ARCH 324 - Structures 2, Winter 2009

von Buelow, Peter


<http://hdl.handle.net/2027.42/64938>
http://hdl.handle.net/2027.42/64938
EXAMPLE: CONCRETE BEAM ANALYSIS WORKING STRESS METHOD

CALCULATE THE BENDING CAPACITY OF THE FOLLOWING BEAM:

3 #9 Bars
(A_s = 3,000 in^2)

\[ E_s = 29,000 \text{ ksi}, \quad f_s = 20 \text{ ksi} \]
\[ E_c = 3,220 \text{ ksi}, \quad f_c = 1.8 \text{ ksi} \]

1. TRANSFORM SECTION:

\[ n = \frac{E_s}{E_c} = \frac{29,000}{3,220} = 9 \]

\[ A_{TR} = (3 \text{ in}^2)(9) = 27 \text{ in}^2 \]

2. DETERMINE NEUTRAL AXIS OF EFFECTIVE SECTION:

\[ A_c \bar{x}_c = A_s \bar{x}_s \]

\[ (12'' \times x) \left(\frac{x}{2}\right) = (27 \text{ in}^2)(17-x) \]

\[ 6x^2 = 459 - 27x \]
\[ 0 = 6x^2 + 27x - 459 \]
\[ 0 = x^2 + 4.5x - 76.5 \]

SOLVE WITH QUADRATIC EQUATION:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
2. **Neutral Axis, Cont'd...**

\[
x = -4.5 \pm \frac{(4.5)^2 - 4(1)(-10.5)}{2(1)} = 6.78'' \quad \text{or} \quad -11.28'' \\
\text{(ignore negative)} \\
\text{Answer}
\]

3. **Find Transformed Moment of Inertia:**

\[
I_c = \frac{bd^3}{3} = \frac{(12'')(6.78'')^3}{3} = 1247 \text{in}^4
\]

\[
I_s = A \bar{x}^2 = (27 \text{in}^2)(10.22'')^2 = 2820 \text{in}^4
\]

(assume minute thickness of steel → no \( \frac{A \bar{x}^2}{12} \) ) \( I_{tr} = 4067 \text{in}^4 \)

4. **Resisting Capacity of Concrete and Steel:**

**Concrete:** \( f_c = 1.8 \text{ksi} \)

\[
M = \frac{f_c I_{tr}}{c} = \frac{(1.8 \text{ksi})(4067 \text{in}^4)}{6.78''} = 1080 \text{in.k} = 90 \text{k}
\]

**Steel:** \( f_s = 20 \text{ksi} \)

\[
M = \frac{f_s I_{tr}}{c n} = \frac{(20 \text{ksi})(4067 \text{in}^4)}{(10.22'')(9)} = 884 \text{in.k} = 74 \text{k}
\]

90.774 \( \rightarrow \) **Steel governs**

**Beam Capacity = 74 k**

5. **Find Actual Stresses in Steel & Concrete:**

**Steel:** Moment-resisting capacity governs

Stress in steel = allowable = 20 ksi

**Concrete:** \( f = \frac{M_c}{I_{tr}} = \frac{(74 \text{k} \times 12'') (6.78'')}{4067 \text{in}^4} = 1.48 \text{ksi} \)