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The beam is underreinforced, so \( f_c \) is the actual stress and not the allowable.

\[
M_{\text{Bend}} = 50 \ k \cdot \text{lb} \\
f_s = 20 \ k \text{si} \\
\frac{f_s}{n} = \frac{20 \text{ksi}}{8.9} = 2.247 \text{ ksi}
\] 

\[
R_c = \frac{f_c B x}{2} \\
R_T = A_s f_s
\]

\[
M_{\text{Resisting}} = M_{\text{Bend}} = 50 \ k \cdot \text{lb} = 600 \ k \cdot \text{lb} = R_c (20 - \frac{x}{3}) = R_T (20 - \frac{x}{3})
\]

By similar triangles from stress diagram:

\[
\frac{f_c}{x} = \frac{2.247 \text{ ksi}}{20 - x} \\
\implies f_c = \frac{2.247 x}{20 - x}
\]

by substitution

\[
R_c = \frac{f_c B x}{2} = \left( \frac{2.247 x}{20 - x} \right) (12)(x) = \frac{13.482 x^2}{20 - x}
\]

\[
600 \ k \cdot \text{lb} = R_c \left( 20 - \frac{x}{3} \right) = \left( \frac{13.482 x^2}{20 - x} \right) \left( 20 - \frac{x}{3} \right)
\]

\[
12000 - 600x = 269.64 x^2 - 4.494 x^2
\]

Solve by trial and error \( x \approx 6 \) in.

\[
600 \ k \cdot \text{lb} = R_T \left( 20 - \frac{x}{3} \right) = A_s f_s \left( 20 - \frac{x}{3} \right)
\]

\[
600 \ k \cdot \text{lb} = (A_s)(20 \text{ ksi})(20 - \frac{x}{3})
\]

\[
A_s = 1.64 \text{ in}^2
\]