Architecture 324
Structures II

Column Analysis and Design

- Failure Modes
- End Conditions and Lateral Bracing
- Analysis of Wood Columns
- Design of Wood Columns
- Analysis of Steel Columns
- Design of Steel Columns

Leonhard Euler (1707 – 1783)

Euler Buckling (elastic buckling)

$$P_{cr} = \frac{\pi^2 AE}{\left(\frac{KL}{r}\right)^2}$$

$$r = \sqrt{\frac{I}{A}}$$

- A = Cross sectional area (in^2)
- E = Modulus of elasticity of the material (lb/in^2)
- K = Stiffness (curvature mode) factor
- L = Column length between pinned ends (in.)
- r = radius of gyration (in.)

$$f_{cr} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \leq F_{cr}$$

Source: Emanuel Handmann (wikimedia commons)

Failure Modes

- **Short Columns** – fail by crushing
  (“compression blocks or piers” Engel)

$$f_c = \frac{P}{A} \leq F_c$$

  – f_c = Actual compressive stress
  – A = Cross-sectional area of column (in^2)
  – P = Load on the column
  – F_c = Allowable compressive stress per codes

- **Intermediate Columns** – crush and buckle
  (“columns” Engel)

- **Long Columns** – fail by buckling
  (“long columns” Engel)

$$f_{cr} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \leq F_{cr}$$

  – E = Modulus of elasticity of the column material
  – K = Stiffness (curvature mode) factor
  – L = Column length between pinned ends (in.)
  – r = radius of gyration = $(I/A)^{1/2}$

Slenderness Ratio

- Radius of Gyration: a geometric property of a cross section

$$r = \sqrt{\frac{I}{A}} \qquad I = Ar^2$$

  - r = Radius of Gyration
  - I = Moment of Inertia
  - A = Cross-sectional Area

$r_x = 0.999$

$r_y = 0.433$

- Slenderness Ratios:

$$\frac{L_x}{r_x} \qquad \frac{L_y}{r_y}$$

The larger ratio will govern.
Try to balance for efficiency

End Support Conditions

K is a constant based on the end conditions

l is the actual length

l_e is the effective length

$l_e = Kl$

K= 1.0

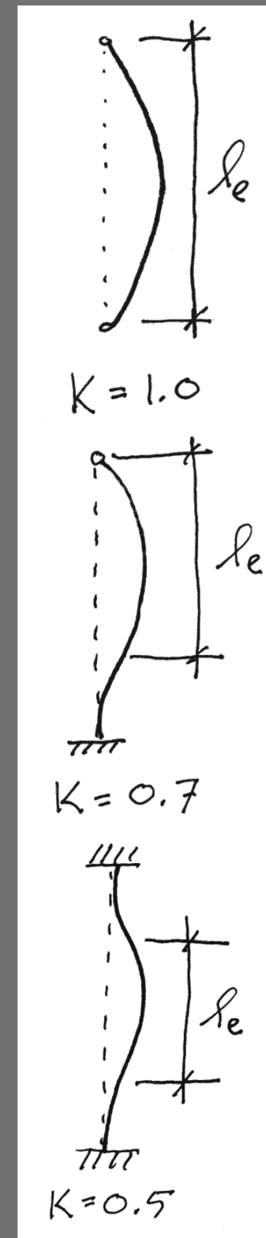
Both ends pinned.

K= 0.7

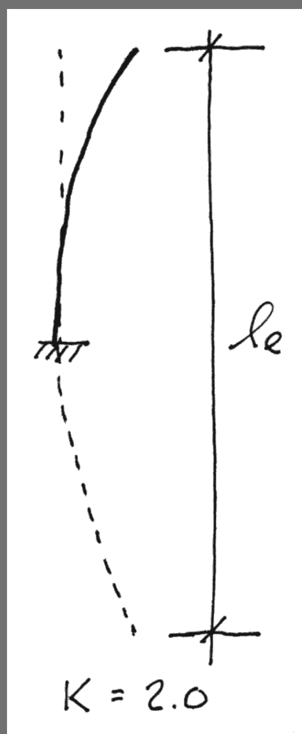
One end free, one end fixed.

K= 0.5

Both ends fixed.



K= 2.0



One end pinned, one end fixed.

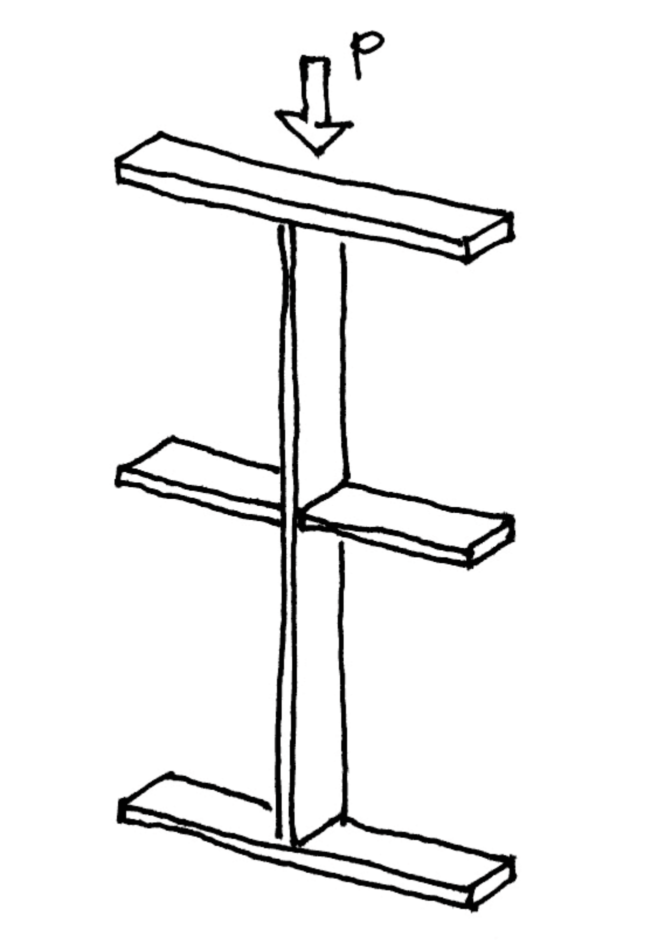
Analysis of Wood Columns

Data:

- Column – size, length
- Support conditions
- Material properties – F_c , E

Required:

- P_{crit} for buckling and crushing



1. Calculate slenderness ratio; largest ratio governs.
2. Check slenderness against upper limit.
3. Calculate P_{crit} for buckling using Euler's equation:
4. Calculate P_{max} for crushing:

   $P_{max} = F_c \; A$
5. Smaller of P_{crit} or P_{max} will fail first.

Example Problem :

Analysis

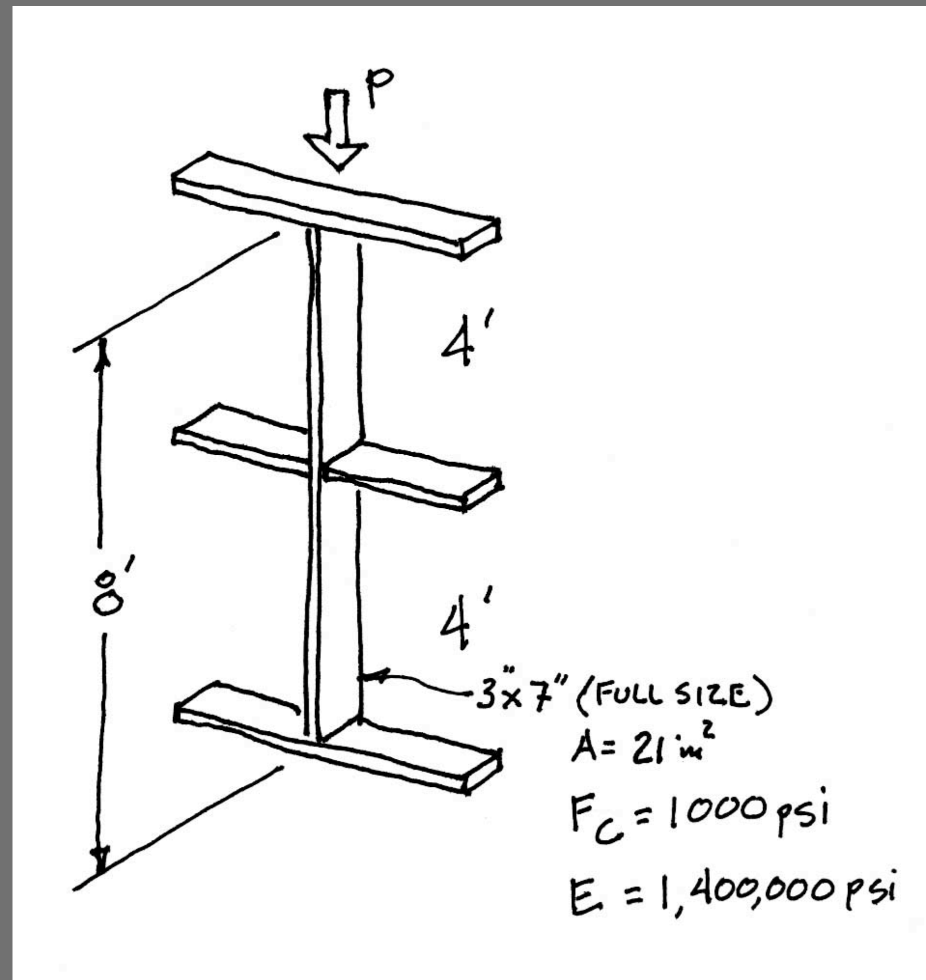
Data: section 3”x7” Full Dimension

F_c = 1000 psi

E = 1,400,000 psi

Find: $P_{critical}$ for buckling and crushing.

Determine the mode of failure for the wood column.

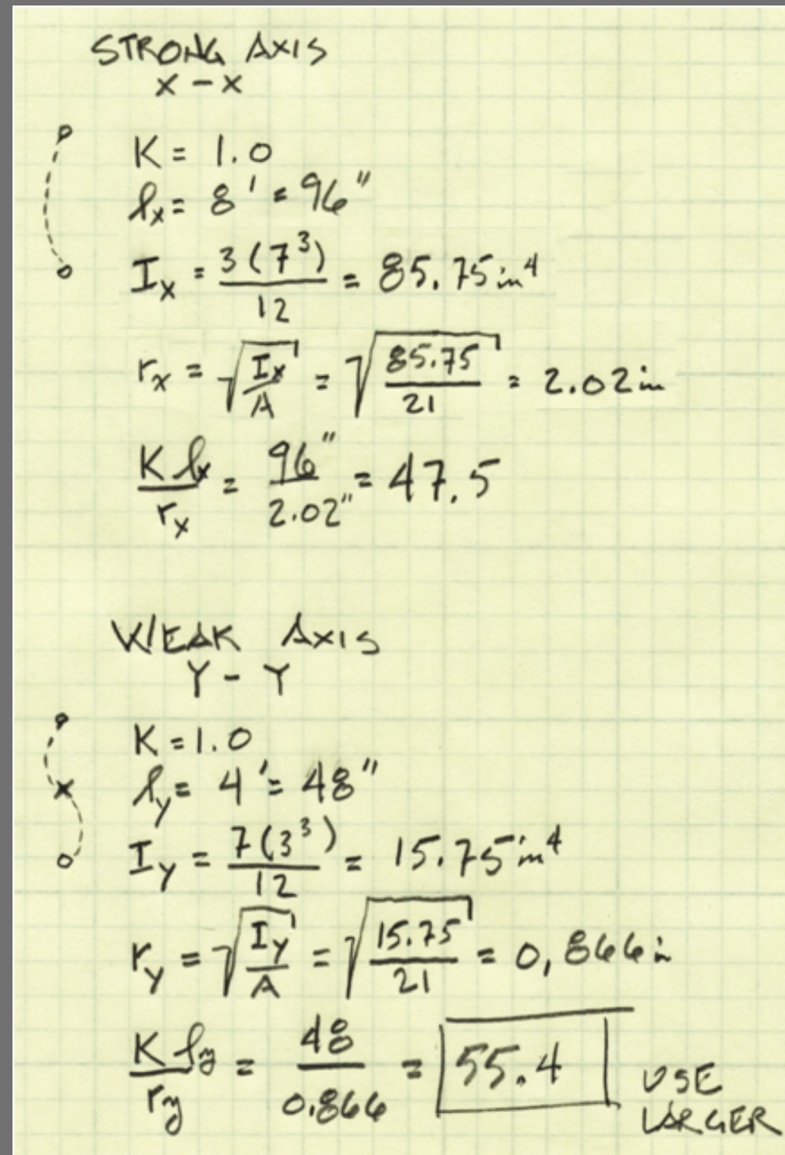


Example Problem : Analysis (cont.)

1. Calculate slenderness ratios for each axis.

The larger (more slender) controls.

2. Upper limits are usually given by codes.



Example Problem : Analysis (cont.)

3. Calculate critical Euler buckling load.

$$P_{cr} = \frac{\pi^2 A E}{\left(\frac{K\ell}{r}\right)^2} = \frac{\pi^2 (21)(1\,400\,000)}{(55.4)^2}$$

$$P_{cr} = 94\,500\ \#$$

5. Calculate crushing load.

$$P = FA = 1000(21) = 21\,000\ \#$$

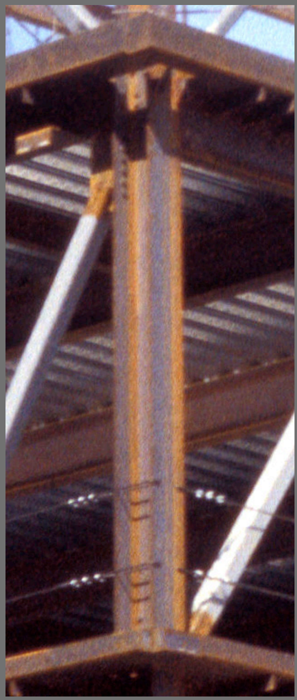
7. Smaller of the two will fail first and control.

$$21000 < 94500 \quad \therefore \quad \text{USE } 21\,000\ \#$$

Analysis of Steel Columns
by Engel

Data:
- Column – size, length
- Support conditions
- Material properties – F_y
- Applied load - P_{actual}

Required:
- $P_{actual} < P_{allowable}$



$$f_{cr} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$

1. Calculate slenderness ratios.
   The largest ratio governs.
2. Check slenderness ratio against upper limit of 200
3. Use the controlling slenderness ratio to find the critical Euler buckling stress, f_{cr}.
4. Apply some Factor of Safety (like 3) to f_{cr}.
5. Determine yield stress limit, F_y.
6. $F_{allowable}$ is the lesser stress: (f_{cr} / F.S.) or F_y
7. Compute allowable capacity: $P_{allowable} = F_{allow}$ A.
8. Check column adequacy:
   $P_{actual} < P_{allowable}$

Design of Steel Columns
by Engel

Data:

- Column – length
- Support conditions
- Material properties – F_y
- Applied load - P_{actual}

Required:

- Column – section

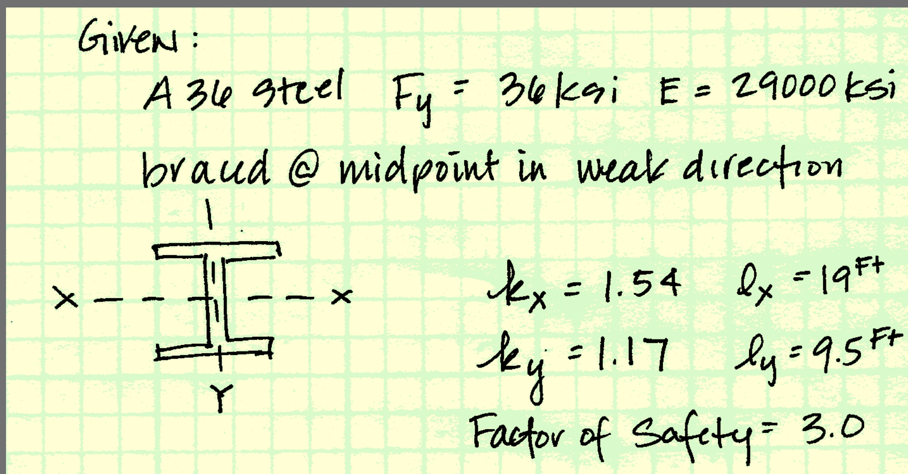
1. Use the Euler equation to solve for Ar^2 which is equal to I for both x and y axis.
2. Enter the section tables and find the least weight section that satisfies **BOTH** I_x and I_y.
3. Check the slenderness ratios are both < 200.
4. Calculate the actual Euler stress f_{cr} for the final section.
5. $F_{allowable}$ is the lesser stress: f_{cr} / F.S. or F_y
6. Compute allowable capacity: $P_{allowable} = F_{allow}$ A.

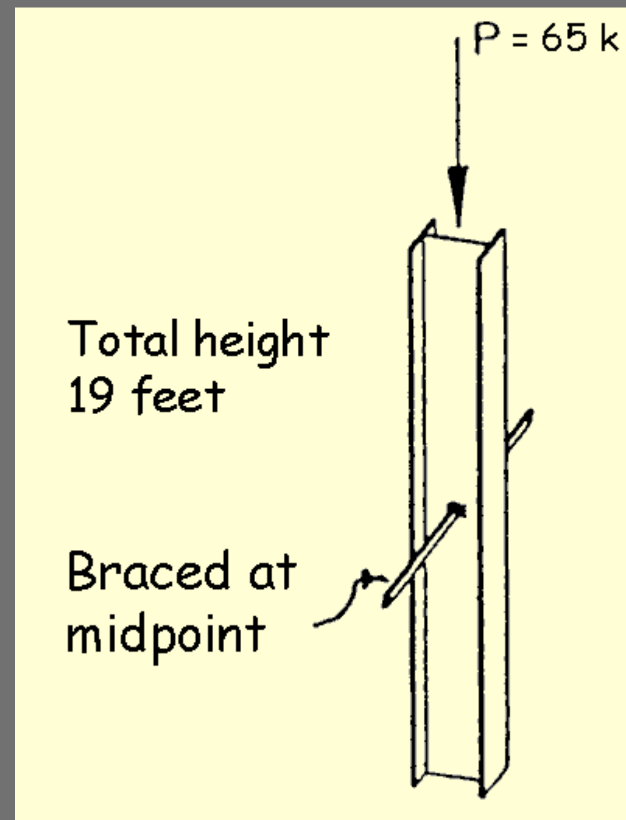
$$I_x = \frac{P(K_x l_x)^2}{\pi^2 E} \times F.S.$$

$$I_y = \frac{P(K_y l_y)^2}{\pi^2 E} \times F.S.$$

Example Problem : Design

Select a steel section that can carry the given load.





Example Problem : Design (cont.)

X-X (strong) Axis

$$I_x = \frac{P(k_x l_x)^2 (F.S.)}{\pi^2 E}$$

$$= \frac{(65^k)(1.54 \times 19^{ft} \times 12''/')^2 (3.0)}{\pi^2 (29000 \text{ ksi})}$$

$$I_x = 134 \text{ in}^4$$

Y-Y (weak) axis

$$I_y = \frac{P(k_y l_y)^2 (F.S.)}{\pi^2 E}$$

$$= \frac{(65^k)(1.17 \times 9.5^{ft} \times 12''/')^2 (3.0)}{\pi^2 (29000 \text{ ksi})}$$

$$I_y = 12.12 \text{ in}^4$$

Pick Section: **W10 x 26** $\quad I_x = 144 \text{ in}^4 \quad I_y = 14.075 \text{ in}^4$

W16 x 31 $\quad I_x = 144 \text{ in}^4 \quad I_y = 12.48 \text{ in}^4$

↑ deeper, heavier.

Select column that meets requirement: lightest weight

Example Problem : Design (cont.)

- Determine the controlling slenderness (larger controls)
- Find the actual buckling stress, fcr
- Compare to allowable stress, $F_{allowable}$ is lesser of :
  fcr/F.S. or Fy
- Determine safe allowable load, $P_{allowable} = F_{allowable}\ A$

SLENDERNESS RATIOS:

X-X

$$\frac{k l_x}{r_x} = \frac{1.54(19')(12''/')}{4.35''}$$

$80.7 < 200$ ✓

Y-Y

$$\frac{k_y l_y}{r_y} = \frac{1.17(9.5')(12''/')}{1.36}$$

$98.1 < 200$ ✓ OK

↑ MORE SLENDER CONTROLS

EULER STRESS

$$f_{cr} = \frac{\pi^2 E}{\left(\frac{kl}{r}\right)^2} = \frac{\pi^2(29000^{KSI})}{98.1^2} = 29.75\ KSI$$

$$F_{ALLOW} = f_{cr}/F.S. = 29.75/3.0 = 9.9\ KSI < F_y$$

∴ USE W10 x 26

$$P_{ALLOW} = F_{ALLOW} A = 9.9(7.61) = 75.3^K > 65^K\ ✓$$

Determining K factors by AISC

Sidesway Inhibited:
- Braced frame
- 1.0 > K > 0.5

Sidesway Uninhibited:
- Un-braced frame
- unstable > K > 1.0

If Ic/Lc is large
and Ig/Lg is small
The connection is more pinned

If Ic/Lc is small
and Ig/Lg is large
The connection is more fixed

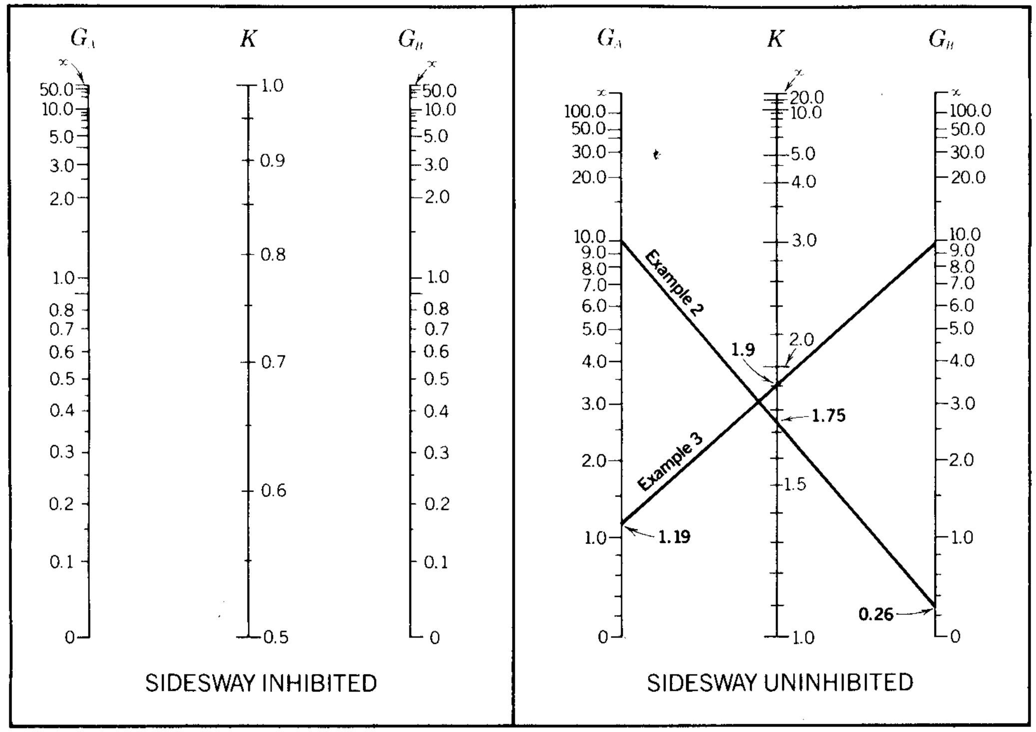


Figure 1. The subscripts A and B refer to the joints at the two ends of the column section being considered. G is defined as

SSRC CHARTS
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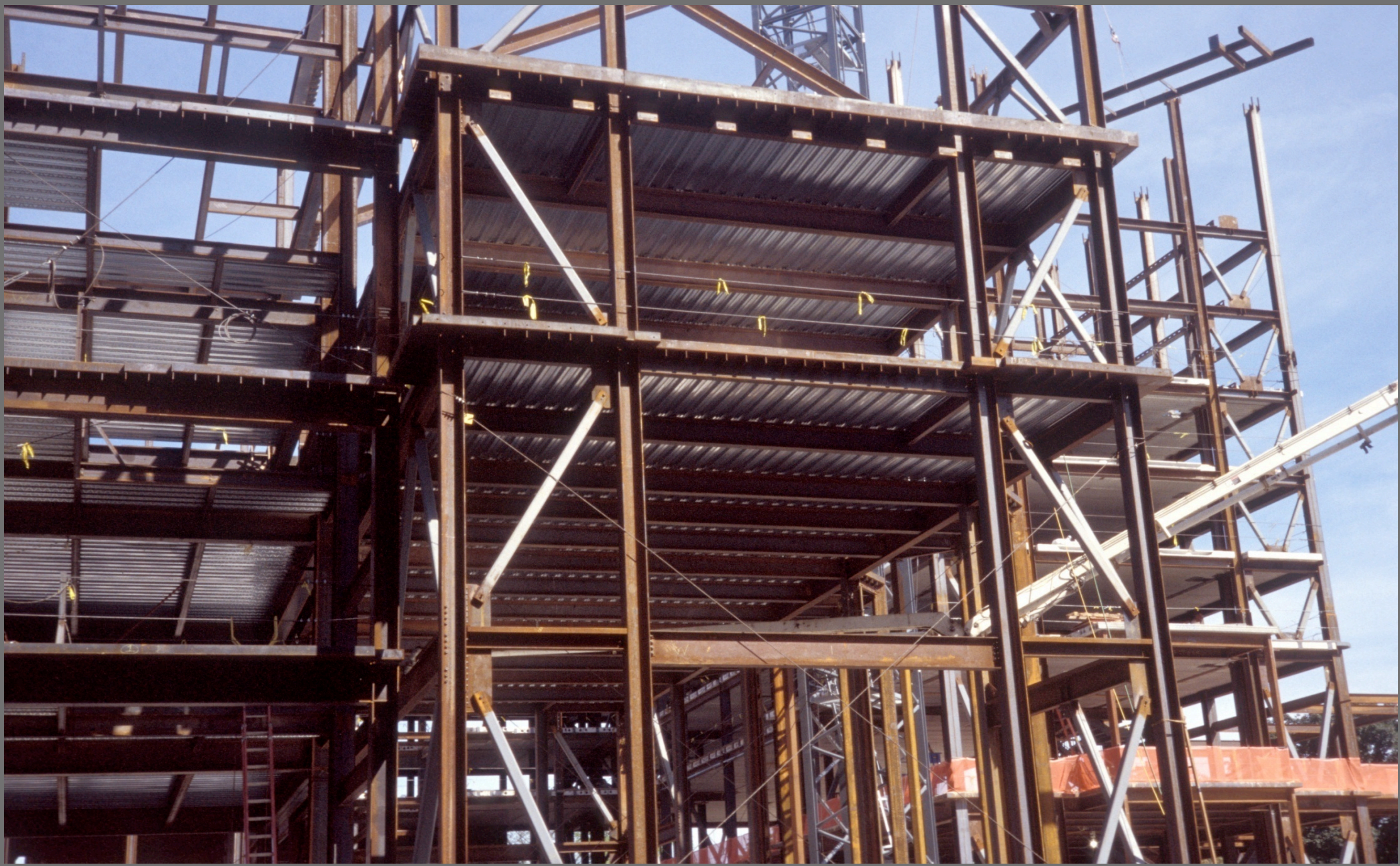
$$G = \frac{\Sigma (I_c/L_c)}{\Sigma (I_g/L_g)}$$

in which Σ indicates a summation of all members rigidly connected to that joint and lying in the plane in which buckling of the column is being considered. I_c is the moment of inertia and L_c the unsupported length of a column section, and I_g is the moment of inertia and L_g the unsupported length of a girder or other restraining member. I_c and I_g are taken about axes perpendicular to the plane of buckling being considered.

For column ends supported by but not rigidly connected to a footing or foundation, G is theoretically infinity, but, unless actually designed as a true friction free pin, may be taken as "10" for practical designs. If the column end is rigidly attached to a properly designed footing, G may be taken as 1.0. Smaller values may be used if justified by analysis.

Source: American Institute of Steel Construction, Manual of Steel Construction, AISC 1980

Steel Frame Construction



Analysis of Steel Columns by AISC-ASD

Data:

- Column – size, length
- Support conditions
- Material properties – F_y
- Applied load - P_{actual}

Required:

- $P_{actual} < P_{allowable}$

1. Calculate slenderness ratios. largest ratio governs.
2. In AISC Table look up F_a for given slenderness ratio.
3. Compute: $P_{allowable} = F_a \; A$.
4. Check column adequacy: $P_{actual} < P_{allowable}$

Table C-36
Allowable Stress
For Compression Members of 36-ksi Specified Yield Stress Steel[a]

$F_y = 36$ ksi

Kl/r	F_a (ksi)	Kl/r	F_a (ksi)	Kl/r	F_a (ksi)	Kl/r	F_a (ksi)	Kl/r	F_a (ksi)
1	21.56	41	19.11	81	15.24	121	10.14	161	5.76
2	21.52	42	19.03	82	15.13	122	9.99	162	5.69
3	21.48	43	18.95	83	15.02	123	9.85	163	5.62
4	21.44	44	18.86	84	14.90	124	9.70	164	5.55
5	21.39	45	18.78	85	14.79	125	9.55	165	5.49
6	21.35	46	18.70	86	14.67	126	9.41	166	5.42
7	21.30	47	18.61	87	14.56	127	9.26	167	5.35
8	21.25	48	18.53	88	14.44	128	9.11	168	5.29
9	21.21	49	18.44	89	14.32	129	8.97	169	5.23
10	21.16	50	18.35	90	14.20	130	8.84	170	5.17
11	21.10	51	18.26	91	14.09	131	8.70	171	5.11
12	21.05	52	18.17	92	13.97	132	8.57	172	5.05
13	21.00	53	18.08	93	13.84	133	8.44	173	4.99
14	20.95	54	17.99	94	13.72	134	8.32	174	4.93
15	20.89	55	17.90	95	13.60	135	8.19	175	4.88
16	20.83	56	17.81	96	13.48	136	8.07	176	4.82
17	20.78	57	17.71	97	13.35	137	7.96	177	4.77
18	20.72	58	17.62	98	13.23	138	7.84	178	4.71
19	20.66	59	17.53	99	13.10	139	7.73	179	4.66
20	20.60	60	17.43	100	12.98	140	7.62	180	4.61
21	20.54	61	17.33	101	12.85	141	7.51	181	4.56
22	20.48	62	17.24	102	12.72	142	7.41	182	4.51
23	20.41	63	17.14	103	12.59	143	7.30	183	4.46
24	20.35	64	17.04	104	12.47	144	7.20	184	4.41
25	20.28	65	16.94	105	12.33	145	7.10	185	4.36
26	20.22	66	16.84	106	12.20	146	7.01	186	4.32
27	20.15	67	16.74	107	12.07	147	6.91	187	4.27
28	20.08	68	16.64	108	11.94	148	6.82	188	4.23
29	20.01	69	16.53	109	11.81	149	6.73	189	4.18
30	19.94	70	16.43	110	11.67	150	6.64	190	4.14
31	19.87	71	16.33	111	11.54	151	6.55	191	4.09
32	19.80	72	16.22	112	11.40	152	6.46	192	4.05
33	19.73	73	16.12	113	11.26	153	6.38	193	4.01
34	19.65	74	16.01	114	11.13	154	6.30	194	3.97
35	19.58	75	15.90	115	10.99	155	6.22	195	3.93
36	19.50	76	15.79	116	10.85	156	6.14	196	3.89
37	19.42	77	15.69	117	10.71	157	6.06	197	3.85
38	19.35	78	15.58	118	10.57	158	5.98	198	3.81
39	19.27	79	15.47	119	10.43	159	5.91	199	3.77
40	19.19	80	15.36	120	10.28	160	5.83	200	3.73

[a]When element width-to-thickness ratio exceeds noncompact section limits of Sect. B5.1, see Appendix B5.
Note: $C_c = 126.1$

AMERICAN INSTITUTE OF STEEL CONSTRUCTION

Source: American Institute of Steel Construction, Manual of Steel Construction, AISC 1980

Design of Steel Columns with AISC-ASD Tables

Data:

- Column – length
- Support conditions
- Material properties – F_y
- Applied load - P_{actual}

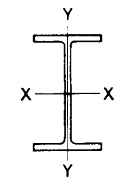
Required:

- Column Size

1. Enter table with height.
2. Read allowable load for each section to find the smallest adequate size.
3. Tables assume weak axis buckling. If the strong axis controls the length must be divide by the ratio r_x/r_y
4. Values stop in table (black line) at slenderness limit, KL/r = 200

F_y = 36 ksi
F_y = 50 ksi

COLUMNS
W shapes
Allowable axial loads in kips



Designation		W8				W6					
Wt./ft		28		24		25		20		15	
F_y		36	50	36	50	36	50	36	50	36†	50†
Effective length in ft KL with respect to least radius of gyration r_y	0	178	248	153	212	159	220	127	176	96	133
	6	155	208	133	178	136	182	109	145	81	108
	7	150	198	129	170	131	173	105	137	78	102
	8	144	188	124	161	126	163	100	129	75	96
	9	138	178	118	152	120	152	95	121	71	89
	10	132	166	113	142	114	141	90	112	67	82
	11	125	154	107	132	107	129	85	102	62	74
	12	118	142	101	121	100	117	79	92	58	66
	13	111	128	95	109	93	103	73	81	53	57
	14	103	114	88	97	85	90	67	70	48	49
	15	95	100	81	85	77	78	60	61	43	43
	16	87	88	74	74	69	69	54	54	38	38
	17	78	78	66	66	61	61	47	47	33	33
	18	69	69	59	59	54	54	42	42	30	30
	19	62	62	53	53	49	49	38	38	27	27
	20	56	56	48	48	44	44	34	34	24	24
	22	46	46	39	39	36	36	28	28	20	20
	24	39	39	33	33	31	31	24	24	17	17
	25	36	36	30	30	28	28	22	22		
	26	33	33	28	28						
	27	31	31								
Properties											
U		3.23	3.23	3.27	3.27	2.38	2.07	2.43	1.86	1.93	1.45
P_{wo} (kips)		48	67	39	54	47	65	35	49	26	36
P_{wi} (kips/in.)		10	14	9	12	12	16	9	13	8	12
P_{wb} (kips)		93	110	59	70	170	200	91	107	63	74
P_{fb} (kips)		49	68	36	50	47	65	30	42	15	21
L_c (ft)		6.9	5.9	6.9	5.8	6.4	5.4	6.4	5.4	6.3	5.4
L_u (ft)		17.5	12.6	15.2	10.9	20.0	14.4	16.4	11.8	12.0	8.7
A (in.^{2})		8.25		7.08		7.34		5.87		4.43	
I_x (in.^{4})		98.0		82.8		53.4		41.4		29.1	
I_y (in.^{4})		21.7		18.3		17.1		13.3		9.32	
r_y (in.)		1.62		1.61		1.52		1.50		1.45	
Ratio r_x/r_y		2.13		2.12		1.78		1.77		1.77	
B_x Bending factors		0.340		0.339		0.440		0.438		0.456	
B_y Bending factors		1.244		1.258		1.308		1.331		1.424	
$a_x/10^6$		14.63		12.34		7.97		6.19		4.33	
$a_y/10^6$		3.23		2.73		2.53		1.97		1.39	
$F'_{ex}(K_xL_x)^2/10^2$ (kips)		123		121		75.9		73.1		68.3	
$F'_{ey}(K_yL_y)^2/10^2$ (kips)		27.2		26.9		24.0		23.3		21.8	

†Flange is noncompact; see discussion preceding column load tables.
Note: Heavy line indicates *Kl/r* of 200.

Source: American Institute of Steel Construction, Manual of Steel Construction, AISC 1980