Physics 140 – Fall 2007
lecture #20: 13 Nov

Ch 12 topics:

- Newton's law of gravitation
- gravitational potential energy
- orbital mechanical energy
- escape velocity, circular velocity
- shell theorem

MaPhys sets 11 and 12 open today:
11 closes next Tuesday (20 Nov)
12 closes Tuesday, 27 Nov

Source: NASA

Stress and Strain

A solid will behave somewhat like a spring in response to competing forces, or stress, acting on it. It can stretch or shrink (or bend), depending on how forces are applied.

Consider these simple ways of applying stress to a bar.

applied forces	type of stress	what the bar does
	tension	stretch
	compression	shrink
	shear	bend or twist

Define the ***stress*** on a bar of length L_0 and cross-sectional area A as the **force per unit area**, ***F*/*A***, acting on the bar.

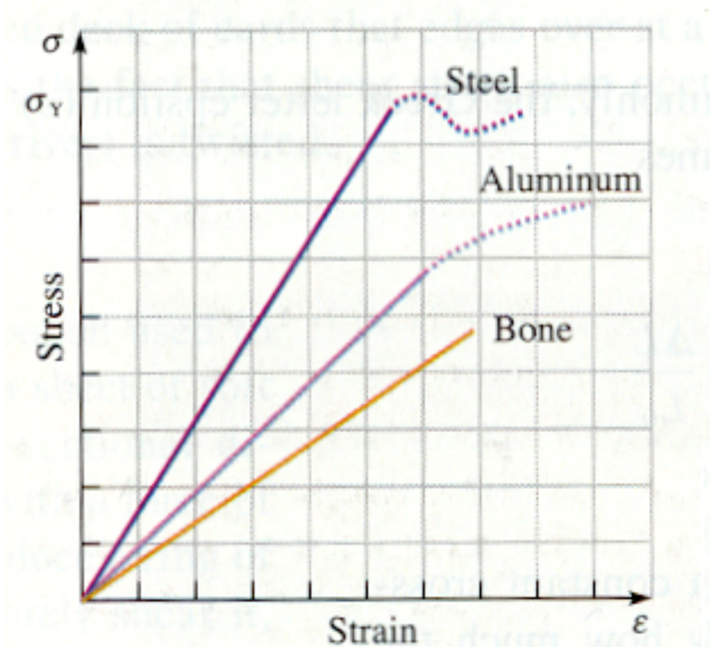
The bar will deform, or ***strain***, under the applied stress, stretching or shrinking by ΔL or bending a distance Δx.

As long as the stress is not too large, the ***fractional strain,*** $\Delta L/L_0$ or $\Delta x/L_0$, is ***linearly proportional to the stress.***

$$F/A = Y\ (\Delta L / L_0) \quad \text{(tension or compression)}$$

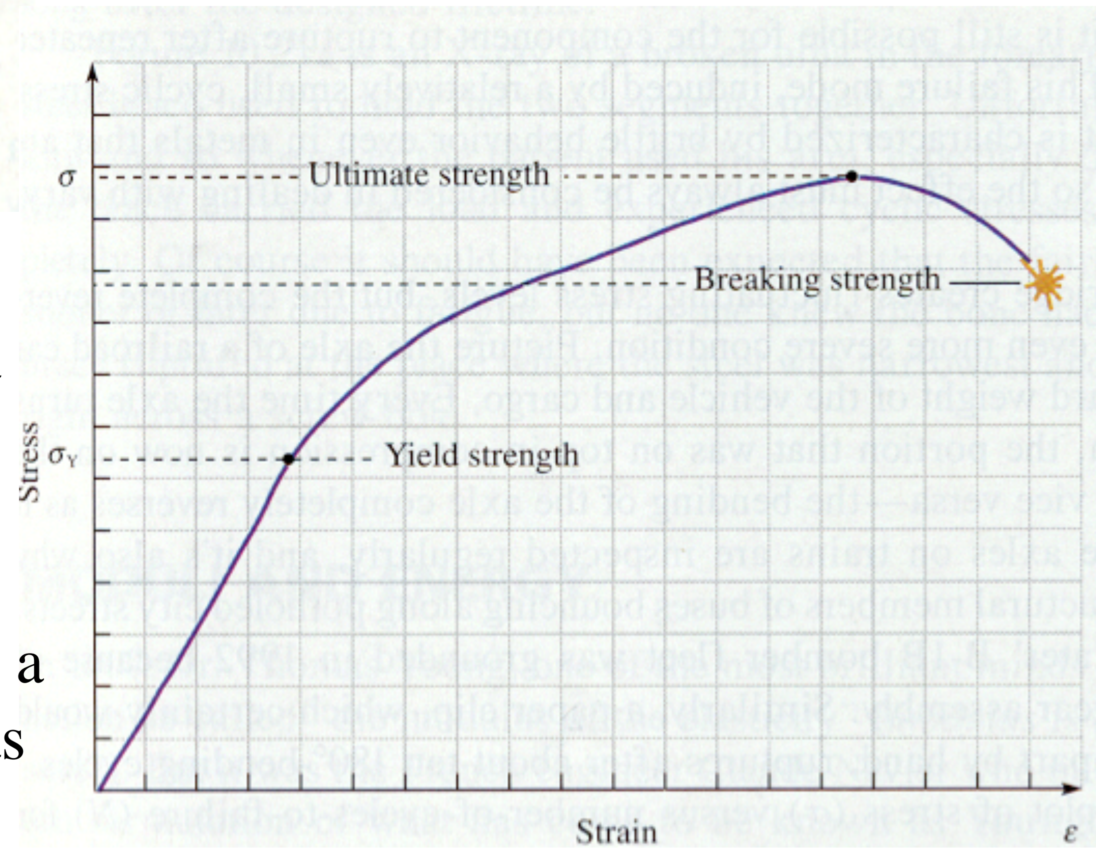
$$F/A = S\ (\Delta x / L_0) \quad \text{(shear)}$$

Here, Y, Young's modulus, and S, the shear modulus, are constants that reflect the stiffness of the bar's material.
Both Y and S are measured in units of force per area (N/m^2). The SI unit of this measure, which is also the unit of pressure, is known as the *Pascal,* 1 Pa = 1 N/m^2.



The stress-strain behavior depends on the material.

The strain is linearly proportional to stress up to a limit (the Yield strength). Too much stress leads to permanent deformation and breaking. The highest stress a material can take is known as its Ultimate strength.



Hydraulic stress and volume strain

If a solid of initial volume V_0 is moved to an environment in which the surrounding pressure (applied perpendicular force per area) changes by an amount ΔP, then the solid will change in volume by a fractional amount, $\Delta V/V_0$, that is linearly proportional to the change in pressure

$$\Delta P = -B \; (\Delta V / V_0) \quad \text{(volume deformation)}$$

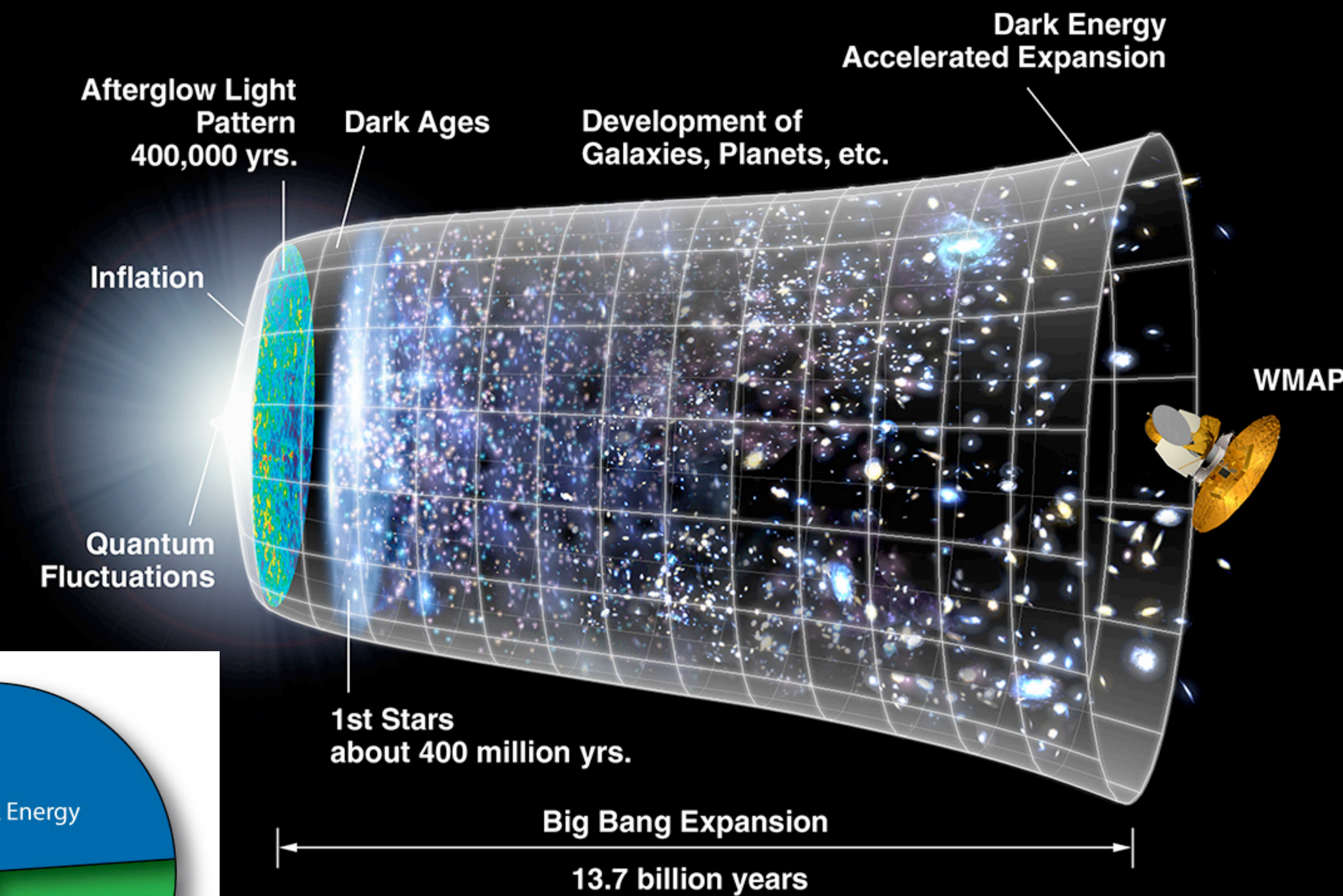
where B is the <u>bulk modulus</u>.

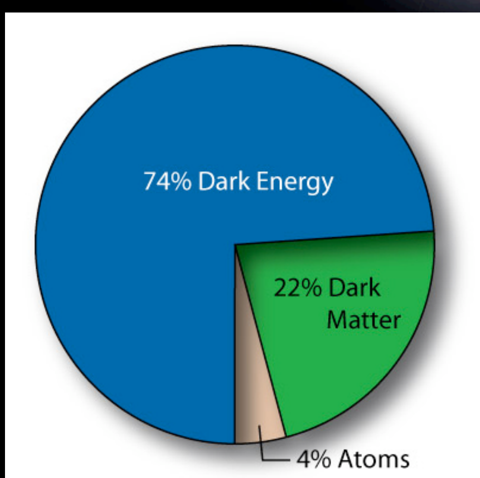
The sign of the above equation reflects the fact that an increase in surrounding pressure leads to a decrease in volume, and vice-verse.

Material	Young's Modulus, Y (Pa)	Bulk Modulus, B (Pa)	Shear Modulus, S (Pa)
Aluminum	7.0×10^{10}	7.5×10^{10}	2.5×10^{10}
Brass	9.0×10^{10}	6.0×10^{10}	3.5×10^{10}
Copper	11×10^{10}	14×10^{10}	4.4×10^{10}
Iron	21×10^{10}	16×10^{10}	7.7×10^{10}
Lead	1.6×10^{10}	4.1×10^{10}	0.6×10^{10}
Nickel	21×10^{10}	17×10^{10}	7.8×10^{10}
Steel	20×10^{10}	16×10^{10}	7.5×10^{10}

Heavy metal. Applying 100kg of weight (1000 N) to a bar of length x and cross-sectional area 0.01m^2 leads to typical strain length of

$$\delta x = (10^5\,\text{Pa}/\text{modulus})\,x \approx 10^{-6}\,x$$





Source: NASA/WMAP Science Team

NASA/WMAP Science Team

The moon does not come crashing down onto Earth because

1. the net force on it is zero.
2. it is in Earth’s gravitational field.
3. it is beyond the main pull of Earth’s gravity.
4. it is being pulled by the Sun and other planets as well as by Earth.
5. its orbit about Earth has angular momentum.
6. it doesn’t obey the laws of gravity.

Newton's law of gravity

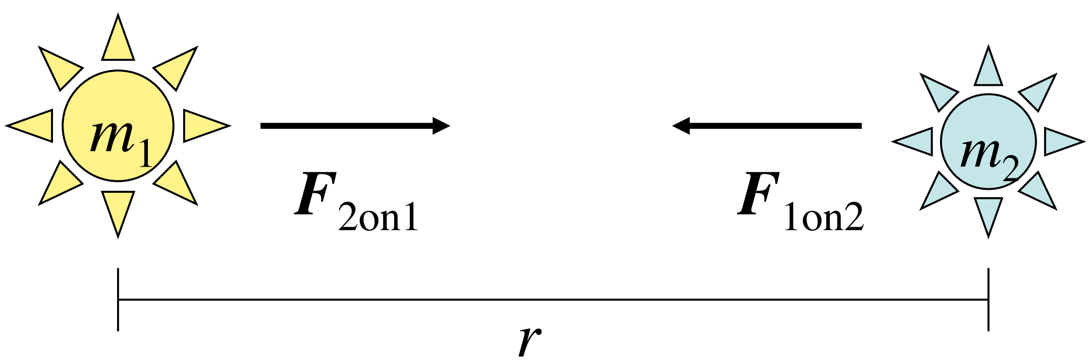
Two bodies of mass m_1 and m_2 separated by a distance r exert on each other an *attractive gravitational force* of magnitude

$$F = G\frac{m_1 m_2}{r^2}$$

where $G = 6.67 \times 10^{-11}\ \mathrm{Nm^2/kg^2}$ is Newton's ***gravitational constant***.

The attractive nature means that the forces are directed so as to pull the bodies together.

Being of equal magnitude and opposite direction, the forces on the two bodies form a Newton's third-law pair.



Remember $\boldsymbol{F}_{\text{AonB}}$ represents "the force that object A exerts on object B".

Cavendish experiment: see gravity in the classroom!

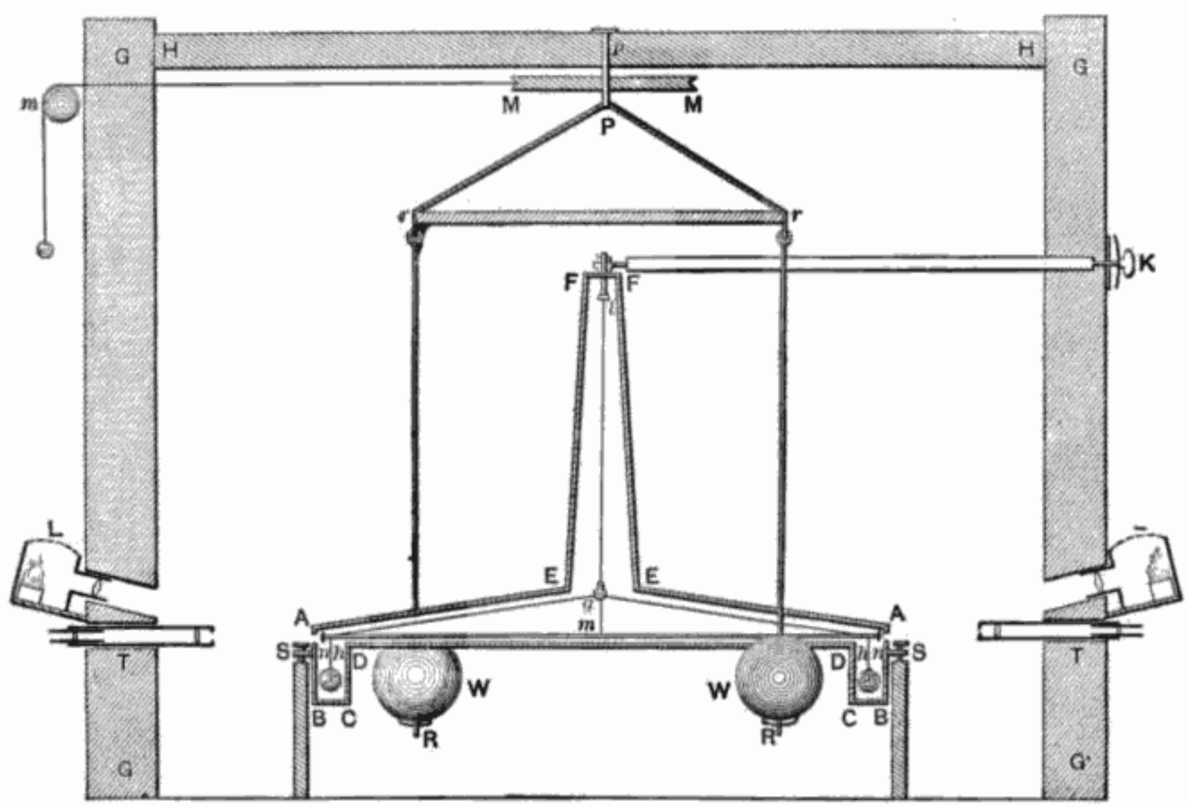


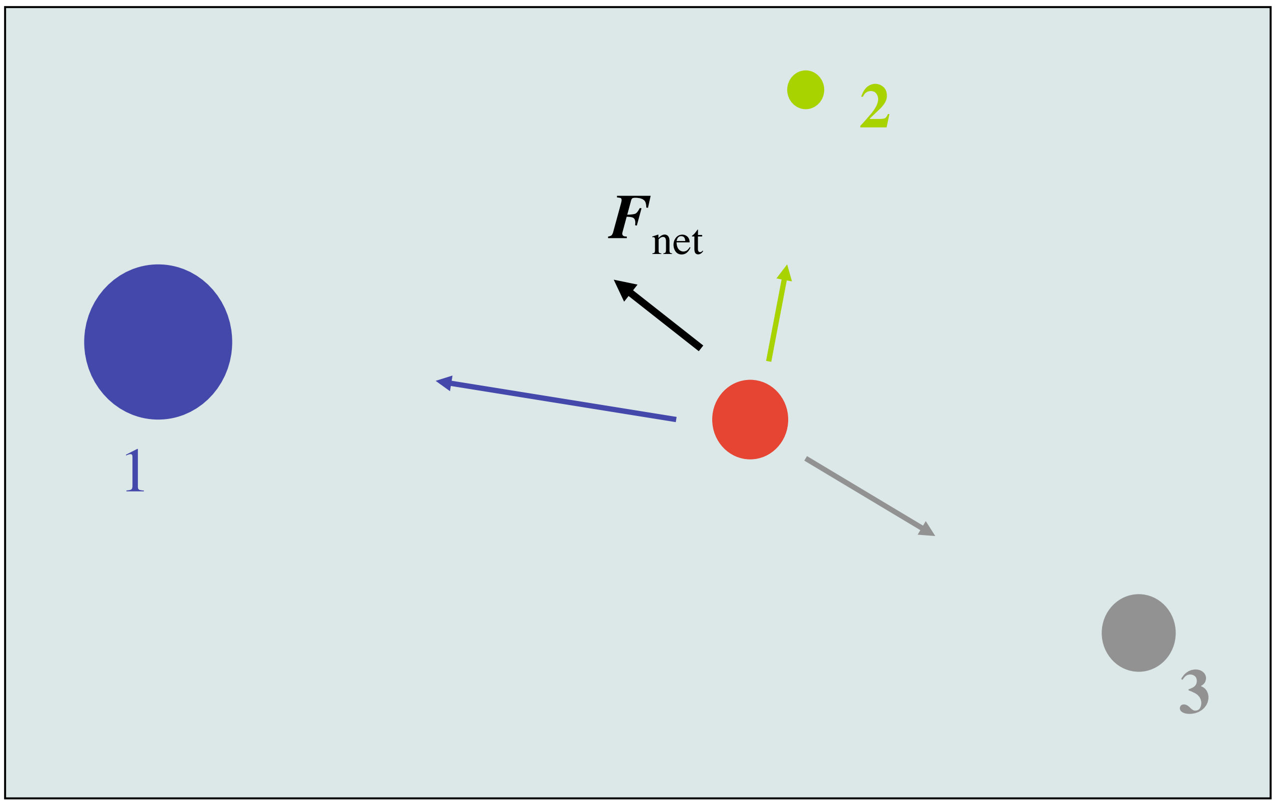
Fig. 1

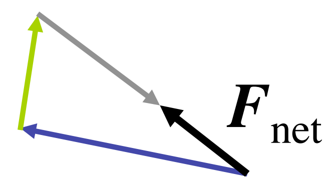
A light beam drifts (slowly!) across the wall as the wire twists.

Source: Philosophical Transactions of the Royal Society of London, (part II) 88 p.469-526 (21 June 1798)

Law of superposition

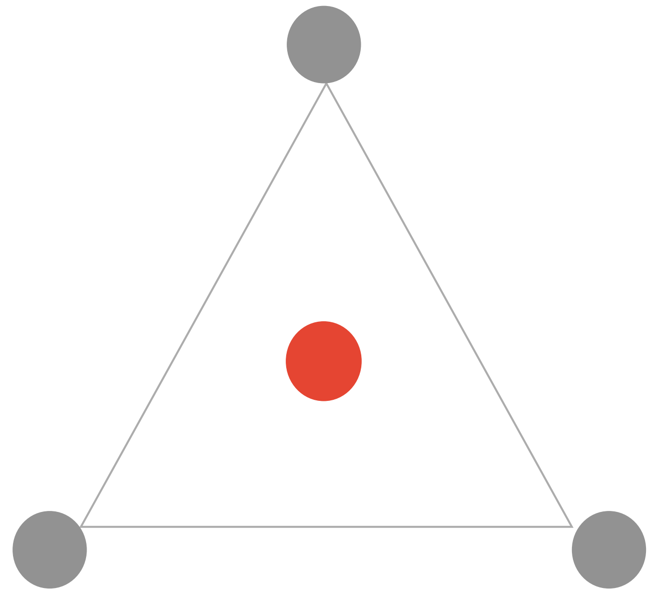
Like all situations involving multiple forces acting on a body, the net force is found by performing the vector sum all the forces. The red body below feels three attractive forces in different directions.





$$\boldsymbol{F}_{\text{net}} = \boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{F}_3$$

The red mass lies in space at the center of an equilateral triangular arrangement of three *identical* grey masses. The net gravitational force acting on the central mass must



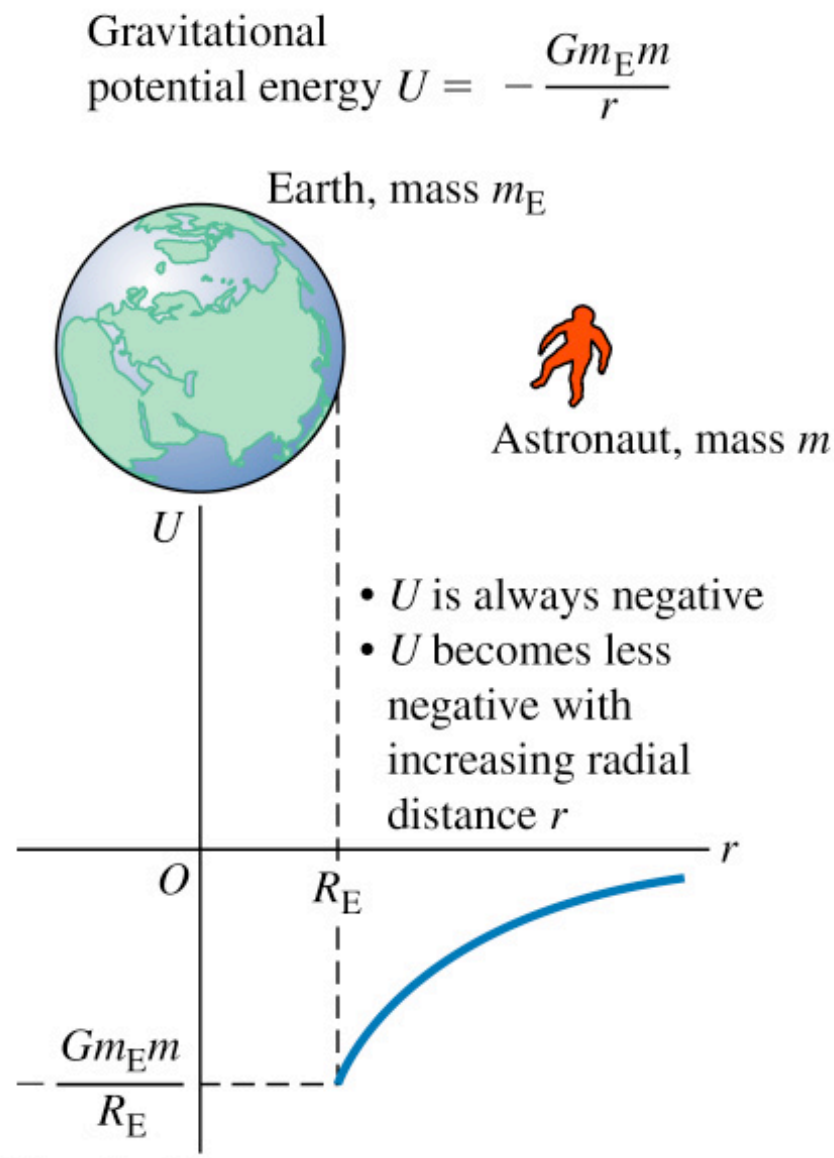
1. act upward in the diagram.
2. act downward.
3. act to the right.
4. act to the left.
5. be zero.

gravitational potential energy

Gravity is a ***conservative force***. When two masses move apart or come together, the work done by gravity depends *only on the overall change in radial separation*, not the detailed path taken.

From $F = -dU(r)/dr$, we find the ***gravitational potential energy*** must have the form

$$U(r) = -G\frac{m_1 m_2}{r}$$



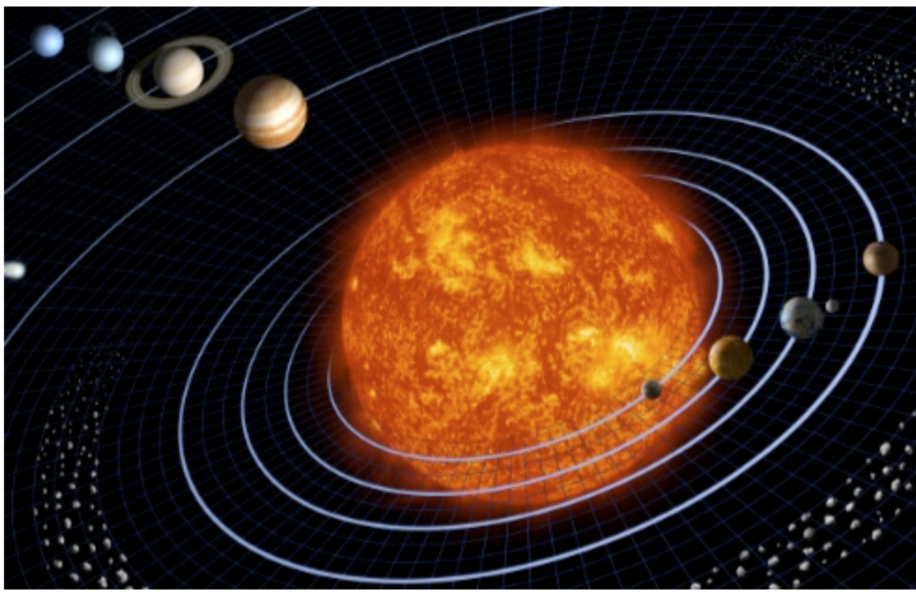
Source: Pearson Education, Inc.

http://hyperphysics.phy-astr.gsu.edu/HBASE/gpot.html

escape and circular velocities

The mechanical energy $E_{\text{mec}}=K+U$ is conserved if no other forces act, so an object launched upward from the surface of a planet of mass M and radius R with sufficient kinetic energy will be able to "escape to infinity" if the initial speed exceeds a critical *escape speed*

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$



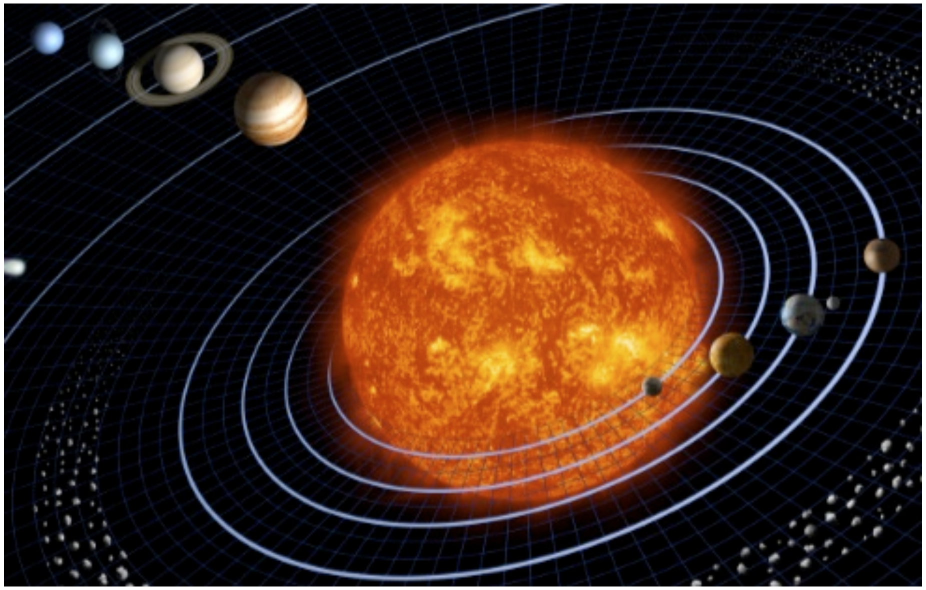
Source: NASA

A satellite in a *circular orbit* of radius r above a planet of mass M will move with speed

$$v_{circ} = \sqrt{\frac{GM}{r}}$$

A satellite orbits the Earth with speed v in a circular orbit of radius r. At what radius would the satellite orbit with speed $v/3$?

1. $2r$
2. $3r$
3. $4r$
4. $6r$
5. $9r$

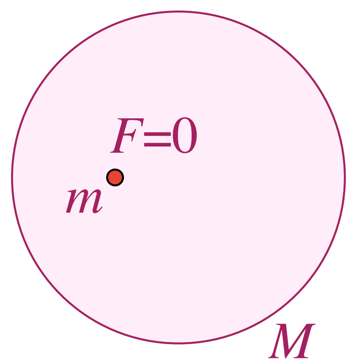


Source: NASA

shell theorem

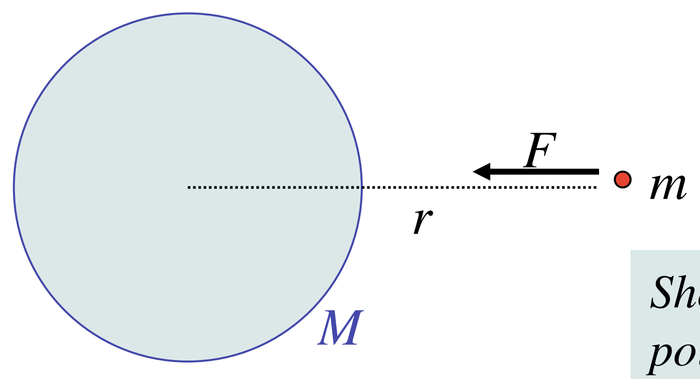
Imagine a ***thin spherical shell*** in three dimensions (like a layer of an onion, a cross section of which is shown below) with mass M concentrated at radius R from its center. Then

- inside the shell, the net gravitational force is zero



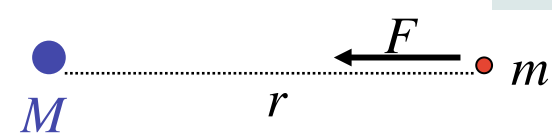
inside

- outside the shell, the gravitational force is the same as that due to a point mass located at the center of the shell.



Shell `feels like' a point mass from outside,

$F = GMm/r^2$



outside