2009-01

ARCH 324 - Structures 2, Winter 2009

von Buelow, Peter

<http://hdl.handle.net/2027.42/64938>
http://hdl.handle.net/2027.42/64938
Example: Concrete Beam Analysis
Working Stress Method

Calculate the bending capacity of the following beam:

3 - #9 bars
(A_s = 3.00 in^2)

12" 17" 20"

E_s = 29,000 ksi, f_s = 20 ksi
E_c = 3,220 ksi, f_c = 1.8 ksi

1. Transform section:

n = \frac{E_s}{E_c} = \frac{29,000}{3,220} = 9

A_{tr} = (3 in^2)(9) = 27 in^2

2. Determine neutral axis of effective section:

A_c \overline{x}_c = A_s \overline{x}_s

(12"*x) \left( \frac{x}{2} \right) = (27 in^2)(17-x)

6x^2 = 459 - 27x

0 = 6x^2 + 27x - 459

0 = x^2 + 4.5x - 76.5

Solve with quadratic equation:

x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
2. Neutral Axis, Cont'd...

\[ x = \frac{-4.5 \pm \sqrt{(4.5)^2 - 4(1)(-70.5)}}{2(1)} = 6.78" \quad \text{OR} = 41.25" \quad \text{ANSWER} \]

(IGNORE NEGATIVE)

3. Find transformed moment of inertia:

\[ I_c = \frac{bd^3}{3} = \frac{12"(6.78\text{in})^3}{3} = 1247 \text{in}^4 \]

\[ I_s = A_x^2 = (27\text{in}^2)(10.22\text{in})^2 = 2820 \text{in}^4 \]

(ASSUME MINUTE THICKNESS OF STEEL → NO \( \frac{bh^3}{12} \)) \[ I_{tr} = 4067 \text{in}^4 \]

4. Resisting capacity of concrete and steel:

**Concrete:** \( f_c = 1.8 \text{ksi} \)

\[ M = \frac{f_c I_{tr}}{c} = \frac{(1.8 \text{ksi})(4067 \text{in}^4)}{6.78\text{in}} = 1080 \text{in} \cdot \text{k} = 90 \text{k} \]

**Steel:** \( f_s = 20 \text{ksi} \)

\[ M = \frac{f_s I_{tr}}{c} = \frac{(20 \text{ksi})(4067 \text{in}^4)}{(10.22\text{in})(9)} = 884 \text{in} \cdot \text{k} = 74 \text{k} \]

90.774 → Steel governs

**Beam capacity = 74 k**

5. Find actual stresses in steel & concrete:

**Steel:** Moment-resisting capacity governs

Stress in steel = allowable = \( 20 \text{ksi} \)

**Concrete:** \( f = \frac{M_c}{I_{tr}} = \frac{(74 \text{ k} \times 12\text{in})(6.78\text{in})}{4067 \text{in}^4} = 1.48 \text{ksi} \)