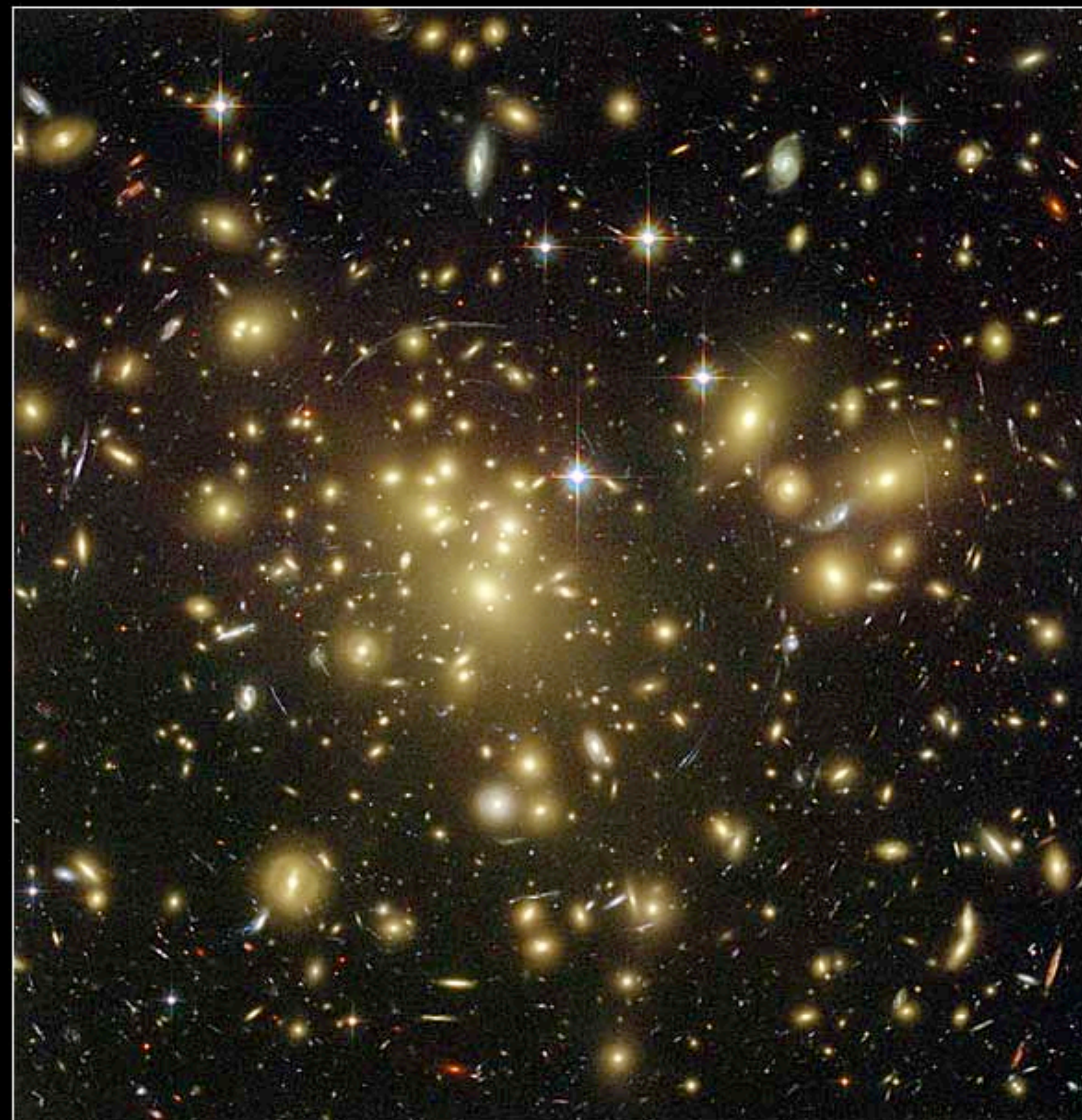
Physics 140 – Fall 2007

15 November: lecture #21

Image of Darth Vader removed

Galaxy Cluster Abell 1689 HST ▪ ACS



NASA, N. Benitez (JHU), T. Broadhurst (Hebrew Univ.), H. Ford (JHU), M. Clampin(STScI), G. Hartig (STScI), G. Illingworth (UCO/Lick Observatory), the ACS Science Team and ESA STScI-PRC03-01a

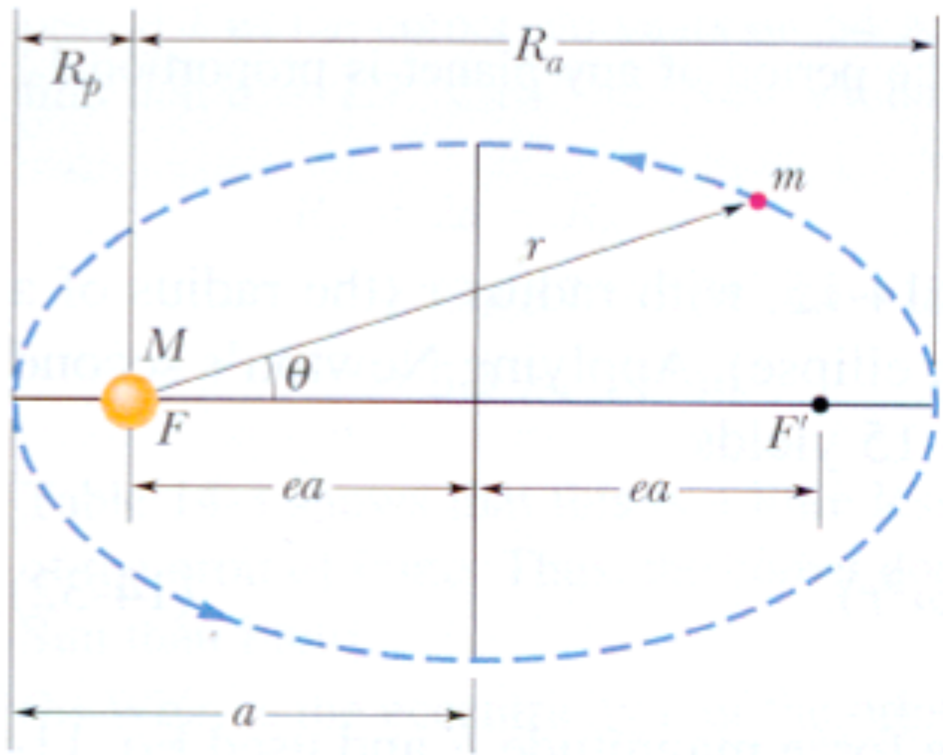
Approximating the earth as a sphere of uniform density, at what radius *inside* the earth is the gravitational acceleration equal to the value that would be felt a height of $3R_E$ *above Earth's surface*?

1. $R_E/ 2$
2. $R_E/ 3$
3. $R_E/ 4$
4. $R_E/ 9$
5. $R_E/ 16$

Kepler's laws of planetary motion

1) Planets move in *ellipses of semi-major axis* a with the sun at a focus.

perihelion -
$R_p = a(1-e)$
closest point in sun-planet orbit



aphelion -
$R_a = a(1+e)$
farthest point in sun-planet orbit

An ellipse has **eccentricity** e, where ea is the distance from the center to a focus.

Java applet: http://www.walter-fendt.de/ph11e/keplerlaw2.htm

2) Planetary orbits sweep out *equal areas in equal times*.

This law reflects the fact that gravity is a **central force**. Since gravity acts along the radial direction connecting two bodies, it produces no torque on either. For a planet of mass *m*, the *angular momentum of the orbit is conserved* and determines the rate of area *A* swept out by its orbit

$$\frac{dA}{dt} = \frac{L}{2m}$$

3) The *square of the orbital period* is proportional to the *cube of the semi-major axis* (and inversely to the Sun's mass *M*)

$$T^2 = \frac{4\pi^2}{GM} a^3$$

A collection of circular orbits around Earth

Radius	Period	Description	Speed
r_E	1.4 hr	Orbiting at surface	7900 m/s
r_E + 200 km	1.5 hr	Low orbit (space shuttle)	7790 m/s
$r_E + 2\ r_E$	7.3 hr	Intermediate orbit	4540 m/s
$r_E + 5.6\ r_E$	1 day	Geosynchronous orbit	3090 m/s
$r_E + 19\ r_E$	5.3 days	Distant orbit	1770 m/s
r_{moon}	27.5 days	Lunar orbit	1025 m/s

mechanical energy and orbit families

Consider an asteroid of mass m in (an arbitrary) orbit around a much larger planet of mass M. The *mechanical energy* of the two-body system

$$E_{\text{mec}} = K + U = \tfrac{1}{2} m v^2 - G \frac{Mm}{r}$$

is a conserved quantity that determines the nature of the orbit.

Different ***families of orbits*** result from different signs of E_{mec}.

family	$\mathbf{E_{mec}}$	**eccentricity *e***	**orbit**
bound	< 0	< 1 (0)	ellipse (circle)
just unbound	$= 0$	$= 1$	parabola
really unbound	> 0	> 1	hyperbola

Bound (negative energy) orbits

If two bodies of masses m and M are in a gravitationally bound orbit, the ***mechanical energy*** determines the ***size of the orbit***, defined as the *semi-major axis a*, of the two-body system

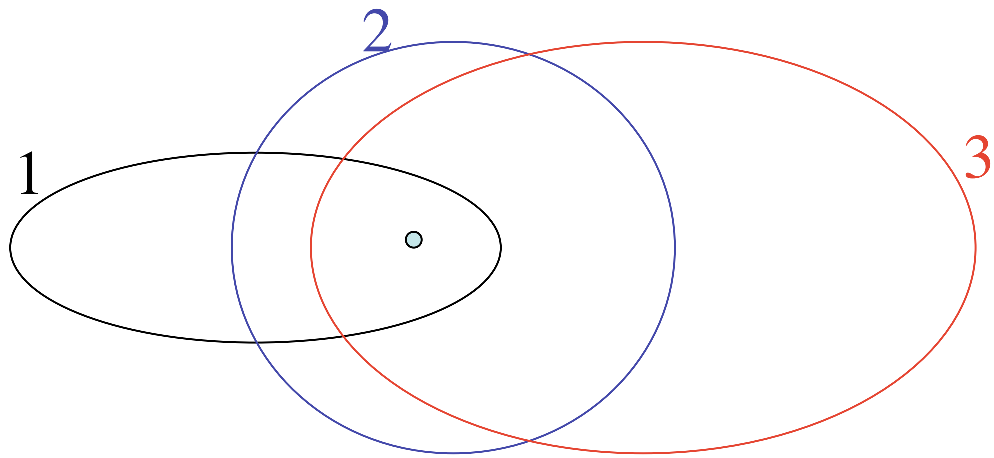
$$E_{\mathrm{mec}} = -\frac{GMm}{2a}$$

while the ***angular momentum L*** determines the ***shape of the orbit,*** defined by the *eccentricity e*

$$L^2 = GMm^2\, a\, (1-e^2)$$

For a set of bodies in circular orbits around a large mass M, the square of the orbital speed decreases inversely with distance r

$$v^2 = GM / r$$



Which orbit has the ***smallest*** angular momentum?

→ **1.**

2.

3.

4. more information is needed

Images of Lensing removed

http://cfa-www.harvard.edu/~bmcleod/castle.html