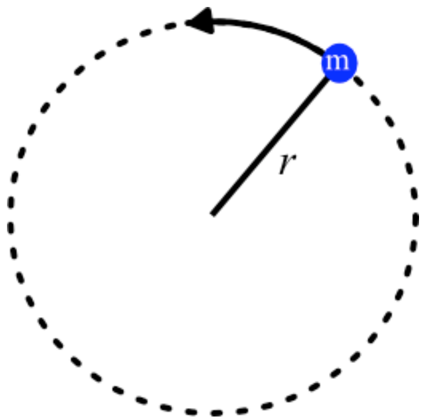
Physics 140 – Fall 2007
lecture #15 : 25 Oct



Ch 9 topics:
- rotational kinematics
- rotational kinetic energy
- moment of inertia

- exam #2 is next Thursday, 1 November, 6:00-7:30pm
- covers Chapters 6-8
- practice exam on CTools site -> Exams & Grading

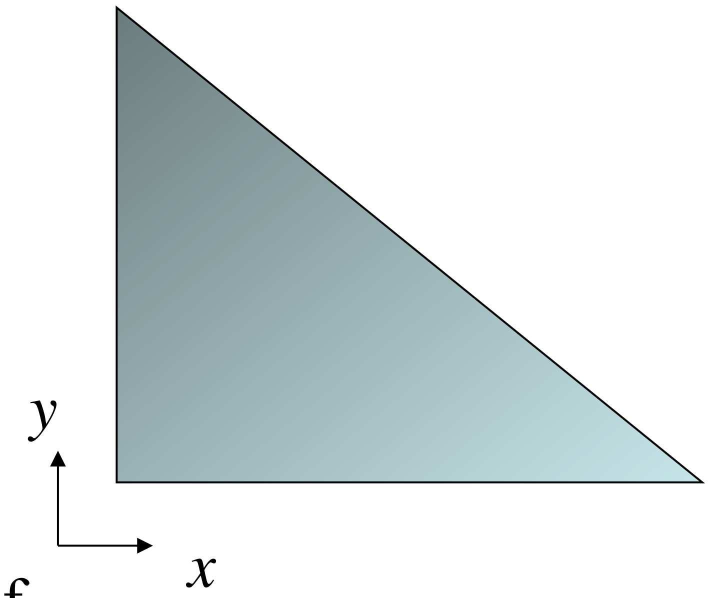
  bring **two** 3x5 notecards, calculator, #2 pencils

- review next Monday evening, 29 October, 8:00-9:30pm

Center of Mass of an extended, non-uniform object

An object with surface mass density $\sigma(\boldsymbol{r})$, shown here in 2D, has a total mass given by the integral

$$M = \iint_{\text{object}} \mathrm{dx}\, \mathrm{dy}\, \boldsymbol{\sigma}(\mathbf{r})$$



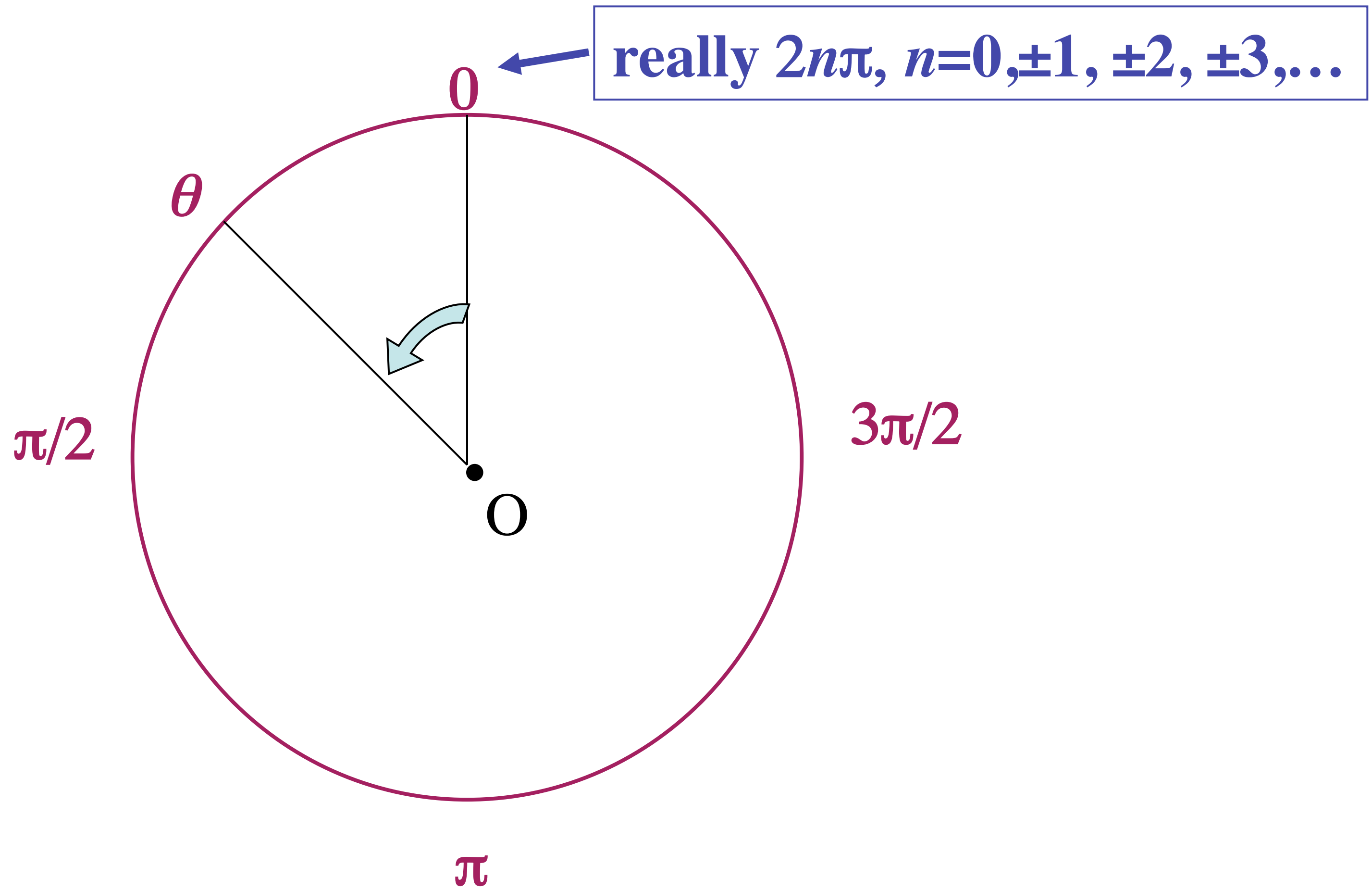
Its center of mass is defined by integrals of the mass-weighted positions:

$$x_{\text{com}} = \frac{1}{M} \iint_{\text{object}} \mathrm{dx}\, \mathrm{dy}\, \boldsymbol{\sigma}(\mathbf{r})\, x$$

$$y_{\text{com}} = \frac{1}{M} \iint_{\text{object}} \mathrm{dx}\, \mathrm{dy}\, \boldsymbol{\sigma}(\mathbf{r})\, y$$

Rotational kinematics

To describe rotational motion, we begin with the ***angular position*** θ (in **radians**) measured relative to an (arbitrary) reference angle.



Rotational kinematics

A change in angular position, $\Delta\theta$, during a time interval Δt implies a non-zero average ***angular velocity***

$$\omega_{\text{avg}} = \Delta\theta \,/\, \Delta t$$

A change in angular velocity, $\Delta\omega$, defines an average ***angular acceleration***

$$\alpha_{\text{avg}} = \Delta\omega \,/\, \Delta t$$

The limit Δt -> 0 defines instantaneous measures for these

$$\omega = \mathrm{d}\theta \,/\, \mathrm{d}t$$

$$\alpha = \mathrm{d}\omega \,/\, \mathrm{d}t$$

The snapshots of the ball are shown at fixed time intervals. Note how the angular displacement between images grows in time, implying an angular acceleration $\alpha>0$.

Relations to translational kinematics

The magnitudes of translational quantities - displacement, velocity and tangential acceleration (l, v, a_{tan}) - are equal to the angular equivalent measures (θ, ω, α) multiplied by the distance r from the rotation axis.

At fixed r, the components of:
displacement
tangential velocity
tangential accel. a_{tan}

TABLE 8.2 Linear and angular kinematic parameters

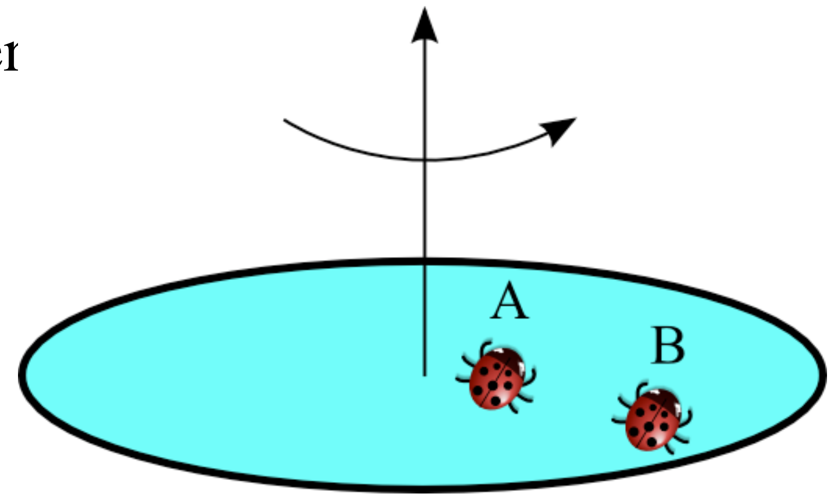
Equation	Units	
$l = r\theta$	l (m)	θ (rad)
$v = r\omega$	v (m/s)	ω (rad/s)
$a_T = r\alpha$	a_T (m/s^2)	α (rad/s^2)

Kinematic equations for rotation

The kinematic equations developed in Chapter 2 for translational motion also apply to rotational motion. (see Table 9.1 in Y&F)

Rotational Motion (α = constant)	Linear Motion (a = constant)
$\omega = \omega_0 + \alpha t$	$v = v_0 + at$
$\theta = \frac{1}{2}(\omega_0 + \omega)\,t$	$x = \frac{1}{2}(v_0 + v)\,t$
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	$x = v_0 t + \frac{1}{2}at^2$
$\omega^2 = \omega_0^2 + 2\alpha\theta$	$v^2 = v_0^2 + 2ax$

Two ladybugs rest without slipping on a rotating platter that is increasing its angular velocity. Ladybug A is closer to the rotation axis than bug B.



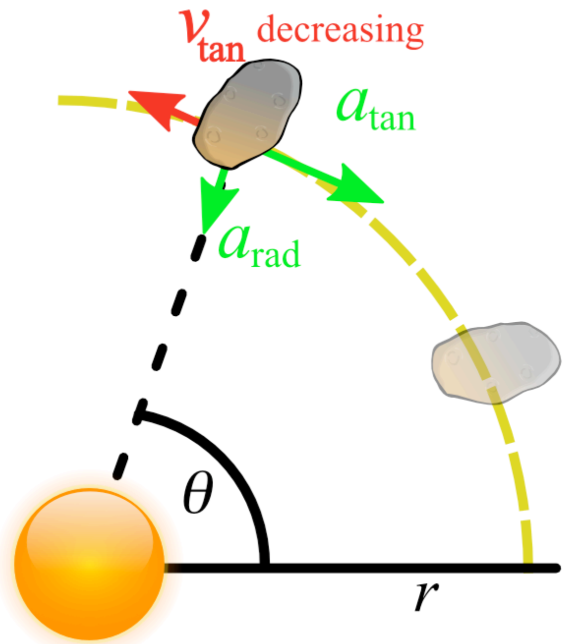
Which statement correctly describes the relationship between the bugs' angular accelerations (α) and centripetal accelerations (a_{rad})?

1) $\alpha_A > \alpha_B$ and $a_{rad,A} > a_{rad,B}$
2) $\alpha_A < \alpha_B$ and $a_{rad,A} < a_{rad,B}$
3) $\alpha_A = \alpha_B$ and $a_{rad,A} < a_{rad,B}$
4) $\alpha_A = \alpha_B$ and $a_{rad,A} = a_{rad,B}$
5) $\alpha_A = \alpha_B$ and $a_{rad,A} > a_{rad,B}$

Negotiating circular motion at tangential speed v_t around a circular arc of radius r still requires a *radial component* of acceleration

$$a_{rad} = v_t^2 / r = \omega^2 r$$

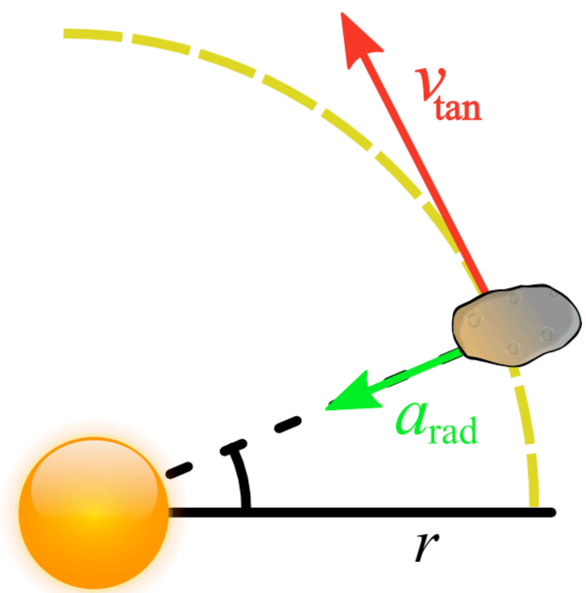
directed towards the center of the circle. This component changes the direction of the velocity, keeping it tangent to the circle.



The *tangential component* of acceleration

$$a_{tan} = \alpha r$$

changes the speed, $\frac{dv_t}{dt} = a_{tan}$.



Rotational kinetic energy and moment of inertia

A set of masses m_i *uniformly rotating* with angular velocity ω *about some fixed axis A* possesses a kinetic energy defined by

$$K = \tfrac{1}{2}\sum_i m_i v_i^2 = \tfrac{1}{2}\sum_i m_i r_i^2 \omega^2$$

where r_i is the distance from the i^{th} mass to the rotation axis.

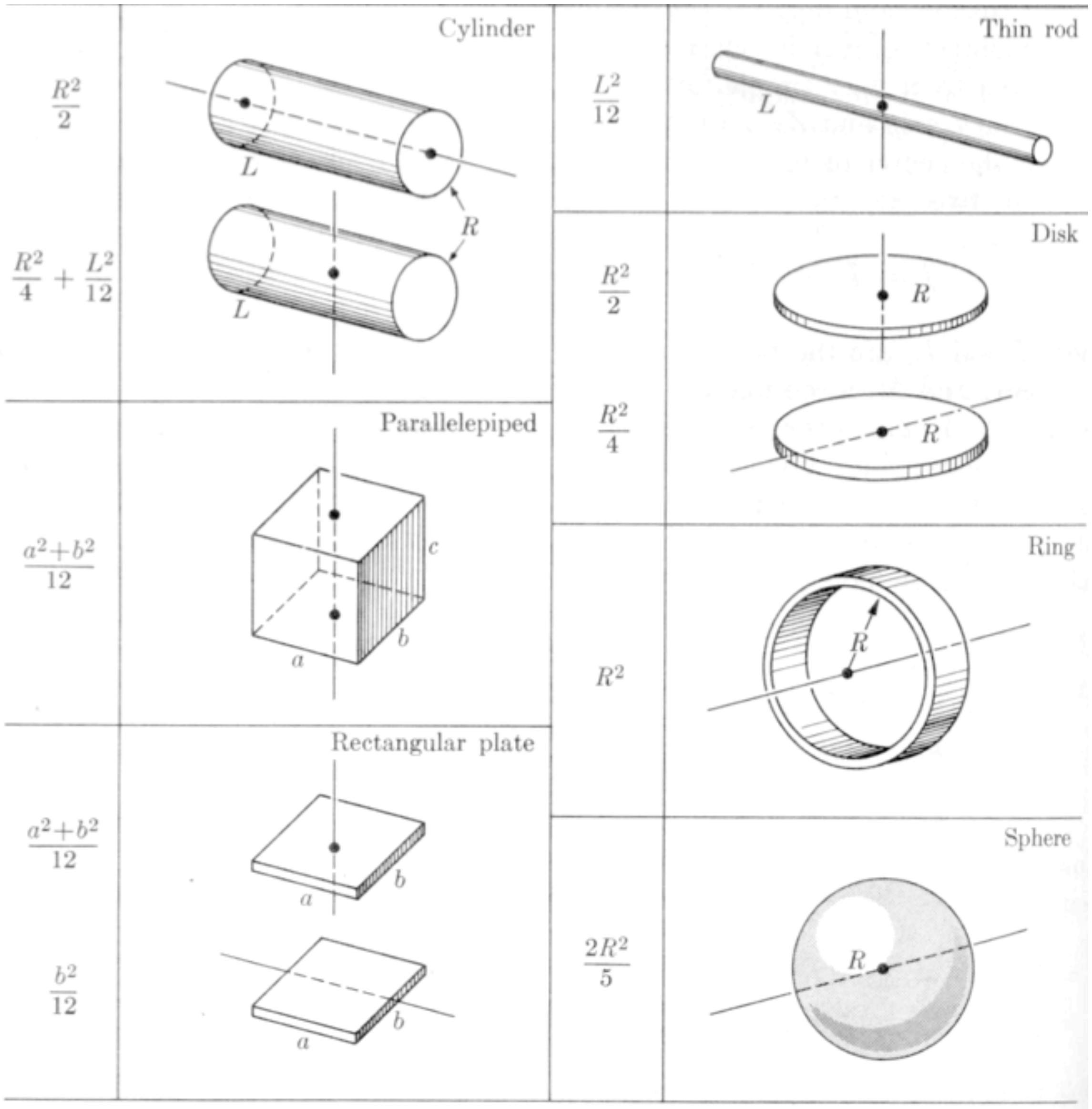
For such a set of mass, or for a continuous body, we define the ***moment of inertia I*** *about the specified axis A* as

$$I = \sum_i m_i r_i^2$$

Then the rotational kinetic energy can be written as

$$K = \tfrac{1}{2} I\omega^2$$

A given object has only one mass m, but **many** moments of inertia I, ***depending on the location and orientation of the rotation axis***.



Note: this graphic assumes an object of unit mass (M=1).

Refer to Table 9.2 in YF for a similar list.

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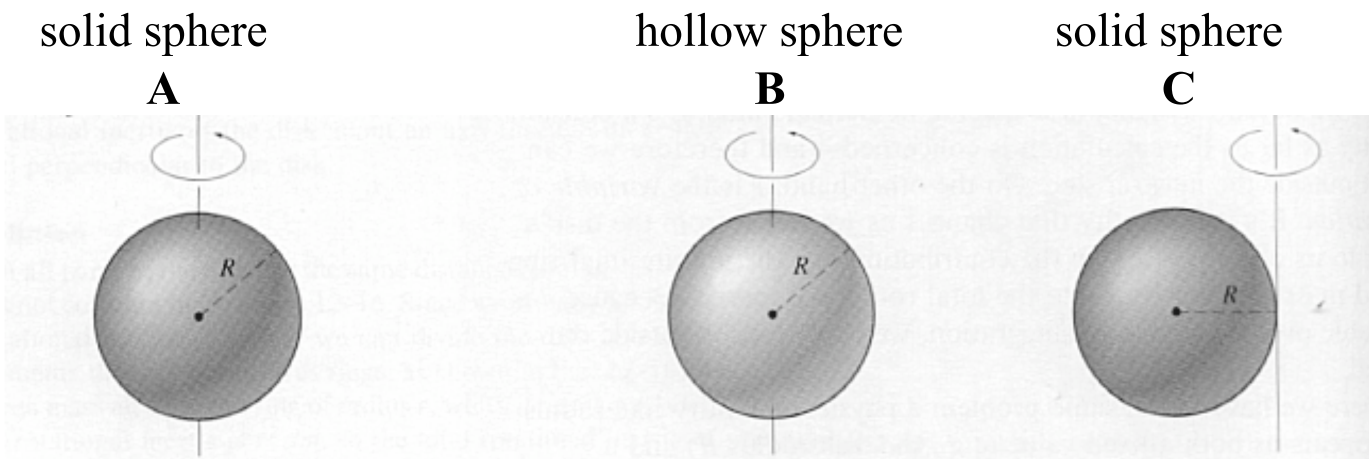

Source: Undetermined

The three spheres above have the same mass *M* and the same radius *R*. Sphere B is hollow, A and C are solid. Sphere C rotates about an axis adjacent to its edge while spheres A and B rotate about their centers. All rotate at the same angular velocity. Rank the spheres according to their rotational kinetic energy, largest to smallest.

1. A, B, C
2. B, A, C
3. A, C, B
4. C, B, A