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ARCH 324 - Structures 2, Winter 2009

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Combined Stress

- Axial vs. Eccentric Load
- Combined Stress
- Interaction Formulas

Source: Caspar David Friedrich from “Man und Frau den Mond betrachtend” 1830-35
Alte Nationalgalerie, Berlin
Axial Stress

- Loads pass through the centroid of the section
- Member is straight
- Load less than buckling load

- $f = \frac{P}{A}$
Eccentric Loads

- Load offset from centroid
- \[ M = P \, e \]
- Total load = \( P + M \)

Interaction formula

\[
f = \frac{P}{A} \pm \frac{Mc}{I}
\]

\[
\frac{f_a}{F_a} \pm \frac{f_b}{F_b} \leq 1.0
\]

Source: University of Michigan, Department of Architecture
Combined Stress

- Stresses combine by superposition
- Values add or subtract by sign

\[
\begin{align*}
\text{axial loaded - uniform compressive stress.} & \quad \text{small eccentricity - linearly varying stress.} & \quad \text{large eccentricity - tensile stress on part of cross section.}
\end{align*}
\]
Example

**CANOPY CONSTRUCTION PROJECTING FROM FACE OF BUILDING.**
The supporting cable is pin-connected on the centroidal axis of the steel beam.

**FOR THE W14x34:**
Determine the magnitude and location of the max. compressive and tensile unit stresses.

Reactions at face of building.

**Determine external reactions**

\[ \Sigma M_A = 0 = -B_B \cdot 6' + 10^k(8') + 10^k(16') + 5^k(24') \]
\[ B_B = 60^k \]

\[ \Sigma M_B = 0 = -A_H(6') + 10^k(8') + 10^k(16') + 5^k(24') \]
\[ A_H = 60^k \]

**Check** \[ \Sigma F_H = 0 = 60^k - 60^k \]

\[ F_{BD} = 60^k \]

\[ \Sigma F_V = 0 = 15^k - 10^k - 10^k - 5^k + B_V \]
\[ B_V = 10^k \]
Example

Determine internal member forces: Axial and Flexural

Determine axial and flexural stresses

Use interaction formula to determine combined stresses at key locations (e.g. extreme fibers)

\[ W14 \times 34 \]

\[ A = 10.0 \text{ in}^2 \]

\[ S_x = 48.4 \text{ in}^3 \]

**FORCE**:

\[ \text{Axial} = 40 \text{k} \]
\[ \text{Flexural} = M = PL/3 = 10 \text{k}(8) = 80 \text{k} \]

**STRESS**:

\[ \text{Axial} = f_a = \frac{P}{A} = \frac{40}{10} = 4.0 \text{ksi} \]
\[ \text{Flexural} = f_b = \frac{M}{S} = \frac{80}{48.4} = 1.67 \text{ksi} \]

**COMBINED STRESS**

**TOP SIDE**:

\[ f_a + f_b = 4.0 + 1.67 = 5.67 \text{ksi (comp)} \]

**BOTTOM SIDE**:

\[ f_a - f_b = 4.0 - 1.67 = 2.33 \text{ksi (tens)} \]
Rafters Flexure + Axial

\[
f = \frac{P}{A} \pm \frac{Mc}{I}
\]

\[
P = 22.5 \text{ #}
\]
\[
A = 5.25 \text{ in}^2
\]
\[
f_c = \frac{22.5}{5.25} = 4.28 \text{ psi}
\]

\[
M = \frac{PL}{4} = 39\text{#} \times 96" / 4 = 936" - \text{#}
\]
\[
c = 1.5" / 2 = 0.75"
\]
\[
I_y = 3.5(1.5^3)/12 = 0.984 \text{ in}^4
\]
\[
f_b = 963 (0.75)/0.984 = 713.14
\]

\[
f = f_c + f_b = 717.4 \text{ psi}
\]

Or

\[
M = \frac{PL}{4} = 45\text{#} \times 83.14" / 4 = 936" - \text{#}
\]
\[
f_b = 963 (0.75)/0.984 = 713.14
\]
Other Examples
Second Order Stress
“P Delta Effect”

1. Eccentric load causes bending moment
2. Bending moment causes deflection, $\Delta$
3. $P \times \Delta$ causes additional moment
4. The additional $P\Delta$ moment causes additional deflection, $\Delta_1$
Treehouse

- Form Exploration
Treehouse

- Details
  - Pinned at ends
  - Radius bent at joints
  - ‘Continuous’ sections
  - Welded seams
Treehouse

- Details
  - Pinned at ends
  - Radius bent at joints
  - ‘Continuous’ sections
  - Welded seams
Treehouse

- Erection
  - 3 sections
  - Trucked to site
  - Erected with mobile crane
  - One day process
Treehouse

- Erection
  - 3 sections
  - Trucked to site
  - Erected with mobile crane
  - One day process
Treehouse

- Finished
Treehouse

- Finished
Treehouse

- Finished
Treehouse

- Finished