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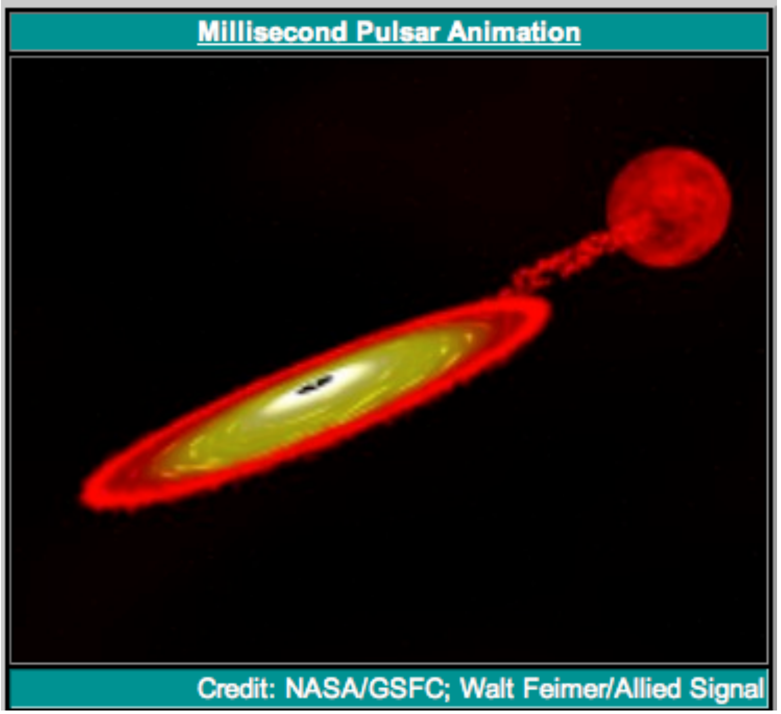
Physics 140 – Fall 2007

lecture #18: 6 Nov

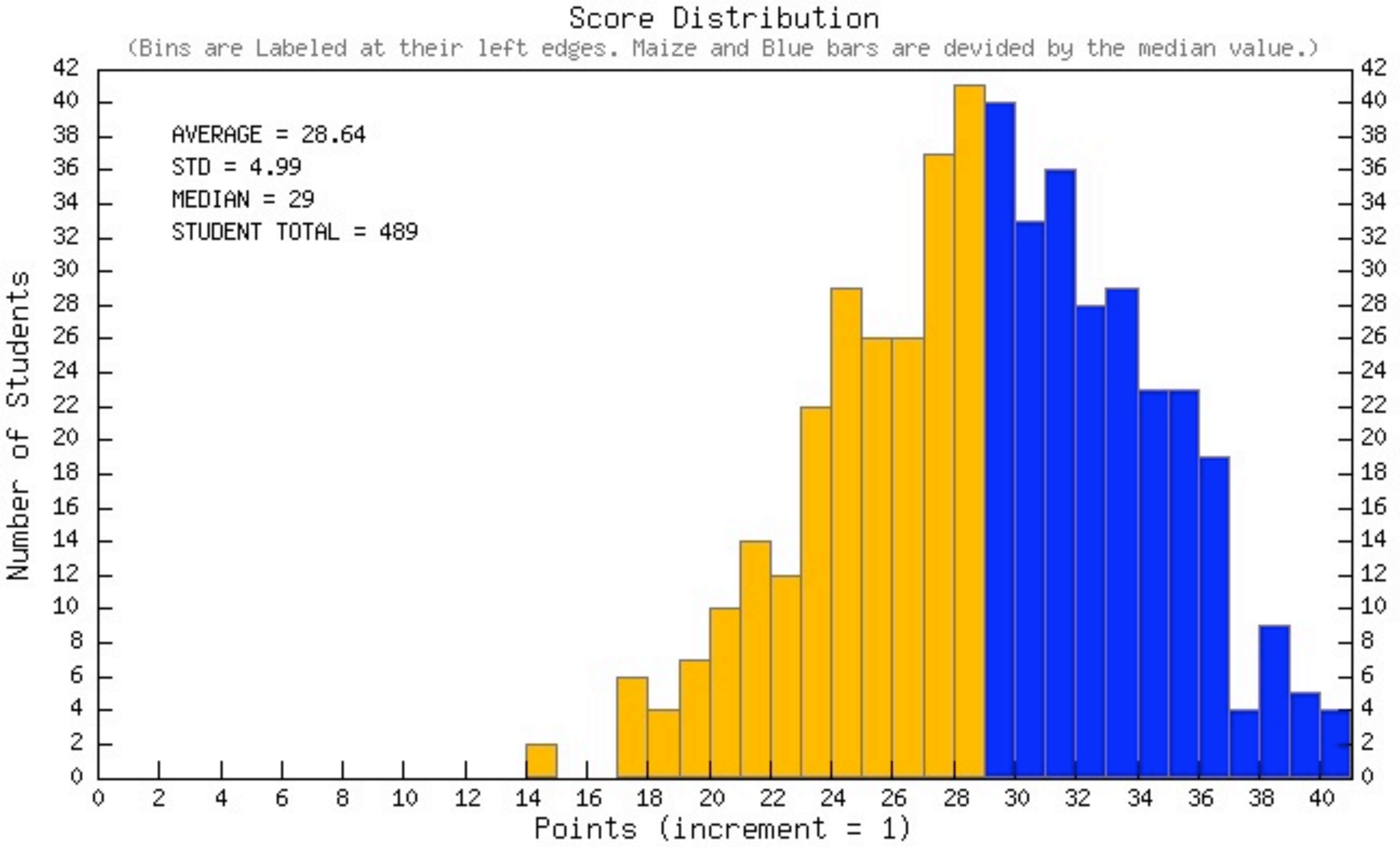
Ch 10 topics:

- angular momentum
- conservation of angular momentum
- precession (gyroscopic motion)



Source: NASA

http://heasarc.gsfc.nasa.gov/docs/xte/Snazzy/Movies/millisecond.html

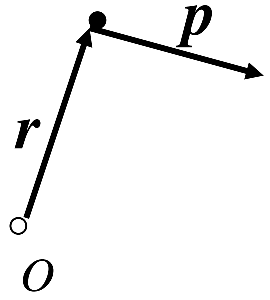

Score Distribution
(Bins are Labeled at their left edges. Maize and Blue bars are devided by the median value.)
AVERAGE = 28.64
STD = 4.99
MEDIAN = 29
STUDENT TOTAL = 489
Number of Students
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40
42
Points (increment = 1)
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40

Angular momentum

A particle at location ***r*** (measured from point O) moving with momentum ***p****=m****v*** has an *angular momentum* defined as the vector product

$$\vec{l} = \vec{r} \times \vec{p}$$

$$= m(\vec{r} \times \vec{v})$$



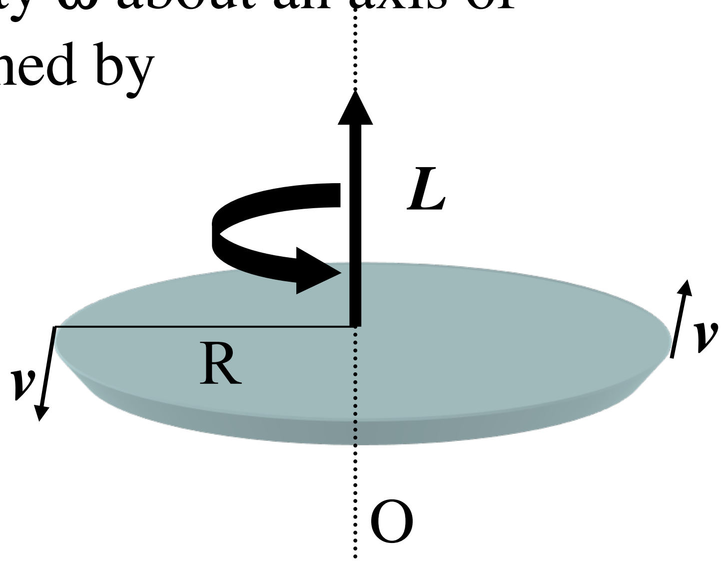
As in the case for linear momentum, a system of particles has a net angular momentum given by the sum of the individual particle contributions

$$\vec{L} = \sum_i \vec{l}_i$$

Angular momentum of a solid body

A solid body rotating with angular velocity **ω** about an axis of symmetry O has angular momentum defined by

$$\vec{L} = I\vec{\omega}$$

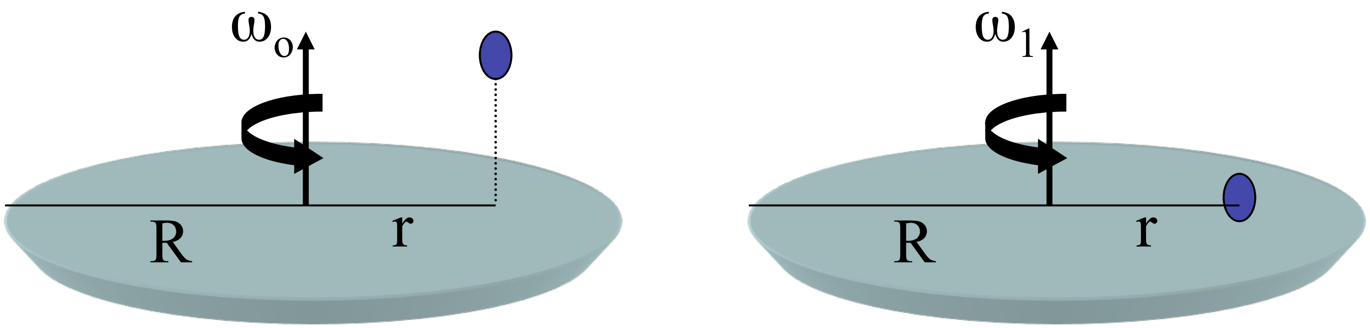


where *I* is the moment of inertia measured about the rotation axis O.

<u>If no external torque acts</u> on the body while internal forces cause its moment of inertia to change, then the angular velocity will react in a manner that <u>*conserves angular momentum*</u>

$$L_f = L_i$$

$$I_f\omega_f = I_i\omega_i$$



A turntable (thin disk) of radius R and mass M spins slowly with angular velocity ω_o about its center. A heavy ostrich egg of mass $8M$ is gently lowered onto the disk and thereafter spins with the turntable. How far from the rotation axis r should the egg be placed in order for the final angular velocity ω_1 of the turntable+egg to be half of the initial value, $\omega_1 = \omega_o/2$?

1. R
2. $3R/4$
3. $R/2$
4. [✓ $R/4$]
5. $R/16$

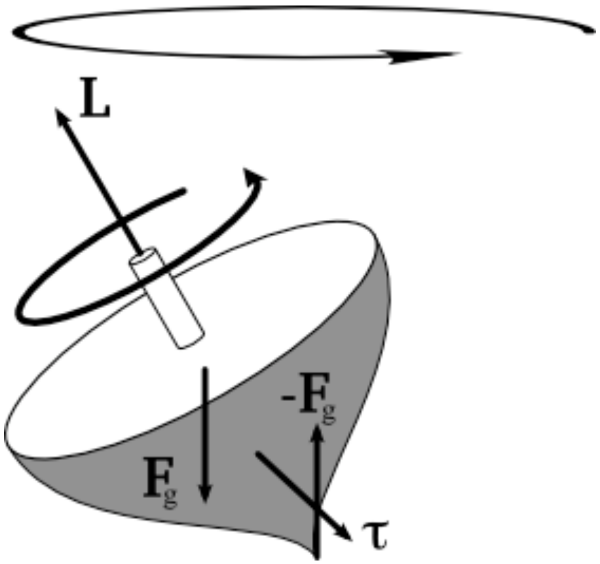
Newton’s second law (last time!)

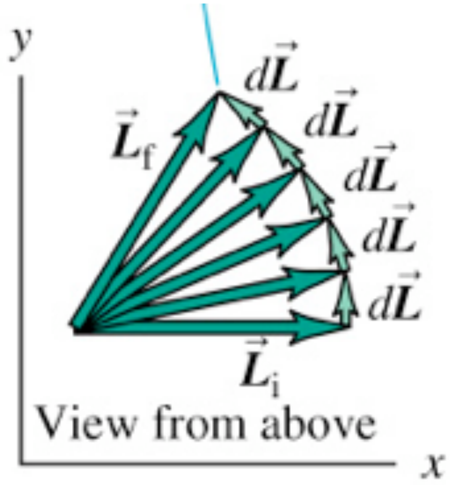
Torque is the time rate of change of angular momentum

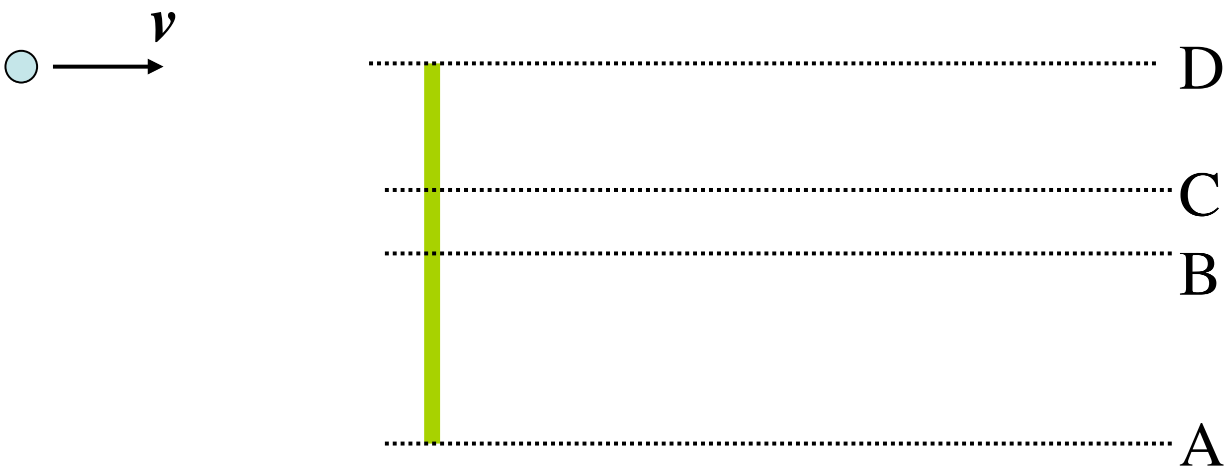
$$\sum \vec{\boldsymbol{\tau}} = \frac{d\vec{L}}{dt}$$

- When ***no net torque acts***, then a system ***conserves its angular momentum***. In particular, a pair of colliding objects conserves the net angular momentum, $\boldsymbol{L}_{\text{after}} = \boldsymbol{L}_{\text{before}}$, if $\Sigma\boldsymbol{\tau}=0$.
- Torque ***parallel to*** the angular momentum causes the system to ***change its angular velocity*** (spin up or down).
- Torque ***perpendicular to*** the angular momentum causes the system to ***change the direction of its angular momentum***, a phenomenon known as ***precession***.

Precession





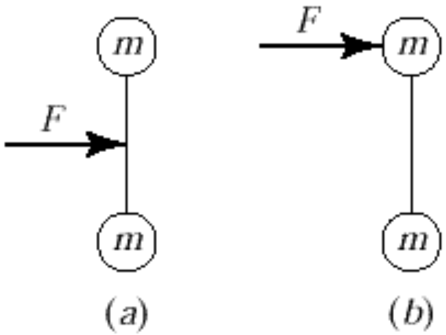


A ball of putty of mass m slides with velocity $\boldsymbol{v}$ across a frictionless ice toward a uniform rod of length L and mass $2m$ that is not anchored to the ice (it is free to translate and rotate). After the putty and rod collide inelastically, about which line will the system rotate?

1. A
2. B
3. [✓ C]
4. D
5. none of the above

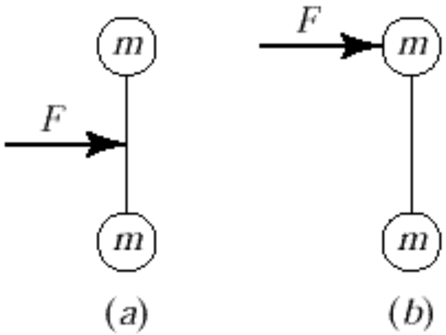
Comparison of linear and rotational motion

Quantity	**Linear Motion**	**Rotational Motion**
displacement	$\boldsymbol{x}$	θ
velocity	$\boldsymbol{v}$	ω
acceleration	$\boldsymbol{a}$	α
inertia	m	$I \sim (\text{constant})mr^2$
kinetic energy	$K_{\text{trans}} = 1/2\ mv^2$	$K_{\text{rot}} = 1/2\ I\omega^2$
momentum	$\boldsymbol{p} = m\boldsymbol{v}$	$\boldsymbol{L} = I\omega$
2nd Law (dynamics)	$\Sigma\ \boldsymbol{F} = \mathrm{d}\boldsymbol{p}/\mathrm{d}t$	$\Sigma\ \tau = \mathrm{d}L/\mathrm{d}t$
work	$W = \mathrm{F}_{\vert\vert}\ \Delta \mathrm{x}$	$W = \tau\ \Delta\theta$
conservation law	$\Delta\boldsymbol{p} = 0$ if $\Sigma\boldsymbol{F}_{\text{ext}}=0$	$\Delta\boldsymbol{L} = 0$ if $\Sigma\tau_{\text{ext}}=0$
impulse	$\boldsymbol{F}\Delta t = \Delta\boldsymbol{p}$	$\tau\ \Delta t = \Delta\boldsymbol{L}$



Two dumbbells rest on a horizontal, frictionless surface (top view shown above). A force F is applied to each dumbbell for a short time interval Δt, either: (a) at the center or (b) at one end. After the impulses are applied, how do the center-of-mass velocities of the dumbbells compare?

1. (a) is greater than (b)
2. (b) is greater than (a)
3. [✓ no difference]
4. can’t tell



Two dumbbells rest on a horizontal, frictionless surface (top view shown above). A force F is applied to each dumbbell for a short time interval Δt, either: (a) at the center or (b) at one end. After the impulses are applied, how do the total kinetic energies of the dumbbells compare?

1. (a) is greater than (b)
2. [✓ (b) is greater than (a)]
3. no difference
4. can’t tell

lesson from the dumbbells about dynamical evolution

1. The sum of external forces describes the linear, center-of-mass motion of a system.

$$\sum \vec{F} = \frac{d\vec{P}}{dt}$$

2. The sum of external torques about the COM describes the system's rotation about its COM.

$$\sum \vec{\tau} = \frac{d\vec{L}}{dt}$$

3. The work-KE theorem applies to both types of motion.

$$\Delta K_{\text{tot}} = W_{\text{tot}} = W_{\text{trans}} + W_{\text{rot}}$$