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COMPLIANCE AT THE END EFFECTOR OF AN ELECTRO-HYDRAULICALLY CONTROLLED ROBOT[1]

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ABSTRACT

A general mathematical model for a robot, where each link is driven by electrohydraulic servo valve and actuator, is developed. Based on this model, several notions for compliance of the end effector of the robot due to changes in the external load on the end effector are defined. Both, global and local, dynamic and static compliance notions are introduced. A formula for the closed loop local, static compliance of the end effector is derived in terms of the robot stiffness, the hydraulic servo valve leakages, the geometry of the robot, and the feedback gains. It is shown that effective feedback control design involves consideration of a tradeoff between good closed loop stability properties and reduction of the closed loop end effector compliance.

1. INTRODUCTION

Most heavy duty industrial robots are driven by electrohydraulic servo actuators, controlled by independent position feedback loops. It is well known that such robots may have high compliance at the end effector (i.e. their stiffness or rigidity is low) with respect to variations in the external load on the robot at the end effector [2]. Such a poor compliance property arises from several sources, but the primary source for hydraulically driven robots is the leakage of the fluid within each of the joint actuators. This source of compliance can be only marginally influenced by the particular design of the hydraulic servo valves and actuators. Moreover, the interaction between the motion of the multiple robotic links plus the hydraulic compliances at each joint make the characterization of the overall compliance of the robot at the end effector a difficult problem.

One approach to reducing the robotic compliance at the end effector would be use of a controller based on direct measurements of the load. Such load feedback control is known to allow improvement of positioning accuracy in a variety of situations [7,8,12,13,16,17]. A related approach, but one that is more suitable for electro-hydraulically driven robots, is to use a controller based on feedback of the pressures in the hydraulic cylinders.

Specifically, it is suggested that an effective controller for an electrohydraulic driven articulated robot should depend on both link displacement feedbacks and cylinder hydraulic pressure feedbacks from the joint actuators. Since the cylinder pressures are directly related, through the geometry, to the joint torques and hence to the load on the end effector it is expected that use of the pressure feedbacks should provide the opportunity for improved control. Although there is a substantial body of material available on control of simple hydraulic servo systems [4,10,15] and on control of articulated robots [5,6,11] there has been scant research on the control of articulated robots using electrohydraulic joint actuators.

Mathematical models are developed for describing the motion of articulated robots driven by electrohydraulic joint actuators. The models include the effects of the hydraulic actuator dynamics, the link dynamics and the geometrical constraints between the actuators and the links. Such models can serve as a basis for the design of effective controllers, based on link displacement and actuator pressure feedbacks. Further, the models can be used to evaluate the advantages of using pressure feedbacks as a part of the controller in terms of improved compliance properties of the robot at the end effector. A centralized controller for all actuators should take into account the multivariable and nonlinear coupling between the links, the actuator compliance characteristics, as well as the specific motion control objectives for the end effector.

2. CHARACTERIZATION OF END EFFECTOR COMPLIANCE

A general form for the equations of motion of a wide class of articulated robots with n links and n joints are

$$M(\Theta)\Theta + H(\Theta,\Theta) = T$$

where $\Theta \epsilon R^n$ is the vector of link angles and $T \epsilon R^n$ is the vector of external torques at the joints, [3,11]. Here $M(\Theta)$ represents an inertia matrix function and $H(\Theta,\dot{\Theta})$ represents a vector function which defines the Coriolis and gravitational terms. In our case the external joint torques are of the form

$$T = T_a + T_L$$

where T_a are the torques supplied by the hydraulic actuators

$$T_a = G(\Theta)f_a$$

where f_a is the vector of actuator forces (i.e. cylinder pressures multiplied by cylinder areas) and $G(\Theta)$ depends on the geometry between the links and the actuators. Also T_L are the load torques experienced at the joints

$$T_L = T_L(F_L, \Theta),$$

where F_L represents the load force at the end effector. It is assumed that the load torques at the joints vanish if the load force is zero, i.e. $T_L(0,\Theta) = 0$.

A general mathematical model for a wide class of electrohydraulic servos is given by [9,10]

$$K_h^{-1}\dot{f}_a + G_p f_a + G^T(\Theta)\Theta = u$$

Here u represents the normalized currents to the servo torque motors and, as before, f_a represents the vector of actuator forces. This model includes the effects of fluid compressibility through the constant actuator stiffness matrix K_h and the effects of fluid leakage around the actuator pistons through the constant leakage sensitivity matrix G_p. The simplified model for the electrohydraulic actuators does not take into account friction forces in the cylinders or nonlinear high frequency dynamics of the servo valves and actuators; it is felt that such effects are not crucial to a careful study of the compliance effects at the end effector and hence they are ignored.

Thus the open loop model of an articulated robot driven by multiple electrohydraulic joint actuators is given by the equations

$$M(\Theta)\ddot{\Theta} + H(\Theta,\dot{\Theta}) = G(\Theta)f_a + T_L(F_L,\Theta)$$

$$K_h^{-1} f_a + G_p f_a + G^T(\Theta)\Theta = u.$$

The inertial displacement of the end effector of the robot is denoted by the vector z and depends on the geometry of the robot and the link positions through

$$z = Z(\Theta).$$

These equations are representative of a large class of robots driven by electrohydraulic actuators.

Assuming that $F_L \epsilon R^3$ and $z \epsilon R^3$, definitions of the robot compliance at the end effector are now given. First the notion of ***open loop static compliance*** at the end effector is introduced. Assume that the servo currents are constant and given by $\overline{u}$. Suppose that if there is no load force, $F_L = 0$, then the robot is in equilibrium with constant joint angles $\overline{\Theta}$ and constant actuator forces $\overline{f}_a$ where

$$H(\overline{\Theta}, \Theta) = G(\overline{\Theta})\overline{f}_a$$

$$G_p \overline{f}_a = \overline{u}.$$

Now assume that the load force $F_L \neq 0$ is constant; then the robot is in equilibrium with constant joint angles $\hat{\Theta}$ and constant actuator forces $\hat{f}_a$ where

$$H(\hat{\Theta}, \Theta) = G(\hat{\Theta})\hat{f}_a + T_L(F_L, \hat{\Theta})$$

$$G_p \hat{f}_a = \overline{u}.$$

The implicit relation

$$F_L \rightarrow Z(\hat{\Theta}) - Z(\overline{\Theta})$$

defines the open loop static compliance of the robot at the end effector. This compliance is a 3×3 nonlinear matrix function.

Next, the notion of *open loop dynamic compliance* at the end effector is introduced. Assume that the servo currents are given by the specified time functions $\bar{u}(t), t \geq 0$, Suppose that if there is no load force, $F_L = 0$, then the robot motion is described by joint angle functions $\bar{\Theta}(t)$ and actuator forces $\bar{f}_a(t), t \geq 0$, where

$$M(\bar{\Theta})\ddot{\bar{\Theta}} + H(\bar{\Theta},\dot{\bar{\Theta}}) = G(\bar{\Theta})\bar{f}_a$$

$$K_h^{-1}\dot{\bar{f}}_a + G_p\bar{f}_a + G^T(\bar{\Theta})\dot{\bar{\Theta}} = \bar{u}.$$

Now assume the load force $F_L \neq 0$ is constant; then the robot motion is described by joint angle functions $\hat{\Theta}(t)$ and actuator forces $\hat{f}_a(t), t \geq 0$, where

$$M(\hat{\Theta})\ddot{\hat{\Theta}} + H(\hat{\Theta},\hat{\Theta}) = G(\hat{\Theta})\hat{f}_a + T(F_l,\hat{\Theta})$$

$$K_h^{-1}\hat{f}_a + G_p\hat{f}_a + G^T(\hat{\Theta})\hat{\Theta} = \bar{u}.$$

The implicit relation

$$F_L \rightarrow Z(\Theta(t)) - Z(\Theta(t))$$

defines the *open loop dynamic compliance* of the robot at the end effector. This compliance is a 3×3 nonlinear and time varying matrix function.

It is further possible to define ***local compliance*** of the robot at the end effector through (Frechet) derivatives of the defined nonlinear compliance maps, evaluated at $F_L = 0$.

The above definitions of open loop compliance at the end effector are easily modified in the case where the input servo currents are not a priori specified but rather are determined according to a feedback relation. Thus the notions of *closed loop static compliance* and *closed loop dynamic compliance* are obtained. Our interest is to consider the closed loop compliance issues in the case that the controller is of the general feedback form

$$u = G_d(\Theta) + G_f(f_a)$$

depending on the joint displacements Θ and the actuator forces f_a (or equivalently the cylinder pressures). Such a controller form is sufficiently general to include controllers designed using recently developed techniques [1,5,14]. For consistency with the previous notation the feedback functions $G_d(\Theta)$ and $G_f(f_a)$ should satisfy

$$\bar{u} = G_d(\bar{\Theta}) + G_f(\bar{f}_a).$$

Our choice of controller based on feedback of link displacements plus actuator forces (or cylinder pressures) is motivated by the effectiveness of load feedback generally and by the theoretical results of our previous work in [9]. However, similar control approaches based on link acceleration feedbacks or on dynamic "high pass filtered" actuator force (or cylinder pressure) feedbacks can also be examined.

Our objective is to develop a procedure for determining the robot compliance in terms of the characteristics of the mechanical configuration of the links and actuators and their dynamic characteristics. Our procedure is based on the development and use of linearized equations suitable for an analytical characterization of local compliance effects. A related objective is to use such a characterization as a basis for the design of feedback controllers which, to some extent, are able to improve the compliance properties of the closed loop. As suggested earlier, the key to improved closed loop compliance at the end effector is felt to be the intelligent use of cylinder pressure feedback loops and joint position feedback loops as a basis for the control logic.

3. CHARACTERIZATION OF LOCAL STATIC END EFFECTOR COMPLIANCE

Consider the problem of characterizing the "local" static compliance of an electrohydraulic driven articulated robot. If the nonlinear link dynamic equations are linearized about the equilibrium values for the joint angles $\overline{\Theta}$ and actuator forces $\overline{f}_a$ the resulting linear equations are of the form

$$M\Delta\ddot{\Theta} + K\Delta\Theta = B\Delta f_a + CF_L$$

where M,K,B,C are suitably defined Jacobian matrices, evaluated at the equilibrium values. In general the matrices M and K are symmetric and positive definite. The corresponding linearized equations for the actuator forces are

$$K_h^{-1}\Delta\dot{f}_a + G^p\Delta f_a + B^T\Delta\Theta = \Delta u.$$

The change in the position of the end effector, based on a linearized approximation, is

$$\Delta z = D\Delta\Theta$$

where D is a Jacobian matrix evaluated at the equilibrium. In the above equations

$$\Delta\Theta = \Theta - \overline{\Theta}$$

$$\Delta f_a = f_a - \overline{f}_a$$

$$\Delta u = u - \overline{u}$$

$$\Delta z = Z(\Theta) - Z(\overline{\Theta}).$$

Now suppose that the closed loop control is of the form

$$u = \overline{u} + G_d(\overline{\Theta} - \Theta) + G_f(\overline{f}_a - f_a)$$

so that

$$\Delta u = -G_d \Delta\Theta - G_f \Delta f_a$$

for constant diagonal feedback gain matrices G_d and G_f . Consequently, the closed loop equations are

$$M\Delta\ddot{\Theta} + K\Delta\Theta = B\Delta f_a + CF_L$$

$$K_h^{-1}\Delta\dot{f}_a + (G_p + G_f)\Delta f_a + G_d\Delta\Theta + B^T\Delta\Theta = 0$$

$$\Delta z = D\Delta\Theta.$$

Thus, the conditions for equilibrium are that

$$K\Delta\Theta = B\Delta f_a + CF_L$$

$$(G_p + G_f)\Delta f_a + G_d\Delta\Theta = 0.$$

Hence the variation in the end effector position can be expressed in terms of the load force F_L as

$$\Delta z = D[K + B(G_p + G_f)^{-1}G_d]^{-1}CF_L$$

Thus the constant matrix

$$D[K + B(G_p + G_f)^{-1}G_d]^{-1}C$$

defines the "local" closed loop static compliance of the end effector.

4. LIMITS ON THE LOCAL STATIC END EFFECTOR COMPLIANCE

It is clear that to reduce this robot compliance the feedback gain matrices should be chosen so that $(G_p + G_f)^{-1}G_d$ is as large as possible. But the feedback gain matrices must also be chosen so that the closed loop is locally asymptotically stable, and this stabilization constraint limits the possible reduction in the robot compliance. In [9] it is shown that a sufficient condition for local asymptotic stability of such a closed loop system is that the effective actuator transfer function matrix

$$H_a(s) = [K_h^{-1}s + G_f]^{-1}[I + \frac{1}{s} G_d]$$

be strictly positive real. Under the stated assumptions this requirement is that there is $\delta > o$ such that

$$\frac{1}{2} [H_a(jw - \delta) + H_a^T(-jw - \delta)]$$

is nonnegative definite for all real w .

One particularly simple form of feedback control is the case of decentralized control for which the gain matrices G_d and G_f are diagonal matrices. Since G_p is diagonal in this case the above stabilization constraint can be written as

$$K_h^{-1} > (G_p + G_f)^{-1} G_d .$$

Hence the matrix

$$D[K + BK_h^{-1}]^{-1}C$$

defines a limit to the possible local static compliance of the closed loop.

It is clear that the selection of feedback gain matrices G_f and G_d involves consideration of a trade-off between closed loop stability (or speed of response) and closed loop compliance reduction.

5. CONCLUSIONS

A general mathematical formulation of the robot dynamics, including a simple form for the hydraulic servo valve/actuator dynamics, has been given. Based on that formulation several notions of compliance of the robot end effector, with respect to load changes at the end effector, have been presented. A rather simple characterization of the local static end effector compliance has been derived; this compliance is shown to depend on the link and actuator geometry, the local robot stiffness, the actuator leakages, and the feedback gain matrices. By making use of a previous stabilization result, the control design trade-off between closed loop stability and compliance reduction at the end effector is clarified.

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