ARCH 324 - Structures 2, Winter 2009

von Buelow, Peter

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Reinforced Concrete by
Ultimate Strength Design

• LRFD vs. ASD
• Failure Modes
• Flexure Equations
• Analysis of Rectangular Beams
• Design of Rectangular Beams
• Analysis of Non-rectangular Beams
• Design of Non-rectangular Beams
Allowable Stress – WSD (ASD)

\[ f_{\text{actual}} \leq (F.S.)F_{\text{failure}} \]

- Actual loads used to determine stress
- Allowable stress reduced by factor of safety

Ultimate Strength – (LRFD)

- Loads increased depending on type load
  \( \gamma \) Factors: \( DL=1.4 \) \( LL=1.7 \) \( WL=1.3 \)
  \( U=1.4DL+1.7LL \)
- Strength reduced depending on type force
  \( \phi \) Factors: flexure=0.9 shear=0.85 column=0.7

\[ M_u \leq \phi M_n \]

Examples:

WSD

\[ f_b \leq 0.45 f'_c \]

\[ f_v \leq 0.1\sqrt{f'_c} \]
Strength Measurement

• Compressive strength
  – 12”x6” cylinder
  – 28 day moist cure
  – Ultimate (failure) strength
  
• Tensile strength
  – 12”x6” cylinder
  – 28 day moist cure
  – Ultimate (failure) strength
  – Split cylinder test
  – Ca. 10% to 20% of $f'c$

Photos: Source: Xb-70 (wikipedia)
Failure Modes

\[ \rho = \frac{A_s}{bd} \]

- No Reinforcing
  - Brittle failure

- Reinforcing < balance
  - Steel yields before concrete fails
  - Ductile failure

- Reinforcing = balance
  - Concrete fails just as steel yields

- Reinforcing > balance
  - Concrete fails before steel yields
  - Sudden failure

\[ \rho_{\text{min}} = \frac{200}{f_y} \]

\[ \rho_{\text{max}} = 0.75 \rho_{\text{bal}} \]

\[ \rho_{\text{bal}} = \left( \frac{0.85 \beta_1 f'_c}{f_y} \right) \left( \frac{87000}{87000 + f_y} \right) \]

\[ \rho > \rho_{\text{max}} \quad \text{SuddenDeath!!} \]

Source: Polyparadigm (wikipedia)
\( \beta_1 \)

\( \beta_1 \) is a factor to account for the non-linear shape of the compression stress block.

\[ a = \beta_1 c \]

<table>
<thead>
<tr>
<th>( f'_c )</th>
<th>( \beta_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.85</td>
</tr>
<tr>
<td>1000</td>
<td>0.85</td>
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<td>0.65</td>
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<tr>
<td>10000</td>
<td>0.65</td>
</tr>
</tbody>
</table>

Image Sources: University of Michigan, Department of Architecture
Flexure Equations

actual stress block

\[ C = T \]
\[ 0.85f'_c ab = A_s f_y \]

solving for \( a \),
\[ a = \frac{A_s f_y}{0.85f'_c b} = \frac{\rho f_y d}{0.85f'_c} \]

\[ \rho = \frac{A_s}{bd} \]

ACI equivalent stress block

\[ M_n = T\left(d - \frac{a}{2}\right) = A_s f_y \left(d - \frac{a}{2}\right) \]

\[ M_u = \phi M_n \]

\[ M_u = \phi M_n = \phi A_s f_y \left(d - \frac{a}{2}\right) \]

\[ M_u = \phi A_s f_y d \left(1 - 0.59 \frac{\rho f_y}{f_c'}\right) \]
Balance Condition

From similar triangles at balance condition:

\[
\frac{c}{d} = \frac{0.003}{0.003 + \left(\frac{f_y}{E_s}\right)} = \frac{0.003}{0.003 + \left(\frac{f_y}{29 \times 10^6}\right)}
\]

\[
c = \frac{87,000}{87,000 + f_y}d
\]

Use equation for \(a\). Substitute into \(c = a/\beta_1\)

\[
a = \frac{\rho f_y d}{0.85 f'_c'}
\]

\[
c = \frac{a}{\beta_1} = \frac{\rho f_y d}{0.85 \beta_1 f'_c'}
\]

Equate expressions for \(c\):

\[
\frac{\rho f_y d}{0.85 \beta_1 f'_c'} = \frac{87,000}{87,000 + f_y}d
\]

\[
\rho_b = \left(\frac{0.85 \beta_1 f'_c'}{f_y}\right) \left(\frac{87,000}{87,000 + f_y}\right)
\]

Table A.8 Balanced Ratio of Reinforcement \(\rho_b\) for Rectangular Sections with Tension Reinforcement Only

<table>
<thead>
<tr>
<th>(f_y)</th>
<th>(f'_c)</th>
<th>(2,500) psi</th>
<th>(3,000) psi</th>
<th>(4,000) psi</th>
<th>(5,000) psi</th>
<th>(6,000) psi</th>
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</thead>
<tbody>
<tr>
<td>(17.2 MPa)</td>
<td>(20.7 MPa)</td>
<td>(27.6 MPa)</td>
<td>(34.5 MPa)</td>
<td>(41.4 MPa)</td>
<td>(49.2 MPa)</td>
<td>(57.1 MPa)</td>
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<tr>
<td>Grade 40</td>
<td>(\rho_b)</td>
<td>2500</td>
<td>3000</td>
<td>4000</td>
<td>5000</td>
<td>6000</td>
</tr>
<tr>
<td>40,000 psi</td>
<td>(0.75 \rho_b)</td>
<td>0.0309</td>
<td>0.0371</td>
<td>0.0495</td>
<td>0.0582</td>
<td>0.0655</td>
</tr>
<tr>
<td>(0.50 \rho_b)</td>
<td>0.0115</td>
<td>0.0186</td>
<td>0.0247</td>
<td>0.0291</td>
<td>0.0328</td>
<td></td>
</tr>
<tr>
<td>Grade 50</td>
<td>(\rho_b)</td>
<td>2500</td>
<td>3000</td>
<td>4000</td>
<td>5000</td>
<td>6000</td>
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<tr>
<td>50,000 psi</td>
<td>(0.75 \rho_b)</td>
<td>0.0229</td>
<td>0.0275</td>
<td>0.0367</td>
<td>0.0432</td>
<td>0.0486</td>
</tr>
<tr>
<td>(0.50 \rho_b)</td>
<td>0.0115</td>
<td>0.0186</td>
<td>0.0247</td>
<td>0.0291</td>
<td>0.0328</td>
<td></td>
</tr>
<tr>
<td>Grade 60</td>
<td>(\rho_b)</td>
<td>2500</td>
<td>3000</td>
<td>4000</td>
<td>5000</td>
<td>6000</td>
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<tr>
<td>60,000 psi</td>
<td>(0.75 \rho_b)</td>
<td>0.0178</td>
<td>0.0214</td>
<td>0.0285</td>
<td>0.0335</td>
<td>0.0377</td>
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<tr>
<td>(0.50 \rho_b)</td>
<td>0.0089</td>
<td>0.0143</td>
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<td>0.0216</td>
<td>0.0243</td>
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<tr>
<td>Grade 75</td>
<td>(\rho_b)</td>
<td>2500</td>
<td>3000</td>
<td>4000</td>
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<tr>
<td>75,000 psi</td>
<td>(0.75 \rho_b)</td>
<td>0.0129</td>
<td>0.0155</td>
<td>0.0207</td>
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<td>0.0274</td>
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<tr>
<td>(0.50 \rho_b)</td>
<td>0.0097</td>
<td>0.0116</td>
<td>0.0155</td>
<td>0.0182</td>
<td>0.0205</td>
<td></td>
</tr>
</tbody>
</table>

Image Sources: University of Michigan, Department of Architecture

University of Michigan, TCAUP

Structures II

Slide 8/26
Rectangular Beam Analysis

Data:
- Section dimensions – b, h, d, (span)
- Steel area - As
- Material properties – f'c, fy

Required:
- Strength (of beam) Moment - Mn
- Required (by load) Moment – Mu
- Load capacity

1. Find $\rho = \frac{A_s}{bd}$
   (check $\rho_{\text{min}} < \rho < \rho_{\text{max}}$)
2. Find a
3. Find Mn
4. Calculate Mu <= $\phi$ Mn
5. Determine max. loading (or span)

\[ a = \frac{A_s f_y}{0.85 f'_c b} \quad \text{or} \quad \frac{\rho f_y d}{0.85 f'_c} \]

\[ M_n = A_s f_y \left( d - \frac{a}{2} \right) \]

\[ M_u \leq \phi M_n \]

\[ M_u = \frac{(1.4 w_{DL} + 1.7 w_{LL}) l^2}{8} \]

\[ 1.7 w_{LL} = \frac{M_u}{l^2} - 1.4 w_{DL} \]
Rectangular Beam Analysis

Data:
• dimensions – b, h, d, (span)
• Steel area - As
• Material properties – f_c, f_y

Required:
• Required Moment – Mu

1. Find $\rho = \frac{A_s}{bd}$
( check $\rho_{min} < \rho < \rho_{max}$)
2. Find a

\[ a = \frac{A_{s}f_{y}}{0.85 f_{c} b} = \frac{2.37 \times 60000}{0.85 \times 4000 \times 12} = 3.49 \]

3. Find Mn

\[ M_{n} = A_{s}f_{y}(d - \frac{a}{2}) \]

4. Find Mu

\[ M_{u} = 0.9 \times 2.37 \times 60000 \times (17.5 - \frac{3.49}{2}) \]

\[ M_{u} = 2017000 \text{ in-lb} \]

\[ M_{u} = 168 \text{ ft-lb} \]
Slab Analysis

Data:
- Section dimensions – h, span take b = 12”
- Steel area - As
- Material properties – f’c, fy

Required:
- Required Moment – Mu
- Maximum LL in PSF

\[
\begin{align*}
f_y &= 60 \text{ ksi (Gr 60)} \\
f'_c &= 3000 \text{ psi} \\
\gamma' &= 150 \text{pcf}
\end{align*}
\]

\[
A_s = \frac{12\,"}{18\,"} \left(0.79\,\text{in}^2\right) = 0.526\,\text{in}^2/\text{ft}
\]
Slab Analysis

1. Find a
2. Find force T
3. Find moment arm z
4. Find strength moment Mn

\[ d = \frac{A_s f_y}{0.85 f'c b} = \frac{0.5267 (60)}{0.85 (3) (12)} = 1.033'' \]

\[ T = A_s f_y = 0.5267 (60) = 31.6' \]

\[ E = d - \frac{a}{2} = 9.75 - \frac{1.033}{2} = 9.23 '' \]

\[ M_n = T E = 31.6 (9.23) = 291.8 \text{ k-ft} \]

\[ = 24317' \text{ k-ft} \]
Slab Analysis

5. Find slab DL

6. Find Mu

7. Determine max. loading

\[
\begin{align*}
W_{DL} &= \gamma_c \frac{\text{AREA}_{m}^2}{144} = 150 \frac{11(12)}{144} = 137.5 \text{ PSF} \\
W_{UPL} &= 1.4 (W_{DL}) = 1.4(137.5) = 192.5 \\
W_{UL} &= 1.7 (W_{UL}) \\
M_u &= \frac{(W_{UPL} + W_{UL})L^2}{8} = 4 \text{ MNN} \\
0.9(243.17) &= \left[192.5 + 1.7(W_{UL})\right]\frac{18^2}{8} \\
W_{LL} &= 204.6 \text{ PSF}
\end{align*}
\]
Rectangular Beam Design

Data:
• Load and Span
• Material properties – f’c, fy
• All section dimensions – b and h

Required:
• Steel area - As

1. Calculate the dead load and find Mu
2. d = h – cover – stirrup – db/2 (one layer)
3. Estimate moment arm jd (or z) ≅ 0.9 d and find As
4. Use As to find a
5. Use a to find As (repeat…)
6. Choose bars for As and check ρ max & min
7. Check Mu<ϕ Mn (final condition)

8. Design shear reinforcement (stirrups)
9. Check deflection, crack control, steel development length.

\[ M_u = \frac{(1.4w_{DL} + 1.7w_{LL})l^2}{8} \]

\[ A_s = \frac{M_u}{\phi f_y d - \frac{a}{2}} \]

\[ a = \frac{A_s f_y}{0.85 f'_c b} \]

\[ M_n = A_s f_y \left( d - \frac{a}{2} \right) \]
Rectangular Slab Design

Data:
- Load and Span
- Material properties – f′c, fy

Required:
- All section dimensions – h
- Steel area - As

1. Calculate the dead load and find Mu
2. Estimate moment arm jd (or z) ≈ 0.9 d and find As
3. Use As to find a
4. Use a to find As (repeat…)

DATA: ONE-WAY FLOOR SLAB - SPAN = 18 FEET
fy = 60,000 psi
f′c = 3000 psi
λ0,k = 150 kips
ρ = 0.5
max = 0.008

REQUIRED: h and As

Assume h
h = \frac{f}{20} = 10.8 \text{ in. use 11”}

Calculate Loads
DL = 137.5 \text{ kips}
LL = 200.0 \text{ kips}
w = 1.4(137) + 1.7(200) = 540

Calculate Mu
Mu = \frac{wL^2}{8} = 21.7 \text{ k-ft}

Initial As Trial
As = \frac{Mu}{f_y (\frac{d - 0.3}{2})} = \frac{21.7 \times 12}{0.9(60)(9)} = 5.36

Initial a
a = \frac{As f_y}{0.85 f′c b} = \frac{5.36(60)}{0.85(3)(12)} = 1.05 \text{”}
Rectangular Slab Design

3. Use As to find a
4. Use a to find As (repeat…)
5. Choose bars for As and check As min & As max
6. Check Mu<\phi Mn (final condition)

7. Check deflection, crack control, steel development length.

\[
A_s = \frac{M_u}{fy(d - \frac{a}{2})} = \frac{21.7 \times 12}{0.9(60)(10 - \frac{1.25}{2})} = 1.508
\]

\[a = \frac{508(60)}{0.85(12)} = 998\]

\[
A_s = \frac{21.7 \times 12}{0.9(60)(10 - \frac{1.25}{2})} = 1.507 \text{ in } \approx 1.508
\]
Quiz 9

Can \( f = \frac{M_c}{I} \) be used in Ult. Strength concrete beam calculations? (yes or no)

HINT:

- WSD stress
- Ult. Strength stress

Source: University of Michigan, Department of Architecture
Rectangular Beam Design

Data:
- Load and Span
- Some section dimensions – b or d
- Material properties – f’c, fy

Required:
- Steel area - As
- Beam dimensions – b or d

1. Choose $\rho$ (e.g. 0.5 $\rho_{\text{max}}$ or 0.18$f’c$/$fy$)
2. Estimate the dead load and find $M_u$
3. Calculate $bd^2$
4. Choose $b$ and solve for $d$
   - $b$ is based on form size – try several to find best
5. Estimate $h$ and correct weight and $M_u$
6. Find $As = \rho bd$
7. Choose bars for $As$ and determine spacing and cover. Recheck $h$ and weight.
8. Design shear reinforcement (stirrups)
9. Check deflection, crack control, steel development length.

\[ M_u = \frac{(1.4w_{DL} + 1.7w_{LL})l^2}{8} \]

\[ bd^2 = \frac{M_u}{\phi fy (1 - 0.59 \rho \left( f'f \right))} \]

\[ As = \rho bd \]
Rectangular Beam Design

Data:
• Load and Span
• Material properties – $f'_c$, $f_y$

Required:
• Steel area - $A_s$
• Beam dimensions – $b$ and $d$

1. Estimate the dead load and find $M_u$
2. Choose $\rho$ (e.g. 0.5 $\rho_{max}$ or 0.18$f'_c$/fy)

\[
\text{Factored LL} = P_{L} = 1.7 (L) = 1.7(20) = 34 \text{ k}
\]
\[
\text{Factored DL} = W_0 = 1.4 \text{ (applied load + beam weight estimate)} = 1.4(2 + .6) = 3.64 \text{ k/ft}
\]
\[
M_u = P_{L} d + \frac{W_0 d^2}{8} = 34(10) + \frac{3.64 \times 10^2}{8} = 340 + 409.5 = 749.5 \text{ k-ft}
\]
\[
M_u = 8994 \text{ 000 in-ft}
\]
\[
\rho = \frac{0.18 f'_c}{f_y} = 0.009
\]
Rectangular Beam Design cont

3. Calculate $bd^2$

$$bd^2 = \frac{Mu}{\phi f_y (1 - 0.59 \phi (f_y/f_c))}$$

$$bd^2 = \frac{8994}{(0.9)(0.09)(60)(1 - 0.59(0.09)(60/3))}$$

$$bd^2 = 20.705 \text{ in}^3$$

4. Choose $b$ and solve for $d$
   
   $b$ is based on form size.
   
   try several to find best

\[
\begin{align*}
\text{Possibilities} & \quad b \quad \times \quad d \\
14'' & \quad 38.5'' \\
16'' & \quad 35.97'' \\
18'' & \quad 33.9''
\end{align*}
\]
5. Estimate h and correct weight and Mu
6. Find As = ρ bd
7. Choose bars for As and determine spacing and cover. Recheck h and weight.
8. Design shear reinforcement (stirrups)
9. Check deflection, crack control, steel development length.
Non-Rectangular Beam Analysis

Data:
- Section dimensions – b, h, d, (span)
- Steel area - As
- Material properties – f'c, fy

Required:
- Required Moment – Mu (or load, or span)

1. Draw and label diagrams for section and stress
   1. Determining b effective (for T-beams)
   2. Locate T and C (or C₁ and C₂)
2. Set T=C and write force equations (P=FA)
   1. T = As fy
   2. C = 0.85 f'c Ac
3. Determine the Ac required for C
4. Working from the top down, add up area to make Ac
5. Find moment arms (z) for each block of area
6. Find Mn = [未知] Cz
7. Find Mu = φ Mn  φ =0.90
8. Check As min < As < As max

Source: University of Michigan, Department of Architecture
Analysis Example

Given:  
- $f'_c = 3000$ psi  
- $f_y = 60$ ksi  
- $A_s = 6$ in$^2$

Req’d:  
- Capacity, $\mu$

1. Find $T$

2. Find $C$ in terms of $A_c$

3. Set $T = C$ and solve for $A_c$

\[
T = A_s f_y = 6 \text{ in}^2 (60000 \text{ psi})
\]
\[
T = 360,000 \text{ kN} = 360 \text{ k}
\]

\[
C = 0.85 f'_c A_c = 0.85 (3000 \text{ psi}) A_c \text{ in}^2
\]
\[
C = (2550 A_c) = (2.55 A_c) \text{ k}
\]

\[
T = C
\]
\[
360 \text{ k} = 2.55 A_c \text{ k}
\]

\[
A_c = 142 \text{ in}^2
\]
Example

4. Draw section and determine areas to make Ac

5. Solve C for each area in compression.
Example

6. Determine moment arms to areas, z.

7. Calculate Mn by summing the Cz moments.

8. Find $M_u = \frac{M_n}{z}$

$z_1 = 22 - 1.5 = 20.5''$
$z_2 = 22 - (3+2.5) = 16.5''$
$z_3 = 22 - (8+2) = 12.0''$

$M_n = \sum Cz$
$M_n = (C_1 z_1) + (C_2 z_2) + (C_3 z_3)$
$M_n = 2509 + 1262 + 1959$
$M_n = 5730$
$M_u = \phi M_n = 0.9(5730) = 5157 \text{ k}-f$
### Other Useful Tables:

#### Table A.1 Values of Modulus of Elasticity for Normal-Weight Concrete

<table>
<thead>
<tr>
<th>Bar No.</th>
<th>Customary Units</th>
<th>SI Units</th>
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<tbody>
<tr>
<td></td>
<td>Diameter (in.)</td>
<td>Cross-sectional Area (in.²)</td>
</tr>
<tr>
<td>3</td>
<td>0.375</td>
<td>0.11</td>
</tr>
<tr>
<td>4</td>
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<td>0.20</td>
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<td>5</td>
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<td>0.31</td>
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<tr>
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<td>0.44</td>
</tr>
<tr>
<td>7</td>
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<td>0.60</td>
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<tr>
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<td>1.128</td>
<td>1.00</td>
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<tr>
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<td>1.27</td>
</tr>
<tr>
<td>11</td>
<td>1.410</td>
<td>1.56</td>
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<tr>
<td>12</td>
<td>1.693</td>
<td>2.25</td>
</tr>
<tr>
<td>13</td>
<td>2.257</td>
<td>4.00</td>
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</tbody>
</table>

#### Table A.2 Designations, Areas, Perimeters, and Weights of Standard Bars

<table>
<thead>
<tr>
<th>Bar No.</th>
<th>Customary Units</th>
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<td>1.27</td>
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<td>1.56</td>
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<td>1.693</td>
<td>2.25</td>
</tr>
<tr>
<td>13</td>
<td>2.257</td>
<td>4.00</td>
</tr>
</tbody>
</table>

#### Table A.4 Areas of Groups of Standard Bars (in.²)

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