ARCH 324 - Structures 2, Winter 2009

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 Reinforced Concrete by Ultimate Strength Design

• LRFD vs. ASD
• Failure Modes
• Flexure Equations
• Analysis of Rectangular Beams
• Design of Rectangular Beams
• Analysis of Non-rectangular Beams
• Design of Non-rectangular Beams
Allowable Stress – WSD (ASD)

\[ f_{\text{actual}} \leq (F.S.)F_{\text{failure}} \]

- Actual loads used to determine stress
- Allowable stress reduced by factor of safety

Ultimate Strength – (LRFD)

- Loads increased depending on type load
  \( \gamma \) Factors: DL=1.4  LL=1.7  WL=1.3
  \( U=1.4DL+1.7LL \)
- Strength reduced depending on type force
  \( \phi \) Factors: flexure=0.9  shear=0.85  column=0.7

\[ M_u \leq \phi M_n \]

Examples:

WSD

\[ f_b \leq 0.45 f'_c \]
\[ f_v \leq 0.1 \sqrt{f'_c} \]

Ultimate Strength

\[ M_u \leq 0.9 M_n \]
\[ V_u \leq 0.85 V_n \]
\[ P_u \leq 0.70 P_n \]
Strength Measurement

- Compressive strength
  - 12”x6” cylinder
  - 28 day moist cure
  - Ultimate (failure) strength

- Tensile strength
  - 12”x6” cylinder
  - 28 day moist cure
  - Ultimate (failure) strength
  - Split cylinder test
  - Ca. 10% to 20% of $f'c$

Photos: Source: Xb-70 (wikipedia)
Failure Modes

\[ \rho = \frac{A_s}{bd} \]

- No Reinforcing
  - Brittle failure
- Reinforcing < balance
  - Steel yields before concrete fails
  - Ductile failure
- Reinforcing = balance
  - Concrete fails just as steel yields
- Reinforcing > balance
  - Concrete fails before steel yields
  - Sudden failure

\[ \rho_{\text{min}} = \frac{200}{f_y} \]

\[ \rho_{\text{max}} = 0.75 \rho_{\text{bal}} \]

\[ \rho_{\text{bal}} = \left( \frac{0.85 \beta_1 f'_c}{f_y} \right) \left( \frac{87000}{87000 + f_y} \right) \]

\[ \rho > \rho_{\text{max}} \quad \text{SuddenDeath!!} \]

Source: Polyparadigm (wikipedia)
\( \beta_1 \) is a factor to account for the non-linear shape of the compression stress block.

\[
a = \beta_1 c
\]

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<th>( \beta_1 )</th>
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\( f_c' \) (ksi) vs. \( \beta_1 \)

Image Sources: University of Michigan, Department of Architecture
Flexure Equations

actual stress block

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<table>
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<th>Actual Stress Block</th>
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<tbody>
<tr>
<td>0.85f'_c</td>
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<tr>
<td>a = \beta_c c</td>
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<tr>
<td>C = 0.85f'_c ab</td>
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ACI equivalent stress block

```
<table>
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<tbody>
<tr>
<td>0.85f'_c</td>
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<tr>
<td>T = A_s f_y</td>
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\[ C = T \]
\[ 0.85f'_c ab = A_s f_y \]

Solving for \( a \),
\[ a = \frac{A_s f_y}{0.85f'_c b} = \frac{\rho f_y d}{0.85f'_c} \]

\[ \rho = \frac{A_s}{bd} \]

\[ M_n = T \left( d - \frac{a}{2} \right) = A_s f_y \left( d - \frac{a}{2} \right) \]

\[ M_u = \phi M_n \]

\[ M_u = \phi M_n = \phi A_s f_y \left( d - \frac{a}{2} \right) \]

\[ M_u = \phi A_s f_y d \left( 1 - 0.59 \frac{\rho f_y}{f'_c} \right) \]
Balance Condition

From similar triangles at balance condition:

\[
\frac{c}{d} = \frac{0.003}{0.003 + \left(\frac{f_y}{E_s}\right)} = \frac{0.003}{0.003 + \left(\frac{f_y}{29 \times 10^6}\right)}
\]

\[
c = \frac{87,000}{87,000 + f_y} d
\]

Use equation for \(a\). Substitute into \(c = a/\beta_1\):

\[
a = \frac{\rho f_y d}{0.85 f'_{c'}}
\]

\[
c = \frac{a}{\beta_1} = \frac{\rho f_y d}{0.85 \beta_1 f'_{c'}}
\]

Equate expressions for \(c\):

\[
\frac{\rho f_y d}{0.85 \beta_1 f'_{c'}} = \frac{87,000}{87,000 + f_y}
\]

\[
\rho_b = \left(\frac{0.85 \beta_1 f'_{c'}}{f_y}\right) \left(\frac{87,000}{87,000 + f_y}\right)
\]

| Table A.8 Balanced Ratio of Reinforcement \(\rho_b\) for Rectangular Sections with Tension Reinforcement Only |
|-----------------------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \(f_y\)   | \(f'_{c'}\) | 2,500 psi (17.2 MPa) | 3,000 psi (20.7 MPa) | 4,000 psi (27.6 MPa) | 5,000 psi (34.5 MPa) | 6,000 psi (41.4 MPa) |
| Grade 40 40,000 psi (275.8 MPa) | \(\rho_b\) | \(\beta_1 = 0.85\) | \(\beta_1 = 0.85\) | \(\beta_1 = 0.85\) | \(\beta_1 = 0.80\) | \(\beta_1 = 0.75\) |
|         | 0.0309       | 0.0371           | 0.0495           | 0.0582           | 0.0655           |
|         | 0.0232       | 0.0278           | 0.0371           | 0.0437           | 0.0492           |
|         | 0.0155       | 0.0186           | 0.0247           | 0.0291           | 0.0328           |
| Grade 50 50,000 psi (344.8 MPa) | \(\rho_b\) | \(\beta_1 = 0.85\) | \(\beta_1 = 0.85\) | \(\beta_1 = 0.85\) | \(\beta_1 = 0.80\) | \(\beta_1 = 0.75\) |
|         | 0.0229       | 0.0275           | 0.0367           | 0.0432           | 0.0486           |
|         | 0.0172       | 0.0206           | 0.0275           | 0.0324           | 0.0365           |
|         | 0.0115       | 0.0138           | 0.0184           | 0.0216           | 0.0243           |
| Grade 60 60,000 psi (413.7 MPa) | \(\rho_b\) | \(\beta_1 = 0.85\) | \(\beta_1 = 0.85\) | \(\beta_1 = 0.85\) | \(\beta_1 = 0.80\) | \(\beta_1 = 0.75\) |
|         | 0.0178       | 0.0214           | 0.0285           | 0.0335           | 0.0377           |
|         | 0.0134       | 0.0161           | 0.0214           | 0.0252           | 0.0283           |
|         | 0.0089       | 0.0107           | 0.0143           | 0.0168           | 0.0189           |
| Grade 75 75,000 psi (517.1 MPa) | \(\rho_b\) | \(\beta_1 = 0.85\) | \(\beta_1 = 0.85\) | \(\beta_1 = 0.85\) | \(\beta_1 = 0.80\) | \(\beta_1 = 0.75\) |
|         | 0.0129       | 0.0155           | 0.0207           | 0.0243           | 0.0274           |
|         | 0.0097       | 0.0116           | 0.0155           | 0.0182           | 0.0205           |
|         | 0.0065       | 0.0078           | 0.0104           | 0.0122           | 0.0137           |
Rectangular Beam Analysis

Data:
- Section dimensions – b, h, d, (span)
- Steel area - As
- Material properties – f’c, fy

Required:
- Strength (of beam) Moment - Mn
- Required (by load) Moment – Mu
- Load capacity

1. Find \( \rho = \frac{A_s}{bd} \)
   (check \( \rho_{\text{min}} < \rho < \rho_{\text{max}} \))
2. Find a
3. Find Mn
4. Calculate Mu \( \leq \phi M_n \)
5. Determine max. loading (or span)

\[
a = \frac{A_s f_y}{0.85 f'_c b} \quad \text{or} \quad \frac{\rho f_y d}{0.85 f'_c}
\]

\[
M_n = A_s f_y \left( d - \frac{a}{2} \right)
\]

\[
M_u \leq \phi M_n
\]

\[
M_u = \frac{(1.4w_{DL} + 1.7w_{LL})l^2}{8}
\]

\[
1.7w_{LL} = \frac{M_u}{l^2} - 1.4w_{DL}
\]
Rectangular Beam Analysis

Data:
- dimensions – b, h, d, (span)
- Steel area - As
- Material properties – f'c, fy

Required:
- Required Moment – Mu

1. Find \( \rho = \frac{As}{bd} \)
   (check \( \rho_{\text{min}} < \rho < \rho_{\text{max}} \))
Rectangular Beam Analysis cont.

2. Find a

\[ a = \frac{A_s f_y}{0.85 f_c b} = \frac{(2.37)(60000)}{0.85(4000)(12)} = 3.49 \]

3. Find \( M_n \)

\[ M_n = A_s f_y (d - \frac{a}{2}) \]

4. Find \( M_u \)

\[ M_u = \phi A_s f_y (d - \frac{a}{2}) \]

\[ M_u = 0.9(2.37)(60000)(17.5 - \frac{3.49}{2}) \]

\[ M_u = 2017000 \text{ in-lb} \]

\[ M_u = 168 \text{ ft-k} \]
Slab Analysis

Data:
- Section dimensions – h, span  
  take b = 12”
- Steel area - As
- Material properties – f’c, fy

Required:
- Required Moment – Mu
- Maximum LL in PSF

\[ f_y = 60 \text{ ksi (GR 60)} \]
\[ f'c = 3000 \text{ PSI} \]
\[ c' = 150 \text{ PCF} \]
Slab Analysis

1. Find a
2. Find force T
3. Find moment arm z
4. Find strength moment Mn

\[ a = \frac{A_s f_y}{0.85 f_c' b} = \frac{0.5267 (60)}{0.85 (3) (12)} = 1.033'' \]

\[ d = \frac{0.85 f_c'}{0.85 f_c' a b} \]

\[ z = d - \frac{a}{2} = 9.75 - \frac{1.033}{2} = 9.23'' \]

\[ T = A_s f_y = 0.5267 (60) = 31.6 \text{ kN} \]

\[ M_n = Tz = 31.6 (9.23) = 291.8 \text{ kNm} \]

\[ = 24317'\text{ft-lb} \]
Slab Analysis

5. Find slab DL
6. Find Mu
7. Determine max. loading

\[ w_{DL} = h_c \frac{\text{AREA}^2}{144} = 150 \frac{11(12)}{144} = 137.5 \text{ PSF} \]

\[ w_{UPL} = 1.4 \cdot (w_{DL}) = 1.4(137.5) = 192.5 \]

\[ w_{ULL} = 1.7 \cdot (w_{UL}) \]

\[ M_u = \frac{(w_{UPL} + w_{ULL}) L^2}{8} = 4 \cdot M_n \]

\[ 0.9(24317) = \frac{[192.5 + 1.7(243.17)] 18^2}{8} \]

\[ w_{LL} = 204.6 \text{ PSF} \]
Rectangular Beam Design

Data:
- Load and Span
- Material properties – f’c, fy
- All section dimensions – b and h

Required:
- Steel area - As

1. Calculate the dead load and find Mu
2. \[d = h – \text{cover} – \text{stirrup} – \frac{d_s}{2}\] (one layer)
3. Estimate moment arm jd (or z) \(\approx 0.9 \, d\) and find As
4. Use As to find a
5. Use a to find As (repeat…)
6. Choose bars for As and check \(\rho\) max & min
7. Check Mu<\(\phi\) Mn (final condition)

8. Design shear reinforcement (stirrups)
9. Check deflection, crack control, steel development length.
Rectangular Slab Design

Data:
- Load and Span
- Material properties – $f'_{c}, f_{y}$

Required:
- All section dimensions – $h$
- Steel area - $A_{s}$

1. Calculate the dead load and find $M_u$
2. Estimate moment arm $j_d$ (or $z$) $\cong 0.9 \, d$ and find $A_{s}$
3. Use $A_{s}$ to find $a$
4. Use $a$ to find $A_{s}$ (repeat...)

**DATA:**
- ONE-WAY FLOOR SLAB - SPAN = 18 FEET
  - $f_{y} = 60,000 \text{ psi}$
  - $f_{c}' = 3,000 \text{ psi}$
  - $20k = 150 \text{ kft}$
  - $\rho = \frac{1}{2} \rho_{max} = 0.008$

**REQUIRED:** $h$ and $A_{s}$

**ASSUME:** $h$
- $h = \frac{f_{c}'}{20} = 10.8$ use 11”

**CALCULATE LOADS**
- $DL = 137.5 \text{ kips}$
- $LL = 200.0 \text{ kips}$
- $W = 1.4(137) + 1.7(200) = 540$

**CALCULATE $M_u$**
- $M_u = \frac{wL^{2}}{8} = 217 \text{ k-ft}$

**INITIAL $A_{s}$ TRIAL**
- $A_{s} = \frac{M_{u}}{f_{y}(d - \frac{a}{2})}$
  - $A_{s} = \frac{217 \times 12}{.9(60)(9)} = 19.56$
- $A_{s} = 0.9d = 0.9(11 - .75 - .25) = 9”$

**INITIAL $a$**
- $a = \frac{A_{s}f_{y}}{.85f_{c}'} = \frac{9.536(60)}{.85(3)(12)} = 1.05”$
Rectangular Slab Design

3. Use As to find a
4. Use a to find As (repeat…)
5. Choose bars for As and check As min & As max
6. Check $M_u < \phi M_n$ (final condition)

7. Check deflection, crack control, steel development length.

\[
\begin{align*}
A_s &= \frac{M_u}{fy(d-\frac{a}{2})} = \frac{21.7 \times 12}{60(0.10 - \frac{1.5}{2})} = 1.508 \\
\phi &= \frac{508(60)}{0.85(3)(12)} = 0.998 \\
A_s &= \frac{21.7 \times 12}{0.9(60)(10 - \frac{1.5}{2})} = 1.507 \text{ in } \approx 1.508 \\
\end{align*}
\]

**BAR SIZE & SPACING**

- Using #4 bar
  \[
  \frac{0.507}{12''} : \frac{0.20}{5''} \\
  s = 4.7'' \\
  \text{ Use } \#4 @ 4'' o.c.
  \]
- Alternate - for max. s of 18''
  \[
  \frac{0.507}{12''} : \frac{0.20}{18''} \\
  s = \frac{0.761 \times 8}{0.79} @ 18'' o.c.
  \]

$A_{S_{\min}} = 0.018 bh = 1.24 < 1.5 \checkmark$
Quiz 9

Can \( f = \frac{Mc}{I} \) be used in Ult. Strength concrete beam calculations? (yes or no)

HINT:

WSD stress

Ult. Strength stress

Source: University of Michigan, Department of Architecture
Rectangular Beam Design

Data:
- Load and Span
- Some section dimensions – b or d
- Material properties – f’c, fy

Required:
- Steel area - As
- Beam dimensions – b or d

1. Choose $\rho$ (e.g. 0.5 $\rho_{max}$ or 0.18f’c/fy)
2. Estimate the dead load and find $M_u$
3. Calculate $bd^2$
4. Choose b and solve for d
   - b is based on form size – try several to find best
5. Estimate h and correct weight and $M_u$
6. Find $As = \rho bd$
7. Choose bars for As and determine spacing and cover. Recheck h and weight.
8. Design shear reinforcement (stirrups)
9. Check deflection, crack control, steel development length.

\[
M_u = \frac{(1.4w_{DL} + 1.7w_{LL})l^2}{8}
\]

\[
bd^2 = \frac{M_u}{\phi \rho f_y \left(1 - 0.59 \rho \left(\frac{f_y}{f_c}\right)\right)}
\]

\[
A_s = \rho bd
\]
Rectangular Beam Design

Data:
- Load and Span
- Material properties – f'c, fy

Required:
- Steel area - As
- Beam dimensions – b and d

1. Estimate the dead load and find Mu
2. Choose $\rho$ (e.g. 0.5 $\rho_{\text{max}}$ or 0.18$f'c/fy$)

\[
\text{Factored LL} = P_e = 1.7(L) = 1.7(20) = 34 \text{ k}
\]

\[
\text{Factored DL} = W_o = 1.4(\text{Applied load + beam weight estimate}) = 1.4(2.6) = 3.64 \text{ k/ft}
\]

\[
M_u = P_e d + \frac{W_o \rho^2}{8} = 34(10) + \frac{3.64 \times 30^2}{8} = 340 + 409.5 = 749.5 \text{ k-ft}
\]

\[
M_u = 8994000 \text{ in.-lb}
\]

\[
\rho = \frac{0.18f'c}{fy} = 0.009
\]
Rectangular Beam Design cont

3. Calculate \( bd^2 \)

\[
bd^2 = \frac{Mu}{f_y (1-0.59\rho (f_y/f_c))}
\]

\[
bd^2 = \frac{8994}{(0.9)(0.009)(60)(1-0.59(0.009)(60/3))}
\]

\[
bd^2 = 20705 \text{ in}^3
\]

4. Choose \( b \) and solve for \( d \)

\( b \) is based on form size. Try several to find best

Possibilities:

- \( 14" \times 38.5" \)
- \( 16" \times 35.97" \)
- \( 18" \times 33.9" \)
5. Estimate $h$ and correct weight and $Mu$

6. Find $A_s = \rho bd$

7. Choose bars for $A_s$ and determine spacing and cover. Recheck $h$ and weight.

8. Design shear reinforcement (stirrups)

9. Check deflection, crack control, steel development length.

---

**Table A.4 Areas of Groups of Standard Bars (in.²)**

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<th>Bar No.</th>
<th>2</th>
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<th>4</th>
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</table>


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University of Michigan, TCAUP
Non-Rectangular Beam Analysis

Data:
- Section dimensions – b, h, d, (span)
- Steel area - As
- Material properties – f’c, fy

Required:
- Required Moment – Mu (or load, or span)

1. Draw and label diagrams for section and stress
   1. Determining b effective (for T-beams)
   2. Locate T and C (or C₁ and C₂)
2. Set T=C and write force equations (P=FA)
   1. T = As fy
   2. C = 0.85 f’c Ac
3. Determine the Ac required for C
4. Working from the top down, add up area to make Ac
5. Find moment arms (z) for each block of area
6. Find Mn = Cz
7. Find Mu = φ Mn     φ =0.90
8. Check As min < As < As max

Source: University of Michigan, Department of Architecture
Analysis Example

Given:  
\( f'c = 3000 \text{ psi} \)  
\( f_y = 60 \text{ ksi} \)  
\( A_s = 6 \text{ in}^2 \)

Req’d:  
Capacity, \( \mu_u \)

1. Find \( T \)
2. Find \( C \) in terms of \( A_c \)
3. Set \( T = C \) and solve for \( A_c \)

\[
T = A_s f_y = 6 \text{ in}^2 (60000 \text{ psi}) = 360000 \text{ k}\text{N}
\]

\[
C = 0.85 f'c A_c = 0.85 (3000 \text{ psi}) A_c \text{ in}^2
\]

\[
C = (2550 A_c)^{1/\kappa} = (2.55 A_c)^{1/\kappa}
\]

\[
T = C
\]

\[
360000 \kappa = 2.55 A_c \kappa
\]

\[
A_c = 142 \text{ in}^2
\]

Source: University of Michigan, Department of Architecture
Example

4. Draw section and determine areas to make $A_c$

5. Solve $C$ for each area in compression.

\[
A_c = 142 \text{ in}^2 = A_{c1} + A_{c2} + A_{c3}
\]
\[
142 = 48 + 30 + A_{c3}
\]
\[
A_{c3} = 44 \text{ in}^2
\]
\[
C_1 = 48(2.55) = 122.4 \text{ k}
\]
\[
C_2 = 30(2.55) = 76.5 \text{ k}
\]
\[
C_3 = 44(2.55) = 113.2 \text{ k}
\]
Example

6. Determine moment arms to areas, $z$.

7. Calculate $M_n$ by summing the $C_z$ moments.

8. Find $M_u = \frac{M_n}{\text{area}}$.
Other Useful Tables:

### Table A.1 Values of Modulus of Elasticity for Normal-Weight Concrete

<table>
<thead>
<tr>
<th>Customary Units</th>
<th>SI Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f'_c$ (psi)</td>
<td>$E'_c$ (psi)</td>
</tr>
<tr>
<td>3,000</td>
<td>3,140,000</td>
</tr>
<tr>
<td>3,500</td>
<td>3,390,000</td>
</tr>
<tr>
<td>4,000</td>
<td>3,620,000</td>
</tr>
<tr>
<td>5,000</td>
<td>3,850,000</td>
</tr>
<tr>
<td>5,000</td>
<td>4,050,000</td>
</tr>
</tbody>
</table>

### Table A.2 Designations, Areas, Perimeters, and Weights of Standard Bars

<table>
<thead>
<tr>
<th>Bar No.</th>
<th>Customary Units</th>
<th>SI Units</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Diameter (in.)</td>
<td>Cross-sectional Area (in.$^2$)</td>
</tr>
<tr>
<td>3</td>
<td>0.375</td>
<td>0.11</td>
</tr>
<tr>
<td>4</td>
<td>0.500</td>
<td>0.20</td>
</tr>
<tr>
<td>5</td>
<td>0.625</td>
<td>0.31</td>
</tr>
<tr>
<td>6</td>
<td>0.750</td>
<td>0.44</td>
</tr>
<tr>
<td>7</td>
<td>0.875</td>
<td>0.60</td>
</tr>
<tr>
<td>8</td>
<td>1.000</td>
<td>0.79</td>
</tr>
<tr>
<td>9</td>
<td>1.128</td>
<td>1.00</td>
</tr>
<tr>
<td>10</td>
<td>1.270</td>
<td>1.27</td>
</tr>
<tr>
<td>11</td>
<td>1.410</td>
<td>1.56</td>
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<tr>
<td>14</td>
<td>1.693</td>
<td>2.25</td>
</tr>
<tr>
<td>18</td>
<td>2.257</td>
<td>4.00</td>
</tr>
</tbody>
</table>

### Table A.3 Areas of Groups of Standard Bars (in.$^2$)

<table>
<thead>
<tr>
<th>Number of Bars</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>4</td>
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<tr>
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<tr>
<td>10</td>
</tr>
<tr>
<td>14</td>
</tr>
<tr>
<td>18</td>
</tr>
</tbody>
</table>


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