HIDDEN ACTION 1: CONTRACTING MODEL WITH ASYMMETRIC INFORMATION

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1. Quick Review: Effort contracting with symmetric information

N.B. Throughout I abbreviate "Principal" as "P", and "Agent" as "A".

Recall the base model from last class (see Table 1 for definitions of the notation):

$$\max_{e,\{t(x_i)\}} \sum_{i=1}^{n} p_i(e)\pi(x_i - t(x_i))$$

$$\text{s.t.} \sum_{i=1}^{n} p_i(e)u(t(x_i)) - v(e) \geq U_0 \qquad \text{(IR)}$$

Table 1. Contracting model notation

e	effort
$t(x_i)$	transfer to agent when output is 'x_i
$p_i(e)$	prob. of state ' i' when effort is e
$v(e)$	disutility of effort
$u()$	agent's utility function
$\pi()$	principal's utility function

The first-order conditions with respect to the transfers at each value of x_i are::

$$\lambda^* = \frac{\pi'(x_i - t^*(x_i))}{u'(t^*(x_i))} \text{ for all } i \qquad \text{(FOC1)}$$

Results:

(1) The outcome (with symmetric information) is Pareto efficient: Holding fixed agent utility, max P's utility. Can change distribution of surplus by changing U_0.

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(2) (FOC1) says ratio of marginal utilities (the MRS) should be constant: usual condition for Pareto efficient outcome
(3) When principal risk-neutral, she completely insures agent (who gets constant t^*).
(4) Suppose A is risk-neutral and P is not. Then P sells the firm to A for a price equal to expected profit of owning the firm less the amount necessary for A to participate.
(5) Marginal expected profit from increased effort must be equal to the transfer increment P pays A to compensate for increased disutility of effort.

2. Hidden Action

Now assume that the agent's effort is not observable (or at least, not verifiable). Then, effort cannot be included as a term of the contract, because it cannot be enforced.

Consider the timing of the problem (the "moves" of the "game"); see Figure 1. Understanding the timing of a strategic interaction is the first crucial step to thinking strategically. We'll show this by thinking logically through the problem of undertaking this project.

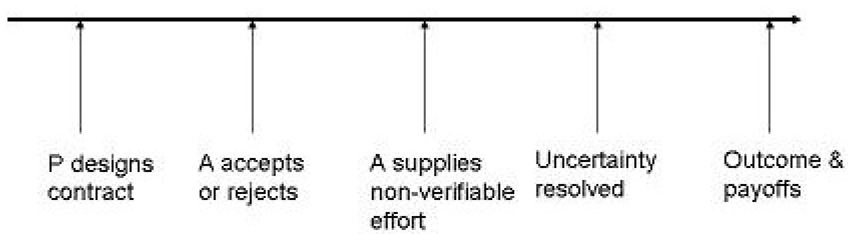

Figure 1. Timing of the canonical hidden action problem

When are strategic decisions made? Reviewing the timing from finish to start:

- The last event occurs when output is measured, and payments made, all according to the contract terms.
- The penultimate move is made by nature: the resolution of output uncertainty.
- The third event is the agent's strategic choice about how much effort to make.
- The second event is the agent's strategic choice about whether to enter he agreement.
- The first event is the principal's strategic choice about what agreement terms to offer.

We're primarily interested in the design problem: what terms should the principal offer? But to answer that, P has to think about what A will do, given the terms offered. So the solution method is to first figure out A's response to various contract terms, then design the terms.

3. The Incentive-Compatibility Constraint (IC)

The agent chooses effort at stage 3. At this point in time, the contract is signed, there is no way to verify effort, and so A can choose whatever effort level she would like. A's freedom to choose the level of effort is a constraint on how well P can do. We call this the incentive-compatibility (IC) constraint.

$$e^* = \arg\max_e \{\sum_{i=1}^{n} p_i(e)u(t(x_i)) - v(e)\} \tag{IC}$$

Remark Notice that the (IC) is A's effort supply function. We will use this characterization of it below.

Remark We are not asserting that all aspects of effort are impossible to verify. Several indicators of effort may be observable, and some of these may be verifiable. For example, whether A shows up to the office often can be verified. Whatever components of effort can be verified can of course be included in the contract. For example, the contract may say if A doesn't show up at least a certain percentage of days, A will be fired. What we are modeling in this example is limited to components of effort that cannot be verified (how many neurons were firing per second?).

4. The Participation Constraint (PC)

At stage 2, the agent decides whether to enter the agreement or not. This is another constraint on the contract that P designs: the agreement must be good enough for A to agree to participate.

$$\sum_{i=1}^{n} p_i(e^*)u(t(x_i)) - v(e^*) \geq U_0 \tag{PC}$$

5. Principal's design problem

The principal designs the terms of the contract, knowing that she is constrained by the agent's behavior in stages 2 and 3. The easiest way to formulate this is to say that P chooses the effort level, and the transfer (as a function of verifiable output), subject to the PC and IC constraints on what effort will be. That is, P can tell the agent how much to work, as long as that amount is consistent with the agent's optimal behavior given by PC and IC. Thus, this is the same problem we solved last class (with symmetric information), except that we've added the IC constraint that the assigned effort level has to be consistent with the agent's own utility maximizing behavior.

$$\max_{e,\{t(x_i)\}} \sum_{i=1}^{n} p_i(e)\pi(x_i - t(x_i)) \text{ s.t. (PC), (IC)}$$

Remark It is important to understand what variables the contract designer can choose, subject to which constraints. With symmetric contracting, the principal could specify the effort level, e, because effort was verifiable. Now, with asymmetric information, the principal cannot verify effort, yet we still have the principal choosing the effort level when designing the contract. How is this? The key is that the principal can choose any level of effort by the agent *that is consistent with the agent's self interest.* Hence the helpfulness of recognizing that the (IC) above is the agent's effort supply function: it represents the effort the agent will choose to supply given a particular configuration of incentives. This is similar to the way in which we model a monopolist as choosing the quantity sold: the monopolist can choose how much the consumer will buy as long as the quantity and price are compatible with the consumer's demand function.

6. Solving the problem with only two possible effort levels

To get the design exactly right (exactly optimal) can be quite difficult if there are many choices of effort level. Most real world agreements are attempts to get close to optimal. We can learn a lot about what a good agreement would look like by studying a simpler version of the problem, with only two possible effort levels (high and low). Further, assume P is risk-neutral. Here are the specifics to modify the model:

$$e \in \{e_L, e_H\}$$

$$v(e_H) > v(e_L)] \quad \text{(the agent prefers less work to more)}$$

Let outputs be ordered: $x_1 < x_2 < \ldots < x_n$. Let's be more precise about the relationship between effort and likelihood of good outcomes. One model proceeds from the fact that:

$$Prob(x > x_k) = 1 - \sum_{i=1}^{k} p_i(e)$$

so, for high effort to improve likelihood of good (high) outcomes, make the (useful, but somewhat restrictive) assumption that:

$$1 - \sum_{i=1}^{k} p_i^H > 1 - \sum_{i=1}^{k} p_i^L \text{ for all } k$$

(known as *first order stochastic dominance*).

6.1. **Solution: Low effort case.** If P only wants to induce low effort (e_L), it's easy:

- Pay fixed amount t_L, agent then works e_L
- Set t_L so the agent is just willing to participate (i.e., so the (PC) is an equality): $t_L = u^{-1}(U_0 + v(e_L))$

Remark This is the same solution as for symmetric information for risk-neutral P. Why? Because the asymmetry doesn't matter if the P only wants to induce minimal effort.

Why would P ever want only minimal effort from A? P has to give A incentive to work harder. If the necessary incentive is high enough, it will exceed the increase in output value, For example, why don't we make employees work 80 hour weeks?

Example Suppose a firm is paying is contract programmers a fixed wage, which will only induce "low" effort. Should the firm switch to a wage based on output (e.g., lines of code), to induce higher effort? Against the benefits, the firm must consider the costs of moving to a non-fixed wage contract. First, it will be more costly to monitor. Second, the metric (lines of code) may not be very good signal of actual effort.

Remark This static model misses an important dynamic feature of contracts: future wages. If effort low now, may not get good raise next year.

6.2. **Solution: High effort case.** We now see that if P does want to induce higher than minimum effort, it must provide an incentive. In this model, transfer must depend on something verifiable that is correlated with effort, i.e., output. Thus, the (IC) becomes

$$\sum_i p_i^H u(t(x_i)) - v(e^H) \geq \sum_i p_i^L u(t(x_i)) - v(e^L) \text{ or,}$$

$$\sum_i [p_i^H - p_i^L] u(t(x_i)) \geq v(e^H) - v(e^L) \tag{IC}$$

Interpretation: Agent will work e^H if expected utility gain exceeds disutility of higher work.

Solve contract design problem now (max principal's profits subject to (PC) and the new (IC)). The FOC are:

$$\frac{p_i^H}{u'(t(x_i))} = \lambda_{PC}\, p_i^H + \lambda_{IC}[p_i^H - p_i^L] \text{ for all } i = 1, \ldots, n \tag{3.5}$$

(Equation number (3.5) corresponds to the same-numbered equation in the MS-PC text.)

Whew! Let's manipulate these a bit to get some principles of good contract design. First, sum (3.5) over $i = 1, \ldots, n$, to get:

$$\lambda_{PC} = \sum_i \frac{p_i^H}{u'(t(x_i))} > 0 \tag{3.6}$$

6.3. **Results.**

Result 1. *So, the value of the (PC) constraint (the "shadow price", or how much this constraint hurts the principal) is positive, which implies that the constraint must be binding (an equality). Once again, the terms of the transfer should be set so that the agent is just willing to work.*

From manipulating the FOC in a different way we can get two more interesting results:

Result 2. *(See MS-PC text for proof) The value of the (IC) constraint is also positive, indicating that the principal is worse off when there is a hidden information problem.*

Result 3. *(Proof below) The relationship between the transfer and observable output depends directly on the likelihood ratio, $\frac{p_i^L}{p_i^H}$. The lower is this ratio,* the more informative *output is about the level of effort made. Therefore, the lower this ratio, the higher is the transfer for the corresponding level of output.*

Proof of Result 3: Rearrange FOC (3.5) to get

$$\frac{1}{u'(t(x_i))} = \lambda_{PC} + \lambda_{IC}[1 - \frac{p_i^L}{p_i^H}] \tag{3.7}$$

We know that both λs are positive constants (these are the costs of both constraints; see Results (1) and (2)). Let $LR = \frac{p_i^L}{p_i^H}$ (the likelihood ratio). A lower LR means the right-hand side is higher. To make the LHS higher, the denominator must be smaller. But u' is a decreasing function (diminishing marginal utility, or for expected utility, risk aversion implied by concavity), so to make the demoninator smaller, $t(x_i)$ must be higher.

6.4. **Hidden action: Interpreting the results.** Result 3 is the key, and we'll interpret the heck out of it:

- What does it mean for LR to be lower? It means that p_i^H is larger relative to p_i^L . In other words, there is a bigger gap, meaning that effort makes a bigger difference on the changing the probabilities for various levels of output. (Example: Compared to base case ratio $LR_i = 1$, which means effort has no effect on the likelihood of seeing this output, consider$p_i^L = 0.1$ and $p_i^H = 0.9$. Clearly making high effort makes it considerably more likely to see output x_i.

- The likelihood ratio is telling us how much information is contained in seeing output x_i , in the Hirshleifer and Riley sense! How much does seeing a particular x_i change our beliefs about whether the agent gave low or high effort? A very small LR has a lot of information: in the example above ($LR = 1/9$), if we see that particular output we become pretty darn sure that effort was high.
- The transfer payment does not depend directly on the level of output. This is probably a bit surprising, but if we solve (3.7) for t, x_i does not appear on the right hand side. Remember, with no hidden information, the optimal transfer is a constant. The only reason to vary the transfer now is to provide incentives. And we want to provide incentives that are correlated with information about likely effort, and that comes from LR.
- Thus, the transfer is not necessarily monotonically increasing in output level though in some sense it is on average, because on average higher effort makes higher output more likely, so on average higher output is evidence of higher effort.
- The agent bears some risk (different transfers in different states). This is an inefficiency (since P risk neutral and A risk averse). It's part of the cost of providing incentives.
- The more risk averse is agent, the higher the transfers must be on average (and the lower is the principal's profit): since A bears some risk, and the (PC) binds (just getting U_0), the more risk averse, the higher average payment has to be to keep A getting U_0 .

Other summary results from the analysis:

- If the cost of incentives high enough, P will just fall back on the low-effort, fixed-payment contract.
- We saw that LR is information and is valuable (the principle pays more for more informative LRs). In general, any signal that provides information (a better estimate of agent's effort) should be used in a contract: reducing uncertainty about agent's effort moves back toward symmetric (efficient) situation. **Example:** Base compensation in part on results from other agents, if those results contain information on the state of nature affecting the first agent (helping to sort out effect of effort versus random nature).
- Since information is valuable (reduces agency cost, increases profits), principal is willing to pay something for it. Justifies some degree of spending on information monitoring and control systems.
- Severe punishments: If very good signal available, impose a severe penalty if that signal is negative.
  - Parking ticket paid? Costly to monitor action, so often the action is hidden. However, if occasional monitoring finds unpaid meter: death penalty. The idea is that the severe threat will mean that everyone always pays their parking tickets.

- Actual: Mass transit in Europe. Mostly honor system, occasional spot checks. But if caught without fare ticket, very severe penalties (e.g., $300).

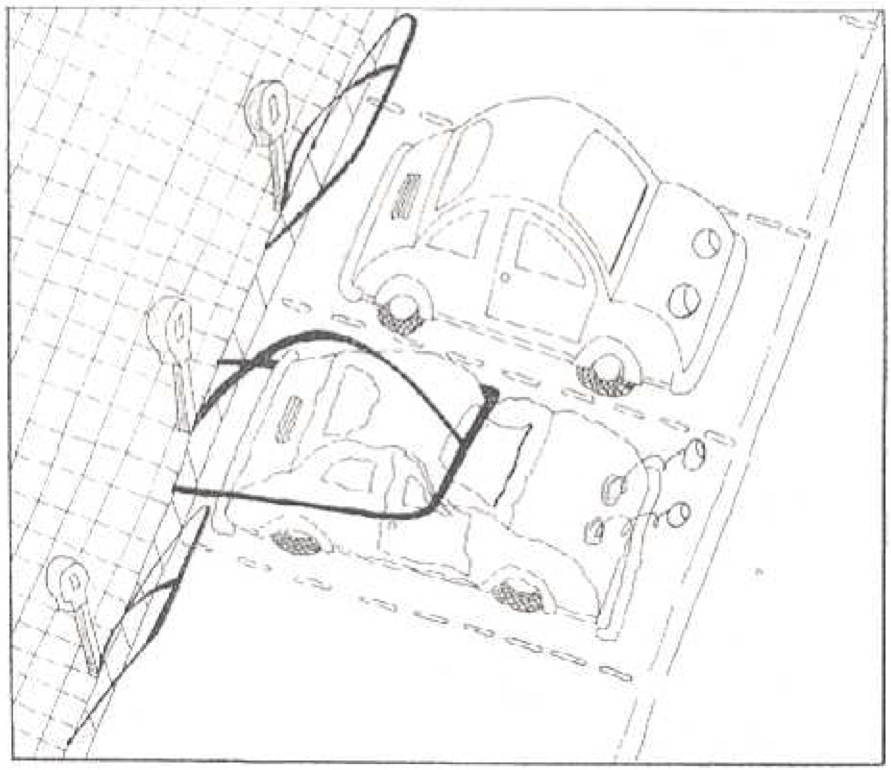

FIGURE 2. Incentives for effort to feed the parking meter

- There is a connection here between hidden action contracts and third-degree price discrimination: In 3rd degree p.d. (differentiation by group), prices vary with different group identities to the extent the group revealed information about willingness to pay. Thus, being a student / senior / other might reveal a lot about WTP for movies, but red / blond / brown / black hair might not reveal much information about WTP, so prices don't vary with hair color.