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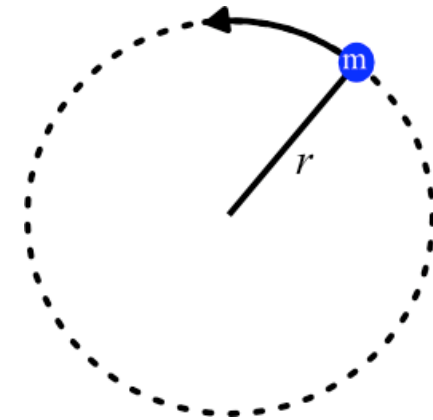
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Physics 140 – Fall 2007

lecture #15 : 25 Oct

Ch 9 topics:

- rotational kinematics
- rotational kinetic energy
- moment of inertia

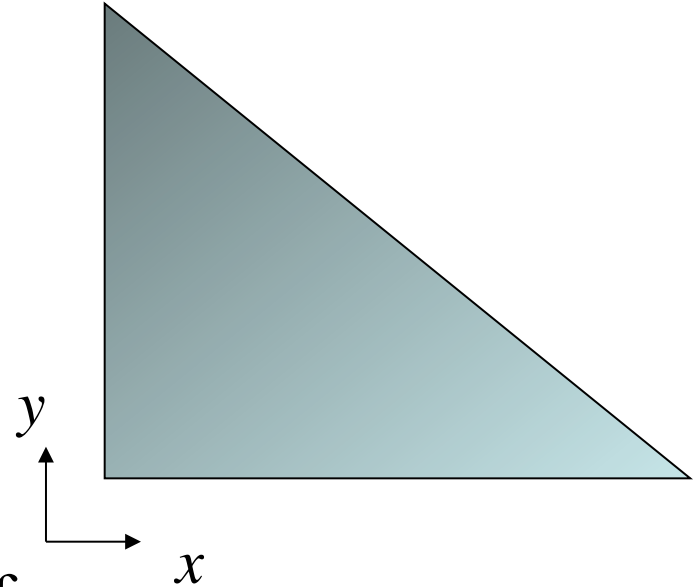


- exam #2 is next Thursday, 1 November, 6:00-7:30pm
- covers Chapters 6-8
- practice exam on CTools site -> Exams & Grading
bring two 3x5 notecards, calculator, #2 pencils
- review next Monday evening, 29 October, 8:00-9:30pm

Center of Mass of an extended, non-uniform object

An object with surface mass density $\sigma(\mathbf{r})$, shown here in 2D, has a total mass given by the integral

$$M = \iint_{\text{object}} dx dy \sigma(\mathbf{r})$$

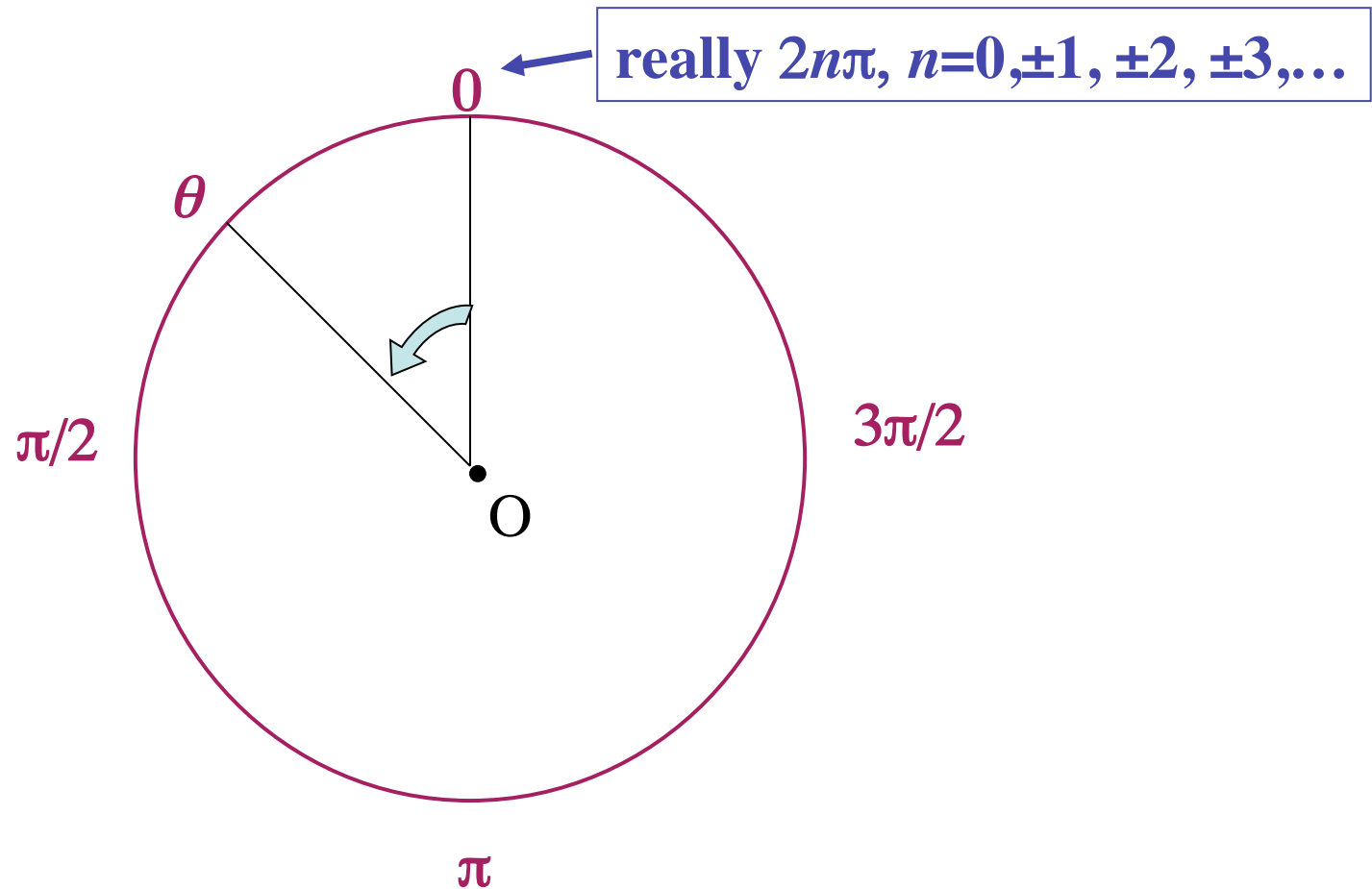


Its center of mass is defined by integrals of the mass-weighted positions:

$$x_{\text{com}} = \frac{1}{M} \iint_{\text{object}} dx dy \sigma(\mathbf{r}) x$$
$$y_{\text{com}} = \frac{1}{M} \iint_{\text{object}} dx dy \sigma(\mathbf{r}) y$$

Rotational kinematics

To describe rotational motion, we begin with the *angular position* θ (in **radians**) measured relative to an (arbitrary) reference angle.



Rotational kinematics

A change in angular position, $\Delta\theta$, during a time interval Δt implies a non-zero average *angular velocity*

$$\omega_{\text{avg}} = \Delta\theta / \Delta t$$

A change in angular velocity, $\Delta\omega$, defines an average *angular acceleration*

$$\alpha_{\text{avg}} = \Delta\omega / \Delta t$$

The limit $\Delta t \rightarrow 0$ defines instantaneous measures for these

$$\omega = d\theta / dt$$

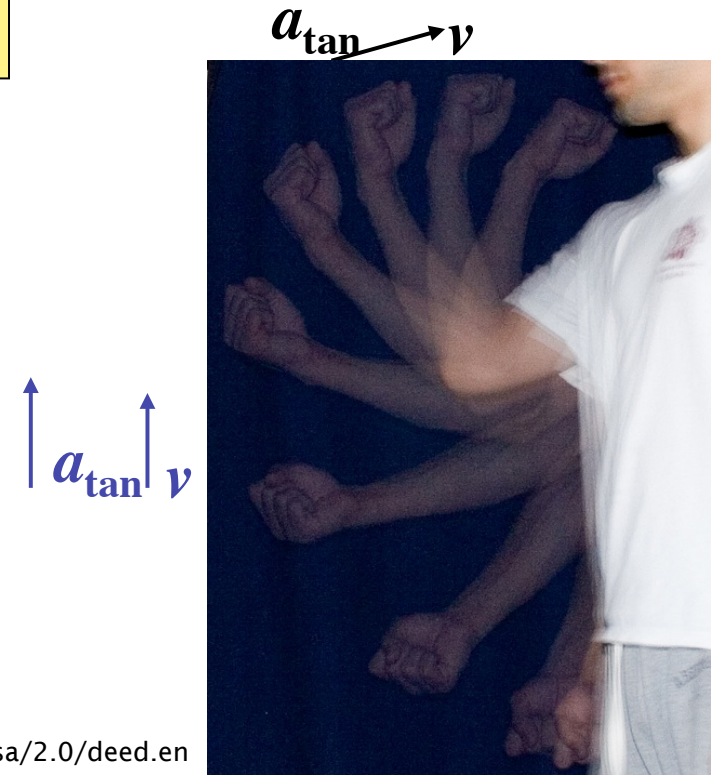
$$\alpha = d\omega / dt$$

The snapshots of the ball are shown at fixed time intervals. Note how the angular displacement between images grows in time, implying an angular acceleration $\alpha > 0$.



Relations to translational kinematics

The magnitudes of translational quantities - displacement, velocity and tangential acceleration (l, v, a_{tan}) - are equal to the angular equivalent measures (θ, ω, α) multiplied by the distance r from the rotation axis.



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At fixed r , the components of:
 displacement
 tangential velocity
 tangential accel. a_{tan}

TABLE 8.2 Linear and angular kinematic parameters

Equation	Units	
$l = r\theta$	l (m)	θ (rad)
$v = r\omega$	v (m/s)	ω (rad/s)
$a_{\text{T}} = r\alpha$	a_{T} (m/s ²)	α (rad/s ²)

Kinematic equations for rotation

The kinematic equations developed in Chapter 2 for translational motion also apply to rotational motion. (see Table 9.1 in Y&F)

Rotational Motion
($\alpha = \text{constant}$)

Linear Motion
($a = \text{constant}$)

$$\omega = \omega_0 + \alpha t$$

$$\theta = \frac{1}{2} (\omega_0 + \omega) t$$

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$v = v_0 + at$$

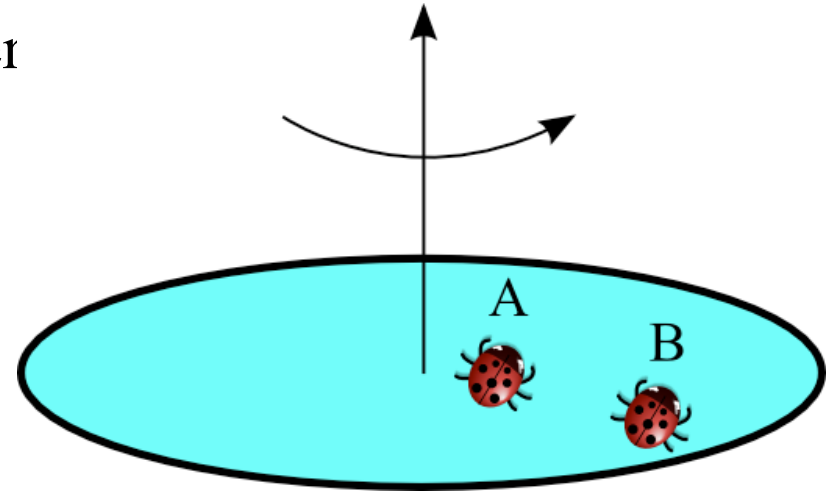
$$x = \frac{1}{2} (v_0 + v) t$$

$$x = v_0 t + \frac{1}{2} at^2$$

$$v^2 = v_0^2 + 2ax$$

Two ladybugs rest without slipping on a rotating platter that is increasing its angular velocity. Ladybug A is closer to the rotation axis than bug B.

Which statement correctly describes the relationship between the bugs' angular accelerations (α) and centripetal accelerations (a_{rad})?



- 1) $\alpha_A > \alpha_B$ and $a_{\text{rad},A} > a_{\text{rad},B}$
- 2) $\alpha_A < \alpha_B$ and $a_{\text{rad},A} < a_{\text{rad},B}$
- 3) $\alpha_A = \alpha_B$ and $a_{\text{rad},A} < a_{\text{rad},B}$
- 4) $\alpha_A = \alpha_B$ and $a_{\text{rad},A} = a_{\text{rad},B}$
- 5) $\alpha_A = \alpha_B$ and $a_{\text{rad},A} > a_{\text{rad},B}$

Negotiating circular motion at tangential speed v_t around a circular arc of radius r still requires a radial component of acceleration

$$a_{\text{rad}} = v_t^2 / r = \omega^2 r$$

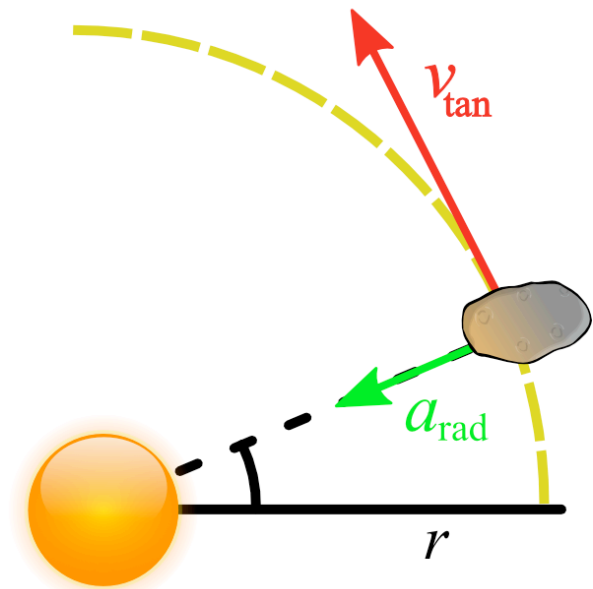
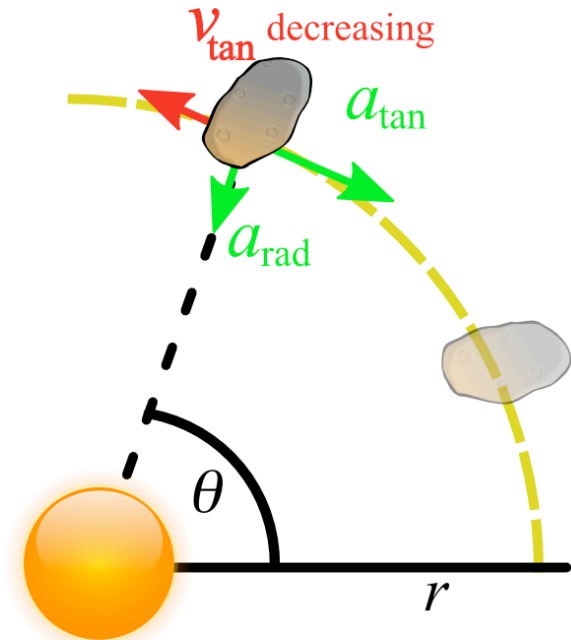
directed towards the center of the circle.

This component changes the direction of the velocity, keeping it tangent to the circle.

The tangential component of acceleration

$$a_{\text{tan}} = \alpha r$$

changes the speed, $\frac{dv_t}{dt} = a_{\text{tan}}$.



Rotational kinetic energy and moment of inertia

A set of masses m_i *uniformly rotating* with angular velocity ω *about some fixed axis A* possesses a kinetic energy defined by

$$K = \frac{1}{2} \sum_i m_i v_i^2 = \frac{1}{2} \sum_i m_i r_i^2 \omega^2$$

where r_i is the distance from the i^{th} mass to the rotation axis.

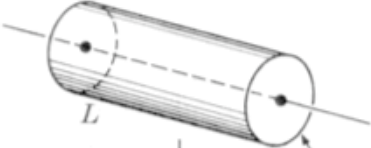

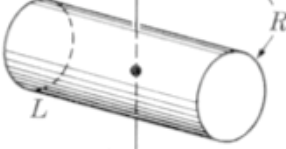

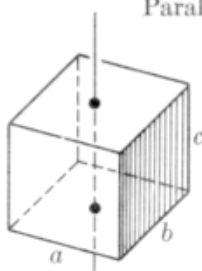

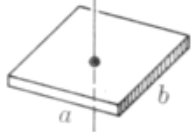
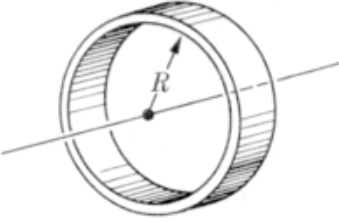
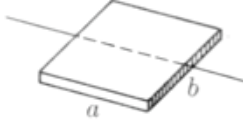
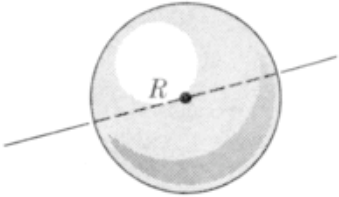
For such a set of mass, or for a continuous body, we define the *moment of inertia I about the specified axis A* as

$$I = \sum_i m_i r_i^2$$

Then the rotational kinetic energy can be written as

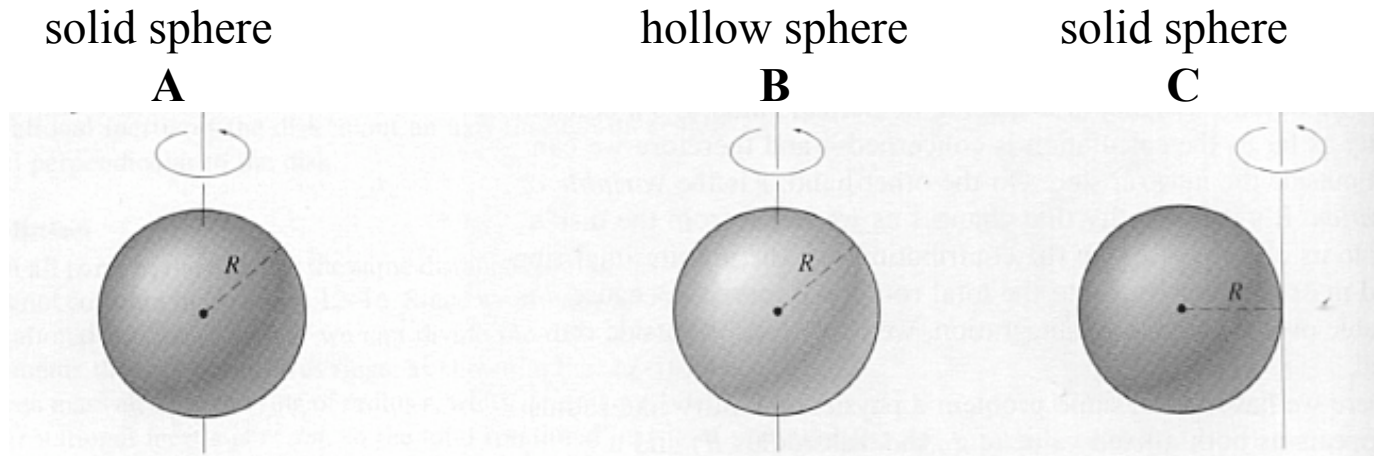
$$K = \frac{1}{2} I \omega^2$$

A given object has only one mass m , but **many** moments of inertia I , *depending on the location and orientation of the rotation axis.*

$\frac{R^2}{2}$	<p>Cylinder</p> 	$\frac{L^2}{12}$	<p>Thin rod</p> 
$\frac{R^2}{4} + \frac{L^2}{12}$		$\frac{R^2}{2}$	<p>Disk</p> 
$\frac{a^2+b^2}{12}$	<p>Parallelepiped</p> 	$\frac{R^2}{4}$	
$\frac{a^2+b^2}{12}$	<p>Rectangular plate</p> 	R^2	<p>Ring</p> 
$\frac{b^2}{12}$		$\frac{2R^2}{5}$	<p>Sphere</p> 

Note: this graphic assumes an object of unit mass ($M=1$).

Refer to Table 9.2 in YF for a similar list.



Source: Undetermined

The three spheres above have the same mass M and the same radius R . Sphere B is hollow, A and C are solid. Sphere C rotates about an axis adjacent to its edge while spheres A and B rotate about their centers. All rotate at the same angular velocity. Rank the spheres according to their rotational kinetic energy, largest to smallest.

1. A, B, C
2. B, A, C
3. A, C, B
4. C, B, A

