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Physics 140 – Fall 2007

lecture #11 : 9 Oct

Ch 7 topics:

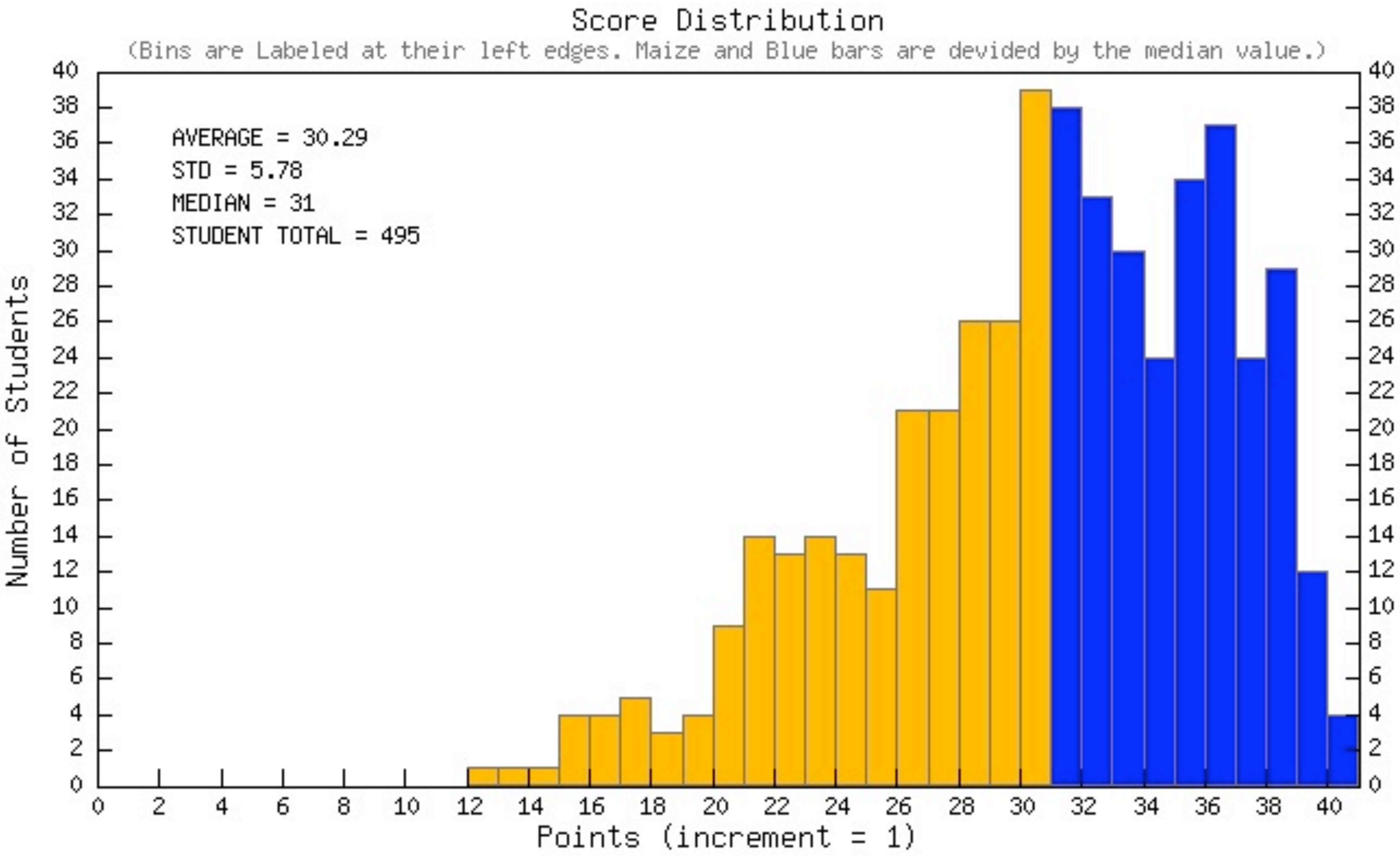
- energy conservation (E_{mec} change due to non-conservative forces)
- force from the derivative of potential energy

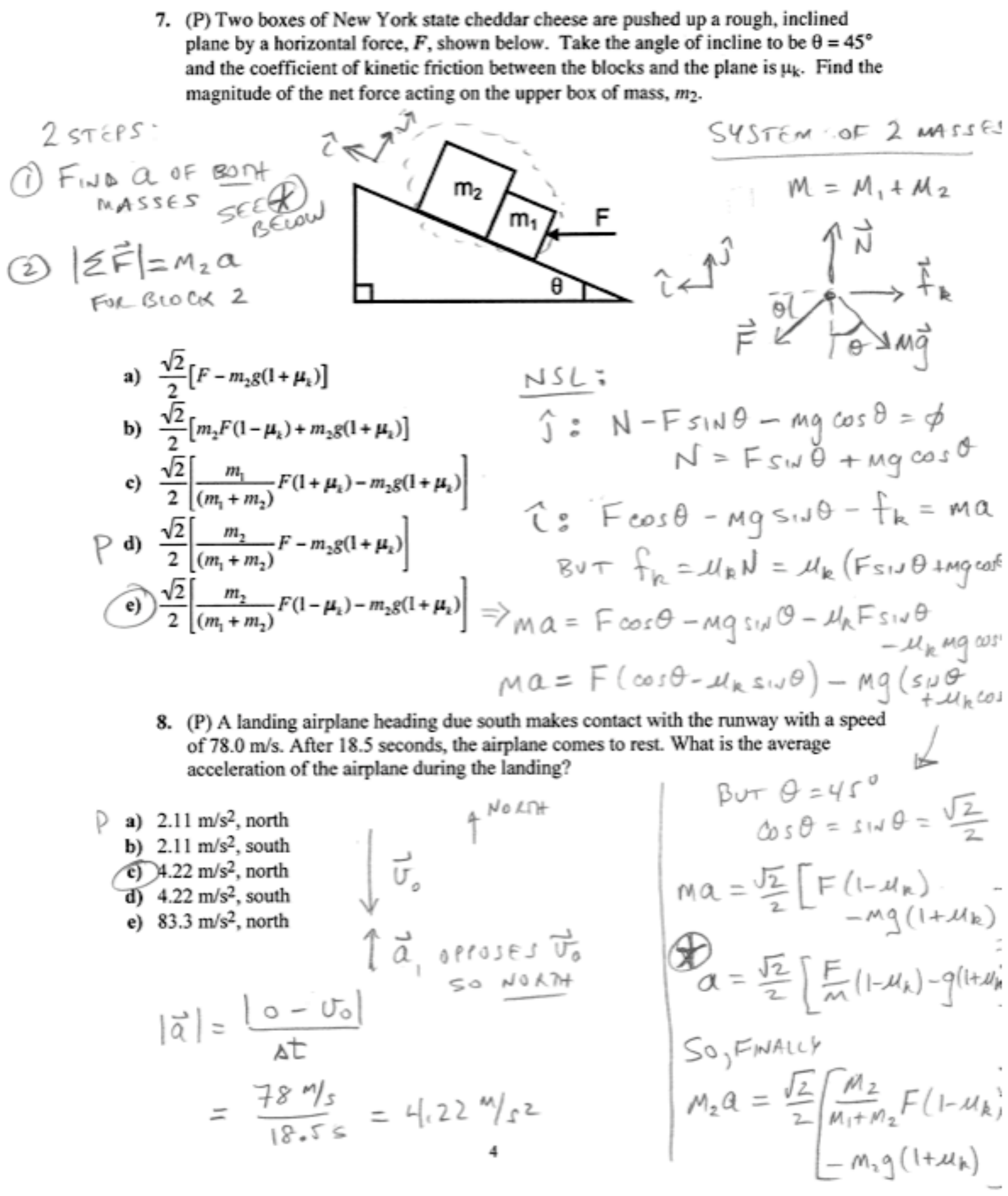
Announcements:

- MaPhys set #5 extension (due this Thursday @ 11:59 pm)
- set #6 due next Thursday

no classes! next Monday & Tuesday
Fall break: 15,16 October

Exam 1 score distribution (on scale 0-40)





7. (P) Two boxes of New York state cheddar cheese are pushed up a rough, inclined plane by a horizontal force, F, shown below. Take the angle of incline to be $\theta = 45°$ and the coefficient of kinetic friction between the blocks and the plane is μ_k. Find the magnitude of the net force acting on the upper box of mass, m_2.

2 STEPS:

① FIND a OF BOTH MASSES SEE ⊛ BELOW

② $|\Sigma \vec{F}| = M_2 a$ FOR BLOCK 2

SYSTEM OF 2 MASSES

$M = M_1 + M_2$

a) $\frac{\sqrt{2}}{2}[F - m_2 g(1+\mu_k)]$

b) $\frac{\sqrt{2}}{2}[m_2 F(1-\mu_k) + m_2 g(1+\mu_k)]$

c) $\frac{\sqrt{2}}{2}\left[\frac{m_1}{(m_1+m_2)}F(1+\mu_k) - m_2 g(1+\mu_k)\right]$

P d) $\frac{\sqrt{2}}{2}\left[\frac{m_2}{(m_1+m_2)}F - m_2 g(1+\mu_k)\right]$

[✓ e)] $\frac{\sqrt{2}}{2}\left[\frac{m_2}{(m_1+m_2)}F(1-\mu_k) - m_2 g(1+\mu_k)\right]$

NSL:

$\hat{\jmath}: N - F\sin\theta - Mg\cos\theta = 0$

$N = F\sin\theta + Mg\cos\theta$

$\hat{\imath}: F\cos\theta - Mg\sin\theta - f_k = Ma$

BUT $f_k = \mu_k N = \mu_k(F\sin\theta + Mg\cos$

$\Rightarrow Ma = F\cos\theta - Mg\sin\theta - \mu_k F\sin\theta - \mu_k Mg\cos$

$Ma = F(\cos\theta - \mu_k\sin\theta) - Mg(\sin\theta + \mu_k\cos$

8. (P) A landing airplane heading due south makes contact with the runway with a speed of 78.0 m/s. After 18.5 seconds, the airplane comes to rest. What is the average acceleration of the airplane during the landing?

P a) 2.11 m/s², north

b) 2.11 m/s², south

[✓ c)] 4.22 m/s², north

d) 4.22 m/s², south

e) 83.3 m/s², north

NORTH

$\vec{v}_0$

$\vec{a}$, OPPOSES $\vec{v}_0$ SO NORTH

$|\vec{a}| = \frac{|0 - v_0|}{\Delta t}$

$= \frac{78 \text{ m/s}}{18.5 \text{ s}} = 4.22 \text{ m/s}^2$

BUT $\theta = 45°$

$\cos\theta = \sin\theta = \frac{\sqrt{2}}{2}$

$Ma = \frac{\sqrt{2}}{2}\left[F(1-\mu_k) - Mg(1+\mu_k)\right.$

⊛ $a = \frac{\sqrt{2}}{2}\left[\frac{F}{M}(1-\mu_k) - g(1+\mu_k\right.$

SO, FINALLY

$M_2 a = \frac{\sqrt{2}}{2}\left[\frac{M_2}{M_1+M_2}F(1-\mu_k) - M_2 g(1+\mu_k)\right.$

34% correct response rate

12. (P) Two blocks, A and C below, lie on a horizontal, frictionless surface, with C atop A. The coefficient of static friction between A and C is μ_S. A very light (massless) rope tied to block A connects to a hanging mass B across a massless, frictionless pulley. What is the largest hanging mass m_B that allows A and C to slide together without separating?

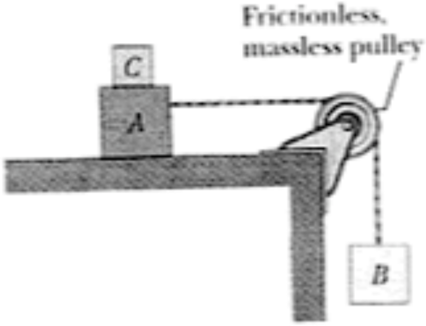
40% correct response rate

STATIC FRICTION ACCELERATES BLOCK C UP TO MAXIMUM:

$$m_c a = f_{s,MAX} = \mu_s N$$

$$M_c a = \mu_s m_c g$$

$$\boxed{a_{MAX} = \mu_s g} \; \circledast$$



ALL BLOCKS ACCELERATE DUE TO M_B'S WEIGHT

NSL:

$$(M_A + M_B + M_C)\,a = M_B g$$

$$a = \left(\frac{M_B}{M_A + M_B + M_C}\right) g$$

a) $(m_A + m_C)\,\mu_S$

P b) $(m_A + m_C)\,\mu_S / (1 + \mu_S)$

c) $(m_A + m_C) / (1 - \mu_S)$

d) $m_A\,\mu_S / (1 + \mu_S)$

[✓ e)] $(m_A + m_C)\,\mu_S / (1 - \mu_S)$

NOW APPLY MAX. ACCELERATION $\circledast$

$$a \le a_{MAX}$$

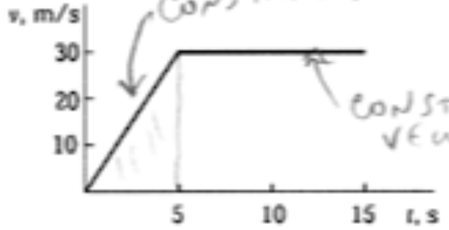
$$\left(\frac{M_B}{M_A + M_B + M_C}\right)\cancel{g} \le \mu_s \cancel{g}$$

$$M_B \le \mu_s (M_A + M_B + M_C)$$

$$M_B (1 - \mu_s) \le \mu_s (M_A + M_C)$$

$$\boxed{M_B \le \left(\frac{\mu_s}{1 - \mu_s}\right)(M_A + M_C)}$$

2. The graph below shows the instantaneous velocity of a car during 15 s of its motion. The distance traveled by this car during this 15-s interval is



CONSTANT ACCELERATION: $a = \frac{30\,m/s}{5s} = 6\,m/s^2$

CONSTANT VELOCITY: $v = 30\,m/s$

[✓ a) 375 m]
b) 75 m
c) 450 m
d) 300 m
e) 30 m

$$(x - x_0) = \frac{1}{2}a(5s)^2 + v(15s - 5s)$$

$$= 75m + 300m$$

$$= \underline{375m}$$

98% correct response rate

Conservation of energy

Energy is a deeply fundamental concept of physics. Conservation of energy is a simply a fact of nature with no compellingly simple theoretical explanation.

The general law of energy conservation states that

all changes in the mechanical energy of a system can be accounted for by either

<u>internal sources of energy acting within it</u>

or

<u>external forces acting upon it.</u>

The Unfair Poker Game Analogy…

When other (non-conservative) forces operate on a system, then its mechanical energy will **not** remain constant.

During some time interval Δt, the mechanical energy will change *due to the work done by other forces*

$$\Delta E_{\text{mec}} = \Delta K + \Delta U = W_{\text{other}}$$

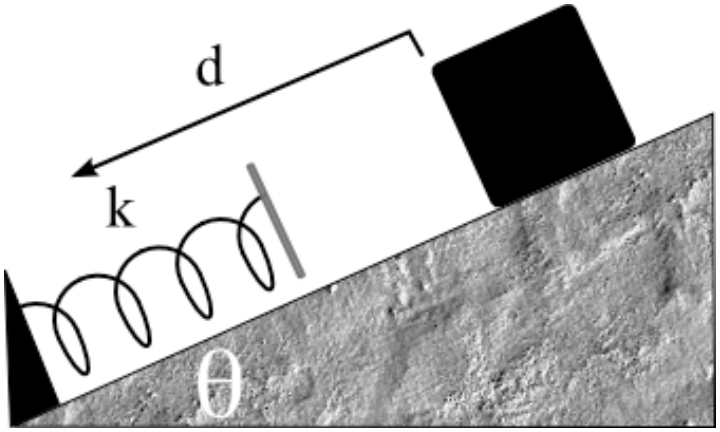
For problems involving gravity (U_g) and springs (U_g), another way to express this change is

$$(K + U_g + U_s)_{\text{initial}} + W_{\text{other}} = (K + U_g + U_s)_{\text{final}}$$

This is a bit like a two-player (A and B) poker game in which money is either added or removed from the table over time.

$$\mathbf{C_A + C_B = C_{total}(\mathit{t}) \quad or \quad \Delta C_A + \Delta C_B = \Delta C_{total}(\mathit{t})}$$

A carton of mass m, packed with the latest issues of *Salami News,* slides down a rough incline toward a spring with spring constant k. The box and incline have coefficient of kinetic friction μ_k. After the box has slid a distance d along the incline, what is the change in mechanical energy of the carton+spring+Earth system?

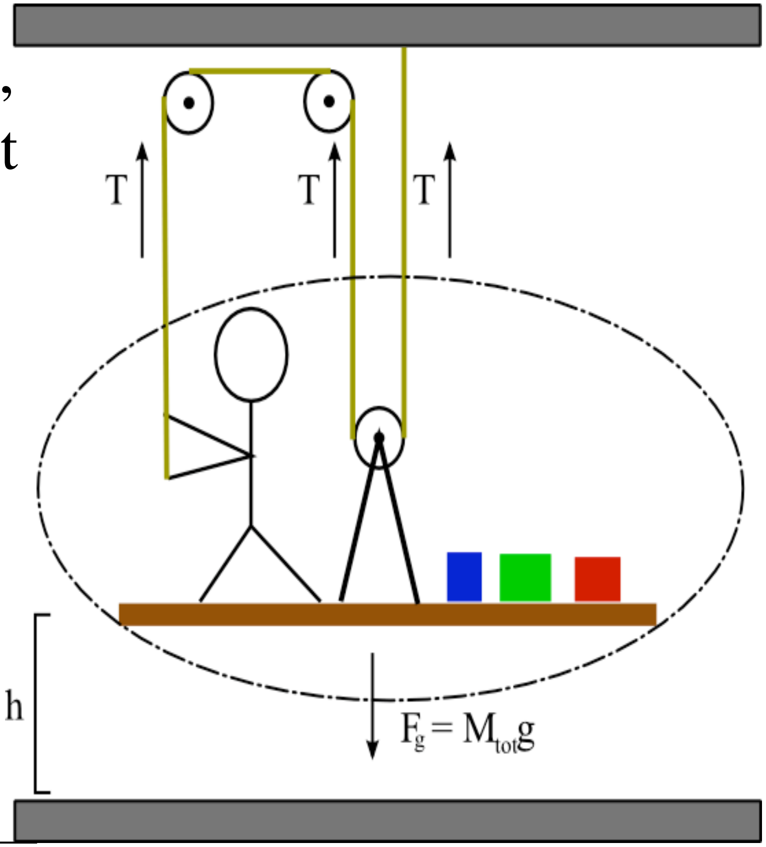


1. $\Delta E_{\text{mec}} = 0$
2. → $\Delta E_{\text{mec}} = -\mu_k mgd\cos\theta$
3. $\Delta E_{\text{mec}} = \mu_k mgd\cos\theta$
4. $\Delta E_{\text{mec}} = mgd\sin\theta$
5. $\Delta E_{\text{mec}} = -mgd\sin\theta$
6. more information is needed

Imagine that a man+box+platform system, of mass M_{tot}, is moving upward at constant speed.

As the platform moves upward a distance h, the mechanical energy of the system, E_{mec}, ***increases*** due to the work done by the tension in the attached ropes

$$\Delta E_{\text{mec}} = W_{\text{ropes}},$$
$$M_{\text{tot}} g h = 3Th.$$



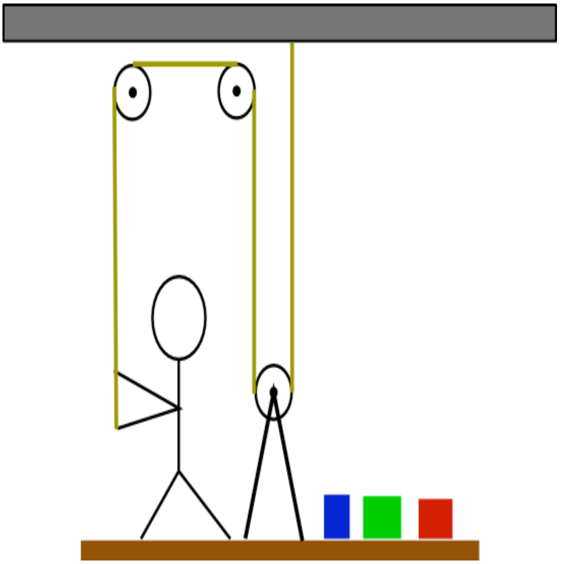
A different pull (higher or lower T) than the case above would produce a change in kinetic energy,

$$\Delta K = (3T - M_{\text{tot}} g)\, h,$$

that arises because the work by the ropes is either larger or smaller than the change in gravitational potential energy of the system. The platform will then accelerate upward or downward, respectively.

Q. Why isn't there a potential energy of string/rope tension?

Ans. #1: In general, the tension in a string or rope is not precisely constant in time. The work done b tension is therefore not a simple function of displacement. This variability is one way that tension differs from near-Earth gravity and the (ideal) spring force, both of which are reliably described by simple formulas.

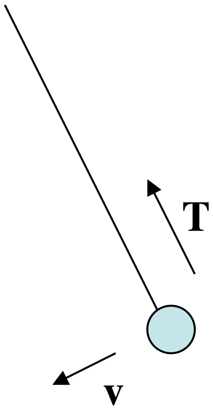


(a)

Ans. #2: Tension sometimes does work and sometimes doesn't. Referring to the graphics at right,

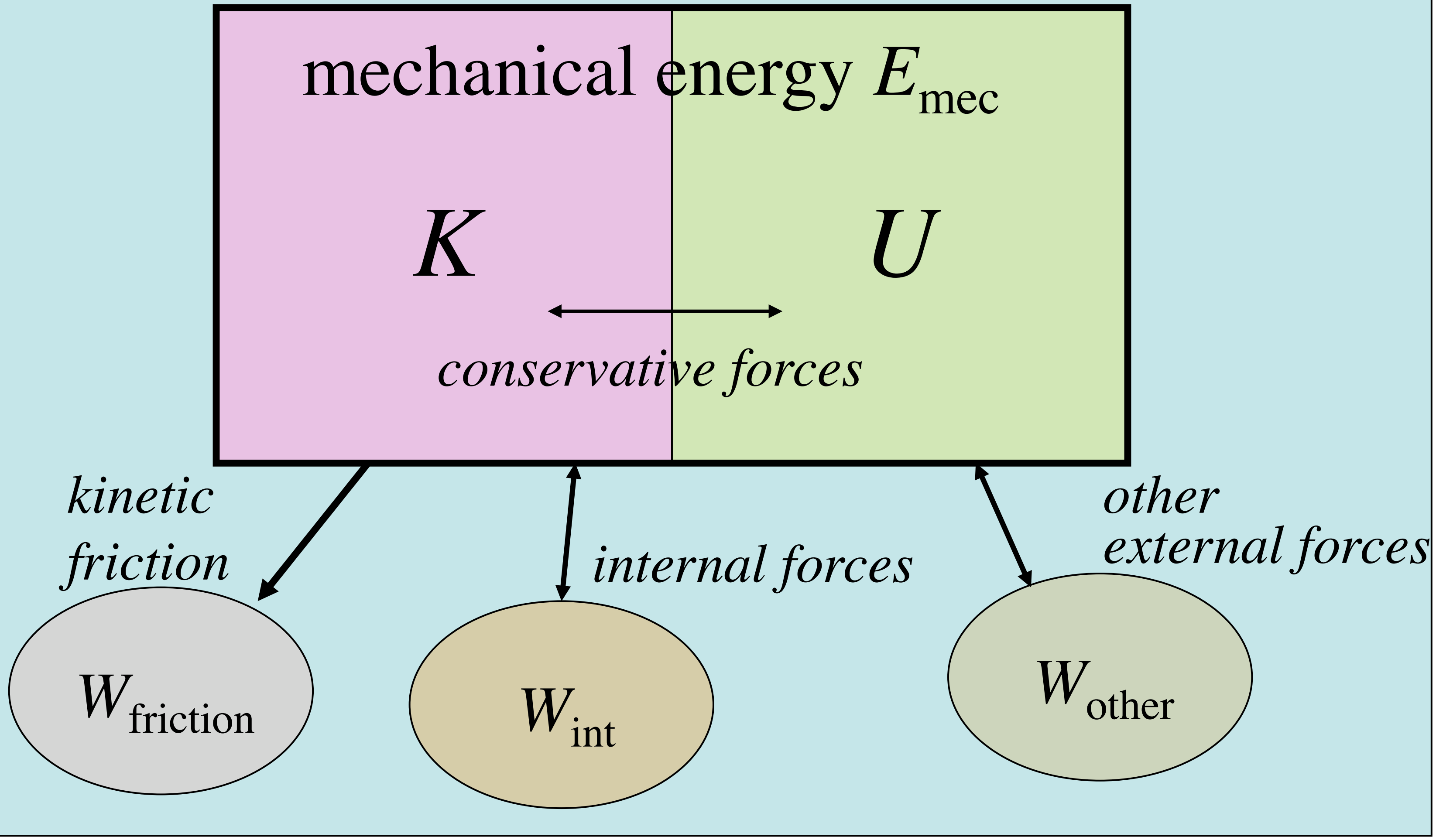
case (a): tension does non-zero work moving the platform+man+box system, since the displacement and tension are aligned.

case (b): in a simple pendulum, tension does no work because the displacement and tension are perpendicular at all points in the trajectory.



(b)

ENERGY



mechanical energy E_{mec}

K

U

conservative forces

kinetic friction

internal forces

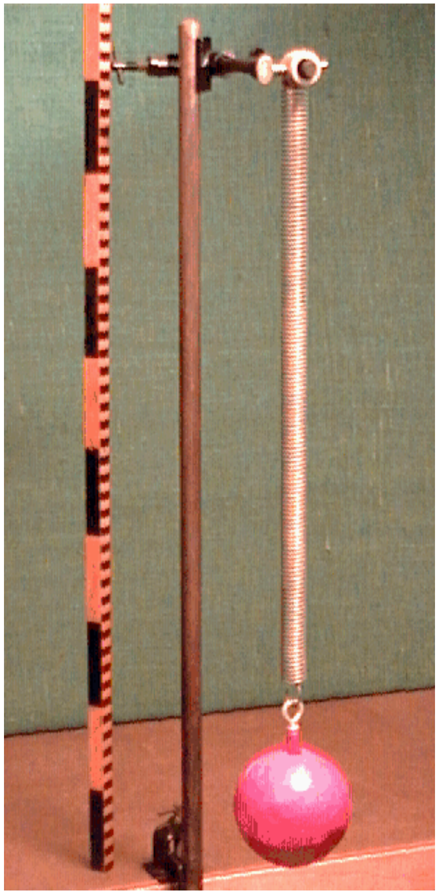
other external forces

W_{friction}

W_{int}

W_{other}

Consider the case of a *vertical* spring-mass system that is initially displaced upward from the spring’s rest length by an amount *d* and released from rest. When compared to the case of a (frictionless) *horizontal* spring-mass system compressed by an initial amount *d* and released from rest, what will be different in the vertical case?



1. The range of motion will be larger.
2. The range of motion will be smaller.
3. The peak velocity will be larger.
4. The peak velocity will be smaller.

→ 5. Both 1 and 3.

6. Both 2 and 4.

Force from potential energy

Given the form of a one-dimensional potential energy curve $U(x)$, the force associated with this potential is given by the derivative

$$F(x) = -\frac{dU(x)}{dx}$$

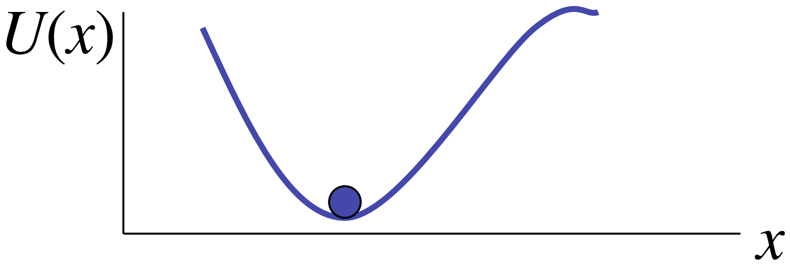
Familiar examples are:

spring force $U(x) = \frac{1}{2}kx^2 \Leftrightarrow F(x) = -kx$

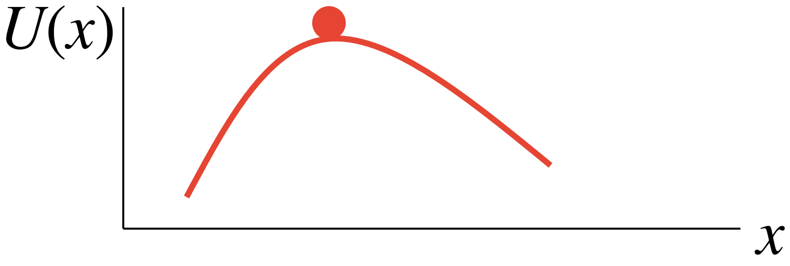
gravity $U(y) = mgy \Leftrightarrow F(y) = -mg$

Points of equilibrium are locations where the net force is zero. There are two classes of such points -

stable **: minima of $U(x)$**



unstable **: maxima of $U(x)$**



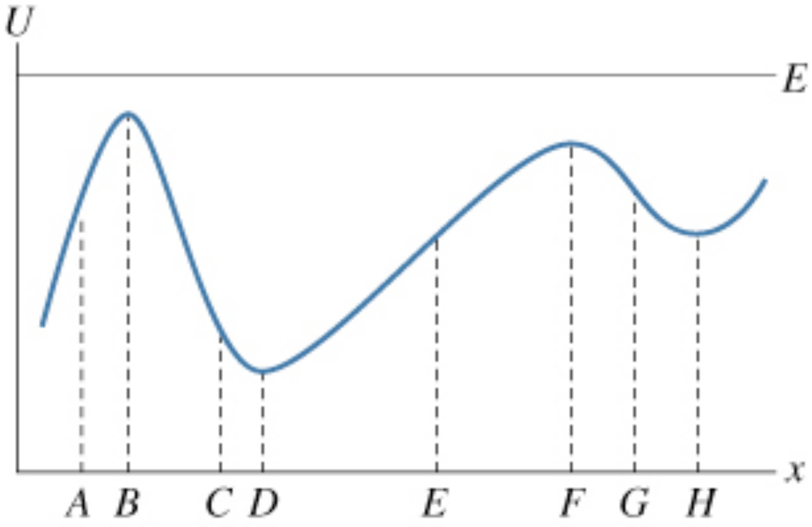
You graduate and obtain gainful employment debugging video game software. The game you're working on involves a one-dimensional potential energy of the form

$$U(x) = -32/x + 15\,x^2$$

and your task is to compute the force $F(x)$ as a function of position. What is the answer?

1. $F(x) = 32/x + 30x$
[✓ 2. $F(x) = -32/x^2 - 30x$]
3. $F(x) = 32/x^2 + 5x$
4. $F(x) = 64/x^2 - 5x$
5. $F(x) = -64/x^2 - 30x$

The curve at right shows the potential energy curve of a particular experimental setup. How many locations of stable equilibrium exist in the interval shown?



1. 1
2. [✓ 2]
3. 3
4. 4
5. 5
6. zero