BASE (SYMMETRIC INFORMATION) MODEL FOR CONTRACT THEORY

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1. Preliminaries

Principal and agent enter a relationship. Assume:

- They have access to the same information (including agent effort)
- All information is verifiable (a contract contingent on observable information can be enforced by courts)

The relationship will have a random outcome $x \in X$ (a finite set):

- x is a random variable with prior distrib known to both principal and agent, and the distribution depends on agent effort e
- $\Pr[x = x_i|e] = p_i(e)$, for $i \in \{1, 2, \ldots, n\}$
- $p_i(e) > 0$ for all e, i: Cannot rule out any result for any given effort level (no perfect backward inferences). (NB: This matters for future analyses, when effort is "hidden" (unobservable or unverifiable by principal). Doesn't matter for the model below.)

Principal's utility function is $\pi(x - t)$ where t is the transfer (payment) *to the agent*.

- $\pi' > 0$, $\pi'' \leq 0$ (concave utility $\implies$ risk neutral or risk averse).
- Profit doesn't depend directly on agent effort or state of nature, but only the outcome.

Agent's utility function is $U(t, e) = u(t) - v(e)$, which depends on transfer and effort.

- Additive separability means risk-aversion is independent of effort level (not a crucial assumption).
- Utility is concave in transfer, convex in effort: $u'(t) > 0$,$u''(t) \leq 0$, $v'(e) > 0$, $v''(e) \geq 0$.

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- Agent has free will: will not sign contract unless the expected utility is at least as high as his next best opportunity: *reservation utility* U_0

There is a clear conflict of interest:

(1) principal cares about result, agent doesn't (directly)
(2) principal not directly interested in effort, agent is
(3) but effort makes better results more likely (NB: Macho-Staedler and Perez-Castrillo state this but don't make the assumption explicit.)

The transfer t is the design variable for the contract. This is what the principal can set to try to reconcile the conflict: principal wants effort, pays agent t for effort. How big should t be? On what should t depend?

2. The optimal transfer

With symmetric verifiable info, the principal's problem is to choose e required, and offer $\{t(x_i)\}$, $i = 1, \ldots, n$, to maximize surplus (expected profit), subject to agent willing to take the contract ($\mathrm{E}[U(x, e)] > U_0$).

The principal can specify the level of effort require *and* the contract terms because effort is verifiable. Thus, the principal can make a "take it or leave it" offer of the form "work e hours for me, and I'll pay you $t(x)$, or don't work for me at all." It's up to the agent to decide whether to accept this offer (the participation constraint).[1]

$$\max_{e, \{t(x_i)\}} \sum_{i=1}^{n} p_i(e)\pi(x_i - t(x_i))$$

$$\text{s.t.} \sum_{i=1}^{n} p_i(e)u(t(x_i)) - v(e) \geq U_0 \tag{IR}$$

Set up the Lagrangian and take first order condition for $t(x_i)$:

$$\mathcal{L} = \sum_{i=1}^{n} p_i(e)\pi(x_i - t(x_i)) - \lambda \sum_{i=1}^{n} p_i(e)u(t(x_i)) - v(e) - U_0$$

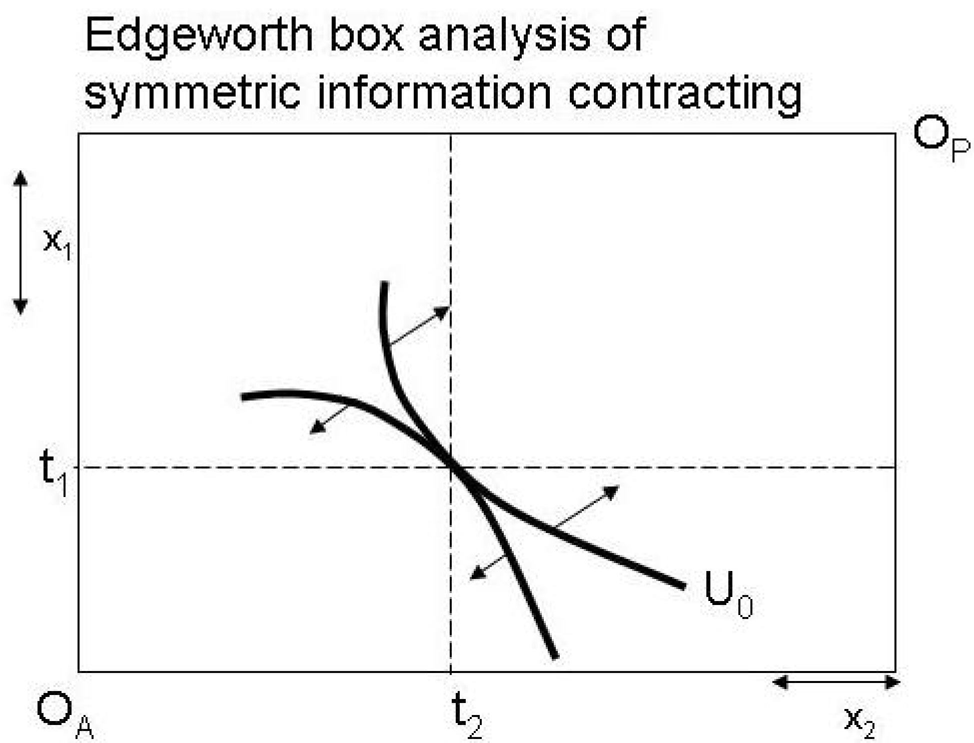
$$\lambda^* = \frac{\pi'(x_i - t^*(x_i))}{u'(t^*(x_i))} \text{ for all } i \tag{FOC1}$$

[1] When we get to hidden action, of course, the principal will not be able to specify *both* the effort level and the transfer terms.

What can we say about the solution? First, the participation constraint (IR) must bind: if not, the principal could reduce the transfer in every state by $\mathrm{E}[U] - U_0$.

What do we know from (FOC1)?

(1) The outcome (with symmetric information) is Pareto efficient: Holding fixed agent U, maximize the principal's utility, $\pi(x - t)$. Can change distribution of surplus by changing U_0.
(2) (FOC1) says ratio of marginal utilities (the MRS) should be constant: usual condition for Pareto efficient outcome.



(3) If π' is constant (P is risk-neutral) then $u'(t^*(x_i)) =$ constant for all i, since the ratio is a constant (λ^*). If u' is not constant for all transfers (that is, if the agent is *not risk neutral*), then the solution when π' is constant requires that all *transfers*

be constant, that is $t^*(x_i) = t^*(x_j)$ for all i, j. In short, when the principal is risk-neutral, she completely insures the agent (who gets constant t^*).

(4) Indeed, with P risk-neutral, we can solve for the optimal transfer explicitly, since (IR) is binding $(\mathrm{E}[U] = U_0) : t^* = u^{-1}(U_0 + v(e^*))$. That is, it's the cash equivalent of U_0 plus the amount necessary to compensate A for disutility of working.

(5) Suppose A is risk-neutral (u' constant) and P is not. Then $\pi'(x_i - t^*(x_i))$ is constant, so $t^*(x_i) = x_i - k$ for some constant k. Essentially, P gives entire output to A, and A pays a constant "franchise fee" k back. Or, put another way, P sells the firm to A for price p. Since (IR) must be satisfied:

$$\sum_{i=1}^{n} p_i(e^*)(x_i - k) = U_0 + v(e^*), \text{so}$$

$$k = \sum_{i=1}^{n} p_i(e^*)x_i - U_0 - v(e^*).$$

That is, k (the price of buying the firm) is set equal to expected profit of owning the firm less the amount necessary for A to participate. When both are risk averse, the optimal contract lies somewhere in-between: share risk so that both have some uncertainty about their outcome (but A still gets only U_0).

3. The optimal effort

The choice of effort affects the probability of different output levels (that is, of course, the whole point). Without strong restrictions on the effect that effort has on distribution of output, *expected utility might not be concave*: and that means *the first order conditions might not provide a maximum.*[2]

Main intuitive result (necessary, not sufficient): the expected profits from an increase in effort must be equal to the transfer increment P pays A to compensate for increased disutility of effort.

Effort: I mentioned above that solving for effort is complex (because the problem may not be concave). Let's look at a simple case that is easy to solve: no uncertainty. Assume that output depends on effort according to $x = f(e)$, with $f' > 0$, $f'' < 0$. Then principal solves:

$$\max_{e,\{t(e)\}} f(e) - t(e)$$
$$\text{s.t. } t(e) - v(e) \geq U_0$$

[2]The principal, or the analyst cannot "insist" that the relationship between effort and output satisfy the conditions necessary for expected utility to be concave: how effort affects output is more or less a given for a particular problem. It's true, the principal might design her production process to change the way that effort affects output, to some extent, but you wouldn't do this just to make the math easier!

(since $x = f(e)$ we now express everything in terms of e for simplicity). Suppose the principal offers a linear wage contract with fixed payment, $t(e) = we + K$. Then the agent chooses how much to work according to:

$$\max_{e} we + K - v(e)$$

which has FOC:

$$w^* = v'(e).$$

Now the principal chooses the parameters of the wage function:

$$\max_{w,K} f(e) - we - K$$

$$\text{s.t. } we + K - v(e) \geq U_0. \qquad \text{(IR)}$$

Since K only makes principal worse off, it is chosen to be as small as possible so that (IR) is binding (satisfied as an equality). Then we can substitute the (IR) constraint into the profit function and optimize:

$$\max_{w,K} f(e) - v(e) - U_0$$

which has FOC:

$$f'(e) = v'(e) = w^*$$

where the last equality is from the solution of the agent's effort choice problem. So, the principal sets the variable component of compensation equal to the marginal product of effort, $f'(e)$.

Summary: wage equals marginal product. Fixed payment just enough to make the worker willing to work overall ($K = U_0 + v(e) - we$). Output is efficient ($f'(e) = v'(e)$), which is the usual efficiency condition that marginal product equal marginal cost.

4. Questions for thought

What change in the information conditions above might lead to a result in which the agent would get expected utility from the contract that is greater than U_0?

(1) If U_0 were unknown by P, then P and A would have to bargain over how much rent to give to A and the outcome will generally be $\mathrm{E}[U] \geq U_0$. That is, because P does not know A's walking away point (U_0), P can't push A all the way down to the minimal level of utility. How much better A will do depends on A's bargaining power.
(2) If e is unobservable (or unverifiable), then the same idea: they would bargain. Although P knows A's walking away point (U_0), P doesn't know how close she is pushing A to that point because she doesn't know exactly how much effort e that A will exert. This situation (asymmetric information about e, the hidden action problem) is the topic of next week's analysis.

The amount by which $\mathrm{E}[U] > U_0$ is A's information rent. The more valuable is the unknown information, and the more bargaining power A has, the more information rent A will receive (the higher will be $\mathrm{E}[U]$).