Patent Licensing Contract Example: Hidden Characteristics

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SI 680

April 3, 2008

Preliminaries

These notes present the patent licensing contract example from Ines Macho-Stadler and J. David Perez-Castrillo, *An Introduction to Economics of Information: Incentives and Contracts*, (Oxford University Press, New York, 2nd edition, 2001), pp. 149–153.

Suppose a patent owner with a cost-reducing innovation can profit only by licensing it to a manufacturer, and that there is a manufacturer who is a monopolist in her product market who can use the innovation in her production process.

Suppose the owner considers licensing contracts with a fixed payment (non-negative) F, and a (non-negative) royalty of ϵ per unit of production, so a contract is defined by $\{F, \epsilon\}$.

Assumptions, notation

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- total cost with innovation after license payments $= F + (c + e)Q$
- Demand is $D(p)$
- Profit (gross of fixed payment), as a function of average cost:

$$\pi(x) = [p(x) - x]D(p(x))$$

where

$$p(x) = \arg\max_p [p - x]D(p)$$

which has the necessary condition (MR = MC):

$$[p - x]D'(p) + D(p) = 0 \tag{1}$$

Symmetric information problem

Simplify notation: let $D(x) = D(p(x))$ Seller:

$$\max_{\{F,\epsilon\}} F + \epsilon D(c+\epsilon)$$

$$\begin{aligned} \text{s.t. } \pi(c+\epsilon) - F &\geq \pi(c^0) \quad \text{(PC)} \\ \epsilon &\geq 0 \\ F &\geq 0 \end{aligned}$$

Solving symmetric information problem

First, it is clear that the (PC) binds. Suppose not, and assume seller chooses optimal ϵ^*. Then seller can increase F without violating (PC), and increasing F increases the objective function, so cannot be at an optimum for F until (PC) binds.

Now we can show that optimal royalty is $\epsilon = 0$. Since (PC) binds, $F = \pi(c+\epsilon) - \pi(c^0)$. Substitute into the objective function and do an unconstrained optimization (we'll confirm the non-negativity constraints hold at the end).

$$
\begin{aligned}
&\max F + \epsilon D(p(c+\epsilon)) \\
&\qquad = \max \pi(c+\epsilon) - \pi(c^0) + \epsilon D(p(c+\epsilon)) \\
&\qquad = \max [p(c+\epsilon) - (c+\epsilon)] D(p(c+\epsilon)) - \pi(c^0) + \epsilon D(p(c+\epsilon)) \\
&\qquad = \max [p(c+\epsilon) - c] D(p(c+\epsilon)) - \pi(c^0)
\end{aligned}
$$

Solving symmetric (cont.)

The necessary condition is

$$\frac{\partial p}{\partial \epsilon} D(p) + p \frac{\partial D(p)}{\partial p} \frac{\partial p}{\partial \epsilon}$$

which we can rearrange as

$$\frac{\partial p}{\partial \epsilon} \left[D(p) + p \frac{\partial D(p)}{\partial p} \right]$$

but from (1) the term in square brackets is equal to $(c + \epsilon) D'(p)$ (MR $=$ MC), which is negative (since demand curves slope down). Therefore, the seller should set ϵ as small as possible: given the non-negativity constraint, $\epsilon = 0$.

Finally, with $\epsilon = 0$, we have $F = \pi(c) - \pi(c^0)$. Since $c < c^0$, and it is easy to show (and it is intuitively obvious) that profit is decreasing in cost, $F > 0$, which satisfies its non-negativity constraint.

Optimal symmetric information contract

Set the royalty (ϵ) to zero, and set the fixed payment (F) to extract all of the buyer's surplus from using the innovation (all of the incremental profit, $F = \pi(c) - \pi(c^0)$).

(The buyer's surplus is the incremental downstream profit from using the lower cost production process.)

Symmetric information: special case

Consider a special case: the new production cost can be either Good or Bad (though both are better than the old cost):

$$c^G < c^B < c^0$$

The optimal symmetric information contract can now be expressed as two contracts, one of which will be offered by the principal depending on which cost the innovation delivers:

$$\text{Good: } \{\epsilon^{G*} = 0, F^{G*} = \pi(c^G) - \pi(c^0)\} \tag{2}$$

$$\text{Bad: } \{\epsilon^{B*} = 0, F^{B*} = \pi(c^B) - \pi(c^0)\} \tag{3}$$

Asymmetric information problem

Now suppose the buyer has private information: only she knows for sure which level of cost the innovation yields in her production process.

The seller has a prior belief that there is a probability q that the cost will be c^G.

Seller's problem

$$\max_{\{F^G,\epsilon^G,F^B,\epsilon^B\}} q\left[F^G+\epsilon^G D(c^G+\epsilon^G)\right]+(1-q)\left[F^B+\epsilon^B D(c^B+\epsilon^B)\right]$$

s.t.

$$\pi(c^G+\epsilon^G)-F^G-[\pi(c^G+\epsilon^B)-F^B]\geq 0 \qquad \text{(IC-G; }\mu)$$

$$\pi(c^B+\epsilon^B)-F^B-[\pi(c^B+\epsilon^G)-F^G]\geq 0 \qquad \text{(IC-B; }\lambda)$$

$$\pi(c^G+\epsilon^G)-F^G-\pi(c^0)\geq 0 \qquad \text{(PC-G; }\rho)$$

$$\pi(c^B+\epsilon^B)-F^G-\pi(c^0)\geq 0 \qquad \text{(PC-B; }\delta)$$

$$\left\{F^G,F^B,\epsilon^G,\epsilon^B\right\}\geq 0 \qquad (\alpha^G,\alpha^B,\beta^G,\beta^B)$$

Step 1

We can show that (PC-G) does not bind (so $\rho = 0$):

$$\begin{aligned} \pi^B &\geq \pi^0 && \text{(from (PC-B))} \\ \pi^G &\geq \pi(c^G + \epsilon^B) - F^B && \text{(from (IC-G))} \\ \pi(c^G + \epsilon^B) - F^B &> \pi(c^B + \epsilon^B) - F^B && \text{(by } \tfrac{\partial \pi}{\partial c} < 0\text{)} \end{aligned}$$

So, $\pi^G > \pi^B \geq \pi^0 \implies \rho = 0$, and (PC-G) binds.

Go to Problem

Step 2

$$\frac{\partial \mathcal{L}}{\partial F^G} : q - \mu + \lambda + \alpha^G = 0 \iff \mu = q + \lambda + \alpha^G > 0 \qquad (4)$$

because $q > 0, \{\lambda, \alpha^G\} \geq 0$. (4) implies that (IC-G) is binding, so:

$$F^G = \pi(c^G + \epsilon^G) - \pi(c^G + \epsilon^B) + F^B \qquad (5)$$

Step 3

$$\frac{\partial \mathcal{L}}{\partial F^B} : (1-q)+\mu-\lambda-\delta+\alpha^B = 0 \iff 1+\alpha^G+\alpha^B = \delta > 0 \qquad (6)$$

by using (4), and the fact that $\{\alpha^G, \alpha^B\} \geq 0$. $\delta > 0$ means (PC-B) is binding, so:

$$F^B = \pi(c^B + \epsilon^B) - \pi(c^0) \qquad (7)$$

Then, by substituting (7) into (5):

$$F^G = \pi(c^G + \epsilon^G) - \pi(c^G + \epsilon^B) + \pi(c^B, \epsilon^B) - \pi(c^0) \qquad (8)$$

Step 4a

First, we're going to need to know by how much changing the royalty (ϵ) changes profits:

$$\pi(c+\epsilon) = [p(c+\epsilon) - (c+\epsilon)]\, D(p(c+\epsilon))$$

$$\begin{aligned} \frac{\partial \pi}{\partial \epsilon} &= \left(\frac{\partial p}{\partial \epsilon} - 1\right) D + (p - (c+\epsilon)) \frac{\partial D}{\partial p} \frac{\partial p}{\partial \epsilon} \\ &= \frac{\partial p}{\partial \epsilon} \left[D + (p - (c+\epsilon) D'\right] - D \qquad (9) \end{aligned}$$

But substituting the first-order condition for optimal pricing in (1), we have:

$$\frac{\partial \pi}{\partial \epsilon} = -D(p(c+\epsilon)) \qquad (10)$$

Step 4b

Using (10), we have $\partial \mathcal{L}/\partial \epsilon^G$:

$$qD(c^G+\epsilon^G)+q\epsilon^G D'(c^G+\epsilon^G)-\mu D(c^G+\epsilon^G)+\lambda D(c^B+\epsilon^G)+\beta^G = 0$$

From (4) we have $q-\mu+\lambda = -\alpha^G$, so

$$(q-\mu+\lambda)D(c^G+\epsilon^G) = -\alpha^G D(c^G+\epsilon^G)$$

and substituting into $\partial \mathcal{L}/\partial \epsilon^G$ we get

$$\beta^G = -q\epsilon^G D'(c^G+\epsilon^G)+\alpha^G D(c^G+\epsilon^G)+\lambda\left[D(c^G+\epsilon^G)-D(c^B+\epsilon^G)\right]$$

Step 4c

Using this result for β^G we can show that $\epsilon^G = 0$ (no royalty in Good contract):

$$\beta^G = -q\epsilon^G D'(c^G+\epsilon^G)+\alpha^G D(c^G+\epsilon^G)+\lambda\left[D(c^G+\epsilon^G)-D(c^B+\epsilon^G)\right]$$

If $\beta^G > 0$, then $\epsilon^G = 0$ by KT conditions.

If $\beta^G = 0$: We know $-qD' > 0$, $D > 0$, and $D(c^G+\epsilon^G) - D(c^B, \epsilon^G) > 0$ (the latter because optimal price is increasing in cost, so equilibrium output is decreasing in cost, making $D(c^B) < D(c^G)$). Since each term is non-negative and all of the multipliers are non-negative, it must be that

$$\epsilon^G = \alpha^G = \lambda = 0$$

Result 1

Result

The Good contract royalty is zero, $\epsilon^G = 0$.

This is generally true for hidden characteristics contracts, and is sometimes referred to as "No distortion at the top".

Step 5a

$$\frac{\partial \mathcal{L}}{\partial \epsilon^B} : (1-q)D(c^B+\epsilon^B) + (1-q)\epsilon^B D'(c^B+\epsilon^B) + \mu D(c^G+\epsilon^B) - \lambda D(c^B+\epsilon^B) - \delta D(c^B+\epsilon^B) + \beta^B = 0 \quad (11)$$

Using the same trick we used with $\partial \mathcal{L}/\partial \epsilon^G$, but substituting this time from (6), we get:

Go to (6)

$$\mu\left[D(c^G+\epsilon^B) - D(c^B+\epsilon^B)\right] - \alpha^B D(c^B+\epsilon^B) + (1-q)\epsilon^B D'(c^B+\epsilon^B) + \beta^B = 0 \quad (12)$$

Step 5b

Using this result:

$$\mu\left[D(c^G+\epsilon^B)-D(c^B+\epsilon^B)\right]-\alpha^B D(c^B+\epsilon^B)+ \\ (1-q)\epsilon^B D'(c^B+\epsilon^B)+\beta^B=0 \quad (13)$$

Suppose $\epsilon^B=0$. From (7)

$$F^B=\pi(c^B)-\pi(c^0)>0$$

which implies that $\alpha^B=0$ (since the non-negativity constraint on F^B is not binding). Then, using (13) we have

$$\mu\left[D(c^G+\epsilon^B)-D(c^B+\epsilon^B)\right]+\beta^B=0$$

but this is a contradiction since $\mu>0$ by (4) and we know $\beta^B\geq 0$.

Therefore, $\epsilon^B>0$.

Result 2

Result

The Bad contract royalty is positive, $\epsilon^B > 0$*.*

This is generally true for hidden characteristics contracts, and is sometimes referred to as "Distortion at the bottom".

Result 3

Result

The Good contract up-front payment is higher than for the Bad contract: $F^G > F^B$.

Proof.

From (8), $F^G = \pi(c^G) - \pi(c^G + \epsilon^B) + \pi(c^B + \epsilon^B) - \pi(c^0)$.

From (7), $F^B = \pi(c^B + \epsilon^B) - \pi(c^0)$.

Since $\epsilon^B > 0$, $\pi(c^G) > \pi(c^G + \epsilon^B) \implies F^G > F^B$. □

Result 4

Result

The up-front payment for the Good contract when there is asymmetric information is less than when information is symmetric, $F^G < F^{G*}$.

Proof.

$F^{G*} = \pi(c^G) - \pi(c^0)$, so

$$F^{G*} - F^G = \pi(c^G + \epsilon^B) - \pi(c^B + \epsilon^B) > 0$$

because profits are decreasing in unit cost. □

Result 5

Result

The up-front payment for the Bad contract when there is asymmetric information is less than when information is symmetric, $F^B < F^{B*}$.

Proof.

$F^B = \pi(c^B + \epsilon^B) - \pi(c^)) < \pi(c^B) - \pi(c^0) = F^{B*}$ because profits are decreasing in unit cost. □

Summary

Here's what we found about the optimal (asymmetric information) patent licensing contract for this problem:

1. The optimal contracts are separating: The Good type selects $\{F^G, 0\}$, while the Bad type selects $\{F^B, \epsilon^B\}$ (see Results 1, 2).

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5. $\epsilon^B > 0 \implies$ efficiency distorted at bottom (to make it less attractive to the Good type to masquerade as a Bad type, so that we can get more rent out of the more productive type) (see Result 2).
6. $\epsilon^G = 0 \implies$ efficient at the top (don't want to distort the most productive) (see Result 1).

Contracts

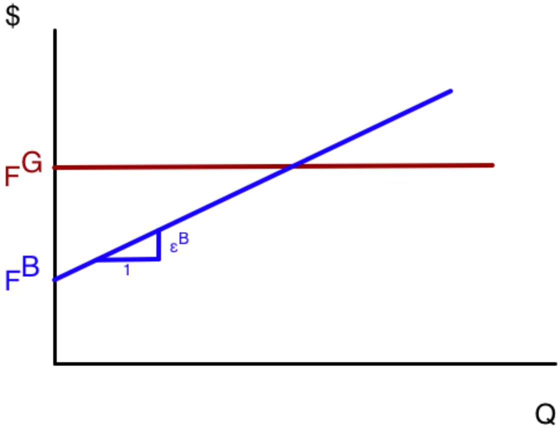

Figure: The asymmetric information contract parameters