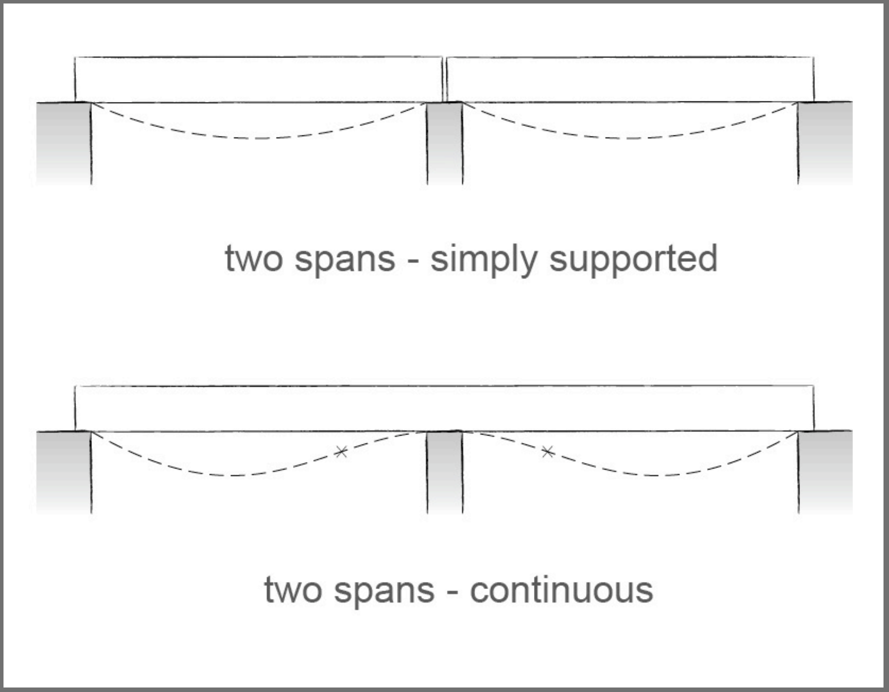
Structural Continuity

Millennium Bridge, London
Foster and Partners + Arup

- Continuity in Beams
- Deflection Method
- Slope Method
- Three-Moment Theorem

Continuous Beams

- Continuous over one or more supports
  - Most common in monolithic concrete
  - Steel: continuous or with moment connections
  - Wood: as continuous beams, e.g. long Glulam spans
- Statically indeterminate
  - Cannot be solved by the three equations of statics alone
  - Internal forces (shear & moment) as well as reactions are effected by movement or settlement of the supports



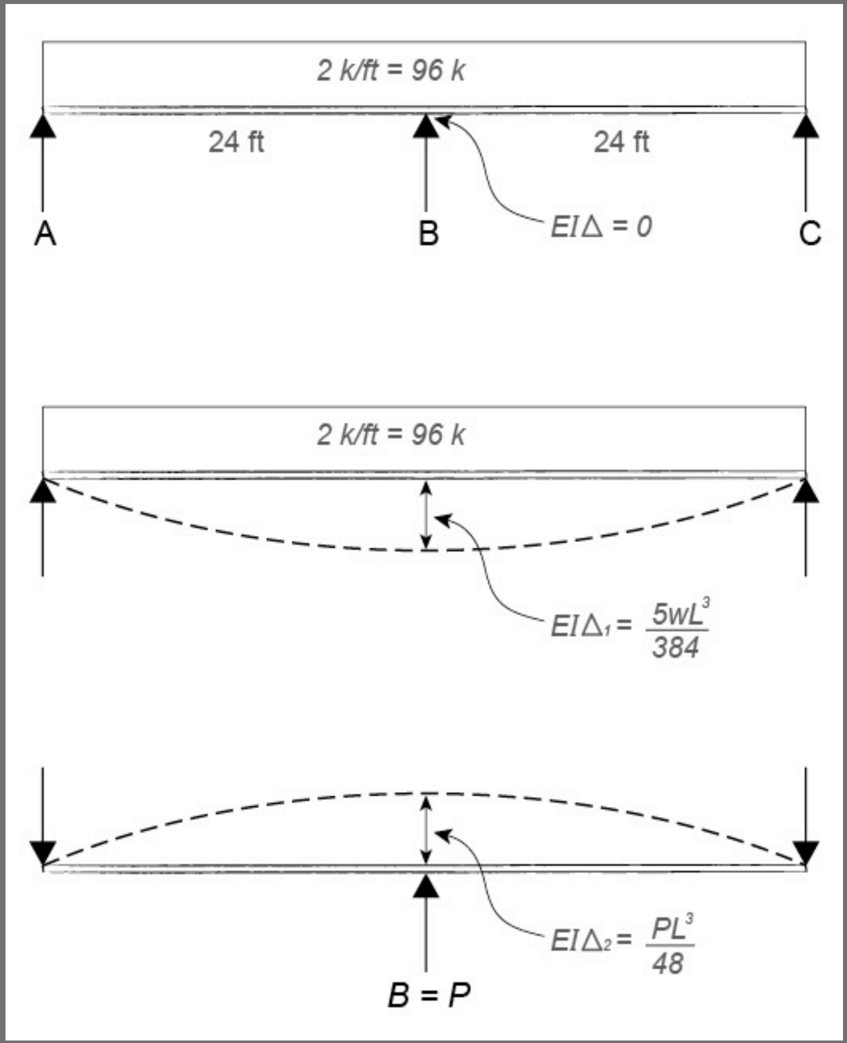
Source: University of Michigan, Department of Architecture

Deflection Method

- Two continuous spans
- Symmetric Load and Geometry

Procedure:

1. Remove the central support.
2. Calculate the central deflection for each load case as a simple span.
3. Set the resulting central deflection equal to the central reaction "deflection" upward, bringing the total central deflection back to zero.
4. Solve the resulting equation for the central reaction force.
5. Calculate the remaining two end reactions
6. Draw shear and moment diagrams as usual.

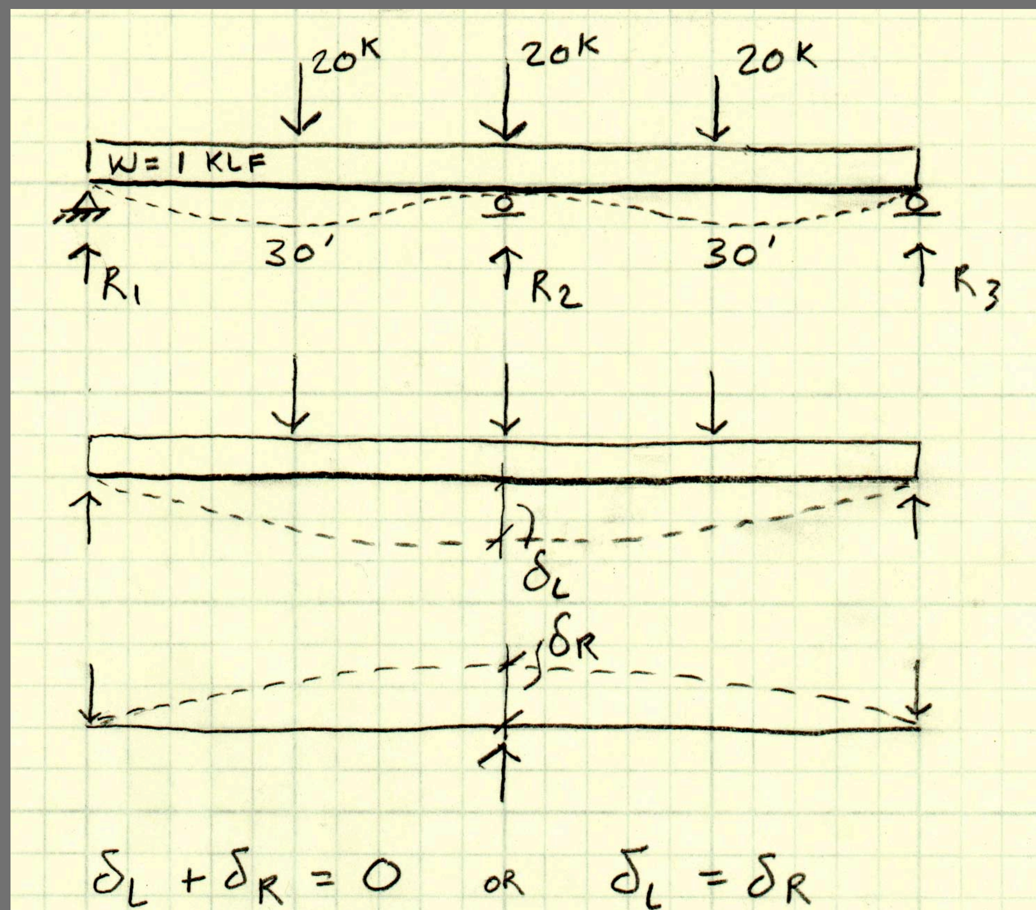


Source: University of Michigan, Department of Architecture

$$EI\Delta_1 + EI\Delta_2 = 0$$

Deflection Method Example:

Replace redundant reaction with point load, bringing reaction deflection to zero.



Deflection Method Example cont.:

Write deflection equations and solve for middle reaction. Solve remaining reactions by summation of forces.

$$\delta_L + \delta_R = 0 \quad \text{OR} \quad \delta_L = \delta_R$$

$$\delta_L = \frac{5w\ell^4}{384\,EI} + \frac{19\,P\ell^3}{384\,EI} = \frac{5(1)(60)^4 + 19(20)(60)^3}{384\,EI}$$

$$\delta_L = 382500/EI = \delta_R = \frac{R_2\ell^3}{48\,EI} = R_2 4500/EI$$

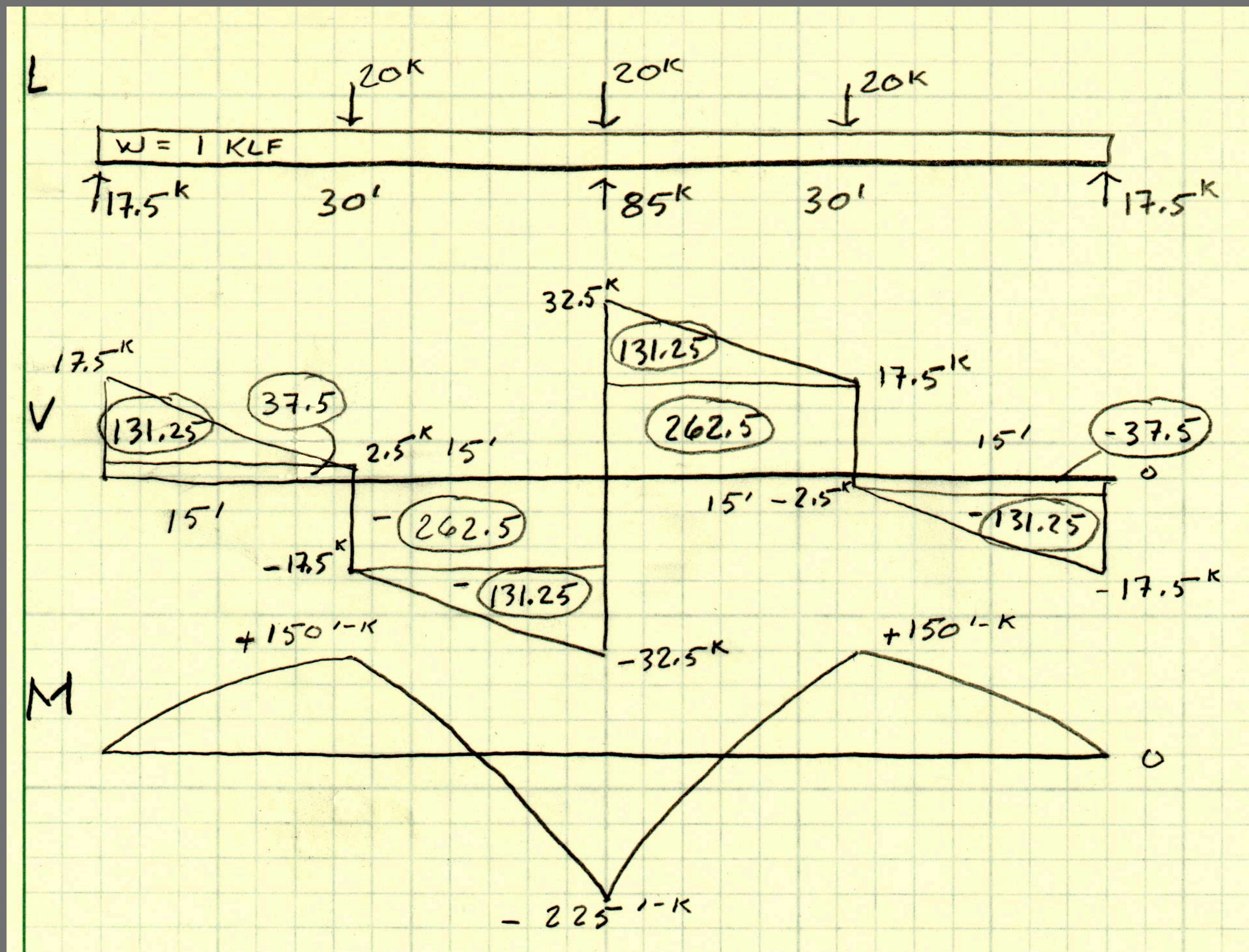
$$R_2 = 85^K$$

$$\Sigma F_V = 0 = R_1 + R_3 + 85 - 60 - 60 = 0$$

$$R_1 = R_3 = 17.5^K$$

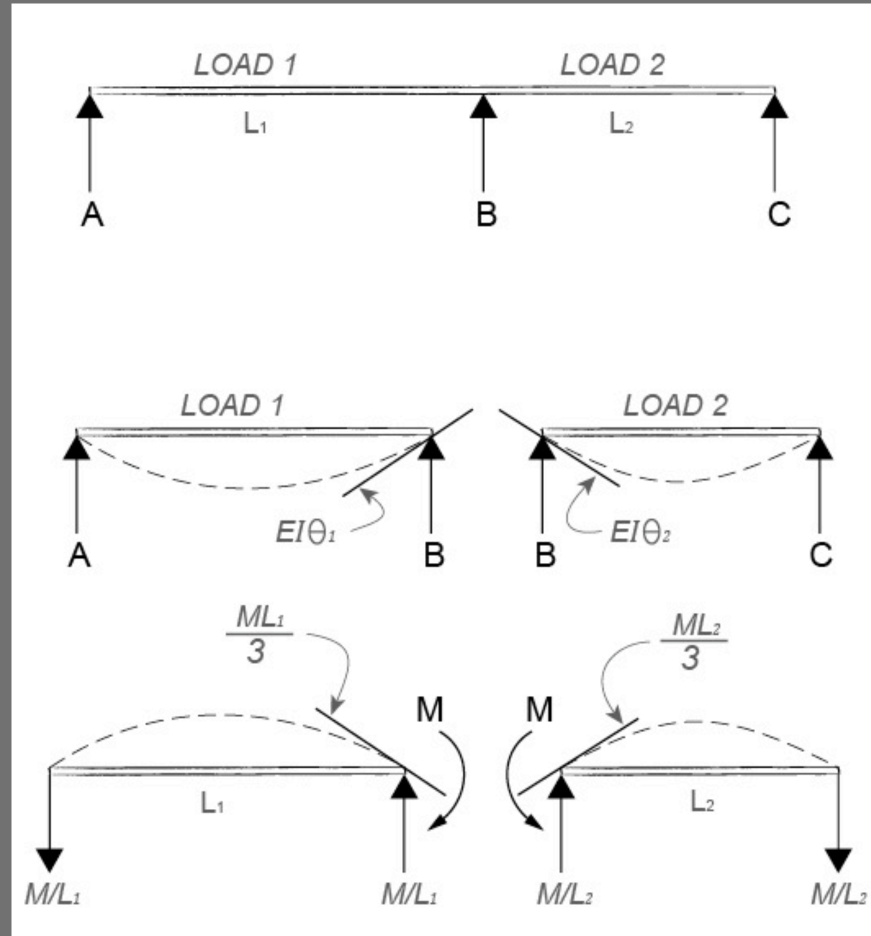
Deflection Method Example cont.:

Complete the shear and moment diagrams.



Slope Method

- Two continuous spans
- Non-Symmetric Load and Geometry



Source: University of Michigan, Department of Architecture

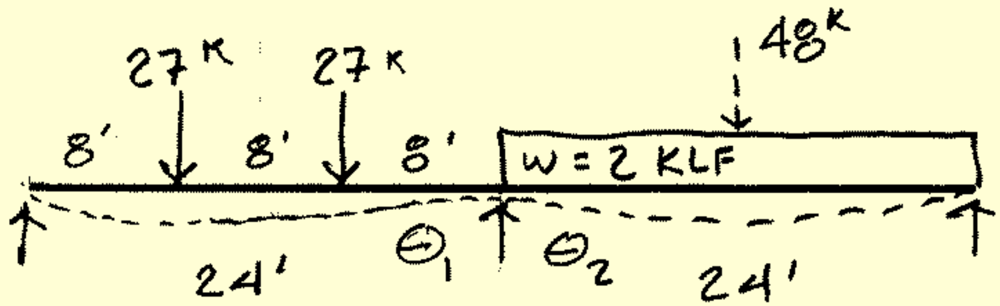
Procedure:

1. Break the beam into two halves at the interior support.
2. Calculate the interior slopes of the two simple spans.
3. Use the Slope Equation to solve for the negative interior moment.
4. Find the reactions of each of the simple spans plus the M/L reactions caused by the interior moment.
5. Add all the reactions by superposition.
6. Draw the shear and moment diagrams as usual.

$$M = \frac{3}{L_1 + L_2}\left[EI\Theta_1 + EI\Theta_2\right]$$

Example of Slope Method:

Use slope formula to solve interior moment



$$EI\theta_1 = \frac{PL^2}{9} = \frac{27(24)^2}{9} = 1728$$

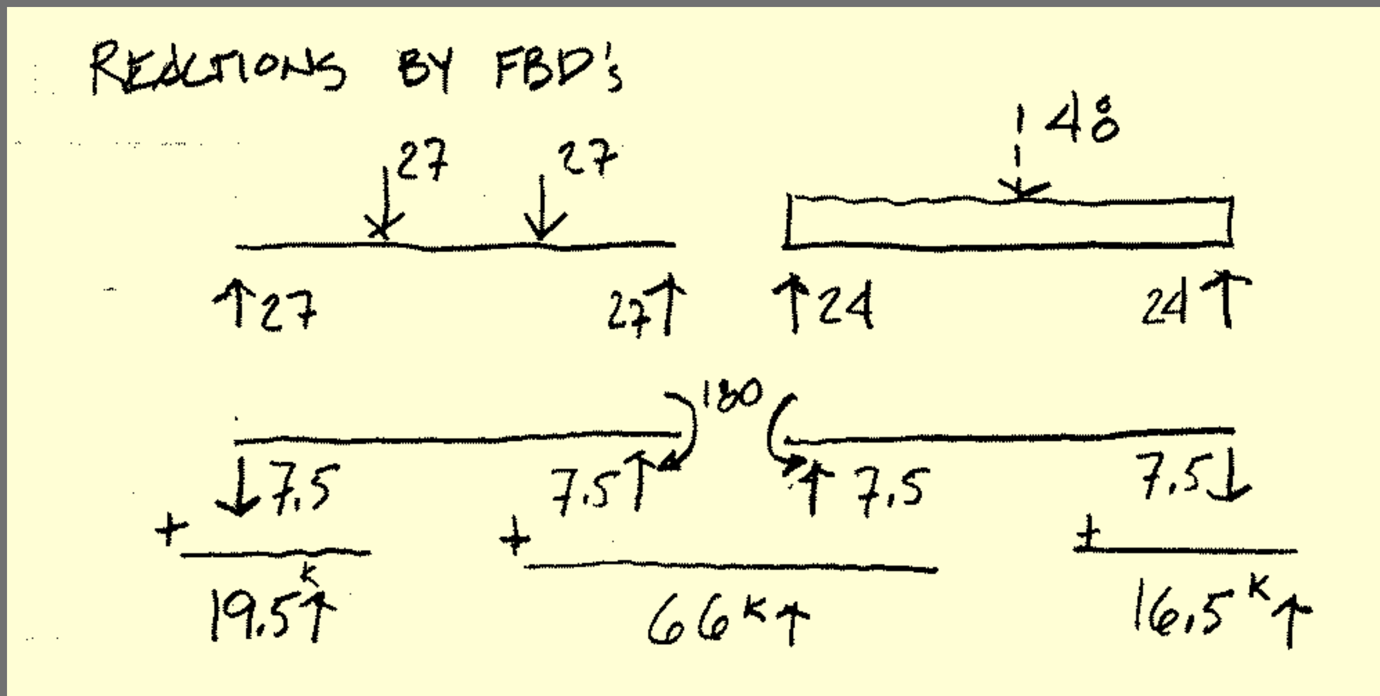
$$EI\theta_2 = \frac{Wl^2}{24} = \frac{48(24)^2}{24} = 1152$$

$$M = \frac{3}{L_1 + L_2}\left[EI\theta_1 + EI\theta_2\right] = \frac{3}{48}\left[2880\right]$$

$$M = 180^{K\prime}$$

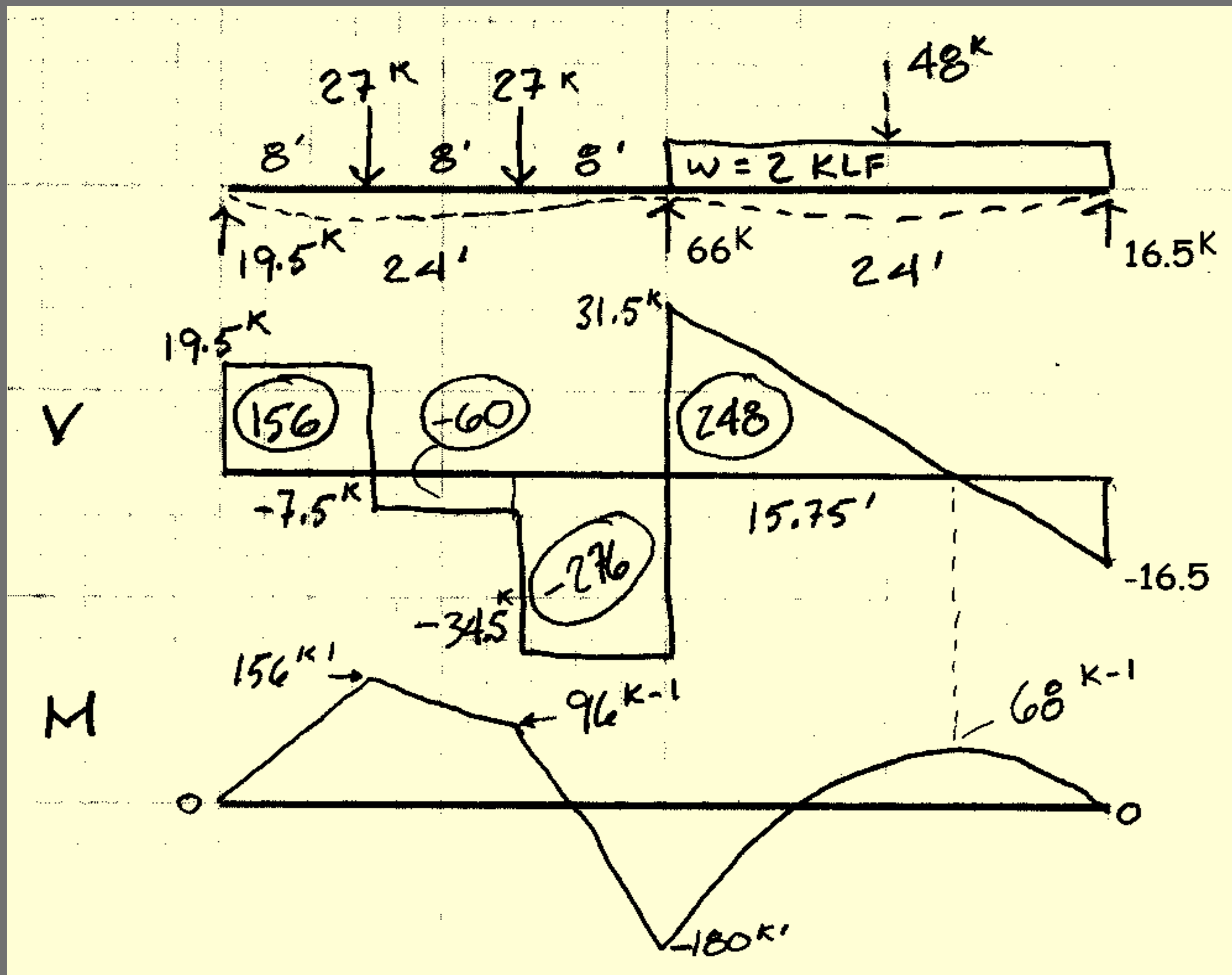
Example of Slope Method cont.:

Solve the end reactions by superposition of FBD’s



Example of Slope Method cont.:

Construct load, shear and moment diagrams.

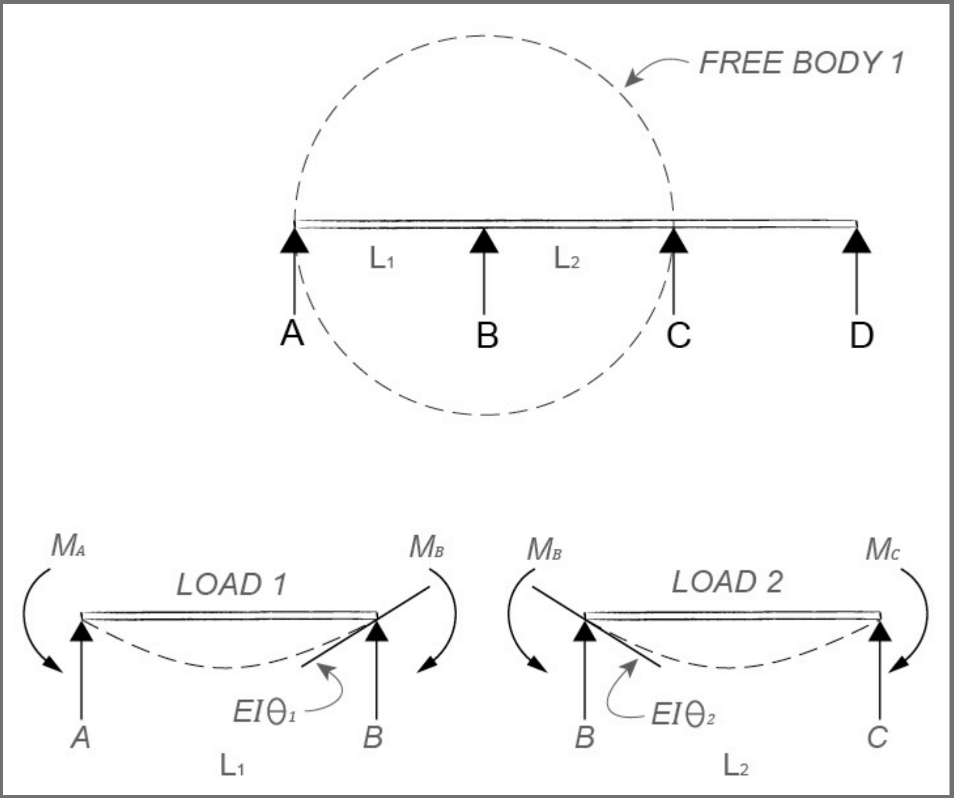


Three-Moment Theorem

- Any number of spans
- Symmetric or non-symmetric

Procedure:

1. Draw a free body diagram of the first two spans.
2. Label the spans L1 and L2 and the supports (or free end) A, B and C as show.
3. Use the Three-Moment equation to solve for each unknown moment, either as a value or as an equation.



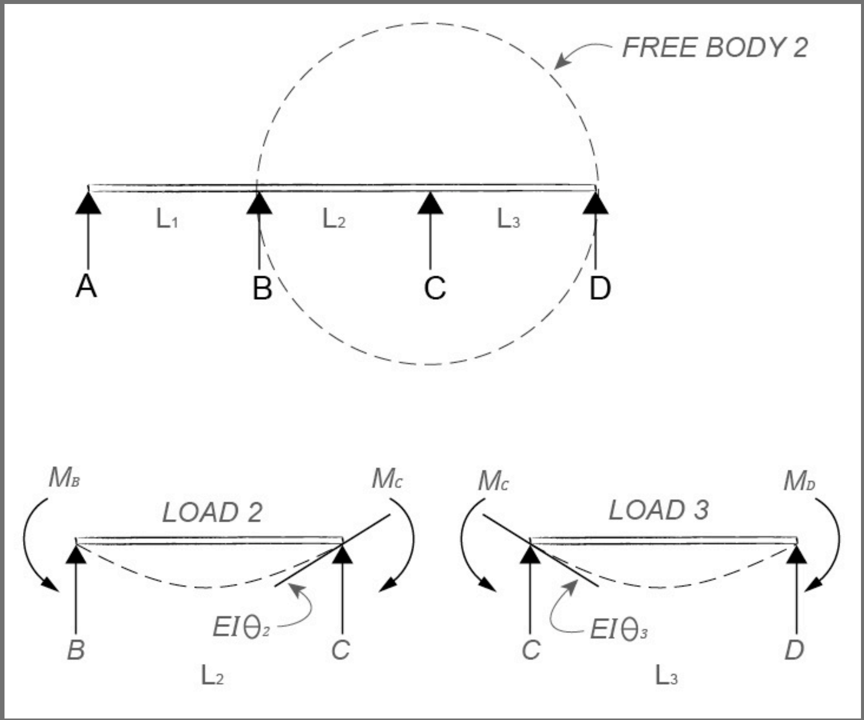
Source: University of Michigan, Department of Architecture

$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = 6[EI\Theta_1 + EI\Theta_2]$$

Three-Moment Theorem (cont.)

Procedure:

4. Move one span further and repeat the procedure.
5. In a 3 span beam, the mid-moment from step 3 above (B), could now be solved using the two equations from step 4 and 3 together, by writing 2 equations with 2 unknowns.
6. Repeat as needed, always moving one span to the right and writing a new set of moment equations.
7. Solve 2 simultaneous equations for 3 spans, or 3 equations for more than 3 spans, to get the interior moments.
8. Once all interior moments are known, solve for reactions using free body diagrams of individual spans.
9. Draw shear and moment diagrams as usual. This will also serve as a check for the moment values.



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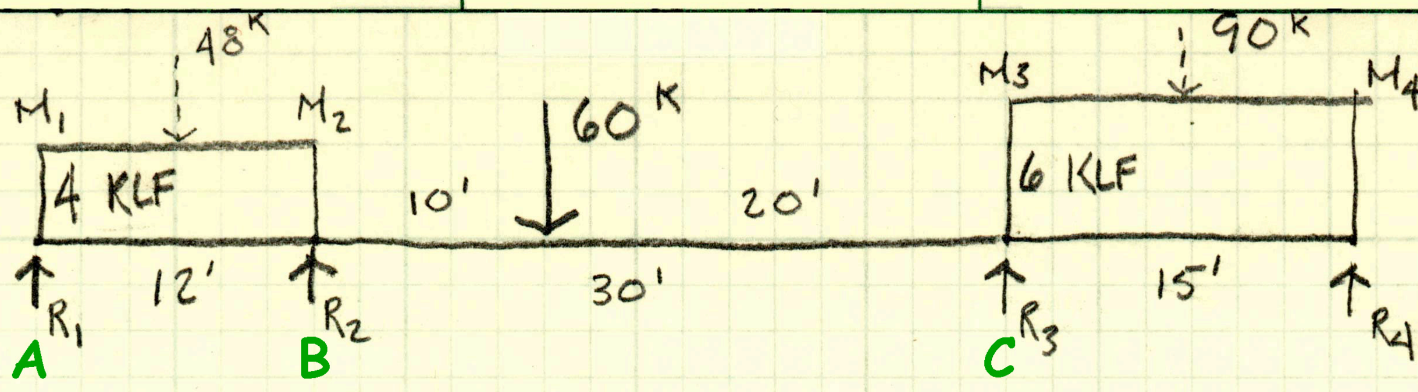
$$M_B L_2 + 2M_C\left(L_2 + L_3\right) + M_D L_3 = 6\left[EI\Theta_2 + EI\Theta_3\right]$$

Three-Moment Theorem Example



FIRST EQUATION (AT 1, 2 AND 3):

$M_A = 0 \qquad L_1 = 12 \qquad EI\theta_1 = \frac{wl^2}{24} = \frac{48(12)^2}{24} = 288$

$M_B = M_2 \qquad L_2 = 30$

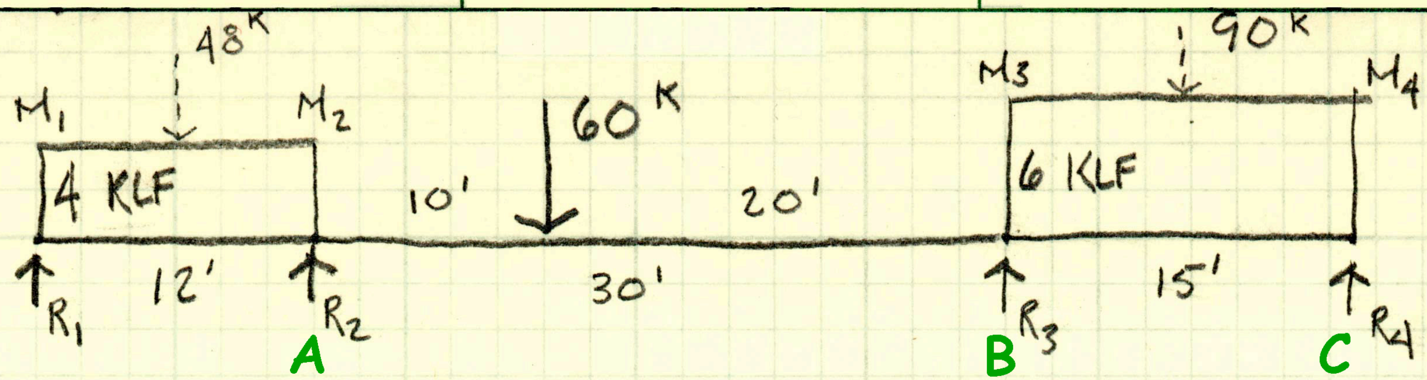
$M_C = M_3 \qquad L_1 + L_2 = 42 \qquad EI\theta_2 = \frac{5Pl^2}{81} = \frac{5(60)(30)^2}{81} = 3333.3$

$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = 6[EI\theta_1 + EI\theta_2]$$

$$0(12) + 2M_2(42) + M_3(30) = 6[288 + 3333.3]$$

$$M_2 = 258.667 - 0.35714\, M_3$$

Three-Moment Theorem Example (cont.)



SECOND EQUATION (AT 2, 3 AND 4):

$M_A = M_2 \qquad L_1 = 30 \qquad EI\theta_1 = \frac{4Pl^2}{81} = \frac{4(60)(30)^2}{81} = 2666.67$

$M_B = M_3 \qquad L_2 = 15$

$M_C = 0 \qquad L_1 + L_2 = 45 \qquad EI\theta_2 = \frac{wl^2}{24} = \frac{90(15^2)}{24} = 843.75$

$$M_2(30) + 2M_3(45) + 0(15) = 6[2666.67 + 843.75]$$

$$M_2 = 702.084 - 3M_3$$

SIMULTANIOUS SOLUTION:

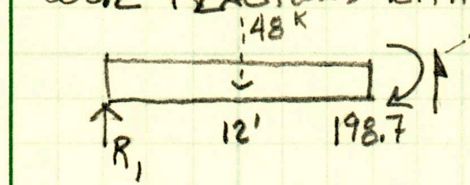
$$258.667 - .35714\, M_3 = 702.084 - 3M_3$$

$$M_3 = 443.417 / 2.6428 = 167.779^{K-'}$$

$$M_2 = 702.084 - 3(167.779) = 198.746^{K-'}$$

Three-Moment Theorem Example (cont.)

SOLVE REACTIONS WITH FBD'S

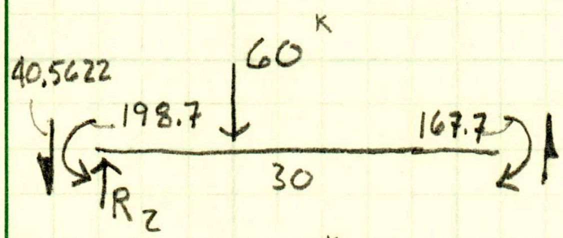


$$\Sigma M_{@R_2} = R_1(12) - 48(6) + 198.7 = 0$$

$$R_1 = \underline{7.4378^K}$$

$$\Sigma F_V = 7.4378 - 48 + V = 0$$

$$V = 40.5622$$

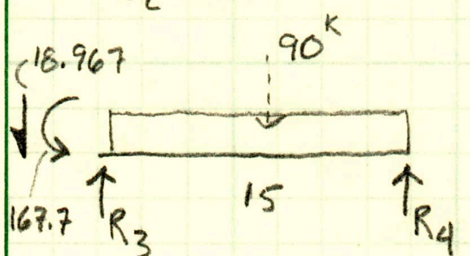


$$\Sigma M_{@R_3} = R_2(30) - 40.5622(30) - 60(20) - 198.75 + 167.78 = 0$$

$$R_2 = \underline{81.594^K}$$

$$\Sigma F_V = -40.5622 + 81.594 - 60 + V = 0$$

$$V = 18.967$$



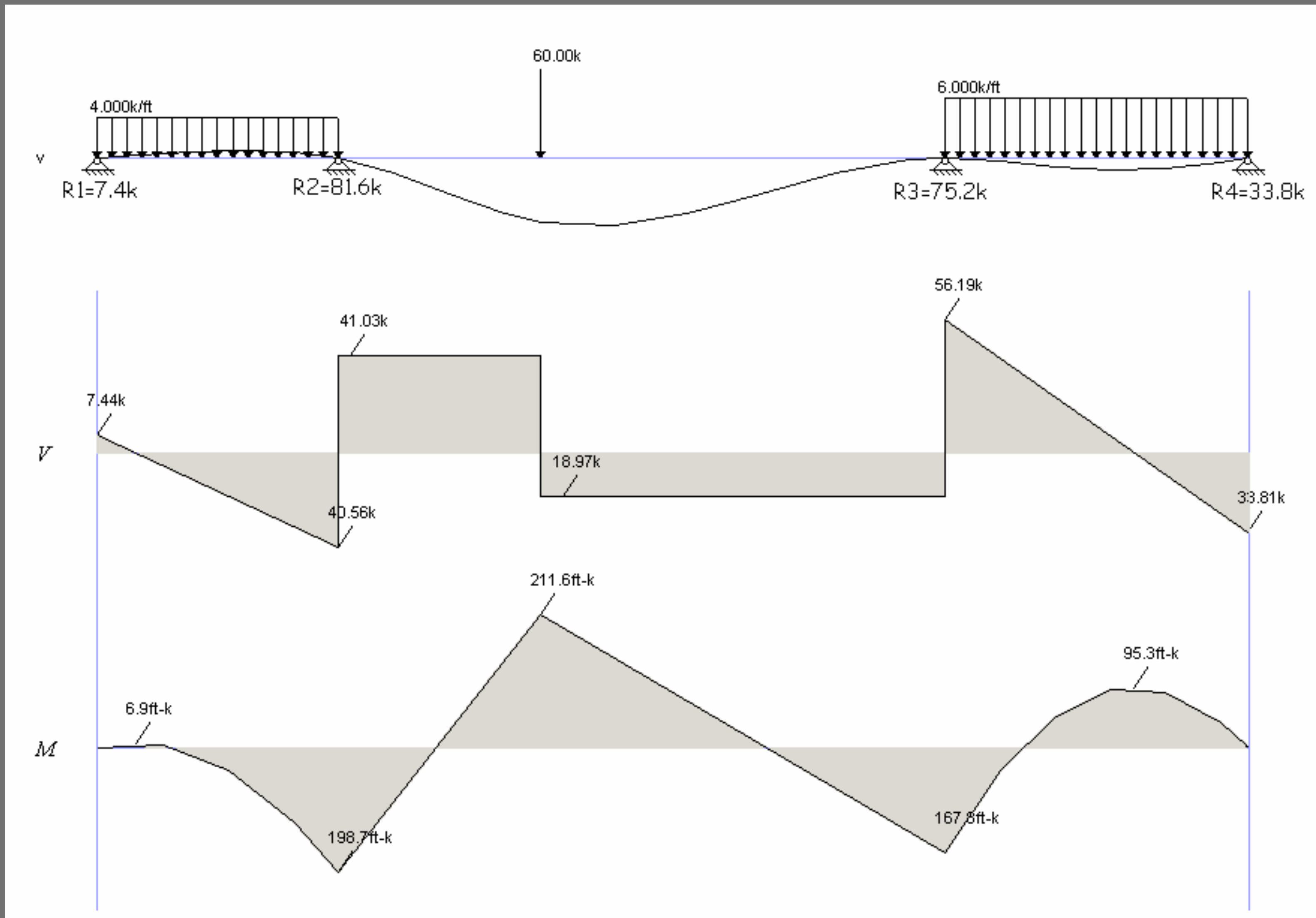
$$\Sigma M_{@R_4} = -167.78 - 18.967(15) + R_3(15) - 90(7.5) = 0$$

$$R_3 = \underline{75.1531^K}$$

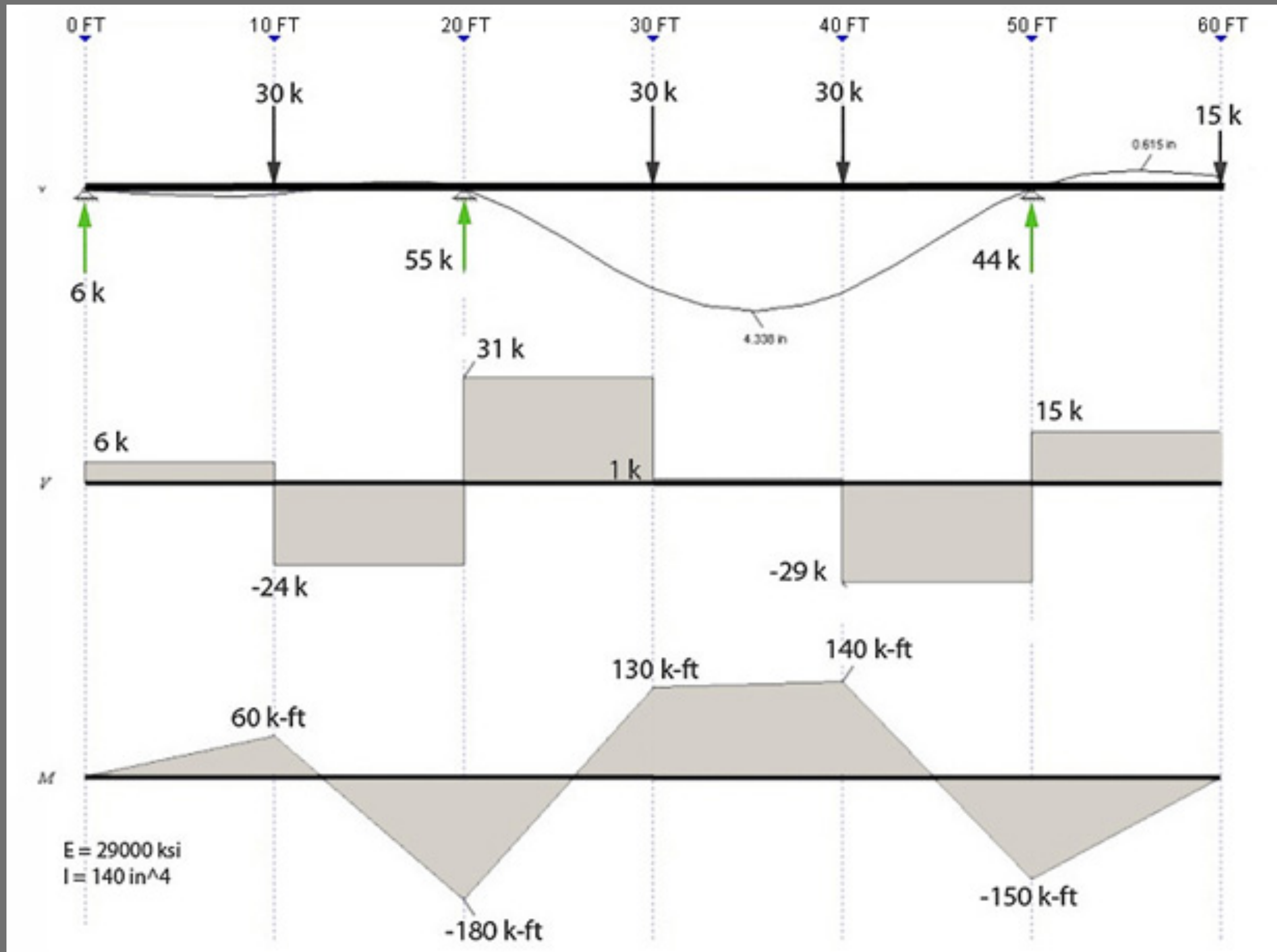
$$\Sigma M_{@R_3} = -167.78 + 90(7.5) - R_4(15) = 0$$

$$R_4 = \underline{33.8147^K}$$

Three-Moment Theorem Example (cont.)



Three-Moment Theorem – 2 Spans



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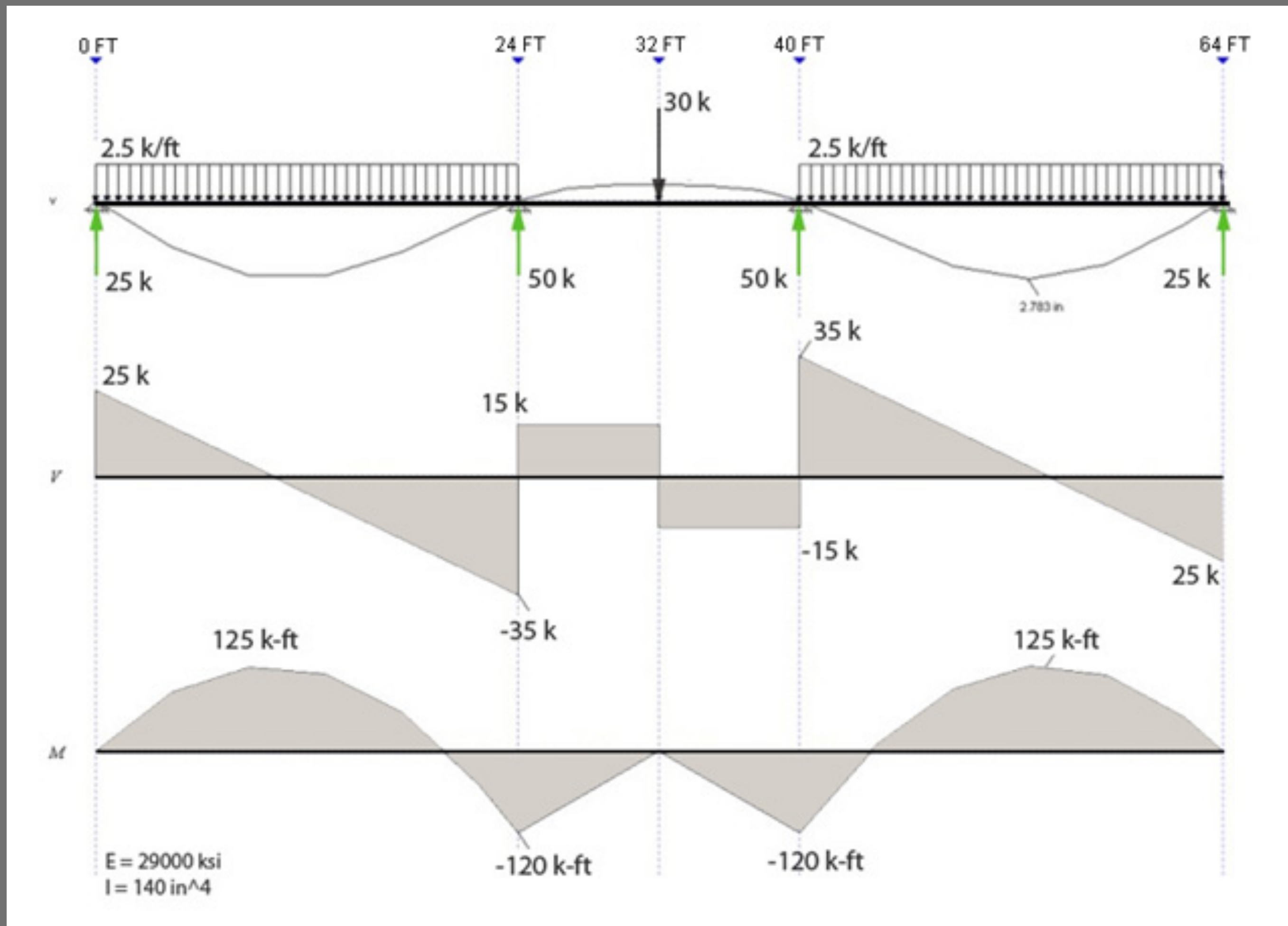
$$M_A L_1 + 2M_B\left(L_1 + L_2\right) + M_C L_2 = 6\left[EI\Theta_1 + EI\Theta_2\right]$$

Three-Moment Theorem – 3 Spans



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$$M_A L_1 + 2M_B\left(L_1 + L_2\right) + M_C L_2 = 6\left[EI\Theta_1 + EI\Theta_2\right]$$