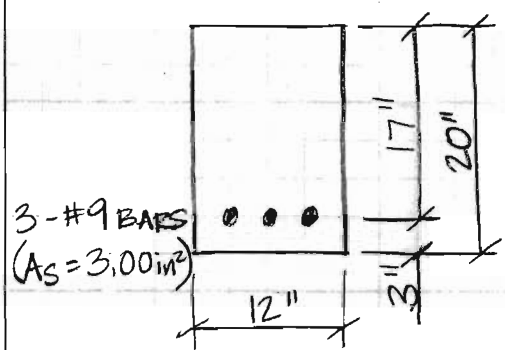
EXAMPLE: CONCRETE BEAM ANALYSIS
WORKING STRESS METHOD

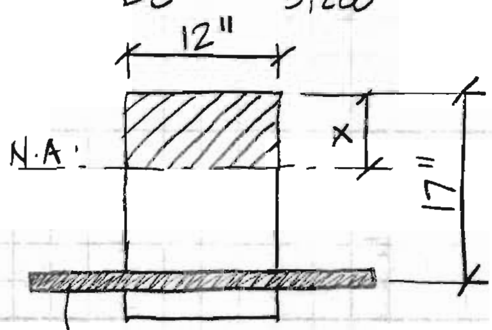
CALCULATE THE BENDING CAPACITY OF THE FOLLOWING BEAM:



$E_S = 29{,}000$ ksi, $f_s = 20$ ksi
$E_C = 3{,}220$ ksi, $f_c = 1.8$ ksi

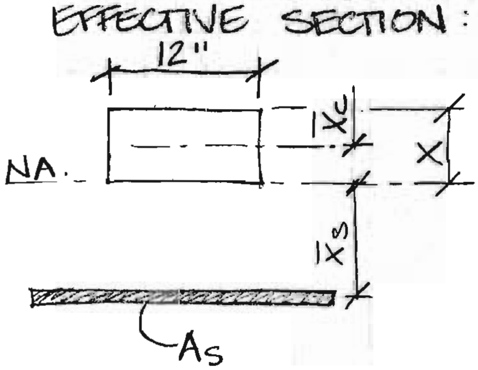
1. TRANSFORM SECTION:

$$n = \frac{E_S}{E_C} = \frac{29{,}000}{3{,}220} = 9$$



$A_{TR} = (3\,in^2)(9) = 27\,in^2$

2. DETERMINE NEUTRAL AXIS OF EFFECTIVE SECTION:



$$A_C \bar{x}_C = A_S \bar{x}_S$$

$$(12''*x)\left(\frac{x}{2}\right) = (27\,in^2)(17-x)$$

$$6x^2 = 459 - 27x$$

$$0 = 6x^2 + 27x - 459$$

$$0 = x^2 + 4.5x - 76.5$$

SOLVE WITH QUADRATIC EQUATION: $0 = ax^2 + bx + c$

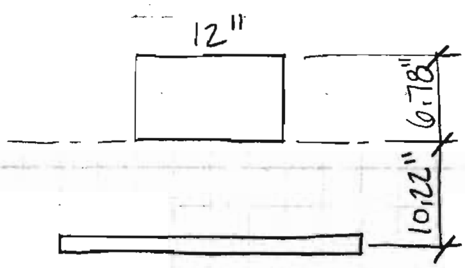
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. NEUTRAL AXIS, CONT'D...

$$x = \frac{-4.5 \pm \sqrt{(4.5)^2 - 4(1)(-76.5)}}{2(1)} = 6.78'' \quad \text{or} \quad \sout{-11.28}$$

(IGNORE NEGATIVE ANSWER)

3. FIND TRANSFORMED MOMENT OF INERTIA:



$$I_c = \frac{bd^3}{3} = \frac{(12'')(6.78'')^3}{3} = 1247 \text{ in}^4$$

$$I_s = A\bar{x}^2 = (27 \text{ in}^2)(10.22'')^2 = 2820 \text{ in}^4$$

(ASSUME MINUTE THICKNESS OF STEEL → NO $\frac{bh^3}{12}$)

$$I_{TR} = 4067 \text{ in}^4$$

4. RESISTING CAPACITY OF CONCRETE AND STEEL:

CONCRETE: $f_c = 1.8$ ksi

$$M = \frac{f_c I_{TR}}{c} = \frac{(1.8 \text{ ksi})(4067 \text{ in}^4)}{6.78''} = 1080''\cdot k = \underline{90'k}$$

STEEL: $f_s = 20$ ksi

$$M = \frac{f_s I_{TR}}{cn} = \frac{(20 \text{ ksi})(4067 \text{ in}^4)}{(10.22'')(9)} = 884''\cdot k = \underline{74'k}$$

$90 > 74$ → STEEL GOVERNS

BEAM CAPACITY = 74'k

5. FIND ACTUAL STRESSES IN STEEL & CONCRETE:

STEEL: MOMENT-RESISTING CAPACITY GOVERNS

STRESS IN STEEL = ALLOWABLE = $\underline{20 \text{ ksi}}$

CONCRETE: $f = \frac{Mc}{I_{TR}} = \frac{(74'k * 12''/')(6.78'')}{4067 \text{ in}^4} = \underline{1.48 \text{ ksi}}$