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Architecture 324  
Structures II

Column Analysis and Design

- Failure Modes
- End Conditions and Lateral Bracing
- Analysis of Wood Columns
- Design of Wood Columns
- Analysis of Steel Columns
- Design of Steel Columns
Leonhard Euler (1707 – 1783)

Euler Buckling (elastic buckling)

\[ P_{cr} = \frac{\pi^2 AE}{\left( \frac{KL}{r} \right)^2} \]

\[ r = \sqrt{\frac{I}{A}} \]

- A = Cross sectional area (in\(^2\))
- E = Modulus of elasticity of the material (lb/in\(^2\))
- K = Stiffness (curvature mode) factor
- L = Column length between pinned ends (in.)
- r = radius of gyration (in.)

\[ f_{cr} = \frac{\pi^2 E}{\left( \frac{KL}{r} \right)^2} \leq F_{cr} \]

Source: Emanuel Handmann (wikimedia commons)
Failure Modes

- **Short Columns** – fail by crushing
  ("compression blocks or piers" Engel)
  \[
f_c = \frac{P}{A} \leq F_c
\]
  - \(f_c\) = Actual compressive stress
  - \(A\) = Cross-sectional area of column (in\(^2\))
  - \(P\) = Load on the column
  - \(F_c\) = Allowable compressive stress per codes

- **Intermediate Columns** – crush and buckle
  ("columns" Engel)

- **Long Columns** – fail by buckling
  ("long columns" Engel)
  \[
f_{cr} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \leq F_{cr}
\]
  - \(E\) = Modulus of elasticity of the column material
  - \(K\) = Stiffness (curvature mode) factor
  - \(L\) = Column length between pinned ends (in.)
  - \(r\) = radius of gyration = \((I/A)^{1/2}\)
Slenderness Ratio

- Radius of Gyration: a geometric property of a cross section
  \[ r = \sqrt{\frac{I}{A}} \]
  \[ I = Ar^2 \]
  - \( r \) = Radius of Gyration
  - \( I \) = Moment of Inertia
  - \( A \) = Cross-sectional Area

- Slenderness Ratios:
  \[ \frac{L_x}{r_x} \]
  \[ \frac{L_y}{r_y} \]

  The larger ratio will govern.
  Try to balance for efficiency

\( r_x = 0.999 \)
\( r_y = 0.433 \)
End Support Conditions

$K$ is a constant based on the end conditions

- $I$ is the actual length
- $l_e$ is the effective length

$l_e = Kl$

$K= 0.5$

Both ends fixed.

$K= 1.0$

Both ends pinned.

$K= 0.7$

One end free, one end fixed.

$K= 2.0$

One end pinned, one end fixed.
Analysis of Wood Columns

Data:
• Column – size, length
• Support conditions
• Material properties – $F_c$, $E$

Required:
• $P_{\text{crit}}$ for buckling and crushing

1. Calculate slenderness ratio; largest ratio governs.
2. Check slenderness against upper limit.
3. Calculate $P_{\text{crit}}$ for buckling using Euler’s equation:
4. Calculate $P_{\text{max}}$ for crushing:
   $$P_{\text{max}} = F_c \cdot A$$
5. Smaller of $P_{\text{crit}}$ or $P_{\text{max}}$ will fail first.
Example Problem:

Analysis

Data: section 3"x7" Full Dimension

\[ F_c = 1000 \text{ psi} \]
\[ E = 1,400,000 \text{ psi} \]

Find: \( P_{\text{critical}} \) for buckling and crushing.
Determine the mode of failure for the wood column.
Example Problem: Analysis (cont.)

1. Calculate slenderness ratios for each axis.

   The larger (more slender) controls.

2. Upper limits are usually given by codes.
Example Problem: Analysis (cont.)

3. Calculate critical Euler buckling load.

\[ P_{cr} = \frac{\pi^2 AE}{(K \frac{L}{r})^2} = \frac{\pi^2 (21)(1400,000)}{(55.4)^2} \]

\[ P_{cr} = 94,500 \] kN

5. Calculate crushing load.

\[ P = FA = 1000 \times 21 = 21,000 \] kN

21,000 < 94,500 \therefore \text{Use} 21,000 \] kN

7. Smaller of the two will fail first and control.
Analysis of Steel Columns
by Engel

Data:
- Column – size, length
- Support conditions
- Material properties – $F_y$
- Applied load - $P_{\text{actual}}$

Required:
- $P_{\text{actual}} < P_{\text{allowable}}$

2. Check slenderness ratio against upper limit of 200
3. Use the controlling slenderness ratio to find the critical Euler buckling stress, $f_{\text{cr}}$.
4. Apply some Factor of Safety (like 3) to $f_{\text{cr}}$.
5. Determine yield stress limit, $F_y$.
6. $F_{\text{allowable}}$ is the lesser stress: $(f_{\text{cr}} / \text{F.S.})$ or $F_y$
7. Compute allowable capacity: $P_{\text{allowable}} = F_{\text{allow}} \ A$.
8. Check column adequacy: $P_{\text{actual}} < P_{\text{allowable}}$

$$f_{\text{cr}} = \frac{\pi^2 E}{(\frac{KL}{r})^2}$$
Design of Steel Columns
by Engel

Data:
• Column – length
• Support conditions
• Material properties – $F_y$
• Applied load - $P_{actual}$

Required:
• Column – section

1. Use the Euler equation to solve for $Ar^2$ which is equal to $I$ for both x and y axis.
2. Enter the section tables and find the least weight section that satisfies BOTH $I_x$ and $I_y$.
3. Check the slenderness ratios are both < 200.
4. Calculate the actual Euler stress $f_{cr}$ for the final section.
5. $F_{allowable}$ is the lesser stress: $f_{cr} / F.S.$ or $F_y$
6. Compute allowable capacity: $P_{allowable} = F_{allow} \times A$.  

\[
I_x = \frac{P(K_x I_x)^2}{\pi^2 E} \times F.S.
\]
\[
I_y = \frac{P(K_y I_y)^2}{\pi^2 E} \times F.S.
\]
Example Problem: Design

Select a steel section that can carry the given load.

Given:
A 36 steel \( F_y = 36 \text{ ksi} \) \( E = 29000 \text{ ksi} \)
braced @ midpoint in weak direction

\( k_x = 1.54 \) \( d_x = 19 \text{ ft} \)
\( k_y = 1.17 \) \( d_y = 9.5 \text{ ft} \)
Factor of Safety = 3.0

Total height 19 feet
Braced at midpoint

\( P = 65 \text{ k} \)
Example Problem: Design (cont.)

X-X (strong) axis

\[ I_x = \frac{P (lx - l_x)^2 (F.S.)}{\Pi^2 E} \]
\[ = \frac{(0.5^k)(1.54 \times 19.4 \times 12'' \times 12''/)}{\Pi^2 (29000 \text{ ksi})} (3.0) \]
\[ I_x = 134 \text{ in}^4 \]

Y-Y (weak) axis

\[ I_y = \frac{P (ly - l_y)^2 (F.S.)}{\Pi^2 E} \]
\[ = \frac{(0.5^k)(1.17 \times 9.5 \times 12'' \times 12''/)}{\Pi^2 (29000 \text{ ksi})} (3.0) \]
\[ I_y = 12.12 \text{ in}^4 \]

Pick section: \[ W10 \times 26 \]

\[ I_x = 144 \text{ in}^4 \quad I_y = 14.075 \text{ in}^4 \]

W16 \times 31 \quad I_x = 144 \text{ in}^4 \quad I_y = 12.48 \text{ in}^4

Select column that meets requirement: lightest weight.
Example Problem: Design (cont.)

- Determine the controlling slenderness (larger controls)
- Find the actual buckling stress, \( f_{cr} \)
- Compare to allowable stress, \( F_{allowable} \) is lesser of: \( f_{cr}/F.S. \) or \( F_y \)
- Determine safe allowable load, \( P_{allowable} = F_{allowable} A \)

![Slenderness Ratios]

\[
\begin{align*}
X-X & \quad Y-Y \\
\frac{k_xl_x}{l_x} & = \frac{1.54(19')(12'')}{4.35''} & \frac{k_yl_y}{l_y} & = \frac{1.17(9.5')(12'')}{1.36} \\
80.7 & < 200 & 98.1 & < 200 \checkmark
\end{align*}
\]

More slender controls

**Euler Stress**

\[
f_{cr} = \frac{\pi^2 E}{(\frac{K}{F})^2} = \frac{\pi^2(29,000 \text{ ksi})}{98.1^2} = 29.75 \text{ ksi}
\]

\[
F_{allow} = \frac{f_{cr}}{F.S.} = \frac{29.75}{3.0} = 9.9 \text{ ksi} < F_y
\]

\[\therefore \text{ use W10 x 26}\]

\[
P_{allow} = F_{allow} A = 9.9 (7.61) = 75.3 > 65^k \checkmark
\]
Determining K factors by AISC

Sidesway Inhibited:
Braced frame
$1.0 > K > 0.5$

Sidesway Uninhibited:
Un-braced frame
unstable $> K > 1.0$

If $I_{c}/L_{c}$ is large
and $I_{g}/L_{g}$ is small
The connection is more pinned

If $I_{c}/L_{c}$ is small
and $I_{g}/L_{g}$ is large
The connection is more fixed

Figure 1. The subscripts $A$ and $B$ refer to the joints at the two ends of the column section being considered. $G$ is defined as

$$G = \frac{\sum (I_{c}/L_{c})}{\sum (I_{g}/L_{g})}$$

in which $\sum$ indicates a summation of all members rigidly connected to that joint and lying in the plane in which buckling of the column is being considered. $I_{c}$ is the moment of inertia and $L_{c}$ the unsupported length of a column section, and $I_{g}$ is the moment of inertia and $L_{g}$ the unsupported length of a girder or other restraining member. $I_{c}$ and $I_{g}$ are taken about axes perpendicular to the plane of buckling being considered.

For column ends supported but not rigidly connected to a footing or foundation, $G$ is theoretically infinity, but, unless actually designed as a true friction free pin, may be taken as “10” for practical designs. If the column end is rigidly attached to a properly designed footing, $G$ may be taken as 1.0. Smaller values may be used if justified by analysis.

Source: American Institute of Steel Construction, Manual of Steel Construction, AISC 1980
Steel Frame Construction
Analysis of Steel Columns by AISC-ASD

Data:
- Column – size, length
- Support conditions
- Material properties – $F_y$
- Applied load - $P_{\text{actual}}$

Required:
- $P_{\text{actual}} < P_{\text{allowable}}$

1. Calculate slenderness ratios. largest ratio governs.
2. In AISC Table look up $F_a$ for given slenderness ratio.
3. Compute: $P_{\text{allowable}} = F_a \cdot A$.
4. Check column adequacy: $P_{\text{actual}} < P_{\text{allowable}}$

---

### Table C-36
Allowable Stress
For Compression Members of 36-ksi Specified Yield Stress Steel

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<thead>
<tr>
<th>$k_I$</th>
<th>$F_a$ (ksi)</th>
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<tr>
<td>1</td>
<td>21.56</td>
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<td>40</td>
<td>19.19</td>
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</table>

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Source: American Institute of Steel Construction, Manual of Steel Construction, AISC 1980

University of Michigan, TCAUP
Structures II
Slide 18/19
Design of Steel Columns with AISC-ASD Tables

Data:
- Column – length
- Support conditions
- Material properties – $F_y$
- Applied load - $P_{\text{actual}}$

Required:
- Column Size

1. Enter table with height.
2. Read allowable load for each section to find the smallest adequate size.
3. Tables assume weak axis buckling. If the strong axis controls the length must be divide by the ratio $r_x/r_y$
4. Values stop in table (black line) at slenderness limit, $KL/r = 200$

---

### Tables

<table>
<thead>
<tr>
<th>Designation</th>
<th>W8</th>
<th>W6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wt./ft</td>
<td>36</td>
<td>50</td>
</tr>
<tr>
<td>$F_y$</td>
<td>28</td>
<td>24</td>
</tr>
<tr>
<td>$F_y$</td>
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<td></td>
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<td>0</td>
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<td>1</td>
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<td>16</td>
<td>46</td>
<td>46</td>
</tr>
<tr>
<td>17</td>
<td>39</td>
<td>39</td>
</tr>
<tr>
<td>Effective length in ft, with respect to least radius of gyration $r_y$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_e$</td>
<td>36</td>
<td>36</td>
</tr>
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</table>

### Properties

<table>
<thead>
<tr>
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<th>W8</th>
<th>W6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U$</td>
<td>3.23</td>
<td>3.23</td>
</tr>
<tr>
<td>$P_{\text{ax}}$ (kips)</td>
<td>48</td>
<td>39</td>
</tr>
<tr>
<td>$P_{\text{ax}}$ (kips/ft)</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>$P_{\text{ax}}$ (kips)</td>
<td>93</td>
<td>110</td>
</tr>
<tr>
<td>$P_{\text{ax}}$ (kips)</td>
<td>49</td>
<td>66</td>
</tr>
<tr>
<td>$L_e$ (ft)</td>
<td>5.9</td>
<td>5.9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>W8</th>
<th>W6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$ (in.$^2$)</td>
<td>8.25</td>
<td>7.08</td>
</tr>
<tr>
<td>$I_x$ (in.$^4$)</td>
<td>96.0</td>
<td>82.8</td>
</tr>
<tr>
<td>$I_y$ (in.$^4$)</td>
<td>21.7</td>
<td>18.3</td>
</tr>
<tr>
<td>$I_{\text{ax}}$ (in.$^4$)</td>
<td>1.62</td>
<td>1.61</td>
</tr>
<tr>
<td>Ratio $I_x/I_y$</td>
<td>2.13</td>
<td>2.12</td>
</tr>
<tr>
<td>$F_{\text{b}}$ (ksi)</td>
<td>0.340</td>
<td>0.339</td>
</tr>
<tr>
<td>$F_{\text{b}}$ (ksi)</td>
<td>1.244</td>
<td>1.258</td>
</tr>
<tr>
<td>$a_{\text{cr}}$ (in.$^4$)</td>
<td>14.62</td>
<td>12.34</td>
</tr>
<tr>
<td>$a_{\text{cr}}$ (ksi)</td>
<td>3.23</td>
<td>2.73</td>
</tr>
<tr>
<td>$F_{\text{ax}}$ ($K_L L_e^2 f_y^2$ ) (kips)</td>
<td>123</td>
<td>121</td>
</tr>
<tr>
<td>$F_{\text{ax}}$ ($K_L L_e^2 f_y^2$ ) (kips)</td>
<td>27.2</td>
<td>26.9</td>
</tr>
</tbody>
</table>

*Note: Heavy line indicates $K/\sigma$ of 200.

Source: American Institute of Steel Construction, Manual of Steel Construction, AISC 1980