PHYSICS 140 - General Physics 1, Fall 2007

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<http://hdl.handle.net/2027.42/64964>
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Approximating the earth as a sphere of uniform density, at what radius inside the earth is the gravitational acceleration equal to the value that would be felt a height of $3R_{E}$ above Earth’s surface?

1. $R_{E}/ 2$
2. $R_{E}/ 3$
3. $R_{E}/ 4$
4. $R_{E}/ 9$
5. $R_{E}/ 16$
Kepler’s laws of planetary motion

1) Planets move in *ellipses of semi-major axis* \( a \) with the sun at a focus.

An ellipse has *eccentricity* \( e \), where \( ea \) is the distance from the center to a focus.

**perihelion** - 
\[ R_p = a(1-e) \]
closest point in sun-planet orbit

**aphelion** - 
\[ R_a = a(1+e) \]
farthest point in sun-planet orbit
2) Planetary orbits sweep out **equal areas in equal times**.

This law reflects the fact that gravity is a **central force**. Since gravity acts along the radial direction connecting two bodies, it produces no torque on either. For a planet of mass $m$, the **angular momentum of the orbit is conserved** and determines the rate of area $A$ swept out by its orbit

$$ \frac{dA}{dt} = \frac{L}{2m} $$

3) The **square of the orbital period** is proportional to the **cube of the semi-major axis** (and inversely to the Sun’s mass $M$)

$$ T^2 = \frac{4\pi^2}{GM} a^3 $$

[Java applet: http://www.walter-fendt.de/ph11e/keplerlaw2.htm]
### A collection of circular orbits around Earth

<table>
<thead>
<tr>
<th>Radius</th>
<th>Period</th>
<th>Description</th>
<th>Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_E$</td>
<td>1.4 hr</td>
<td>Orbiting at surface</td>
<td>7900 m/s</td>
</tr>
<tr>
<td>$r_E + 200$ km</td>
<td>1.5 hr</td>
<td>Low orbit (space shuttle)</td>
<td>7790 m/s</td>
</tr>
<tr>
<td>$r_E + 2 r_E$</td>
<td>7.3 hr</td>
<td>Intermediate orbit</td>
<td>4540 m/s</td>
</tr>
<tr>
<td>$r_E + 5.6 r_E$</td>
<td>1 day</td>
<td>Geosynchronous orbit</td>
<td>3090 m/s</td>
</tr>
<tr>
<td>$r_E + 19 r_E$</td>
<td>5.3 days</td>
<td>Distant orbit</td>
<td>1770 m/s</td>
</tr>
<tr>
<td>$r_{moon}$</td>
<td>27.5 days</td>
<td>Lunar orbit</td>
<td>1025 m/s</td>
</tr>
</tbody>
</table>
Consider an asteroid of mass $m$ in (an arbitrary) orbit around a much larger planet of mass $M$. The mechanical energy of the two-body system

$$E_{\text{mec}} = K + U = \frac{1}{2}mv^2 - G\frac{Mm}{r}$$

is a conserved quantity that determines the nature of the orbit.

Different families of orbits result from different signs of $E_{\text{mec}}$.

<table>
<thead>
<tr>
<th>family</th>
<th>$E_{\text{mec}}$</th>
<th>eccentricity $e$</th>
<th>orbit</th>
</tr>
</thead>
<tbody>
<tr>
<td>bound</td>
<td>$&lt; 0$</td>
<td>$&lt; 1$ (0)</td>
<td>ellipse (circle)</td>
</tr>
<tr>
<td>just unbound</td>
<td>$= 0$</td>
<td>$= 1$</td>
<td>parabola</td>
</tr>
<tr>
<td>really unbound</td>
<td>$&gt; 0$</td>
<td>$&gt; 1$</td>
<td>hyperbola</td>
</tr>
</tbody>
</table>
Bound (negative energy) orbits

If two bodies of masses $m$ and $M$ are in a gravitationally bound orbit, the *mechanical energy* determines the *size of the orbit*, defined as the *semi-major axis* $a$, of the two-body system

$$E_{\text{mec}} = -\frac{GMm}{2a}$$

while the *angular momentum* $L$ determines the *shape of the orbit*, defined by the *eccentricity* $e$

$$L^2 = GMm^2 \ a \ (1-e^2)$$

For a set of bodies in *circular orbits* around a large mass $M$, the square of the orbital speed decreases inversely with distance $r$

$$v^2 = \frac{GM}{r}$$
Which orbit has the *smallest* angular momentum?

1. 
2. 
3. 
4. more information is needed
http://cfa-www.harvard.edu/~bmcleod/castle.html