THE UNIVERSITY OF MICHIGAN

INDUSTRY PROGRAM OF THE COLLEGE OF ENGINEERING

DYNAMIC DESIGN OF REINFORCED CONCRETE CHIMNEYS

L. C. Maugh
W. S. Rumman

# TABLE OF CONTENTS

# LIST OF TABLES

# LIST OF FIGURES

# DYNAMIC DESIGN OF REINFORCED CONCRETE CHIMNEYS

L. C. Maugh[1]
W. S. Rumman[2]

## INTRODUCTION

The present trend toward the construction of reinforced concrete chimneys up to 1000 feet in height, and perhaps higher in the future, provides ample reasons for a re-evaluation of design criteria for such structures. The need for tall chimneys is primarily due to increasing atmospheric pollution which is now causing serious problems even in locations outside of metropolitan districts. This discussion of chimney design, however, will be confined to the solution of a few structural problems although the economic problems involved in the prevention of atmospheric pollution as well as outage time in the electrical generating units are important ones. In other words, it is essential that no interruption of plant production be caused by functional or structural troubles with chimneys and, therefore, a conservative design should be made.

Two structural problems that trouble most designers of tall chimneys involve the determination of dynamic forces and the corresponding stresses due to earth and wind vibrations. Only the earth movements caused by seismic action and the possible occurrence of a periodic forcing function due to the wind will be considered in this paper.

---

[1] Professor of Civil Engineering, University of Michigan, Ann Arbor, Michigan.

[2] Associate Professor of Civil Engineering, University of Michigan, Ann Arbor, Michigan.

# DESIGN FOR SEISMIC ACTION

In the preliminary design stage, empirical formulas for deter-
mining seismic forces can be very helpful if these formulas have been
obtained from an analytical study of the behavior of similar stacks.
For most chimneys a preliminary design based upon static wind loads
and empirical seismic forces will provide dimensions that are sufficient-
ly accurate for final design purposes. Only the outside diameters, the
thicknesses of the concrete, the modulus of elasticity of the concrete
and the magnitude and location of any supported weight are required.
The exact reinforcing steel ratio is not important at this stage of the
design.

After a preliminary design has been selected the modal charac-
teristics (displacements, bending moments, and shears) can be calculated
for as many modes of vibration as is desired. The writers have found
that the first three modes of vibration are sufficient. Assuming that
the values of the displacements, bending moments and shears for each
mode have been tabulated at  n  selected locations, the following pro-
cedure for estimating the seismic design moments is recommended.

Select several accelerograms, which have been obtained from
actual earthquakes and have been reduced to digital form[3], to represent
the ground motion. One advantage of using several accelerograms is the
smoothing of any unusual peaks and valleys in the response that may be
obtained from a single earthquake and, perhaps more important, to pro-
vide a more general representation of ground motion.

---

[3] Berg, G.V., and Thomaides, S.S., "Punched Card Accelerograms of Strong
Motion Earthquakes", The University of Michigan Research Institute,
Report No. 2881-1-P, September, 1959.

From these accelerograms and the modal properties, the fundamental equation of motion can be solved directly or a response spectrum can be established for the selected accelerograms for use with any chimney. At present the writers use a direct solution of the fundamental equation which provides the maximum shears and bending moments at as many sections as desired along the vertical axis of the chimney for any number of accelerograms. By plotting the results for each earthquake a comparison of the average curve with the upper and lower bounds, as shown in Figure 1, is clearly established.

Up to this point the numerical operations have been fairly well defined but, as in any design, there comes a time when judgment must enter. At this stage of the design, decisions must be made with regard to the following problems:

(a) By what factor K should the average of the maximum seismic shears and moments be multiplied to give design values for use in both a working stress design and a maximum stress design that will be discussed later. Both types of design are essential if a consistent factor of safety is desired throughout the chimney. Obviously the choice of the K factor will be influenced by the severity of the selected earthquakes, the past seismic history of the area in which the site is located, the soil characteristics at the site, and the opinions of qualified people. The uncertain nature of the problem is well expressed by a statement in the 1955 code of the American Standards Association for "Minimum Design Loads in Buildings and Other Structures", which says "There is no way of predicting the time and place of a destructive earthquake in either a seismic or monseismic area."
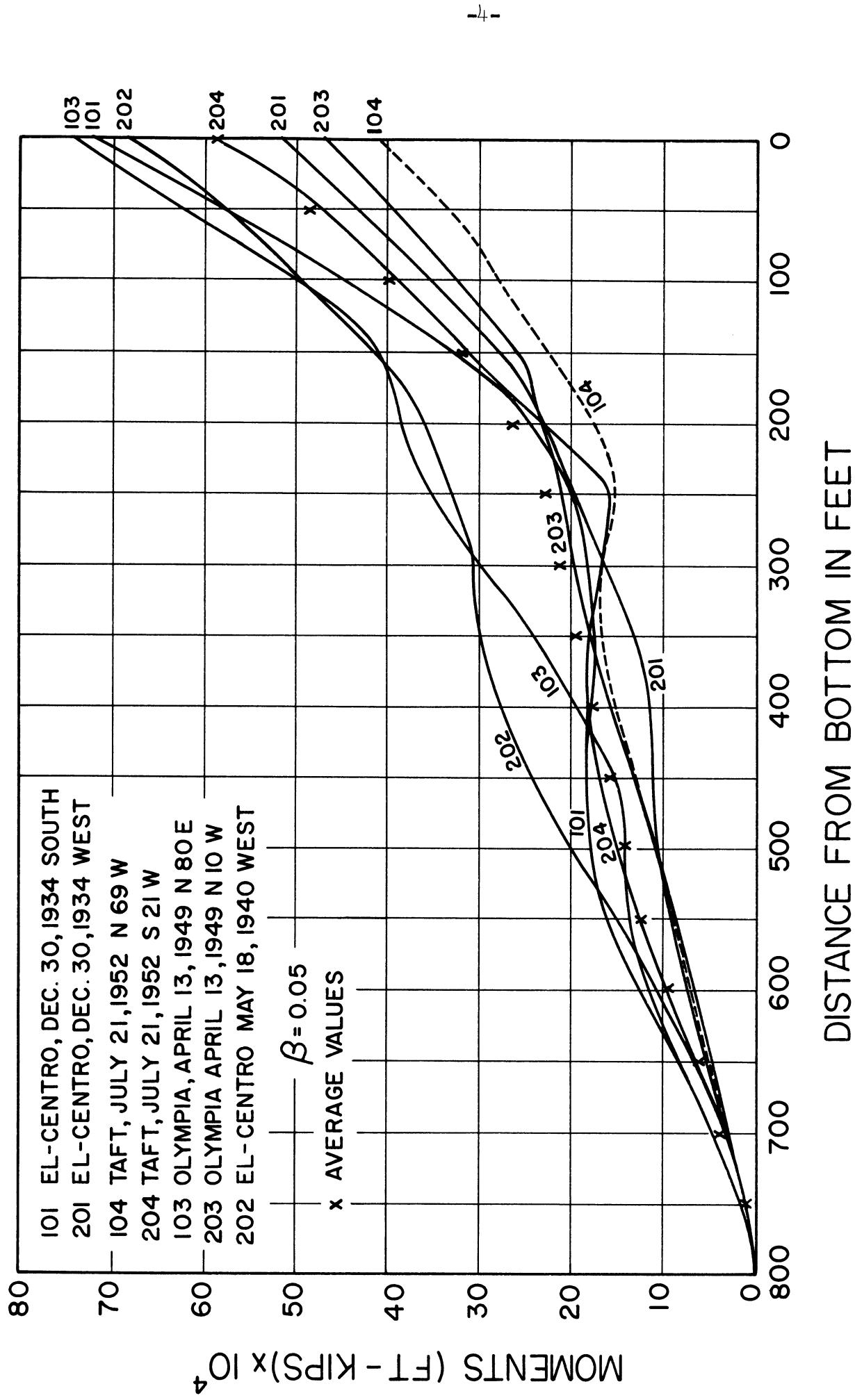
Figure 1. Earthquake Maximum Moment Curves.

(b) What allowable unit stresses should be used in the rein-
forcing steel and in the concrete for both working stress and maximum
stress designs? In the working stress design the designer can obtain
valuable assistance from the ACI Specification for the Design and Con-
struction of Reinforced Concrete Chimneys but in a maximum stress de-
sign he is definitely "on his own". However, it is recommended that
the formulas for computing stresses as presented in the ACI Specifica-
tion be used for both cases. The proposed design procedure will be
illustrated by giving the numerical results for an 800 ft. reinforced
concrete chimney whose outside diameters, thicknesses and vertical steel
ratios are tabulated in Table 1 for 17 equally spaced sections. As
this chimney had an independent liner no additional mass except a small
amount for access stairways was involved.

The normalized mode shapes and the moments and shears associa-
ted with each shape are given in Table 2 for the first three modes of vi-
bration. A value of $E$ equal to $3.75 \times 10^6$ psi was used in these cal-
culations. No allowance for base rotation was made. For this chimney
the circular frequency $\omega_1$ is 2.25 radians per sec. giving a fundamen-
tal period of 2.79 secs. per cycle.

The fundamental equation of motion is solved by the modal analy-
sis technique whereby:

$$Y(x,t) = \sum_{j=1}^{3} \phi_j (x) \cdot q_j (t) \qquad (1)$$

$$V(x,t) = \sum_{j=1}^{3} V_j (x) \cdot q_j (t) \qquad (2)$$

$$M(x,t) = \sum_{j=1}^{3} M_j (x) \cdot q_j (t) \qquad (3)$$

## TABLE 1

### CHIMNEY DATA

| Distance from Bottom | Outside Diameter (ft.) | Thickness (ft.) | Vertical Steel ratio, p |
|---|---|---|---|
| 800 | 36.5600 | .6354 | .00250 |
| 750 | 38.5900 | .6510 | .00412 |
| 700 | 40.6200 | .6667 | .00715 |
| 650 | 42.6500 | .6823 | .00895 |
| 600 | 44.6800 | .6979 | .00981 |
| 550 | 46.7100 | .7135 | .00974 |
| 500 | 48.7400 | .7292 | .00906 |
| 450 | 50.7700 | .7813 | .00820 |
| 400 | 52.8000 | .8333 | .00930 |
| 350 | 54.8300 | 1.0417 | .00835 |
| 300 | 56.8600 | 1.2500 | .00750 |
| 250 | 58.8900 | 1.4167 | .00685 |
| 200 | 60.9200 | 1.5833 | .00615 |
| 150 | 62.9600 | 1.7083 | .00557 |
| 100 | 65.0000 | 1.8333 | .00496 |
| 50 | 65.0000 | 2.8700 | .00400 |
| 0 | 65.0000 | 2.6667 | .00304 |

TABLE 2

MODE SHAPES, MOMENTS AND SHEARS

| Distance from base (ft.) | First Mode | | | Second Mode | | | Third Mode | | |
|---|---|---|---|---|---|---|---|---|---|
| | Shape (ft.) | Moments (ft-k) | Shears (k) | Shape (ft.) | Moments (ft-k) | Shears (k) | Shape (ft.) | Moments (ft-k) | Shears (k) |
| 800 | 1.0000 | 0 | | 1.0000 | 0 | | 1.0000 | 0 | |
| 750 | .8897 | 2092 | 42 | .6814 | 28419 | 568 | .4707 | 152372 | 3047 |
| 700 | .7802 | 8250 | 123 | .3726 | 102392 | 1479 | -.0072 | 488992 | 6732 |
| 650 | .6729 | 18256 | 200 | .0906 | 203254 | 2017 | -.3592 | 824837 | 6717 |
| 600 | .5695 | 31827 | 271 | -.1472 | 311221 | 2159 | -.5345 | 1002241 | 3548 |
| 550 | .4721 | 48636 | 366 | -.3281 | 407098 | 1918 | -.5286 | 924367 | -1537 |
| 500 | .3823 | 68313 | 394 | -.4452 | 474021 | 1338 | -.3790 | 578992 | -6927 |
| 450 | .3017 | 90479 | 443 | -.4982 | 498778 | 495 | -.1513 | 25742 | -11065 |
| 400 | .2316 | 114826 | 487 | -.4943 | 471065 | -554 | .0801 | -618913 | -12893 |
| 350 | .1722 | 141081 | 525 | -.4459 | 383869 | -1744 | .2558 | -1205085 | -11723 |
| 300 | .1229 | 169113 | 561 | -.3699 | 229386 | -3090 | .3479 | -1564471 | -7188 |
| 250 | .0827 | 198717 | 592 | -.2813 | 5789 | -4472 | .3567 | -1545529 | 379 |
| 200 | .0510 | 229567 | 617 | -.1923 | -279631 | -5708 | .2996 | -1071871 | 9473 |
| 150 | .0275 | 261306 | 635 | -.1129 | -613857 | -6685 | .2048 | -157898 | 18279 |
| 100 | .0116 | 293584 | 646 | -.0514 | -980298 | -7329 | .1050 | 1093849 | 25035 |
| 50 | .0029 | 326121 | 651 | -.0139 | -1363476 | -7664 | .0307 | 2542501 | 28973 |
| 0 | 0 | 358760 | 653 | 0 | -1753602 | -7803 | 0 | 4079806 | 30746 |

Frequencies:  $\omega_1 = 2.2506$ radians/sec.

$\omega_2 = 8.6209$ radians/sec.

$\omega_3 = 20.8583$ radians/sec.

in which $Y(x,t), V(x,t),$ and $M(x,t)$ are the displacements, shears and moments in the chimney; $x$ represents the distance along the chimney, and $t$ refers to the time. $\phi_j(x)$ is the mode shape in the j-th mode and $V_j(x)$ and $M_j(x)$ are the shears and moments associated with it as illustrated in Table 2.

The value of $q_j(t)$, for any mode $j$, and time $t$ is obtained from the following equation:

$$\ddot{q}_j(t) + 2\beta\omega_j\dot{q}_j(t) + \omega_j^2 q_j(t) = \frac{-a(t)\int_0^L m(x)\ \phi_j(x)\ dx}{\int_0^L m(x)\ \phi_j^2(x)\ dx} \quad (4)$$

in which $\beta$ is the fraction of critical damping, $\omega_j$ is the frequency in radians/second, $a(t)$ is the acceleration of the earthquake, and $m(x)$ refers to the mass/unit length.

The solution of Equation (4) is accomplished by using a third order Runge-Kutta process in which the time interval for this example is not greater than 0.012 seconds. At any time during the duration of the earthquake the values of $q_1$, $q_2$ and $q_3$ as obtained from Equation (4) are then used to determine the values $Y(x,t)$, $V(x,t)$, and $M(x,t)$ from Equations (1), (2), and (3).

The maximum moments for seven different accelerograms, together with the average values, are shown in Figure 1. The particular earthquakes used which are numbered on the graph are those of strong motion earthquakes. It is apparent from the curves in Figure 1 that each earthquake has a somewhat individual effect along the chimney and therefore an average of several values is much more representative. A value of $\beta$, the fraction of critical damping, of .05 was used in the solution. The average maximum shear curve for the same earthquakes is shown in Figure 2.
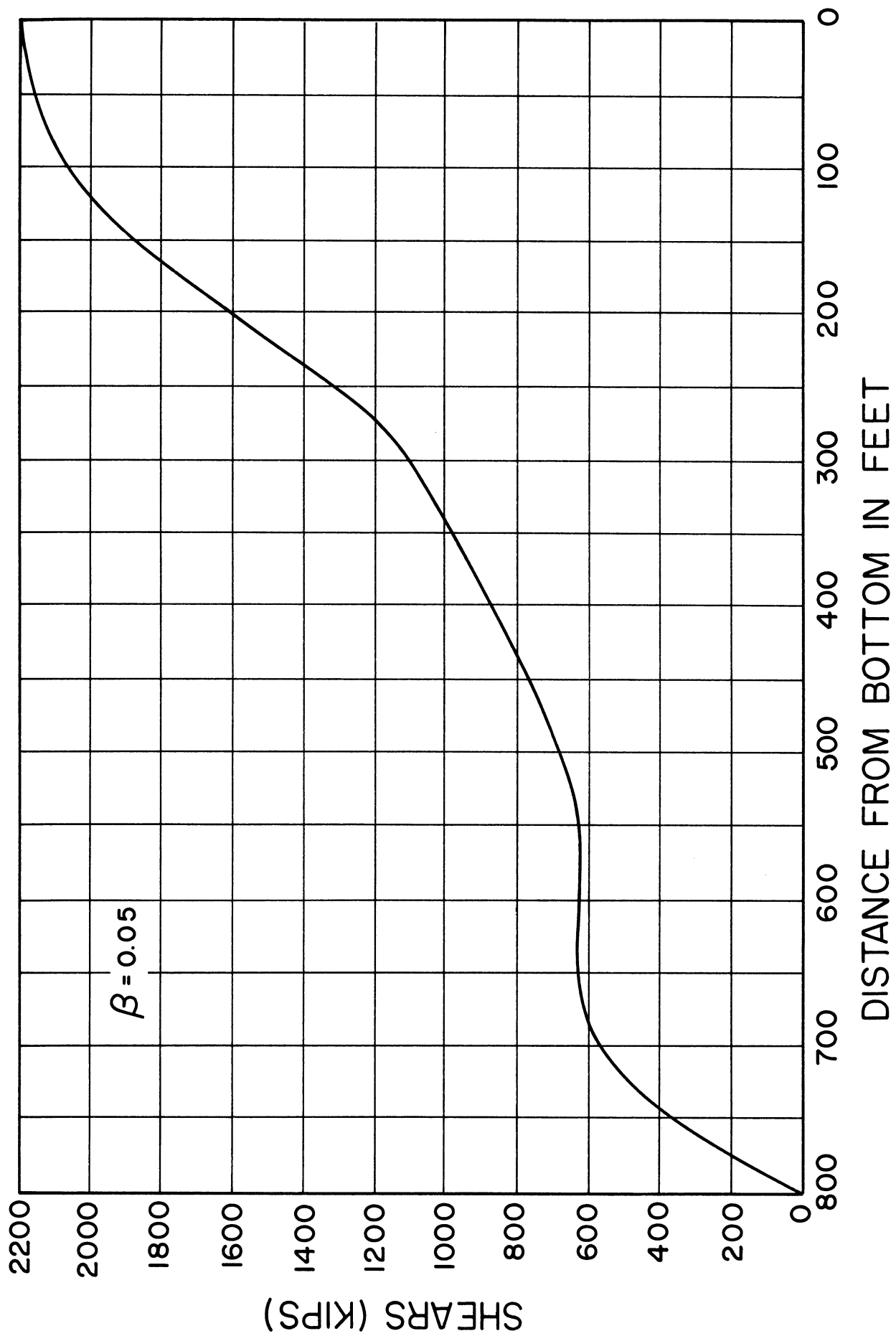
Figure 2. Average of Maximum Shears Due to Seven Earthquakes.

Figure 3 gives typical design moment curves based upon K values of .9 and 1.4 times the average of the maximum seismic moment values, the first for the working stress design and the second for the maximum stress. A load factor of about 1.5 is therefore provided.

Before discussing the magnitude of the steel and concrete stresses due to these moments, another source of dynamic magnification, that is wind resonant vibration will be briefly reviewed.

## DESIGN FOR A RESONANT WIND VIBRATION

Considerable evidence has been presented in technical literature to substantiate the existence of wind forces that tend to produce transverse oscillations in slender vertical members. These oscillations are perpendicular to the main movement of the wind. Actual structures are subjected to such variable wind conditions, however, that identical structures built on different topographical sites may undergo greatly different behavior. One structure might suffer serious damage while an identical structure at another site develops no trouble whatever.

An excellent summary of the characteristics of wind forces acting upon tall slender structures is presented in the 1961 Report of the ASCE Task Committee on Wind Forces.[4] Under the heading "Self-Excited Oscillations," this committee quotes Den Hartog as follows.[5] "In a self-excited oscillation, the alternating force that sustains the motion is created or controlled by the motion itself; if the motion stops, the alternating force disappears."

---

[4] Trans., Am. Soc. of Civil Eng., Vol. 126 (1961).

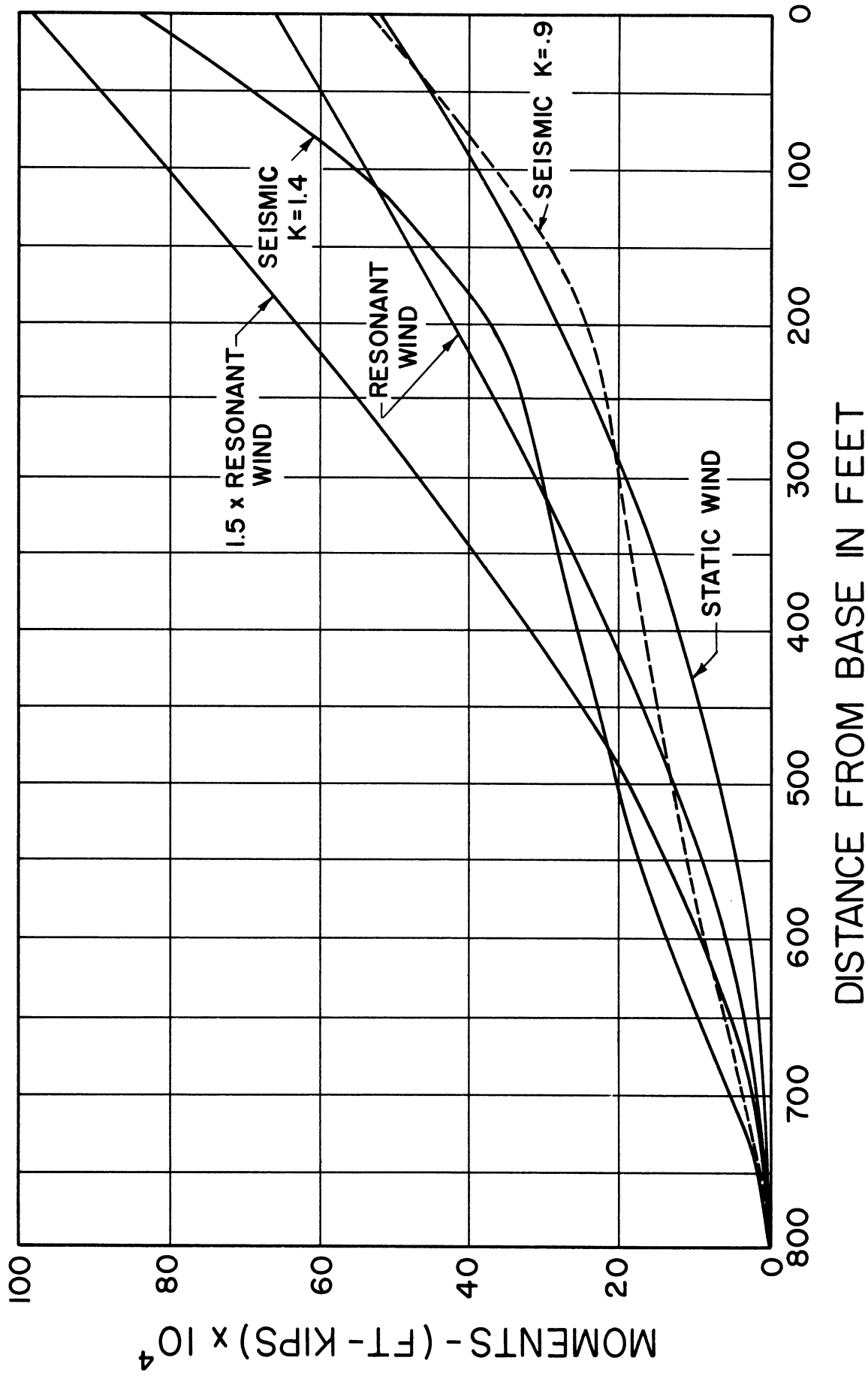[5] "Mechanical Vibrations" by J. P. Den Hartog, p. 282.

Figure 3. Design Moments.

The self-excitation of structures is an extremely complicated phenomenon but, in general, appears to produce oscillations that have the same frequency as the fundamental frequency of the structure. This agreement is not, however, sufficient to assume that the forcing function is necessarily periodic; in fact, for a large Reynolds number, the fluctuating wind forces are more likely to be random than periodic with respect to time. However, the assumption that maximum amplitudes will be obtained when there is sustained periodicity of the vortice action with the same period as the chimney is on the safe side and will be accepted as a reasonable basis for estimating the maximum lateral wind moments and shears. Furthermore, for tapered chimneys, experimental results indicate that the resonant wind velocity is influenced primarily by the upper portion.[6]

Assuming that the chimney is oscillating perpendicular to the wind in the fundamental mode, the critical wind velocity $V_c$ can be estimated from the equation

$$V_c = \frac{F\, D_c}{S} = 5\, F\, D_c \qquad (5)$$

In Equation (5), F is the frequency in cycles per second and S is the Strouhal number which after considerable survey of the literature has been selected as 0.2. As already mentioned, $D_c$ should apparently be selected from the upper portion of the chimney; the writers have usually selected a value of the outer diameter between H/6 and H/3 from the top where H is the height of the chimney above grade.

---

[6] "Wind Effects on Buildings and Structures", Proceedings of the Conference held at the National Physical Laboratory, Teddington, Middlesex, June 1963, Volume II., P. 806.

The periodic exciting force of the wind is assumed as

$$f = C_L \left(\frac{\rho V_c^2}{2}\right) D \sin \omega t = A D \sin \omega t \tag{6}$$

where:

$f$ = resultant transverse force per unit height of chimney

$\rho$ = mass density of the air

$V_c$ = critical velocity of the wind

$D$ = outside diameter at any elevation

$\omega$ = natural frequency of the mode considered (rad/sec.)

$C_L$ = lift coefficient

$A = C_L \dfrac{\rho V_c^2}{2}$ = constant.

Although the outside diameter $D$ varies, the critical velocity $V_c$ is assumed constant as it is apparently governed by the upper part of the chimney. This assumption of uniform velocity throughout the stack is considered satisfactory for the investigation even if not in strict accordance with meteorological data. $C_L$ is another uncertain value which undoubtedly varies for different surface and wind conditions. If several stacks are in a row the exciting force may be increased beyond the value for a single chimney. At present the authors use an apparently conservative value of $C_L$ = .66 but, when more experimental results are available, this value may be considerably altered. When the forcing function has been established the shears and moments accompanying such a time variation can be computed in a similar manner to the determination of seismic forces.

Due to the forcing function, f , of the wind the displacements, the shears and moments in the chimney vibrating in the first mode are given by

$$Y(x,t) = \phi_1(x), \; q_1(t) \tag{7}$$

$$V(x,t) = V_1(x), \; q_1(t) \tag{8}$$

$$M(x,t) = M_1(x), \; q_1(t) \tag{9}$$

where $q_1(t)$ is obtained from the following equation:

$$\ddot{q}_1 + 2\,\beta\,\omega_1\,\dot{q}_1 + \omega_1{}^2\,q_1 = \dfrac{\displaystyle\int_o^L f(x,t)\,\phi_1\,dx}{\displaystyle\int_o^L m\,\phi_1{}^2\,dx} \tag{10}$$

Equation (10) can be reduced to the form

$$\ddot{q}_1 + 2\,\beta\,\omega_1\,\dot{q}_1 + \omega_1^2\,q_1 = C'\,\sin\omega_1 t \tag{11}$$

where

$$C' = \dfrac{A\displaystyle\int_o^L D\,\phi_1\,dx}{\displaystyle\int_o^L m\,\phi_1{}^2\,dx} = \text{constant}$$

It will be found that Equation (11) is satisfied by a value of

$$q_1 = -\dfrac{C'\cos\omega_1 t}{2\beta\omega_1{}^2} \tag{12a}$$

or

$$q_1\,(\text{max}) = -\dfrac{C'}{2\beta\omega_1{}^2} \tag{12b}$$

All maximum displacements, moments, and shears throughout the chimney can now be found (see Equations (7), (8) and (9)) by multiplying the values for the first mode by the above value of $q_{1(max.)}$ given by Equation (12b).

The same chimney that was used in the numerical example for determining seismic forces will now be used to illustrate the calculation of the wind resonant forces. The critical velocity $V_c$ will be determined from the equation:

$$V_c = 5 F_1 D_c = \frac{(5)(2.25)(46)}{2\pi} = 82.3 \text{ ft/sec. or } 56.1 \text{ mph.}$$

$$A = C_L \frac{\rho V^2}{2} = (0.66)(.00238)\left(\frac{82.3}{2}\right)^2 = 5.3 \text{ lbs./ft.}^2$$

$$\int_0^L D \phi_1 \, dx = 11,680 \text{ ft.}^2$$

$$\int_0^L m \phi_1^2 \, dx = 67,172 \frac{\text{lbs/sec}^2}{\text{ft.}}$$

$$C' = \frac{(5.3)(11,680)}{67.172} = 0.932 \frac{\text{ft}}{\text{sec}^2}$$

$$q_1(max) = \frac{(0.923)}{(2)(.05)(2.25)^2} = 1.82 \text{ ft.}$$

The bending moments and shears can now be obtained by multiplying the values in Table 2 for the first mode by $q_1 = 1.82$. These bending moments are shown in Figure 3. Another curve giving 1.5 times these values is used for the maximum stress design.

# WORKING STRESS AND MAXIMUM STRESS DESIGN

In this paper the term maximum stress design is used to des-
ignate a design that will keep the maximum tensile stress in the rein-
forcing steel and the compressive stress in the concrete within certain
designated upper limits when the bending moments used in the ordinary
working stress designs are multiplied by some load factor LF. The same
basic assumptions and formulas for calculating stresses are used in
both solutions and therefore no ultimate strength conditions are utilized.
For the maximum stress condition  a  tensile stress near the yield point
of the steel can be permitted and a compressive stress in the concrete
of approximately .8 $f'_c$ .

As the vertical load is not multiplied by any load factor the
unit stresses increase more rapidly than do the bending moments. This
condition is illustrated in Figures 4 and 5 which show the variation in
the steel stress at a typical section of a chimney. It is apparent from
these curves that when the value of the steel ratio is .0025, the usual
minimum, the tensile stress in the steel may increase 300 percent for a
33 percent increase in the bending moment. If only a working stress de-
sign is used the load factor LF may have large variations throughout the
chimney.

The tensile and compressive stresses for the chimney used in
the preceding examples which are shown graphically in Figures 6 and 7,
indicate that seldom does a particular loading condition govern the
design throughout the height of a chimney. In the example given the
earthquake stresses govern the design in the upper half of the chimney
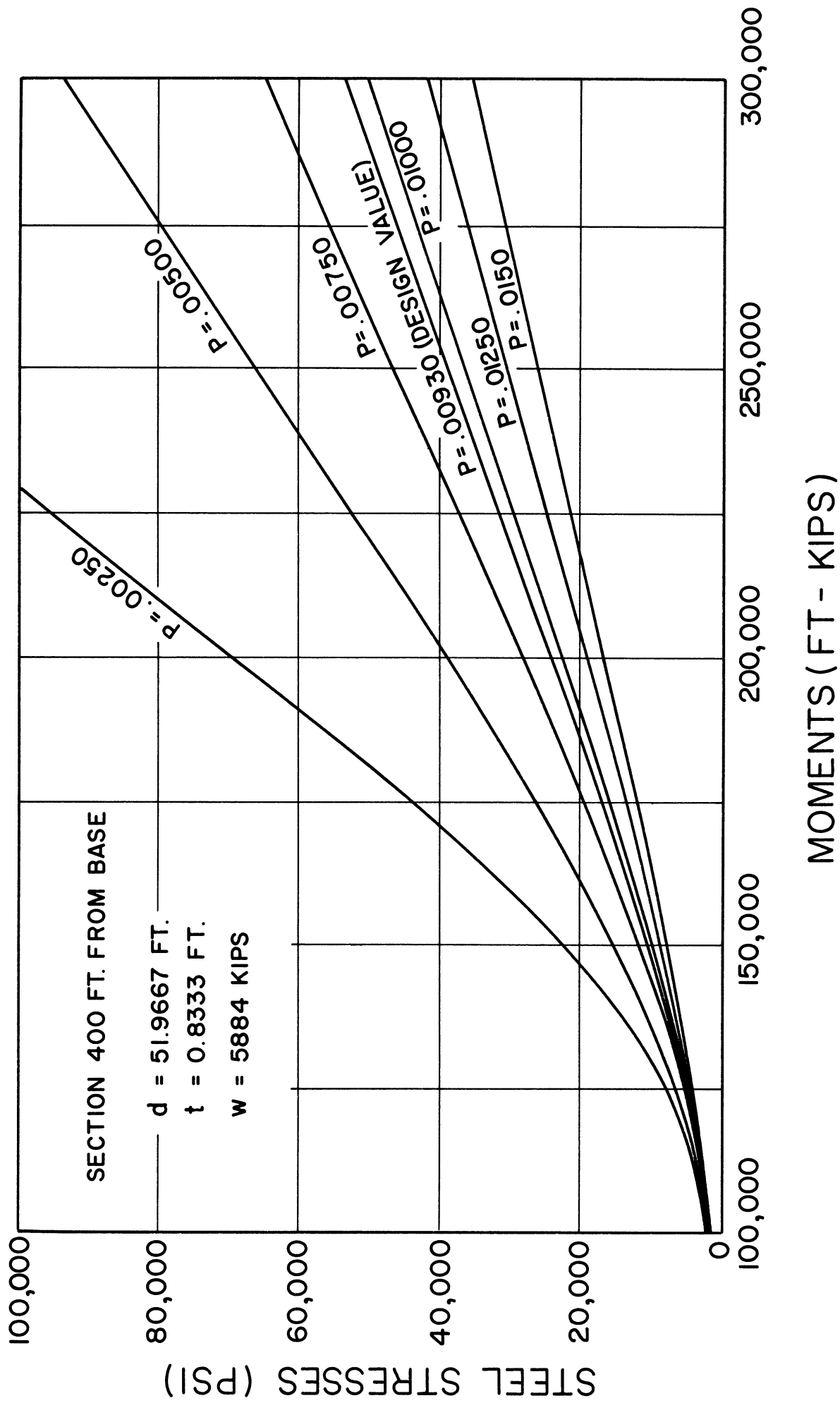and the resonant wind in the lower part. In many chimneys the earthquake

SECTION 400 FT. FROM BASE

d = 51.9667 FT.
t = 0.8333 FT.
w = 5884 KIPS

P=.00250

P=.00500

P=.00750

P=.00930 (DESIGN VALUE)

P=.01000

P=.01250

P=.0150

STEEL STRESSES (PSI)

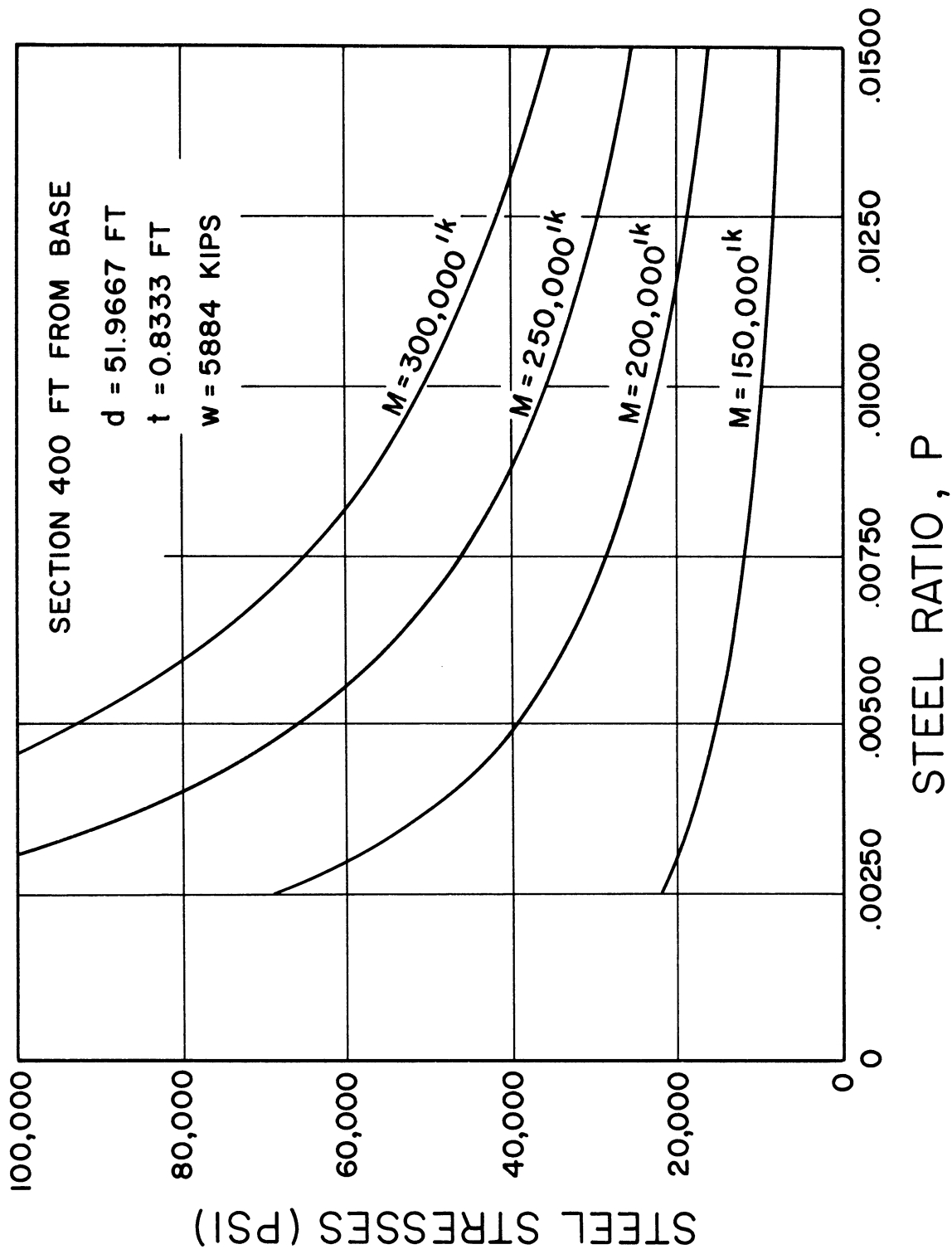MOMENTS (FT-KIPS)

Figure 4. Steel Stresses vs. Moments.
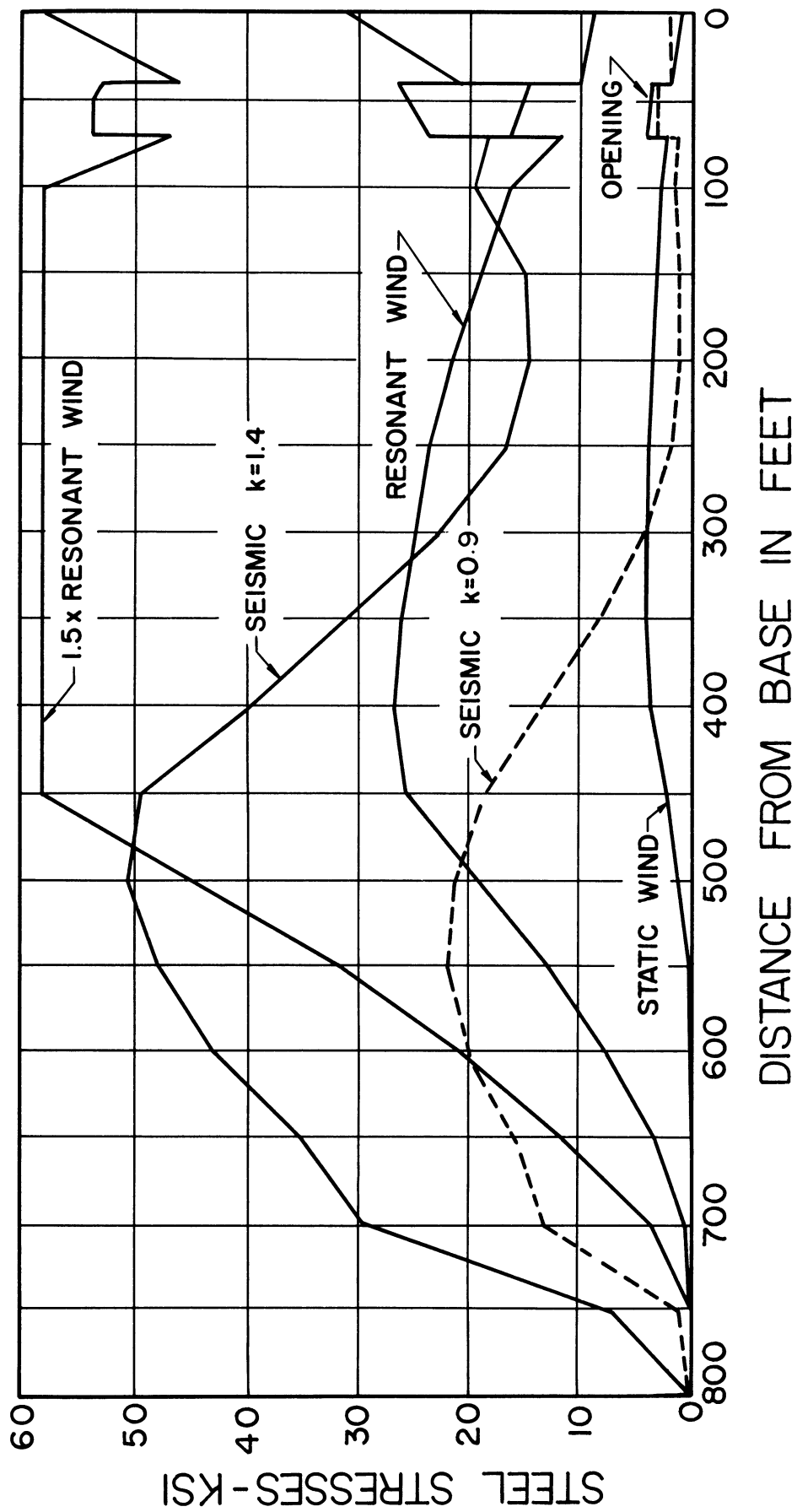
Figure 5. Steel Stresses vs. Steel Ratio.
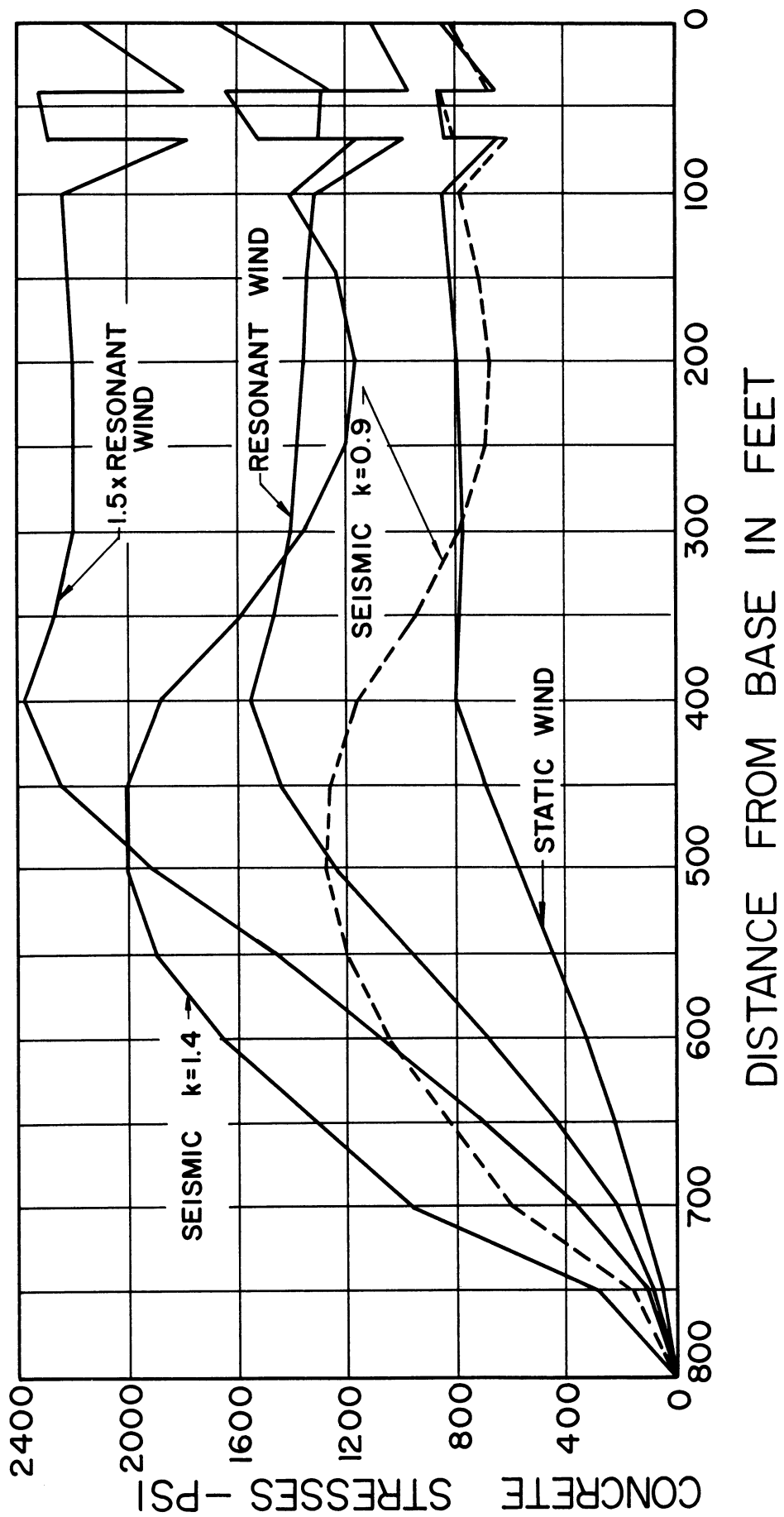
Figure 6. Steel Stresses

Figure 7.  Concrete Stresses.

stresses govern in the upper part and the static wind stresses in the lower portion. This condition is illustrated in Figure 8 for an 825 ft. chimney which has a supported steel liner. For this chimney the resonant wind moments were relatively small and did not govern the design. However, in the lower 350 ft. of the chimney the maximum steel stress of 50,000 psi (allowed for A432 steel) due to 1.5 times the static wind moments governed the design. In terms of wind velocity the load factor is $(1.5)^{1/2}$ or 1.22. These results again emphasize the importance of considering the non-linear nature of the relationship between stresses and bending moments in the reinforced concrete chimneys.

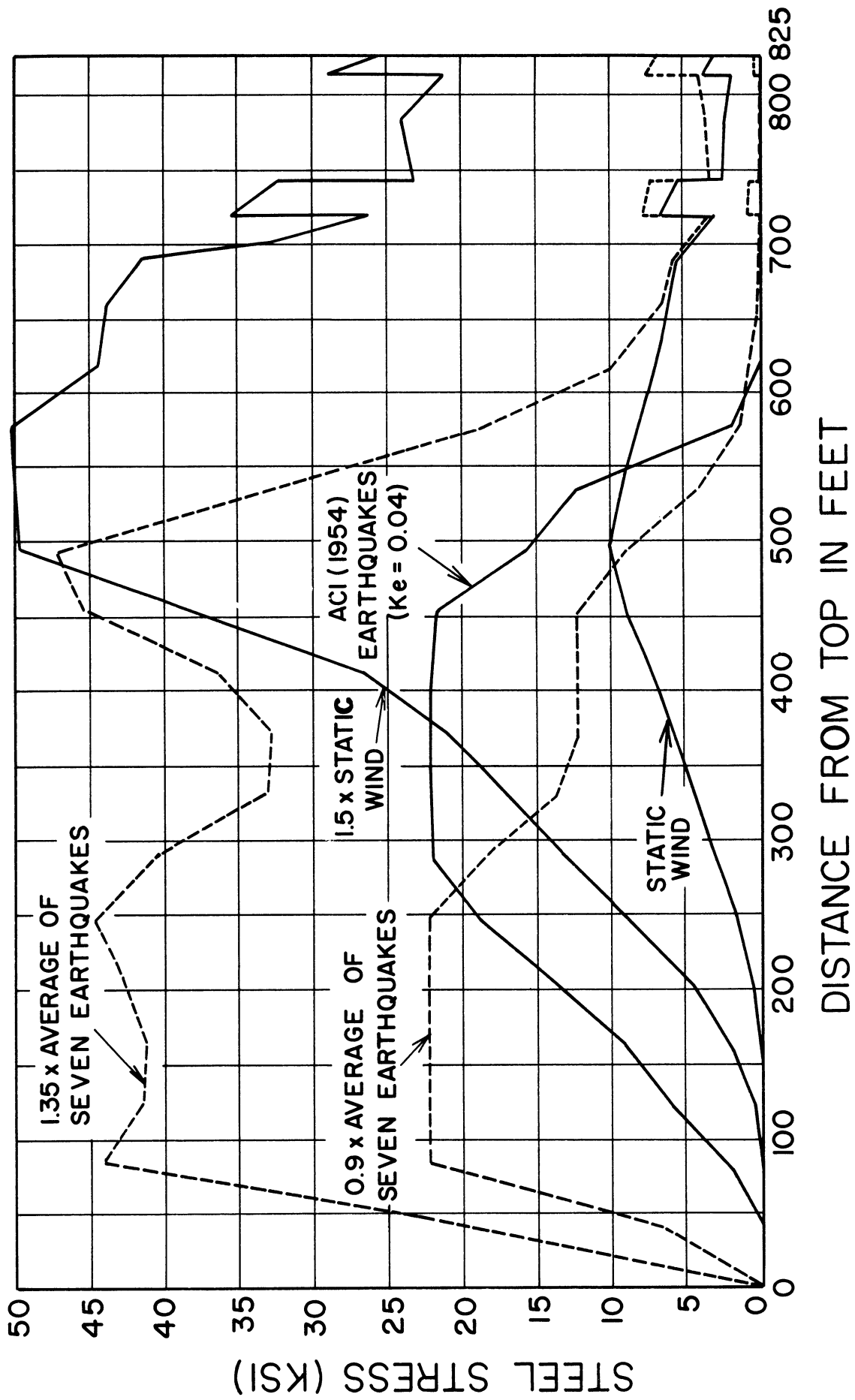A comparison of the steel stresses for the proposed seismic design with

Figure 8. Steel Stresses (A 432 Steel).

stresses govern in the upper part and the static wind stresses in the low-
er portion. This condition is illustrated in Figure 8 for an 825 ft. chim-
ney which has a supported steel liner. For this chimney the resonant wind
moments were relatively small and did not govern the design. However, in
the lower 350 ft. of the chimney the maximum steel stress of 50,000 psi
(allowed for A432 steel) due to 1.5 times the static wind moments governed
the design. In terms of wind velocity the load factor is $(1.5)^{1/2}$ or 1.22.
These results again emphasize the importance of considering the non-linear
nature of the relationship between stresses and bending moments in rein-
forced concrete chimneys.

A comparison of the steel stresses for the proposed seismic de-
sign with $K = 0.9$ and the procedure specified in the A.C.I. Specifica-
tions for the Design and Construction of Reinforced concrete chimneys
(1954) is given in Figure 8. For this chimney the top portion would be
underdesigned by the A.C.I. 1954 requirements.

## CONCLUSIONS

Tall reinforced concrete chimneys can now be designed for seis-
mic and wind forces so as to reduce greatly the need for empirical formulas.
The solutions for these problems, as discussed in this paper, will provide
the following information:

(a) Values of shape, shear, and bending moments for any number
of modes of free vibration. The use of the first three modes is recommend-
ed.

(b) The maximum shear and bending moment diagrams from selected
accelerograms that have been obtained from ground measurements for actual
earthquakes. Perferably some information concerning the visual damage

resulting from these earthquakes should be available. For design purposes

the authors recommend the use of three or more accelerograms for strong

motion earthquakes.

(c) The average of the maximum shears and bending moments at

any section should be multiplied by a factor  K  to take into account

the regional differences in past seismic action as well as site charac-

teristics. However, undue optimism that is based on a relatively short

seismic history of the region or upon recorded damage to one or two chim-

neys is not recommended. The authors, believe that any high chimney should

be designed for a minimum value of .7 for  K  unless the accelerograms

used have been obtained from stronger motion earthquakes than the ones

used in this paper.

(d) The calculation of design moments for assumed periodic

wind forces have been discussed in this paper. As the resulting design

moments and shears are inversely proportional to the assumed value of  $\beta$ ,

the fraction of critical damping, this quantity deserves careful consid-

eration.

(e) The non-linear variation of the tensile stresses in the

vertical reinforcing steel and compressive stress in the concrete with

respect to changes in the bending moment is an important factor in the

design. Therefore to maintain a consistent factor of safety the stresses

should be calculated for both the working load conditions and for a load

factor times these working loads. The latter has been designated as a

maximum stress design.

(f) Although shearing stresses are not discussed in this paper

they can be checked with sufficient accuracy be means of standard formulas

for homogeneous thin walled cross-sections.