2008-09

SI 563 - Game Theory, Fall 2008

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http://hdl.handle.net/2027.42/64940
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Bargaining

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Fall 2008

Some material in this lecture drawn from http://gametheory.net/lectures/level.pl
Bargaining Problems

(Watson Chapter 18)
Bargaining: Value Creation and Division

• Value creation
  – Trade creates value
  – Gains from trade

• Value division
  – Parties jointly decide how to divide the value
  – Bargaining strengths
  – Negotiation procedures
  – Greater contracting environment
• Example: partnership formation
  – Players 1, 2
  – If form partnership, payoff vector (4, 6)
  – If not, payoff vector (2, 2)

• Bargaining set: set of alternatives for a given bargaining problem
  \[ V = \{(4, 6), (2, 2)\} \]

• Default outcome (or disagreement point)
  \[ d = (2, 2) \]
• Monetary transfer, $t$
• Outcome, $z$
  $z=1$: forming partnership
  $z=0$: no partnership
• Transferable utility
  $u_1 = v_1(z) + t$
  $u_2 = v_2(z) - t$
• Efficient outcomes: max joint value
The bargaining set in the partnership example

$u_2 \quad 10$

(4+t, 6-t)

(4,6)

(2+t, 2-t)

(6,4)

d

d_2 = 2

d_1 = 2

u_1

10
Joint Value and Surplus

- For any \( z \) and \( t \), **joint value** is
  \[
  [v_1(z) + t] + [v_2(z) - t] = v_1(z) + v_2(z)
  \]

- **Surplus** of an agreement is defined as the difference between the joint value of the contract and the default:
  \[
  v_1(z) + v_2(z) - d_1 - d_2
  \]

- **Bargaining power**: bargaining weight

\( \pi_i \): proportion of surplus obtained by player \( i \)
Standard Bargaining Solution

• Efficient outcome:
  
  maximum payoff  \[ v^* = v_1(z) + v_2(z) \]

• Players negotiate over the surplus:
  \[ v^* - d_1 - d_2 \]

• Standard bargaining solution (Nash)
  \[ u_1 = d_1 + \pi_1(v^* - d_1 - d_2) \]
Simple Bargaining Games

Using Noncooperative Game Theory
(Watson Chapter 19)
What determines a player’s bargaining power (weight)?

- Importance of rules:
  The rules of the game determine the outcome

- Diminishing pies:
  The importance of patience

- Estimating payoffs:
  Trust your intuition
Ultimatum Games: Power to the Proposer

• Consider the following bargaining game (over a cake):
• I name a take-it-or-leave-it split.
• If you accept, we trade
• If you reject, no one eats!
• Under perfect information, there is a simple SPNE
Suppose I can only propose three divisions, (my share, your share):

- \((\frac{1}{4}, \frac{3}{4})\)
- \((\frac{1}{2}, \frac{1}{2})\)
- \((\frac{3}{4}, \frac{1}{4})\)

• Draw the extensive form

• Solve for the SPNE
Ultimatum Bargaining: continuous version

Bargaining set; disagreement point
Ultimatum Bargaining: continuous version

(Player i’s payoff is listed first.)

Player j: accept if $m > 0$;
Player i: offer the smallest possible $m$.
SPNE: $\{m=0; \text{accept all offers}\}$
Proposer keeps all profits.
Cake Cutting: changing the rules

- Suppose I get to cut the cake in one of three different ways (as before)
- And you get to pick which part is yours
- Draw the extensive form
- Solve for the SPNE
Two-Period Alternating Offer Games: Power to the Patient

- In general, bargaining takes on a “take-it-or-counteroffer” procedure.
- If time has value, both parties prefer trade earlier to trade later.
- E.g. Labor negotiations – Later agreements come at a price of strikes, work stoppages, etc.
- Delays imply less surplus left to be shared among the parties.
Two Stage Bargaining

• Bargaining over division of a cake

• I offer a proportion, $m$, of the cake to you

• If rejected, you may counteroffer (and $\delta$ of the cake remains, the rest melts)

• Discount factor: $\delta$

• Payoffs:
  » In first period: $1-m$, $m$
  » In second period: $\delta(1-m)$, $\delta m$
Bargaining set and disagreement point for 2-stage game
Extensive Form
Backward Induction

• Since period 2 is the final period, this is just like a take-it-or-leave-it offer:
  – You will offer me the smallest piece that I will accept, leaving you with all of $\delta$ and leaving me with almost 0

• What do I do in the first period?
Backward Induction

• Give you at least as much surplus
• Your surplus if you accept in the first period is 1-m

• Accept if:
  Your surplus in 1st period \geq Your surplus in 2nd period

\[ m \geq \delta \]
Backward Induction

• If there is a second stage, you get $\delta$ and I get 0.

• You will reject any offer in the first stage that does not offer you at least $\delta$.

• In the first period, I offer you $\delta$.

• Note: the more patient you are (the slower the cake melts) the more you receive now!
First or Second Mover Advantage?

• Are you better off being the first to make an offer, or the second?
Example: Cold Day

- If δ=4/5 (20% melts)

- Period 2: You offer a division of 1,0
  » You get all of remaining cake = 0.8
  » I get 0 = 0

- In the first period, I offer 80%
  » You get 80% of whole cake = 0.8
  » I get 20% of whole cake = 0.2

Source: Mike Shor, gametheory.net
Example: Hot Day

- If $\delta=1/5$  (80% melts)
- Period 2: You offer a division of 1,0
  - You get all of remaining cake $= 0.2$
  - I get 0 $= 0$
- In the first period, I offer 20%
  - You get 20% of whole cake $= 0.2$
  - I get 80% of whole cake $= 0.8$

Source: Mike Shor, gametheory.net
First or Second Mover Advantage?

• When players are impatient (hot day)
  First mover is better off
  – Rejecting my offer is less credible since we both lose a lot

• When players are patient (cold day)
  Second mover better off
  – Low cost to rejecting first offer

• Either way – if both players think through it, deal struck in period 1

Source: Mike Shor, gametheory.net
Don’t Waste Cake

• In any bargaining setting, strike a deal as early as possible!

• Why doesn’t this happen?
  – Reputation building
  – Lack of information

Source: Mike Shor, gametheory.net
Uncertainty in Civil Trials

- Plaintiff sues defendant for $1M
- Legal fees cost each side $100,000
- If each agrees that the chance of the plaintiff winning is $\frac{1}{2}$:
  - Plaintiff: \$500K - $100K = $400K
  - Defendant: -$500K - $100K = -$600K
- If simply agree on the expected winnings, $500K, each is better off

Source: Mike Shor, gametheory.net
Uncertainty in Civil Trials

• What if both parties are too optimistic?

• Each thinks that his or her side has a $\frac{3}{4}$ chance of winning:
  » Plaintiff: $750K - $100K = $ 650K
  » Defendant: $250K - $100K = $-350K

• No way to agree on a settlement!

Source: Mike Shor, gametheory.net
Lessons

• Rules of the bargaining game uniquely determine the bargaining outcome

• Which rules are better for you depends on patience, information

• What is the smallest acceptable piece? Trust your intuition

• Delays are always less profitable: Someone must be wrong

Source: Mike Shor, gametheory.net
Homework Assignment

• Chapter 19: #1, 2, 7, 8