Physics 140 – Fall 2007
lecture #19: 8 Nov

Ch 11 topics:

- static equilibrium
- stress and strain

static equilibrium

An object that is neither translating (in a particular inertial reference frame) nor rotating is in ***static equilibrium***. For such an object:

1. The sum of the external forces must vanish,

$$\sum \vec{F} = 0$$

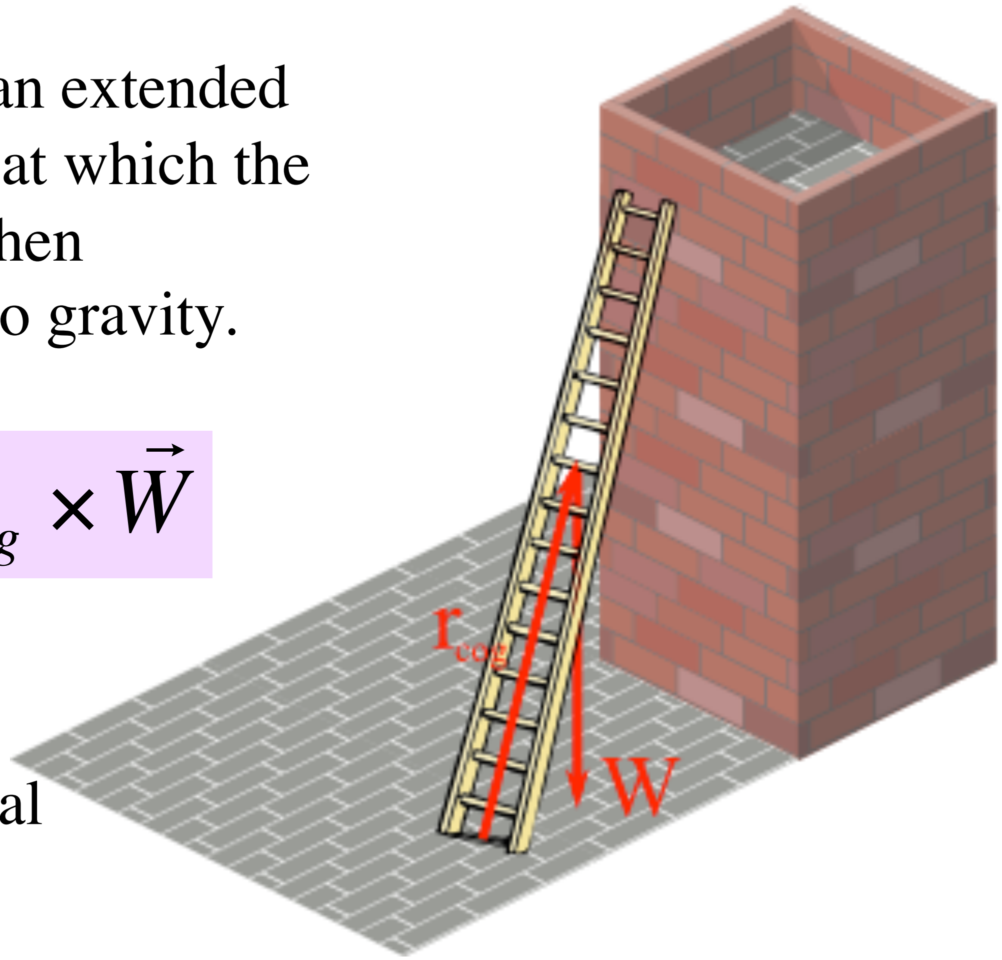
2. The sum of the external torques ***about any point*** must vanish,

$$\sum \vec{\tau} = 0$$

center of gravity = center of mass

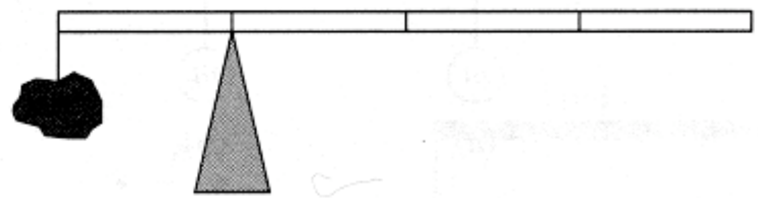
The center-of-gravity of an extended body, $\boldsymbol{r}_{\text{cog}}$, is the location at which the weight effectively acts when calculating a torque due to gravity.

$$\vec{\boldsymbol{\tau}}_{grav} = \vec{r}_{cog} \times \vec{W}$$



In near-Earth gravity, the center of gravity is identical to the center of mass.

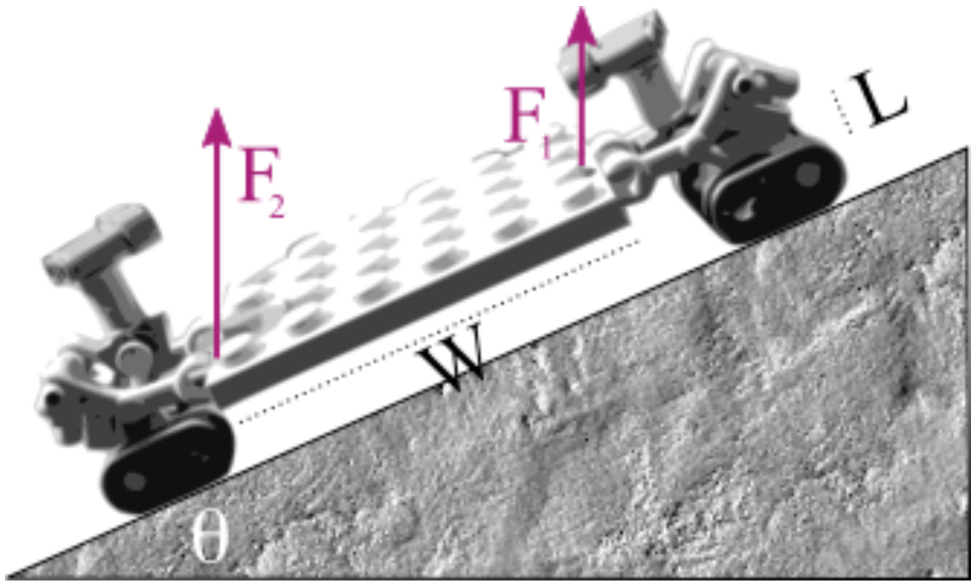
$$\vec{r}_{\text{cog}} M g = \sum \vec{r}_i m_i g \;\; \text{implies} \;\; \vec{r}_{\text{cog}} = \frac{1}{M} \sum \vec{r}_i m_i \equiv \vec{r}_{\text{com}}$$



A rock of mass 0.25 kg is suspended by a very light string from one end of a uniform (1m-long) meter stick. If the rock-stick system balances on a pivot at the 0.25m mark, what is the mass of the meter stick?

1. 1 kg
2. 0.5 kg
3. 0.25 kg
4. 0.125 kg
5. impossible to determine

Two workers are hauling a tall case of select goat cheeses at constant speed up an inclined ramp. They lift vertically on either end of with forces needed to keep the board in equilibrium.



At what incline angle θ will the force exerted by the first (front) worker become zero?

1. $\sin\theta = L/W$
2. $\sin\theta = W/L$
3. $\tan\theta = L/W$
4. $\tan\theta = W/L$

Stress and Strain

A solid will behave somewhat like a spring in response to competing forces, or stress, acting on it. It can stretch or shrink (or bend), depending on how forces are applied.

Consider these simple ways of applying stress to a bar.

applied forces	type of stress	what the bar does
	tension	stretch
	compression	shrink
	shear	bend or twist

Define the ***stress*** on a bar of length L_0 and cross-sectional area A as the **force per unit area**, ***F*/*A***, acting on the bar.

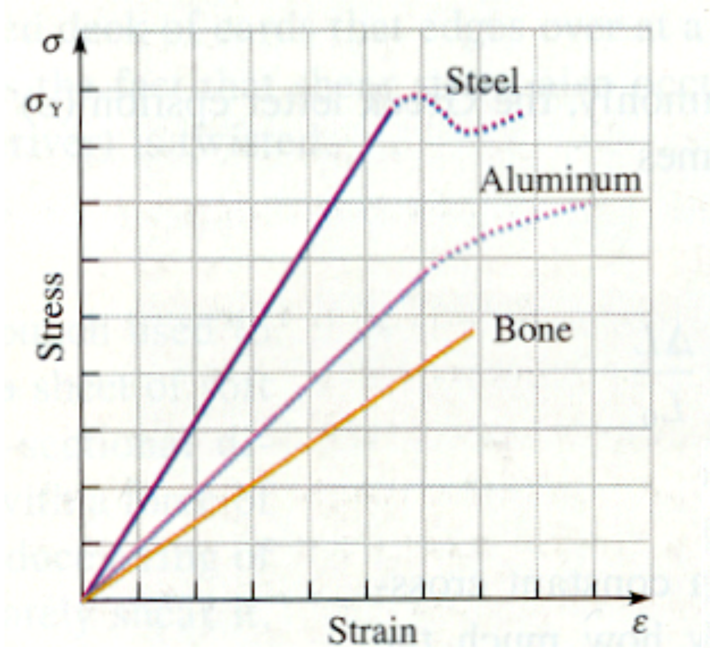
The bar will deform, or ***strain***, under the applied stress, stretching or shrinking by ΔL or bending a distance Δx.

As long as the stress is not too large, the ***fractional strain,*** $\Delta L/L_0$ or $\Delta x/L_0$, is ***linearly proportional to the stress.***

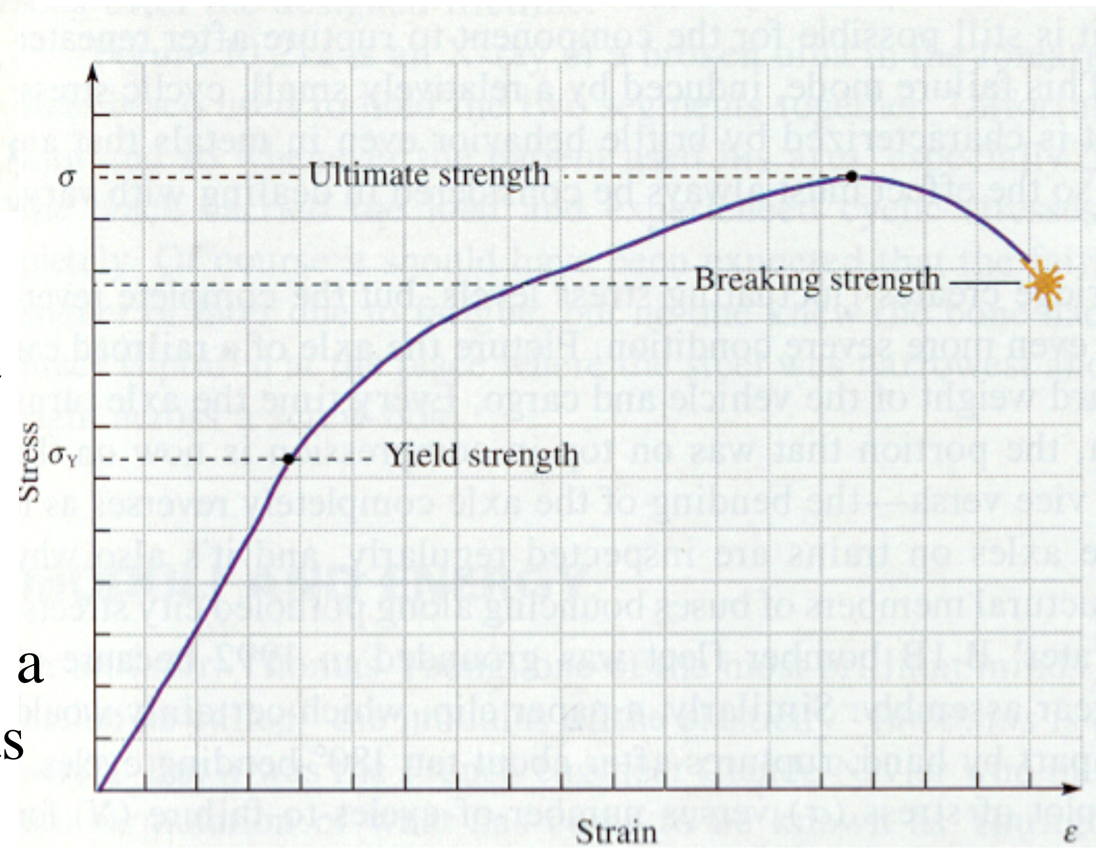
$$F/A = Y\ (\Delta L / L_0) \quad \text{(tension or compression)}$$

$$F/A = S\ (\Delta x / L_0) \quad \text{(shear)}$$

Here, Y, Young's modulus, and S, the shear modulus, are constants that reflect the stiffness of the bar's material.
Both Y and S are measured in units of force per area (N/m^2). The SI unit of this measure, which is also the unit of pressure, is known as the *Pascal,* $1\ \text{Pa} = 1\ \text{N/m}^2$.



The stress-strain behavior depends on the material.



The strain is linearly proportional to stress up to a limit (the Yield strength). Too much stress leads to permanent deformation and breaking. The highest stress a material can take is known as its Ultimate strength.

Hydraulic stress and volume strain

If a solid of initial volume V_0 is moved to an environment in which the surrounding pressure (applied perpendicular force per area) changes by an amount ΔP, then the solid will change in volume by a fractional amount, $\Delta V/V_0$, that is linearly proportional to the change in pressure

$$\Delta P = -B\ (\Delta V / V_0) \quad \text{(volume deformation)}$$

where B is the <u>bulk modulus</u>.

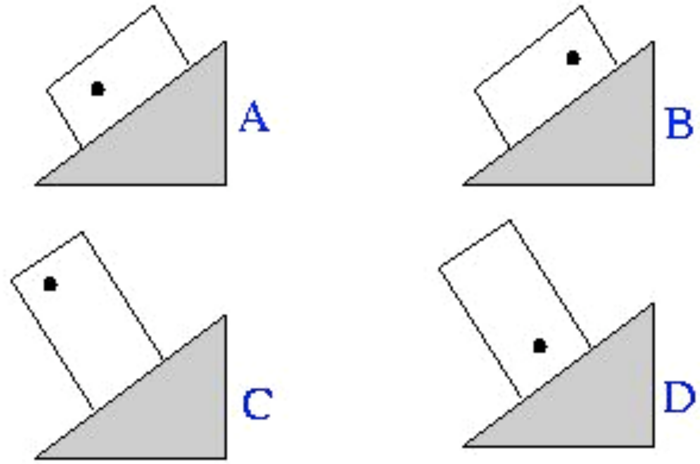
The sign of the above equation reflects the fact that an increase in surrounding pressure leads to a decrease in volume, and vice-verse.

Material	Young's Modulus, Y (Pa)	Bulk Modulus, B (Pa)	Shear Modulus, S (Pa)
Aluminum	7.0×10^{10}	7.5×10^{10}	2.5×10^{10}
Brass	9.0×10^{10}	6.0×10^{10}	3.5×10^{10}
Copper	11×10^{10}	14×10^{10}	4.4×10^{10}
Iron	21×10^{10}	16×10^{10}	7.7×10^{10}
Lead	1.6×10^{10}	4.1×10^{10}	0.6×10^{10}
Nickel	21×10^{10}	17×10^{10}	7.8×10^{10}
Steel	20×10^{10}	16×10^{10}	7.5×10^{10}

The radius of a solid sphere of some material is found to shrink by 0.001% when placed in a pressure chamber under 8 atmospheres of pressure. If a sphere made out of the same material, but with twice the radius of the first, were placed in the same chamber under 8 atmospheres of pressure, what would be the fractional decrease in its radius?

1) 0.0005%
2) 0.001%
3) 0.002%
4) 0.004%
5) 0.008%

A box, with its center-of-mass off-center as indicated by the dot, is placed on a rough inclined plane (so rough that the box does not slide). In which of the four orientations shown, if any, will the box tip over?



1. A
2. B
3. C
4. D