

M-flation: inflation from matrix valued scalar fields

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Abstract. We propose an inflationary scenario, *M-flation*, in which inflation is driven by three $N \times N$ hermitian matrices Φ_i , $i = 1, 2, 3$. The inflation potential of our model, which is strongly motivated from string theory, is constructed from Φ_i and their commutators. We show that one can consistently restrict the classical dynamics to a sector in which the Φ_i are proportional to the $N \times N$ irreducible representations of $SU(2)$. In this sector our model effectively behaves as an N-flation model with $3N^2$ number of fields and the effective inflaton field has a super-Planckian field value. Furthermore, the fine-tunings associated with unnaturally small couplings in the chaotic type inflationary scenarios are removed. Due to the matrix nature of the inflaton fields there are $3N^2 - 1$ extra scalar fields in the dynamics. These have the observational effects such as production of iso-curvature perturbations on cosmic microwave background. Moreover, the existence of these extra scalars provides us with a natural preheating mechanism and exit from inflation. As the effective inflaton field can traverse super-Planckian distances in the field space, the model is capable of producing a considerable amount of gravity waves that can be probed by future CMB polarization experiments such as PLANCK, QUIET and CMBPOL.

Keywords: inflation, string theory and cosmology, cosmological perturbation theory

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1 Introduction

Recent observations, specially the five years Wilkinson Microwave Anisotropy Probe (WMAP5) data [1] strongly support inflation as the theory of early Universe and structure formation. In their simplest forms, models of inflation are constructed from a scalar field, the inflaton field, which is minimally coupled to gravity. The potential is flat enough, so that a period of slow-roll inflation is achieved. These simple models of inflation, not only solve

the problems associated with the standard big bang cosmology such as the horizon problem, the flatness problem and the monopole problem, but also generate quantum perturbations which seed the structures in the Universe. The slow-roll models of inflation produce perturbations on cosmic microwave background (CMB) which are almost scale-invariant, adiabatic and Gaussian which are in very good agreement with the current data. Theoretically, on the other hand, inflation still remains a paradigm and one can construct many inflationary models compatible with the current data. There have been many attempts to embed inflation in string theory, for a review see [2].

Theories of multi-field inflation, in which one deals with more than one scalar field, have also been studied [3], for a review see [4]. In the multiple field inflationary models one can perform a rotation in the field space of scalar fields where the inflaton field is evolving along a trajectory while the remaining fields are orthogonal to it. These extra fields, like the inflaton itself, have quantum fluctuations which once stretched to super-Hubble scales can become classical and can therefore contribute to the power spectrum of iso-curvature as well as curvature perturbations, the details of which depends on the post inflationary dynamics and the reheating scenario. A specific possibility in the multi-field inflation is the idea of assisted inflation [5]: while the potential is too steep for an individual field to support inflation, the collective effect of a large number of scalar fields leads to enough number of e-folds. Similar idea was exploited in N-flation [6] and Cascade inflation [7, 8] to obtain inflation from several potentials that are individually too steep to sustain inflation. In these models, although the “effective” inflaton field gets a super-Planckian field value for chaotic $m^2\phi^2$ and $\lambda\phi^4$ inflationary scenarios, due to the large number of fields, each physical field remains sub-Planckian. Moreover, for the case of the $\lambda\phi^4$ inflationary theory this can be used to resolve problem of unnaturally small coupling [9, 10]. Also following the Lyth bound [11], due to the large excursions of the effective inflaton in comparison with M_P , in these models one expects to obtain a considerable amount of gravity waves.

In this work we take a different view and promote the inflaton fields to general $N \times N$ hermitian matrices and hence these models will be called Matrix Inflation or *M-flation* for short. In this sense M-flation is a special case of multi-field inflation. Working with matrices, besides the simple products of the fields, we can also consider commutators of matrices. In our class of M-flation models we consider three $N \times N$ matrices, Φ_i , $i = 1, 2, 3$ and the potential is taken to be quadratic in the Φ_i or in their commutator $[\Phi_i, \Phi_j]$. Therefore, in the class of models we consider the potential term for Φ_i can have three types of terms: $\text{Tr} [\Phi_i, \Phi_j]^2$, $\text{Tr} \epsilon_{ijk} \Phi_i [\Phi_j, \Phi_k]$ and $\text{Tr} \Phi_i^2$. As we will argue, this class of potentials is well-motivated from string theory and brane dynamics.

As we will see despite the simple form of the potential constructed from these matrices and their commutators our model has a rich dynamics. We argue that M-flation can solve the fine-tunings associated with standard chaotic inflationary scenarios. Furthermore, like any multi-field inflation model, there would be iso-curvature perturbations as well as the usual adiabatic perturbations. This can have significant observational consequences for the CMB observations [1, 12]. Moreover, we argue that our model has an embedded efficient preheating mechanism.

The outline of the paper is as follows. In section 2, we provide the setup through introducing the action and show that, with the appropriate initial conditions, for the sector in which the Φ_i fields fall into irreducible $N \times N$ representations of $SU(2)$, the theory effectively and at the classical level, behaves like a single field inflation. In section 3, we discuss simple models of inflation such as chaotic, symmetry breaking and inflection point inflation which are

constructed from our M-flation. In section 4, we work out the mass spectrum of the remaining $3N^2 - 1$ scalar fields, the iso-curvature modes, which are not classically turned on during inflation (these fields will be generically called Ψ -modes). In sections 5 and 6 we consider quantum mechanical excitations of Ψ -modes and their effects. In section 5, we compute the power spectrum of fluctuations of the inflaton and the $3N^2 - 1$ iso-curvature modes. In section 6, we focus on quantum mechanical creation of the sub-Hubble modes. This particle creation takes energy away from the inflaton field. We show that our model naturally contains this mechanism [13] which is the basis of the preheating scenarios [14, 15]. In section 7, we give string theory motivations behind the M-flation models, arising from dynamics of multiple D3-branes in specific flux compactifications. The last section is devoted to discussion and outlook. In the appendix we have gathered some technical details of slow-roll inflation.

2 M-flation scenario, the setup

As explained before, we start with an inflationary setup in which the inflaton fields are taken to be $N \times N$ non-commutative hermitian matrices. We start from the following action

$$S = \int d^4x \sqrt{-g} \left(\frac{M_P^2}{2} R - \frac{1}{2} \sum_i \text{Tr} (\partial_\mu \Phi_i \partial^\mu \Phi_i) - V(\Phi_i, [\Phi_i, \Phi_j]) \right), \quad (2.1)$$

where the reduced Planck mass is $M_P^{-2} = 8\pi G$ with G being the Newton constant and the signature of the metric is $(-, +, +, +)$. Note that the i -index counts the number of matrices and is not denoting the matrix elements; the matrix element indices are suppressed here. Also, V represents our potential constructed from the $N \times N$ matrices Φ_i and their commutators $[\Phi_i, \Phi_j]$. The kinetic energy for Φ_i has the standard form and it is assumed that the Φ_i matrices are minimally coupled to gravity.

As we will discuss in section 7, the potential $V(\Phi_i, [\Phi_i, \Phi_j])$ can be motivated from dynamics of branes in string theory where up to leading terms in Φ_i and $[\Phi_i, \Phi_j]$, in specific string theory backgrounds, the potential takes the form

$$V = \text{Tr} \left(-\frac{\lambda}{4} [\Phi_i, \Phi_j] [\Phi_i, \Phi_j] + \frac{i\kappa}{3} \epsilon_{jkl} [\Phi_k, \Phi_l] \Phi_j + \frac{m^2}{2} \Phi_i^2 \right), \quad (2.2)$$

where $i = 1, 2, 3$ and hence we are dealing with $3N^2$ real scalar fields. The Tr is over $N \times N$ matrices, and here and below the summation over repeated i, j indices is assumed. λ is a dimensionless number while κ and m are constants with dimensions of mass. We take λ , κ and m^2 to be positive. Note that the potential (2.2) is quadratic in powers of $[\Phi_i, \Phi_j]$ and Φ_i . The action (2.1) together with the potential (2.2) are invariant under $U(N)$ (acting on the matrices) and $SU(2)$ (acting on i, j indices) which are both global symmetries.

Starting with an FRW background

$$ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2, \quad (2.3)$$

the equation of motions are

$$H^2 = \frac{1}{3M_P^2} \left(-\frac{1}{2} \text{Tr} (\partial_\mu \Phi_i \partial^\mu \Phi_i) + V(\Phi_i, [\Phi_i, \Phi_j]) \right) \quad (2.4a)$$

$$\ddot{\Phi}_l + 3H\dot{\Phi}_l + \lambda [\Phi_j, [\Phi_l, \Phi_j]] + i\kappa \epsilon_{ljk} [\Phi_j, \Phi_k] + m^2 \Phi_l = 0, \quad (2.4b)$$

$$\dot{H} = -\frac{1}{2M_P^2} \sum_i \text{Tr} \partial_\mu \Phi_i \partial^\mu \Phi_i, \quad (2.4c)$$

where $H = \dot{a}/a$ is the Hubble expansion rate.

2.1 Truncation to the SU(2) sector

As discussed, Φ_i are $N \times N$ matrices and we are hence dealing with $3N^2$ real scalar fields which are generically coupled to each other. This makes the analysis of the model in the most general form very difficult, if not impossible. Noting the specific form of the the potential (2.2) and that i, j indices are running from 1 to 3, there is the possibility of consistently restricting the classical dynamics to a sector in which we are effectively dealing with a single scalar field. This sector, which will be called the SU(2) sector, is obtained for matrix configurations of the form

$$\Phi_i = \hat{\phi}(t)J_i, \quad i = 1, 2, 3, \quad (2.5)$$

where J_i are the basis for the N dimensional irreducible representation of the SU(2) algebra

$$[J_i, J_j] = i \epsilon_{ijk} J_k, \quad \text{Tr}(J_i J_j) = \frac{N}{12}(N^2 - 1) \delta_{ij}. \quad (2.6)$$

Since both Φ_i and J_i are hermitian, we conclude that $\hat{\phi}$ is a real scalar field.

Plugging these into the action (2.1), we obtain

$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R + \text{Tr} J^2 \left(-\frac{1}{2} \partial_\mu \hat{\phi} \partial^\mu \hat{\phi} - \frac{\lambda}{2} \hat{\phi}^4 + \frac{2\kappa}{3} \hat{\phi}^3 - \frac{m^2}{2} \hat{\phi}^2 \right) \right], \quad (2.7)$$

where $\text{Tr} J^2 = \sum_{i=1}^3 \text{Tr}(J_i^2) = N(N^2 - 1)/4$.

Interestingly enough, this represents the action of chaotic inflationary models with a non-standard kinetic energy. Upon the field redefinition

$$\hat{\phi} = (\text{Tr} J^2)^{-1/2} \phi = \left[\frac{N}{4}(N^2 - 1) \right]^{-1/2} \phi, \quad (2.8)$$

the kinetic energy for the new field ϕ becomes standard, while the potential for it becomes

$$V_0(\phi) = \frac{\lambda_{\text{eff}}}{4} \phi^4 - \frac{2\kappa_{\text{eff}}}{3} \phi^3 + \frac{m^2}{2} \phi^2, \quad (2.9)$$

where

$$\lambda_{\text{eff}} = \frac{2\lambda}{\text{Tr} J^2} = \frac{8\lambda}{N(N^2 - 1)}, \quad \kappa_{\text{eff}} = \frac{\kappa}{\sqrt{\text{Tr} J^2}} = \frac{2\kappa}{\sqrt{N(N^2 - 1)}} \quad (2.10)$$

2.2 Consistency of the truncation to the SU(2) sector

The SU(2) sector seems to be a special sector of the M-flation action in which the theory becomes tractable and very simple. However, we need to make sure that this truncation to the SU(2) sector is indeed consistent with the classical dynamics of the model and that we can consistently turn off the other $3N^2 - 1$ fields. In order to do this, we define

$$\Psi_i = \Phi_i - \hat{\phi} J_i \quad (2.11)$$

where, as before,

$$\hat{\phi} = \frac{4}{N(N^2 - 1)} \text{Tr}(\Phi_i J_i),$$

and hence $\text{Tr}(\Psi_i J_i) = 0$. In other words, Ψ_i is defined such that it has no components along J_i . Using $[J_i, J_j] = i\epsilon_{ijk} J_k$, we may rewrite the potential (2.2) in terms of $\hat{\phi}$ and Ψ_i as

$$V = V_0(\hat{\phi}) + V_{(2)}(\hat{\phi}, \Psi_i) \quad (2.12)$$

where V_0 , after the field re-definition (2.8), is given by (2.9).

Using (2.6), (2.11) and the fact that $\text{Tr}(\Psi_i J_i) = 0$, one can show that $V_{(2)}$ does not have any linear terms in Ψ_i , i.e.

$$V_{(2)}(\hat{\phi}, \Psi_i = 0) = 0, \quad \left(\frac{\delta V_{(2)}}{\delta \Psi_i} \right)_{\Psi_i=0} = 0. \quad (2.13)$$

This leads to the important result that the ϕ field does not source the Ψ_i fields. Explicitly, in the equations of motion (2.4b) if we start with the initial conditions $\Psi_i = 0$, $\dot{\Psi}_i = 0$ and $\hat{\phi} \neq 0$, Ψ_i will always remain zero and will hence not contribute to the classical background inflationary dynamics at all. This means that we are consistent in considering ϕ as the sole field driving the inflation. The remaining Ψ_i modes, however, as we shall see in next sections, will be excited at the level of perturbations. So, we have $3N^2 - 1$ Ψ modes besides the background ϕ inflationary field. In the language of [3] this corresponds to a straight inflationary trajectory in the field space of $3N^2$ scalar fields, where the inflationary trajectory is along the ϕ direction and there would be $3N^2 - 1$ iso-curvature perturbations perpendicular to the adiabatic trajectory. We should stress that this result will not remain valid if initially Ψ_i fields are also turned on.

One may wonder about the special role of the $SU(2)$ generators J_i among other $N \times N$ matrices in our analysis and whether a similar reduction to a sector other than the $SU(2)$ sector is also possible. Traces of the fact that $SU(2)$ sector is special is already built in the initial construction of the potential (2.2) with three Φ_i , $i = 1, 2, 3$ (3 is the dimension of the $SU(2)$ algebra) and that in the cubic term in the action the structure constant of $SU(2)$ ϵ_{ijk} appears. To see this more explicitly, let us consider the more generic decomposition for Φ_i matrices

$$\Phi_i = \Upsilon_i + \Xi_i, \quad (2.14)$$

such that $\text{Tr}(\Upsilon_i \Xi_i) = 0$, we take both Υ_i and Ξ_i to be hermitian. The potential will again have two parts, $V = V_0(\Upsilon_i) + V_{(1)}(\Upsilon, \Xi)$, where $V_0(\Upsilon_i) = V(\Xi_i = 0)$ and

$$V_{(1)}(\Upsilon_i, \Xi_i) = \text{Tr} \left[\left(-\lambda[\Upsilon_i, [\Upsilon_i, \Upsilon_k]] + i\kappa\epsilon_{ijk}[\Upsilon_i, \Upsilon_j] \right) \Xi_k \right] + \mathcal{O}(\Xi^2).$$

In order the Υ -sector to decouple, the above expression in the bracket should vanish for any Ξ_i . As explained above, if Υ_i is proportional to J_i this condition obviously holds. In general, however, this can happen if the Υ -dependent term in parenthesis is proportional to Υ_k . This condition can be satisfied if $[\Upsilon_i, \Upsilon_j] = f_{ijk} \Upsilon_k$ for some functions f_{ijk} which means that the three Υ_i matrices should form a Lie-algebra of dimension three. The only non-trivial solution is then Υ_i forming an $SU(2)$ algebra.¹ Note, however, that the representation for the $SU(2)$ algebra is not fixed by this requirement and Υ_i could also form reducible $N \times N$ representation of $SU(2)$.

Motivated by this unique property of the $SU(2)$ sector, from now on we assume that the background inflationary trajectory is along the ϕ direction, while the other $3N^2 - 1$ fields are only excited at the level of (quantum) perturbations.

¹There is also the trivial choice of $f_{ijk} = 0$, corresponding to choosing three Abelian subgroups of $U(N)$. Working with commuting matrices, however, kills all the interesting inflationary dynamics.

3 Various inflation models resulting from M-flation scenario

The $SU(2)$ sector, which is governed by the potential (2.9), depending on the values of the parameters λ_{eff} , κ_{eff} and m^2 , leads to different inflationary models. These models have been studied in the literature, which we will review below. The important advantage of our M-flation scenario, as we will show, is the scaling properties between the original parameters of the model appearing in the action and the effective dressed parameters appearing in (2.9), which allow us to remove the unnaturalness and fine-tuning of these parameters.

3.1 Chaotic inflation

If we set $\lambda = \kappa = 0$, we obtain the simple quadratic chaotic potential and in this case our model is nothing but an N-flation model [6] with $3N^2$ fields. To fit the CMB observations and obtain right number of e-foldings, one needs that $m \sim 10^{12}$ GeV and a super-Planckian field variation for effective inflaton field ϕ during inflation, $\Delta\phi \sim 10M_P$. In the context of effective field theory this may sound problematic. However, as in the N-flation model [6], note that $\hat{\phi}$ is our physical field and $\Delta\hat{\phi} \sim \Delta\phi/N$. For a sufficiently large value of N one can arrange that $\Delta\hat{\phi} \ll M_P$ and one can avoid the problem with super-Planckian field values.

On the other hand, if $m = \kappa = 0$, we obtain the quartic $\lambda_{\text{eff}}\phi^4/4$ chaotic inflation. To fit the COBE normalization and obtain right number of e-folds, one requires $\lambda_{\text{eff}} \sim 10^{-14}$ and $\Delta\phi \gtrsim 10M_P$. In the context of a single scalar field, these are viewed as severe fine-tunings in the model. In our case, however, we see that both of these can be relaxed. Assuming that $\lambda \sim 1$ dictated by the naturalness of the theory, to obtain the above value for λ_{eff} one needs to have $N \sim 10^5$. In the context of string theory studied in section 7, where N is viewed as the number of coincident branes, this is easily achieved in light of recent developments in the flux compactification. With this value of N for the physical field $\hat{\phi}$ one obtains $\Delta\hat{\phi} \sim 10^{-7}M_P$ and can hence safely bypass the problem of super-Planckian excursion of the field. In both of these examples, inflation ends when $\hat{\phi}(t) \rightarrow 0$ which means that the matrices Φ_i commute with each other.

In both examples, and for next two examples below, due to super-Planckian value of $\Delta\phi$ during inflation, a considerable amount of gravity waves can be produced which can be detected in future gravity wave observations such as PLANCK [16, 17], CMBPOL [18] and QUIET [19].

3.2 Symmetry breaking inflation

Now consider the general case where none of the coefficients in $V_0(\phi)$ is zero. We study two interesting example here. The first example is when $V_0(\phi)$ is positive definite and has two degenerate minima. This corresponds to $\kappa = 3m\sqrt{\lambda}/2$ and the potential has the form

$$V_0 = \frac{\lambda_{\text{eff}}}{4}\phi^2(\phi - \mu)^2 \quad (3.1)$$

where $\mu \equiv \sqrt{2}m/\sqrt{\lambda_{\text{eff}}}$. As mentioned, the potential has global minima at $\phi = 0$ and $\phi = \mu$. In the language of the brane construction (*cf.* section 7), the minimum at $\phi = \mu$ corresponds to super-symmetric vacua when N D3-branes blow up into a giant D5-brane in the presence of background RR field $C^{(6)}$ which in our notation is represented by κ . The minimum at $\phi = 0$, on the other hand, corresponds to the trivial solution when matrices become commutative. If we allow the field ϕ to take negative values, then the potential is

symmetric under $\phi \rightarrow -\phi + \mu$ and it represents the standard double well potential, justifying the name “symmetry breaking” inflation.

Potential (3.1) is well studied in the literature and is in the form of symmetry breaking potential and “hilltop inflation” [20, 21]. In appendix A we briefly look at the predictions from this potential compared to WMAP5 data. It is assumed that inflation lasts for 60 number of e-folds, $N_e = 60$, and the scalar spectral index, $n_s = 0.96$, from WMAP5 central value. Furthermore, the COBE normalization is set to $\delta_H \simeq 2.41 \times 10^{-5}$.

Depending on the initial value of the inflaton field, ϕ_i , the inflationary period is divided in three categories.

(a) $\phi_i > \mu$

Suppose inflation starts when $\phi_i > \mu$. With $N_e = 60$, $\delta_H \simeq 2.41 \times 10^{-5}$ and $n_s = 0.96$, one obtains

$$\phi_i \simeq 43.57 M_P, \quad \phi_f \simeq 27.07 M_P, \quad \mu \simeq 26 M_P. \quad (3.2)$$

and

$$\lambda_{\text{eff}} \simeq 4.91 \times 10^{-14}, \quad m \simeq 4.07 \times 10^{-6} M_P, \quad \kappa_{\text{eff}} \simeq 9.57 \times 10^{-13} M_P. \quad (3.3)$$

(b) $\mu/2 < \phi_i < \mu$ This is an example of hill-top inflation when the inflaton field is between the local maximum at $\phi = \mu/2$ and the “supersymmetric minimum” at $\phi = \mu$. To fit the above observational constraints one obtains

$$\phi_i \simeq 23.5 M_P, \quad \phi_f \simeq 35.03 M_P, \quad \mu \simeq 36 M_P. \quad (3.4)$$

and

$$\lambda_{\text{eff}} \simeq 7.18 \times 10^{-14}, \quad m \simeq 6.82 \times 10^{-6} M_P, \quad \kappa_{\text{eff}} \simeq 1.94 \times 10^{-12} M_P. \quad (3.5)$$

(c) $0 < \phi_i < \mu/2$

Due to symmetry $\phi \rightarrow -\phi + \mu$ this inflationary region has the same properties as $\mu/2 < \phi_i < \mu$ above. If we allow for negative values of ϕ , then the inflationary prediction for $\phi < 0$ region is the same as in region 1 above.

In all these examples, to fit the COBE normalization, one obtains $N \sim 10^5$ and $\Delta\hat{\phi} \sim 10^{-7} M_P$ during inflation so the issues with super-Planckian field range is resolved.

3.3 Inflection point inflation

Another class of models widely studied in the literature coming from the potential (2.9) is when the potential has an inflection point [22]. This happens when $\kappa = \sqrt{2\lambda} m$ or equivalently, $\kappa_{\text{eff}} = m\sqrt{\lambda_{\text{eff}}}$. Denoting the inflection point by ϕ_0 , the potential near ϕ_0 is approximately given by

$$V(\phi) \simeq V(\phi_0) + \frac{1}{3!} V'''(\phi_0) (\phi - \phi_0)^3 \quad (3.6)$$

where

$$V(\phi_0) = \frac{m^2}{12} \phi_0^2, \quad V'''(\phi_0) = \frac{2m^2}{\phi_0}. \quad (3.7)$$

The CMB observables are given by [22]

$$n_s \simeq 1 - \frac{4}{N_e}, \quad \delta_H \simeq \frac{2}{5\pi} \frac{\lambda_{\text{eff}} M_P}{m} N_e^2. \quad (3.8)$$

Choosing $N_e = 60$, one obtains $n_s \simeq 0.93$ which is within 2σ error bar of WMAP5 but is somewhat to its lower end. One may add small modification to the coefficient of potential [22] such that the potential around the inflection point is slightly modified. This in turn can result in a higher value of n_s . On the other hand, from COBE normalization, one obtains

$$\lambda_{\text{eff}} \sim 10^{-8} \frac{m}{M_P}. \quad (3.9)$$

In a conservative limit that $m \lesssim M_P$, this yields $\lambda_{\text{eff}} \lesssim 10^{-8}$. Starting with $\lambda \sim 1$, this corresponds to $N \gtrsim 10^3$.

If we look into the amplitude of the gravity wave, determined by quantity r which is the ratio of gravitational perturbation amplitude to scalar perturbation amplitude, one obtains

$$r = 8M_P^2 \left(\frac{V'}{V} \right)^2 = \frac{2}{9} \left(\frac{m}{\sqrt{\lambda_{\text{eff}}}} \right)^6 N_e^{-4}. \quad (3.10)$$

Combining this with n_s and δ_H , we obtain

$$\lambda_{\text{eff}} = \left(\frac{9r}{32} \right)^{1/3} \left(\frac{5\pi}{8} \delta_H \right)^2 (1 - n_s)^{8/3}. \quad (3.11)$$

The upper bound, $r < 0.22$, from WMAP5 implies that $\lambda_{\text{eff}} \lesssim 10^{-13}$ and $N \gtrsim 10^5$ which is stronger than the bound above.

4 Mass spectrum of Ψ_i modes in M-flation

Having studied the SU(2) sector and the resulting inflationary models, we review our model, noting that the other $3N^2 - 1$ fields encoded in Ψ_i , although not contributing to the classical inflationary dynamics, do have quantum fluctuations and will hence affect the cosmological perturbation analysis. To compute these effects we need to have the mass spectrum of the $3N^2 - 1$ modes coming from the Ψ_i .

To this end, starting from (2.11) we expand the action up to the second order in Ψ_i . Given the orthogonality condition, $\text{Tr}(\Psi_i J_i) = 0$, the kinetic term readily takes the standard form $\frac{1}{2} \text{Tr}(\partial_\mu \Psi_i \partial^\mu \Psi_i)$. After a slightly lengthy but straightforward computation the potential to second order in Ψ_i is obtained as

$$V_{(2)} = \text{Tr} \left[\frac{\lambda}{2} \hat{\phi}^2 \Omega_i \Omega_i + \frac{m^2}{2} \Psi_i \Psi_i + \left(-\frac{\lambda}{2} \hat{\phi}^2 + \kappa \hat{\phi} \right) \Psi_i \Omega_i \right] \quad (4.1)$$

where

$$\Omega_k \equiv i\epsilon_{ijk} [J_i, \Psi_j]. \quad (4.2)$$

From the above form we see that if we have the eigenvectors (eigen-matrices) of the Ω_i we can compute the spectrum of Ψ_i in terms of $\hat{\phi}$ -field (to be viewed as the inflaton). Finding the eigenvectors of Ω_i is mathematically the same problem as finding the vector

spherical harmonics. (For a detailed discussion see e.g. [23], section 5.2.) If we denote the Ω eigenvalues by ω , i.e.

$$\Omega_i = \omega \Psi_i, \quad (4.3)$$

we obtain

$$V_{(2)} = \left(\frac{\lambda_{\text{eff}}}{4} \phi^2 (\omega^2 - \omega) + \kappa_{\text{eff}} \omega \phi + \frac{m^2}{2} \right) \text{Tr} \Psi_i \Psi_i. \quad (4.4)$$

If we have the possible values of ω we can read off the effective (ϕ -dependent) mass of the Ψ_i modes

$$\begin{aligned} M^2 &= \frac{\lambda_{\text{eff}}}{2} \phi^2 (\omega^2 - \omega) + 2\kappa_{\text{eff}} \omega \phi + m^2 \\ &= V_0'' (\omega + 1)^2 - \frac{V_0'}{\phi} (4\omega + 3)(\omega + 2) + \frac{6V_0}{\phi^2} (\omega + 1)(\omega + 2), \end{aligned} \quad (4.5)$$

where V_0' and V_0'' denote the first and second derivatives of the potential V_0 with respect to the inflaton ϕ . For later convenience it is also useful to write the expression for the mass during inflation in terms of the Hubble expansion rate and the slow-roll parameters ϵ and η

$$\frac{M^2}{3H^2} = \left[\eta(\omega + 1)^2 - \text{sgn}(V_0') \sqrt{2\epsilon} \frac{M_P}{\phi} (4\omega + 3)(\omega + 2) + 6 \frac{M_P^2}{\phi^2} (\omega + 1)(\omega + 2) \right], \quad (4.6)$$

where $\text{sgn}(V_0')$ represents the sign of V_0' and as usual the slow-roll parameters are defined by

$$\epsilon = \frac{M_P^2}{2} \left(\frac{V_0'}{V_0} \right)^2, \quad \eta = M_P^2 \frac{V_0''}{V_0}. \quad (4.7)$$

Following the analysis of [23], we find that ω can take three values:

- “The zero modes” $\omega = -1$. This happens for modes of the form

$$\Psi_i = [J_i, \Lambda],$$

with Λ being an arbitrary matrix. For these modes the expression for the mass simplifies to

$$M^2 = \frac{V_0'}{\phi}. \quad (4.8)$$

Therefore, at the minimum values for ϕ where V_0' vanishes these modes become massless. (This justifies the name “massless modes”.)

Noting that Λ is an arbitrary matrix there are N^2 of such modes, all with the same mass.

- “The α modes”: $\omega = -(l + 1)$, $l \in \mathbb{Z}$, $0 \leq l < N$, with the mass

$$M_l^2 = \frac{\lambda_{\text{eff}}}{2} (l + 1)(l + 2) \phi^2 - 2\kappa_{\text{eff}} (l + 1) \phi + m^2. \quad (4.9)$$

Each mode for a given l has a multiplicity of $2l + 1$. Therefore, there are N^2 of α -modes.

- “The β modes”: $\omega = l$, $l \in \mathbb{Z}$, $0 < l < N$, with the mass

$$M_l^2 = \frac{\lambda_{\text{eff}}}{2} l(l - 1) \phi^2 + 2\kappa_{\text{eff}} l \phi + m^2. \quad (4.10)$$

Each mode for a given l has a multiplicity of $2l + 1$. Therefore, there are $N^2 - 1$ of β -modes.

As expected, there are altogether $3N^2 - 1$ zero, α , and β modes. For the zero modes where $\omega = -1$, one observes that $M^2/3H^2 \sim \sqrt{\epsilon}M_P/\phi \sim 0.01$. So there are N^2 of such light zero modes. For α and β modes, only those with $l \lesssim \epsilon^{-1/2}, \eta^{-1/2} \sim 10$ are light, while the modes with higher values of l are heavy.

5 Power spectra in the presence of Ψ_i modes

With the mass spectrum for Ψ_i modes computed in the previous section we can compute the power spectra of the adiabatic and the iso-curvature perturbations. Here we are dealing with a $3N^2$ real scalar field inflationary system. Our inflationary background is along the ϕ direction. The remaining $3N^2 - 1$ scalars are frozen classically during inflation and are excited only quantum mechanically. Correspondingly, the modes are classified as the adiabatic perturbation, the one which is tangential to the classical inflationary background, and the iso-curvature modes, the $3N^2 - 1$ perturbations which are perpendicular to the background inflationary trajectory. In this respect our model is similar to the model studied in [24] where the potential has an $O(N)$ symmetry such that the inflaton field is the radial direction while the remaining $N - 1$ angular directions are iso-curvature perturbations.

The formalism to calculate the adiabatic and iso-curvature (entropy) power spectra was systematically developed in [3]. Here we shall repeat those analysis for our system of $3N^2$ scalar fields.

5.1 Linear perturbations

The perturbed metric in the longitudinal gauge is

$$ds^2 = -(1 + 2\Phi)dt^2 + a(t)^2(1 - 2\Phi)d\mathbf{x}^2 \quad (5.1)$$

where $\Phi(t, \mathbf{x})$ is the gravitational potential and should not be mistaken with Φ_i which represents our matrix fields. Similarly, the linear perturbed scalar fields are $\delta\phi(t, \mathbf{x})$ and $\Psi_i(t, \mathbf{x})$.

We find it is much easier to work with the Ψ_i modes with definite mass spectrum. As discussed in the previous section, these fall into three classes denoted by $\Psi_{r,lm}$ where $r = 0, \alpha, \beta$ stand, respectively, for zero-mode, α -mode and β -mode and m is running from one to $D_{r,l}$, the degeneracy factor of each $\Psi_{r,lm}$ mode, with $D_0 = N^2$ and $D_{\alpha,l} = D_{\beta,l} = 2l+1$. In this notation, the Lagrangian for the scalar fields with potential $V = V_0(\phi) + V_{(2)}(\phi, \Psi_i)$ from (4.4) is

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\partial_\mu\Psi_{r,lm}^*\partial^\mu\Psi_{r,lm} - V_0(\phi) - \frac{1}{2}M_{r,l}^2(\phi)\Psi_{r,lm}^*\Psi_{r,lm}, \quad (5.2)$$

where summation over repeated r and l, m indices is assumed.

Define the gauge invariant Mukhanov-Sasaki variable Q_ϕ

$$Q_\phi \equiv \delta\phi + \frac{\dot{\phi}}{H}\Phi. \quad (5.3)$$

The perturbed Klein-Gordon equations for ϕ and $\Psi_{r,lm}$ are

$$\begin{aligned} \ddot{Q}_\phi + 3H\dot{Q}_\phi + \frac{k^2}{a^2}Q_\phi + \left[V_{0,\phi\phi} - \frac{1}{a^3M_P^2} \left(\frac{a^3}{H} \dot{\phi}^2 \right) \right] Q_\phi &= 0 \\ \ddot{\Psi}_{r,lm} + 3H\dot{\Psi}_{r,lm} + \left(\frac{k^2}{a^2} + M_{r,l}^2(\phi) \right) \Psi_{r,lm} &= 0. \end{aligned} \quad (5.4)$$

Here k is the momentum number in the Fourier space. One interesting aspect of these equations is that the adiabatic and iso-curvature modes completely decouple and they do not source each other. This is a result of our initial conditions in which $\Psi_{r,lm}$ fields are turned off. Because of this, the equation of motion for $\Psi_{r,lm}$ is that of a scalar field in a homogeneous and isotropic expanding background but with a time dependent mass.

These equations are accompanied by Einstein equations

$$\begin{aligned}
 3H(\dot{\Phi} + H\Phi) + \left(\dot{H} + \frac{k^2}{a^2}\right)\Phi &= -\frac{1}{2M_P^2} \left[\dot{\phi}\delta\dot{\phi} + V_{0,\phi}\delta\phi\right] \\
 \dot{\Phi} + H\Phi &= \frac{1}{2M_P^2}\dot{\phi}\delta\phi.
 \end{aligned}
 \tag{5.5}$$

Interestingly enough, $\Psi_{r,lm}$ fields do not show up in perturbed Einstein equations due to our assumption that they are absent at the background dynamics. Physically, this means that the modes $\Psi_{r,lm}$ do not carry energy up to linear order in perturbation theory. Since they do not couple to gravitational potential and the inflaton perturbations, they have no gravitational effect, justifying the name iso-curvature perturbations.

Consider the normalized curvature perturbation \mathcal{R} and the normalized iso-curvature perturbations $\mathcal{S}_{r,lm}$

$$\mathcal{R} \equiv \frac{H}{\dot{\phi}}Q_\phi, \quad \mathcal{S}_{r,lm} \equiv \frac{H}{\dot{\phi}}\Psi_{r,lm}.
 \tag{5.6}$$

Using the Einstein equations one can show

$$\dot{\mathcal{R}} = \frac{H}{\dot{H}}\frac{k^2}{a^2}\Phi.
 \tag{5.7}$$

This indicates that on arbitrary large scales where $k \rightarrow 0$, the curvature perturbation is conserved. This is similar to single field inflationary system.

Here we assume that initially $\delta\phi$ and $\Psi_{r,lm}$ are random, Gaussian and adiabatic fields which are excited quantum mechanically from vacuum and

$$\begin{aligned}
 \langle Q_{\phi\mathbf{k}}^* Q_{\phi\mathbf{k}'} \rangle &= \frac{2\pi^2}{k^3} P_{Q_\phi} \delta^3(\mathbf{k} - \mathbf{k}') \\
 \langle \Psi_{r,lm\mathbf{k}}^* \Psi_{r',l'm'} \mathbf{k}' \rangle &= \frac{2\pi^2}{k^3} P_{\Psi_{r,l}} \delta_{rr'} \delta_{ll'} \delta_{mm'} \delta^3(\mathbf{k} - \mathbf{k}') \\
 \langle Q_{\phi\mathbf{k}}^* \Psi_{r,lm\mathbf{k}'} \rangle &= 0,
 \end{aligned}
 \tag{5.8}$$

where P_{Q_ϕ} and $P_{\Psi_{r,l}}$ are the primordial power spectra of the scalar fields. We note that the last equation above also holds during the inflationary stages, indicating that there is no cross-correlation between adiabatic and iso-curvature modes. Physically, this means that they do not source each other as can be seen from (5.4).

5.2 Power spectra from Hubble-crossing to end of inflation

As usual, it is instructive to express the Klein-Gordon equations (5.4) in terms of variables u and $v_{r,m}$ defined by

$$u \equiv aQ_\phi, \quad v_{r,lm} \equiv a\Psi_{r,lm}.$$

Going to conformal time $dt = a d\tau$, (5.4) is transformed into Schrödinger equation form

$$\frac{d^2 u}{d\tau^2} + \left[k^2 - \frac{2 - 3\eta + 9\epsilon}{\tau^2} \right] u = 0 \quad (5.9a)$$

$$\frac{d^2 v_{r,lm}}{d\tau^2} + \left[k^2 - \frac{2 - 3\eta_{r,l} + 3\epsilon}{\tau^2} \right] v_{r,lm} = 0 \quad (5.9b)$$

where $\eta_{r,l} = M_{r,l}^2(\phi)/3H^2$ given by (4.6).

The initial conditions deep inside the Hubble radius are given by the Bunch-Davies vacua. We evolve equations in (5.9) till the time of horizon crossing τ_* at which $k = (aH)_*$. Here and below the subscript $*$ indicates that the quantities are calculated at the time of Hubble crossing during inflation. Following the standard inflationary prescriptions e.g. [25], up to the first order in slow-roll parameters one has

$$\begin{aligned} u &= \frac{\sqrt{\pi|\tau|}}{2} e^{i(1+2\nu_R)\pi/4} H_{\nu_R}^{(1)}(k|\tau|) \\ v_{r,lm} &= \frac{\sqrt{\pi|\tau|}}{2} e^{i(1+2\nu_{S_{r,lm}})\pi/4} H_{\nu_{S_{r,lm}}}^{(1)}(k|\tau|) \end{aligned} \quad (5.10)$$

where $H^{(1)}(x)$ is the Hankel function of the first kind and ν_R and $\nu_{S_{r,lm}}$ are given by

$$\nu_R = \frac{3}{2} + 3\epsilon - \eta, \quad \nu_{S_{r,lm}} = \frac{3}{2} + \epsilon - \eta_{r,l}. \quad (5.11)$$

Correspondingly, using (5.6), the power spectra of the scalar curvature perturbations and iso-curvature perturbations, $P_{\mathcal{R}}$ and $P_{S_{r,lm}}$, at the time of Hubble exit are given by

$$\begin{aligned} P_{\mathcal{R}}|_* &\simeq \left(\frac{H^2}{2\pi\dot{\phi}} \right)_*^2 [1 + (-2 + 6C)\epsilon - 2C\eta]_* \\ P_{S_{r,lm}}|_* &\simeq \left(\frac{H^2}{2\pi\dot{\phi}} \right)_*^2 [1 + (-2 + 2C)\epsilon - 2C\eta_{r,l}]_* \end{aligned} \quad (5.12)$$

Here $C = \Gamma'(3/2)/\Gamma(3/2) + \ln 2 \simeq 0.7296$ where $\Gamma(x)$ is the Gamma function.

Few e-folds after the mode of interest has left the Hubble radius, one can neglect the term k^2/a^2 in (5.4). Furthermore, it would be more instructive to use the number of e-folds before the end of inflation, N_e , as the clock. Using the relation $dN_e = Hdt$, (5.4) is transformed into

$$\frac{dQ_\phi}{Q_\phi} \simeq (2\epsilon - \eta) dN_e, \quad \frac{d\Psi_{r,lm}}{\Psi_{r,lm}} \simeq -\eta_{r,l} dN_e. \quad (5.13)$$

To calculate the power spectra as a function of N_e , for \mathcal{R} and $S_{r,lm}$ defined as in (5.6), we need to incorporate the evolution of the pre-factor $H/\dot{\phi}$ which is given by

$$\frac{d}{dN_e} \left(\frac{H}{\dot{\phi}} \right) \simeq -(2\epsilon - \eta) \left(\frac{H}{\dot{\phi}} \right). \quad (5.14)$$

Combining equations (5.13) and (5.14) one obtains

$$P_{\mathcal{R}}(N_e) \simeq P_{\mathcal{R}}|_* \quad (5.15)$$

$$P_{S_{r,lm}}(N_e) \simeq P_{S_{r,lm}}|_* \exp \left[-2 \int_0^{N_e} dN'_e B_{r,l}(N'_e) \right], \quad (5.16)$$

where $B_{r,l}(N_e) \equiv 2\epsilon - \eta + \eta_{r,l}$ and using (4.6) one obtains

$$B_{r,l}(N_e) \simeq 2\epsilon + \omega(2 + \omega)\eta - \text{sgn}(V'_0) \frac{\sqrt{2\epsilon} M_P}{\phi} (4\omega + 3)(\omega + 2) + \frac{6M_P^2}{\phi^2} (\omega + 1)(\omega + 2)$$

where ω takes values 0, $-l - 1$ and l respectively for zero, α and β modes (*cf.* section 4).

As explained previously, due to conservation of \mathcal{R} on super-Hubble scales, $P_{\mathcal{R}}$ remains unchanged after Hubble exit as shown in (5.15). On the other hand, the evolution of power spectra for the iso-curvature modes, $P_{\mathcal{S}_{r,lm}}(N_e)$, as shown in (5.16), depends on the dynamics of the inflationary background and the eigenvalues ω .

5.3 Iso-curvature vs. entropy perturbations

As we saw in the previous section, the $\Psi_{r,lm}$ perturbations do not couple to the inflaton field and the gravitational potential so they do not contribute to the primordial curvature perturbations during inflation which justify the name iso-curvature for these modes. Physically, this means that up to the first order in perturbation theory, $\Psi_{r,lm}$ fields do not carry energy during inflation. As showed in section 2.2 this has the origin in our initial conditions for the $\Psi_{r,lm}$ fields that they are absent in classical background dynamics.

In the literature, the terminologies “iso-curvature perturbations” and “entropy perturbations” are usually used interchangeably. However, one can easily check that $\Psi_{r,lm}$ perturbations do not induce entropy perturbations during inflation. To see this explicitly, let us look at non-adiabatic components of pressure, δp_{nad} , defined as [4]

$$\delta p_{\text{nad}} = \delta p - \frac{\dot{p}}{\dot{\rho}} \delta \rho, \quad (5.17)$$

where ρ and p are the background energy density and pressure respectively and $\delta \rho$ and δp represent their first order variations. Using Einstein equations (5.5) one can show that

$$\delta p_{\text{nad}} \simeq -4M_P^2 \frac{k^2}{a^2} \Phi, \quad (5.18)$$

which is the same as the standard single-field inflation result. On super-Hubble scale one observes that δp_{nad} vanishes and there is no entropy perturbation. This is similar to our earlier result (5.7) that $\dot{\mathcal{R}} \simeq 0$ on super-Hubble scale. In this work, in order to emphasize that there is no entropy perturbations in our setup, we use the terminology iso-curvature perturbations throughout.

In general multiple-field inflation, the iso-curvature perturbations perpendicular to the classical inflationary trajectory do produce entropy perturbations and $\delta p_{\text{nad}} \neq 0$. These entropy perturbations source the adiabatic perturbation and have non-zero energy at the linear order of perturbations so they also source the Einstein equations and contribute to the primordial curvature perturbations. More specifically

$$\dot{\mathcal{R}} = \frac{H}{\dot{H}} \frac{k^2}{a^2} \Phi + 2 \sum_{r, lm} \dot{\theta}_{r,lm} \mathcal{S}_{r,lm}. \quad (5.19)$$

The above equation is a generalization of the two-field result of [3], the sum in the last term is over $3N^2 - 1$ entropy modes and $\theta_{r,lm}$ represent the angle between $\Psi_{r,lm}$ and the inflaton trajectory, ϕ in our case, in the space of $3N^2$ scalar fields. As discussed above, if we start with

the initial conditions $\Psi_i = 0$, $\dot{\Psi}_i = 0$, ϕ behaves as the sole inflaton field and in the language of [3], the background inflationary trajectory in $\phi - \Psi_i$ phase space is flat, i.e. $\dot{\theta}_{r,lm} = 0$. Therefore, these iso-curvature perturbations do not feed the curvature perturbation and there is no cross-correlation between the iso-curvature and adiabatic perturbations.

However, if we start with an arbitrary initial condition in $\phi - \Psi_i$ field space, Ψ_i are not frozen classically, the inflation trajectory is curved and the inflaton field has a component along the $\Psi_{r,lm}$ direction and $\dot{\theta}_{r,lm} \neq 0$. This in turn produces entropy perturbations during inflation which also source the cosmic perturbations.

Having this said, however, there are two mechanisms to create entropy perturbations in our setup. The first way is to consider second order perturbation in $\Psi_{r,lm}$. In second order perturbation theory, $\Psi_{r,lm}$ carries energy [27] and couple to both Einstein equations and the inflaton field perturbation $\delta\phi$. This in turn leads to entropy perturbations during inflation. However, the amplitude of these entropy perturbations are much smaller than the first order adiabatic perturbations coming from $\delta\phi$. The second mechanism to create entropy perturbations from iso-curvature perturbations $\Psi_{r,lm}$ can happen during preheating and/or reheating era. Similar idea was used in [24] through asymmetric preheating.

5.4 Curvature and iso-curvature perturbations power spectra for specific examples

Using the general formulation presented in previous subsections we calculate the power spectra at the end of inflation for chaotic, symmetry breaking and the inflection point inflationary potentials.

5.4.1 Chaotic inflation $\frac{m^2}{2}\phi^2$

When $\lambda = \kappa = 0$, the action takes the form

$$S = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\partial_\mu\Psi_{r,lm}^*\partial^\mu\Psi_{r,lm} - \frac{1}{2}m^2[\phi^2 + \Psi_{r,lm}^*\Psi_{r,lm}] . \quad (5.20)$$

The $3N^2$ modes all have equal masses. Interestingly, this closely resembles the chaotic inflation studied in [24, 26]. The potential (5.20) has $O(3N^2)$ symmetry for the fields ϕ and $\Psi_{r,lm}$. The background inflation field is ϕ and the number of e-folds in terms of the initial value of the inflaton field ϕ_i is given by

$$N_e \simeq \frac{\phi_i^2}{4M_P^2} . \quad (5.21)$$

Correspondingly, the amplitude of the adiabatic curvature perturbation is

$$P_{\mathcal{R}} \simeq \frac{1}{6\pi^2} \left(\frac{mN_e}{M_P} \right)^2 . \quad (5.22)$$

For $N_e = 60$, to fit the WMAP5 normalization $P_{\mathcal{R}} \simeq 2.41 \times 10^{-9}$, one requires $m \simeq 6.304 \times 10^{-6} M_P$. For such values of m , we have calculated numerically the amplitudes of $3N^2 - 1$ iso-curvature spectra at the end of inflation, i.e. where $\epsilon = 1$, which turns out to be $P_{S_{r,lm}} \simeq 8.426 \times 10^{-14}$ at today's Hubble scale. The spectral indices of adiabatic and iso-curvature spectra at such scale, respectively, are $n_{\mathcal{R}} \simeq 0.966$ and $n_{\Psi_{r,lm}} \simeq 0.9998$, which coincide with the analytic results of [24], see figure 1. One can lower the value of m — which in turn lowers the amplitude of adiabatic perturbations — and generate the difference

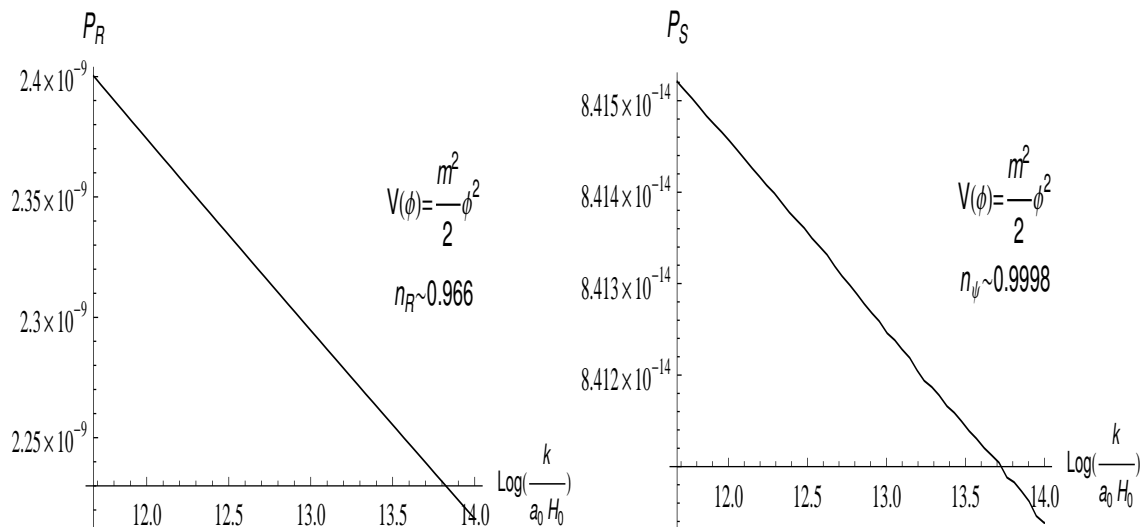


Figure 1. Left and right graphs respectively show the curvature and isocurvature spectra for chaotic inflation with potential $\frac{m^2}{2}\phi^2$

by transforming the iso-curvature fluctuations to curvature ones through an asymmetric mechanism of preheating [24]. The ratio $P_{\mathcal{S}_{r,lm}}/P_{\mathcal{R}}$ for the mode that exit the horizon 60 e-folds before the end of inflation is graphed as a function of N_e , see **figure 2**. Note that as in this case all the $\Psi_{r,lm}$ modes have the same mass and hence $\eta_{r,l}$, $P_{\mathcal{S}_{r,lm}}/P_{\mathcal{R}}$ are independent of r and l, m . When the mode is inside the Hubble radius the spectra are almost equal and the ratio is one. However, around the Hubble-exit, the iso-curvature mode starts to decay following (5.16). For the mode that exits the Hubble radius 60 e-folds before the end of inflation, $k_{60} = e^{-60} a_e H_e$, according to (5.16) the ratio decays like

$$\frac{P_{\mathcal{S}_{r,lm}}(N_e)}{P_{\mathcal{R}}|_*} \simeq (1 - N_e/60)^2, \quad (5.23)$$

where N_e is the number of e-folds the mode spends outside the Hubble radius before the end of inflation. As can be seen in **figure 2**, the analytic result is in a good agreement with the numerical one.

For this model, the amplitude of tensor fluctuations of the adiabatic mode at today's Hubble scale is $P_T(k_{60}) \simeq 3.1856 \times 10^{-10}$. The corresponding tensor/scalar ratio, r , is $r \simeq 0.132$. The tensor spectral index, $n_T \equiv d \ln P_T / d \ln k$, is $n_T \simeq -0.0165$. Future CMB probes such as PLANCK [16], or exclusive polarization probes such as CMBPOL [18] or QUIET [19] should be able to test this scenario.

5.4.2 Chaotic inflation $\frac{\lambda_{\text{eff}}}{4}\phi^4$

When $m = \kappa = 0$, the potential energy $V = V_0(\phi) + V_{(2)}(\phi, \Psi_i)$ is

$$V = \frac{\lambda_{\text{eff}}}{4}\phi^4 + \frac{\lambda_{\text{eff}}}{4}(\omega^2 - \omega)\phi^2 \Psi_{r,lm}^* \Psi_{r,lm}. \quad (5.24)$$

To match the amplitude of adiabatic perturbations with WMAP result at horizon scale, $\lambda_{\text{eff}} \simeq 1.6315 \times 10^{-13}$. For such value of λ_{eff} , the scalar spectral index for the adiabatic mode is $n_{\mathcal{R}} \simeq 0.949$.

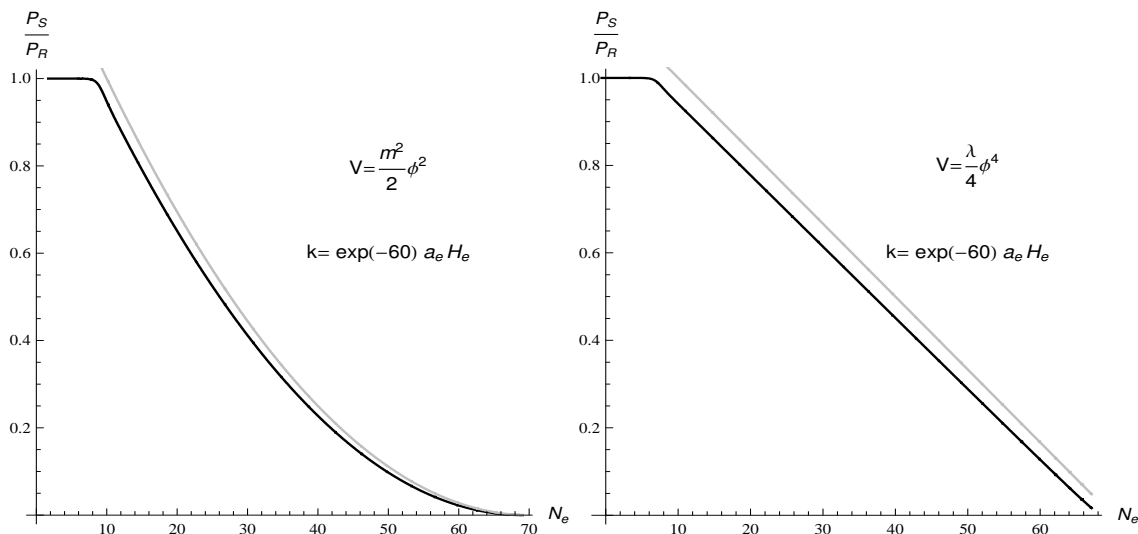


Figure 2. Left graph shows the ratio $P_{S_{r,lm}}(N_e)/P_{\mathcal{R}}|_*$ vs. N_e for the mode that exit the Hubble radius 60 e-folds before the end of chaotic inflation with $V = \frac{1}{2}m^2\phi^2$. Right graph shows the evolution of $P_{S_{\beta,1m}}(N_e)/P_{\mathcal{R}}|_*$ for $l = 1$ β -mode in $\lambda_{\text{eff}}\phi^4/4$ potential. Black and gray curves respectively demonstrate the numerical and analytic results.

In $\lambda_{\text{eff}}\phi^4/4$ chaotic model, the masses of iso-curvature modes are different. The lowest mass belongs to the $l = 1$ β -mode whose mass is equal to zero, $M_{\beta,1}^2(\phi) = 0$. The corresponding iso-curvature spectrum amplitude at the end of inflation is equal to $P_{S_{\beta,1m}} \simeq 3.949 \times 10^{-11}$ at today's Hubble scale. The corresponding spectral index is $n_{\Psi_{\beta,1m}} \simeq 0.966$. The relatively larger value of iso-curvature perturbations could be attributed to the fact that the iso-curvature spectrum for this mode decays linearly with the number of e-folds it spends outside the horizon (see below).

The next modes in the tower of masses are the zero mode, $l = 0$ α -mode and $l = 2$ β -mode whose masses are equal to $M_{\beta,2}^2(\phi) = \lambda_{\text{eff}}\phi^2$. Their corresponding amplitudes are $P_{S_{\beta,2m}} \simeq 4.449 \times 10^{-13}$ at today's horizon scale. Taking the multiplicity of α and β modes into account, there are $N^2 + 6$ iso-curvature modes with such an amplitude. The corresponding spectral index is $n_{\Psi_{\beta,1m}} \simeq 0.9828$.

For the $l = 1$ α -mode with the mass $M_{\beta,3}^2(\phi) = 3\lambda_{\text{eff}}\phi^2$, which is equal to the mass of the $l = 3$ β -mode, the iso-curvature spectrum amplitude is $P_{S_{\beta,3m}} \simeq 3.967 \times 10^{-18}$ at today's Hubble scale. The corresponding spectral index is equal to $n_{\Psi_{\beta,3m}} \simeq 1.016$, which indicates a blue spectrum.

In general mass of a $l \geq 1$ α -mode is identical to the $l+2$ β -mode. Therefore, there are $4l+6$ iso-curvature modes with identical spectra, all of which have a blue tilt. Increasing the value of l for α -mode and β -mode, the amplitude of heavy iso-curvature modes decreases quickly. This can be understood from (5.16), which yield

$$\frac{P_{S_{r,lm}}(N_e)}{P_{\mathcal{R}}|_*} \simeq (1 - N_e/60)^{1 + \frac{\omega^2 - \omega}{2}} = \begin{cases} (1 - N_e/60)^2 & \text{zero modes} \\ (1 - N_e/60)^{(l^2+3l+4)/2} & \alpha - \text{modes} \\ (1 - N_e/60)^{(l^2-l+2)/2} & \beta - \text{modes,} \end{cases} \quad (5.25)$$

where again N_e is the number of e-folds the mode spends outside the Hubble radius before

inflation ends. In particular, for $l = 1$ β -mode our analytic result indicates that the ratio $P_{\mathcal{S}_{\beta,1m}}(N_e)/P_{\mathcal{R}}|_*$ decreases linearly with N_e , which is verified numerically as shown in figure 2.

Besides a considerable iso-curvature/adiabatic ratio of 1.638% for $l = 1$ β -mode, another signature of the model is its observable gravity waves. For this model the amplitude of tensor spectrum for adiabatic perturbations at Hubble scale today is $P_T(k_0) \simeq 6.3176 \times 10^{-10}$, i.e. $r \simeq 0.26$, whose spectral index is $n_T \simeq -0.033$. This model is currently on the verge of becoming ruled out.

5.4.3 Symmetry breaking inflation

We consider $\phi > \mu$ and $\mu/2 < \phi < \mu$ cases separately.

(a) $\phi > \mu$

In this case neither of the parameters of the potential, λ , κ and m are zero. To match the observational constraints from WMAP5, the above parameters have to take the following values given in (3.3):

$$\lambda_{\text{eff}} \simeq 4.91 \times 10^{-14}, \quad m \simeq 4.074 \times 10^{-6} M_P, \quad \kappa_{\text{eff}} \simeq 9.574 \times 10^{-13} M_P.$$

The scalar spectral index for the adiabatic perturbations is $n_{\mathcal{R}} \simeq 0.959$.

The lowest masses in the tower of $\Psi_{r,lm}$ iso-curvature modes belong to the zero mode and the $l = 0$ α -mode, whose mass is $M_{\alpha,0}^2(\phi) = \lambda_{\text{eff}}\phi^2 - 2\kappa_{\text{eff}}\phi + m^2$. The amplitudes and spectral indices of these two iso-curvature spectra are respectively $P_{\mathcal{S}_{\alpha,0m}} \simeq 1.162 \times 10^{-11}$ and $n_{\Psi_{\alpha,0m}} \simeq 0.981$.

Next in the tower of iso-curvature modes is the $l = 1$ α -mode whose mass is equal to $M_{\alpha,1}^2(\phi) = 3\lambda_{\text{eff}}\phi^2 - 4\kappa_{\text{eff}}\phi + m^2$. Its amplitude and index are, respectively, $P_{\mathcal{S}_{\alpha,1m}} \simeq 6.966 \times 10^{-15}$ and $n_{\Psi_{\alpha,1m}} \simeq 1.01$.

The $l = 1$ β -mode with mass equal to $2\kappa_{\text{eff}}\phi + m^2$ stands next in the tower. The amplitude of this mode is equal to $P_{\mathcal{S}_{\beta,1m}} \simeq 8.842 \times 10^{-18}$. The corresponding iso-curvature spectrum for this mode has a blue tilt but an almost scale-invariant spectrum, with $n_{\Psi_{\beta,1m}} \simeq 1.002$. As before, the next iso-curvature modes have negligible amplitudes at Hubble scale and therefore their contributions could be ignored.

The amplitude of tensor spectrum at Hubble scale is $P_T(k_{60}) \simeq 4.84 \times 10^{-10}$, $r \simeq 0.2$ with the spectral index $n_T \simeq -0.025$. Planck [16] should be able to verify this model.

(b) $\mu/2 < \phi < \mu$

Here to satisfy the constraints from the amplitude and spectral index from WMAP5, one has to adjust the the parameters as in (3.5):

$$\lambda_{\text{eff}} \simeq 7.187 \times 10^{-14}, \quad m \simeq 6.824 \times 10^{-6} M_P, \quad \kappa_{\text{eff}} \simeq 1.940 \times 10^{-12} M_P.$$

The index of the adiabatic spectrum for such values of parameters is $n_{\mathcal{R}} \simeq 0.961$.

Again the least massive iso-curvature modes are the zero mode and the $l = 0$ α -mode. Their amplitude and spectral index are, respectively, $P_{\mathcal{S}_{\alpha,0m}} \simeq 1.46 \times 10^{-11}$ and $n_{\Psi_{\alpha,0m}} \simeq 0.987$. The next biggest iso-curvature amplitude belongs to the $l = 1$ α -mode whose amplitude and spectral index are $P_{\mathcal{S}_{\alpha,1m}} \simeq 9.99 \times 10^{-13}$ and $n_{\Psi_{\alpha,1m}} \simeq 0.988$, respectively. The $l = 1$ β -mode stands in the next rank with

an amplitude of $P_{S_{\beta,1m}} \simeq 6.558 \times 10^{-16}$ with $n_{\Psi_{\beta,1m}} \simeq 1.054$. Next iso-curvature modes have larger masses and therefore, as before, their amplitudes are negligible in comparison with the adiabatic one.

The amplitude of tensor spectrum at Hubble scale is $P_T(k_0) \simeq 1.307 \times 10^{-11}$, i.e. $r \simeq 0.048$ with the spectral index $n_T \simeq -0.006$. Such gravity wave spectrum could be detected by CMBPOL [18] or QUIET [19]. The tensor spectrum is very close to being scale-invariant in this case.

5.4.4 Inflection point inflation

A possible parameter set that satisfies the observational constraints are:

$$\lambda_{\text{eff}} \simeq 4.8 \times 10^{-14}, \quad m \simeq 10^{-6} M_P, \quad \kappa_{\text{eff}} \simeq 1.94 \times 10^{-12} M_P. \quad (5.26)$$

For this parameter set the scalar spectral index is $n_{\mathcal{R}} \simeq 0.93$ which is within 2σ error bar of WMAP5 but is somewhat to its lower end.

The lowest mass mode belongs to the $l = 1$ α -mode whose mass is $M_{\alpha,1}^2(\phi) = 3\lambda_{\text{eff}}\phi^2 - 2\kappa_{\text{eff}}\phi + m^2$. Its corresponding spectrum amplitude and spectral index are $P_{S_{\alpha,1m}} \simeq 1.38 \times 10^{-10}$ and $n_{\Psi_{\alpha,1m}} \simeq 0.932$, at Hubble scales which is interestingly very close to the spectral index of the adiabatic spectrum. This mode has the highest ratio of $P_{S_{\alpha,1m}}(N_e)/P_{\mathcal{R}}|_* \simeq 5.7\%$. The next modes in the series are the zero mode and the $l = 0$ α -mode whose corresponding amplitudes and spectral indices are respectively $P_{S_{\alpha,0m}} \simeq 1.7551 \times 10^{-16}$ and $n_{\Psi_{\alpha,0m}} \simeq 1.0012$. Other iso-curvature modes have completely negligible spectra in comparison with the adiabatic one.

The amplitude of the tensor spectral index is negligible for the above set of parameters in this model, $P_T \simeq 1.168 \times 10^{-13}$, which is scale-invariant with the precision of 10^{-6} .

6 End of inflation and preheating

As discussed while the ϕ field is turned on during inflation, the other fields $\Psi_{r,lm}$ are also present. Although not turned on at the onset of inflation (by the choice of initial conditions) and hence due to the specifics of the classical dynamics of our model remain zero during inflation, the presence of $\Psi_{r,lm}$ fields can be felt through quantum effects. These quantum effects show up in two different contexts; one is of course through the power spectrum of the quantum fluctuations of these fields at the super-horizon scales, the iso-curvature modes which were discussed in some detail in the previous section. The other quantum effect is the possibility of creation of the $\Psi_{r,lm}$ particles, due to the coupling to the inflaton field ϕ . If the pair creation mechanism is “efficient enough” this will eventually back react on the classical dynamics of the system. This effect, if too efficient during inflation and before completion of the needed 60 e-folds, can tamper the whole M-fiation scenario.² Recalling the large number of $\Psi_{r,lm}$ modes ($3N^2 - 1$), their collective effect on the inflaton field could be very large ending inflation too fast. While if activated only toward the end of inflation it will be a positive feature of our model, providing us with a mechanism to end inflation while transferring the potential energy of the inflaton field into the kinetic energy of the $\Psi_{r,lm}$ fields, a preheating scenario [14, 15, 25, 29].

²This “slow-down” effect via particle creation, although potentially harmful for the standard slow-roll inflation, can be used as the mechanism to render an otherwise fast-roll inflationary scenario which does not give enough e-folds, to an effectively slow-roll inflation with enough number of e-folds. The recent publication [28] discusses this possibility.

In the first subsection, we first show that particle creation during slow-roll inflation is not harmful to our M-flation model. In the next subsection we explore the possibility of the particle creation as the basis for a natural and inherent preheating scenario in our M-flation model.

6.1 Particle creation during slow-roll inflation is not harmful to M-flation

In this section we show that quantum production of $\Psi_{r,lm}$ modes during inflation is not large enough to derail the slow-roll M-flation. This is done at two steps, first we compute the particle creation rate during inflation and then study the back reaction of the $\Psi_{r,lm}$ modes on the dynamics of the inflaton ϕ .

6.1.1 Quantum production of $\Psi_{r,lm}$ modes during inflation

The Lagrangian governing the dynamics of our model, up to the second order in $\Psi_{r,lm}$ fields, and the corresponding equations of motion are given respectively by (5.2) and (5.4). As explained before, the time-dependence in $M_{r,l}(\phi)$ leads to $\Psi_{r,lm}$ particle creation from the vacuum [13–15, 25, 29]. (Note that the expression (4.5) for $M_{r,l}^2(\phi)$ depends on r and l as well as the momentum number k .)

By replacing

$$\chi_{r,lm\ k} = a^{3/2} \Psi_{r,lm\ k}, \quad (6.1)$$

the equation of $\Psi_{r,lm}$ field takes the form of

$$\ddot{\chi}_k + \Omega_{k,rl}(t)^2 \chi_k = 0, \quad (6.2)$$

which is an oscillator with a time dependent frequency, $\Omega_{k,rl}^2$

$$\Omega_{k,rl}^2 = \frac{k^2}{a^2} - \frac{9}{4}H^2 \left(1 - \frac{2}{3}\epsilon\right) + M_{r,l}^2(\phi), \quad (6.3)$$

where $\epsilon \equiv -\dot{H}/H^2$ which in the slow-roll limit reduces to its conventional form (4.7).

During inflation and when $\Omega^2 < 0$ and χ is either the inflaton or iso-curvature perturbations, χ modes have an imaginary frequency. As is well-known $\Omega^2 < 0$ during inflation (e.g. see discussions and analysis of section 4) happens for the “super-horizon” modes. These imaginary frequency modes are those which follow a classical dynamics and contribute to the power spectrum of curvature or iso-curvature (entropy) modes.

For particle creation inside the horizon which is what we will be mainly concerned with here and can happen if the time variation of Ω^2 is not negligible, we should focus on the $\Omega^2 > 0$ regime. Let us suppose that we have the solution to (6.2) for the $\Omega^2 > 0$ regime. Our goal is to compute the number density of the χ_k particles produced during inflation and for that we need to compare the solutions at $t = -\infty$ (the onset of inflation) to $t = +\infty$ (the end of inflation). Denoting the former by χ_k^- and the latter by χ_k^+ , one may expand

$$\chi_k^- = A_k \chi_k^+ + B_k \chi_k^{*+}. \quad (6.4)$$

The A_k and B_k coefficients maybe thought as the Bogoliubov transformation parameters. The number density of χ_k mode produced is then equal to $|B_k|^2$. For a detailed discussion on this matter see [14] or appendix B of [38].

Except for some specific cases (e.g. see [25, 30]), however, it is not possible to solve (6.2) and we are hence forced to use approximations. One of the approximations which is usually

employed (e.g. see [15, 25, 29]) is the stationary phase approximation, which if (6.2) viewed as the Schrodinger equation, it is the WKB approximation. This approximation is valid if

$$\dot{\Omega}_{k,rl} \lesssim \Omega_{k,rl}^2 \quad , \quad \Omega_{k,rl}^2 > 0 . \quad (6.5)$$

In this regime one may expand Ω^2 around its minima as

$$\Omega_{k,rl}^2 = \Omega_{0k,rl}^2 + \Gamma_{k,rl}^2 (t - t_k)^2 + \mathcal{O}((t - t_k)^3), \quad \dot{\Omega}_{k,rl}|_{t=t_k} = 0 . \quad (6.6)$$

where t_k is where $\dot{\Omega}_{k,rl}$ vanishes. Note that within our assumptions $\Omega_{0k,rl}^2$ and $\Gamma_{k,rl}^2$ are both positive. (If $\Omega_{k,rl}^2$ has several minima one should sum over all of them.)

In the WKB approximation the number density of the particles produced for each mode k is

$$\langle \chi_k | \chi_k \rangle = |B_k|^2 = e^{-\pi \frac{\Omega_{0,k}^2}{\Gamma_k}} . \quad (6.7)$$

As we see the particle creation is effective if $\frac{\Omega_{0,k}^2}{\Gamma_k}$ is of order one and not large.

In order to be specific and to get a better theoretical understanding of the analysis we shall focus on the specific $\lambda_{\text{eff}}\phi^4/4$ model for which κ_{eff} and m^2 both vanish. In this case

$$M^2 = \nu \lambda_{\text{eff}} \phi^2 , \quad (6.8)$$

where $\nu = 1, \frac{1}{2}l(l+1), \frac{1}{2}l(l-1)$ respectively for zero, α and β modes. Moreover,

$$\epsilon = \frac{2}{3}\eta = 8 \left(\frac{M_P}{\phi} \right)^2 .$$

In the leading order in ϵ, η

$$\begin{aligned} \frac{k^2}{a^2 H^2} &= \frac{1}{2H^4} \Gamma_{k,rl}^2 = \frac{3}{4} \epsilon (3 - \nu \epsilon) , \\ \Omega_{0k,rl}^2 &= \frac{3}{2} H^2 \left(\nu \epsilon - \frac{3}{2} \right) . \end{aligned} \quad (6.9)$$

The creation of particles can then happen in the window

$$\frac{3}{2} < \nu \epsilon < 3 . \quad (6.10)$$

That is, it is not possible for zero modes and for α and β modes with l less or of order $1/\sqrt{\epsilon}$. Recalling the expression (6.7) and the exponential suppression by the factor of Ω_0^2/Γ , the particle creation is effective in the region

$$\frac{\Omega_{0k,rl}^2}{\Gamma_{k,rl}} = \frac{\nu \epsilon - \frac{3}{2}}{\sqrt{2\epsilon(1 - \frac{\nu\epsilon}{3})}} \lesssim 1 . \quad (6.11)$$

The above can be satisfied if ν is close to its lower bound $\frac{3}{2\epsilon}$. However, recalling that ν is integer-valued and that for large ν , $\nu \simeq l^2/2$, the number of allowed l 's is very limited and as a result $\nu\epsilon$ can be tuned around the center value $3/2$ with accuracy of order $\sqrt{\epsilon}$. That is, for ν around

$$\nu \epsilon = \frac{3}{2} + \delta \sqrt{\epsilon} , \quad (6.12)$$

with δ being an order one number, the particle creation is effective.

As the last step we compute the energy which is carried by the $\Psi_{r,lm}$ particles in α and β modes produced during inflation. For that we need to integrate over the number density (6.7), explicitly

$$N_{\Psi} = \sum_{r,l} D_{r,l} \int \frac{d^3k}{(2\pi)^3} \frac{1}{a^3 \Omega_{0k,rl}} e^{-\pi \frac{\Omega_{0k,rl}^2}{\Gamma_{k,rl}}}, \quad (6.13)$$

where sum over r only runs over α and β modes and $D_{r,l}$ is the degeneracy of the modes which is equal to $2l + 1$ for α and β modes. Recalling that the particle creation is effective in the range,

$$\frac{k}{a} = H \sqrt{\frac{9}{8}\epsilon}, \quad \frac{k\Delta k}{a^2} = \frac{3}{8} H^2 \epsilon^{3/2} \delta \quad \Omega_0 = H \sqrt{\frac{3}{2}\delta} \epsilon^{1/4},$$

and for $\nu\epsilon = \frac{3}{2}$, $D_{r,l}$ is roughly $2\sqrt{3/\epsilon}$. Given the above one can perform the integral

$$N_{\Psi} \simeq \frac{9}{8\pi^2} H^2 \epsilon^{5/4} \delta^{1/2} \cdot e^{-\pi\delta}. \quad (6.14)$$

We would like to remark that as Ω_0^2/Γ is of order unity one may still trust the WKB approximation.

6.1.2 Back reaction on the inflationary dynamics

The created Ψ particles during slow-roll inflation will back react on the dynamics of the inflaton ϕ . Their back reaction can be traced through their effect in the equation of motion of the inflaton. Strictly speaking the back reaction effects we want to consider arise from the one loop correction to the inflaton potential. Noting the action (5.2) and the form of the potential V_0 , these corrections are

$$\ddot{\phi} + 3H\dot{\phi} + V_0' + \Delta V' = 0, \quad (6.15)$$

where³

$$\Delta V' = 3\lambda_{\text{eff}}\phi\langle\phi^2\rangle - 2\kappa_{\text{eff}}\langle\phi^2\rangle + \frac{1}{2} \sum_{r,lm} \langle\Psi_{r,lm}^* \Psi_{r,lm}\rangle \frac{dM_{r,l}^2(\phi)}{d\phi} \quad (6.16)$$

where $\langle\Psi_{r,lm}^* \Psi_{r,lm}\rangle$ during inflation should be replaced with N_{Ψ} (6.14). The $\langle\phi^2\rangle$ during slow-roll inflation where ϵ, η are very slowly varying (and are basically constant), is negligible and can be dropped.⁴ This term, however, may become large toward the end of inflation

³Note that at the one loop level besides $\Delta V'$ terms one should consider the running of the coupling constants λ_{eff} , κ_{eff} and m^2 . Since we are dealing with quadratic potentials which are renormalizable, one may compute these loop corrections. Due to the large number of fields these one loop effects could be large. In our case, however, we can just use the renormalized values for these parameters. We also note that among the specific cases that we discussed, the $\lambda\phi^4$ theory at one loop level will receive a contribution of the form $\delta m^2\phi^2$. The condition of having an inflection point, $\kappa_{\text{eff}}^2 = m^2\lambda_{\text{eff}}$ will not be preserved by the quantum corrections (at one loop level). The symmetry breaking potential, however, will preserve its form. Besides the renormalization of the couplings there is also the Coleman-Weinberg corrections. In our discussion, however, we will not consider this. The symmetry breaking case can be viewed as the bosonic part of the potential in a supersymmetric theory, for which the Coleman-Weinberg correction is absent.

⁴The $\langle\phi^2\rangle$ is essentially the same $\langle\Psi\Psi\rangle$ as the zero Ψ modes for which the ν factor, which causes the enhancement in the α and β modes of Ψ , is absent. Therefore, the $\langle\phi^2\rangle$ is negligible compared to $\langle\Psi_{r,lm}^2\rangle$ term.

when both ϵ and $\dot{\epsilon}/H$ can become order one. This is the regime we consider in the next subsection. During the slow-roll inflation therefore, $\Delta V' = \frac{1}{2}N_\Psi dM^2(\phi)/d\phi$.

Let us focus on the $\lambda_{\text{eff}}\phi^4/4$ theory for which the computations of 6.1.1 was mainly carried out. In this case the back-reaction equation of motion for the inflaton ϕ , is

$$\ddot{\phi} + 3H\dot{\phi} + \lambda_{\text{eff}}\phi^3 + \nu\lambda_{\text{eff}}N_\Psi\phi = 0. \quad (6.17)$$

As the computation of N_Ψ was carried out assuming the slow-roll approximation is valid, we stress that the above equation is hence only trustable during slow-roll inflation.

To check whether the last term is harmful to the (slow-roll) inflationary dynamics we compare the last term to contribution of the main potential driving inflation:

$$\frac{\lambda_{\text{eff}}\nu\phi \cdot N_\Psi}{\lambda_{\text{eff}}\phi^3} = \frac{27}{128\pi^2} \left(\frac{H}{M_P}\right)^2 \cdot \epsilon^{5/4} \delta^{1/2} e^{-\pi\delta}. \quad (6.18)$$

Recalling the WMAP bound $H/M_P < 10^{-4}$, the above expression is of course much smaller than one during slow-roll inflation. We therefore conclude that the particle creation is not going to destroy our M-flaton model.

6.2 Particle creation and the preheat scenario

We argued that during the slow-roll inflation particle creation, and hence its back-reaction on the dynamics of the ϕ field is not large. However, particle creation can become important when ϵ, η are of order one. In this section we explore this region. The equations we employ are of course the equation of motion for $\Psi_{r,lm}$ modes (5.4) and the modified equation for the inflaton field (6.15).

We follow the line of analysis performed in [30]. That is, in first step we ignore the ΔV term in (6.15), i.e. we study classical dynamics of the inflaton field ϕ when ϵ is of order one and then study the dynamics of the $\Psi_{r,lm}$ fields using this solution for ϕ as background, and finally, we include ΔV in the equation of ϕ . The inflaton potential we start with (2.9) is a generic quartic potential and the analytic treatment of the equations is not possible for generic values of parameters $\lambda_{\text{eff}}, \kappa_{\text{eff}}, m^2$. The analysis for the chaotic case, i.e. when only λ_{eff} (or when m^2) is non-zero has been carried out in some detail in [15, 30–32]. In our model, however, the case with only non-zero m^2 does not have the quartic coupling to the preheat field(s) and hence does not involve a preheating model. We therefore focus on the $\lambda\phi^4/4$ theory.

For the $\lambda\phi^4/4$ theory, the potential for the fields is

$$\begin{aligned} V(\phi, \Psi_{r,lm}) &= \frac{1}{4}\lambda_{\text{eff}}\phi^4 + \frac{1}{2}\lambda_{\text{eff}}\phi^2 \sum_{r,lm} \frac{1}{2}(\omega^2 - \omega)\Psi_{r,lm}^* \Psi_{r,lm} \\ &= \frac{1}{4}\lambda_{\text{eff}}\phi^4 + \frac{1}{2}\lambda_{\text{eff}}\phi^2 \sum_{m=1}^{N^2} |\Psi_{0m}|^2 \\ &\quad + \frac{1}{2}\lambda_{\text{eff}}\phi^2 \left[\sum_{l=0}^{N-1} \frac{l(l+1)}{2} \sum_{m=1}^{2l+1} |\Psi_{\alpha, lm}|^2 + \sum_{l=1}^{N-1} \frac{l(l-1)}{2} \sum_{m=1}^{2l+1} |\Psi_{\beta, lm}|^2 \right] \end{aligned} \quad (6.19)$$

where in the second line we have decomposed $\Psi_{r,lm}$ into zero, α and β modes for which ω respectively takes values $-1, -(l+1)$ and l . We remark that the potential (6.19) is an approximation to the potential term we start with (2.1) (for the $\kappa = 0, m^2 = 0$ case) to order Ψ^2 .

As shown in [30], the ϕ equation of motion toward the end of inflation is an (anharmonic) oscillator around $\phi = 0$, the amplitude of the solution is decreasing as $t^{-1/2}$. In this regime the ϕ field is effectively living in a radiation dominated background with $1/H = 2t$. It appears that the equation for ϕ takes a simple form in the conformal time, with Jacobi (elliptic) cosine function as its solution [30]. The equation for the Ψ modes, too, can be solved explicitly in our case. In fact in the notations of [30], the equation for all three zero, α and β modes is of the form of $g^2/\lambda = n(n+1)/2$ (with $n = 1, l, l-1$ respectively for zero, α and β modes) for which most of the calculations can be performed analytically. (Note that g^2/λ of [30] coincides with the parameter ν (6.8) in our model.) As discussed in [30] for these specific values of g^2/λ we have the significant property that there is an enhancement in the parametric resonance leading to considerable creation of zero, α and β modes. As discussed in [30] one can distinguish two even and odd n cases. For the odd n (i.e. for our zero modes, odd l α -mode, and even l β -mode) the particle creation is peaked around zero momentum k modes. For the even n modes, however, the particle creations is peaked around momenta $k^2 = \frac{3}{2}H_{\text{inf}}^2\epsilon\sqrt{\frac{g^2}{2\lambda}}$, where ϵ is the computed for the beginning of the slow-roll inflation and H_{inf} is the Hubble during inflation. For low n , the Floquet index $\mu_k \propto \ln n_k$ (n_k is the number density of the produced particles at momentum k) is around 0.15 for odd n and around 0.5 for even n . Therefore, among the low n modes the main contribution to preheating is coming from odd n [30]. As discussed in [30] the bigger k is, the more energy can be transferred from the inflationary sector to the Ψ sector and a more efficient preheat mechanism. This means that α and β modes with large l , l of order N , make the biggest contribution, this is despite the fact that the zero modes have a larger degeneracy (of order N^2) compared to the degeneracy of order N for the large l modes. All in all, due to the existence of the large l modes, and for large N in our model we expect to have a very efficient preheating model. The computations for the modes with large g^2/λ has been carried out in [30] and the only point which is different in our case is that their result should be multiplied with the degeneracy factor $2l+1$.

As we argued we have an efficient preheat mechanism in our model. As a very crude estimate of the preheat temperature in our M-flation setup we may hence use an instant efficient preheating that all the energy of the inflaton field has gone to effectively massless zero modes by the end of inflation, leading to

$$N^2 T^4 \sim 3H^2 M_P^2, \quad (6.20)$$

where N^2 estimates the number of species and T is the preheat temperature. As we see, this is as if we have effectively an instant preheating model in which the maximum temperature achieved is lowered by $1/\sqrt{N}$. As a rough estimate taking H saturating its current bound $H \sim 10^{-5}M_P$ and $N \sim 10^5$ then the preheat temperature becomes of order $T \sim 10^{13}$ Gev. Reducing the preheat temperature to below GUT scale is in principle a positive feature, as it removes the problem with overproduction of gravitinos.

7 Motivation from string theory

Here we argue that our M-flation setup presented in section 2 with non-commutative matrices and potential in the form of (2.2) is strongly motivated from string theory.

In the context of string theory, the world-volume theory of N coincident p-branes is described by a (supersymmetric) $U(N)$ gauge theory. In this system, the transverse positions

of the branes, Φ_I , $I = p + 1, \dots, 9$, which from the world-volume theory are scalars in the adjoint representation of $U(N)$, are hence $N \times N$ matrices. For the case of our interest, $p = 3$, there are six such scalars. The DBI action for the system of N coincident D3-branes in the background RR six form flux (sourced by a distribution of D5-branes) is given by (e.g. see [33])

$$S = \frac{1}{(2\pi)^3 l_s^4 g_s} \int d^4x \text{STr} \left(1 - \sqrt{-|g_{ab}|} \sqrt{|Q_J^I|} + \frac{i g_s}{2 \cdot 2\pi l_s^2} [X^I, X^J] C_{IJ0123}^{(6)} \right) \quad (7.1)$$

Here l_s is the string scale and g_s is the perturbative string coupling. The operator STr on a product of matrices is the trace of their symmetrized product. The induced metric on branes, g_{ab} , is given by $g_{ab} = G_{MN} \partial_a X^M \partial_b X^N$, where X^M indicates the ten-dimensional positions of the branes and G_{MN} is the ten dimensional background metric. Here the indices $I, J = 4, 5, \dots, 9$ represent the coordinates perpendicular to the branes world-volume, the indices $a, b = 0, 1, 2, 3$ represent the brane world-volume coordinates and the capital letters $M, N = 0, 1, \dots, 9$ indicate the ten-dimensional coordinates. The matrix Q_J^I is due to non-commutativity properties of the system given by

$$Q^{IJ} = \delta^{IJ} + \frac{i}{2\pi l_s^2} [X^I, X^J], \quad (7.2)$$

and $C_{IJ0123}^{(6)}$ is a rank-6 antisymmetric Ramond-Ramond (RR) field which has two legs along the direction transverse to the D3-brane.

We consider the ten-dimensional IIB supergravity background

$$ds^2 = -2dx^+ dx^- - \hat{m}^2 \sum_{i=1}^3 (x^i)^2 (dx^+)^2 + \sum_{I=1}^8 dx_I dx_I \quad (7.3)$$

$$C_{+123ij} = \frac{2\hat{\kappa}}{3} \epsilon_{ijk} x^k$$

where i, j indices, which are ranging over 1, 2, 3, parameterize three out of six transverse directions to D3-brane and x^I include three spatial directions along the brane and five of the transverse directions to D3-branes. With $\hat{m}^2 = 4g_s^2 \hat{\kappa}^2 / 9$ the above background, with constant dilaton, is a solution to supergravity equations of motion. This background is very similar to the background of [34] (see also appendix D of [35] for a discussion on the Matrix model on the above background).

If we turn-off fluctuations along the directions transverse to the branes and the x^i directions (this may be done if we compactify these three directions on a T^3 of very small radius), fix the light-cone gauge on the D3-branes, expand the action and keep up to order four in X^I , we obtain

$$S = \frac{1}{(2\pi)^3 l_s^4 g_s} \int d^4x \text{Tr} \left[\frac{-1}{2} \partial_\mu X_i \partial^\mu X_i - V(X) \right] \quad (7.4)$$

$$V = -\frac{1}{4 \cdot (2\pi l_s^2)^2} [X_i, X_j][X_i, X_j] + \frac{i g_s \hat{\kappa}}{3 \cdot 2\pi l_s^2} \epsilon^{ijk} X_i [X_j, X_k] X_i + \frac{1}{2} \hat{m}^2 X_i^2.$$

If we redefine

$$X_i = \sqrt{(2\pi)^3 g_s} l_s^2 \Phi_i \quad (7.5)$$

and upon addition of the four-dimensional Einstein gravity, the above action takes the form of (2.1) with the potential (2.2) once we identify the parameters as

$$\lambda = 2\pi g_s, \quad \hat{\kappa} = \frac{\kappa}{g_s \cdot \sqrt{2\pi g_s}}, \quad \hat{m}^2 = m^2. \quad (7.6)$$

Although from the brane theory viewpoint we need to choose \hat{m}^2 and $\hat{\kappa}$ such that (7.3) is a solution to supergravity, namely $\lambda m^2 = 4\kappa^2/9$, since we presented the brane theory by the way of motivation, at the level of M-flation action we may relax this condition and take λ , κ and m^2 as independent parameters. It is also worth noting that $\lambda m^2 = 4\kappa^2/9$ corresponds to the “symmetry breaking” inflation potential (3.1). Furthermore, the minimum $\phi = \mu$ for the potential (3.1) corresponds to the supersymmetric background where N D3-branes blow-up into a giant D5-brane.

It is worth noting that in the brane theory setting the $U(N)$ symmetry appears as a gauge symmetry, while in our model we took it to be a global symmetry. Promoting $U(N)$ to a gauge symmetry does not change our analysis of the $SU(2)$ sector and the corresponding inflationary dynamics. Due to the gauge symmetry, however, not all the Ψ_i modes are physical. Among them the zero modes can be removed by the gauge transformations and hence in the theory where $U(N)$ is gauged only we deal with α and β modes. Thus in the gauge symmetry case, from the first $N^2 + 1$ iso-curvature modes in the mass tower, only one ($l = 0$ α -mode) remains, which has an amplitude of order few percent of curvature spectrum. However this will not change the analysis of section 6.1, as the main contribution were coming from α or β modes of l in the window (6.10) which are also present in the gauged theory. The analysis of preheating mechanism of section 6 will remain valid because again zero modes do not have the main contribution.

As a result of motion of D3-branes in the background $C^{(6)}$ flux, two of the directions transverse to D3-branes blow-up into (fuzzy) two sphere, which in the large N limit behaves as a D5-brane with world-volume $R^4 \times S^2$ [33]. In this geometric picture our inflaton field ϕ is nothing but the radius of this two-sphere. In this sense the effective inflaton field ϕ in our M-flation scenario is closely related to the inflaton in the “giant inflaton” model of [36] (see also [37]).

8 Discussion

In this work we have presented a new inflationary scenario, the M-flation, in which inflation is driven by matrix valued scalar fields. M-flation, hence, falls into the general class of multi-field inflation models and shares positive features of N-flation. Specifically, we used M-flation to obtain a super-Planckian (large field) field variation during inflation. This leads to a considerable amount of gravity waves which can be detected in future gravity wave observations such as PLANCK [16, 17], CMBPOL [18] and QUIET [19]. Moreover, as we discussed, due to the scalings with powers of N , the dimension of the matrices, M-flation bears a solution to the fine-tuning problem of the coupling in the $\lambda\phi^4/4$ chaotic inflation.

Here we focused on a special class of M-flation scenarios with potentials of the form (2.2) where the potential is quadratic in powers of Φ_i and their commutators. Within our three parameter family of the potentials there are interesting special models of inflation which were analyzed in section 3. One may, however, start with other forms for the potential. Specifically, if one starts from the string theory realization of the scenario, then from action (7.1) one obtains higher power corrections in terms of Φ_i and $[\Phi_i, \Phi_j]$. Furthermore, the kinetic energy

may also have a non-trivial form and one may combine the idea of M-flation with a DBI non-standard kinetic energy [39]. It would be interesting to see the spectrum of the adiabatic and iso-curvature perturbations for this case of “DBI M-flation”.

One of the observable effects of multi-field inflation models is that besides the usual power spectrum of the adiabatic fluctuations $P_{\mathcal{R}}$, we also have a non-zero power spectrum for the iso-curvature perturbation, $P_{\mathcal{S}_{r,lm}}$. We analyzed the ratio $P_{\mathcal{S}_{r,lm}}/P_{\mathcal{R}}$ for various inflationary models up to the end of inflation. Inside the Hubble radius this ratio is close to unity but once the mode leaves the Hubble radius this ratio decays quickly towards the end of inflation. As shown, our analytical estimates of $P_{\mathcal{S}_{r,lm}}/P_{\mathcal{R}}$ are in good agreement with the numerical results. In order to relate this ratio to the observed values from CMB, however, we should also supplement our M-flation scenario with a reheating mechanism.

As we discussed in section 5.3, the iso-curvature perturbations $\Psi_{r,lm}$ do not produce entropy perturbations nor couple to curvature perturbations. This is due to our initial conditions resulting in the fact that they are classically frozen during inflation. This in turn implies that they do not carry energy up to leading order in perturbation theory. However, they can contribute to entropy perturbations at second order in perturbation theory or through preheating mechanism. Via the same mechanisms, the iso-curvature perturbations $\Psi_{r,lm}$ can produce non-Gaussianities which are under intense observational investigations.

As was discussed in section 6 our model naturally contains a preheating sector (essentially the $\Psi_{r,lm}$ -modes) which due to the large number of these modes works very efficiently, taking away the energy stored in the inflaton field. In order to complete our model, we need to have a reheating model via which the energy of the Ψ -modes is transferred into the Standard Model particles. This is postponed to future works. As a possibility for reheating mechanism in string theory setup, where Φ_i represents the collective positions of N D3-branes, we may imagine that the Standard Model of particle physics are confined to the branes in the forms of open strings gauge fields $A_\mu^{(a)}$. The question of reheating would be how to transfer energy from the Φ_i fields, more precisely from the $\Psi_{r,lm}$ modes, to the open string gauge fields $A_\mu^{(a)}$.

In this work we have restricted the analysis to a particular solution where $\Psi_{r,lm}$ fields are absent in classical inflationary dynamics. The inflaton field ϕ is the projection of Φ_i along the $N \times N$ irreducible representation of $SU(2)$, the J_i matrices. In the field space of $\phi - \Psi_{r,lm}$ this corresponds to a straight inflationary background. In general, one may consider an arbitrary initial condition where $\Psi_{r,lm}$ fields are turned on. In the field space of $\phi - \Psi_{r,lm}$ this gives a complicated curved inflationary trajectory. One such possibility with a more controlled dynamics is to take J_i to form a reducible $N \times N$ representation of $SU(2)$ which consists of n irreducible blocks. In this case the classical inflationary dynamics of our theory reduces to that of n decoupled scalar fields, each with generic quartic potential. In this case the iso-curvature perturbations would be non-adiabatic and a significant amount of entropy perturbations can be created during inflation. Similarly, one expects a considerable amount of non-Gaussianities to be produced in this case. It would be interesting to build an specific model of M-flation where $\Psi_{r,lm}$ fields are turned on during inflation and calculate non-Gaussianities and entropy perturbations produced and compare them with the observational bounds.

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A Symmetry breaking inflation

Here we study inflation from the symmetry breaking potential in some details. Suppose inflation starts when $\phi_i > \mu$. The total number of e-folds N_e is obtained by

$$\begin{aligned} M_P^2 N_e &= \int_{\phi_f}^{\phi_i} \frac{d\phi V}{V'} \\ &= \frac{1}{8}(y - x) - \frac{\mu^2}{32} \ln \left(\frac{\mu^2 + 4y}{\mu^2 + 4x} \right) \end{aligned} \quad (\text{A.1})$$

where ϕ_f is the endpoint of inflation and for later convenience we have defined $x = \phi_f(\phi_f - \mu)$ and $y = \phi_i(\phi_i - \mu)$. As usual, define the slow-roll parameters

$$\epsilon = \frac{1}{2} M_P^2 \left(\frac{V'}{V} \right)^2, \quad \eta = M_P^2 \frac{V''}{V}. \quad (\text{A.2})$$

where $'$ denotes derivative with respect to ϕ . Inflation ends when $\epsilon = 1$, which is used to fix x and ϕ_f

$$x = 4M_P^2 + M_P \sqrt{16M_P^2 + 2M_P^2 \mu^2} \quad (\text{A.3})$$

The scalar spectral index at ϕ_i is $n_{\mathcal{R}} - 1 = 2\eta - 6\epsilon$ at ϕ_i which can be used to eliminate y

$$\frac{y}{M_P^2} = \frac{12 + \sqrt{144 + 8(1 - n_{\mathcal{R}}) \frac{\mu^2}{M_P^2}}}{1 - n_{\mathcal{R}}} \quad (\text{A.4})$$

Plugging these values for x and y in (A.1), we find an equation for μ/M_P . Solving this equation numerically with $N_e = 60$ and $n_s = 0.96$ from WMAP5 central value, one obtains $\mu/M_P \sim 26$. This in turn yields $\phi_i \simeq 44M_P$ and $\phi_f \simeq 28M_P$.

The COBE normalization, can be used to fix the value of λ_{eff}

$$\delta_H = \frac{1}{\sqrt{75}\pi} \frac{V^{3/2}}{M_P^3 V'} = \frac{\lambda_{\text{eff}}^{1/2}}{4\sqrt{75}\pi} \frac{\phi_i^2 (\phi_i - \mu)^2}{(2\phi_i - \mu) M_P^3}. \quad (\text{A.5})$$

Using $\delta_H \simeq 2 \times 10^{-5}$ and the above values for ϕ_i and μ , one obtains $\lambda_{\text{eff}} \simeq 10^{-14}$. This corresponds to $N \sim 10^5$ as in chaotic inflation case.

It is also instructive to look into gravity wave amplitudes, determined by quantity r , defined as the ratio of gravitational perturbation amplitude to scalar perturbation amplitude at ϕ_i :

$$\begin{aligned} r &= \frac{8}{3}(1 - n_{\mathcal{R}}) + \frac{16}{3} M_P^2 \frac{V''}{V} \\ &= 4(1 - n_{\mathcal{R}}) + 32 \frac{M_P^2}{y}. \end{aligned} \quad (\text{A.6})$$

Using the above values for ϕ_i and μ , one obtains $r \lesssim 0.2$ which is consistent with the upper bound $r < 0.22$ from WMAP5.

The analysis when inflation takes place in regions $\mu/2 < \phi_i < \mu$ and $0 < \phi_i < \mu/2$ is similar to the above.

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