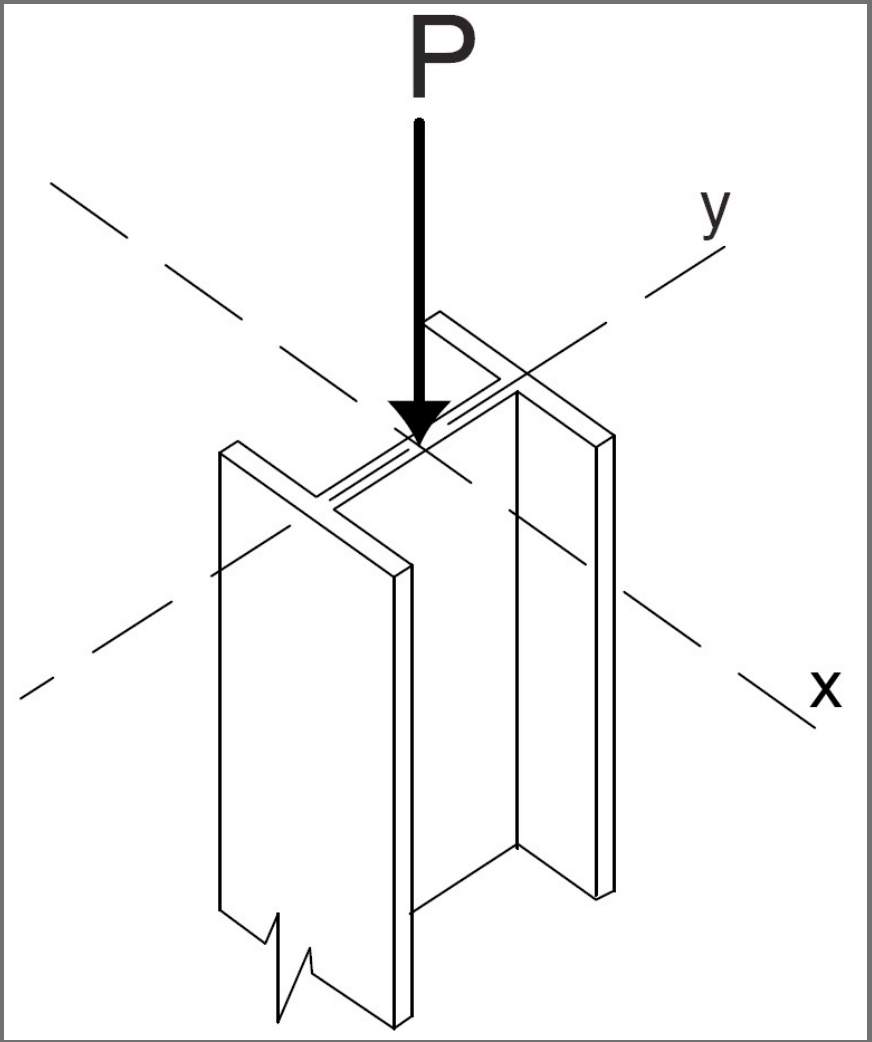
Combined Stress

- Axial vs. Eccentric Load
- Combined Stress
- Interaction Formulas

Source: Caspar David Friedrich from “Man und Frau den Mond betrachtend” 1830-35
Alte Nationalgalerie, Berlin

Axial Stress

- Loads pass through the centroid of the section
- Member is straight
- Load less than buckling load
- **f = P/A**



Source: University of Michigan, Department of Architecture

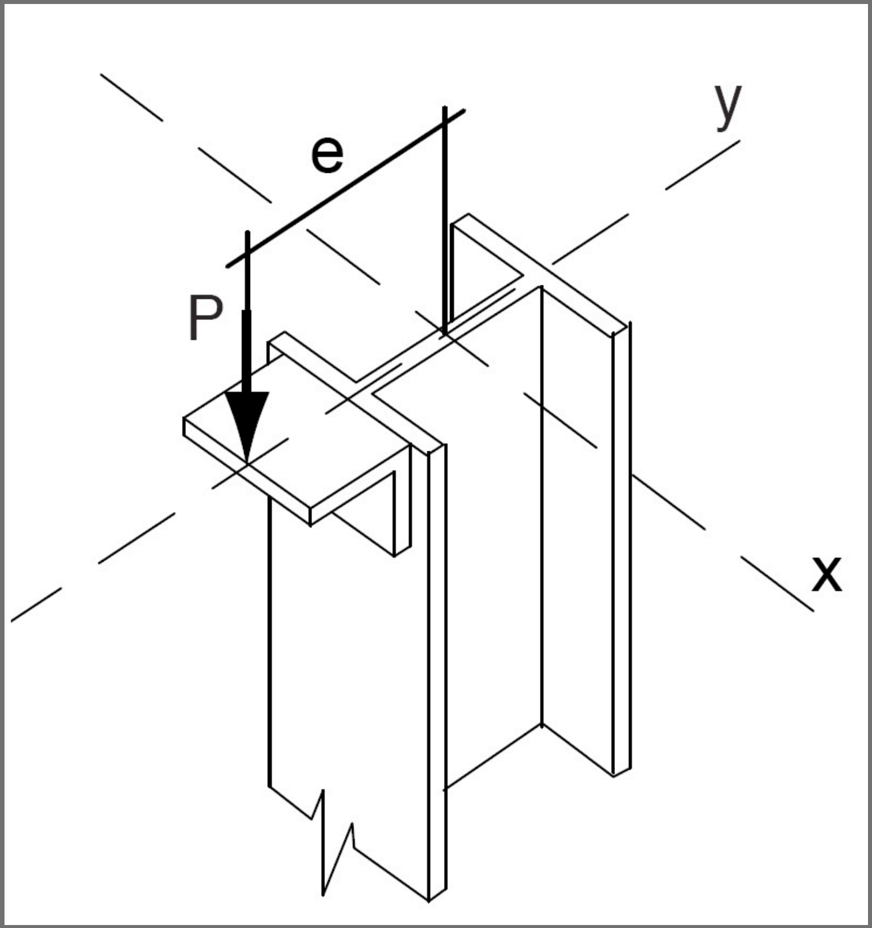
Eccentric Loads

- Load offset from centroid
- M = P e
- Total load = P + M

Interaction formula

$$f = \frac{P}{A} \pm \frac{Mc}{I}$$

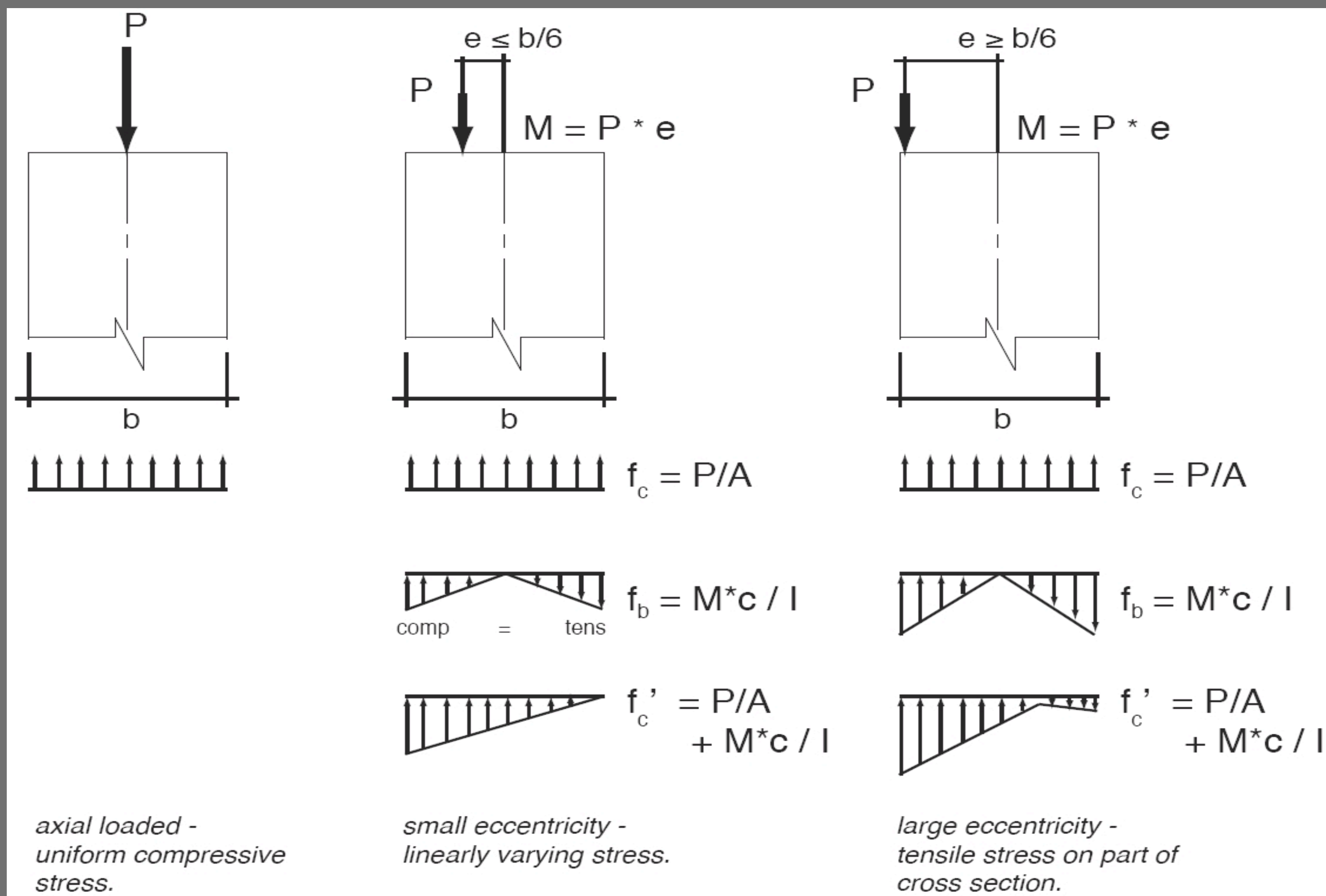
$$\frac{f_a}{F_a} \pm \frac{f_b}{F_b} \leq 1.0$$



Source: University of Michigan, Department of Architecture

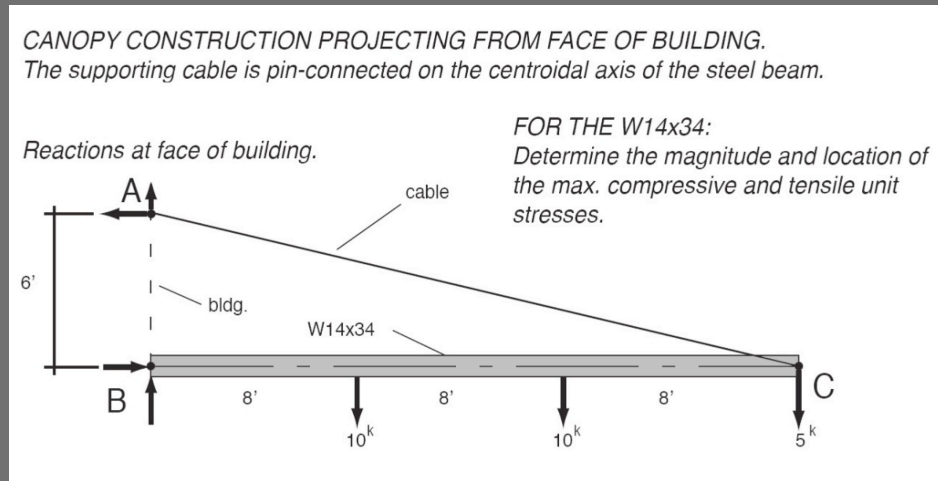
Combined Stress

- Stresses combine by superposition
- Values add or subtract by sign



Source: University of Michigan, Department of Architecture

Example



Source: University of Michigan, Department of Architecture

Determine external reactions

$$\Sigma M_A = 0 = -B_H 6' + 10^k(8') + 10^k(16') + 5^k(24')$$

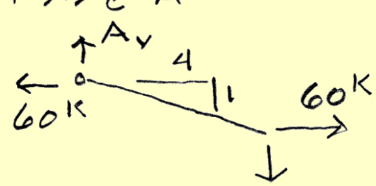
$$B_H = 60^k$$

$$\Sigma M_B = 0 = -A_H(6') + 10^k(8') + 10^k(16') + 5^k(24')$$

$$A_H = 60^k$$

CHECK $\Sigma F_H = 0 = 60^k - 60^k$ ✓

FBD @ A



$$\frac{60}{4} : \frac{A_V}{1}$$

$$A_V = 15^k$$

$$\Sigma F_V = 0 = 15^k - 10^k - 10^k - 5^k + B_V$$

$$B_V = 10^k$$

Example

Determine internal member forces: Axial and Flexural

Determine axial and flexural stresses

Use interaction formula to determine combined stresses at key locations (e.g. extreme fibers)

W14x34 $A = 10.0\ in^2$

$S_x = 48.6\ in^3$

FORCE:

$AXIAL = 60^K$

$FLEXURAL = M = PL/3 = 10^K(8') = 80^{'-K}$

STRESS:

$AXIAL = f_a = \frac{P}{A} = 60^K / 10 in^2 = 6.0\ KSI$

$FLEXURAL: f_b = M/S = 80^{'-K}(12)/48.6 in^3 = 19.75\ KSI$

COMBINED STRESS

TOP SIDE:

$f_a + f_b = 6.0 + 19.75 = 25.75\ KSI$ (COMP)

BOTTOM SIDE:

$f_a - f_b = 6.0 - 19.75 = -13.75\ KSI$ (TENS)

Rafters Flexure + Axial

$$f = \frac{P}{A} \pm \frac{Mc}{I}$$

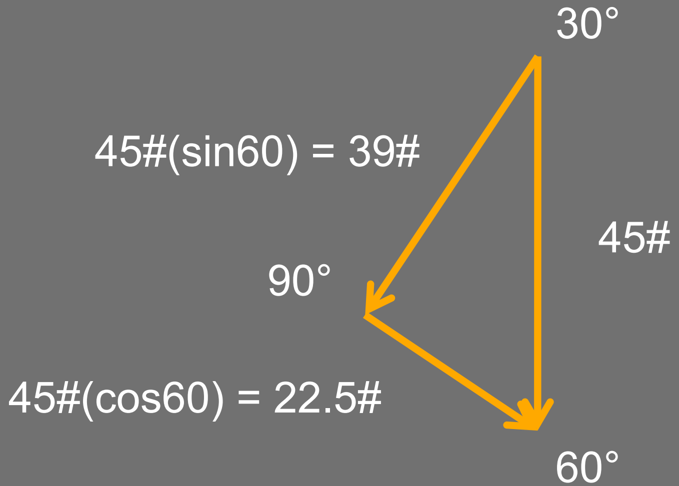
P = 22.5 #
A = 5.25 in^2
f_c = 22.5 / 5.25 = 4.28 psi

M = PL/4 = 39# x 96″ /4 = 936 ″- #
c = 1.5″ /2 = 0.75″
I_y = 3.5(1.5^3)/12 = 0.984 in^4
f_b = 963 (0.75)/0.984 = 713.14

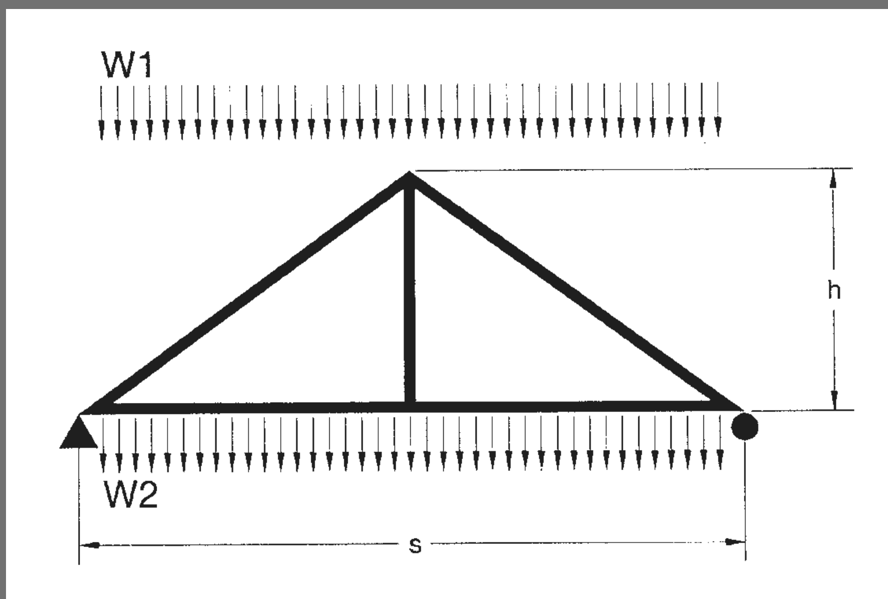
f = fc + fb = 717.4 psi

Or

M = PL/4 = 45# x 83.14″ /4 = 936 ″- #
f_b = 963 (0.75)/0.984 = 713.14

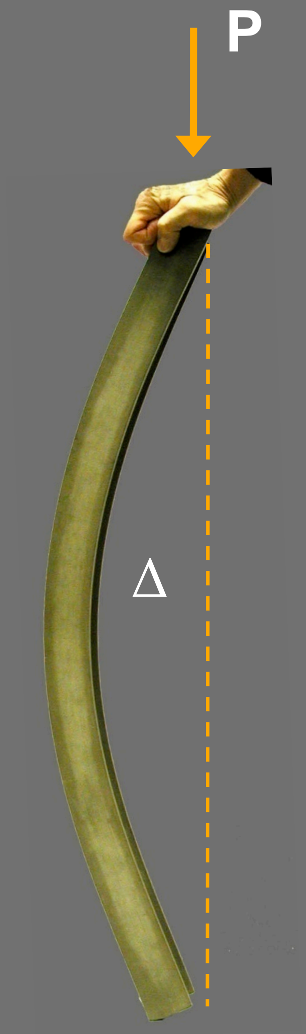


Other Examples



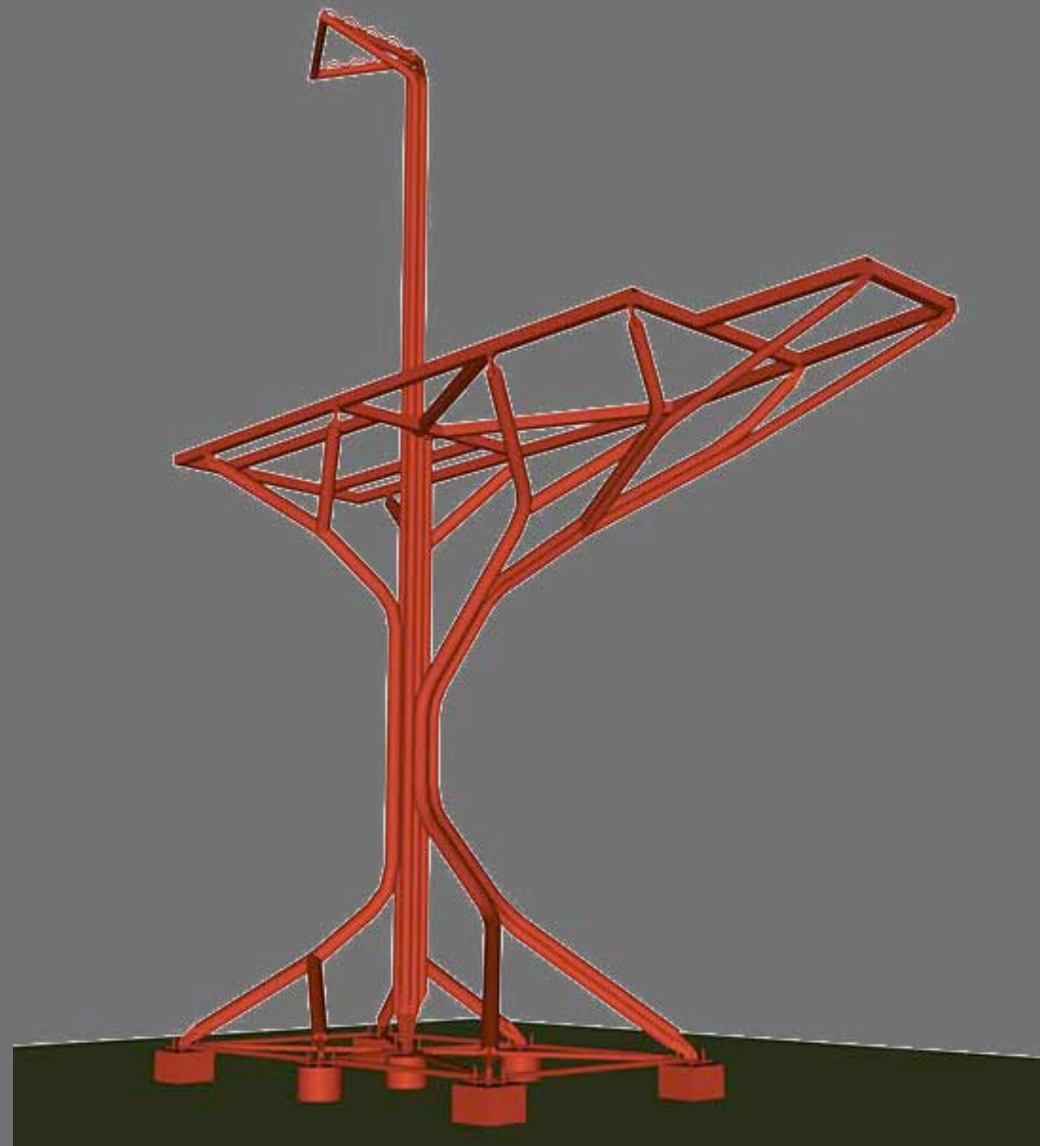
Second Order Stress
"P Delta Effect"

1. Eccentric load causes bending moment
2. Bending moment causes deflection, Δ
3. P x Δ causes additional moment
4. The additional PΔ moment causes additional deflection, Δ_1



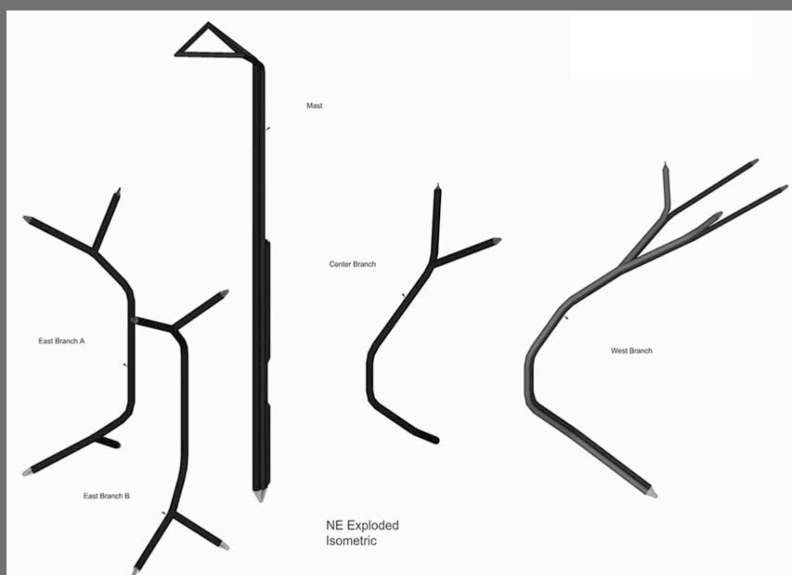
Treehouse

- Form Exploration



Treehouse

- Details
  - Pinned at ends
  - Radius bent at joints
  - ‘Continuous’ sections
  - Welded seams



Treehouse

- Details
  - Pinned at ends
  - Radius bent at joints
  - ‘Continuous’ sections
  - Welded seams

Treehouse

- Erection
  - 3 sections
  - Trucked to site
  - Erected with mobile crane
  - One day process

Treehouse

- Erection
  - 3 sections
  - Trucked to site
  - Erected with mobile crane
  - One day process

Treehouse

- Finished

Treehouse

- Finished

Treehouse

- Finished

Treehouse

- Finished