ARCH 324 - Structures 2, Winter 2009

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Combined Stress

- Axial vs. Eccentric Load
- Combined Stress
- Interaction Formulas

Source: Caspar David Friedrich from “Man und Frau den Mond betrachtend” 1830-35
Alte Nationalgalerie, Berlin
Axial Stress

- Loads pass through the centroid of the section
- Member is straight
- Load less than buckling load

- $f = \frac{P}{A}$

Source: University of Michigan, Department of Architecture
Eccentric Loads

- Load offset from centroid
- $M = P \cdot e$
- Total load $= P + M$

Interaction formula

$$f = \frac{P}{A} \pm \frac{M c}{I}$$

$$\frac{f_a}{F_a} \pm \frac{f_b}{F_b} \leq 1.0$$

Source: University of Michigan, Department of Architecture
Combined Stress

- Stresses combine by superposition
- Values add or subtract by sign

\[
P \Rightarrow f_c = \frac{P}{A}
\]

\[
\text{comp} = \frac{M^*c}{l}
\]

\[
f_c' = \frac{P}{A} + \frac{M^*c}{l}
\]

\[
f_b = \frac{M^*c}{l}
\]

axial loaded - uniform compressive stress.

small eccentricity - linearly varying stress.

large eccentricity - tensile stress on part of cross section.

Source: University of Michigan, Department of Architecture
Example

**CANOPY CONSTRUCTION PROJECTING FROM FACE OF BUILDING.**
The supporting cable is pin-connected on the centroidal axis of the steel beam.

**Reactions at face of building.**

FOR THE W14x34:
Determine the magnitude and location of the max. compressive and tensile unit stresses.

Source: University of Michigan, Department of Architecture

**Determine external reactions**

\[
\sum M_A = 0 = -B_h(6') + 10^k(8') + 10^k(16') + 5^k(24')
\]

\[
B_h = 60^k
\]

\[
\sum H_B = 0 = -A_h(6') + 10^k(8') + 10^k(16') + 5^k(24')
\]

\[
A_h = 60^k
\]

**CHECK** \( \sum F_H = 0 = 60^k - 60^k \)

**FBID E A**

\[
\sum F_V = 0 = 15^k - 10^k - 10^k - 5^k + B_v
\]

\[
B_v = 10^k
\]
Example

Determine internal member forces: Axial and Flexural

Determine axial and flexural stresses

Use interaction formula to determine combined stresses at key locations (e.g. extreme fibers)

\[ \text{W14 x 3A} \quad A = 10.0 \text{ in}^2 \]
\[ S_x = 48.4 \text{ in}^3 \]

**Force:**

- Axial: \( F_{axial} = 60 \text{k} \)
- Flexural: \( F_{flexural} = M/L = PL/3 = 10 \text{k}(8') = 80 \text{k} \)

**Stress:**

- Axial: \( f_{axial} = \frac{P}{A} = \frac{60}{10 \text{ in}^2} = 6.0 \text{ksi} \)
- Flexural: \( f_{flexural} = \frac{M}{I} = \frac{80 \text{k}}{48.4 \text{ in}^3} = 19.75 \text{ksi} \)

**Combined Stress**

**Top Side:**

\[ f_a + f_b = 6.0 + 19.75 = 25.75 \text{ ksi (comp)} \]

**Bottom Side:**

\[ f_a - f_b = 6.0 - 19.75 = -13.75 \text{ ksi (tens)} \]
Rafters  Flexure + Axial

\[ f = \frac{P}{A} \pm \frac{Mc}{I} \]

\( P = 22.5 \# \)
\( A = 5.25 \text{ in}^2 \)
\( f_c = \frac{22.5}{5.25} = 4.28 \text{ psi} \)

\( M = \frac{PL}{4} = \frac{39 \# \times 96”}{4} = 936 ”- \# \)
\( c = \frac{1.5”}{2} = 0.75” \)
\( l_y = 3.5(1.5^3)/12 = 0.984 \text{ in}^4 \)
\( f_b = \frac{963(0.75)}{0.984} = 713.14 \)

\( f = f_c + f_b = 717.4 \text{ psi} \)

Or

\( M = \frac{PL}{4} = \frac{45 \# \times 83.14”}{4} = 936 ”- \# \)
\( f_b = \frac{963(0.75)}{0.984} = 713.14 \)

\( 45\#(\sin 60) = 39\# \)
\( 45\#(\cos 60) = 22.5\# \)
Other Examples
Second Order Stress
“P Delta Effect”

1. Eccentric load causes bending moment
2. Bending moment causes deflection, $\Delta$
3. $P \times \Delta$ causes additional moment
4. The additional $P\Delta$ moment causes additional deflection, $\Delta_1$
Treehouse

- Form Exploration
Treehouse

- Details
  - Pinned at ends
  - Radius bent at joints
  - ‘Continuous’ sections
  - Welded seams
Treehouse

• Details
  – Pinned at ends
  – Radius bent at joints
  – ‘Continuous’ sections
  – Welded seams
Treehouse

- Erection
  - 3 sections
  - Trucked to site
  - Erected with mobile crane
  - One day process
Treehouse

- Erection
  - 3 sections
  - Trucked to site
  - Erected with mobile crane
  - One day process
Treehouse

- Finished
Treehouse

- Finished
Treehouse

- Finished
Treehouse

- Finished