Architecture 324

Structures II

Reinforced Concrete - WSD

- Material Properties
- Stress in Beams
- Transformed Sections
- Analysis by WSD
- Design by WSD

Constituents of Concrete

- Sand
- Aggregate
- Cement
- Water

Fine aggregate
(Sand)
≤ 1/4"

~ 3/8" aggregate

limestone aggregate
~ 1.5"

Constituents of Concrete

- Sand
- Aggregate
- Water

- Cement
  - Limestone
  - Cement rock
  - Clay
  - Iron ore
  - + (after firing and grinding)
  - gypsum

Cement Types

- **Type 1**
  normal portland cement. Type 1 is a **general use** cement.
- **Type 2**
  is used for structures in water or soil containing moderate amounts of **sulfate**, or when heat build-up is a concern.
- **Type 3**
  high early strength. Used when high strength are desired at very early periods.
- **Type 4**
  low heat portland cement. Used where the amount and rate of heat generation must be kept to a minimum.
- **Type 5**
  Sulfate resistant portland cement. Used where water or soil is high in alkali.
- Types IA, IIA and IIIA are cements used to make **air-entrained** concrete.

Workability

- Measured by the inches of “slump” of a molded cone of fresh mix.
  - range 1” to 4” with vibration
  - 2” to 6” without vibration
- Water/Cement Ratio
  - range 0.4 to 0.7
  - for strength: higher is weaker
  - for workability: higher is better
- Cement Content
  - LBS per cubic yard
  - range 400-800
  - dependent on aggregate
  - increases cost

Reinforcing

- Grade = Yield strength
  - gr. 40 is 40 ksi
  - gr. 60 is 60 ksi

- Size in 1/8 inch increments
  - #4 is ½ inch dia.
  - #6 is ¾ inch dia.

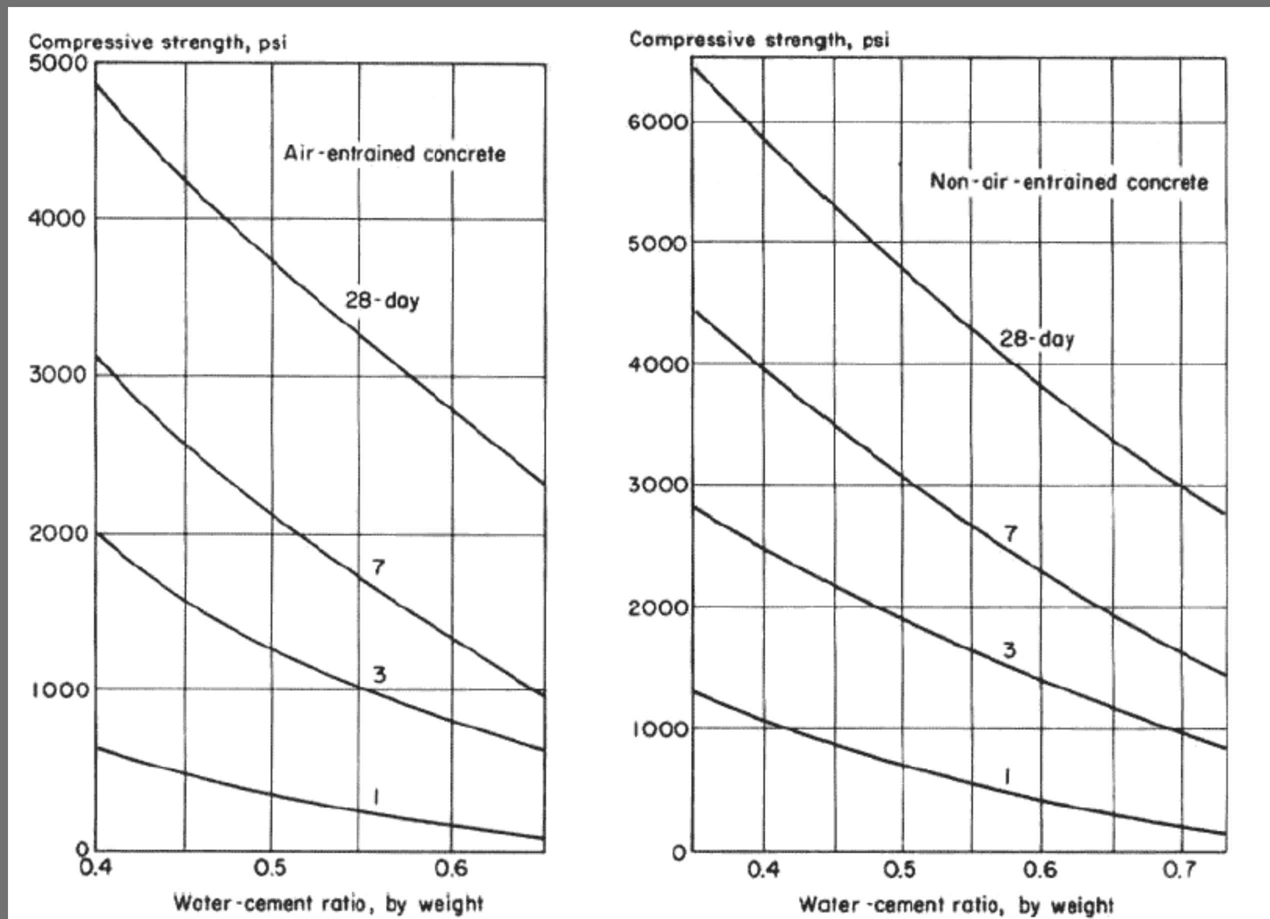
- Deformation Patterns
  - add to bond with concrete

- Spacing
  - between bars
    - Bar diameter
    - 1”
    - 5/4 x max agg.
  - between layers
    - 1”
  - coverage
    - 3” against soil
    - 1.5”-2” exterior
    - 3/4” interior

Reinforcement of Weidatalbrücke

Curing

Strength increases with age. The "design" strength is 28 days.

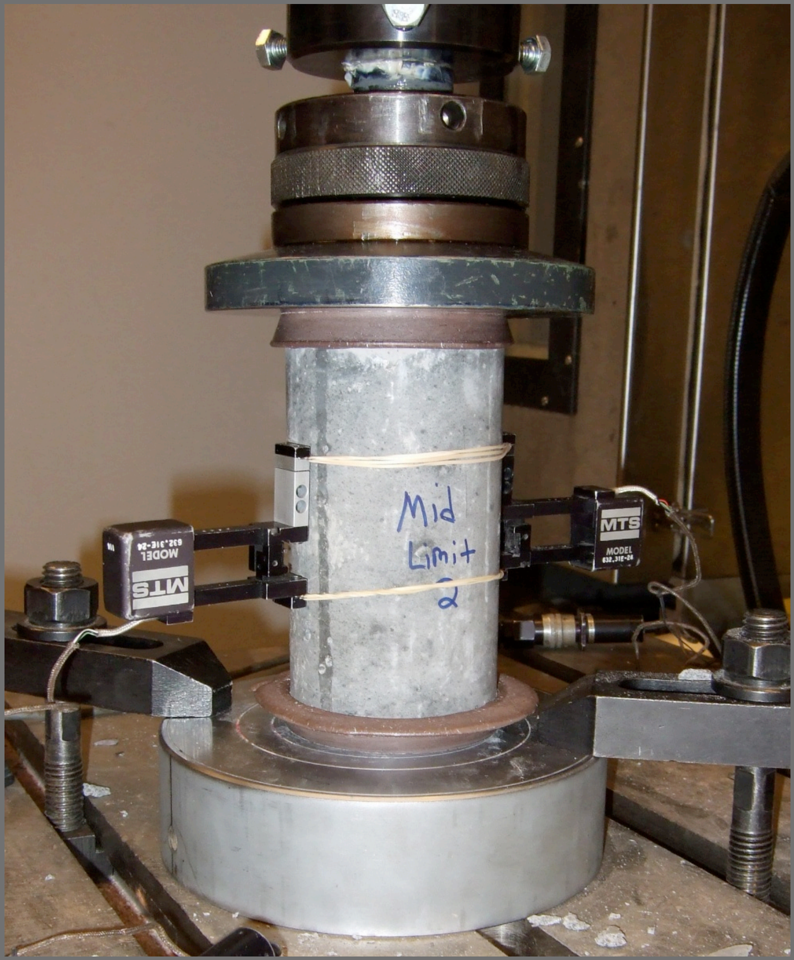


Source: Portland Cement Association

Strength Measurement

- Compressive strength f'_c
  - 12”x6” cylinder
  - 28 day moist cure
  - Ultimate (failure) strength

- Tensile strength f'_t
  - 12”x6” cylinder
  - 28 day moist cure
  - Ultimate (failure) strength
  - Split cylinder test
  - Ca. 10% to 20% of f’c



Photos: Source: Xb-70 (wikipedia)

Young's Modulus

- Depends on density and strength

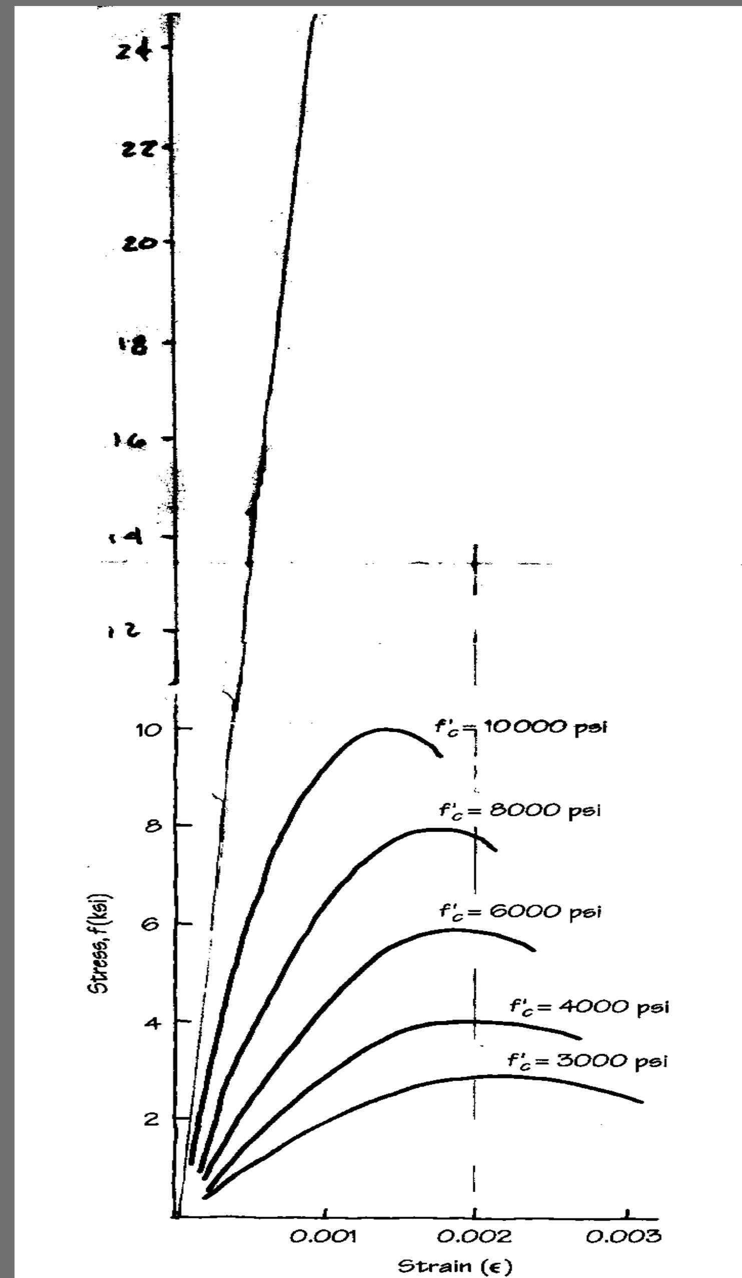
$$E_c = w_c^{1.5} 33\sqrt{f_c^{'}}$$

- For normal (144 PCF) concrete

$$E_c = 57000\sqrt{f_c^{'}}$$

- Examples

f'c	E
3000 psi	3,140,000 psi
4000 psi	3,620,000 psi
5000 psi	4,050,000 psi

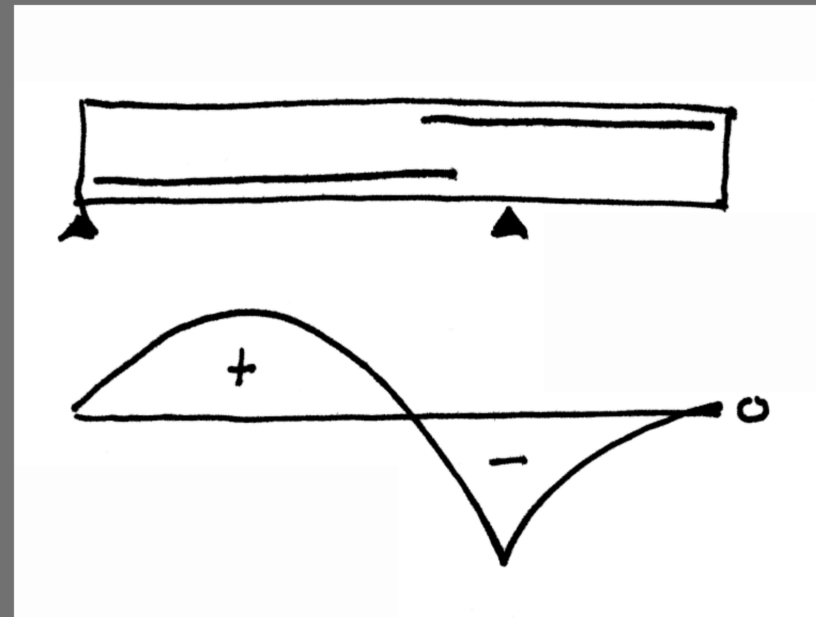


Source: Ronald Shaeffer

Flexure and Shear in Beams

Reinforcement must be placed to resist these tensile forces

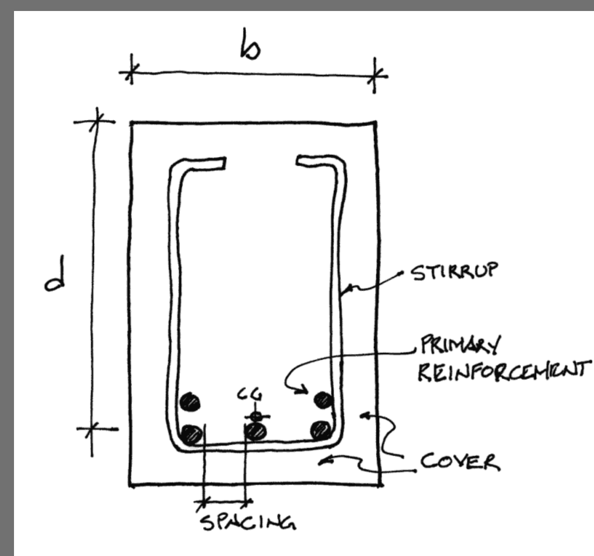
In beams continuous over supports, the stress reverses (negative moment). In such areas, tensile steel is on top.



Shear reinforcement is provided by vertical or sloping stirrups.

Cover protects the steel.

Adequate spacing allows consistent casting.



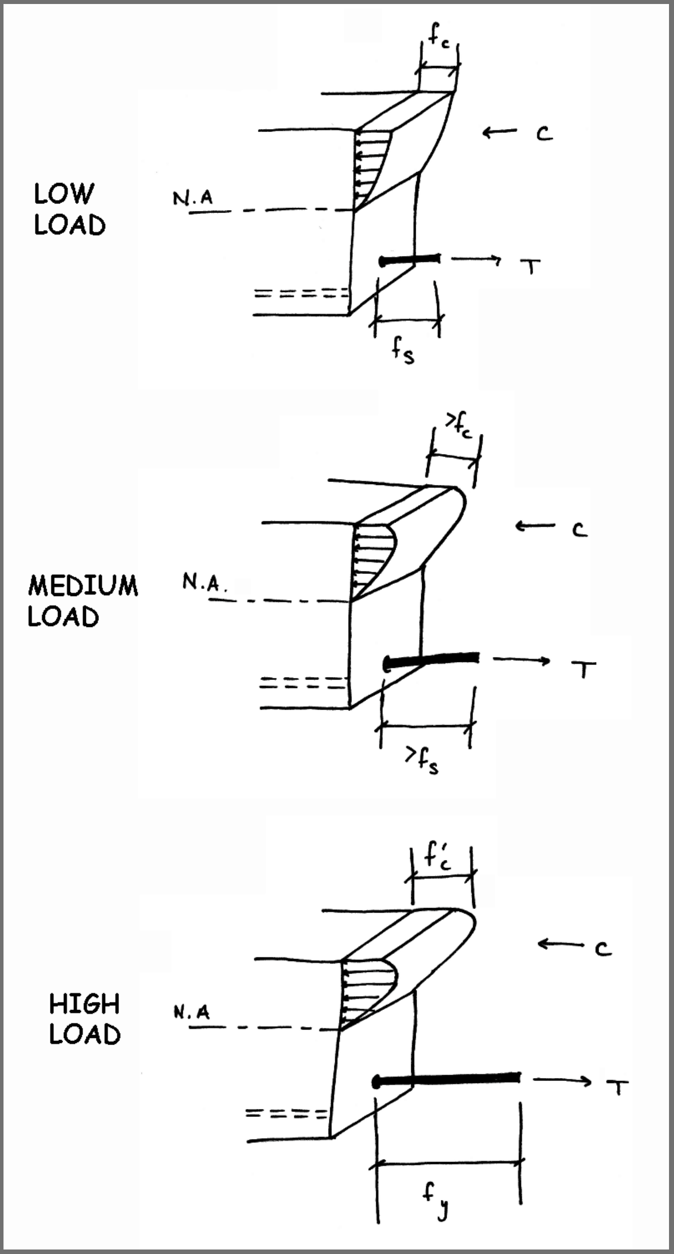
Flexure – WSD Method

- Assumptions:
  - Plane sections remain plane
  - Hooke's Law applies
  - Concrete tensile strength is neglected
  - Concrete and steel are totally bonded

- Allowable Stress Levels
  - Concrete = 0.45f'c
  - Steel = 20 ksi for gr. 40 or gr. 50
    = 24 ksi for gr. 60

- Transformed Section
  - Steel is converted to equivalent concrete.

$$n = \frac{E_s}{E_c}$$



Source: University of Michigan, Department of Architecture

Flexure Analysis

Procedure:

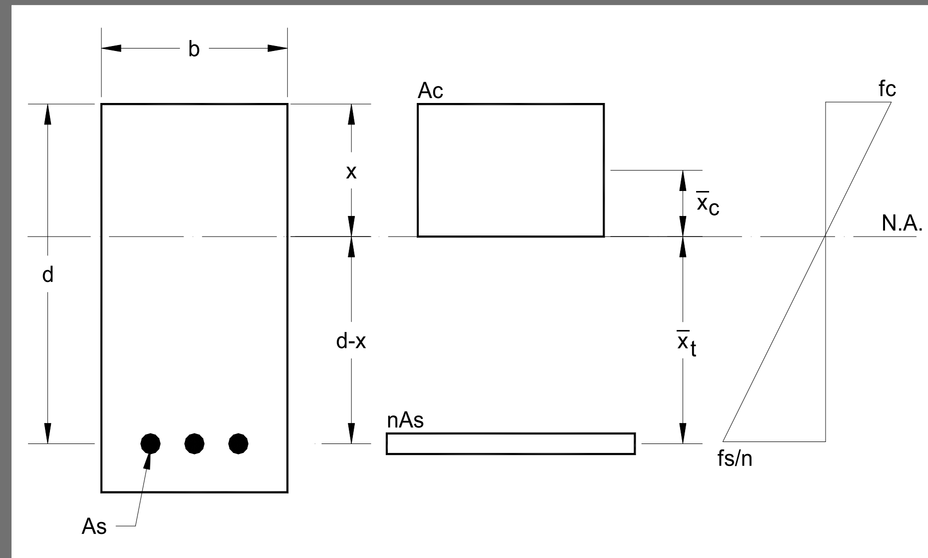
1. Assume the section is cracked to the N.A
2. Determine the modular ratio:

$$n = \frac{E_s}{E_c}$$

3. Transform the area of steel to equivalent concrete, nAs
4. Calculate the location of the N.A. using the balanced tension and compression to solve for x.

$$A_c \bar{x}_c = A_t \bar{x}_t$$

5. Calculate the transformed Moment of Inertia.
6. Calculate a maximum moment based first on the allowable conc. stress and again on the allowable steel stress.
7. The lesser of the two moments will control.



$$A_c \bar{x}_c = A_t \bar{x}_t$$

$$bx\frac{x}{2} = nA_s(d - x)$$

$$\frac{b}{2}x^2 + nA_s x - nA_s d = 0$$

$$I_{tr} = \frac{bx^3}{3} + nA_s(d - x)^2$$

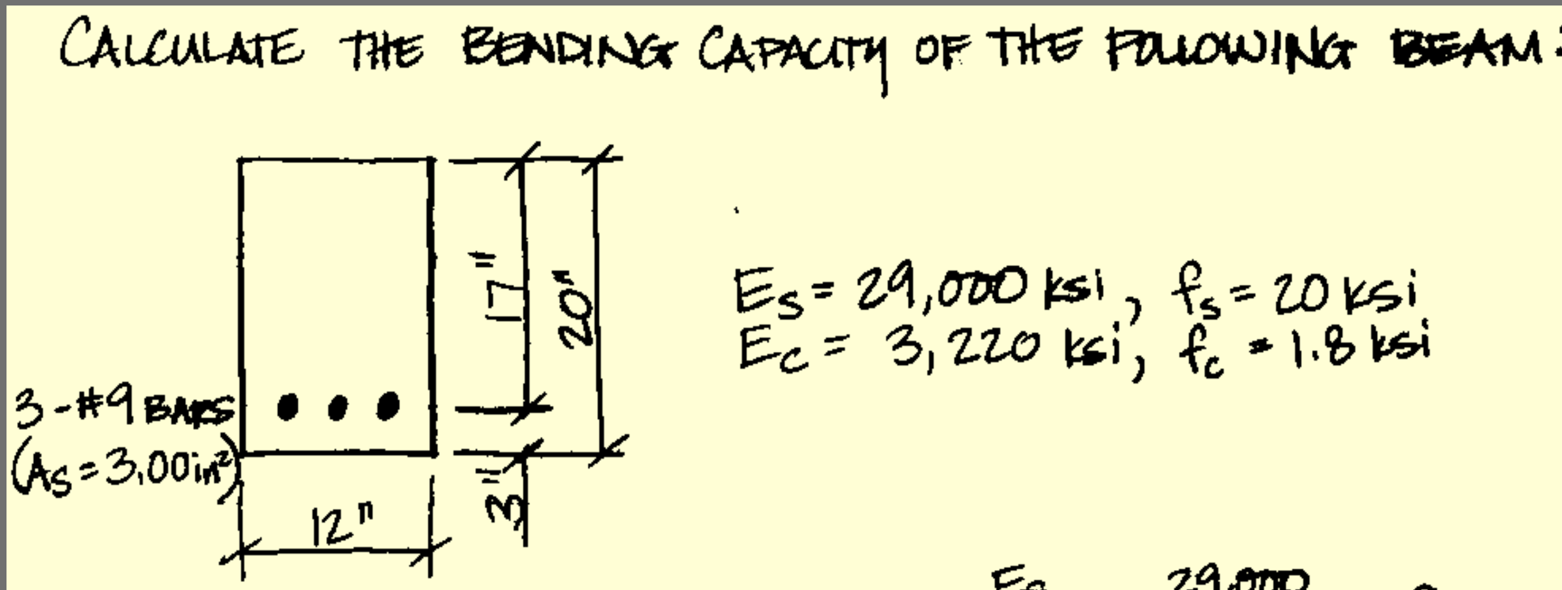
$$M_c = \frac{f_c I_{tr}}{c_c}$$

$$c_c = x$$

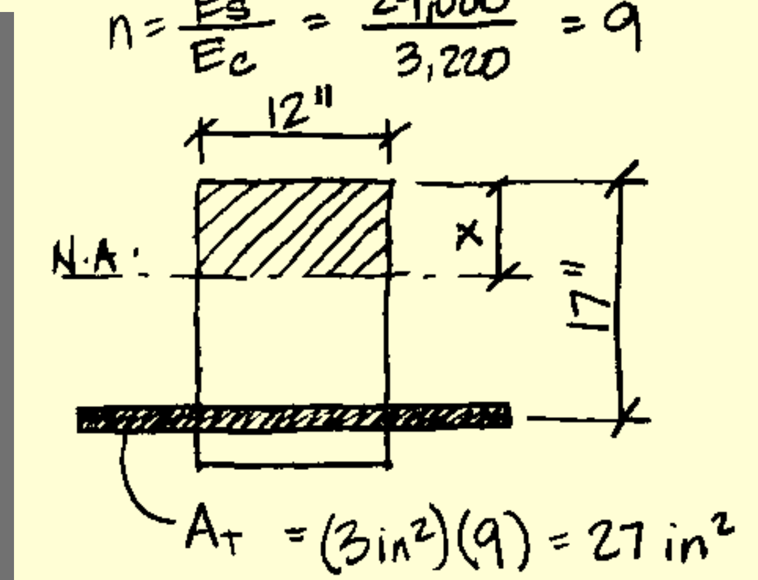
$$M_s = \frac{f_s I_{tr}}{nc_t}$$

$$c_t = d - x$$

Example – Flexure Analysis



1. Assume the section is cracked to the N.A.
2. Determine the transformation ratio, n
3. Transform the area of steel to equivalent concrete, nAs

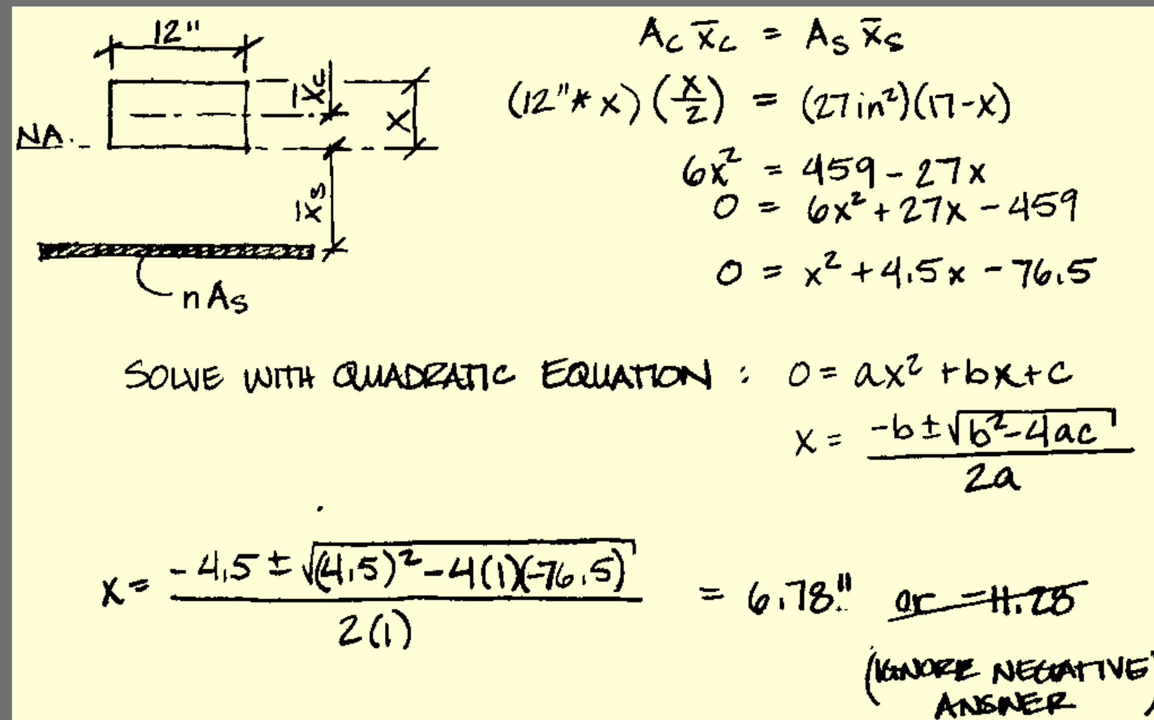


Source: University of Michigan, Department of Architecture

Example – Flexure Analysis cont.

4. Calculate the N.A. using the balanced tension and compression to solve for x.

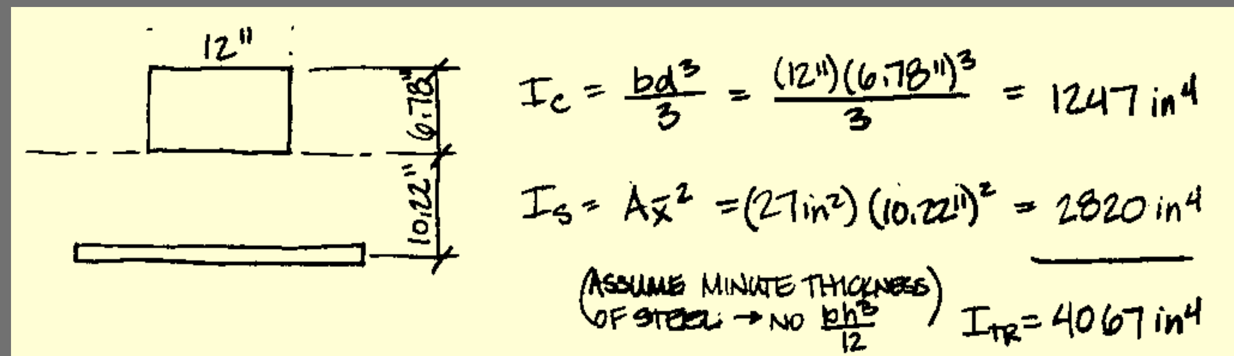
$A_c x_c = A_t x_t$



Source: University of Michigan, Department of Architecture

Example - Flexure Analysis cont.

5. Calculate the transformed Moment of Inertia.



Source: University of Michigan, Department of Architecture

Example – Flexure Analysis cont.

6. Calculate a maximum moment based first on the allowable concrete stress and again on the allowable steel stress.
7. The lesser of the two moments will control.

CONCRETE: $f_c = 1.8\ \text{ksi}$

$$M = \frac{f_c I_{TR}}{c} = \frac{(1.8\ \text{ksi})(4067\ \text{in}^4)}{6.78''} = 1080''\cdot k = 90'k$$

STEEL: $f_s = 20\ \text{ksi}$

$$M = \frac{f_s I_{TR}}{cn} = \frac{(20\ \text{ksi})(4067\ \text{in}^4)}{(10.22'')(9)} = 884''\cdot k = 74'k$$

$90 > 74$ → STEEL GOVERNS

BEAM CAPACITY = 74'k

Source: University of Michigan, Department of Architecture

STEEL: MOMENT-RESISTING CAPACITY GOVERNS

STRESS IN STEEL = ALLOWABLE = 20 ksi

CONCRETE: $$f = \frac{Mc}{I_{TR}} = \frac{(74'k * 12''/')(6.78'')}{4067\ \text{in}^4} = 1.48\ \text{ksi}$$

Source: University of Michigan, Department of Architecture

Effect of ρ

The behavior of the beam at failure (mode of failure) is determined by the relative amount of steel present – measured by ρ.

$$\rho = \frac{A_s}{bd}$$

ρ = 0
No steel used. Brittle (sudden) failure.

ρ min
Just enough steel to prevent brittle failure

$$\rho_{\min} = \frac{200}{f_y}$$

$$\rho = \frac{0.18 f_c'}{f_y}$$

$$\rho_{\max} = \rho_{balanced}$$

ρ < ρ balance
Steel fails first – ductile failure (desirable)

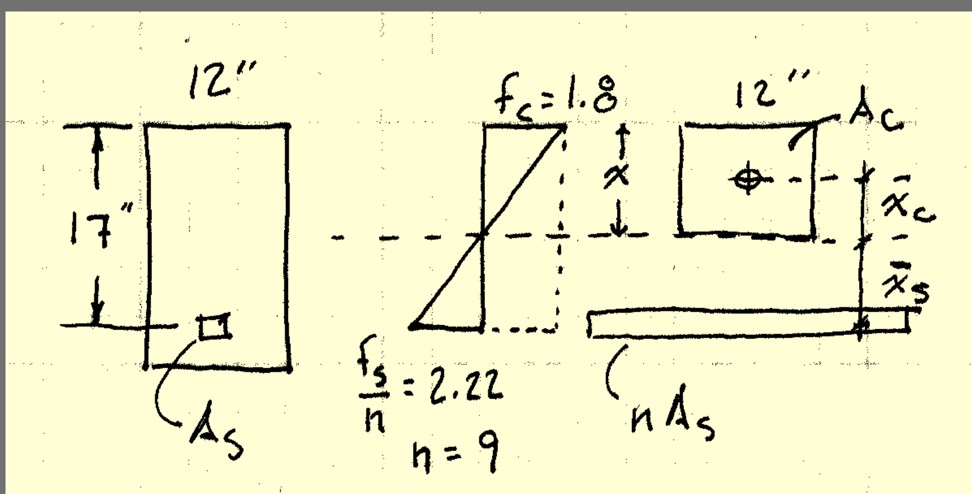
ρ balance = ρ max
Steel and concrete both stressed to allowable limit

ρ > ρ balance
Concrete fails first – brittle failure (not desirable)

Calculate ρ balance

Procedure:

1. Draw stress diagram using allowable stresses fc and fs/n
2. Use similar triangles to find x and bar_x$_s$
3. Find bar_x$_c$ = x/2
4. Use moments of areas on transformed section to solve for As.
5. Calculate ρ bal = As/bd



Stress Triangles:

$$\frac{1.8 + 2.22}{17''} : \frac{1.8}{x} : \frac{2.22}{\bar{x}_s}$$

$$x = 7.612'' \quad \bar{x}_s = 9.388''$$

$$\bar{x}_c = x/2 = 3.806''$$

Moments of Areas:

$$A_c \bar{x}_c = n A_s \bar{x}_s$$

$$A_s = \frac{A_c \bar{x}_c}{n \bar{x}_s} = 4.11 \text{ in}^2$$

Balanced ρ

$$\rho_{bal} = \frac{A_s}{bd} = \frac{4.11}{12(17)} = 0.02017$$

“Internal Couple” Method

- Uses the internal force couple T & C to determine the moment
- Defines factors k and j that can be used to find depth of stress block and moment arm of couple
- Provides equations for analysis or design.

$$\rho = \frac{A_s}{bd}$$

$$n = \frac{E_s}{E_c}$$

$$k = \sqrt{2\rho n + (\rho n)^2} - \rho n$$

$$j = 1 - \frac{k}{3}$$

$$f_c = \frac{2M}{bd^2 kj}$$

$$f_s = \frac{M}{A_s jd}$$

Analysis:

$$M = Cjd$$

$$M = \frac{bkdf_c}{2} jd$$

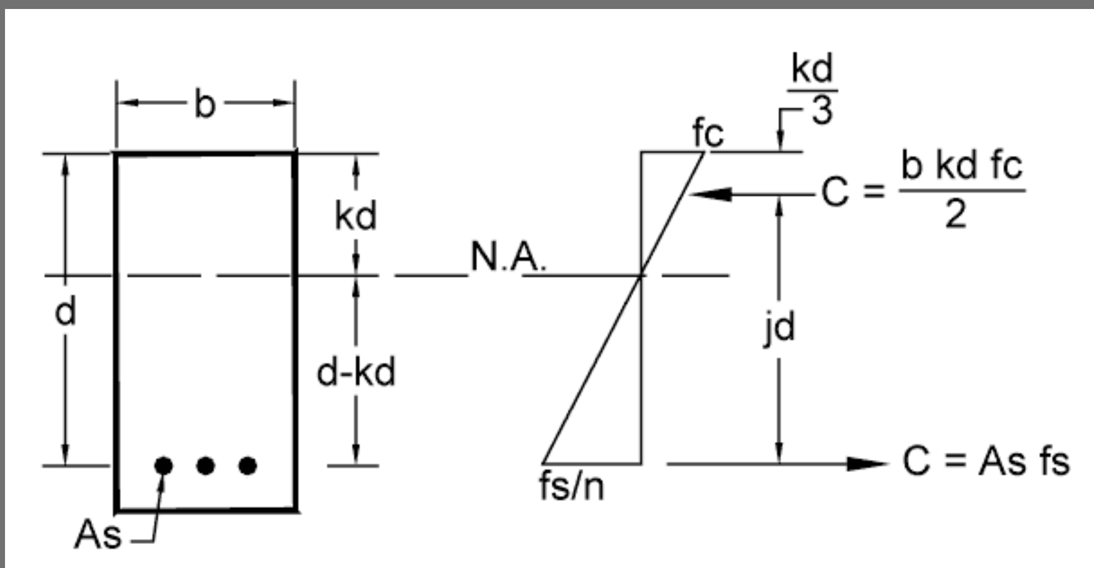
$$M = Tjd$$

$$M = A_s f_s jd$$

Design:

$$A_s = \frac{M}{f_s jd}$$

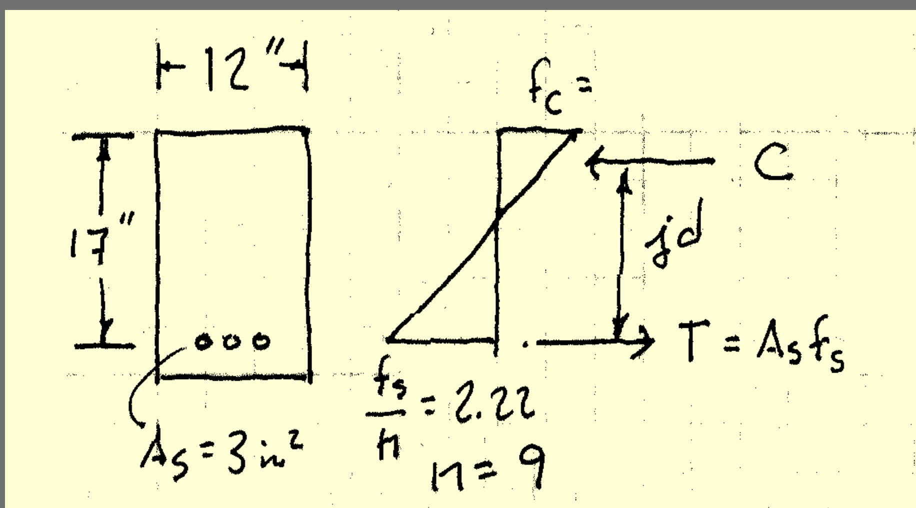
$$bd^2 = \frac{2M}{f_c kj}$$



Analysis by “Internal Couple”

Example :

1. Find ρ =As/bd
2. Find k
3. Calculate j
4. Calculate either force T or C
5. Calculate M using either T or C



$$\rho = \frac{A_s}{bd} = \frac{3.0}{(12)(17)} = 0.0147$$

$$\rho n = 0.0147(9) = 0.1324$$

$$k = \sqrt{2(0.1324) + 0.1324^2} - 0.1324$$

$$k = 0.3989$$

$$kd = 0.3989(17) = 6.78''$$

$$j = 1 - \frac{k}{3} = 1 - \frac{0.3989}{3}$$

$$j = 0.867$$

Assume steel controls:

$$T = A_s f_s = 3.0(20) = 60^k$$

$$M = T\,jd = 60(0.867)(17)$$

$$= 884.4''^{-k} = 73.7'^{-k}$$

Flexure Design

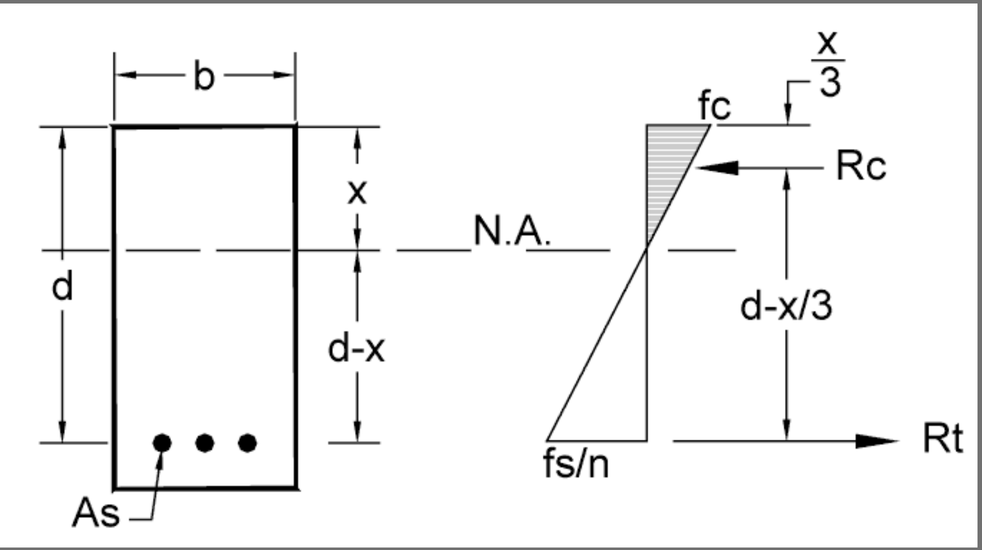
Procedure:

1. Determine load conditions.
   - choose material grade, f'c
   - calculate n = Es/Ec
   - estimate size, choose b and estimate d

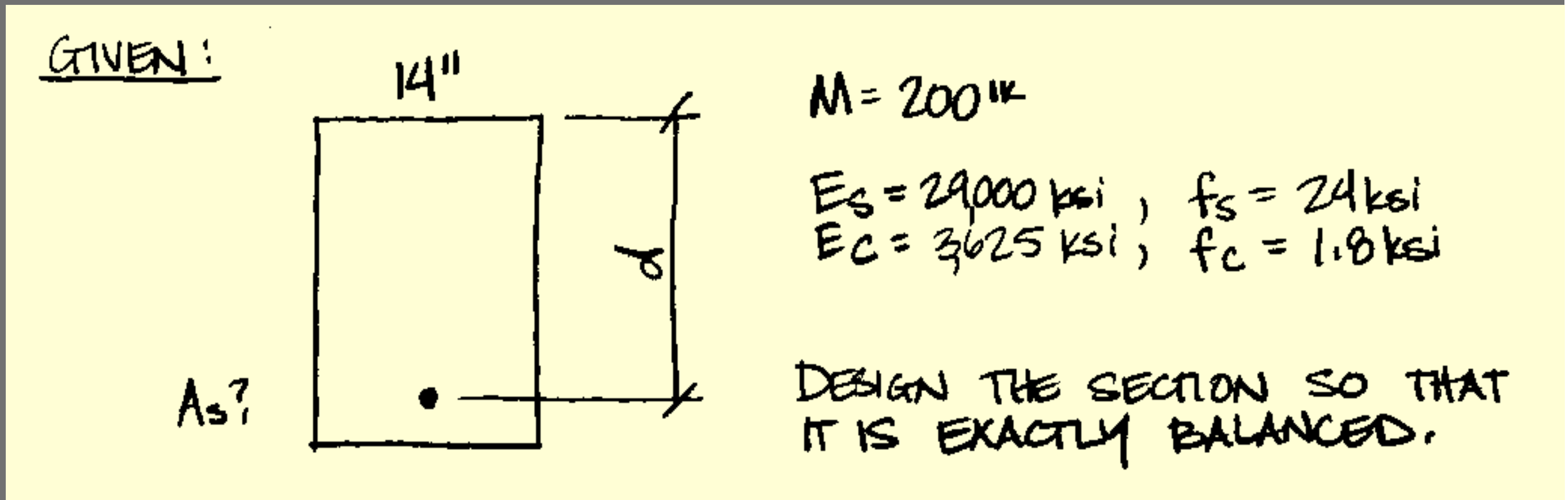
$$\frac{1}{2} \cong \frac{b}{d} \cong \frac{2}{3}$$

   - determine loads (+ member DL)
   - calculate moment

2. Choose a target steel ratio, ρ.
3. Sketch the stress diagram with force couple.
4. Calculate *d* based on the required moment.
5. Calculate As.
6. Choose bar sizes and spacing.
7. Choose beam size and revise (back to step 1 with new b, d and ρ) .



Example – Flexure Design



Source: University of Michigan, Department of Architecture

1. As a simplification the moment is given = 200 ft-k.

   d will be determined based on the moment.

f'c is given as 4000 psi

   n is found = 8.

MODULAR RATIO: $n = \frac{E_s}{E_c} = \frac{29{,}000}{3{,}625} = 8$

Example – Flexure Design cont.

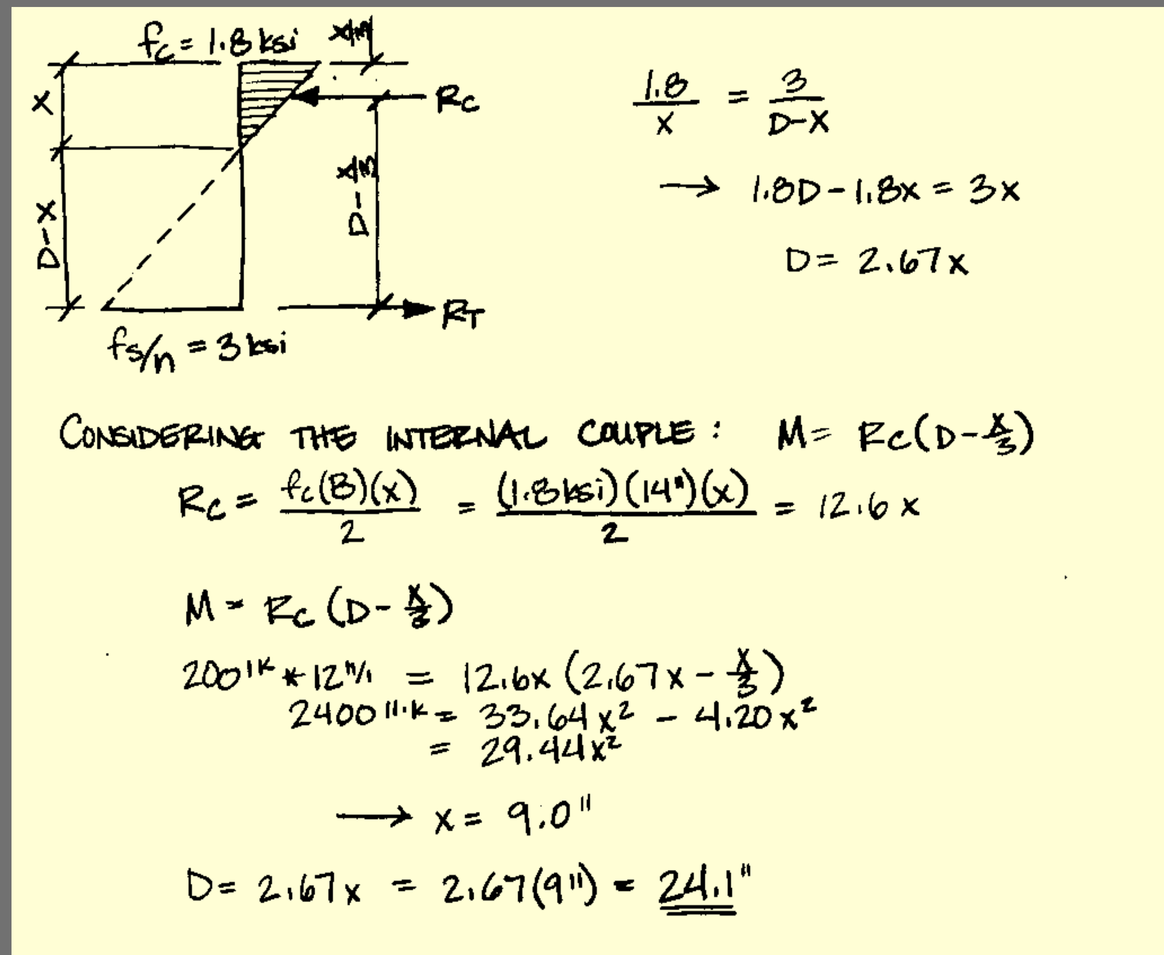
2. Steel ratio, As/bd is taken as balanced for this problem.
3. Using similar triangles, determine depth of reinforcement, D in relationship to depth of compression zone, x.

   Calculate the compression zone resultant, R_c in terms of x

   $R_c = f_c Bx/2$
4. Use the internal moment couple

   $M = R_c(D-x/3)$

   to solve for x and D.



Source: University of Michigan, Department of Architecture

Example - Flexure Design cont.

5. Calculate As using

   $R_c = R_t$ and

   $R_t = A_s f_s$

$$R_c = \frac{f_c (B)(x)}{2} = 12.6\,x$$
$$= 12.6(9.0")$$
$$= 113.4^k$$

$R_T = R_c$

$R_T = A_s f_s$

$113.4^k = A_s(24\,ksi)$ → **$A_s = 4.73\ in^2$**

7. Choose bar sizes and spacing.
   - Area >= As
   - c.g. = D
   - must be symetric
   - minimum spacing

8. Choose cover, recalculate dead load, iterate with new moment.

ASTM STANDARD REINFORCING BARS

Bar size, no.	Nominal diameter, in.	Nominal area, in.^{2}	Nominal weight, lb/ft
3	0.375	0.11	0.376
4	0.500	0.20	0.668
5	0.625	0.31	1.043
6	0.750	0.44	1.502
7	0.875	0.60	2.044
8	1.000	0.79	2.670
9	1.128	1.00	3.400
10	1.270	1.27	4.303
11	1.410	1.56	5.313
14	1.693	2.25	7.650
18	2.257	4.00	13.600

Source: ACI-318-05