the design scenarios with the Model 2 demand model and either the empirical or the equilibrium cost model.

Figure 8.1(b) shows how the sales-weighted fuel consumption rating for a given firm (Hyundai) changes with change in the designed vehicle. The fuel consumption rating decreases with decreasing designed-vehicle fuel-consumption for all scenarios. The firm sales-weighted fuel consumption rating for the BLP95 scenario is much lower than the other scenarios. This is again indicative of the sales mix under the BLP95 scenario that is heavily weighted towards less expensive vehicles with higher fuel economies.

Figure 8.1(c) shows the fuel consumption of the midsize crossover segment. The reporting is given in average-vehicle-mile-traveled gallons. What this means is that we can calculate a projected annual fuel consumption of the new midsize crossover vehicle sales by assigning an average vehicle-miles-traveled to the segment car buyers. For example, if the average-vehicle-miles-traveled per new car buyer were 12,000 miles/year and the segment per average-vehicle-mile-traveled fuel consumption was 16,000 gallons, then the projected annual fuel consumption of the new midsize crossover sales would be $12,000 \times 16,000 = 192 \times 10^6$ gallons of gasoline.

We can also calculate an average-vehicle-mile-traveled fuel consumption for the entire US fleet. These results for each scenario are presented in Figure 8.1(d). The increasing aggregate fuel consumption in the Model 2 scenarios resulting from de-

[1]Here profit is listed with negative values as a convention so that the improving direction is toward the bottom left corner of the plot

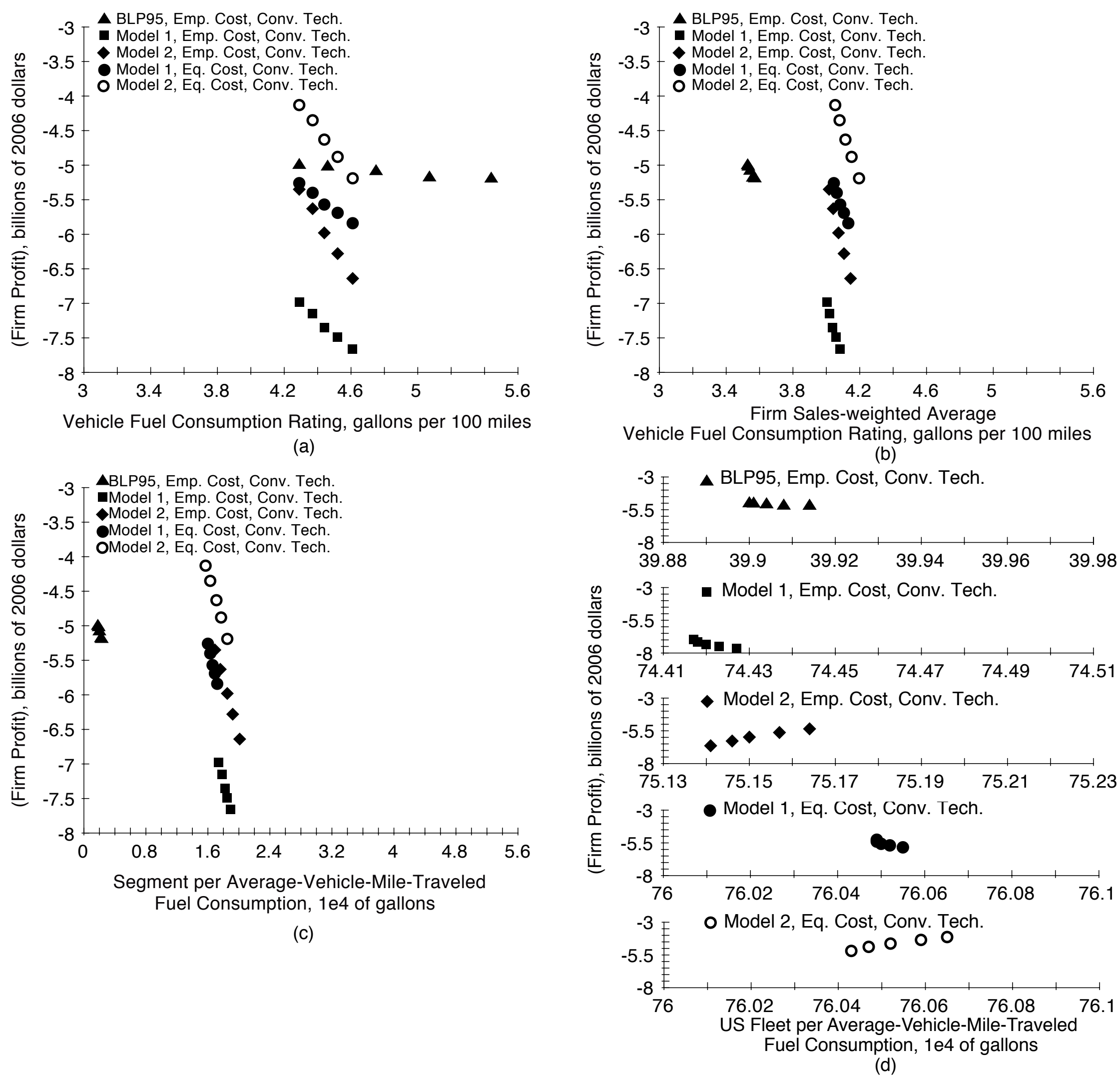


Figure 8.1: Pareto results for a single conventional designed vehicle with Model 2 demand model and equilibrium cost model (a) Vehicle fuel consumption rating in gallons per 100 miles versus negative firm profit; (b) Firm sales-weighted vehicle fuel consumption rating in gallons per 100 miles versus negative firm profit; (c) Segment per average-vehicle-mile-traveled fuel consumption in 10,000's of gallons versus negative firm profit; (d) US fleet per average-vehicle-mile-traveled fuel consumption in 10,000's of gallons versus negative firm profit;

creasing the fuel consumption for the designed vehicle beyond the maximum profit value indicates that potential buyers substitute away from the designed vehicle to a vehicle that on average is less fuel efficient. In the other scenarios they substitute on average to a vehicle that is more fuel efficient.

8.3 Single Designed Vehicle with GTDI Powertrain

Results are presented in this section for various design scenarios where a single firm has been selected as the designing firm and a single midsize crossover vehicle with a GTDI gasoline engine has been selected as the designed vehicle. The subsections are organized in order to facilitate comparison between problem formulation differences in the design scenarios. Design scenarios are compared across demand and cost models (Section 8.3.1) and across designing firms (Section 8.3.2). We report the values of the design variables, the vehicle attributes including price, the designed vehicle sales volume, and the designing firm's profit for each scenario. We present the Pareto sets for the profit versus fuel consumption tradeoff for each of the different designing firms scenarios in the discussion in Section 8.3.3. Comparisons of the other environmental impact metrics are presented in Section 8.7

8.3.1 Comparing Results across Demand and Cost Models

Table 8.10 presents the design variable values for design scenarios with a single designed vehicle and a single designing firm (Hyundai). Table 8.11 presents the vehicle attributes for design scenarios with a single designed vehicle and a single designing firm (Hyundai), and Table 8.12 presents the simulated market outcomes. Each of the scenarios reported is a maximum profit scenario. In other words, there was no constraint set on vehicle fuel consumption. The table rows are labeled by demand model (BLP95, Model 1, Model 2) and by cost model (Emp, Eq). Examining the design variables in Table 8.10 shows that there are large differences between the optimal vehicle designs under the BLP95 model versus the new models. There are small differences in the optimal vehicle designs between Model 1 and Model 2 and between the empirical and equilibrium cost models.

The BLP95 vehicle has a large engine and small body compared to the Model 1 and Model 2 vehicles similar to the conventional technology case in Section 8.2.1

Table 8.10: Maximum profit design variable values for BLP95, Model 1, and Model 2 with GTDI technology and empirical cost model (Emp) or equilibrium cost model (Eq)

Model	x_B	x_{FD}	x_{L103}	x_{H101}	x_{W105}	x_{BtS}	x_{L101}
BLP95, Emp	92.0	2.50	4711	1704	1890	1.18	2825
Model 1, Emp	76.0	3.61	5008	1704	1982	1.07	3048
Model 2, Emp	75.4	3.63	5008	1704	1982	1.05	3048
Model 1, Eq	75.5	3.55	5008	1704	1982	1.03	3048
Model 2, Eq	76.0	3.59	5008	1704	1982	1.06	3048

Table 8.11: Maximum profit vehicle attributes for BLP95, Model 1, and Model 2 with GTDI technology and empirical cost model (Emp) or equilibrium cost model (Eq)

Model	z_{MPG}	$10z_{hp}/z_{VM}$	$z_L z_W$	z_{hp}	z_{VM}	z_{Edisp}	z_{060}	z_{3050}	z_{65T}	$z_{MPG_{city}}$	$z_{MPG_{hwy}}$	z_{CVI}	z_{TS}
BLP95, Emp	24.3	0.86	1.38	304	3539	2.07	7.12	4.89	7.1	23.7	25.0	29	136
Model 1, Emp	24.8	0.54	1.54	207	3862	1.29	8.88	5.76	5.0	24.8	24.9	45	115
Model 2, Emp	24.8	0.53	1.54	204	3860	1.28	8.88	5.75	5.0	24.8	24.9	45	116
Model 1, Eq	24.8	0.53	1.54	205	3866	1.31	8.63	5.78	5.0	24.8	24.9	45	115
Model 2, Eq	24.8	0.54	1.54	207	3863	1.30	8.87	5.77	5.0	24.8	24.9	45	115

Table 8.12: Maximum profit market performance for BLP95, Model 1, and Model 2 with GTDI technology and empirical cost model (Emp) or equilibrium cost model (Eq)

Model	p_j	$P_j \times M$	$\pi_{j,f}$	$M \sum_{j=1}^{J_f} P_{j,f}$	π_f
BLP95, Emp	30,360	11,391	75,348,000	538,489	4,730,949,000
Model 1, Emp	51,020	121,240	3,235,017,000	471,844	8,674,810,000
Model 2, Emp	46,753	189,624	4,256,152,000	510,093	8,140,953,000
Model 1, Eq	19,719	177,863	3,651,427,000	407,409	7,727,542,000
Model 2, Eq	19,087	299,493	5,243,011,000	519,755	8,234,667,000

leading to a lower fuel economy rating and higher power to weight ratio as shown in Table 8.11. The design differences between the Model 1 and Model 2 scenarios led to small differences in the engine displacement, the power to weight ratio, and other performance metrics.

Referencing the market outcomes found in Table 8.12 shows that the BLP95 model suggests a low price and achieves very low sales volume compared with the Model 1 and Model 2 results with the same empirical cost model. The empirical cost model for Model 1 and Model 2 projects higher prices, but lower sales volumes and profits compared to the equilibrium cost model. Model 2 consistently prices lower than Model 1 and achieves higher sales volume and higher profits on the designed model. Model 2 also projects higher overall firm profits than Model 1 for the equilibrium cost model, but lower overall firm profits for the equilibrium model. The BLP95 demand model favors Hyundai in terms of total sales, and the Model 1 demand model with the empirical cost model again projects the highest firm profits similar to the conventional technology case.

8.3.2 Comparing Results across Designing Firms

Table 8.13 presents the design variable values for design scenarios with a single designed vehicle and a single designing firm. All scenarios employ the Model 1 demand model and the equilibrium cost model. However, the first three rows represent three different designing firms (Hyundai, Toyota, or General Motors). Table 8.14 presents the vehicle attributes for the same design scenarios as Table 8.13 and Table 8.15 presents the simulated market outcomes for the same scenarios. Each of the scenarios reported is a maximum profit scenario. In other words, there was no constraint set on vehicle fuel consumption. The table rows are labeled by designing firm (Hyundai, Toyota, General Motors). Examining the design variables in Table 8.13 shows that there are small differences in design variables across designing firms in the engine

Table 8.13: Maximum profit design variable values for equilibrium cost model and Model 1 demand model with GTDI technology comparing different designing firms

Model	x_B	x_{FD}	x_{L103}	x_{H101}	x_{W105}	x_{BtS}	x_{L101}
Hyundai	75.5	3.55	5008	1704	1982	1.03	3048
Toyota	75.5	3.53	5008	1704	1982	1.02	3048
GM	76.0	3.57	5008	1704	1982	1.06	3048

Table 8.14: Maximum profit vehicle attributes for equilibrium cost model and Model 1 demand model with GTDI technology comparing different designing firms

Model	z_{MPG}	$10z_{hp}/z_{VM}$	$z_L z_W$	z_{hp}	z_{VM}	z_{Edisp}	z_{060}	z_{3050}	z_{65T}	$z_{MPG_{city}}$	$z_{MPG_{hwy}}$	z_{CVI}	z_{TS}
Hyundai	24.8	0.53	1.54	205	3866	1.31	8.63	5.78	5.0	24.8	24.9	45	115
Toyota	24.8	0.53	1.54	205	3868	1.32	8.60	5.78	5.0	24.8	24.9	45	115
GM	24.8	0.54	1.54	207	3864	1.30	8.86	5.77	5.0	24.8	24.9	45	115

Table 8.15: Maximum profit market performance for equilibrium cost model and Model 1 demand model with GTDI technology comparing different designing firms

Model	p_j	$P_j \times M$	$\pi_{j,f}$	$M\sum_{j=1}^{J_f} P_{j,f}$	π_f
Hyundai	19,719	177,863	3,651,427,000	407,409	7,727,542,000
Toyota	19,551	390,685	9,589,568,000	252,351	4,505,983,000
GM	10,896	378,403	8,387,379,000	3,951,519	110,900,784,000

bore, final drive ratio, and engine bore to stroke ratios for different designing firms. These differences were not large enough to change the fuel economy, and only the GM scenario reports a different power to weight ratio at the reported precision in Table 8.14.

The market performance is very similar to the conventional technology case. Hyundai and Toyota vehicles report very similar prices in Table 8.15. However, the Toyota vehicle has more than double the sales volume. The GM vehicle has the lowest price and a sales volume in between the Hyundai and Toyota vehicle.

8.3.3 Discussion

powertrain difference more noticeable perhaps because technology able to achieve a higher fuel economy so closer to a saturated level with fuel economy

The GTDI technology allows the designed vehicle to achieve higher fuel economy while still meeting the gradeability and top speed constraint. The Model 1 and Model 2 demand models push the design in this way while maintaining the largest vehicle possible. The BLP95 scenario produces a smaller powerful vehicle similar to the conventional case. Overall the designs are again very similar across scenarios. One reason that powertrain differences are slightly more noticeable in the GTDI scenarios may be because fuel economy is higher. The marginal benefit of increasing fuel economy may be closer to the marginal benefit of increasing the power to weight ratio for example.

The design solutions again are all highly constrained. The constraints come from the specific requirements to stay within the midsize crossover vehicle class in terms of size and towing capacity. The Model 2, empirical cost model case does not strictly constrained by the top speed constraint. Both Model 1 and Model 2 reward increases in size over the allowable range of size for a midsize crossover vehicle. The market simulations which then take into account the change in demand for the change in cost to produce a larger vehicle all suggested that firms should make the largest vehicle possible. In this case the vehicle size was limited by a practical constraint on the wheelbase design variable, a practical constraint on the vehicle width, and an SAE specification on departure angle that prohibits the tail end of the vehicle protruding too far from the rear wheels.

The other result that was common across the scenarios was that each scenario exhibited a very similar tradeoff solution between power to weight ratio and fuel consumption. This may have resulted because the degrees of freedom of the powertrain variables were taken away by the gradeability constraint and the top speed constraint. Again it appears that for Model 1 and Model 2 high power to weight ratios were not valued as highly as a larger, more fuel efficient, and less expensive vehicles.

The designed vehicle according to the BLP95 model in Section 8.2.1 projects lower sales than the other demand models because consumers according to the BLP95 model are more price sensitive. Sales are then weighted to the very least expensive vehicles. The design constraints on size and towing capability inhibit the designed vehicle from becoming smaller and less costly to build, and so it is not competitive with the least expensive vehicles in the market. Additionally, the BLP95 model scenario predicts higher total sales for Hyundai than in the other scenarios because Hyundai has several small inexpensive models on the market. It appears that for the designed vehicle in the BLP95 scenario the maximum profit design was to differentiate as much as possible from the rest of the Hyundai portfolio and capture what was available of a very small market for high power to weight ratio.

Just as in the conventional technology case the scenarios that combine the empirical or equilibrium cost models with the Model 1 or Model 2 demand model result in grossly inflated profits compared to the real market. The combination of demand and cost model result in per vehicle profit margins that are very large. We conjecture that introducing alternative specific constants would resolve this issue by increasing sales volume of less expensive cars and reducing sales volume of luxury models. The overall effect would be downward pressure on prices in the case of the empirical cost model and higher equilibrium costs in the case of the equilibrium cost model.

Model 2 marginally benefits smaller less expensive cars compared to Model 1 as seen in the predicted market shares plot in Figure 4.3. Shares and designed vehicle profit are therefore higher for Model 2. However, total firm profits are split. In the equilibrium cost case, firm profits are higher for Model 2, and in the empirical cost case profits are higher for Model 1.

The small design differences that were observed between designing firms in Section 8.3.2 may mean that different firms are motivated to design different vehicles based on their specific context. Whether these differences are driven by differences in brand value or by a firm's existing portfolio or both should be a topic of future work. For the given problem formulation these details may be difficult to determine because the design decisions are in a highly constrained space. These points of exploration are in addition to exploring the differences that emerge when firms have different technology and cost capabilities.

Figure 8.2 shows the public versus private tradeoff with various measures for the public tradeoff for the design scenario from Tables 8.13-8.15. Figure 8.2(a) shows the designed vehicle fuel consumption rating in gallons per 100 miles plotted versus the firm's profit[2]. As the designed vehicle is constrained to improve fuel economy beyond the maximum profit design, firm profit decreases. While the initial thought was to use the fuel consumption rating of the designed vehicle to represent the public interest (Figure 8.2(a)), the market simulation results indicate that the aggregate fuel consumption of the US fleet can actually increase rather than decrease when the designed vehicle fuel consumption is reduced. This is the case for all of the design

[2]Here profit is listed as a change from the profit at the maximum profit case. This allows data for all three firms to be presented on the same plots. A positive profit value actually indicates a decrease in profit of the given magnitude. This notation maintains the convention that the improving direction is toward the bottom left corner of the plot

scenarios once the designed vehicle fuel consumption reaches a certain point.

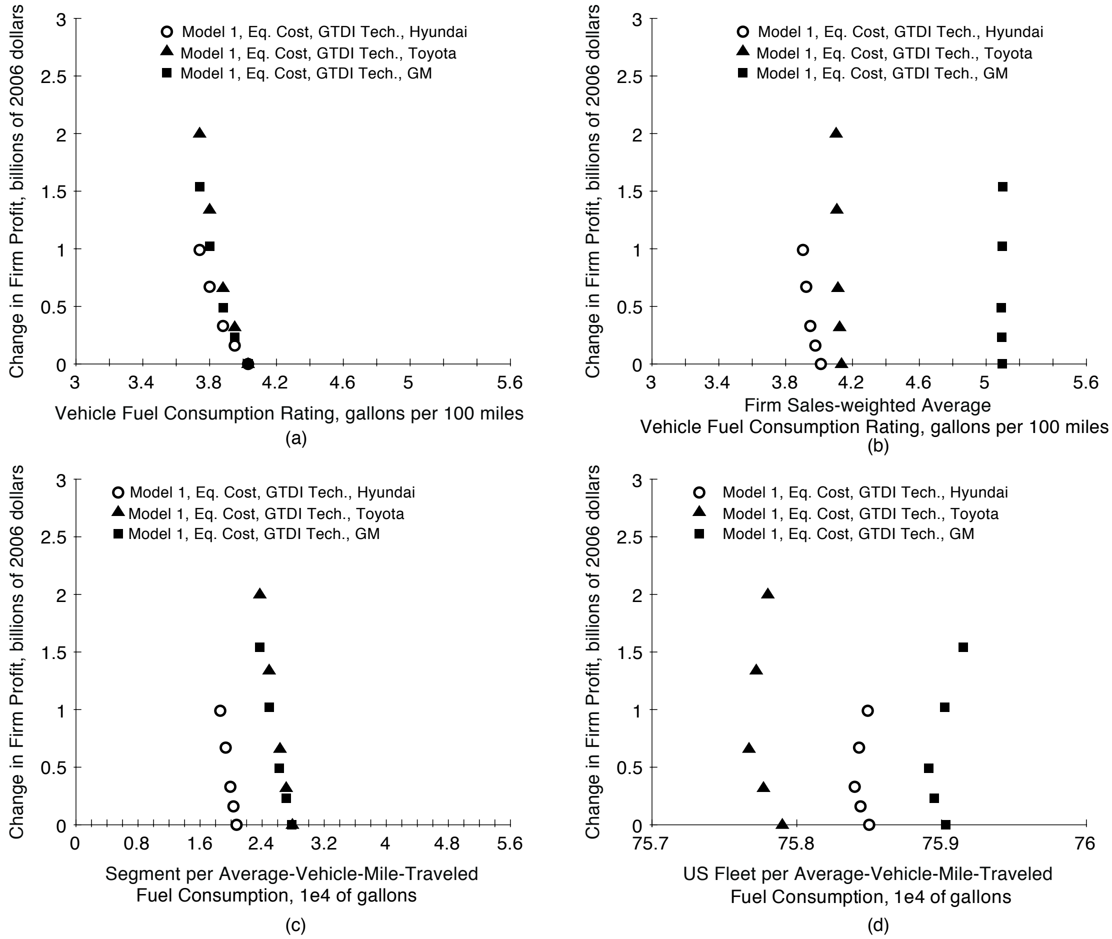


Figure 8.2: Pareto results for a single GTDI designed vehicle with Model 1 demand model and equilibrium cost model; (a) Vehicle fuel consumption rating in gallons per 100 miles versus negative firm profit; (b) Firm sales-weighted vehicle fuel consumption rating in gallons per 100 miles versus negative firm profit; (c) Segment per average-vehicle-mile-traveled fuel consumption in 10,000's of gallons versus negative firm profit; (d) US fleet per average-vehicle-mile-traveled fuel consumption in 10,000's of gallons versus negative firm profit;

Figure 8.2(b) shows how the sales-weighted fuel consumption rating for a given firm (Hyundai, Toyota, or GM) changes with change in the designed vehicle. The fuel consumption rating decreases with decreasing designed-vehicle fuel-consumption for Hyundai, marginally for Toyota, and actually increases very slightly for GM. This shows that the sales-weighted average fuel consumption for Hyundai without sales from the designed vehicle is lower, it is slightly lower for Toyota, and it is higher for GM. It also shows that GM has better substitutes for the designed vehicle than Hyundai, for example. They both lose about 41,000 total vehicle sales in the scenario where the designed vehicle has minimum fuel consumption. This was on a loss of 43,000 designed vehicle sales for Hyundai and 87,000 designed vehicle sales for GM. As a percentage of total sales the GM loss is much smaller (1% vs. 10%).

Figure 8.2(c) shows the fuel consumption of the midsize crossover segment. The reporting is given in average-vehicle-mile-traveled gallons. What this means is that we can calculate a projected annual fuel consumption of the new midsize crossover vehicle sales by assigning an average vehicle-miles-traveled to the segment car buyers. For example, if the average-vehicle-miles-traveled per new car buyer were 12,000 miles/year and the segment per average-vehicle-mile-traveled fuel consumption was 16,000 gallons, then the projected annual fuel consumption of the new midsize crossover sales would be $12,000 \times 16,000 = 192 \times 10^6$ gallons of gasoline. The segment fuel consumption is going down both because of the increased fuel efficiency of the designed vehicle, but also because of the reduction in sales of the designed vehicle. Only a fraction of the consumers that substitute away from the designed vehicle will remain in the narrowly defined midsize crossover segment.

We can also calculate an average-vehicle-mile-traveled fuel consumption for the entire US fleet. The results for each scenario are presented in Figure 8.1(d). The increasing aggregate fuel consumption for all firms beyond the third decrease in designed vehicle fuel consumption results from potential buyers substituting away from the designed vehicle to a vehicle that on average is less fuel efficient. The behavior in this plot can be contrasted with Figure 8.1(d), where for the Model 1 scenarios the US fleet fuel consumption always decreased. We can interpret this by noting that the maximum profit design for the GTDI technology has a much higher fuel economy to begin with than the conventional technology design. When buyers substitute away from this vehicle there reaches a point where the benefit from the (decreasing) sales of the designed vehicle are outweighed by the higher fuel consumption values of the substitute vehicles.

8.4 Single Designed Vehicle with HEV Powertrain

Results are presented in this section for various design scenarios where a single firm has been selected as the designing firm and a single midsize crossover vehicle with an HEV powertrain has been selected as the designed vehicle. The subsections are organized in order to facilitate comparison between problem formulation differences in the design scenarios. Design scenarios are compared across demand and cost models (Section 8.4.1). We report the values of the design variables, the vehicle attributes including price, the designed vehicle sales volume, and the designing firm's profit for each scenario.

8.4.1 Comparing Results across Demand and Cost Models

Table 8.16 presents the design variable values for design scenarios with a single designed vehicle and a single designing firm (Hyundai). Table 8.17 presents the vehicle attributes for design scenarios with a single designed vehicle and a single designing firm (Hyundai), and Table 8.18 presents the simulated market outcomes. Each of the scenarios reported is a maximum profit scenario. In other words, there was no constraint set on vehicle fuel consumption. The table rows are labeled by demand model (BLP95, Model 1, Model 2) and by cost model (Emp, Eq). Examining the design variables in Table 8.16 shows that the only difference between the optimal vehicle designs under the BLP95 model versus the new models are the engine bore and engine bore to stroke ratio. There are no differences in the optimal vehicle designs between Model 1 and Model 2 and between the empirical and equilibrium cost models.

Table 8.16: Maximum profit design variable values for BLP95, Model 1, and Model 2 with HEV technology and empirical cost model (Emp) or equilibrium cost model (Eq)

Model	x_B	x_{FD}	x_{L103}	x_{H101}	x_{W105}	x_{BtS}	x_{L101}	x_{PGR}	x_{BPow}
BLP95, Emp	88.0	4.89	5008	1704	1982	0.92	3048	0.6666	65
Model 1, Emp	93.5	4.89	5008	1704	1982	1.18	3048	0.6666	65
Model 2, Emp	93.5	4.89	5008	1704	1982	1.18	3048	0.6666	65
Model 1, Eq	93.5	4.89	5008	1704	1982	1.18	3048	0.6666	65
Model 2, Eq	93.5	4.89	5008	1704	1982	1.18	3048	0.6666	65

Table 8.17: Maximum profit vehicle attributes for BLP95, Model 1, and Model 2 HEV technology and empirical cost model (Emp) or equilibrium cost model (Eq)

Model	z_{MPG}	$10z_{hp}/z_{VM}$	$z_L z_W$	z_{hp}	z_{VM}	z_{Edisp}	z_{060}	z_{3050}	z_{65T}	$z_{MPG_{city}}$	$z_{MPG_{hwy}}$	z_{CVI}	z_{TS}
BLP95, Emp	29.1	0.24	1.54	97	4066	2.32	7.72	4.64	7.4	33.0	25.4	45	115
Model 1, Emp	30.9	0.27	1.54	110	4039	2.18	7.27	4.35	6.7	34.7	27.2	45	115
Model 2, Emp	30.9	0.27	1.54	110	4039	2.18	7.27	4.35	6.7	34.7	27.2	45	115
Model 1, Eq	30.9	0.27	1.54	110	4039	2.18	7.27	4.35	6.7	34.7	27.2	45	115
Model 2, Eq	30.9	0.27	1.54	110	4039	2.18	7.27	4.35	6.7	34.7	27.2	45	115

Table 8.18: Maximum profit market performance for BLP95, Model 1, and Model 2 with HEV technology and empirical cost model (Emp) or equilibrium cost model (Eq)

Model	p_j	$P_j \times M$	$\pi_{j,f}$	$M \sum_{j=1}^{J_f} P_{j,f}$	π_f
BLP95, Emp	38,321	1,059	10,532,820	537,688	4,717,935,274
Model 1, Emp	63,758	170,858	5,849,719,592	519,168	10,924,720,258
Model 2, Emp	58,840	278,222	8,157,363,147	587,607	11,452,188,308
Model 1, Eq	21,326	370,689	8,595,897,906	588,420	12,332,309,958
Model 2, Eq	25,129	509,693	12,549,391,841	720,574	15,082,089,827

The BLP95 vehicle has a larger engine but with smaller horsepower and the same large body compared to the Model 1 and Model 2 vehicles. This is different than the other two technologies where the BLP95 scenarios had small vehicles and more powerful engines.

Referencing the market outcomes found in Table 8.18 shows that the BLP95 model suggests a lower price and achieves extremely low sales volume compared with the Model 1 and Model 2 results with the same empirical cost model. The empirical cost model for Model 1 and Model 2 projects higher prices, but lower sales volumes and profits compared to the equilibrium cost model. Model 2 prices lower than Model 1 for the empirical cost model but higher than Model 1 for the equilibrium cost model. In both cases the Model 2 scenario achieves higher sales volume and higher profits on the designed model. Model 2 also projects higher overall firm profits than Model 1. The BLP95 demand model no longer favors Hyundai in terms of total sales given the high popularity of the HEV designed vehicle in the other demand model scenarios. The Model 2 equilibrium costs scenario, in particular, has very high HEV sales volume.

An oversight in the HEV simulations was not updating the power to weight ratio to reflect the benefit of the electric machines. The additional effective horsepower for the HEV vehicle because of the electric machines was accounted for in the vehicle mass and vehicle cost calculations. However, it was not added to the vehicle attributes used by the demand models. Correcting for this oversight would lead to higher power to weight ratios and presumably higher demand for these vehicles than already projected.

8.4.2 Discussion

The HEV results are substantially different than the other two technologies. The designed vehicles all have very high performance characteristics in terms of towing and acceleration, and the fuel economy is also very high. The designed vehicles are much more expensive than the other technology cases. However, the combination of high attribute levels more than overcomes the high price for consumers in the Model 1 and Model 2 scenario where sales volumes and profits are the highest of any scenario.

8.5 Three Designing Firms with one Designed Vehicle Each

Results are presented in this section for various design scenarios where three firms have been selected as the designing firms and a single midsize crossover vehicle from each firm is selected as the designed vehicles. Each firm is assigned a particular powertrain technology. The subsections are organized in order to facilitate comparison between problem formulation differences in the design scenarios. Design scenarios are compared across demand models where all firms produce conventional designs (Section 8.5.1) and across demand models and technology assignment (Section 8.5.2). We report the values of the design variables, the vehicle attributes including price, the designed vehicle sales volume, and the designing firm's profit for each scenario. Comparisons of the other environmental impact metrics are presented in Section 8.7

8.5.1 Comparing Results across Demand Models for Conventional Designs

Table 8.19 presents the design variable values for design scenarios with three designed vehicles and three designing firms (Hyundai, Toyota, and GM). Table 8.20 presents the vehicle attributes for design scenarios with three designed vehicles and three designing firms (Hyundai, Toyota, and GM), and Table 8.21 presents the simulated market outcomes. Each of the scenarios reported is a maximum profit scenario. In other words, there was no constraint set on vehicle fuel consumption. The table rows are labeled by demand model (Model 1, Model 2), by cost model (Eq), and by the designing firm (Hyundai, Toyota, Japan). Examining the design variables in Table 8.19 shows that there are only small differences in final drive and engine bore between the designing firms and between the two demand model scenarios. The

Table 8.19: Maximum profit design variable values for Model 1 and Model 2 and equilibrium cost model with Hyundai Conventional, Toyota Conventional, and GM Conventional

Model	x_B	x_{FD}	x_{L103}	x_{H101}	x_{W105}	x_{BtS}	x_{L101}
Model 1, Eq, Hyundai	84.5	3.49	5008	1704	2000	1.00	3048
Model 1, Eq, Toy	84.5	3.48	5008	1704	2000	1.00	3048
Model 1, Eq, GM	84.4	3.46	5008	1704	2000	0.99	3048
Model 2, Eq, Hyundai	84.5	3.49	5008	1704	2000	1.00	3048
Model 2, Eq, Toy	84.5	3.49	5008	1704	2000	1.00	3048
Model 2, Eq, GM	84.4	3.47	5008	1704	2000	0.99	3048

Table 8.20: Maximum profit vehicle attributes for Model 1 and Model 2 and equilibrium cost model with Hyundai Conventional, Toyota Conventional, and GM Conventional

Model	z_{MPG}	$10z_{hp}/z_{VM}$	$z_{L}z_{W}$	z_{hp}	z_{VM}	z_{Edisp}	z_{060}	z_{3050}	z_{65T}	$z_{MPG_{city}}$	$z_{MPG_{hwy}}$	z_{CVI}	z_{TS}
Model 1, Eq, Hyundai	21.7	0.52	1.55	221	4218	2.84	9.28	6.65	5.0	20.4	23.5	46	115
Model 1, Eq, Toy	21.7	0.52	1.55	221	4219	2.84	9.27	6.66	5.0	20.4	23.5	46	115
Model 1, Eq, GM	21.7	0.52	1.55	221	4223	2.86	9.23	6.68	5.0	20.4	23.5	46	115
Model 2, Eq, Hyundai	21.7	0.52	1.55	221	4219	2.84	9.21	6.66	5.0	20.4	23.5	46	115
Model 2, Eq, Toy	21.7	0.52	1.55	221	4219	2.84	9.21	6.66	5.0	20.4	23.5	46	115
Model 2, Eq, GM	21.7	0.52	1.55	221	4221	2.85	9.19	6.67	5.0	20.4	23.5	46	115

Table 8.21: Maximum profit market performance for Model 1 and Model 2 and equilibrium cost model with Hyundai Conventional, Toyota Conventional, and GM Conventional

Model	p_j	$P_j \times M$	$\pi_{j,f}$	$M\sum_{j=1}^{J_f} P_{j,f}$	π_f
Model 1, Eq, Hyundai	28,994	63,956	1,540,975,686	302,428	5,744,786,079
Model 1, Eq, Toy	28,927	139,384	3,931,586,867	2,033,887	47,246,369,081
Model 1, Eq, GM	18,944	133,287	3,292,532,214	3,783,573	105,387,515,091
Model 2, Eq, Hyundai	26,729	88,305	1,641,337,089	319,460	4,931,920,491
Model 2, Eq, Toy	27,493	184,991	4,225,165,675	2,103,691	40,062,423,596
Model 2, Eq, GM	21,760	152,318	3,507,939,900	3,762,234	87,828,354,497

designs are very similar to the single designed vehicle scenarios in Section 8.2.2.

The attribute values for the three designed vehicles in Table 8.20 are also very similar to the respective single designed vehicle cases in Section 8.2.2. The design differences between the Model 1 and Model 2 scenarios led to small differences in the engine displacement, vehicle mass and acceleration times.

Referencing the market outcomes found in Table 8.21 shows that again similar to the single designed vehicle cases, Hyundai and Toyota have similar prices and GM has a much lower price. Toyota and GM have similar sales volumes, and Hyundai has less than half the others' sales volumes. The Model 1 scenario has higher prices and lower sales volume than the Model 2 scenario. The designed vehicle profits are higher in the Model 2 scenario, but the firm profits are higher in the Model 1 scenario.

8.5.2 Comparing Results across Designing Firms for Multiple Technologies

Table 8.22 presents the design variable values for design scenarios with three designed vehicles and three designing firms assigned to a particular technology (Hyundai, Conventional or GTDI; Toyota, GTDI or Conventional; GM, HEV). Table 8.23 presents the vehicle attributesfor design scenarios with three designed vehicles and three designing firms assigned to a particular technology (Hyundai, Conventional or GTDI; Toyota, GTDI or Conventional; GM, HEV), and Table 8.24 presents the simulated market outcomes. Each of the scenarios reported is a maximum profit scenario. In other words, there was no constraint set on vehicle fuel consumption. The table rows are labeled by demand model (Model 1, Model 2), by cost model (Eq, Emp), by the designing firm (Hyundai, Toyota, Japan), and by powertrain technology (Conv, GTDI, HEV).

Examining the design variables in Table 8.22 shows that there are powertrain design differences between each of the three technologies across all scenarios. However,

Table 8.22: Maximum profit design variable values for Model 1 and Model 2 and equilibrium and empirical cost model with Hyundai Conventional or GTDI, Toyota GTDI or Conventional, and GM HEV

Model	x_B	x_{FD}	x_{L103}	x_{H101}	x_{W105}	x_{BtS}	x_{L101}	x_{PGR}	x_{BPow}
Model 1, Eq, Hyundai, Conv	84.5	3.49	5008	1704	2000	1.00	3048	-	-
Model 1, Eq, Toyota, GTDI	76.2	3.56	5008	1704	1982	1.06	3048	-	-
Model 1, Eq, GM, HEV	86.8	4.89	5008	1704	1982	0.90	3048	0.6666	65
Model 2, Eq, Hyundai, Conv	84.5	3.48	5008	1704	2000	1.00	3048	-	-
Model 2, Eq, Toyota, GTDI	76.2	3.57	5008	1704	1982	1.07	3048	-	-
Model 2, Eq, GM, HEV	86.8	4.89	5008	1704	1982	0.90	3048	0.6666	65
Model 1, Emp, Hyundai, Conv	84.6	3.55	5008	1704	2000	1.02	3048	-	-
Model 1, Emp, Toyota, GTDI	76.2	3.60	5008	1704	1982	1.08	3048	-	-
Model 1, Emp, GM, HEV	93.5	4.89	5008	1704	1982	1.18	3048	0.6666	65
Model 1, Emp, Toy, Conv	84.6	3.56	5008	1704	2000	1.02	3048	-	-
Model 1, Emp, Hyundai, GTDI	76.2	3.60	5008	1704	1982	1.08	3048	-	-
Model 1, Emp, GM, HEV	93.5	4.89	5008	1704	1982	1.18	3048	0.6666	65

Table 8.23: Maximum profit vehicle attributes for Model 1 and Model 2 and equilibrium and empirical cost model with Hyundai Conventional or GTDI, Toyota GTDI or Conventional, and GM HEV

Model	z_{MPG}	$10z_{hp}/z_{VM}$	z_{LZW}	z_{hp}	z_{VM}	z_{Edisp}	z_{060}	z_{3050}	z_{65T}	$z_{MPG_{city}}$	$z_{MPG_{hwy}}$	z_{CVI}	z_{TS}
Model 1, Eq, Hyundai, Conv	21.7	0.52	1.55	221	4218	2.84	9.22	6.65	5.0	20.4	23.5	46	115
Model 1, Eq, Toyota, GTDI	24.8	0.54	1.54	208	3865	1.31	8.68	5.77	5.0	24.8	24.9	45	115
Model 1, Eq, GM, HEV	30.0	0.23	1.54	95	4059	2.28	7.03	4.19	7.1	34.1	26.2	45	115
Model 2, Eq, Hyundai, Conv	21.7	0.52	1.55	221	4219	2.84	9.21	6.66	5.0	20.4	23.5	46	115
Model 2, Eq, Toyota, GTDI	24.8	0.54	1.54	209	3865	1.31	8.68	5.77	5.0	24.8	24.9	45	115
Model 2, Eq, GM, HEV	30.0	0.23	1.54	95	4059	2.28	7.03	4.19	7.1	34.1	26.2	45	115
Model 1, Emp, Hyundai, Conv	21.7	0.53	1.55	222	4210	2.80	9.30	6.61	5.0	20.4	23.5	46	115
Model 1, Emp, Toyota, GTDI	24.8	0.54	1.54	209	3862	1.29	8.69	5.76	5.0	24.8	24.9	45	115
Model 1, Emp, GM, HEV	30.9	0.27	1.54	110	4039	2.18	7.27	4.35	6.7	34.7	27.2	45	115
Model 1, Emp, Toy, Conv	21.7	0.53	1.55	222	4210	2.80	9.31	6.61	5.0	20.4	23.5	46	115
Model 1, Emp, Hyundai, GTDI	24.8	0.54	1.54	209	3862	1.29	8.69	5.76	5.0	24.8	24.9	45	115
Model 1, Emp, GM, HEV	30.9	0.27	1.54	110	4039	2.18	7.27	4.35	6.7	34.7	27.2	45	115

Table 8.24: Maximum profit market performance for Model 1 and Model 2 and equilibrium and empirical cost model with Hyundai Conventional or GTDI, Toyota GTDI or Conventional, and GM HEV

Model	p_j	$P_j \times M$	$\pi_{j,f}$	$M \sum_{j=1}^{J_f} \mathbf{P}_f$	π_f
Model 1, Eq, Hyundai, Conv	29,704	56,093	1,391,424,438	276,688	5,402,085,182
Model 1, Eq, Toyota, GTDI	19,373	335,131	8,074,538,823	2,127,392	49,650,438,955
Model 1, Eq, GM, HEV	11,348	645,531	16,099,275,384	4,043,386	114,747,156,007
Model 2, Eq, Hyundai, Conv	27,407	66,301	1,277,502,424	284,014	4,378,984,948
Model 2, Eq, Toyota, GTDI	17,879	474,059	9,333,630,380	2,311,928	43,345,154,383
Model 2, Eq, GM, HEV	14,961	781,293	19,585,545,522	4,023,378	106,474,323,366
Model 1, Emp, Hyundai, Conv	42,661	117,628	2,623,548,443	462,552	8,154,253,003
Model 1, Emp, Toyota, GTDI	56,932	226,586	8,402,626,099	2,090,034	52,426,988,618
Model 1, Emp, GM, HEV	89,517	125,694	7,541,158,043	3,044,062	90,146,215,625
Model 1, Emp, Toy, Conv	50,327	154,130	4,697,656,197	2,048,324	48,676,322,356
Model 1, Emp, Hyundai, GTDI	43,173	166,053	3,788,662,483	501,867	8,976,830,938
Model 1, Emp, GM, HEV	70,560	159,215	6,534,087,962	3,112,107	90,807,674,279

vehicle size is the maximum allowable in all cases. The individual designs for the conventional vehicle correspond well to the single designed cases from Section 8.2.2. The powertrain variables for the GTDI designs shifted slightly from Section 8.3.2, and the HEV designs are similar for the empirical cost model, but different for the equilibrium model compared to Section 8.4.1.

The attribute values for the three designed vehicles in Table 8.23 are also very similar to the respective single designed vehicle cases with the exception of the equilibrium cost model HEV cases. These cases are also different from the single designed vehicle cases because the designing firm for the HEV in the multi-firm case is GM rather than Hyundai as in the single designed vehicle case.

Referencing the market outcomes found in Table 8.24 shows that the sales volume and profit trends described in the previous sections held in the cases of multiple firms with differing technologies. The two Model 1 empirical costs scenarios show a behavior where the Toyota vehicle commands a higher price than the Hyundai vehicle whether it designs a conventional or GTDI vehicle. The GTDI vehicle gains a higher sales volume in both cases, but when Hyundai designs the GTDI vehicle the difference between sales is much less than when Toyota designs the GTDI vehicle. For the equilibrium cost case for both Model 1 and Model 2 Toyota prices the GTDI vehicle below the conventional vehicle from Hyundai.

Examining the behavior of GM with the hybrid vehicle design shows interesting behavior between the two cost models. In the case of the empirical cost model, GM's costs are similar to the other firms. It then prices the HEV much higher than the other firms. The HEV is still popular, but achieves sales volumes on the order of the other vehicles. The equilibrium cost case where GM's costs are much lower than the other firms shows that GM prices the hybrid well below the other vehicles and achieves very large sales volumes.

8.5.3 Discussion

The results for the multi-firm cases are similar to the single designed vehicle cases. The differences in price and shares are reflective of differences in brand value which also led to differences in cost according to equilibrium cost model. Because of this the interaction between the firms in terms of vehicle price and sales volume, the market outcomes are different between the two cost different cost models even though the designs are for the most part similar.

8.6 Comparing Results across Design Scenarios

Across all scenarios there were small differences in powertrain variables between scenarios with different cost models and demand models. The GTDI designed vehicles had larger changes in powertrain variables, and the HEV designed vehicles had larger changes still. There were some small differences between the single designed vehicle scenarios and the multi-firm design scenarios particularly for the GTDI technology. Further work should explore wether this behavior holds in more general cases or if it was related to the highly constrained design space in these problems.

Vehicle size reached its upper bound for all cases except for the conventional and GTDI BLP95 scenarios. We had expected to see differences in vehicle design between the Model 1 and Model 2 demand models and in the cases with multiple random coefficient draws per individual versus the 1 draw cases. However, the expected changes hinged on differences in vehicle size. The footprint size of a midsize crossover is very similar to a midsize sedan. The market simulations indicated that preference for increased size was stronger than increased fuel economy or increased power to weight for both Model 1 with the preference for size linear in footprint and for Model 2 with preference for size quadratic in footprint. We conjecture that based on the elasticity plots for both Model 1 and Model 2 in Figures 4.4, 4.5, 4.8, 4.12, were we to design two seaters, full size vans, or pickup trucks we may have observed differences in design outcomes between demand models and between number of draws. These segments show the most difference in own and cross-elasticity values between the two demand models and between number of draws.

The vehicle attribute tables show that the design variable differences did lead to significant differences in predicted 0-60 mph acceleration times. However, the acceleration times were not used in the demand model. In the past, researchers have noted that the power to weight ratio could serve as a surrogate for some consumer perceived attribute such as acceleration. The fact that the acceleration time and power to weight ratios are not perfect corollaries raises the question of what are the right attributes to include in the demand model, and how are consumers perceiving differences in design.

The enhanced performance capability of GTDI technology allowed the engine displacement to be dropped considerably. The resulting design weighed less and was less costly while improving fuel economy. Power to weight was comparable. In general the GTDI vehicles enjoyed higher sales and profit than the conventional technology vehicles. The trend of improving technology was even greater for the HEV designed vehicles, which enjoyed the largest profits and highest sales volumes.

The differences in market outcomes were much larger than differences in design variables across scenarios. For example, the GTDI and HEV vehicles had higher sales volumes with equilibrium cost model than with empirical cost model. The opposite was true for the conventional technology. The conclusion is then that even when the vehicle design is highly constrained or exhibits similar behavior under different demand models, the details of market outcomes can vary widely depending on the other details of the market simulation including the cost model, the demand model, and how the consumer population was simulated in terms of the number of draws.

The brand coefficients clearly play a role in the price equilibrium game in terms of the price charged by a specific firm and the market share of the vehicle. The brand coefficients are simply very granular alternative-specific constants. We expect the influence of adding alternative-specific to the demand models to also make a large difference in price and sales volume as did the brand coefficients. Beyond the influence of the brand constants it would be interesting to explore the influence of a firm's portfolio on its own prices and sales volume.

The succeeding paragraphs discuss some of the shortcomings of the existing engineering, cost, and demand models.

The engineering model is currently limited to a narrow range of design variable and parameter values. One next step is to enhance the engineering model to accommodate vehicle designs over a range of vehicle classes and performance requirements. The HEV performance model appears overly optimistic in terms of its combined fuel economy and acceleration times compared to current vehicles on the market. The optimism may come from the lack of a missing performance consideration or unrealistic engine properties. Another source for the very high attribute values in the market simulations could be the hybrid cost model. The performance model could be presenting the technology capabilities correctly, however, the electric machine and battery costs could be lower than reality.

The costs for alternative technologies were introduced as incremental to the base vehicle cost. This methodology appears to have worked as expected for the empirical cost model case. However, for the equilibrium cost model case, the base vehicle costs are clearly unrealistic. For the scenarios with homogeneous technologies such as the conventional technology scenarios, the equilibrium cost model contributed to market simulation outcomes with reasonable prices. However, in the cases with mixed technologies, unexpected and unrealistic dynamics unfolded where (what are held as) expensive technology vehicles were priced well below conventional vehicles.

The equilibrium cost model with its negative costs revealed failings in the multi-firm scenarios. The baseline GM vehicle had a negative cost. This meant that even with the added HEV technology costs, GM was able to lower the price of the HEV and capture a very large market share. The empirical model does not exhibit this behavior because all costs were positive and equivalent across brands for equivalent vehicle designs.

In order to consider a firm's portfolio design decisions modifications must be made to the cost structure. For example, the current cost models consider only variable cost. However, the automotive industry is a capital intensive industry that requires significant investment costs for each new vehicle line introduced. Additionally, savings can be captured by sharing components and assembly facilities as a platform for multiple vehicles. A cost model that took these factors into account may be more likely to suggest an interior solution for vehicle size rather than seek to maximize vehicle wheelbase and length. For example, the midsize crossover vehicles typically have a midsize sedan counterpart. The common wheelbase of these vehicles allows them to share many chassis components and a common assembly facility. Were the cost model to account for the additional investment cost required to develop a new vehicle platform, the optimal design decisions would reflect this cost.

The largest shortcomings for the demand models are the inaccuracy of baseline market shares predictions and the resultant high profit margins in the market simulations. As discussed previously, the most straightforward way to resolve these issues is to estimate demand models that incorporate alternative-specific constants for each vehicle alternative.

The power to weight ratios of the optimal vehicle designs appeared to be lower than the power to weight ratios of the vehicles in the market. In general, the optimal vehicle design improved fuel economy at the expense of power to weight ratio compared to the baseline 2006 midsize crossover vehicles. This opens the question of whether power to weight is really valued less than what is produced by the market, or if the demand model specification did not adequately capture the factors that most influence consumer preferences.

The opposite behavior of the US fleet fuel consumption for the different demand models shown in Figure 8.1 leads to the question of which model more accurately reflects the real market. Further works should address how to test different demand models and the respective market simulation outcomes.

8.7 Discussion of Public versus Private Tradeoff

Sections 8.2.3 and 8.3.3 present figures showing the relationship between various measure of the public objective of reduced fuel consumption with the private objective of firm profit. Depending on the demand model, the vehicle technology, and the level of fuel consumption required, a reduction in fuel consumption of the designed vehicle may lead to an increase or decrease in the total fuel consumption of the US fleet. This statement is made assuming a fixed number of vehicles sold in a given year and the average number of vehicle-miles-traveled per individual vehicle owner is the same across all vehicle classes.

The maximum profit design for each corresponds to the maximum sales volume case for each scenario. As fuel consumption is constrained to decrease, sales of the designed vehicle decrease. Given a fixed number of new vehicles sold, the consumers who leave the designed vehicle purchase some other vehicle. The vehicle they purchase could have a fuel consumption rating greater or smaller than the designed vehicle. The US fleet fuel consumption will increase if the substitute vehicle has higher fuel consumption than the designed vehicle. US fleet fuel consumption will decrease if the substitute vehicle has lower fuel consumption. Particularly, in the conventional case in Figure 8.1, for the Model 2 scenarios, the US fleet fuel consumption goes up when the fuel consumption of the designed vehicle increases. What this means is that individuals are substituting to a higher fuel consumption vehicle on average. The opposite behavior is true for the Model 1 scenarios.

Therefore, the market simulation according to Model 2 indicates that having the designed midsize crossover vehicle is helpful in reducing US fleet fuel consumption. This is because the US fleet fuel consumption increases when the designed vehicle sales are reduced. The results for the Model 1 scenario indicate that the US fleet fuel consumption would decrease without the designed vehicle in the market, since US fleet fuel consumption decreases with decreasing designed vehicle sales. The substitution patterns for the two models must then be markedly different. This is a very interesting result given that the two demand models appear to fit the data similarly well based on the estimation fit and the demand model parameter coefficients significance.

There is a clear conflict between selling vehicles and reducing US fleet fuel consumption if we assume that the average vehicle miles traveled is the same for each new vehicle sold. According to some of the design scenarios, a firm could reduce US fleet fuel consumption by redesigning its products to be more fuel efficient than market expectations or by repricing—in both cases making less profit, but this is not a sustainable way for the firm to operate. It is also not clear that such a strategy could be used to the firms advantage in terms of PR value. In both cases, redesigning cars for higher fuel efficiency or raising prices on high fuel consumption vehicles, they would be saying, "We want you to buy fewer of our cars so we use less gas."

In any case, the market simulations appear to indicate that forcing a technology beyond the market is a dangerous game. The Model 2 scenarios for the conventional vehicle give no expectation of improvement of public objective in terms of US fleet fuel consumption by improving fuel economy of a single midsize crossover. It would be interesting to explore the impact of changing fuel consumption for vehicles in other segments.

The GTDI and HEV technologies provide a different approach to the public versus private tradeoff problem. The advanced technologies allow increased performance.

The market simulations indicate that the increased performance can be used to decrease fuel consumption in the case of the maximum profit designed vehicles. The US fleet fuel consumption for the GTDI designed vehicle with Model 1 and the equilibrium cost model is about 75.85e4 gallons per average-vehicle-mile-traveled. The US fleet fuel consumption for the conventional designed vehicle with Model 1 and the equilibrium cost model is about 76.06e4 gallons per average-vehicle-mile-traveled. Introducing the GTDI vehicle reduces US fleet fuel consumption at a profit for the firm.

Because the GTDI maximum profit vehicle has lower fuel consumption than the conventional maximum profit vehicle, the effects on US fuel consumption of decreasing the fuel consumption below the maximum profit design are more pronounced. Examining Figure 8.2 shows that for Model 1, which showed monotonically decreasing US fleet fuel consumption for the conventional case in Figure 8.1, there is a point where US fleet fuel consumption will begin to increase.

The success of the technology strategy hinges on the cost per performance tradeoff of the technology and the underlying consumer demand. If the consumer demand is in the direction of increased performance rather than decreased fuel consumption, the impact of the new technology on fuel consumption will be negligible.

8.8 Summary

This chapter presented a problem formulation for conducting market simulations to examine the vehicle design decisions of profit maximizing firms. Four primary design scenarios were explored, and the impact on the design scenario of changing one of the modeling elements was examined. The results of the market simulation are vehicle design variables, vehicle attributes, vehicle prices, sales volumes, firm profits, and firm, segment, and US fleet fuel consumption. These outputs allow the exploration of the tradeoff between firm profit and fuel consumption.

The first scenario considered the design of a single midsize crossover vehicle using conventional powertrain technology by a single designing firm. Market simulations were run for changes in the demand model, the cost model, the designing firm, the preference for fuel consumption, and the number of random coefficient draws per individual in the simulated population. Vehicle designs for all scenarios were large vehicles with marginal differences in powertrain variables, except for the BLP95 demand model scenarios, which were small vehicles with high power to weight ratios and high fuel consumption. Market outcomes such as vehicle price, sales volume, and firm profit were more varied.

The second scenario considered the design of a single midsize crossover vehicle using GTDI powertrain technology by a single designing firm. Market simulations were run for changes in the demand model, the cost model, and the designing firm. Vehicle designs for all scenarios were large vehicles with marginal differences in powertrain variables, except for the BLP95 demand model scenarios, which were small vehicles with high power to weight ratios and high fuel consumption. Market outcomes such as vehicle price, sales volume, and firm profit were more varied.

The fuel consumption rating for the GTDI vehicles were categorically higher than the conventional vehicles. The effect of decreasing designed vehicle fuel consumption below the maximum profit designed vehicle were mixed depending on the vehicle technology and the demand model. Decreasing fuel consumption for the conventional designed vehicle for the Model 2 demand model case actually increased US fleet fuel consumption.

The third scenario considered the design of a single midsize crossover vehicle using HEV powertrain technology by a single designing firm. Market simulations were run for changes in the demand model, the cost model. Vehicle designs for all scenarios were large vehicles with marginal differences in powertrain variables. The BLP95 demand model scenarios had larger changes in powertrain variables. Market outcomes such as vehicle price, sales volume, and firm profit were more varied.

The fourth scenario considered the design of a three midsize crossover vehicles by three designing firms. Market simulations were run for changes in the demand model for the case where all designing firms designed conventional vehicles. Market simulations were run for changes in the demand model and the cost model and the technology of the designing firm for the case where the designing firms each designed a vehicle with a different technology. Vehicle designs for all scenarios were large vehicles with marginal differences in powertrain variables. Market outcomes such as vehicle price, sales volume, and firm profit were more varied. Particularly, the pricing strategy and the sales volume changed for the HEV designing firm between the empirical and the equilibrium cost models.

In general, the market simulations pushed the vehicle designs to the largest possible size, the smallest gradeability, and the lowest top speed allowed for the midsize crossover segment. The market outcomes of price, sales volume, and profit were much more varied across the scenarios than the vehicle designs.

Improving fuel economy of vehicles beyond market equilibrium may be good for PR, but it may not improve the sales-weighted average vehicle fuel consumption rating because of lost sales and reduces the total fuel consumption attributable to the firm's new car sales mostly by reducing the firm's sales. The action may or may not reduce the fuel consumption of the US fleet given a fixed set of new car buyers.

Areas for improvement include reestimating demand models with alternative-specific constants in order to match historical sales. This could make interpretation with respect to the real market more reliable. Continued exploration of consumer utility specification is desired to better capture consumer decision-making behavior. For example, the vehicle designs were constrained to remain in within a pre-defined class through towing and size constraints rather than through consumer preferences. The equilibrium cost model can be reestimated with the new demand models in order to reach reasonable (nonnegative) costs for each vehicle. Fixed costs, investment costs, and platforming costs should all be considered in defining the producer's decision paradigm. The HEV design and cost model should be revisited for technical and economic validity.

CHAPTER IX

Conclusions

9.1 Summary

This dissertation establishes a methodology for evaluating automotive vehicle design according to private (firm profit) and public (fuel consumption) criteria. The methodology set forth relies on previously-made developments from engineering, economics, and marketing. New work in this dissertation includes the development of several models that can be applied to other similar problems. Specifically, the problem formulation integrates models of demand, cost, and product performance in order to implement a game-theoritic formulation of producer behavior where producers choose the attributes of the products they produce and the prices they will charge in order to maximize profit (Chapter 6). Two variations of a newly estimated mixed-logit discrete choice model of new car buyer purchase behavior are developed for incorporation as demand models (Chapter 4). Three cost model formulations are developed and compared in the context of the problem formulation (Chapter 5). An explicit representation of an automotive manufacturer's technology capability in the form of a comprehensive yet stylized engineering performance model of a midsize crossover vehicle is developed (Chapter 3). A methodology for developing and estimating mixed-logit choice models accessible to the design for market systems researcher is documented (Chapter 4).

Novel measures for analyzing and comparing Pareto sets are developed (Chapter 7). These measures can be applied to studying the public versus private tradeoff in the automotive industry or another industry. The application of these measures was demonstrated for a simple nonlinear programming example and a simplified vehicle design problem.

Hypothetical scenarios were evaluated in order to test the vehicle design problem formulation involving the design of a single vehicle within a price-equilibrium market context and the design of multiple same-segment vehicles within a price-equilibrium market context (Chapter 8). The results of the market simulation are vehicle design variables; vehicle attributes; vehicle prices; sales volumes; firm profits; and firm, segment, and US fleet fuel consumption. These outputs allow the exploration of the tradeoff between firm profit and fuel consumption. Similar models using the same structure could be set up for a different market rather than the automotive vehicle market. Measures for the private and public objectives could be defined and the tradeoffs explored.

The differences in scenario outcomes based on differences in the demand and cost models were explored. The results show that improving the fuel economy of a specific vehicle does not always lead to a reduction in US fleet fuel consumption. This is due to substitution patterns where new car buyers substitute toward less fuel efficient vehicles.

There are several improvements that could be made to enable the market simulation results to be useful at a practical level. We feel that to be useful for real market insights, the market simulations need to predict baseline market shares that start from the real market position and profits should be projected at reasonable levels. This could be accomplished in large measure by including alternative-specific constants for the vehicle alternatives in the choice model estimation. Finally, the difficult underlying question remains that is how well do the models produce realistic behavior as the design scenario deviates from the current market. The proposal of two different demand models that produce very different substitution patterns based on the same market data as shown in this dissertation is a start at addressing this question.

9.2 Contributions

The primary contribution of this dissertation is that disparate developments from several academic fields have been brought together in a single mathematical problem formulation for a large-scale product development problem. The integrated problem formulation will allow study of interdisciplinary issues related to product development in a new way. While ongoing work from other researchers has similarly begun to develop comprehensive problem formulations, one aspect that is new for this problem formulation is that the vehicle design problem is set in the context of full market price equilibrium. This work also demonstrates some of the challenges that must be addressed in such formulations. Specifically, comparing the empirical cost model to the equilibrium cost model shows that the empirical cost model resulted in grossly inflated prices for the given demand model. The equilibrium cost model produced realistic price projections. However, due to the irregularities of the equilibrium cost model the scenarios with that model produced nonintuitive pricing behavior for the case of multiple firms designing different technologies. Scenarios with the empirical cost model, while producing inflated prices, maintained economic intuition. Problems with both cost models may be addressed by developing demand models that match sales data.

The demand models illustrated two points. The first point is that without alternative-specific constants used to match demand model projected sales to actual market sales the market outcomes of the hypothetical scenarios can not be interpreted in the context of the real market. This has been known in marketing for some time.

However, it has become common practice in the design for market systems engineering community to adopt demand models from the literature without employing the alternative-specific constants for those models. The second point is that changes in the functional form of the utility specification can alter substitution patterns–and thus market outcomes–significantly even when both models appear to fit the underlying data similarly well. We showed this by estimating and then employing a model with preference monotonic in vehicle footprint and another model where preference was quadratic in vehicle footprint. In other words individual consumers had an ideal vehicle footprint. While the two differing models did not result in large changes in the designed vehicles, the vehicle price, sales volume, and the overall market fuel consumptions changed much. This result has important implications for the public versus private tradeoff. In one model a certain policy pursued by a firm or public entity could be expected to improve the public objective. However, according to a competing model, the same action could be detrimental. A first clue for effects such as these can be gleaned at the level of the demand model by studying the own- and cross-elasticities for the product attributes in addition to price.

A second contribution of the dissertation is the application of the notion that we can explore the tradeoff between private interests and public interests by simulating market response under different hypothetical scenarios. We can then gain deeper insights by examining the tradeoff relationships between the different scenarios. New local and global measures were defined or reemphasized for application in comparing results of market simulations. The local measures are defined around a single Pareto point. The global measures are defined by the ideal points of each objective and the shape of the Pareto set.

These contributions have extended engineers' capability for developing and applying choice models to design problems by illustrating the impact of choice model specification and the pitfalls of adopting "off-the-shelf" models.

9.3 Future Work

9.3.1 Modeling Framework

Several next steps can be taken to improve the modeling framework. Future work remains to expand the engineering design model to encompass a broader range of vehicle sizes and performance requirements. This is an important piece to fully explore the implications of the differing demand models because the most dramatic differences in elasticities of demand with respect to vehicle footprint occur for the luxury models and for specialty market segments such as the two-seaters or minicompacts and the pickup trucks. The engineering model must be capable of predicting performance for these vehicles.

There remains much to be learned about the relationship between the consumer utility function and the optimal vehicle design result. Perhaps studying the colinearity of attributes such as high fuel economy and small size can be fruitful in understanding how to pose a utility specification that better captures purchase behavior in the context of the vehicle design.

Also, future work should study the difference in optimization outcomes between a model that was estimated without alternative-specific constants and one with alternative-specific constants. We expect that a model estimated with alternative-specific constants will overcome many of the difficulties with unrealistic share predictions and inflated profit margins.

The number of variables and the number of products designed can be increased. Perhaps homotopy techniques can be applied for solving for Pareto points in order to reduce the computational burden of performing a full design optimization with nested price equilibrium calculations.

More work can be done to explore cost models and their implications for design optimization.

Comparing demand model prediction on data from another year is also an obvious next step.

An obvious omission from the modeling framework has been the consideration of the Corporate Average Fuel Economy Standard (CAFE) enforced in the United States. This modeling piece has been avoided on a practical level because it complicates the equilibrium cost model estimation significantly. It also complicates the solution to the price equilibrium game. Another practical reason for avoiding CAFE is that it is difficult to prescribe its effects for a given model year. Several automakers (primarily European manufacturers) do not comply with CAFE and prefer to pay a penalty. US manufacturers have traditionally kept CAFE. However, there is a complicated bookkeeping for CAFE that allows manufacturers to carry over CAFE penalties in one year and then make them up in a succeeding year. Still, Japanese manufacturers historically exceeded the CAFE standard because they produced mostly smaller cars. All of this does not yield easily to a neat mathematical description.

The traditional effect ascribed to CAFE by the US automakers is that they must lower their prices on their most fuel efficient models in order to sell more of these vehicles or raise prices on their least fuel efficient vehicles in order to sell less. Regardless of the producer decisions the demand model estimation should be unaffected. Consumers observe vehicle attributes and prices and make purchase decisions with no consequence from CAFE. In this sense the demand models developed here are reflective of the US market under CAFE.

The missing piece is to have a cost model that reflects producers' costs and then enforce (some) producers to abide by CAFE standards in any hypothetical market scenario. The empirical cost model could be adjusted for CAFE by adjusting the firm margin with respect to the dealer margin for fuel efficient vehicles. In the equilibrium cost case, new maximum profit conditions need to be defined and solved given the CAFE constraint on some firms. Enforcing CAFE during the price equilibrium calculations is more problematic and requires further research into a numerical method that can make this problem tractable.

9.3.2 Enhancing Choice Paradigm

There are several aspects of the choice paradigm implied by the current approach that should be reconsidered. One topic would be to compare exploded-logit with stated consideration as pseudo observations (what we did in this dissertation) vs. single purchase observations with conditional choice sets based on the stated considered vehicles.

There may also be a computationally efficient approach to expanding the vehicle alternatives in the choice set to allow many more covariates by conditioning the choice set in a smart way. This is an important step in the vehicle design problem because being able to identify a greater number of vehicle attributes that influence choice will allow the vehicle design decision space to grow.

There are three additional directions related to demand modeling that are highlighted here for consideration in future work to improve the performance of demand models for design optimization studies.

The first is to consider alternative utility specifications that represent fundamentally different decision-making behaviors. A body of research continues to show that models incorporating heuristic decision rules can perform as well or better than models based on compensatory trades in many decision-making scenarios[*Hauser et al.* (In Review)]. The evaluation methods presented here can be used to test such alternative functional forms for various behavioral hypotheses. Areas for exploration in design research include models with dummy-coded attribute levels similar to [*Michalek et al.* (2005)] that allow highly nonlinear attribute weightings; rigorous exploration of preference thresholds and cutoffs continuing the work of Wassenar [*Wassenaar et al.* (2005)]; and in place of random coefficients, random functional forms or stratified functional forms of utility based on demographics or latent class analysis, which allow different individuals to think about the same choice differently.

The second is to explore modifications to the representation of the the consumer choice paradigm. The choice paradigm employed in conventional econometric demand models of the automotive industry coincides with that of consumers making compensatory choices across the entire new-vehicle fleet when making a purchase decision. Enforcing this paradigm presumes consumers consider all new-vehicle makes and models at the time of purchase. Researchers have suggested that this paradigm is unrealistic from a cognitive-ability perspective, and it excludes a multistage decision process where a consumer's decision rules change from one stage to the next. An alternative two-stage decision process, where the consumer first identifies a subset of all possible alternatives as the consideration set and then makes compensatory trades between the attributes of the consideration set, has shown some promise in the choice modeling literature [*Hauser and Wernerfelt* (1990); *Horowitz and Louviere* (1995)]. Additionally, incorporating the Heckman selection process [*Heckman* (1979)] to reconcile differences between an overall population and a sub-population of product purchasers (as we had with the new-car buyer survey) may yield fruit in treating the outside good as it relates to setting overall market demand.

The third area is to compare fundamental limits of aggregate and disaggregate data as well as revealed and stated-choice data. Individual-choice level or disaggregate data are difficult and costly to obtain. However, when available, analysts derive significant benefit. The theme for this work could be to describe the best demand modeling behavior for design optimization to be expected from an aggregate vs. a disaggregate source, and similarly what is the best modeling behavior that can be derived from revealed choice vs. stated choice data in the context of design.

9.3.3 Cost Modeling Improvements

Cost modeling could be enhanced by expanding the firm's perspective to include annual fixed costs and investment costs. This is particularly important for the automotive industry where products are capital intensive and product decisions are made with a long lead time. It will be a challenge to develop cost model forms that allow incorporation of more supply side information (e.g., economies of scale, investment costs, etc.) and that can maintain compatibility with the existing market equilibrium simulations.

We also lack cost models that can be related back to technology design decisions (i.e., engineering variables), but do not require the extensive proprietary data sources held by manufacturer's. The empirical cost model was a first step to develop suitable models with publicly available data. An extension would be to identify the best methods for estimating costs empirically and the best sources of non-OEM-proprietary data.

We made assumptions about the relationships between dealer and OEM profits. More work can be done to explore these assumptions.

9.3.4 Expand Application Domains

The integration of demand, cost, and design into a maximum-profit-firm-design-decision can be applied beyond the automotive industry. Other transportation and energy related domains to consider would be vehicle and grid integration, consumer electricity consumption, and multi-mode transportation systems. Consumer products and other durable goods are also an obvious extension.

APPENDIX

APPENDIX A

Technical Report

NOTE: The nomenclature in this appendix does not match the nomenclature used throughout the document. Instead notation that largely matches the Matlab code implementation of the model has been preserved.

An engineering model was developed to represent the technical performance of the vehicle. It calculates the characteristics listed in Table A.1 using using AVL Cruise software package [16] powertrain simulations, curve-fits from empirical data, and analytical expressions. Design variables and other model parameters are listed in Table A.2. The gear shifting program is also specified as shown in Figure A.1.

Vehicle characteristics other than powertrain performance characteristics

Engine sizing

`{VehicleCG.m}`

The engine configuration, engine bore, bore to stroke ratio, and the number of cylinders are used to calculate other size related engine characteristics. The

Table A.1: Vehicle characteristics computed by engineering model

Veh. Char.	Input to Demand Model	Units	Description
MPG	B&M, BLP	[mi/gal]	fuel economy
$Acc060$	B&M, BLP	[s]	0-60 mph time
$Styl1$	B&M	[-]	exterior proportions
$Size$	BLP	[in^2]	exterior size
$Acc3050$	-	[s]	30-50 mph accel. time while towing
$Grad65Tow$	-	[%]	max grade at 65 mph while towing
$MaxSpeed$	-	[mph]	vehicle top speed
CVI	-	[ft^3]	cargo volume index behind 2nd row
A147	-	[°]	ramp breakover angle
A107	-	[°]	angle of departure
A106	-	[°]	angle of approach
CG_{long}	-	[mm]	center of gravity in long. direction
CG_{vert}	-	[mm]	center of gravity in vert. direction
$VehMass$	-	[kg]	vehicle curbweight
$GVWR$	-	[kg]	gross vehicle weight rating
$VehM\%_{front}$	-	[%]	percent vehicle mass on front wheels
$EngLength$	-	[mm]	estimated engine length
$FrontArea$	-	[m^2]	frontal area
$CrushSpace$	-	[mm]	crush space-bumper to driver heel
$MaxDecel$	-	[m/s^2]	est. peak decel. in front crash test
$MG1$ (HEV)		[kW]	peak power electric machine 1
$MG2$ (HEV)		[kW]	peak power electric machine 2

Table A.2: Partial list of vehicle parameters

Veh. Char.	Design Var.	Units	Description
$EngBore$	✓	[mm]	engine bore
$EngBoretoStroke$	✓	[-]	bore to stroke ratio
$FinalDrive$	✓	[-]	final drive ratio
L103	✓	[mm]	exterior length
W105	✓	[mm]	exterior width
H101	✓	[mm]	exterior height
L101	✓	[mm]	wheelbase
PGR	✓(HEV)	[-]	planetary gear ratio
$BPow$	✓(HEV)	[kW]	peak battery power
$EngNofCyl$	-	[-]	number of engine cylinders
GearRatio	-	[-]	gear ratios (1-6)
$VehGTankVol$	-	[l]	gas tank volume
L104	-	[mm]	dist. f. bumper to f. axle
H103-1	-	[mm]	dist. f. bumper to f. axle
H103-2	-	[mm]	dist. f. bumper to f. axle
H156	-	[mm]	dist. f. bumper to f. axle
$VehLegRoom1$	-	[mm]	driver legroom
$VehLegRoom2$	-	[mm]	2nd row legroom
$VehHeelPointX$	-	[mm]	longitudinal location of driver heel point
$MidRailThick$	-	[mm]	mid-rail thickness
$MidRailWidth$	-	[mm]	mid-rail width
$VehTurnRad$	-	[mm]	minimum turn radius
$WheelDiam$	-	[mm]	wheel diameter
H108	-	[mm]	tire static rolling radius
C_D	-	[-]	vehicle drag coefficient
$VehSeatCap$	-	[-]	max seating capacity
$PassWeight$	-	[lbm]	average weight of vehicle occupant

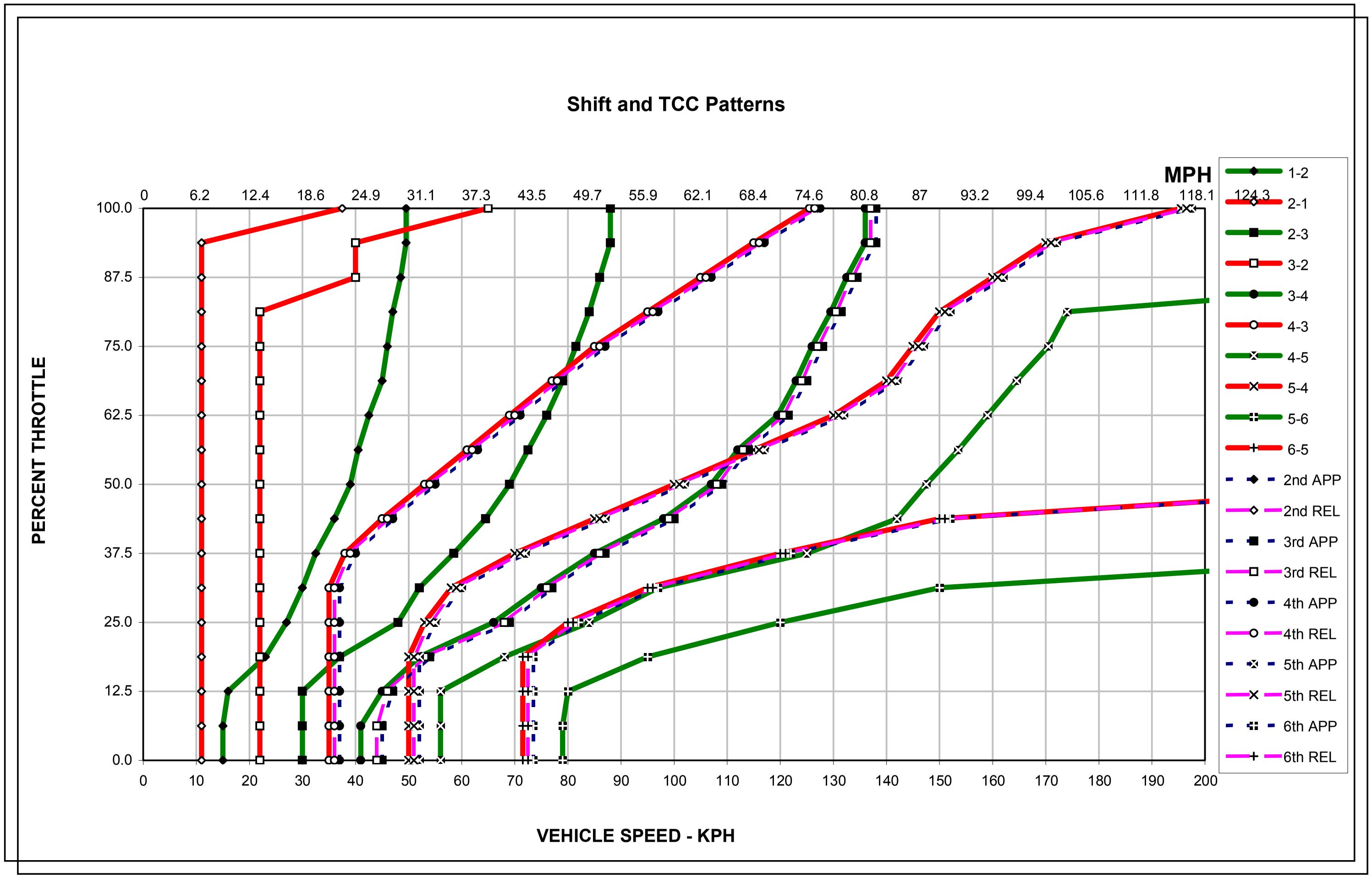


Figure A.1: Six-speed gear shifting program as a function of throttle and vehicle speed. Figure from lecture slides presented in AUTO 501, Fall 2006 by guest lecturer John Maten.

EngDepth equation assumes that the engine is a 60° vee configuration and provides an estimate for the depth of the engine at the engine base. The *EngExpMass* calculation comes from two linear regressions of engine data provided by Mike Anderson of GM in a lecture slide from a guest lecture in AUTO 501, Fall 2006 (See Figure A.2). The estimated length of the engines comes from summing the bore diameters, then using a 10 mm cylinder offset for line, or 26 mm cylinder offset for vee engines[1]. Additional length has been added to account for transmission package.

$$EngStroke\ [\text{mm}] = EngBore/EngBoretoStroke \tag{A.1}$$

$$EngDisp\ [\text{cc}] = EngNofCyl \times EngStroke/10 \times \pi(EngBore/2/10)^2 \tag{A.2}$$

$$EngDepth\ [\text{mm}] = 2EngBore\cos(\pi/180 \times 30) + 50 \tag{A.3}$$

$$\begin{aligned} EngExpMass\ [\text{kg}] &= 2.2046(1-\delta_{EngVee})\left(48.5\frac{EngDisp}{1000}+58\right) \\ &\quad +2.2046\delta_{EngVee}\left(28.57\frac{EngDisp}{1000}+103.57)\right) \end{aligned} \tag{A.4}$$

$$\begin{aligned} EngLength\ [\text{mm}] &= \delta_{EngVee}\left(\frac{EngNofCyl}{2}(EngBore+10)+16+25.4(3+6)\right) \\ &\quad +(1-\delta_{EngVee})\left(EngNofCyl(EngBore+10)-10+25.4(3+6)\right) \end{aligned} \tag{A.5}$$

Powertrain width requirement

{VehicleCG.m}

The maximum tire steering angle and the lateral tire flop distance are used to compute the powertrain width constraint. These values are a function of the minimum vehicle turn radius and the tire radius. Inequality A.8 shows the constraint that the powertrain and wheels must package within the overall vehicle width assuming a transverse engine orientation following Figure A.3[2]. The tire steering angle is the

[1] As per an example given from the Mike Anderson AUTO 501 slides

[2] Anderson, M., K, 2005, Powertrain Design and Integration, Lecture Slides, Automotive Engineering 501, University of Michigan, guest lecture, Ann Arbor, MI, 7.

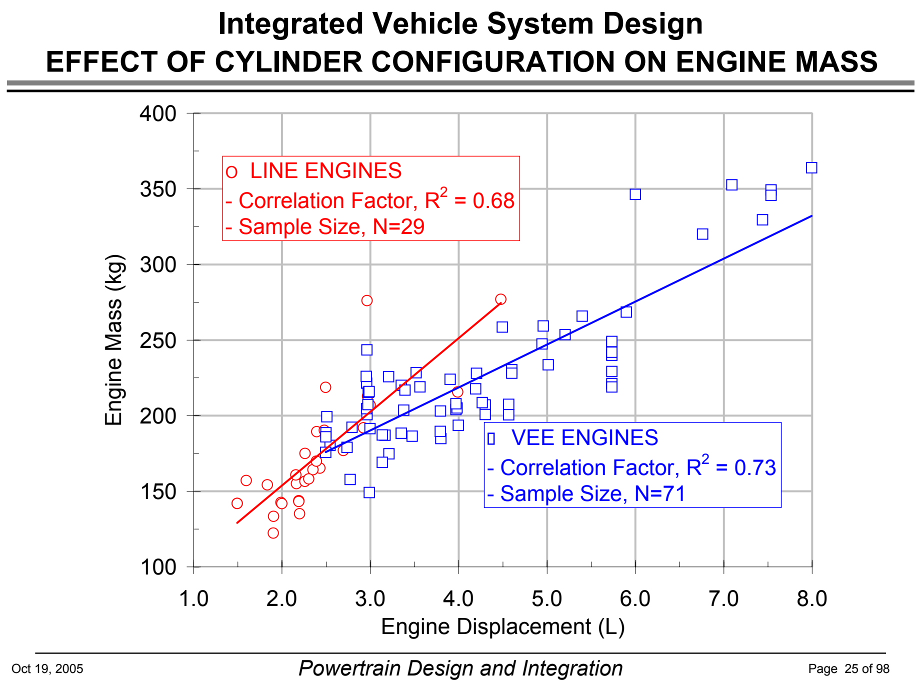


Figure A.2: Engine mass as a function of configuration and displacement. Figures and linear regressions of engine data provided by Mike Anderson of GM in a lecture slide from a guest lecture in AUTO 501, Fall 2006

maximum steering angle of inside wheel required to achieve a given vehicle turning radius[3]. The tire flop is a distance in the vehicle width (transverse) dimension assumed to be width needed for minimum turn radius + 1.5 inches.

$$TireSteerA\ [\text{-}] = \arctan\left(L101/(VehTurnRad - W101/2)\right) \tag{A.6}$$

$$\begin{aligned} TireFlop\ [\text{mm}] &= TireDynRollRad \sin TireSteerA \\ &\quad + \frac{8(25.4)}{2} \cos TireSteerA + 1.5(25.4) \end{aligned} \tag{A.7}$$

$$\begin{aligned} W105 - 254 &\geq 2TireFlop + 2MidRailWidth \\ &\quad + EngLength + 50.8 \end{aligned} \tag{A.8}$$

[3]This equation came from from Dr. Thomas Gillespie guest lecture slides in AUTO 501, Fall 2006

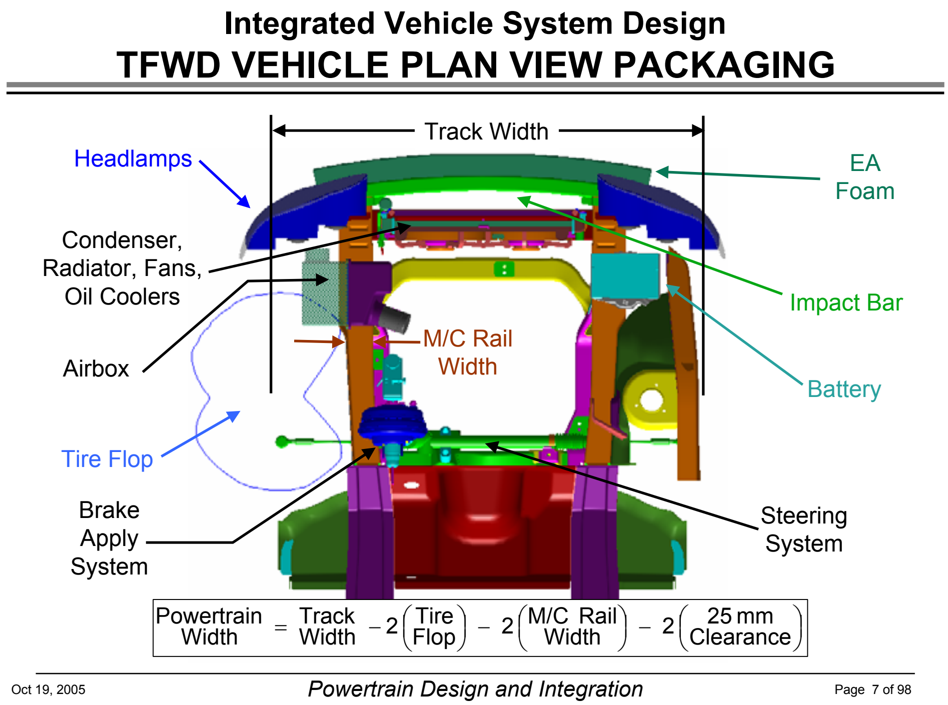


Figure A.3: Powertrain packaging in vehicle width dimension. Figures provided by Mike Anderson of GM in a lecture slide from a guest lecture in AUTO 501, Fall 2006

Other dimension conversions

{VehicleCG.m}

Three other dimension conversions are required for computing other vehicle characteristics or for input into the *AVL Cruise* powertrain simulations. They are the trackwidth $W101$, the vehicle frontal area $VehFrontArea$, and the distance from the front axle to the trailer hitch $VehHitchtoFAxle$.

$$\begin{aligned} W101\ [\text{mm}] &= W105 - 10 \times 25.4 && \text{(A.9)} \\ VehFrontArea\ [\text{m}^2] &= \frac{W105}{1000}\frac{H101}{1000} - \frac{H156}{1000}\frac{W101}{1000} && \text{(A.10)} \\ VehHitchtoFAxle\ [\text{mm}] &= L103 + 4 \times 25.4 - L104 && \text{(A.11)} \end{aligned}$$

Vehicle mass properties

`{VehicleCG.m}`

NOTE: Throughout the Vehicle mass properties section all vehicle dimensions are assumed to be given in inches unless otherwise stated, and all masses are assumed to be given in pounds-mass. This is different than the rest of the report where the default specifications are millimeters and kilograms.

Vehicle mass properties consist of the vehicle curbweight and gross vehicle weight rating, the distribution of mass among vehicle subsystems, the spatial distribution of mass throughout the vehicle, and the wheel assembly inertia. Derivation of each property is given below.

Curbweight and GVWR

`{VehicleCG.m}`

A regression was fit (R^2:0.92) to estimate curbweight for each light-duty truck vehicle class using data for 2005 light-duty trucks from Ward's automotive yearbook [24]. Each of the 8 class was fit with one of three different forms of the regression equation. Here, δ_{FWD}, δ_{AWD}, δ_{4WD}, δ_{RWD} are dummy variables {0,1} for driveline configuration; δ_{Cargo} is a dummy variable for specifying cargo van vs. passenger van; $\delta_{TCrew}, \delta_{TExtended}, \delta_{TRegular}$ are dummy variables for pickup cab configuration. The regression for the CUV class is a function of shadow area $VehPArea$ and engine displacement. The regression for the LxSUV class which includes all sizes of luxury SUVs and CUVs is a function of exterior box volume $VehPVol$ and engine displacement. The regressions for the remaining classes are functions of exterior box volume and $EngExpMass$.

Table A.3 indicates the equation used for each class and the R^2 value for each fit. Table A.4 lists the coefficient values for the vehicle mass equations for each class.

$$\begin{aligned}
VehMass_1 &= C_1\left(\frac{VehPArea}{100}\right)^2 + C_2 VehPArea + C_3(1-\delta_{EngDiesel})\frac{EngDisp}{10} \\
&+ C_4\delta_{EngDiesel}\frac{EngDisp}{10} + C_5\delta_{FWD} + C_6\delta_{AWD} + C_7\delta_{4WD} + C_8\delta_{RWD} \\
&+ C_9\delta_{Cargo} + C_{10}\delta_{TCrew} + C_{11}\delta_{TExtended} + C_{12}\delta_{TRegular} + C_{13} \quad \text{(A.12)} \\
VehPArea &= \frac{L103}{12} \times \frac{W105}{12} \quad \text{(A.13)} \\
VehMass_2 &= C_1\left(\frac{VehPVol}{100}\right)^2 + C_2 VehPVol + C_3(1-\delta_{EngDiesel})EngExpMass \\
&+ C_4\delta_{EngDiesel}\frac{EngDisp}{10} + C_5\delta_{FWD} + C_6\delta_{AWD} + C_7\delta_{4WD} + C_8\delta_{RWD} \\
&+ C_9\delta_{Cargo} + C_{10}\delta_{TCrew} + C_{11}\delta_{TExtended} + C_{12}\delta_{TRegular} + C_{13} \quad \text{(A.14)} \\
VehPVol &= \frac{L103}{12} \times \frac{W105}{12} \times \frac{H101}{12} \quad \text{(A.15)} \\
VehMass_3 &= C_1\left(\frac{VehPVol}{100}\right)^2 + C_2 VehPVol + C_3(1-\delta_{EngDiesel})\frac{EngDisp}{10} \\
&+ C_4\delta_{EngDiesel}\frac{EngDisp}{10} + C_5\delta_{FWD} + C_6\delta_{AWD} + C_7\delta_{4WD} + C_8\delta_{RWD} \\
&+ C_9\delta_{Cargo} + C_{10}\delta_{TCrew} + C_{11}\delta_{TExtended} + C_{12}\delta_{TRegular} + C_{13} \quad \text{(A.16)}
\end{aligned}$$

Gross vehicle weight rating (GVWR) is the total loaded weight of the vehicle including the vehicle, occupants, and other payload that should not be exceeded during operation of the vehicle. While this value is not critical to the vehicle simulations, is a recognized vehicle specification. It serves as a check on the vehicle design to ensure that the expected GVWR is sufficient for the expected vehicle capacity. GVWR is estimated based on class average payload percentages of the GVWR. Table A.5 lists, and Figure A.4 shows graphically, the average fraction of the GVWR that is allo-

Table A.3: Vehicle curbweight regression equation by vehicle class

Class	Eqn.	R^2	Sample
CUV	1	0.92	98
Lpkup	2	0.84	395
LSUV	2	0.94	78
LVAN	2	0.80	56
MSUV	2	0.90	127
Spkup	2	0.70	87
SVAN	2	0.56	70
LxSUV	3	0.83	68

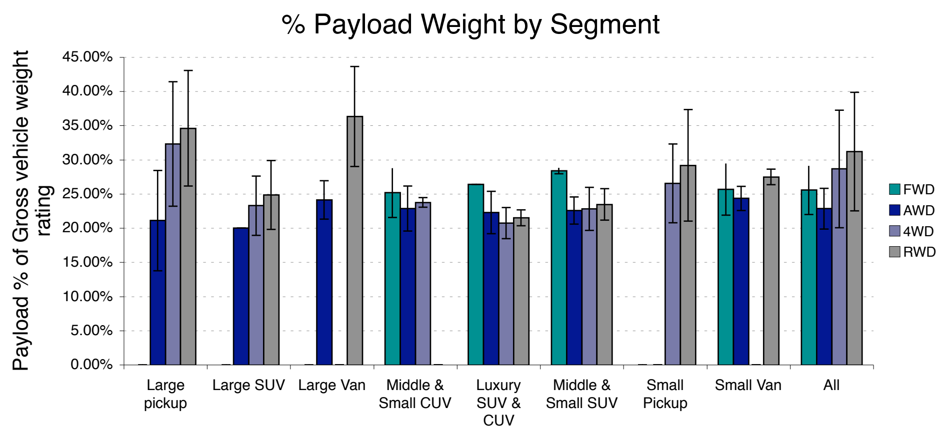


Figure A.4: Payload percent of GVWR by vehicle class and driveline

cated to payload (i.e., passengers and cargo) in terms of vehicle class and driveline configuration.

GVWR is then assigned to a vehicle based on the following equation.

$$GVWR = VehMass/(1 - VehPayload\%) \qquad \text{(A.17)}$$

Payload capacity is determined by taking the difference of GVWR and $VehMass$. Minimum cargo capacity by mass is the difference between the payload capacity and

Table A.4: Vehicle curbweight coefficients by vehicle class

Class		C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}
CUV	C= [	5141	-60.82	1.992	0.0	2001	2180	2106	1936	0.0	0.0	0.0	0.0	2233
Lpkup		-106.6	23.13	3.031	4.263	0.0	-614.8	-647.9	-869.1	0.0	-57.93	-88.76	-54.51	-7140
LSUV		734.3	-99.95	2.000	3.455	0.0	2247	2183	1985	0.0	0.0	0.0	0.0	35870
LVAN		-53.66	12.21	4.842	7.420	0.0	-1913	0.0	-1813	-288.3	0.0	0.0	0.0	-1729
MSUV		-146.1	21.27	2.490	0.0	-1801	-1526	-1583	-1740	0.0	0.0	0.0	0.0	-2650
Spkup		0.0	4.632	2.521	0.0	0.0	197.2	37.73	-162.1	0.0	2216	2000	2002	-1938
SVAN		-744.6	92.44	2.264	0.0	-14820	-14600	0.0	-14890	-366.7	0.0	0.0	0.0	-10590
LxSUV		-53.42	11.60	2.344	3.793	-415.6	-154.6	100.6	-480.0	0.0	0.0	0.0	0.0	-955.4]

Table A.5: Payload capacity percent of GVWR

Class		FWD	AWD	4WD	RWD
CUV	VehPayload%= [	25.2	22.9	23.8	25.2
Lpkup		34.6	21.1	32.3	34.6
LSUV		24.9	20.0	23.3	24.9
LVAN		36.3	24.1	24.1	36.3
MSUV		28.4	22.6	22.8	23.5
Spkup		29.2	26.6	26.6	29.2
SVAN		25.7	24.4	24.4	27.5
LxSUV		26.4	22.3	20.7	21.5]

the average fully occupied passenger weight.

$$VehPayloadMass = GVWR - VehMass \quad (A.18)$$

$$VehCargoFullMass = VehPayloadMass - VehSeatCap \times 150 \text{ lbm} \quad (A.19)$$

Mass breakdown

{VehicleCG.m}

Vehicle mass is distributed between the several vehicle subsystems using estimated vehicle mass, GVWR, and estimated engine mass, and a baseline mass distribution[4] (See Figure A.5). Powetrain mass is estimated and taken from the total vehicle mass, then the remaining mass is divided between the remaining subsystems preserving the same relative ratios as the parition given in Figure A.5. Figure A.6 shows an example applied to the CUV segment. $VehMass$, $EngExpMass$, and $GVWR$ to estimate the remaining mass of the vehicle. Powertrain mass for FWD vehicles excluding engine is assumed to be 5% of vehicle curbweight[5]. Powertrain mass in Figure A.5 is 14% and payload is 33% of GVWR, leaving 53% remaining. Payload mass was 25.2%, and powertrain mass was $0.05 \times VehMass + EngExpMass (\approx 11.4\% VehMass) = 16.4\%/(100 - 25.2\%) = 12.3\%$ of GVWR in the crossover example. The mass of each

[4]Baseline mass distribution comes from a slide presented in AUTO 501 by guest lecturer Dr. Don Malen. He proposed the distribution as"typical of a body-frame integral mid-size vehicle"

[5]Anderson, M., K, 2005, Powertrain Design and Integration, Lecture Slides, Automotive Engineering 501, University of Michigan, guest lecture, Ann Arbor, MI, 25-64.

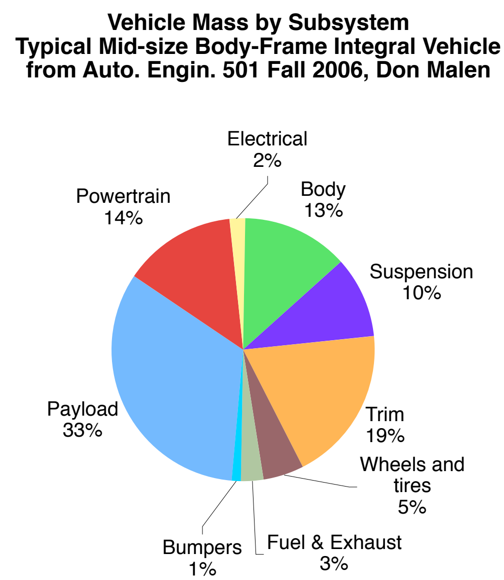


Figure A.5: Mass breakdown by vehicle subsystem

subsystem given as a percentage of the remaining mass is then

$$M_{subsystem}\% = \frac{M_{subsytembase}\%}{100 - M_{payloadbase}\% - M_{pwtrnbase}\%} \times (100 - M_{payload}\% - M_{pwtrn}\%) \quad \text{(A.20)}$$

$$M_{pwtrn}\% = M_{EngExpMass}\% + M_{Driveline}\% \quad \text{(A.21)}$$

The subsystem mass percent breakdown as a percentage of GVWR (as shown in the pie chart on the right in Figure A.6) is then electrical, 2%; body, 15%; suspension, 12%; trim, 22%; wheels, 6%; fuel and exhaust 4%; bumpers, 1%. The vehicle mass is broken down further for some subsystems. $VehGasTMass$ is found from the fuel and exhaust subsystem by

$$VehGasTMass = VehGasTVol \times VehGasDens \quad \text{(A.22)}$$

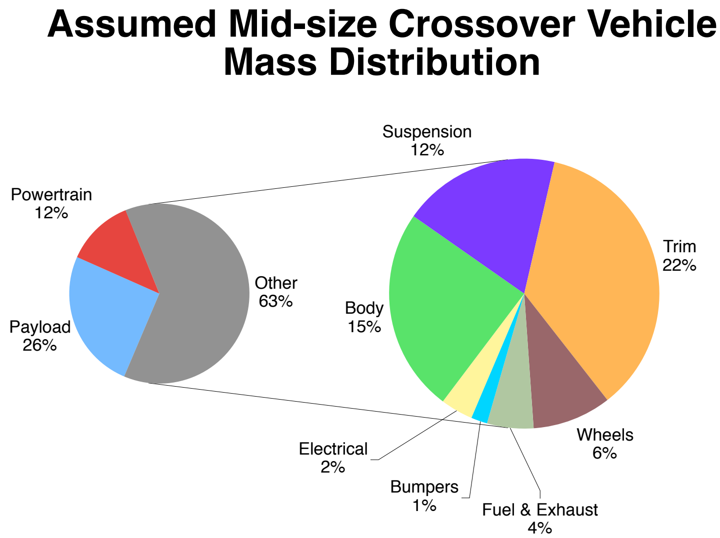


Figure A.6: Mass breakdown by vehicle subsystem for mid-size crossover

The remaining fuel and exhaust mass is assigned to the exhaust system

$$ExhaustMass = M_{fuel\&exhaust}\% - VehGasTMass \tag{A.23}$$

Wheel mass is 1/4 of the wheels subsystem and includes tire and wheel assembly.

$$WheelMass = M_{Wheels}\% \times VehMass/4 \tag{A.24}$$

We assume 20% of driveline mass is located at each axle. and that the remaining driveline mass is concentrated in the transmission box.

$$VehFrontAxleMass = 0.2 \times 0.05 \times VehMass \tag{A.25}$$

$$VehRearAxleMass = 0.2 \times 0.05 \times VehMass \tag{A.26}$$

$$TranMass = M_{Driveline}\% \times VehMass - VehFrontAxleMass - VehRearAxleMass \tag{A.27}$$

Mass distribution

`{VehicleCG.m}`

The model estimates the longitudinal and vertical position of the vehicle center of mass as measured from the ground plane and the frontmost point on a base equipped model. The relationships developed here are derived by estimating mass allocations as described in the previous section and the spatial distribution of this mass as described in this section. Vehicle occupancy, passenger weight, driver and second row legroom, and other vehicle dimensions are inputs.

One purpose of establishing center of mass position is to populate the center of gravity table for the *AVL Cruise* software powertrain simulations. *AVL Cruise* interpolates between the table values to find the values for the test specified vehicle loads. Distributed or point loads, as appropriate, were postulated for each subsystem mass (See Figures A.7 and A.8). Locations of passenger and cargo masses were incorporated for different loading conditions by considering legroom and overall vehicle length. Three load cases (i.e., curbweight-no occupants; ≈half occupants-half cargo; GVWR-full occupants-full cargo) are used to the center of mass positions and hitch heights for each table entry. A sprung mass vertical deflection of 2 inches is assumed between curbweight and GVWR. The relative percent weight on each axle is also

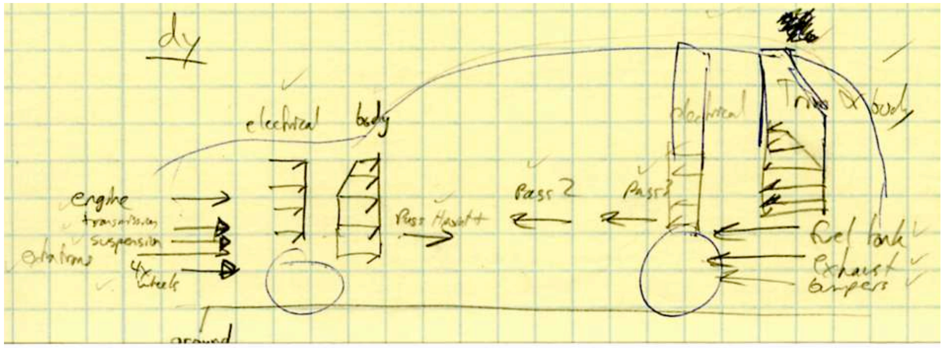


Figure A.7: Sketch of vertical mass distribution

computed. The height from the ground to the top of the wheel (excluding tire), and the height from ground to the top of the tire are give as follows.

$$VehTopofWheel = H108 + 1/2VehWheelDiam \tag{A.28}$$

$$VehTopofTire = H108 + TireDynRollRad \tag{A.29}$$

Legroom dimensions as specified on vehicle comparison websites are 10 inches greater than corresponding SAE dimensions [*SAE International* (2005)]

$$L34 = VehLegRoom1 - 10 \tag{A.30}$$

$$L51\text{-}2 = VehLegRoom2 - 10 \tag{A.31}$$

$$L51\text{-}3 = VehLegRoom3 - 10 \tag{A.32}$$

The distance from the ground $H5$, and the distance from the cabin floor $H30$ to the seating guide reference points (SgRP or H-point) are defined as follows, where the -#

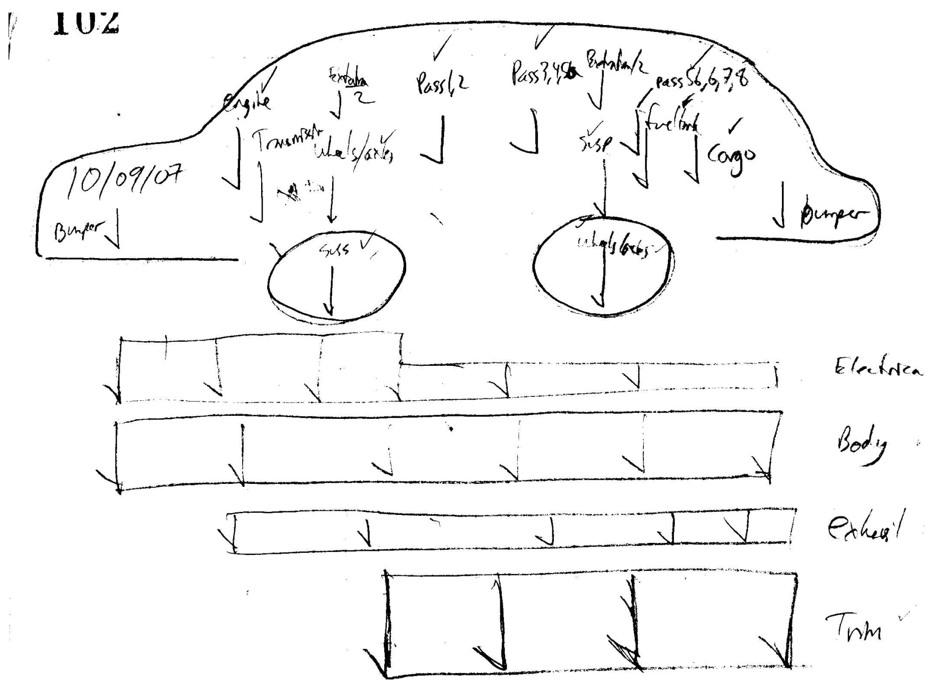


Figure A.8: Sketch of longitudinal mass distribution

corresponds to row number.

$$H\mathit{5\text{-}1} = H\mathit{156} + 18 \quad (A.33)$$

$$H\mathit{5\text{-}2} = H\mathit{5\text{-}1} + 2 \quad (A.34)$$

$$H\mathit{5\text{-}3} = H\mathit{5\text{-}2} + 1 \quad (A.35)$$

$$H\mathit{30} = H\mathit{5\text{-}1} - H\mathit{156} - 5 \quad (A.36)$$

The vertical position of the engine, transmission, front and rear suspension, and additional transmission mass (e.g., associated with AWD or *4*WD vehicles), gas tank, front and rear axles, height at the front of the hood, height at the cowl point, cargo load floor, front and rear bumper, and exhaust plumbing are assumed as follows.

$$EngY = VehTopofTire \tag{A.37}$$

$$TranY = H\mathit{103\text{-}1} + 8 \tag{A.38}$$

$$SuspensionFrontY = VehTopofWheel \tag{A.39}$$

$$SuspensionRearY = VehTopofWheel \tag{A.40}$$

$$ExtraTranMassY = VehTopofWheel \tag{A.41}$$

$$VehGasTY = VehTopofTire \tag{A.42}$$

$$VehFrontAxleY = H\mathit{108} \tag{A.43}$$

$$VehRearAxleY = H\mathit{108} \tag{A.44}$$

$$VehHoodHeight = 0.6H\mathit{101} \tag{A.45}$$

$$VehCowlHeight = 0.7H\mathit{101} \tag{A.46}$$

$$VehCargoY = H\mathit{195} + 1/3(H\mathit{101} - H\mathit{195}) \tag{A.47}$$

$$VehBumperHeightFront = H\mathit{103\text{-}1} + 6 \tag{A.48}$$

$$VehBumperHeightRear = H\mathit{103\text{-}2} + 6 \tag{A.49}$$

$$VehExhaustY = H\mathit{156} + 4 \tag{A.50}$$

The longitudinal position of the engine, transmission, passengers on rows 1-3, rear axle, gas tank, cargo load floor behind row 2 and row 3 are assumed as follows.

$$EngX = 1/2 VehHeelPointX \tag{A.51}$$

$$TranX = EngX + 10 \tag{A.52}$$

$$\begin{aligned} Veh1RX &= VehHeelPointX - \frac{81}{25.4}\cos A47 + \frac{107}{25.4}\cos(90 - A47) \\ &+ \sqrt{L34^2 - \left(H30 - \left(\frac{81}{25.4}\sin A47 + \frac{107}{25.4}\sin(90 - A47)\right)\right)^2} \end{aligned} \tag{A.53}$$

$$\begin{aligned} Veh2RX &= Veh1RX + \frac{76}{25.4} + \frac{306 - 81}{25.4} \\ &+ \sqrt{VehLegRoom2^2 - \left(H5\text{-}2 - H156 - 5 - \frac{107}{25.4}\right)^2} \end{aligned} \tag{A.54}$$

$$\begin{aligned} Veh3RX &= Veh2RX + \frac{76}{25.4} + \frac{306 - 81}{25.4} \\ &+ \sqrt{VehLegRoom3^2 - \left(H5\text{-}3 - H156 - 5 - \frac{107}{25.4}\right)^2} \end{aligned} \tag{A.55}$$

$$VehRearAxleX = L104 + L101 \tag{A.56}$$

$$VehGasTX = VehRearAxleX + VehWheelDiam/2 \tag{A.57}$$

$$VehCargoX2 = Veh2RX + 10 + 1/2(L103 - (Veh2RX + 10)) \tag{A.58}$$

$$VehCargoX3 = Veh3RX + 10 + 1/2(L103 - (Veh3RX + 10)) \tag{A.59}$$

Several subsystems are treated as distributed loads including body, exhaust, trim, and electric. The vertical distributed loads are described as follows.

$$\begin{aligned}
WBodyUnderHood &= (BodyMass \times VehHeelPointX/L103) \\
&\quad /(VehCowlHeight + VehHoodHeight(1-a) - H103 - 1 \\
&\quad +(a-1)/(VehCowlHeight - VehHoodHeight) \\
&\quad (VehCowlHeight^2/2 - VehHoodHeight^2/2)) && \text{(A.60)} \\
bBodyUnderHood &= WBodyUnderHood\left(1 - \frac{VehHoodHeight(a-1)}{VehCowlHeight - VehHoodHeight}\right) && \text{(A.61)} \\
WElectUnderHood &= (0.66ElectMass)/(VehCowlHeight - H103 - 1) && \text{(A.62)} \\
WTrim &= TrimMass/(H101 + VehCowlHeight(1-a) - H156 \\
&\quad +(a-1)/(H101 - VehCowlHeight) \\
&\quad \times(H101^2/2 - VehCowlHeight^2/2)) && \text{(A.63)} \\
bTrim &= WTrim\left(1 - \frac{VehCowlHeight(a-1)}{H101 - VehCowlHeight}\right) && \text{(A.64)} \\
WBodyPostHeel &= (L103 - VehHeelPointX)/L103BodyMass \\
&\quad /(H101 + VehCowlHeight(1-a) - H156 + (a-1) \\
&\quad /(H101 - VehCowlHeight)(H101^2/2 - VehCowlHeight^2/2)) && \text{(A.65)} \\
bBodyPostHeel &= WBodyPostHeel\left(1 - \frac{VehCowlHeight(a-1)}{H101 - VehCowlHeight}\right) && \text{(A.66)} \\
WElectPostHeel &= (0.34ElectMass)/(H101 - H156) && \text{(A.67)}
\end{aligned}$$

The next step is to compute the mass weighted vertical center of mass locations for each distributed load as follows, where $a = 0.25$ is a coefficient splitting trim and body weight in the vertical direction between above and below cowl point.

$$
\begin{aligned}
VGBodyUnderHood &= WBodyUnderHood(1/2(VehHoodHeight^2 - H103\text{-}1^2) \\
&+ \frac{a-1}{VehCowlHeight - VehHoodHeight} \\
&\times (VehCowlHeight^3/3 - VehHoodHeight^3/3)) \\
&+ bBodyUnderHood/2(VehCowlHeight^2 - VehHoodHeight^2)
\end{aligned} \tag{A.68}
$$

$$
VCGElectUnderHood = WElectUnderHood(VehCowlHeight^2/2 - H103 - 1^2/2) \tag{A.69}
$$

$$
\begin{aligned}
VCGTrim &= WTrim(1/2(VehCowlHeight^2 - H156^2) \\
&+ \frac{a-1}{(H101 - VehCowlHeight}(H101^3/3 - VehCowlHeight^3/3)) \\
&+ bTrim/2(H101^2 - VehCowlHeight^2)
\end{aligned} \tag{A.70}
$$

$$
\begin{aligned}
VCGBodyPostHeel &= WBodyPostHeel(1/2(VehCowlHeight^2 - H156^2) \\
&+ \frac{a-1}{H101 - VehCowlHeight}(H101^3/3 - VehCowlHeight^3/3)) \\
&+ bBodyPostHeel/2(H101^2 - VehCowlHeight^2)
\end{aligned} \tag{A.71}
$$

$$
VCGElectPostHeel = WElectPostHeel(H101^2/2 - H156^2/2) \tag{A.72}
$$

The longitudinal distributed loads are described as follows.

$$
WBody = BodyMass/L103 \tag{A.73}
$$

$$
WExhaust = ExhaustMass/(L103 - EngX) \tag{A.74}
$$

$$
\begin{aligned}
WTrimL &= TrimMass/((Veh2RX - VehHeelPointX) \\
&+ (L103 - Veh2RX))
\end{aligned} \tag{A.75}
$$

$$
WElectUnderHoodL = 0.66 ElectMass/VehHeelPointX \tag{A.76}
$$

$$
WElectPostHeelL = 0.34 ElectMass/(L103 - VehHeelPointX)
$$

The mass weighted longitudinal center of mass locations for each distributed load are as follows, where $b = 0.25$ is a coefficient splitting trim weight between seat area and cargo area.

$$LCGBody = WBody(L103^2/2) \quad (A.77)$$

$$LCGExhaust = WExhaust(L103^2/2 - EngX^2/2) \quad (A.78)$$

$$\begin{aligned} LCGTrim &= WTrimL(Veh2RX^2/2 - VehHeelPointX^2/2) \\ &+ b \times WTrimL(L103^2/2 - Veh2RX^2/2) \quad (A.79) \end{aligned}$$

$$LCGElectUnderHood = WElectUnderHoodL(VehHeelPointX^2/2) \quad (A.80)$$

$$LCGElectPostHeel = WElectPostHeelL(L103^2/2 - VehHeelPointX^2/2)$$

The vertical position of the center of mass is found by summing the mass weighted point and distributed loads and then dividing by the total mass of the system according to the following expression.

$$\begin{aligned} CG_{vert} = {} & (EngY \times EngExpMass + H5\text{-}1 \times PassWeight(\delta_{pass1} + \delta_{pass2}) \\ & + H5\text{-}2 \times PassWeight(\delta_{pass3} + \delta_{pass4} + \delta_{pass5}) \\ & + H5\text{-}3 \times PassWeight(\delta_{pass6} + \delta_{pass7} + \delta_{pass8} + \delta_{pass9}) \\ & + VehGasTY \times VehGasTMass \\ & + VehFrontAxleY(2WheelMass + VehFrontAxleMass) \\ & + VehRearAxleY(2WheelMass + VehRearAxleMass) \\ & + VehCargoY \times VehCargoMass + SuspenMass/2(SuspensionFrontY) \\ & + SuspenMass/2(SuspensionRearY) \\ & + ExtraTranMass \times ExtraTranMassY + TranMass \times TranY \\ & + (ExhaustMass) \times VehExhaustY \\ & + (0.5BumperMass)VehBumperHeightFront \\ & + (0.5BumperMass)VehBumperHeightRear \\ & + VertCGBodyUnderHood + VertCGElectUnderHood + VertCGTrim \\ & + VertCGBodyPostHeel + VertCGElectPostHeel) \\ & /(VehMass + [PassWeight]\mathbf{pass} + VehCargoMass) \quad (A.81) \end{aligned}$$

The longitudinal position of the center of mass is found by the following expression.

$$\begin{aligned}
CG_{long} = & (EngX \times EngExpMass + Veh1RX \times PassWeight(\delta_{pass1} + \delta_{pass2}) \\
& +Veh2RX \times PassWeight(\delta_{pass3} + \delta_{pass4} + \delta_{pass5}) \\
& +Veh3RX \times PassWeight(\delta_{pass6} + \delta_{pass7} + \delta_{pass8} + \delta_{pass9}) \\
& +VehGasTX \times VehGasTMass \\
& +L104(2WheelMass + VehFrontAxleMass) \\
& +VehRearAxleX(2WheelMass + VehRearAxleMass) \\
& +VehCargoX \times VehCargoMass + LongCGBody + LongCGExhaust \\
& +LongCGTrimL + LongCGElectUnderHoodL + LongCGElectPostHeelL \\
& +SuspenMass/2(L104) + SuspenMass/2(VehRearAxleX) \\
& +ExtraTranMass/2(L104) + ExtraTranMass/2(VehRearAxleX) \\
& +TranX \times TranMass \\
& +BumperMass/2(VehBumperDepth/2) \\
& +BumperMass/2(L103 - VehBumperDepth/2)) \\
& /(VehMass + [PassWeight]\mathbf{pass} + VehCargoMass) \qquad (A.82)
\end{aligned}$$

The relative mass split between the front and rear wheels can be found by solving for the reaction forces at the wheels.

$$R_2 = VehMass(VehLongCG - L104)/L101 \qquad (A.83)$$

$$R_1 = VehMass - R_2 \qquad (A.84)$$

$$VehM\%_{Front} = 100R_1/VehMass \qquad (A.85)$$

$$VehM\%_{Rear} = 100R_2/VehMass \qquad (A.86)$$

Wheel assembly inertia

{VehicleCG.m}

Wheel assembly inertia is an input to the *AVL Cruise* powertrain simulations. This quantity is estimated by approximating the tire and the wheel as a ring and a wheel respectively with mass evenly distributed throughout.

$$
\begin{aligned}
RubberInertia &= TireMass\left(\frac{TireDynRollRad}{1000}\right)^2 \\
&\times(1-\frac{\frac{TireDynRollRad}{1000}-\frac{25.4VehWheelDiam}{2\times1000}}{\frac{TireDynRollRad}{1000}} \\
&+\frac{1}{2}(\frac{\frac{TireDynRollRad}{1000}-\frac{25.4VehWheelDiam}{2\times1000}}{\frac{TireDynRollRad}{1000}})^2) \qquad (A.87)\\
RimInertia &= \frac{WheelMass-TireMass}{2}\left(\frac{25.4VehWheelDiam}{2\times1000}\right)^2 \qquad (A.88)\\
WheelInertia &= RubberInertia+RimInertia \qquad (A.89)
\end{aligned}
$$

Angle of approach, departure, and ramp breakover

{VehicleCG.m}

SAE standard SAE J689 [*SAE International* (2005)] specifies minimum angles and curb clearances evaluated at the manufacturer's most severe design load, which we take to be GVWR. Flexible bumper components should also be considered. Table A.6 lists the standards. Curbstone clearance height is measured behind the back wheels and in front of the front wheels. The dimensions *H103-1*, *H103-2* are chosen in our model to represent curbstone clearance height. These SAE dimensions are specifically assigned to the height between the ground and the front and rear fascia respectively, but in application may not necessarily represent the shortest clearance height. Fig-

Table A.6: SAE J689 [*SAE International* (2005)] minimum standards for road clearance vehicle geometry

SAE Dim.	Standard	Description
A*106*	16°	Angle of approach
A*107*	13°	Angle of departure
A*147*	12°	Ramp breakover angle
H*103-1*	203mm	Front curbstone clearance height
H*103-2*	203mm	Rear curbstone clearance height

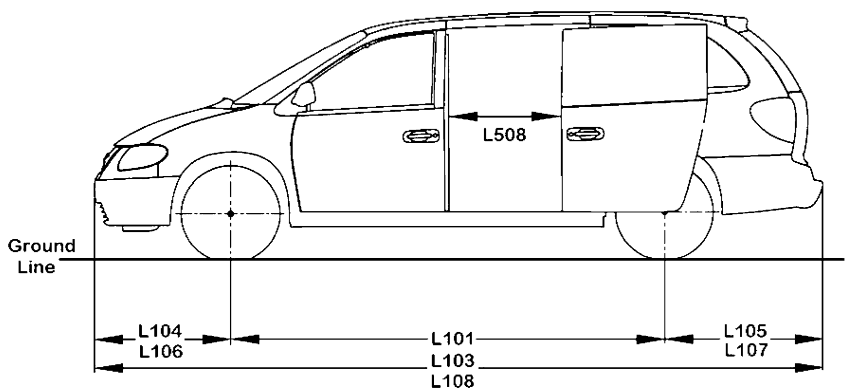


FIGURE 17—SIDE VIEW

Figure A.9: Vehicle side view from SAE J1100. Figure from [*SAE International* (2005)]

ures A.9 & A.10 come from SAE J1100 and label dimensions used to evaluate SAE J689 in our model.

The longitudinal distance measured from the front axle to the lowest point (ground clearance) between the wheels. Although the ground clearance point is typically biased towards the front axle, at present we assume it is located midway between axles. *L105* is the distance from the rear axle to the end of the back bumper.

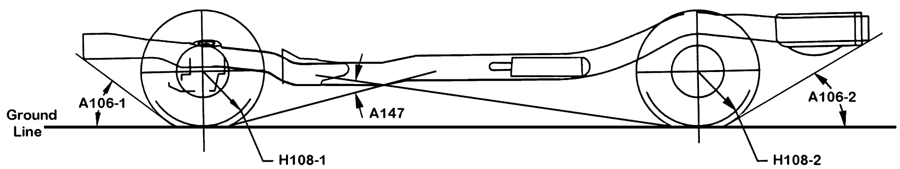


FIGURE 25—SIDE VIEW OF CHASSIS

Figure A.10: Chassis side view from SAE J1100. Figure from [*SAE International* (2005)]

$$l_2 = 1/2 \times L101 \tag{A.90}$$

$$L105 = L103 - L104 - L101 \tag{A.91}$$

Angle of approach

Required inputs for angle of approach calculation are the front ground clearance height *H103-1*, the static loaded tire radius *H108*, the front overhang distance *L104*, and the unloaded tire radius *TireDynRollRad*.

$$L104' = \sqrt{L104^2 + (H108 - H103\text{-}1)^2} \tag{A.92}$$

$$\theta_8' = \arccos\left(TireDynRollRad/L104'\right) \tag{A.93}$$

$$\theta_7 = \arcsin\left((H108 - H103\text{-}1)/L104'\right) \tag{A.94}$$

$$\theta_{0a} = 90 - \theta_7 - \theta_8' \tag{A.95}$$

$$L104_{bit} = TireDynRollRad \sin\theta_{0a} \tag{A.96}$$

$$L104_{part} = L104 - L104_{bit} \tag{A.97}$$

$$L104'_{fa} = \sqrt{L104'^2 - TireDynRollRad^2} \tag{A.98}$$

$$A106 = \arccos\left(L104_{part}/L104'_{fa}\right) \tag{A.99}$$

Angle of departure

Required inputs for angle of departure calculation are rear ground clearance *H103-2*, static loaded tire radius *H108*, distance from rear of vehicle to rear wheel axle *L105*, and the unloaded tire radius $TireDynRollRad$.

$$L105' = \sqrt{L105^2 + (H108 - H103\text{-}2)^2} \quad (A.100)$$

$$\text{if}(\frac{TireDynRollRad}{L105'} > 1)$$

$$A107 = 90 \quad (A.101)$$

else

$$\theta'_{10} = \arccos\left(TireDynRollRad/L105'\right) \quad (A.102)$$

$$\theta_9 = \arcsin\left((H108 - H103\text{-}2)/L105'\right) \quad (A.103)$$

$$\theta_{0d} = 90 - \theta_9 - \theta'_{10} \quad (A.104)$$

$$L105_{bit} = TireDynRollRad \sin\theta_{0d} \quad (A.105)$$

$$L105_{part} = L105 - L105_{bit} \quad (A.106)$$

$$L105'_{rd} = \sqrt{L105'^2 - TireDynRollRad^2} \quad (A.107)$$

$$A107 = \arccos\left(L105_{part}/L105'_{rd}\right) \quad (A.108)$$

Ramp breakover angle

Required inputs for ramp breakover anlge are ground clearance *H156-3*, static loaded wheel radius *H108*, longitudinal distance from ground clearance to front wheel axlel_2, wheelbase *L101*, and the unloaded tire radius $TireDynRollRad$.

$$l_2' = \sqrt{(l_2^2 + (H108 - H156)^2)} \tag{A.109}$$

$$\theta_5' = \arccos(TireDynRollRad/l_2') \tag{A.110}$$

$$\theta_4 = \arcsin((H108 - H156)/l_2') \tag{A.111}$$

$$\theta_0 = 90 - \theta_4 - \theta_5' \tag{A.112}$$

$$l_{2bit} = TireDynRollRad \sin\theta_0 \tag{A.113}$$

$$l_{2part} = l_2 - l_{2bit} \tag{A.114}$$

$$l_{2fb}' = \sqrt{{l_2'}^2 - TireDynRollRad^2} \tag{A.115}$$

$$\theta_6 = \arcsin(l_{2part}/l_{2fb}') \tag{A.116}$$

$$l_1 = L101 - l_2 \tag{A.117}$$

$$l_1' = \sqrt{l_1^2 + (H108 - H156)^2} \tag{A.118}$$

$$\theta_2' = \arccos(TireDynRollRad/l_1') \tag{A.119}$$

$$\theta_1 = \arccos((H108 - H156)/l_1') \tag{A.120}$$

$$\theta_{0r} = 90 - \theta_1 - \theta_2' \tag{A.121}$$

$$l_{1bit} = TireDynRollRad \sin\theta_{0r} \tag{A.122}$$

$$l_{1part} = l_1 - l_{1bit} \tag{A.123}$$

$$l_{1rb}' = \sqrt{{l_1'}^2 - TireDynRollRad^2} \tag{A.124}$$

$$\theta_3 = \arcsin(l_{1part}/l_{1rb}') \tag{A.125}$$

$$A147 = 180 - \theta_6 - \theta_3 \tag{A.126}$$

Rollover

`{Rollover.m}`

The NCAP rollover rating system[6] was used to model a vehicles propensity for rollover. The predicted ratings in the model rely on the static stability factor SSF alone (i.e., no j-hook simulations have been developed). The star rating is a function of a vehicle's static stability factor according to the relationship plotted in Figure A.11. The static stability factor is estimated as follows. A polynomial regression was fit to approximate the relationship between rollover probability and static stability factor shown in Figure A.11. The verticle vehicle center of mass position used in the calculations is for a fully loaded vehicle.

$$
\begin{aligned}
SSF &= \frac{W101 \times VehHeighCG}{2} && \text{(A.127)} \\
RolloverScore &= 16.094SSF^4 - 86.233SSF^3 + 173.3SSF^2 \\
&\quad -155.14SSF + 52.444 && \text{(A.128)} \\
RolloverStar &= 5 - \text{floor}(10 \times RolloverScore) && \text{(A.129)}
\end{aligned}
$$

Cargo Volume

`{CargoVolume.m}`

Cargo volume calculations follow SAE J1100 [*SAE International* (2005)]. This standard includes 11 different formulas for computing a cargo volume index depending on the classification of the vehicle being measured. The pattern for the cargo volume calculations is to take the product of a representative length, width, and height. The various formulations are labeled in Table A.7 The formula included in the model at this point is V-10 CVI-2 (station wagon CVI-maximum behind second

[6]NHTSA, "SaferCar.Gov Rollover - FAQs," http://www.safercar.gov, 2008(2/4/2008)

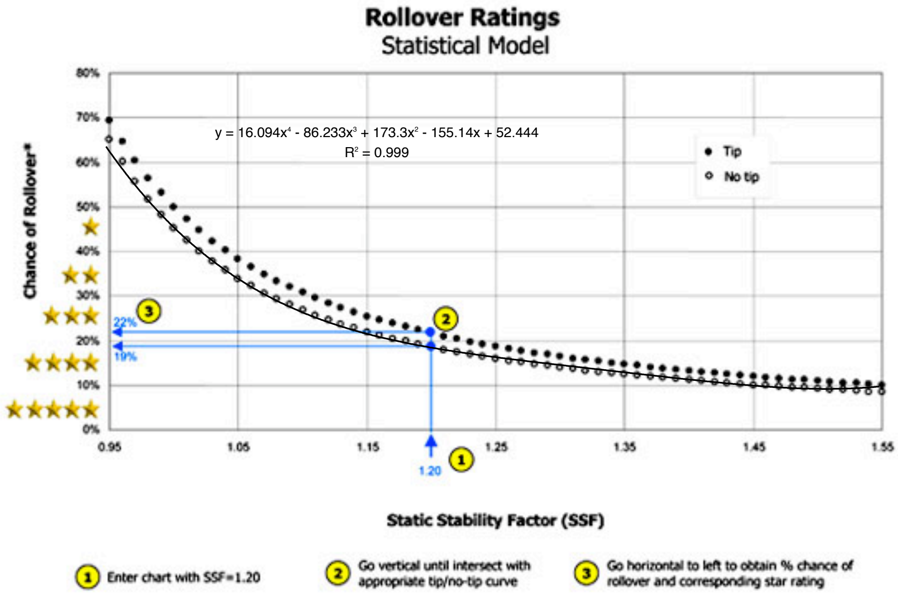


Figure A.11: NCAP rollover crash ratings as a function of static stability factor. Figure from www.safercar.gov

Table A.7: SAE J1100 cargo volume index formulations [*SAE International* (2005)]

Index label	Calculation	Description
V1	Total volume of the individual pieces of luggage plus H-boxes that can be stowed	This measure applies to hatchback and stations wagons if they are partitioned to secure hidden cargo
V2	$(L204\text{-}1) \times W3\text{-}2 \times H201$	Station Wagon CVI-Maximum estimate
V3	$\frac{L208\text{-}1+L209\text{-}1}{2} \times W3\text{-}2 \times H197\text{-}1$	Hatchback CVI-Maximum estimate
V4	measured the same as V1 for any hidden cargo area below the load floor, the rear of the front seat	This measure applies to hatchback and stations wagons if they are partitioned to secure hidden cargo
V5	$L506 \times W500 \times H503$	Open truck and MPV CVI-Maximum
V6	$\frac{L202\text{-}1+L204\text{-}1}{2} \times W3\text{-}2 + W2012 \times \frac{H201+H505}{2}$	Enclosed truck and MPV CVI - maximum behind front seat
V7	$\frac{L202\text{-}2+L204-2}{2} \times \frac{W3-3+W201}{2} \times \frac{H201+H505}{2}$	Enclosed truck and MPV CVI - maximum behind second seat
V9	$\frac{L202\text{-}3+L204\text{-}3}{2} \times \frac{W3\text{-}3+W201}{2} \times H201$	Enclosed truck and MPV - Maximum behind third seat
V10	$L204\text{-}2 \times \frac{W3\text{-}2+W201}{2} \times H201$	Station wagon CVI-maximum behind second seat
V11	$\frac{L208\text{-}2+L209\text{-}2}{2} \times W3\text{-}2 \times H197\text{-}2$	Hatchback cargo volume-maximum behind second seat

seat). Assuming the width of the interior between wheelhouse trim panel is the track width - 20 inches:

$$W201 = W101 - 20 \times 25.4 \tag{A.130}$$

Assuming the interior width at the second row is ≈ track width:

$$W3\text{-}2 = W101 \tag{A.131}$$

Assuming the interior width at third row is ≈ track width - 5 inches:

$$W3\text{-}3 = W101 - 5 \times 25.4 \tag{A.132}$$

Assuming cargo length from rearmost part of row 2 is Veh2RX-23 inches due to 15 inches to back of the seat and the seat recline angle, and 8 inches from rear bumper and door:

$$L204\text{-}2 = L103 - (Veh2RX - (15 + 8))25.4 \tag{A.133}$$

Assuming cargo length from rearmost part of row 3 is Veh3RX-20 inches due to 12 inches from seat recline and 8 inches from rear bumper and door:

$$L204\text{-}3 = L103 - (Veh3RX - (12 + 8))25.4 \tag{A.134}$$

Assuming cargo load floor is at the same height as the top of the tire and there is a 3 inch thick headliner + roof hardware:

$$H201 = H101 - (3 + VehTopofTire)25.4 \tag{A.135}$$

V-10: station wagon CVI-maximum behind second row seat:

$$
\begin{aligned}
CVI - 2m \text{ m}^3 &= \frac{L204\text{-}2 \times \frac{W3\text{-}2+W201}{2} \times H201}{1000^3} \\
CVI - 2 \text{ ft}^3 &= CVI - 2m \times \left(\frac{100}{2.54 \times 12}\right)^3
\end{aligned}
\tag{A.136}
$$

NOTE: The NCAP rollover star ratings, axle weight distributions (Equation A.85), and cargo volume index CVI for several vehicles compared favorably with those predicted by the model giving confidence in the approach. However, the model consistently underestimated both CVI and $RolloverScore$ as compared to sampled midsize crossovers. Cargo volume and rollover constraints were relaxed ($g_4(\min CVI)$ from 32 ft^3 to 29 ft^3, $g_6(\max RolloverScore)$ from 0.1999–a 4-star rating–to 0.21) to account for differences between the model and real world data.

Front crash

```
{FrontCrash.m}
```

The front crash model provides a back of the envelope check on powertrain packaging and vehicle geometry to reasonably ensure an appropriate front crash test score is acheivable. Parameter estimates required to complete the analysis include $VehicleBumperDepth$(8in), $MidRailYStrength$, $SuspenDepth$(5in), $PassWeight$, $EngDepth$, $TestFrontCrushEff$, $VehMass$, $MidRailLoadFactor$, $MidRailWidth$, $VehHeelPointX$, $MidRailThick$, $VehRadDepth$(6in). It should be noted that component dimensions represent the dimension in a longitudinal direction and represent the sum of the "thickness" of the solid elements of the component.

$MidRailLoadFactor$ is set equal to 1/2, implying that the mid-rails absorb 1/2 the total impact. $TestFrontCrushEff$ is a factor related to the buckling efficiency of the mid-rail design. We take a value of 0.7. Outputs predicted are the crush space available given driver heel point and underhood component sizes $CrushSpace$, average load on a single mid-rail $TestMidRailForceAvg$, and peak acceleration in a front crash $AccelTestFront$. $MinCrushSpace$ reflects the minimum required crush space to stop the vehicle in a front impact at a velocity of $TestFrontVel$, taken here to be 35 mph.

Mid-rails are assumed to be square cross-sections. Average force calculations come from a curve-fit supplied by an SAE paper[7] for square tubes with thickness and width constraints: $(width/thick) > 62.5$ and 1.5in $< width <$ 3.75in. Model constraints are placed on the minimum crush space and peak deceleration as shown in Section A. $EngDepth$ is given in millimeters and $VehMass$ is given in kg. $MidRailThick$ and $MidRailWidth$ are given in millimeters. $MidRailYStrength$ is given in N/mm^2. Other dimensions are given in inches and lbm respectively.

$$
\begin{aligned}
CrushSpace &= VehHeelPointX \\
&\quad -(25.4(VehicleBumperDepth + VehRadDepth + SuspenDepth) \\
&\quad +EngDepth) && \text{(A.137)} \\
MinCrushSpace &\quad \left(VehMass + \frac{PassWeight}{kgtolbs}\right) \times (0.44704 TestFrontVel)^2 \\
&\quad / \left(2\frac{1}{MidRailLoadFactor} \times 2TestMidRailForceAvg\right) && \text{(A.138)} \\
TestMidRailForceAvg &= 386(MidRailThick^{1.86}) \times (MidRailWidth^{0.14}) \\
&\quad \times(MidRailYStrength^{0.57}) && \text{(A.139)} \\
AccelTestFront &= \frac{1}{MidRailLoadFactor} \times 2TestMidRailForceAvg \\
&\quad / \left(TestFrontCrushEff\left(VehMass + \frac{PassWeight}{kgtolbs}\right)\right) && \text{(A.140)}
\end{aligned}
$$

[7]H. F. Hahmood and A. Paluszny, “Design of Thin Walled Columns for Crash Energy Management”, Society of Automotive Engineers Paper 811302

Powertrain performance characteristics

Powertrain performance is evaluated by referencing results from vehicle simulations conducted using *AVL Cruise* software. Typical procedure is to model the vehicle powertrain of interest within *Cruise*. Next, determine parameters to vary (i.e., design variables and other parameters of interest such as drag coefficient). Then, execute a design of experiments to obtain simulation results for all powertrain performance measures. Finally, fit the data using polynomials and/or neural nets for implementation with the other elements of the overall model.

Engine characteristics

`{EngScaling.m}`

`{EngScalingTC.m}`

`{EngScalingHEV.m}`

AVL Cruise characterizes engine performance by reference to engine maps derived from experimental results of a baseline engine. Different baseline engine data are used as baselines for fuel consumption maps and full load performance curves as well as between naturally aspirated and turbo-charged direct injection engines. Engine maps are scaled for each design iteration as functions of *EngBore* and *EngBoretoStroke* following established scaling relationships[8].

Using the peak mean piston speed and the engine geometry it is possible to calculate the engine peak RPM.

$$RPM_{peak} = \bar{S}_{p_{peak}} / (2EngStroke \times 10^{-3} - \times 60) \tag{A.141}$$

[8] Chon, D. M., and Heywood, J. B., 2000, Performance Scaling of Spark-Ignition Engines: Correlation and Historical Analysis of Production Engine Data, Technical Paper 2000-01-0565. Anderson, M., K, 2005, Powertrain Design and Integration, Lecture Slides, Automotive Engineering 501, University of Michigan, guest lecture, Ann Arbor, MI, 25-64.

We add 200 RPM to the peak power engine speed as an estimate of maximum engine speed.

At a constant mean piston speed, fuel mass flow rate and the full load performance torque curve are scaled as follows, where N_{cyl} is the number of cylinders of the engine. Scaling mass flow rate in this way assumes that, at a given piston speed, BMEP is constant between the base and the scaled engine. In most cases we assume both base and scaled engines have the same peak BMEP, resulting in Equation A.143 simplifying to be the ratio of the engine displacements (evaluated at a constant mean piston speed).

$$\frac{\dot{m}_{new}}{\dot{m}_{base}} = EngBore^2_{new} N_{cyl_{new}} / EngBore^2_{base} N_{cyl_{base}} \tag{A.142}$$

$$\frac{\tau_{new}}{\tau_{base}} = \frac{BMEP_{Ppeak_{new}} EngBore^3_{new} EngBoretoStroke_{base} N_{cyl_{new}}}{BMEP_{Ppeak_{base}} EngBore^3_{base} EngBoretoStroke_{new} N_{cyl_{base}}} \tag{A.143}$$

Peak power is found by taking the scaled torque at the peak mean piston speed from Equation A.143. Peak power given in horsepower is as follows.

$$P_{peak} = \tau_{new_{peak}} \times RPM_{peak}/5252 \tag{A.144}$$

The motoring curve, representing power losses in the engine at various speeds is updated by scaling power loss by engine displacement ratio. Normalization by constant piston speed is neglected for motoring curve. Engine idle fuel consumption is assumed to be 1.5 liters per hour for the *Cruise* provided 2.5 liter V-6 engine. Idle fuel consumption for unique engine designs is scaled by displacement ratio. The advanced friction module found in *Cruise*, which incorporates engine and valvetrain architecture, based on an SAE paper[9] is used to integrate frictional engine losses into the

[9]Patton, K. J., Nitschke, R. G., and Heywood, J. B., 1989, Development and Evaluation of a Friction Model for Spark-Ignition Engines, SAE Technical Paper.

Table A.8: Engine characteristics for 2.5 l V6 naturally aspirated gasoline engine

Stroke	82.17mm
Bore	80.0mm
Displacement	2478cc
Bore to Stroke	0.97
Compression ratio	10.0
Idle speed	700rpm
Max engine speed	6000rpm

Table A.9: Engine characteristics for 3.5 l V6 Duratec35 naturally aspirated gasoline engine

Stroke	86.6mm
Bore	92.5mm
Displacement	3491cc
Bore to Stroke	1.07
Compression ratio	10.3
Idle speed	640rpm
Max engine speed	6500rpm

simulations.

Naturally aspirated spark-ignition

The fuel consumption map for the conventional vehicle is taken from a 2.5 l, V-6 engine with BMEPPpeak=1068 kPa provided with the software. The full load characteristic is scaled from the Duratec35 engine (BMEPPpeak=1085 kPa) used in the Ford Edge. We assume the peak power brake mean effective pressure of the vehicle engine (1085 kPa) and mean piston speed at peak power out (18.1 m/s) are constant for all designs. Baseline engine details are summarized in Tables A.8-A.10.

Table A.10: Full load performance characteristic for 3.5 l V6 Duratec35 naturally aspirated gasoline engine

RPM	Torque (lb-ft)	Power (hp)	BMEP (kPa)	Piston Speed (m/s)
1500	133	38	649	4.3
2000	197	75	962	5.8
2500	210	100	1025	7.2
3000	233	133	1138	8.7
3500	255	170	1245	10.1
4000	263	200	1284	11.6
4500	251	215	1226	13.0
5000	247	235	1206	14.5
5500	244	256	1191	15.9
6000	228	260	1113	17.3
6250	223	265	1089	18.1
6500	213	264	1040	18.8

Gas turbo direct injection

The GTDI baseline fuel consumption map is adapted from an SAE paper[10]. The full load characteristic is adopted from a “best guess” transcription from the EcoBoost YouTube video hosted by Derek Kuzak. The GTDI engine is assumed to have a peak mean piston speed of 16.7 m/s. Baseline engine details are summarized in Tables A.12 & A.14. The fuel consumption map for the 2.0 l GTDI engine as interpolated by AVL Cruise is shown in Figure A.12

Additionally, AVL Cruise provides modules for modeling turbo-charger behavior. The following figures represent the data included in the GTDI simulations. Figure A.13 was generated by examining load steps from 2 bar to full load for the engine described in SAE 2006-01-1266[11]. The authors presented results at various engine speeds.

[10] Henning Kleeberg, Dean Tomazic, Oliver Lang, and Knut Habermann, ”Future Potential and Development Methods for High Output Turbocharged Direct Injected Gasoline Engines,” SAE Paper 2006-01-0046

[11] W. Bandel, G. K. Fraidl, P. E. Kapus, and H. Sikinger, C. N. Cowland, “The Turbocharged GDI Engine: Boosted Synergies for High Fuel Economy Plus Ultra-low Emission,” SAE Paper 2006-01-1266

Table A.11: Full consumption map for 2.5 V6 naturally aspirated gasoline engine from *AVL Cruise*

RPM	BMEP (bar)	Fuel Consumption (l/h)
700	-4	1.6
700	-0.5	1.61
700	0	1.65
700	1.6	1.88
700	3.16	2.13
700	4.55	2.496
700	5.95	2.964
700	7.32	3.552
700	8.63	4.188
700	9.3	4.788
1200	-4	1.75
1200	-0.5	1.76
1200	0	1.8
1200	0.35	1.93
1200	1.82	2.35
1200	3.32	2.784
1200	4.73	3.516
1200	6.17	4.248
1200	7.59	4.968
1200	8.9	5.868
1200	10.21	6.6
1200	10.88	6.72
1700	-4	1.95
1700	-0.5	1.96
1700	0	2
1700	0.42	2.16
1700	2.06	2.85
1700	3.54	3.72
1700	4.89	4.608
1700	6.33	5.532
1700	7.83	6.516
1700	9.48	7.704
1700	10.93	9.432
1700	12.3	10.62
2550	-4	2.25
2550	-0.5	2.3
2550	0	2.45
2550	0.73	2.976
2550	2.3	4.2
2550	3.8	5.688
2550	5.25	7.008
2550	6.71	8.304
2550	8.21	9.828
2550	9.55	10.992
2550	12.4	15.096
2550	14.29	17.592
3000	-4	2.73
3000	-0.5	2.84
3000	0	3
3000	0.71	3.552
3000	2.42	5.208
3000	3.97	6.852
3000	5.49	8.412
3000	6.97	10.008
3000	8.41	11.508
3000	9.81	13.14
3000	11.29	14.844
3000	14.75	20.916
3600	-4	3.22
3600	-0.5	3.34
3600	0	3.67
continued...		
3600	0.72	4.44
3600	2.41	6.396
3600	3.97	8.472
3600	7.04	12.18
3600	8.59	14.304
3600	10.31	16.572
3600	11.61	18.588
3600	14.68	24.804
3600	15.11	25.356
4150	-4	4.15
4150	-0.5	4.27
4150	0	4.64
4150	0.7	5.412
4150	2.31	7.656
4150	3.82	9.96
4150	5.34	12.18
4150	6.9	14.556
4150	8.53	17.004
4150	9.99	19.5
4150	13.06	26.76
4150	15.11	33.024
5050	-4	5.25
5050	-0.5	5.6
5050	0	6
5050	1.99	9.432
5050	3.62	12.3
5050	5.1	14.856
5050	6.63	17.82
5050	8.12	20.736
5050	9.77	24.696
5050	11.19	28.476
5050	12.53	33.456
5050	14.5	41.784
6000	-4	6.6
6000	-0.5	7.25
6000	0	8
6000	1.21	10.428
6000	2.76	13.86
6000	4.18	17.052
6000	5.53	20.22
6000	6.94	23.916
6000	9.72	33.984
6000	10.88	39.144
6000	12.05	44.664
6000	13.05	49.032

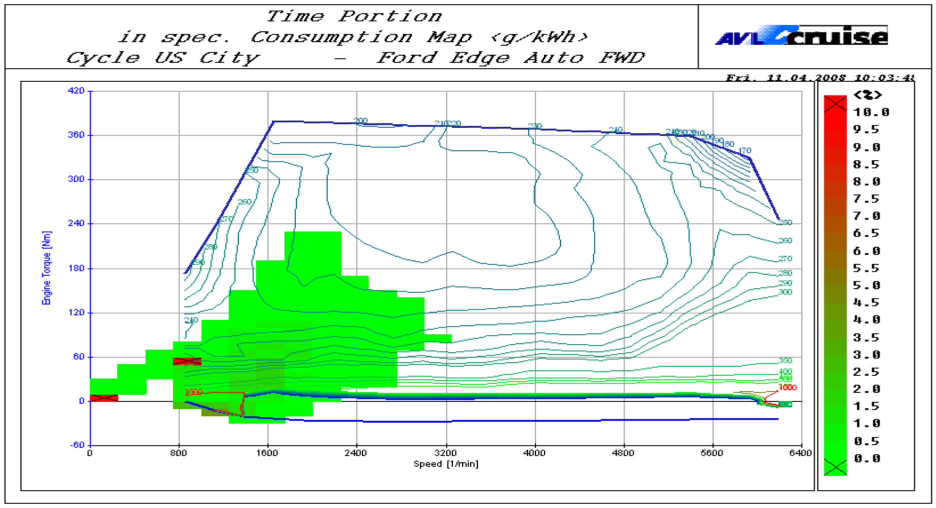


Figure A.12: Fuel consumption map for 2.0 l GTDI engine

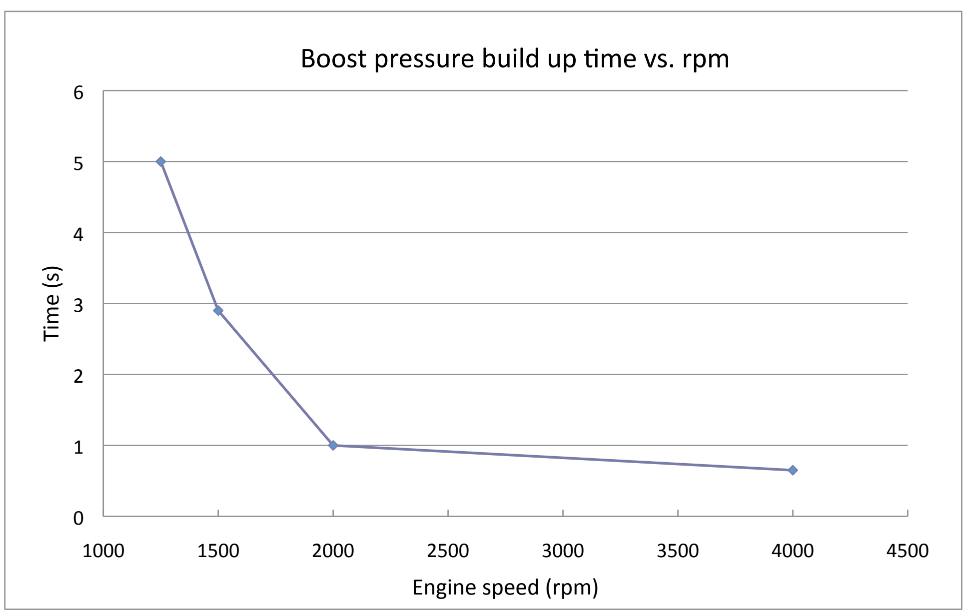


Figure A.13: Boost pressure build up time as a function of engine speed

Table A.12: Engine characteristics for 2.0 l I-4 gas turbo direct injection engine

Stroke	83.34mm
Bore	87.4mm
Displacement	2000cc
Bore to Stroke	1.05
Compression ratio	9.8
Idle speed	850rpm
Max engine speed	6000rpm

Table A.13: Full consumption map for 2.0 l I-4 gas turbo direct injection engine

RPM	Torque (lb-ft)	Power (hp)	BMEP (kPa)	Piston Speed (m/s)
1000	150	29	1278	2.8
1650	280	88	2385	4.6
3000	275	157	2343	8.3
4000	272	207	2317	11.1
5400	265	272	2258	15.0
6000	240	274	2045	16.7

The operating points listed in Table A.15 were adopted for the 2.0 l EcoBoost concept to estimate boost pressure and temperature after compressor from SAE Papers 2005-01-1144 & 2006-01-0046.

The data for boost pressure as a function of engine speed shown in Figure A.14 was modified after receiving feedback from Michael Shelby of Ford from the full load operating points of the compressor as described in SAE 2005-01-1144[12].

Figure A.15 shows the air temperature exiting the turbo-charger and the intercooler for various engine speeds. The air turbo-charger exit temperature data was generated by taking the full load compressor operating points from the engine described in SAE 2005-01-1144, SAE 2006-01-0046. Using the compressor map (also given in the paper) for pressure and isentropic efficiency, temperature was computed using ideal gas theory. The intercooler temperatures were generated by taking an

[12] Oliver Lang, Jos Geiger, Knut Habermann and Michael Wittler, "Boosting and Direct Injection - Synergies for Future Gasoline Engines," SAE Paper 2005-01-1144

Table A.14: Full load performance characteristic for 2.0 l I-4 gas turbo direct injection engine

RPM	BMEP (bar)	Fuel Consumption (l/h)	RPM	BMEP (bar)	Fuel Consumption (l/h)
1020	-2.17	0.51	continued...		
1020	0	0.57	4080	-4	2.89
1020	0.56	0.64	4080	-0.5	2.97
1020	1.67	1.27	4080	0	3.23
1020	1.99	1.4	4080	1.34	6.09
1020	4.61	2.63	4080	2.6	7.92
1020	6.7	3.57	4080	6.3	14.37
1020	17.34	11.41	4080	8.56	18.23
2040	-4	1.22	4080	23.28	49.58
2040	-0.5	1.22	4225	22.62	49.88
2040	0	1.25	4371	8.92	20.34
2040	0.88	2	4662	9.61	23.39
2040	2.1	3.2	4808	0	3.68
2040	2.51	3.51	4808	1.8	7.65
2040	5.79	6.6	4808	4	11.82
2040	7.79	8.29	4808	6.87	18.47
2040	21.46	21.52	4808	19.23	48.27
2404	16.73	18.75	4881	18.55	47.26
2404	18.13	20.31	4954	16.5	42.65
2477	15.74	18.17	5100	-4	3.65
2477	18.8	21.7	5100	-0.5	3.89
2550	-4	1.56	5100	0	4.17
2550	-0.5	1.6	5100	1.38	7.84
2550	0	1.7	5100	2.92	11.11
2550	7.65	10.18	5100	4.25	14.91
2550	15.57	18.9	5100	8.25	23.2
2550	21.43	25.74	5100	11.69	30.67
2768	15.38	19.85	5172	23.16	66.99
2768	20.79	26.82	5245	9.03	26.5
3060	-4	1.9	5245	20.85	61.15
3060	-0.5	1.97	5391	9.83	29.64
3060	0	2.09	5391	18.96	57.17
3060	1.03	3.54	5537	11.77	36.45
3060	2.28	5.19	5537	14.54	45.03
3060	5.69	9.74	6120	-4	4.59
3060	7.93	12.67	6120	-0.5	5.04
3060	21.04	30	6120	0	5.56
3351	15.45	24.14	6120	1.39	9.51
3643	-4	2.24	6120	3.34	15.22
3643	-0.5	2.32	6120	8.32	35.01
3643	0	2.55			
3643	15.75	26.73			
3643	20.88	35.45			
3788	16.43	29.01			
3788	20.2	35.66			

percentage difference from the charger outlet temperature. An initial guess for the temperature difference across the intercooler was taken referencing the intercooler described in SAE 2006-01-0046.

Table A.15: Operating points for sample GTDI engine

P_2/P_1	Engine speed (RPM)	Volume Flow (m^3/s)	η_{isV}	n_{redV} x 10^3 (RPM)
1.27	1000	0.021	0.6	80
1.53	1250	0.028	0.62	106
1.5	1500	0.031	0.65	103
1.52	1750	0.042	0.675	106
1.56	2000	0.05	0.7	112
1.65	3000	0.065	0.715	122
1.62	3500	0.075	0.723	121
1.65	4000	0.085	0.721	124
1.7	4500	0.097	0.715	131
1.8	5000	0.112	0.69	143
1.75	5500	0.12	0.655	143

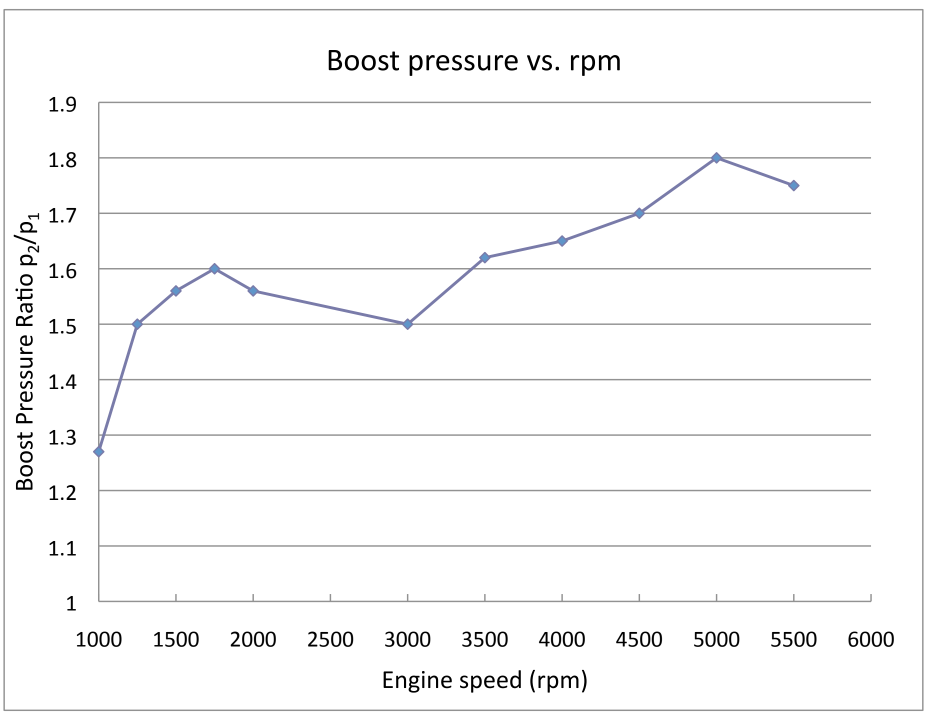


Figure A.14: Relative boost pressure as a function of engine speed

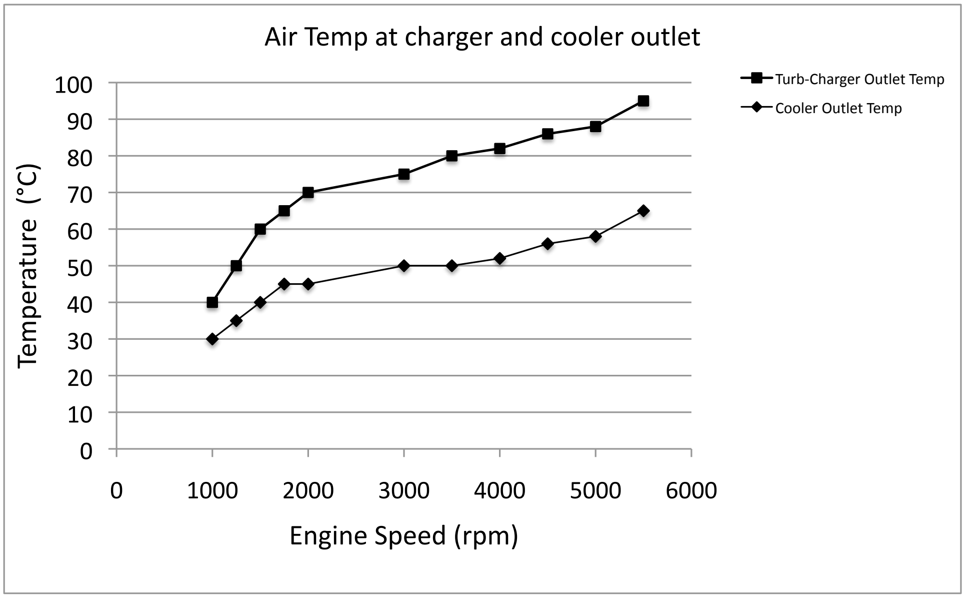


Figure A.15: Turbo-charger and intercooler outlet temperatures at full load for various engine speeds

AVL Cruise simulations

Vehicle simulations using *AVL Cruise* software were configured to represent a standard automatic transmission front wheel drive vehicle with a gasoline engine. In addition to powertrain specifications (i.e., gear ratios, gear shifting schedule, engine number of cylinders, vee or inline configuration, bore, and stroke, valvetrain configuration, and final drive ratio.) *Cruise* also receives other vehicle inputs, including curbweight, frontal area, drag coefficient, tire radius, and center of gravity location under various loads. Over 30 *AVL Cruise* parameters were tuned for midsize crossover vehicles based on data from one 2007 model. All other parameters in *Cruise* were left at the default passenger vehicle levels. Section A shows how model parameters are converted into three of the required *Cruise* inputs.

Five vehicle simulations were executed using *Cruise*: 1) FTP (US urban cycle) and 2) HFET (US highway driving cycle) can be used to estimate either the combined fuel economy rating used for CAFE purposes or combined fuel economy rating according to 2008 EPA MPG-based guidelines[13]. 3) Shifting gears from standstill was used to predict 0-60 acceleration and vehicle top speed; 4) shifting gears from 30-50 mph was used to estimate 30-50 mph time with towing simulated by adding the max trailer weight to the mass of the vehicle; 5) max gradeability estimates the percent grade achievable at 65 mph with towing simulated using the virtual trailer option that allows specification of trailer mass and an estimate of losses.

The engineering model as described above (including a call to the *AVL Cruise* solver to execute the 5 simulations) was integrated using *iSIGHT v10.0* system integration software. Latin hypercube design of experiments were executed for both the naturally aspirated and (1000 runs) the GTDI vehicles (1600 runs). Execution of 1 computer experiment run on a 3.6 GHz Pentium IV processor averaged slightly over 30 seconds. Factors for the naturally aspirated engine vehicle included: C_D (vehicle drag coefficient), *EngBore*, *FinalDrive*, *H101*, *L101*, *L103*, *W105*, and *EngBoretoStroke*. Factors for the GTDI engine vehicle included: . Design runs that resulted in failed *Cruise* runs were removed from the data set. Simulations failed for less than 10% of the runs, and failed runs typically corresponded to cases where a combination of large vehicle size, small engine, and large final drive ratio resulted in inability to achieve 65 mph in the acceleration test or a positive towing grade at 65 mph. The data from the computer experiments was then fit with surrogate models to decrease computational speed for design optimization runs, and to allow integration of the entire model in Matlab, thereby improving portability of the model.

The hybrid electric vehicle simulation is a backwards-looking simulation built by visiting scholar Kukhyun Ahn based on his dissertation work. Similar to above, a DOE was executed (500 runs), and the data was fit with surrogate models for each simulation. Factors in the HEV DOE included: *EngBore*, *FinalDrive*, *H101*, pg_{ratio}, bat_{power}, *L103*, *W105*, and *EngBoretoStroke*. The outputs of the HEV simulation

[13]USEPA, 2006, Fuel Economy Labeling of Motor Vehicles: Revisions to Improve Calculation of Fuel Economy Estimates; Final Rule, Federal Register, 71(248) pp. 77872-77969

were scaled to account for losses not accounted for due to the backward-looking simulator.

Combined fuel economy

`{CombinedFuelEcon.m}`

Combined fuel economy in miles/gallon is computed for both the CAFE reported value and the 2008 window sticker value. The test values $CycUSCity$ and $CycUSHwy$ are given in liters/100 km. The CAFE value is simply the harmonic mean of the tested values according the following relationships.

$$MPGCycUSCity = \frac{1}{\frac{1.609344 CycUSCity}{100 \times 3.786235}} \tag{A.145}$$

$$MPGCycUSHwy = \frac{1}{\frac{1.609344 CycUSHwy}{100 \times 3.786235}} \tag{A.146}$$

$$CombinedMPG = \frac{1}{\frac{0.55}{MPGCycUSCity} + \frac{0.45}{MPGCycUSHwy}} \tag{A.147}$$

The window sticker value is found by adopting the "MPG-based method" from the 2008 guidelines for reporting window sticker fuel economy[14]. This method provides a regression formula for adjusting the city and highway test values according to the following relationships. The new method for reporting window sticker fuel economy presents a different weighting of city vs. highway driving for the combined fuel economy (43% city, 57% highway). However, vehicles in 2008 are being reported using the "MPG-based method" rather than the new tests, and the old ratios (55% city,

[14]USEPA, 2006, Fuel Economy Labeling of Motor Vehicles: Revisions to Improve Calculation of Fuel Economy Estimates; Final Rule, Federal Register, 71(248) pp. 77872-77969

45% highway) have been preserved in reporting.

$$MPGCycUSCity = \frac{1}{0.003259 + \frac{1.18053(0.264115 CycUSCity)}{62.13712}} \quad \text{(A.148)}$$

$$MPGCycUSHwy = \frac{1}{0.001376 + \frac{1.3466(0.264115 CycUSHwy)}{62.13712}} \quad \text{(A.149)}$$

$$CombinedMPG = \frac{1}{\frac{0.55}{MPGCycUSCity} + \frac{0.45}{MPGCycUSHwy}} \quad \text{(A.150)}$$

Range

`{Range.m}`

Vehicle range is estimated based on city and highway fuel economy and gas tank volume. Gas tank volume is converted from liters[3].

$$VehCityRange = MPGCycUSCity \times VehGasTankVol \times 264.17 \quad \text{(A.151)}$$

$$VehHwyRange = MPGCycUSHwy \times VehGasTankVol \times 264.17 \quad \text{(A.152)}$$

Surrogate models

`{Acc0to60.m}`

`{Acc30to50Tow.m}`

`{CityHwyGrad.m}`

`{CityHwyGradTC.m}`

`{Acc0to60TC.m}`

`{TopSpeedTC.m}`

```
{AccelGradHEV.m}
{HEVUSHwy.m}
{HEVSimTopSpeed.m}
```

Surrogate models were obtained from the *Cruise* simulation data to reduce computational expense (See Table A.16), using Latin hypercube experimental designs (1000 Runs NA SI, 1600 Runs GTDI, 500 runs HEV). Failed runs were removed from the data set before surrogate model fitting. For the NA SI and GTDI, satisfactory polynomials were found for both driving cycles, the gradeability simulation, and $Acc3050Tow$ (R^2: 0.998 City, 0.994 Hwy, 0.997 Grade, 0.998 3050). Two neural nets were generated in Matlab, one for $Acc060$ and one for $MaxSpeed$, both of which had R^2 values for the training points and the test points above 0.99. For the HEV model, satisfactory polynomials were found for all powertrain performance characteristics except the $CycUSHwy$, and the $MaxSpeed$. These two outputs were fit with neural nets.

The parameters for each polynomial are given below, grouped by powertrain technology

NA SI

For naturally aspirated spark-ignition vehicles polynomials were fit for $CycUSCity$, $CycUSHwy$, and $Grad65Tow$. The elements of the polynomials are shown in $\mathbf{N}$. The values for each element correspond to the real values for a given design, however they have been scaled between [0,2] based on upper and lower bounds for each element. Values for the polynomial parameters are shown in $\mathbf{C}_{NASI}$. Variable upper and lower bounds are given in Table A.17.

Table A.16: Surrogate models by simulation and powertrain technology

Pwtrn Tech.	Simulation	Model type	R^2
NA SI	$CycUSCity$	Poly	0.998
	$CycUSHwy$	Poly	0.994
	$Acc060$	NN	0.99
	$Acc3050Tow$	NN	
	$Grad65Tow$	Poly	0.997
	$MaxSpeed$	NN	0.99
GTDI	$CycUSCity$	Poly	0.998
	$CycUSHwy$	Poly	0.995
	$Acc060$	NN	
	$Acc3050Tow$	Poly	0.998
	$Grad65Tow$	Poly	0.999
	$MaxSpeed$	NN	
HEV	$CycUSCity$	Poly	0.99
	$CycUSHwy$	NN	
	$Acc060$	Poly	0.999
	$Acc3050Tow$	Poly	0.999
	$Grad65Tow$	Poly	0.99
	$MaxSpeed$	NN	
	$MG1$	Poly	1
	$MG2$	Poly	1

Table A.17: Upper and lower bounds on parameters for naturally aspirated V-6 engines

		EngBore	*Eng Bore to Stroke*	*Final Drive*	*H101*	*L101*	*L103*	*W105*	C_D	
Varlb	= [	86	0.95	1.1	1600	2286	3556	1600	0.33	]
Varub	= [	100	1.18	4.0	1930	3048	5080	2000	0.42	]

$$
\mathbf{N} = [\ 1 \\
xVehDragCoef \\
xEngBore \\
xFinalDrive \\
xH101 \\
xL103 \\
xW105 \\
xEngBoretoStroke \\
xEngBore^2 \\
xFinalDrive^2 \\
xL103^2 \\
xEngBoretoStroke^2 \\
xEngBore \times xFinalDrive \\
xEngBore \times xEngBoretoStroke \\
xFinalDrive \times xL103 \\
xFinalDrive \times xW105 \\
xL103 \times xW105 \\
xFinalDrive^3\ ]
$$

$$\mathbf{C}_{NASI} = [\begin{array}{lll} 8.944 & 5.453 & -2.410 \\ 0.1281 & 0.2881 & -0.2008 \\ 1.371 & 0.7173 & 2.046 \\ 0.0730 & 0.4728 & 3.707 \\ 0.1120 & 0.2498 & -0.1897 \\ -0.0256 & 0.0 & 0.0530 \\ 0.1621 & 0.2968 & -0.1428 \\ -0.5295 & -0.2377 & -1.465 \\ 0.1070 & 0.0 & 0.2098 \\ 0.2054 & 0.1011 & 4.7509 \\ 0.0879 & 0.0544 & 0.0 \\ 0.0603 & 0.0 & 0.1735 \\ 0.090 & 0.0 & 0.6704 \\ -0.126 & 0.0 & -0.3412 \\ 0.0 & 0.0 & -0.3506 \\ 0.0 & 0.0 & -0.2292 \\ 0.1316 & 0.0913 & 0.0 \\ 0.0 & 0.0 & -1.791 \end{array}]$$

$$\begin{bmatrix} CycUSCity \\ CycUSHwy \\ Grad65Tow \end{bmatrix} = \mathbf{C}'_{NASI}\mathbf{N} \tag{A.153}$$

GTDI

For GTDI spark-ignition vehicles polynomials were fit for $CycUSCity$, $CycUSHwy$, $Grad65Tow$, and $Acc3050Tow$. The elements of the polynomials are shown in $\mathbf{G}$. The values for each element correspond to the real values for a given design, however they have been scaled between [0,2] based on upper and lower bounds for each element. Values for the polynomial parameters are shown in $\mathbf{C}_{GTDI}$. Variable upper and lower bounds are given in Table A.18.

Table A.18: Upper and lower bounds on parameters for GTDI I-4 engines

		EngBore	*Eng Bore to Stroke*	*Final Drive*	$Gear_1$	$Gear_2$	$Gear_3$	$Gear_4$	*H101*	*H156*	*L103*	*W105*	C_D	
Varlb	= [	80	0.9	2.5	2.85	1.85	1.40	1.0	1651.0	175.0	4318.0	1727	0.33	]
Varub	= [	92	1.18	5.0	5.0	2.85	1.85	1.4	1803.0	215.0	5080.0	1982.0	0.42	]

Table A.19: Upper and lower bounds on parameters for parallel HEV configuration with I4 engine

		EngBore	*Eng Bore to Stroke*	*Final Drive*	*H101*	*L101*	*L103*	*W105*	C_D	PGR	BPow	
Varlb	= [	88	0.9	3.5	1651.0	2286.0	4319.0	1727	0.33	0.3333	45	]
Varub	= [	92	1.18	5.0	1803.0	3048.0	5080.0	1982.0	0.42	0.6666	65	]

$$
\mathbf{G} = [\ 1,\ xBore,\ xFD,\ xH101,\ xH156,\ xL103,\ xW105,\ xBoretoStroke,\ xVehDrag,\ Gear1,\ Gear2,\ Gear3,\ Gear4,\ xBore^2,\ xFD^2,\ xL103^2,\ xW105^2,\ xEngBoretoStroke^2,\ Gear1^2,\ Gear2,\ Gear4^2,\ xEngBore \times xFinalDrive,\ xEngBore \times xL103,\ xEngBore \times xW105,\ xEngBore \times xEngBoretoStroke,\ b \times g1,\ b \times g2,\ b \times g3,\ b \times g4,\ xFinalDrive \times xL103,\ xFinalDrive \times xW105,\ xFinalDrive \times xEngBoretoStroke,\ fd \times g1,\ fd \times g2,\ fd \times g3,\ fd \times g4,\ xL103 \times xW105,\ xL103 \times xEngBoretoStroke,\ L103 \times g2,\ L103 \times g3,\ xW105 \times xEngBoretoStroke,\ w105 \times g2,\ w105 \times g3,\ xEngBoretoStroke \times g2,\ xEngBoretoStroke \times g3,\ xFD^3,\ Gear2^3\ ]
$$

$$
\mathbf{C}_{GTDI} = [\begin{array}{cccc}
7.5368 & 5.9459 & 2.6084 & 8.1527 \\
0.6725 & 0.4634 & 2.2639 & -1.9052 \\
-0.1582 & 0.4656 & 5.1671 & -3.6308 \\
0.0716 & 0.1517 & -0.0846 & 0.0 \\
0.0 & -0.0353 & 0.0 & 0.0 \\
0.1729 & 0.1058 & 0.0 & 0.2328 \\
0.2423 & 0.3297 & -0.0956 & 0.2386 \\
-0.4781 & -0.3029 & -1.4864 & 1.2364 \\
0.1775 & 0.3742 & -0.2098 & 0.0 \\
-0.0628 & 0.0 & 0.0 & 0.0 \\
0.0291 & 0.0 & 0.0 & -1.8901 \\
0.0 & 0.0 & 1.6158 & 0.0 \\
-0.1092 & 0.0535 & .0 & 0.0 \\
0.076 & 0.0216 & 0.1559 & 0.2151 \\
0.3658 & 0.1388 & -0.3177 & 1.1556 \\
0.0319 & 0.0225 & 0.0 & 0.0267 \\
0.0232 & 0.0 & 0.0 & 0.0 \\
0.0935 & 0.0519 & 0.1971 & 0.0276 \\
0.0273 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.3308 \\
0.0289 & 0.0 & 0.0 & 0.0 \\
0.1525 & 0.093 & 1.0083 & 0.687 \\
0.0 & 0.0 & -0.1223 & -0.0675 \\
0.0 & 0.0 & -0.0918 & -0.0506 \\
-0.1421 & -0.067 & -0.3007 & -0.2292 \\
0.0215 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.3595 \\
0.0 & 0.0 & 0.3651 & 0.0 \\
0.0 & -0.0237 & 0.0 & 0.0 \\
-0.0298 & -0.0299 & -0.2106 & -0.1111 \\
-0.0227 & -0.0241 & -0.1453 & -0.1106 \\
-0.1386 & -0.0595 & -0.5439 & -0.4629 \\
0.0888 & 0.0 & 0.0 & 0. \\
0.0 & 0.0 & 0.0 & 0.4977 \\
0.1203 & 0.0 & 0.6037 & 0.0336 \\
0.0991 & 0.0 & 0.0 & 0.0 \\
0.0796 & 0.0486 & -0.0659 & 0.0607 \\
0.0 & 0.0 & 0.0487 & 0.0459 \\
0.0 & 0.0 & 0.0 & -0.0606 \\
0.0 & 0.0 & -0.0815 & 0.0 \\
0.0 & 0.0 & 0.0 & 0.044 \\
0.0 & 0.0 & 0.0 & -0.0488 \\
0.0 & 0.0 & -0.06780 & .0 \\
0.0 & 0.0 & 0.0 & -0.2344 \\
0.0 & 0.0 & -0.1942 & 0.0 \\
0.0 & 0.0 & 0.0744 & -0.1144 \\
0.0 & 00. & 0.0 & -0.0377
\end{array}]
$$

$$\begin{bmatrix} CycUSCity \\ CycUSHwy \\ Grad65Tow \\ \text{Acc}\mathit{3050}\text{Tow} \end{bmatrix} = \mathbf{C}'_{GTDI}\mathbf{G} \tag{A.154}$$

HEV

The HEV vehicle powertrain simulations were based on data generated from HEV simulations developed by Kuhkyun Ahn. For HEV vehicles polynomials were fit for *CycUSCity*, *Acc060*, *Grad65Tow*, *Acc3050Tow*, *MG2*, and *MG1*. The elements of the polynomials are shown in **H**. The values for each element correspond to the real values for a given design, however they have been scaled between [0,2] based on upper and lower bounds for each element. Values for the polynomial parameters are shown in $\mathbf{C}_{HEV}$. Variable upper and lower bounds are given in Table A.19.

$$
\mathbf{H} = [\ 1,\ xBore,\ xFD,\ xH101,\ xPGR,\ xBPow,\ xL103,\ xW105,\ xBoretoStroke,\ xFD^2,\ xH101^2,\ xPGR^2,\ xBPow^2,\ xL103^2,\ xW105^2,\ xBoretoStroke^2,\ xBore \times xFD,\ xBore \times xBPow,\ xBore \times x_W 105,\ xBore \times xBoretoStroke,\ xFD \times xH101,\ xFD \times xPGR,\ xFD \times xBPow,\ xFD \times xL103,\ xFD \times xW105,\ xFD \times xBoretoStroke,\ xPGR \times xBPow,\ xBPow \times xL103,\ xBPow \times x_W 105,\ xBPow \times xBoretoStroke,\ xL103 \times xW105,\ xL103 \times xBoretoStroke,\ xW105 \times xBoretoStroke,\ xFD^3,\ xH101^3,\ xPGR^3,\ xBoretoStroke^3\ ]
$$

$$
\mathbf{C}_{HEV} = [\begin{array}{rrrrrr}
5.5377 & 7.8296 & 7.0946 & 5.2934 & 10.7896 & 6.2998 \\
0.1535 & 0.6885 & -0.2215 & -0.2005 & 0.4452 & 0.4452 \\
0.0708 & 1.4549 & -0.8854 & -0.4827 & 0.0 & 0.0 \\
0.094 & -0.1504 & 0.0 & 0.0 & 0.0 & 0.0 \\
-0.0007 & 1.1733 & -0.5106 & -0.5057 & 0.0 & 0.0 \\
-0.1178 & -0.0578 & -0.4582 & -0.3931 & 1.0002 & 0.0 \\
0.095 & -0.2434 & 0.2365 & 0.0831 & 0.0 & 0.0 \\
0.2278 & -0.3431 & 0.2709 & 0.0916 & 0.0 & 0.0 \\
-0.3065 & -1.-655 & 0.368 & 0.4034 & -0.9582 & -0.9582 \\
-0.0611 & -0.4024 & 0.1878 & 0.0423 & 0.0 & 0.0 \\
0.0 & 0.1074 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & -0.4385 & 0.0739 & 0.0492 & 0.0 & 0.0 \\
0.0152 & 0.0 & 0.0265 & 0.0233 & 0.0 & 0.0 \\
0.0196 & -0.016 & 0.047 & 0.0183 & 0.0 & 0.0 \\
0.0131 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0345 & 0.3318 & 0.0 & -0.0173 & 0.1076 & 0.1076 \\
0.0 & 0.0274 & 0.0 & 0.0 & 0.0 & 0.0 \\
-0.0164 & 0.0 & 0.0323 & 0.0274 & 0.0 & 0.0 \\
0.0 & -0.0198 & 0.0 & 0.0 & 0.0 & 0.0 \\
-0.0195 & -0.074 & 0.0 & 0.0 & -0.0532 & -0.0532 \\
0.0 & -0.0292 & 0.0 & 0.0 & 0.0 & 0.0 \\
-0.0387 & -0.416 & 0.0647 & 0.0767 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0536 & 0.0379 & 0.0 & 0.0 \\
0.0 & 0.0 & -0.0359 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & -0.0287 & 0.0 & 0.0 & 0.0 \\
0.0 & -0.0468 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0332 & 0.0381 & 0.0 & 0.0 \\
0.0 & 0.0 & -0.0355 & -0.0161 & 0.0 & 0.0 \\
0.0 & 0.0 & -0.0224 & 0.0 & 0.0 & 0.0 \\
0.0181 & 0.0 & -0.0586 & -0.0511 & 0.0 & 0.0 \\
0.0505 & -0.0485 & 0.1044 & 0.0419 & 0.0 & 0.0 \\
0.0 & 0.0392 & 0.0381 & 0.0154 & 0.0 & 0.0 \\
0.0 & 0.0388 & 0.0328 & 0.0155 & 0.0 & 0.0 \\
0.0 & 0.0541 & -0.0256 & 0.0 & 0.0 & 0.0 \\
0.0 & -0.0312 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & 0.0778 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.0 & -0.0629 & 0.0 & 0.0 & 0.0 & 0.0
\end{array}]
$$

$$
\begin{bmatrix} CycUSCity \\ Acc060 \\ Grad65Tow \\ Acc3050Tow \\ MG2 \\ MG1 \end{bmatrix} = \mathbf{C}'\mathbf{N} \tag{A.155}
$$

Engineering Performance Constraints

The constraint set is as follows.

$$g_1 = 5\% - Grad65Tow \leq 0 \tag{A.156}$$

$$g_2 = 13^\circ - \mathit{A107} \leq 0 \tag{A.157}$$

$$g_3 = 12^\circ - \mathit{A147} \leq 0 \tag{A.158}$$

$$g_4 = 29\ \text{ft}^3 - CVI \leq 0 \tag{A.159}$$

$$g_5 = CVI - 60\ \text{ft}^3 \leq 0 \tag{A.160}$$

$$g_6 = \mathit{Rollover} - 0.1 \leq 0 \tag{A.161}$$

$$g_7 = 50\% - 100\,(1 - CG_{long} - \mathit{L104}/\mathit{L101}) \leq 0 \tag{A.162}$$

$$g_8 = Payload + VehMass - GVWR \leq 0 \tag{A.163}$$

$$g_9 = MinCrushSpace - CrushSpace \leq 0 \tag{A.164}$$

$$g_{10} = MaxDecel - 20(9.81\ \text{m/s}^2) \leq 0 \tag{A.165}$$

$$g_{11} = (2TireFlop + 2MidRailWidth + EngLength + 50.8) - (\mathit{W105} - 254) \leq 0 \tag{A.166}$$

$$g_{12} = \mathit{L101} + \mathit{L104} - \mathit{L103} \leq 0 \tag{A.167}$$

$$g_{13} = 115\ \text{mph} - MaxSpeed \leq 0 \tag{A.168}$$

$$g_{14} = MinSitHeight - \mathit{H101} \leq 0 \tag{A.169}$$

where

Constraint		Description
g_1	:	Min towing grade @ 65 mph
g_2	:	Min angle of departure
g_3	:	Min ramp breakover angle
g_4	:	Min cargo volume
g_5	:	Max cargo volume
g_6	:	Max rollover score
g_7	:	Min % weight on front wheels
g_8	:	Min payload capacity
g_9	:	Min front crash crush space
g_{10}	:	Max estimated deceleration
g_{11}	:	Max powertrain width
g_{12}	:	Max wheelbase
g_{13}	:	Min top speed
g_{14}	:	Min available passenger sitting height

Other constraints that have been included in the code, but are not typically employed in the model include:

$$
\begin{aligned}
g_{15} &= MinBoretoStroke - EngBoretoStroke && \text{(A.170)}\\
g_{16} &= EngBoretoStroke - MaxBoretoStroke && \text{(A.171)}\\
g_{17} &= MinEngDisp - EngDisp && \text{(A.172)}\\
g_{18} &= EngDisp - MaxEngDisp && \text{(A.173)}\\
g_{19} &= MinFinalDrive - FinalDrive && \text{(A.174)}\\
g_{20} &= FinalDrive - MaxFinalDrive && \text{(A.175)}\\
g_{21} &= GearBoxRatio(1) - Max1stGear && \text{(A.176)}\\
g_{22} &= Min6thGear - GearBoxRatio(6) && \text{(A.177)}\\
g_{23} &= MinGear1to2 - (GearBRatio(1) - GearBRatio(2)) && \text{(A.178)}\\
g_{24} &= MinGear2to3 - (GearBRatio(2) - GearBRatio(3)) && \text{(A.179)}\\
g_{25} &= MinGear3to4 - (GearBRatio(3) - GearBRatio(4)) && \text{(A.180)}\\
g_{26} &= MinGear4to5 - (GearBRatio(4) - GearBRatio(5)) && \text{(A.181)}\\
g_{27} &= MinGear5to6 - (GearBRatio(5) - GearBRatio(6)) && \text{(A.182)}\\
g_{28} &= MinHwyRange - VehHwyRange && \text{(A.183)}\\
g_{29} &= MinCityRange - VehCityRange && \text{(A.184)}\\
g_{30} &= (TireDynRollRad + 10 * 25.4) - L104 && \text{(A.185)}\\
g_{31} &= MinAofApr - A106 && \text{(A.186)}\\
g_{32} &= MinFrontClear - H103d1d3 && \text{(A.187)}\\
g_{33} &= MinRearClear - H103d2d3 && \text{(A.188)}\\
g_{34} &= MinDriverLegRoom - VehLegRoom1 && \text{(A.189)}\\
g_{35} &= VehLegRoom1 - MaxDriverLegRoom && \text{(A.190)}\\
g_{36} &= Min2ndRLegRoom - VehLegRoom2 && \text{(A.191)}\\
g_{37} &= VehLegRoom2 - Max2ndRLegRoom && \text{(A.192)}\\
g_{38} &= (W101 + 7 * 25.4) - W105 && \text{(A.193)}\\
g_{39} &= Acc3050Tow - Max30to50 && \text{(A.194)}\\
g_{40} &= Acc060 - Max0to60 && \text{(A.195)}\\
g_{41} &= (L104 + TireDynRollRad + 10 * 25.4) \\
&\quad -VehHeelPointX && \text{(A.196)}\\
g_{42} &= 62.5 - MidRailWidth/MidRailThick && \text{(A.197)}\\
g_{43} &= MidRailWidth - 3.75 * 25.4 && \text{(A.198)}\\
g_{44} &= 1.5 * 25.4 - MidRailWidth && \text{(A.199)}\\
g_{45} &= MPGConstraint - CombinedMPG && \text{(A.200)}\\
&\quad \text{if } Powetrain = HEV \\
g_{46} &= MinTopSpeed \\
&\quad -6700/FinalDrive * .358 * 2 * pi/60 * 3.6/1.60934 && \text{(A.201)}
\end{aligned}
$$

where

Constraint		Description
g_{15}	:	Min bore to stroke ratio
g_{16}	:	Max bore to stroke ratio
g_{17}	:	Min engine displacement
g_{18}	:	Max engine displacement
g_{19}	;	Min final drive ratio
g_{20}	:	Max final drive ratio
g_{21}	:	Max 1st gear ratio
g_{22}	:	Min 6th gear ratio
g_{23}	:	Min difference 1st gear ratio to 2nd gear ratio
g_{24}	:	Min difference 2nd gear ratio to 3rd gear ratio
g_{25}	:	Min difference 3rd gear ratio to 4th gear ratio
g_{26}	:	Min difference 4th gear ratio to 5th gear ratio
g_{27}	:	Min difference 5th gear ratio to 6th gear ratio
g_{28}	:	Min vehicle range on highway
g_{29}	:	Min vehicle range in city
g_{30}	:	Min front wheel clearance
g_{31}	:	Min angle of approach
g_{32}	:	Min front ground clearance
g_{33}	:	Min rear ground clearance
g_{34}	:	Min driver leg room
g_{35}	:	Max driver leg room
g_{36}	:	Min 2nd row leg room
g_{37}	:	Max 2nd row legroom
g_{38}	:	Max wheel track width
g_{39}	:	Max 30 to 50 mph acceleration time @tow
g_{40}	:	Max 0 to 60 mph acceleration time
g_{41}	:	Min clearance between heelpoint and tire
g_{42}	:	Min midrail width to thickness ratio
g_{43}	:	Max midrail width
g_{44}	:	Min midrail width
g_{45}	:	Min miles per gallon
g_{46}	:	Max top speed limited by motor in HEV

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