2008-09

SI 563 - Game Theory, Fall 2008

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<http://hdl.handle.net/2027.42/64940>
http://hdl.handle.net/2027.42/64940
SI 563 Lecture 1

Introduction and Representation of Games

Professor Yan Chen
Fall 2008
Agenda

- Game Theory
  » History and applications
  » Definitions and overview
- Representation: Extensive forms
- Strategies
- Representation: Normal forms
Introduction

Game Theory and Applications
(Watson Chapter 1)
It’s Your Move
The ESP Game: Labeling the Web

10 MILLION LABELS COLLECTED

The ESP Game
As seen on CNN and newspapers around the world!

top scores

HOW TO PLAY

New to the ESP Game?
Sign up for FREE!

Already have an account?

Screen Name: ____________________________

Password: ____________________________

Sign In

Did you know?
The ESP Game is helping to label all images on the Web!

Play our new game
 Peekaboom

Terms of Service | FAQ | ESP Image Search | Contact Us | Credits

Funded in part by the National Science Foundation (NSF).
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THE ESP GAME

TWO-PLAYER ONLINE GAME

PARTNERS DON’T KNOW EACH OTHER AND CAN’T COMMUNICATE

OBJECT OF THE GAME:
TYPE THE SAME WORD

THE ONLY THING IN COMMON IS AN IMAGE
THE ESP GAME

PLAYER 1

GUESSING: CAR

GUESSING: HAT

GUESSING: KID

SUCCESS!

YOU AGREE ON CAR

PLAYER 2

GUESSING: BOY

GUESSING: CAR

SUCCESS!

YOU AGREE ON CAR

PICTURE BY: anyjazz65 (flickr)
http://creativecommons.org/licenses/by/2.0/deed.en
What is a game?

• A game is being played whenever people interact with each other
  – Bidding in an auction
  – Pricing: amazon.com
  – Adoption of a new standard
  – Cuban missile crisis

• Interdependence
  – One person’s behavior affect another’s well-being

• What is not a game?
  – N=1: monopoly
  – N= infinity: perfect competition
Three Major Tensions of Strategic Interaction

• Game theory: a theory of strategic interaction
  – Conflict
  – Cooperation

• Three major tensions
  – Conflict between individual and group interests
  – Strategic uncertainty
  – Insufficient coordination
Game Theory: A Short History

• Cournot (1838) and Edgeworth (1881)
• Zermelo (1913): chess-like games can be solved in a (large!) finite number of moves
• von Neumann and Morgenstern (1944)
• Nash, Harsanyi, Selten: 1994 Nobel Prize for solution concepts in non-cooperative game theory
• Aumann and Schelling: 2005 Nobel Prize for game theoretic analysis of conflict and cooperation
Noncooperative vs. Cooperative Game Theory

• Noncooperative game theory
  – Individual decision making
  – Group decision making: specify procedures leading individual decisions to group outcomes
  – Solution concepts: prescriptions and predictions about the outcomes of games

• Cooperative game theory
  – Model joint actions
• Game theory has been applied to sociology, economics, political science, decision theory, law, evolutionary biology, experimental psychology, military strategy, anthropology …

• School of information
  – Incentive-centered design
  – Information policy
  – Social computing
  – HCI and CSCW
  – ARM and LIS
Representing Games

An Overview
Representing Games

- A list of players
- A complete description of what players can do
- A description of what the players know when they act
- A specification of how player actions lead to outcomes
- A specification of player preferences over outcomes
Extensive- and Normal-Form Games

• Two basic types of interactions
  – Sequential: players make alternating moves
  – Simultaneous: players act at the same time
• In most cases interactions are partly sequential and partly simultaneous
• Can be modeled in two ways
  – Extensive-form games
  – Normal-form games
SI 563 Overview

• Games of complete information
  – Normal form games: Nash equilibrium
  – Extensive form games: SPNE
    » Static
    » Repeated

• Games of incomplete information
  – Normal form games: Bayesian Nash equilibrium
  – Extensive form games: perfect Bayesian equilibrium
Representing Games

The Extensive Form

(Watson Chapter 2)
• Set of players
  • CB
  • L
• Set of strategies
  • CB: \{accept, reject\}
  • L: \{pull, not pull\}
• Sequence of actions
• Outcomes
  • CB falls
  • CB kicks the ball
  • Nothing happens

Link to football Peanuts comic:
http://comics.com/peanuts/1952-11-16/
Extensive Form Representation

Charlie Brown falls down

Pull the ball

Accept

Let him kick

Charlie Brown kicks the ball

Reject

Nothing happens

Diagram courtesy: Dr. Tayfun Sönmez
A game tree consists of:

• A series of nodes linked in a sequence
  – Non-terminal node: not an endpoint
  – Terminal node: indicates that game is over

• Branches represent actions

Note: loops (i.e. cycles) are not allowed in game trees.
Two Crucial Elements of Extensive-Form Games

- **Timing** of actions that players may take
- **Information** they have when they must take those actions
  - Information sets
Example: the Bug Game

• A tale of two films (1998)
  – Disney: A bug’s life
  – Dreamwork: Antz

• A model
  – Set of players
    » Jeffrey Katzenberg
    » Michael Eisner (Disney CEO)
  – Set of actions for each player, etc.
Building an extensive form: Katzenberg’s first move

Initial node

K

Stay

Leave
Adding the production decisions

- Produce Antz
- Produce A Bug’s Life
- Not
- Not

K

a

b

K

E

Not

Leave

Stay
Capturing lack of information

- Produce
  - Antz
- A Bug’s Life
- Leave
- b
- E
- Not
- d
- K
- c
- Not
- a
- Stay
- K
- a
- Leave
- b
- E
- Not
- d
- K
- c
- Not

Produce
Antz
A Bug’s Life
Leave
b
E
Not
d
K
c
Not
a
Leave
b
E
Not
d
K
K
a
Leave
b
E
Not
d
K
K
Information Sets

• Information sets summarize a player’s knowledge of prior moves when she must decide.

• If there are more than one nodes in an information set, a player knows that she is in one of the nodes in the information set (but does not know which one).

• Information sets containing only one node are referred to as singletons.
Adding terminal nodes

Initial node

Stay

Leave

Produce A Bug’s Life

Produce Antz

Release early

Not

Terminal modes

Not
The Full Extensive Form

- **Leave**
  - **A Bug’s Life**
    - Produce
      - **K**
        - Release early: 40,110
      - Not
        - **Antz**
          - Produce
            - **K**
              - 13,120
            - Not
              - **e**
                - 0,140
          - Not
            - **d**
              - **Antz**
                - 80,0
              - Not
                - **0,0**
    - Not
      - **35,100**
- **Stay**
  - **K**
    - 0,0
A more compact representation

Labeling branches:
- Differentiate between N and N'
- Conformity within an information set
Example: Cuban Missile Crisis

• Why did the Soviet Union attempt to place offensive missiles in Cuba?
• Why did US respond with a blockade of Cuba?
• Why did the Soviet Union decide to withdraw the missiles?
A Simple Model of the Contest

• Set of players
  – Challenger: player CH
  – Defender: player D

• Preferences
  – Challenger (best to worst)
    » Concession
    » Status quo
    » Back down
    » war
  – Defender
    » Backdown
    » Status quo
    » Concession
    » war
Diagram courtesy: Dr. Tayfun Sönmez
Adding Uncertainty

• If there is uncertainty, we model this by adding Nature (or Chance) as another player
  – It does not have payoffs
  – It chooses different types

• Example: two types of Defenders
  – Resolute type: prefers War to Concession
  – Irresolute type: prefers Concession to War
If Challenger can observe Defender’s type

Diagram courtesy: Dr. Tayfun Sönmez
If Challenger can’t observe Defender’s type:

Diagram courtesy: Dr. Tayfun Sönmez
Example: Rock, Paper, Scissors

- Simultaneous move game
- Normal-form representation:

```
<table>
<thead>
<tr>
<th></th>
<th>Rock</th>
<th>Paper</th>
<th>Scissors</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROCK</td>
<td>0,0</td>
<td>-1,1</td>
<td>1,-1</td>
</tr>
<tr>
<td>PAPER</td>
<td>1,-1</td>
<td>0,0</td>
<td>-1,1</td>
</tr>
<tr>
<td>SCISSORS</td>
<td>-1,1</td>
<td>1,-1</td>
<td>0,0</td>
</tr>
</tbody>
</table>
```

Diagram courtesy: Dr. Tayfun Sönmez
Rock, Paper, Scissors: Extensive Forms

• Sequential moves

• Simultaneous moves

Diagrams courtesy: Dr. Tayfun Sönmez
What if 2 can observe if 1 chooses Rock, but not otherwise?

Diagram courtesy: Dr. Tayfun Sönmez
Example: The Truth Game

- An uneven coin: Heads 80% of the times
- Two players: 1 and 2
- Player 1 flips the coin and observes the results
- Player 1 announces H or T
- Player 2 hears 1’s announcement but cannot observe results of the actual coin flip. 2 announces h or t
- Payoffs
  - 2 receives $10 if answer is true, $0 otherwise
  - 1 receives $20 if 2 announces heads, and an additional $10 if 1 tells the truth about the coin flip
**Payoffs**

- **2** receives $10 if answer is true, $0 otherwise
- **1** receives $20 if 2 announces heads, and an additional $10 if 1 tells the truth

Diagram courtesy: Dr. Tayfun Sönmez
Example: Advertising/Exit

(a) 2 observes 1’s actions:

Firm 1: how much to spend on advertising, [0, $1 million]
Example: Ultimatum Bargaining

Player 1 wishes to sell a painting to player 2. Painting is worth nothing to player 1, 100 to player 2. Seller makes a take-it-or-leave-it offer. If buyer accepts the price, trade at this price. Otherwise, both parties obtain nothing.
Definition: an n-person extensive form game consists of:

- A finite game tree composed of nodes and branches
- A division of nodes over players, chance, and endpoints
- Probability distribution for each chance move
- A division of each player’s nodes into information sets
- A set of outcomes and an outcome to each endpoint
- A payoff (or utility) function for each player over all outcomes

All this is common knowledge to all players
Strategies

(Watson Chapter 3)
• A strategy is a complete contingent plan for a player in the game
  – Complete contingent: describes what she will do at each of her information sets

• Writing strategies for a player i:
  – Find every information set for player i
  – At each information set, find all actions
  – Find all combinations of actions at these information sets
Example: Exit Decisions
(1 info set per player)

Firm 1: Aggressive (A), Passive (P) or Out (O)
Firm 2: Aggressive (A) or Passive (P)

Strategy Sets:
Firm 1: \( S_1 = \{A, P, O\} \)
Firm 2: \( S_2 = \{A, P\} \)
Exercise: finding strategies

1. Find number of Information sets for Players 1 and 2;
2. Find number of actions at each information set;
3. Write down the strategy set for each player.
More Exercises:

(a) $S_1 = \{U, D\}$
$S_2 = \{AC, AE, BC, BE\}$
$S_3 = \{RP, RQ, TP, TQ\}$

(b) $S_1 = \{AW, BW, CW, AZ, BZ, CZ\}$
$S_2 = \{X, Y\}$
Representing Games

The Normal Form
(Watson Chapter 3)
The Normal (Strategic) Form

• A game in normal form consists of
  – A set of players, \(\{1, 2, \ldots, n\}\)
  – Strategy spaces for the players, \(S_1, S_2, \ldots, S_n\)
  – Payoff functions for the players, \(u_1, u_2, \ldots, u_n\)

• Compared to the extensive form, normal form can be
  – More compact
  – For each extensive form, there exists an equivalent normal form representation
**Classic Normal-Form Games**

- **Example: Prisoners’ Dilemma**
  - **Set of players:** \( N = \{\text{Conductor, Tchaikovsky}\} \)
  - **Timing:** simultaneous move
  - **Set of strategies:** \( S_i = \{\text{Confess, Not Confess}\} \)
  - **Set of payoffs:**
    - If one confesses, the other does not: 0, 15 years in jail
    - If both confess: each gets 5 years in jail
    - If neither confess: each gets 1 year in jail
PD: Write down the extensive form representation
Example: Prisoners’ Dilemma

<table>
<thead>
<tr>
<th></th>
<th>Confess</th>
<th>Not Confess</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Conductor</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Confess</td>
<td>-5, -5</td>
<td>0, -15</td>
</tr>
<tr>
<td>Not Confess</td>
<td>-15, 0</td>
<td>-1, -1</td>
</tr>
</tbody>
</table>

Tchaikovsky
Classical Games: Matching Pennies

Zero-sum game: sum of payoffs in each cell is zero
## Classic Game: Coordination

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1, 1</td>
<td>0, 0</td>
</tr>
<tr>
<td>2</td>
<td>0, 0</td>
<td>1, 1</td>
</tr>
</tbody>
</table>

Coordination: want to use the same strategy, (A, A) or (B, B)
Example: traffic rules
Coordination: want to select the same strategy; Prefer to coordinate on A rather than on B.
**Coordination game:** want to go to an event together, with slightly different preferences
### Classic Game: Hawk-Dove/Chicken

<table>
<thead>
<tr>
<th></th>
<th>H</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>0, 0</td>
<td>3, 1</td>
</tr>
<tr>
<td>D</td>
<td>1, 3</td>
<td>2, 2</td>
</tr>
</tbody>
</table>

Coordination game: want to take different strategies
### Classic Game: Pigs

<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>4, 2</td>
<td>2, 3</td>
</tr>
<tr>
<td>S</td>
<td>6, -1</td>
<td>0, 0</td>
</tr>
</tbody>
</table>

**D:** dominant pig  
**S:** submissive pig
Corresponding extensive and normal forms
Beliefs, Mixed Strategies, and Expected Payoffs

(Watson Chapter 4)
Beliefs

• A player’s assessment about the strategies of the others in the game

• Representing beliefs
  – Probabilities
  – Normal form games:
    » probability distribution over the strategies of the other players
    » Example: Prisoner’s Dilemma
### Example: Prisoners’ Dilemma

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</tr>
<tr>
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<td>-5, -5</td>
<td>0, -15</td>
</tr>
<tr>
<td>Not Confess</td>
<td>-15, 0</td>
<td>-1, -1</td>
</tr>
</tbody>
</table>

Tchaikovsky's expected payoff from “Confess”

\[=0.25(-5)+0.75(0) = -1.25\]
Example: Prisoners’ Dilemma

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Conductor

Tchaikovsky
Highlights

• What is a game?

• What is a strategy?

• Key concepts
  – Extensive form
  – Normal form
Homework Assignment

- Chapter 2: #1, 2, 5
- Chapter 3: #2, 3