Architecture 324

Structures II

Combined Materials

- Strain Compatibility
- Transformed Sections
- Flitched Beams

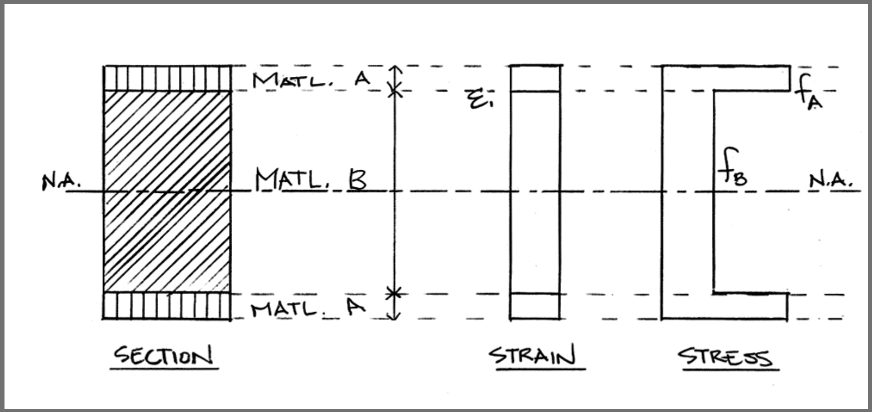
Strain Compatibility

With the two materials bonded together, both will act as one and the deformation in each is the same.

Therefore, the strains will be the same in each under axial load, and in flexure stains are the same as in a solid section, i.e. linear.

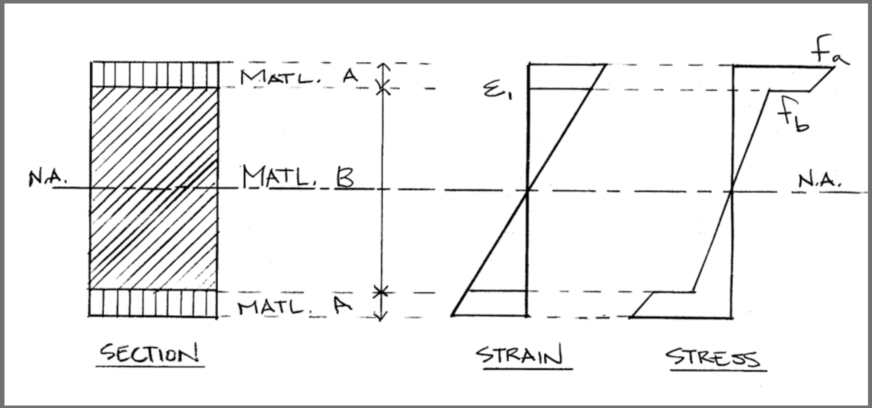
In flexure, if the two materials are at the same distance from the N.A., they will have the same strain at that point. Therefore, the strains are “compatible”.

Axial



Source: University of Michigan, Department of Architecture

Flexure



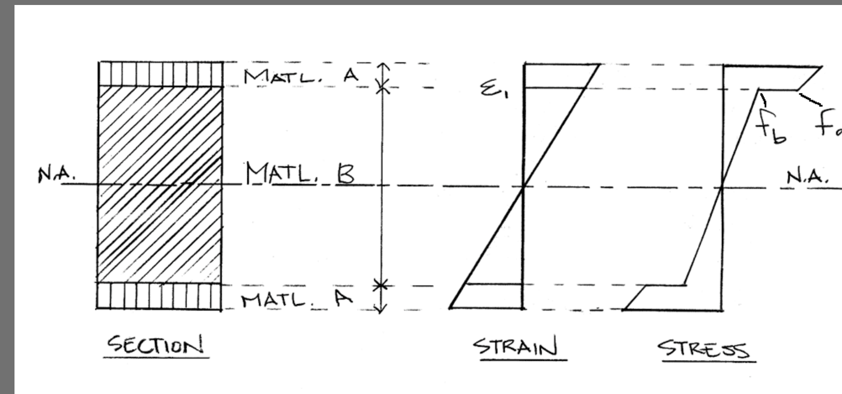
Source: University of Michigan, Department of Architecture

Strain Compatibility (cont.)

The stress in each material is determined by using Young's Modulus

$$\sigma = E\varepsilon$$

Care must be taken that the elastic limit of each material is not exceeded, in either stress or strain.



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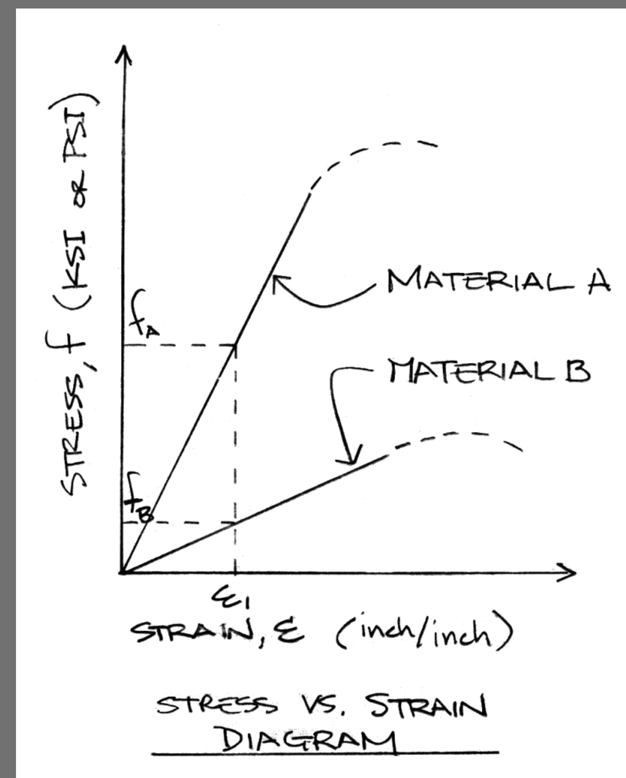


Source: University of Michigan, Department of Architecture

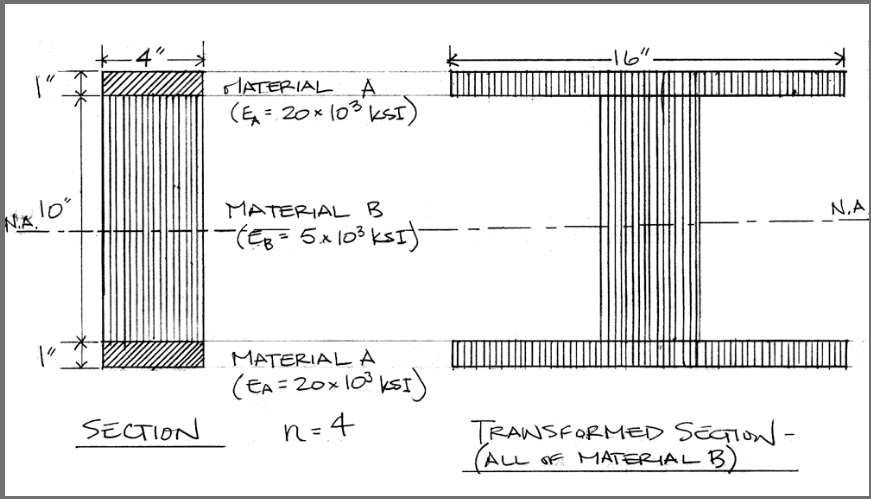
Transformed Sections

Because the material stiffness E can vary for the combined materials, the Moment of Inertia, I , needs to be calculated using a "transformed section".

In a transformed section, one material is transformed into an equivalent amount of the other material. The equivalence is based on the modular ratio, n.

$$n = \frac{E_A}{E_B}$$

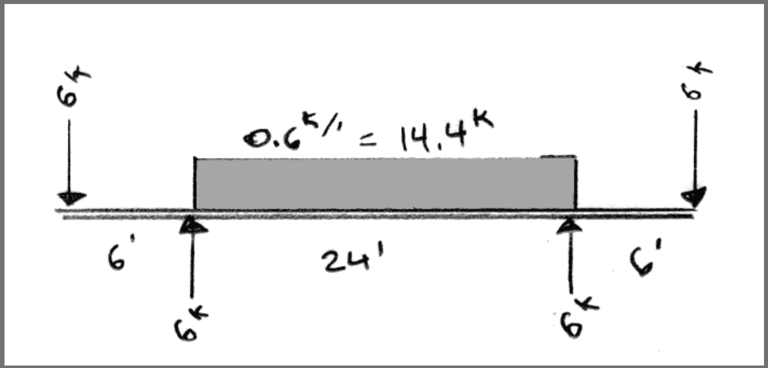
Based on the transformed section, I_{tr} can be calculated, and used to find flexural stress and deflection.



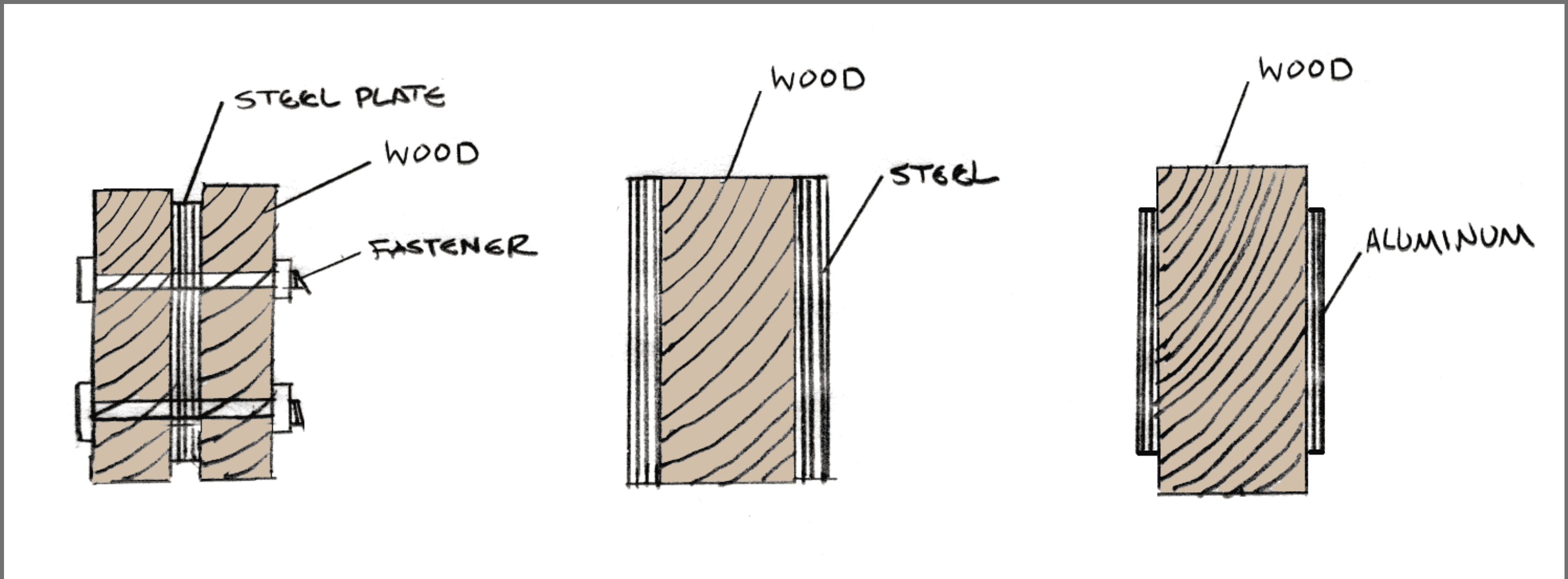
Source: University of Michigan, Department of Architecture

Flitched Beams & Scab Plates

- Compatible with wood structure, i.e. can be nailed
- Lighter weight than steel section
- Less deep than wood alone
- Stronger than wood alone
- Allow longer spans
- Can be designed over partial span to optimize the section (scab plates)



Source: University of Michigan, Department of Architecture



Source: University of Michigan, Department of Architecture

Analysis Procedure:

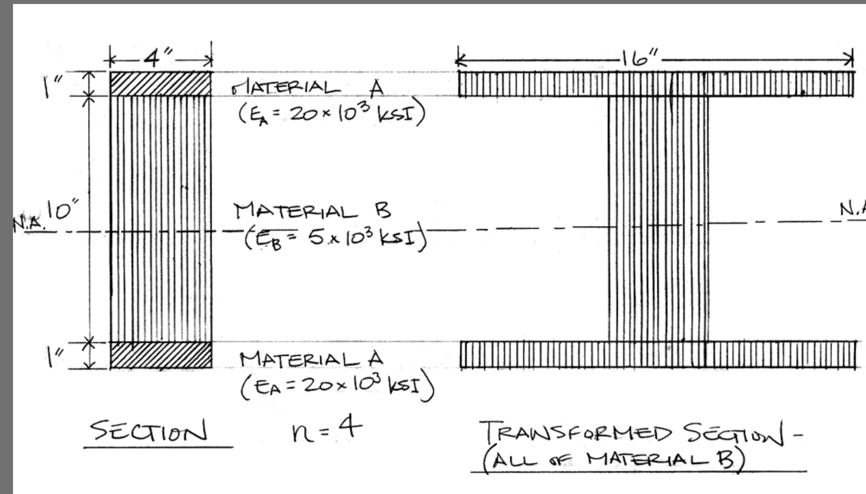
1. Determine the modular ratio(s). Usually the weaker (lower E) material is used as a base.

2. Construct a transformed section by scaling the width of the material by its modular, n.

3. Determine the Moment of Inertia of the transformed section.

4. Calculate the flexural stress in each material using:

$$f_b = \frac{Mc\ n}{I_{tr}}$$



Source: University of Michigan, Department of Architecture

$$n = \frac{E_A}{E_B}$$

Transformed material

Base material

$$I_{tr} = \sum I + \sum Ad^2$$

Transformation equation or solid-void

Analysis Example:

For the composite section, find the maximum flexural stress level in each laminate material.

$$f_b = \frac{\mathrm{M}c\ n}{\mathrm{I}_{tr}}$$

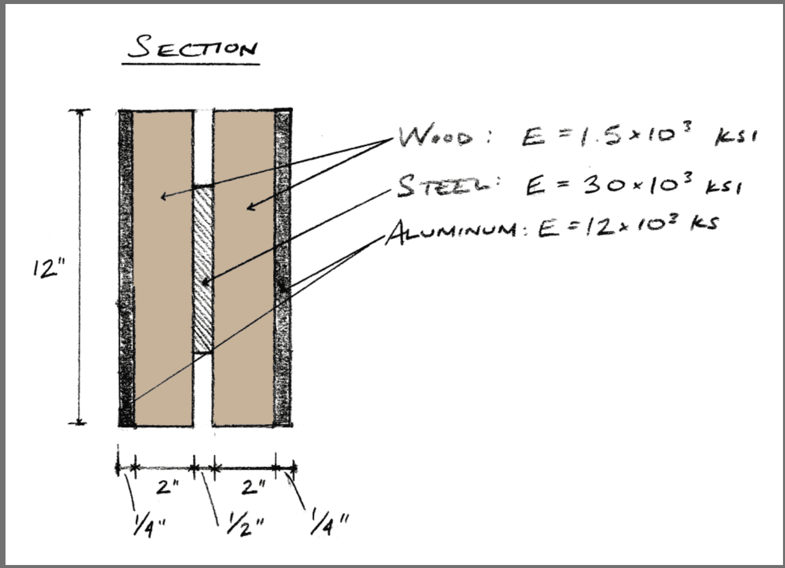
Determine the modular ratios for each material.

Use wood (the lowest E) as base material.

$$n_{WOOD} = \frac{1.5}{1.5} = 1.0$$

$$n_{AL} = \frac{12}{1.5} = 8.0$$

$$n_{ST} = \frac{30}{1.5} = 20.$$



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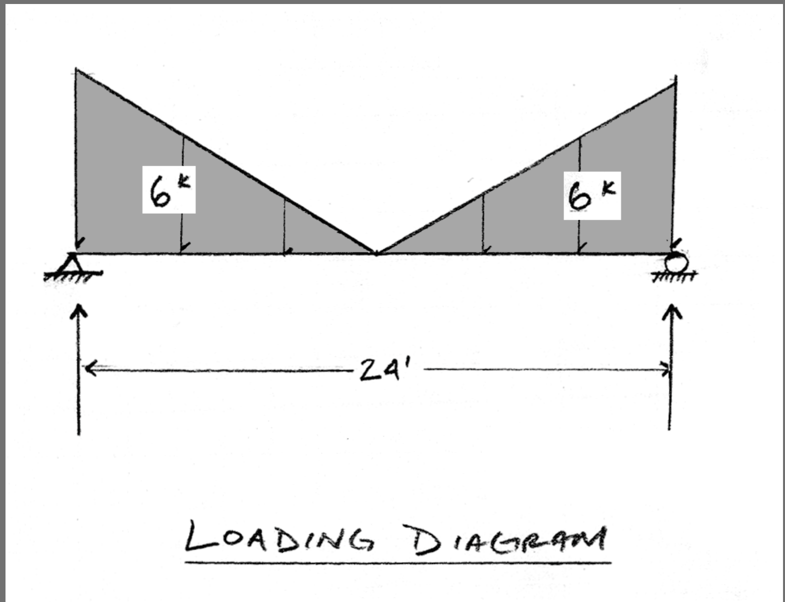

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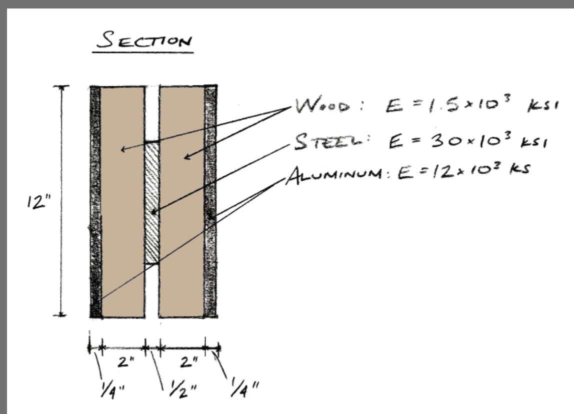
Analysis Example cont.:

Determine the transformed width of each material.

Construct a transformed section.



Source: University of Michigan, Department of Architecture

$$n_{WOOD} = \frac{1.5}{1.5} = 1.0$$

$$n_{AL} = \frac{12}{1.5} = 8.0$$

$$n_{ST} = \frac{30}{1.5} = 20.$$

ALUM.

$t = 1/4''$

$t_{tr} = 1/4 \times n_{AL}$

$= 1/4\,(8.0) = 2.0''$

STEEL

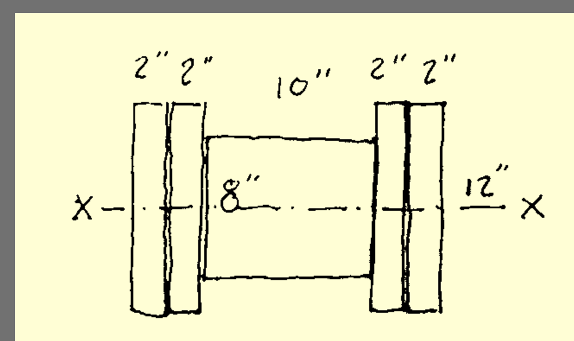
$t = 1/2''$

$t_{tr} = 1/2 \times n_{ST}$

$= 1/2\,(20) = 10.''$

WOOD

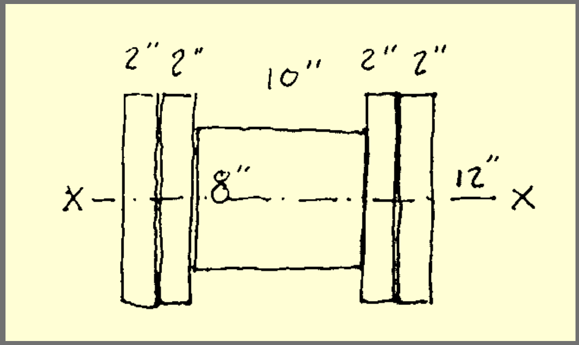
$t = t_{tr} = 2''$



Transformed Section

Analysis Example cont.:

Construct a transformed section.



Calculate the Moment of Inertia for the transformed section.

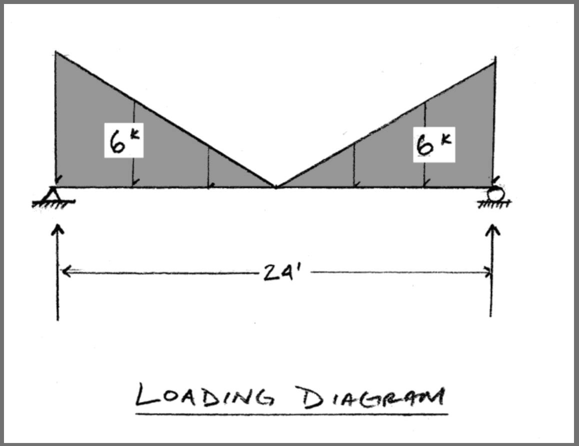
$$I_{tr} = \frac{8 \cdot 12^3}{12} + \frac{10 \cdot 8^3}{12}$$

$$= 1152 + 426$$

$$= 1578 \text{ in}^4$$

Analysis Example cont.:

Find the maximum moment.



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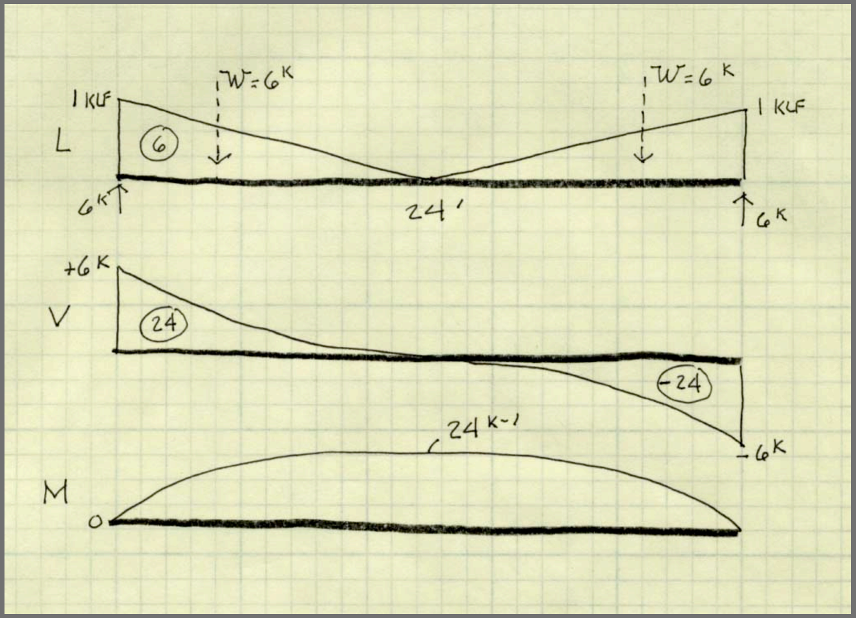
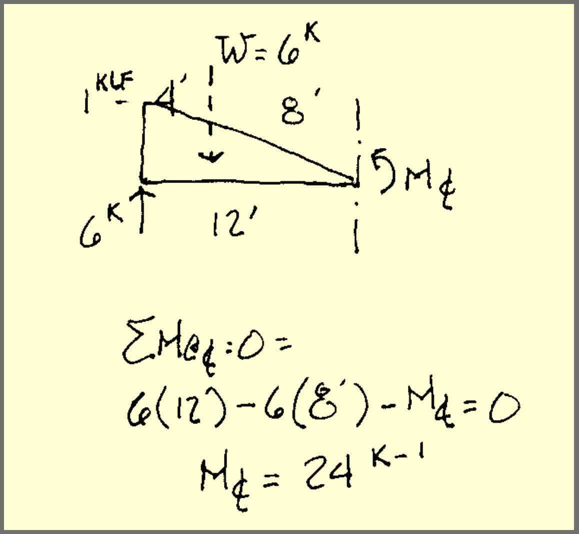




By diagrams or by summing moments

Analysis Example cont.:

Calculate the stress for each material using the modular ratio to convert I.

$$I_{tr} / n = I$$

Compare the stress in each material to limits of yield stress or the safe allowable stress.

$$f_{AL} = \frac{Mc(n)}{I_{tr}} = \frac{24(12)(6'')\,8}{1578}$$

$$= 8.76 \text{ KSI} \quad (f_y \approx 35 \text{ KSI})$$

$$f_{ST} = \frac{Mc(n)}{I_{tr}} = \frac{24(12)(4'')\,20}{1578}$$

$$= 14.6 \text{ KSI} \quad (f_y \approx 36 \text{ KSI})$$

$$f_{WD} = \frac{Mcn}{I_{tr}} = \frac{24(12)(6'')\,1.0}{1578}$$

$$= 1.09 \text{ KSI} \quad (f_y \approx 1.5)$$

Pop Quiz

Material A is A-36 steel (Data Sheet D-4, Gr S-1) E = 29,000 ksi

Material B is aluminum (Data Sheet D-10, Gr A-2) E = 10,000 ksi

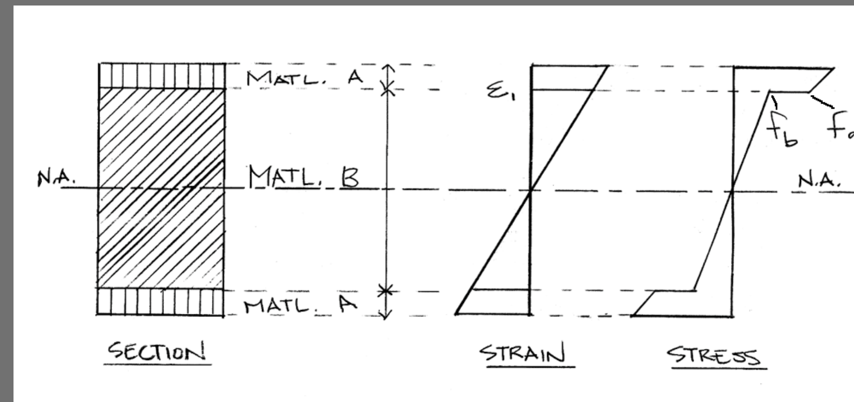
If strain, ε_1 = 0.001

What is the stress in each material at that point?

A steel _______ ksi

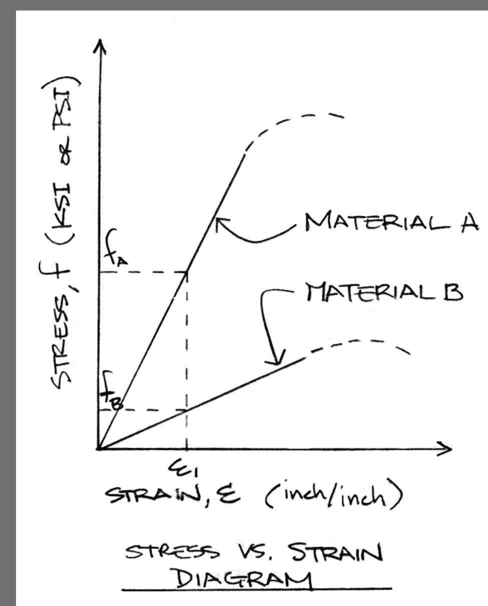
B aluminum ______ ksi

Care must be taken that the elastic limit of each material is not exceeded, in either stress or strain.



Source: University of Michigan, Department of Architecture

$$\sigma = E\varepsilon$$



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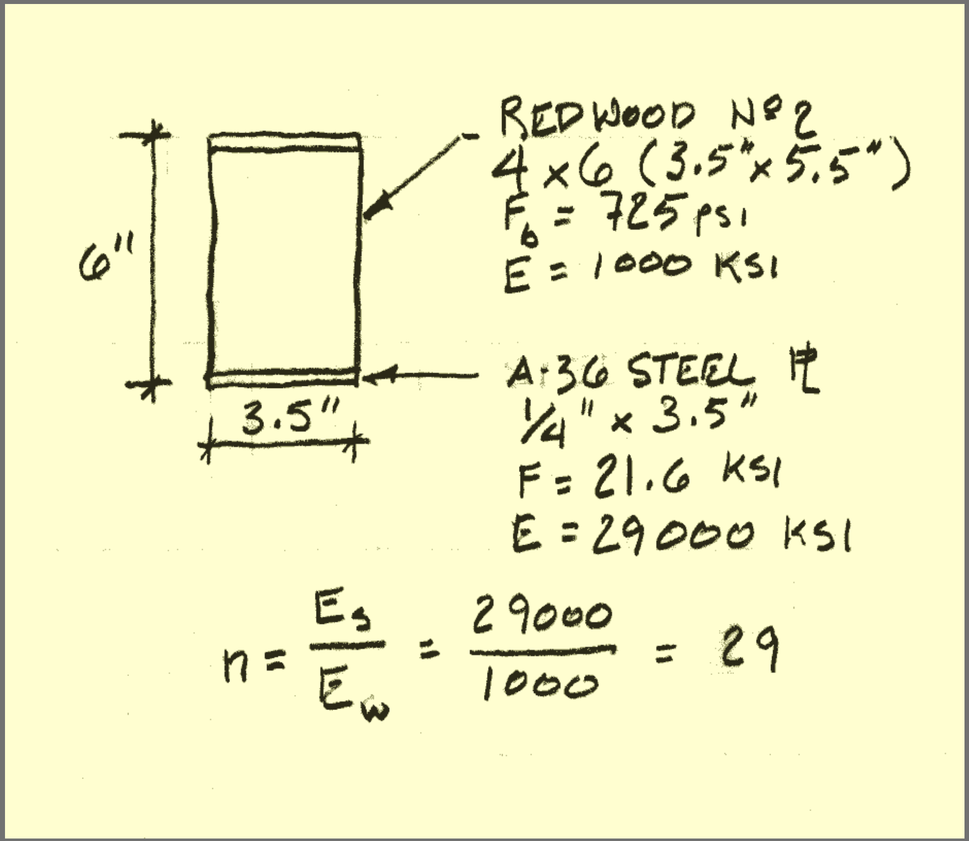
Capacity Analysis

Given

- Dimensions
- Material

Required

- Load capacity



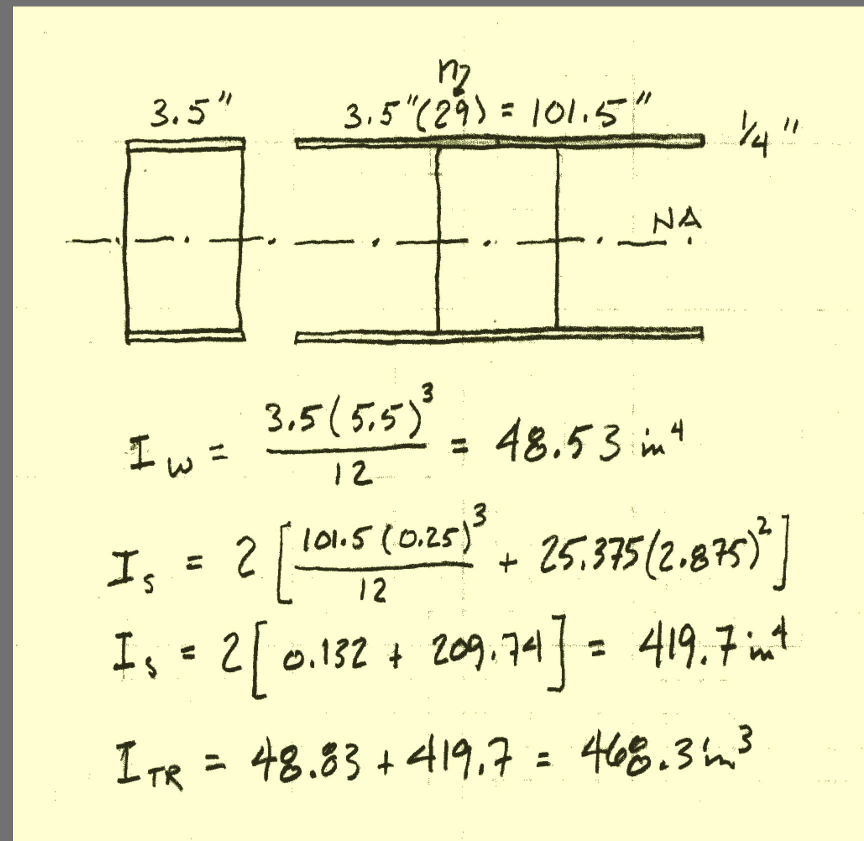
1. Determine the modular ratio.
   It is usually more convenient to transform the stiffer material.

Capacity Analysis (cont.)

2. Construct the transformed section. Multiply all widths of the transformed material by n. The depths remain unchanged.

3. Calculate the transformed moment of inertia, I_{tr} .

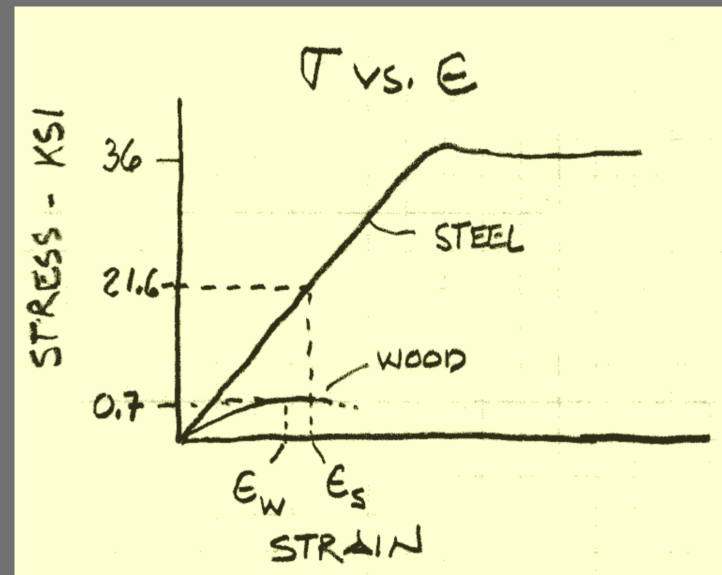
$$I_{tr} = \sum I + \sum A d^2$$



Capacity Analysis (cont.)

4. Calculate the allowable strain based on the allowable stress for the material.

$$\varepsilon_{allow} = \frac{f_{allow}}{E}$$



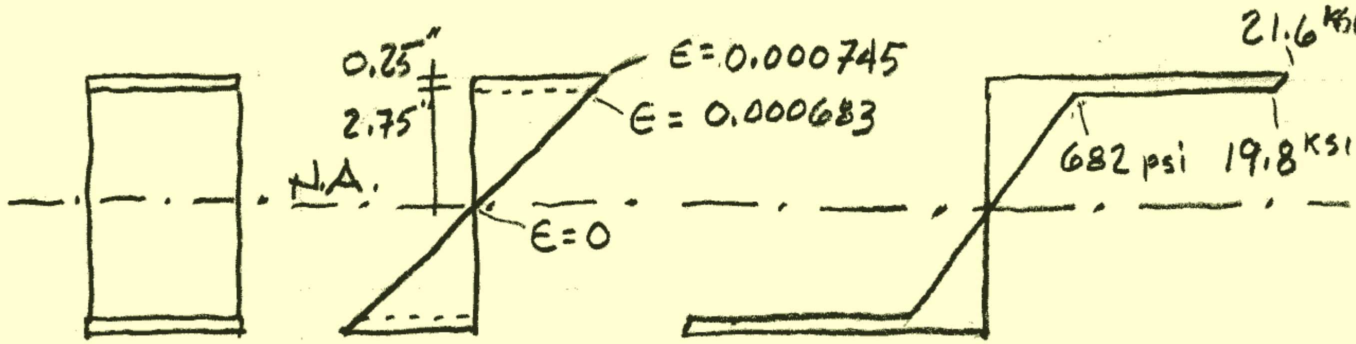
$$\epsilon = \frac{\sigma}{E}$$

$$\epsilon_W = \frac{725}{1000000} = 0.000725$$

$$\epsilon_S = \frac{21.6}{29000} = 0.000745$$

Capacity Analysis (cont.)

5. Construct a strain diagram to find which of the two materials will reach its limit first. The diagram should be linear, and neither material may exceed its allowable limit.



$$\frac{\epsilon}{c} : \frac{0.000745}{3.0''} : \frac{\epsilon_w}{2.75''}$$

$$\epsilon_w = 0.000683$$

$$\sigma = E\epsilon$$

$$f_w = 1000000(0.000683)$$

$$f_w = 682 \text{ psi}$$

$$f_s = 29000(0.000683)$$

$$f_s = 19.8 \text{ ksi}$$

Capacity Analysis (cont.)

6. The allowable moments (load capacity) may now be determined based on the stress of either material. Either stress should give the same moment if the strains from step 5 are compatible (linear).

$$M_s = \frac{f_s I_{TR}}{c n} = \frac{21.6\,(468.3)}{3\,(29)} = 116.2^{\text{k-''}}$$

$$M_w = \frac{f_w I_{TR}}{c} = \frac{0.682\,(468.3)}{2.75''} = 116.1^{\text{k-''}}$$

7. Alternatively, the controlling moment can be found without the strain investigation by using the maximum allowable stress for each material in the moment-stress equation. The lower moment the first failure point and the controlling material.

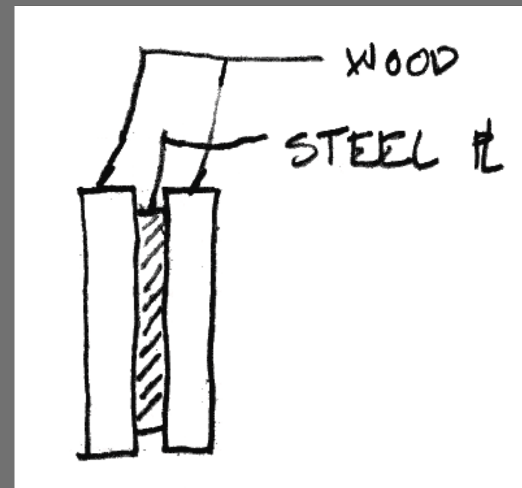
$$M_s = \frac{F_s I_{TR}}{c n} = \frac{21.6\,(468.3)}{3\,(29)} = 116.2^{\text{k-''}} \leftarrow$$

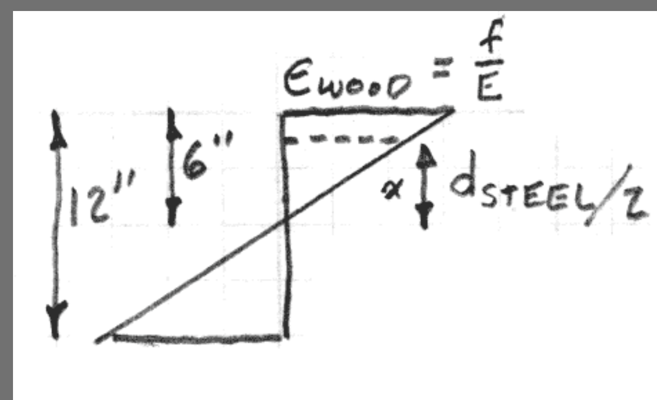
$$M_w = \frac{F_w I_{TR}}{c} = \frac{.725\,(468.3)}{2.75''} = 123.5^{\text{k-''}}$$

Design Procedure:

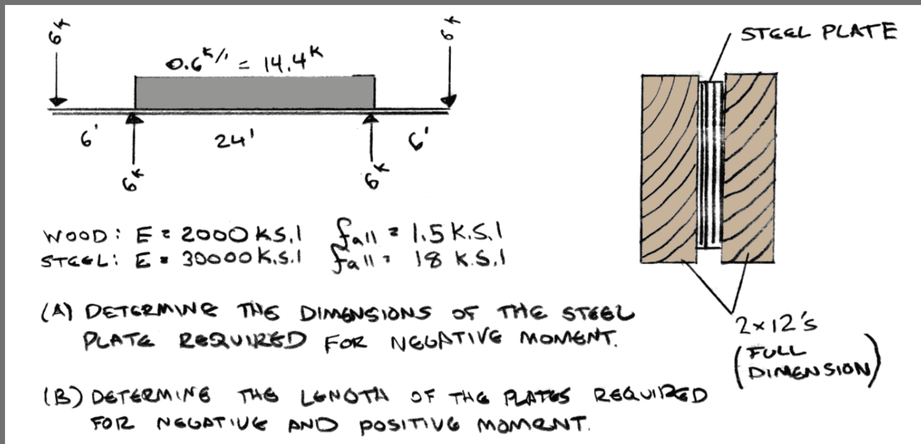
Given: Span and load conditions
Material properties
Wood dimensions
Req'd: Steel plate dimensions

1. Determine the required moment.
2. Find the moment capacity of the wood.
3. Determine the required capacity for steel.
4. Based on strain compatibility with wood, find the largest d for steel where $X_s < X_{allow}$.
5. Calculate the required section modulus for the steel plate.
6. Using d from step 4. calculate b (width of plate).
7. Choose final steel plate based on available sizes and check total capacity of the beam.

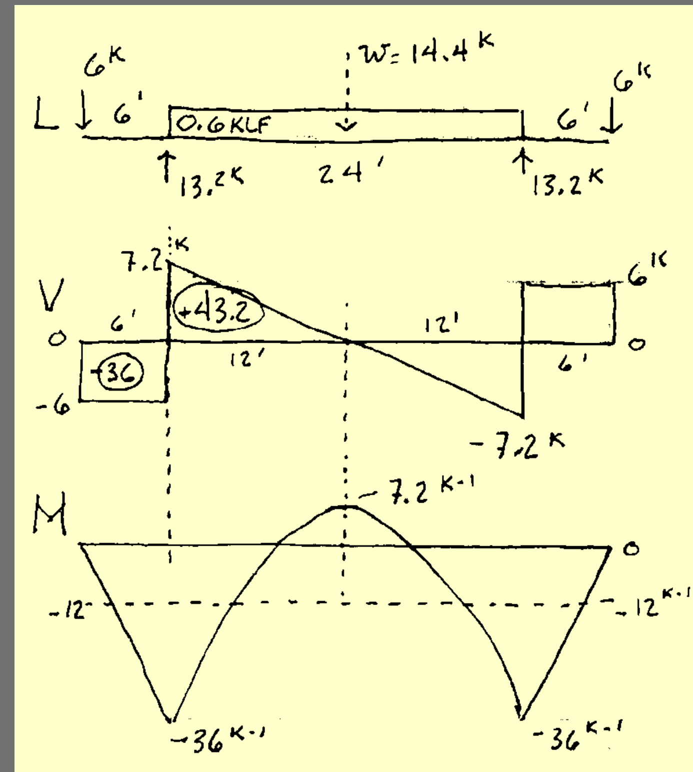




Design Example:



Source: University of Michigan, Department of Architecture



1. Determine the required moment.
2. Find the moment capacity of the wood.
3. Determine the required capacity for steel.

WOOD

$b = 2''$ $d = 12''$

$$S_x = \frac{bd^2}{6} = \frac{2(144)}{6} = 48 \text{ in}^3$$

$\times$ 2 pcs. $S_{WOOD} = 96 \text{ in}^3$

$$M_{WOOD} = F_b S_x$$
$$= 1.5 \text{ ksi } 96 \text{ in}^3 = 144^{k\text{-}''}$$
$$= 12^{k\text{-}1}$$

$$M_{TOTAL} = M_{WOOD} + M_{STEEL} = 36^{k\text{-}1}$$
$$M_{STEEL} = 36^{k\text{-}1} - 12^{k\text{-}1} = 24^{k\text{-}1}$$

Design Example cont:

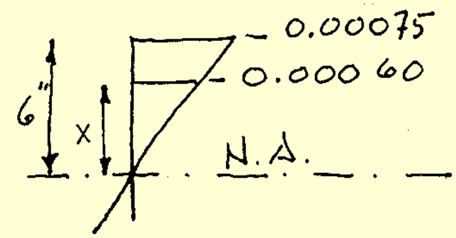
4. Based on strain compatibility with wood, find the largest d for steel where $X_s \leq X_{ALLOW.}$

ALLOWABLE STRAINS

$$\epsilon_w = \frac{f}{E} = \frac{1.5}{2000} = 0.00075$$

$$\epsilon_s = \frac{f}{E} = \frac{18}{30000} = 0.00060$$

STRAIN DIAGRAM



$$\frac{75}{6} : \frac{60}{x} \qquad x = 4.8''$$

$\therefore$ d STEEL PLATE = 9.6"

Design Example cont:

5. Calculate the required section modulus for the steel plate.
6. Using d from step 4. calculate b (width of plate).
7. Choose final steel plate based on available sizes and check total capacity of the beam.

STEEL

$M_{STEEL} = 24^{K-'} = 288^{K-''}$

$F_{ST} = 18^{KSI}$ (GIVEN)

$S_x = \frac{M}{F} = \frac{288}{18} = 16 \text{ in}^3$

STEEL PLATE

S_x REQ'D $= 16 \text{ in}^3 = \frac{bd^2}{6}$

$b = \frac{S_x 6}{d^2} = \frac{16(6)}{9.6^2} = 1.042''$

ROUND TO $\frac{1}{8}'' = 1\frac{1}{8}''$ (SAFE)

OR $1''$ (~OK)

∴ USE

$9.6'' \times 1''$

OR

$9.5'' \times 1\frac{1}{8}''$

Design Example cont:

8. Determine required length and location of plate.

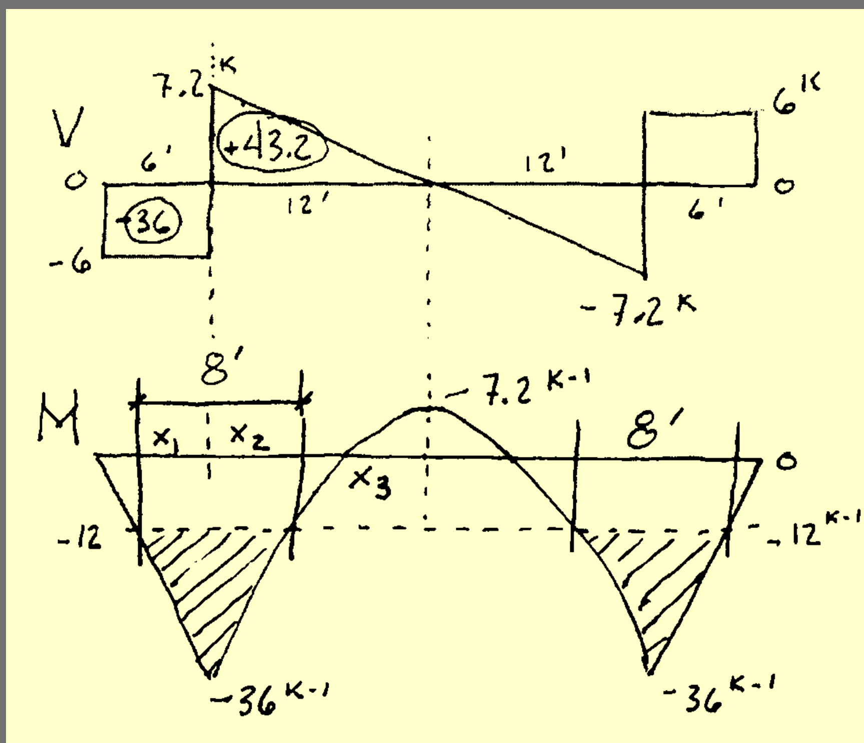
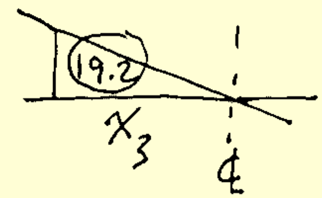


PLATE LENGTH

$\frac{36}{24} \quad \frac{6}{x_1} \quad x_1 = 4'$

SHEAR AREA = 24

$43.2 - 24 = 19.2$



$\frac{bd}{2} = 19.2$

$\frac{x_3\left(\frac{7.2}{12}\ x_3\right)}{2} = 19.2$

$x_3^{\,2} = 64 \quad x_3 = 8'$

$x_2 = 12 - x_3 = 4'$

$\therefore$ PLATE LENGTH = 8'

Applications:

Renovation in Edina, Minnesota

Four 2x8 LVLs, with two 1/2" steel plates. 18 FT span
Original house from 1949
Renovation in 2006
Engineer: Paul Voigt

Applications:

Renovation

Chris Withers House, Reading, UK 2007
Architect: Chris Owens, Owens Galliver
Engineer: Allan Barnes