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PHYSICS 140 - General Physics 1, Fall 2007

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Physics 140 – Fall 2007
lecture #15 : 25 Oct

Ch 9 topics:
• rotational kinematics
• rotational kinetic energy
• moment of inertia

• exam #2 is next Thursday, 1 November, 6:00-7:30pm
• covers Chapters 6-8
• practice exam on CTools site -> Exams & Grading
  bring two 3x5 notecards, calculator, #2 pencils

• review next Monday evening, 29 October, 8:00-9:30pm
Center of Mass of an extended, non-uniform object

An object with surface mass density $\sigma(r)$, shown here in 2D, has a total mass given by the integral

$$M = \iiint_{\text{object}} dx \, dy \, \sigma(r)$$

Its center of mass is defined by integrals of the mass-weighted positions:

$$x_{\text{com}} = \frac{1}{M} \iiint_{\text{object}} dx \, dy \, \sigma(r) x$$

$$y_{\text{com}} = \frac{1}{M} \iiint_{\text{object}} dx \, dy \, \sigma(r) y$$
To describe rotational motion, we begin with the angular position $\theta$ (in radians) measured relative to an (arbitrary) reference angle. Really $2n\pi$, $n=0,1,\pm 2, \pm 3,\ldots$
Rotational kinematics

A change in angular position, $\Delta \theta$, during a time interval $\Delta t$ implies a non-zero average angular velocity

$$\omega_{\text{avg}} = \frac{\Delta \theta}{\Delta t}$$

A change in angular velocity, $\Delta \omega$, defines an average angular acceleration

$$\alpha_{\text{avg}} = \frac{\Delta \omega}{\Delta t}$$

The limit $\Delta t \to 0$ defines instantaneous measures for these

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt}$$

The snapshots of the ball are shown at fixed time intervals. Note how the angular displacement between images grows in time, implying an angular acceleration $\alpha > 0$. 

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The magnitudes of translational quantities - displacement, velocity and tangential acceleration \((l, v, a_{\text{tan}})\) - are equal to the angular equivalent measures \((\theta, \omega, \alpha)\) multiplied by the distance \(r\) from the rotation axis.

At fixed \(r\), the components of:

- displacement
- tangential velocity
- tangential accel. \(a_{\text{tan}}\)

<table>
<thead>
<tr>
<th>Equation</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>(l = r\theta)</td>
<td>(l) (m)</td>
</tr>
<tr>
<td>(v = r\omega)</td>
<td>(v) (m/s)</td>
</tr>
<tr>
<td>(a_T = r\alpha)</td>
<td>(a_T) (m/s²)</td>
</tr>
</tbody>
</table>
The kinematic equations developed in Chapter 2 for translational motion also apply to rotational motion. (see Table 9.1 in Y&F)

### Kinematic equations for rotation

<table>
<thead>
<tr>
<th>Rotational Motion $(\alpha = \text{constant})$</th>
<th>Linear Motion $(a = \text{constant})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega = \omega_0 + \alpha t$</td>
<td>$v = v_0 + at$</td>
</tr>
<tr>
<td>$\theta = \frac{1}{2} (\omega_0 + \omega) t$</td>
<td>$x = \frac{1}{2} (v_0 + v) t$</td>
</tr>
<tr>
<td>$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$</td>
<td>$x = v_0 t + \frac{1}{2} at^2$</td>
</tr>
<tr>
<td>$\omega^2 = \omega_0^2 + 2\alpha \theta$</td>
<td>$v^2 = v_0^2 + 2ax$</td>
</tr>
</tbody>
</table>
Two ladybugs rest without slipping on a rotating platter that is increasing its angular velocity. Ladybug A is closer to the rotation axis than bug B.

Which statement correctly describes the relationship between the bugs’ angular accelerations ($\alpha$) and centripetal accelerations ($a_{rad}$)?

1) $\alpha_A > \alpha_B$ and $a_{rad,A} > a_{rad,B}$
2) $\alpha_A < \alpha_B$ and $a_{rad,A} < a_{rad,B}$
3) $\alpha_A = \alpha_B$ and $a_{rad,A} < a_{rad,B}$
4) $\alpha_A = \alpha_B$ and $a_{rad,A} = a_{rad,B}$
5) $\alpha_A = \alpha_B$ and $a_{rad,A} > a_{rad,B}$
Negotiating circular motion at tangential speed $v_t$ around a circular arc of radius $r$ still requires a **radial component** of acceleration

$$a_{rad} = \frac{v_t^2}{r} = \omega^2 r$$
directed **towards the center** of the circle. This component changes the direction of the velocity, keeping it tangent to the circle.

The **tangential component** of acceleration

$$a_{tan} = \alpha r$$
changes the speed, $\frac{dv_t}{dt} = a_{tan}$.
Rotational kinetic energy and moment of inertia

A set of masses \( m_i \) uniformly rotating with angular velocity \( \omega \) about some fixed axis A possesses a kinetic energy defined by

\[
K = \frac{1}{2} \sum_i m_i v_i^2 = \frac{1}{2} \sum_i m_i r_i^2 \omega^2
\]

where \( r_i \) is the distance from the \( i^{th} \) mass to the rotation axis.

For such a set of mass, or for a continuous body, we define the moment of inertia \( I \) about the specified axis A as

\[
I = \sum_i m_i r_i^2
\]

Then the rotational kinetic energy can be written as

\[
K = \frac{1}{2} I \omega^2
\]
A given object has only one mass $m$, but many moments of inertia $I$, depending on the location and orientation of the rotation axis.

Note: this graphic assumes an object of unit mass ($M=1$).

Refer to Table 9.2 in YF for a similar list.

Source: Undetermined
The three spheres above have the same mass $M$ and the same radius $R$. Sphere B is hollow, A and C are solid. Sphere C rotates about an axis adjacent to its edge while spheres A and B rotate about their centers. All rotate at the same angular velocity. Rank the spheres according to their rotational kinetic energy, largest to smallest.

1. A, B, C  
2. B, A, C  
3. A, C, B  
4. C, B, A