RSD-TR-9-83

DISPLACEMENT CONTROL OF FLEXIBLE STRUCTURES USING ELECTROHYDRAULIC SERVO ACTUATORS[1]

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June 1983

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[1]This work was supported by the National Science Foundation under Grant CEE 8207375 and by the Air Force Office of Scientific Research/AFSC, United States Air Force under AFOSR contract number F49620-82-C-0089.

Abstract

A general approach to the displacement or position control of a nonlinear flexible structure using electrohydraulic servo actuators is developed. Our approach makes use of linear feedback of measured structural displacements plus linear feedback of the actuator control forces; a nonlinear feedforward function of the displacement command is also used for control. Based on a mathematical model of the closed loop, general conditions for closed loop stability are obtained. In the special case that the feedback is decentralized the stabilization conditions are stated in terms of simple inequalities; moreover, the stabilization conditions are robust to structural uncertainties since the conditions do not depend on explicit properties of the structure. Such robustness is a direct consequence of use of force feedback rather than, for example, acceleration feedback. Conditions are also developed for selection of the feedforward control to achieve zero steady state error; but this condition does depend on explicit properties of the structure. The theoretical results developed in the paper should provide a framework for advanced applications of control of mechanical systems using electrohydraulic servo actuators.

1. Introduction

Electrohydraulic servo actuators have been widely used for position control of large loads. Common applications include position control of antennas, of airframe surfaces, and of machine tools. Such applications are characterized by relatively simple load dynamics and use of a single actuator. In applications where substantial stability margin is required, use of actuator force (or pressure) feedback and displacement feedback as inputs to the electrohydraulic servo valve has proved desirable [1,2]. Recent work on use of electrohydraulic servo actuators for position control of robotic devices [3] and for position control of civil engineering structures [4] are characterized by complicated structural load dynamics and use of multiple actuators. Our objective is to demonstrate the feasibility of using a generalized form of actuator force (or pressure) feedbacks and displacement feedbacks as inputs to the multiple electrohydraulic servo valves.

In this work a general mathematical theory is developed for the multivariable displacement control of an arbitrary nonlinear flexible structural load controlled by multiple electrohydraulic servo actuators. Specific mathematical equations are developed to describe the structural load dynamics and the actuator dynamics. Closed loop control is investigated using force and displacement feedback, plus feedforward of the position command. General systems theoretic conditions on the feedback are developed for which the closed loop is guaranteed to be stable; it is shown that the suggested feedback loops result in a particularly robust closed loop system. The general stabilization conditions are considerably simplified to simple inequalities if decentralized feedback control is used. Mathematical conditions are also given for selection of the feedforward control to achieve zero steady state controlled displacement error. Two special cases of the general results, that appear of particular interest in certain applications, are examined.

2. Mathematical Models

Consider a general structural load controlled by multiple electrohydraulic servo actuators. A typical part of the system, showing a part of the structural load connected to one of the electrohydraulic servo actuators, is shown in Figure 1.

The mathematical model of the structural load to be controlled is first described. For simplicity, a finite dimensional model of the structural load dynamics is assumed of the form

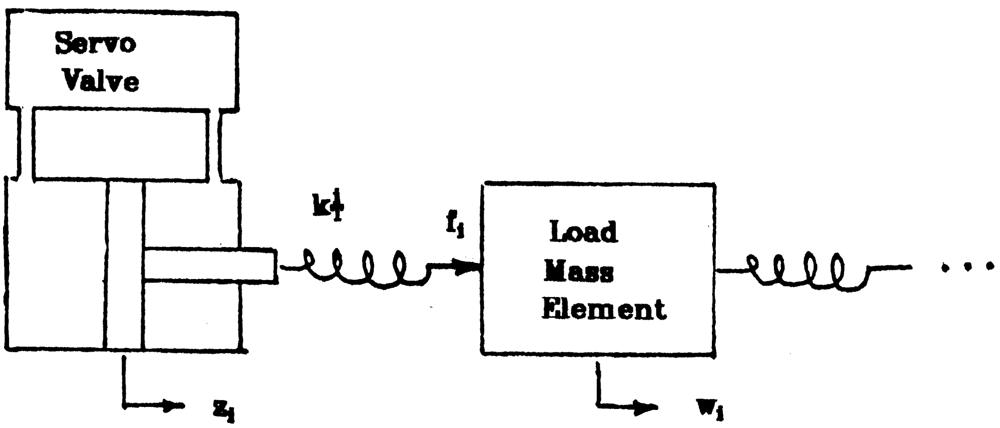

Figure 1

$$M\ddot{x} + F(x) = Bf \tag{1}$$

Here $x = (x_1,\ldots,x_n)$ denotes a generalized structural load displacement vector and $f = (f_1,\ldots,f_m)$ denotes the hydraulic control force vector on the structural load. The $n \times n$ mass matrix M is assumed to be a constant symmetric, positive definite matrix. The $n \times m$ influence matrix B is a dimensionless matrix which reflects the location of the actuators and the geometry of the structural load. Note that the displacement vector $w = (w_1,\ldots,w_m)$ of the structural load at the locations of the actuators is given by

$$w = B^T x \tag{2}$$

where the superscript T denotes matrix transpose. The flexible nonlinear restoring force function $F:R^n \to R^n$ is assumed to be continuously differentiable such that: for given initial displacement $x(0)$ and velocity $\dot{x}(0)$ and given control force function $f(t)$, $t \geq 0$, equation (1) has a unique solution defined for $t \geq 0$. Further, it is assumed that $F:R^n \to R^n$ has **incremental positive stiffness** in the sense that for each $\bar{x} \in R^n$ there is a scalar potential function $Q:R^n \to R^1$ satisfying $Q(0) = 0$ and

$$\mu_1 \, ||x||^2 \leq Q(x) \leq \mu_2 \, ||x||^2, \quad \text{for all } x \in R^n$$

with $\mu_1 \geq 0$, $\mu_2 \geq 0$, such that

$$F(x+\bar{x}) - F(\bar{x}) = \nabla Q(x) \quad \text{for all } x \in R^n \; .$$

One common case is where the structural load dynamics are characterized by an elastic restoring force so that $F(x) = Kx$, with K and $n \times n$ matrix that is symmetric and nonnegative definite. Many structural loads have very small intrinsic damping; in fact, feedback control is often used to actively augment the structural damping. In this work it is assumed that there is no explicit damping in the structural load dynamics.

Conventional characteristics of the electrohydraulic servo actuators are assumed [1]. The force f_i between the i^{th} actuator and the structural load is given by

$$f_i = A_i p_i \ , \qquad i = 1,..,m \tag{3}$$

where A_i is the effective piston area and p_i is the differential pressure in the cylinder of the i^{th} servo valve. The usual flow balance relation for the i^{th} servo valve is assumed

$$A_i \dot{z}_i + \frac{V_i}{B_i} \dot{p}_i = Q_i u_i - L_i p_i , \quad i = 1,\ldots,m \ . \tag{4}$$

Here z_i is the relative displacement of the piston of the i^{th} servo valve and u_i is the current controlling the i^{th} servo valve torque motor. Also V_i is the effective volume of the i^{th} servo valve cylinder, B_i is the fluid bulk modulus, L_i is a leakage parameter and Q_i is an input flow sensitivity parameter; each of these parameters is assumed to have a positive value.

The force f_i between the structural load and the i^{th} actuator is given by

$$f_i = k_f^i (z_i - w_i) \quad , \qquad i = 1,\ldots,m \tag{5}$$

where k_f^i denotes the stiffness of the connection between the structural load and the i^{th} electrohydraulic servo actuator.

Thus equations (1) to (5) form the basic mathematical model of the general structural load influenced by the multiple electrohydraulic servo actuators. These equations can be simplified and expressed in a more compact form. Although there are numerous choices of primary variables that could be selected our subsequent analysis will be based on use of the structural load displacement vector x and the actuator force vector f. Then the above equations can be written in the vector form

$$M\ddot{x} + F(x) = Bf \tag{6}$$

$$K_h^{-1}\dot{f} = -B^T\dot{x} - G^L f + v \ . \tag{7}$$

Here

$$K_h^{-1} = \mathrm{diag}\left[\frac{V_1}{B_1 A_1^2} + \frac{1}{k_f^1}, \ldots, \frac{V_m}{B_m A_m^2} + \frac{1}{k_f^m}\right]$$

denotes the effective compliance between the actuators and the structural load; the leakage sensitivity matrix is

$$G^L = \mathrm{diag}\left[\frac{L_1}{A_1}, \ldots, \frac{L_m}{A_m}\right]$$

and the effective actuator input vector is given by

$$v = \left\{\frac{Q_1}{A_1}u_1, \ldots, \frac{Q_m}{A_m}u_m\right\} .$$

Our subsequent development is based entirely on equations (6) and (7), which form a general mathematical model for a structural load controlled by multiple electrohydraulic servo actuators.

Although the mass of the hydraulic piston has not been explicitly taken into account in this development, it can be handled within the general framework of equations (6) and (7). In particular the piston mass and the transmission stiffness can be included as part of the structural load equation so that the form of equations (6) and (7), including piston mass effects, is preserved.

3. Control Objectives

The specific displacement control objectives are now stated. A general controller consisting of linear feedback of displacement and force and non-linear feedforward of the position command is suggested. The resulting closed loop multivariable system is described.

Suppose that $y = (u_1, \ldots, y_m)$ where

$$y = Cx \tag{8}$$

denotes the output vector displacement of the structural load to be controlled. In particular, if $\bar{y} = (\bar{y}_1, \ldots, \bar{y}_m)$ denotes a constant command or set point displacement vector then the control problem is to select the actuator input vector v so that $y(t) \to \bar{y}$ as $t \to \infty$, for each initial state.

Our approach is to make use of suitably defined feedback to achieve the desired result. In particular, it is assumed that the control is based on feedback of the displacement vector w of the structural load and feedback of the force vector f; such feedback depends on use of displacement transducers suitably mounted on the structural load and of force transducers (load cells) between the actuators and the structural load. A similar development could easily be carried out if feedback of the displacements $z_1, \ldots z_m$ of the servo valve pistons and feedback of the servo vale differential pressures $p_1, \ldots, p_m$ were assumed.

Note that in general the displacement vector y to be controlled is not the displacement vector w fed back to the controller. These two structural load displacement vectors are the same only if $C = B^T$.

The assumed form for the controller, stated here in terms of the effective actuator input vector v, is given by

$$v = G(\bar{y}) - G^d w - G^F f \quad . \tag{9}$$

Thus the controller is defined in terms of a nonlinear feedforward function $G{:}R^m \to R^m$ of the displacement command and a linear feedback function of the measured displacement and force vectors. Here G^d is $m \times m$ displacement feedback gain matrix and G^F is $m \times m$ force feedback gain matrix.

The closed loop equations are obtained by substituting the control relation (9) and equation (8) into equations (6) and (7) to obtain

$$M\ddot{x} + F(x) = Bf \tag{10}$$

$$K_h^{-1}\dot{f} = -B^T\dot{x} + G(\bar{y}) - G^d B^T x - G^f f \tag{11}$$

where $G^f = G^F + G^L$.

The specific control objectives can now be stated more explicitly in terms of the above closed loop. The feedforward function $G{:}R^m \to R^m$ and the feedback gain matrices G^d and G^f are to be selected so that for any constant displacement command vector $\bar{y}$ the solution of equations (10) and (11) satisfy $y(t) \to \bar{y}$ as $t \to \infty$ for each initial state. This implies that the closed loop is globally asymptotically stable. Depending on the specific form of the controller, even if the closed loop is stable there could be a steady state error between the controlled structural displacement and the command displacement. Hence selection of the specific controller should also be made to eliminate, if possible, the steady state error. In many practical situations the structural restoring forces, as defined by the vector function $F:R^n \to R^n$, are not accurately known; hence, the above results should be robust to uncertainties in the structural restoring forces.

In the next selection, guidelines are given for selecting the feedforward function $G:R^m \to R^m$ and the feedback gain matrices G^d and G^f to achieve the desired closed loop properties. The main difficulty is illustrated by the block diagram of the closed loop, in Figure 2, where explicit coupling between the structural load dynamics and the actuator dynamics is clear.

4. Analysis of Closed Loop Properties

In this section the basic theoretical result of the paper is presented. An interpretation of the result in systems theoretic terms is given. The general result is then specialized to the case where control is based on decentralized feedback.

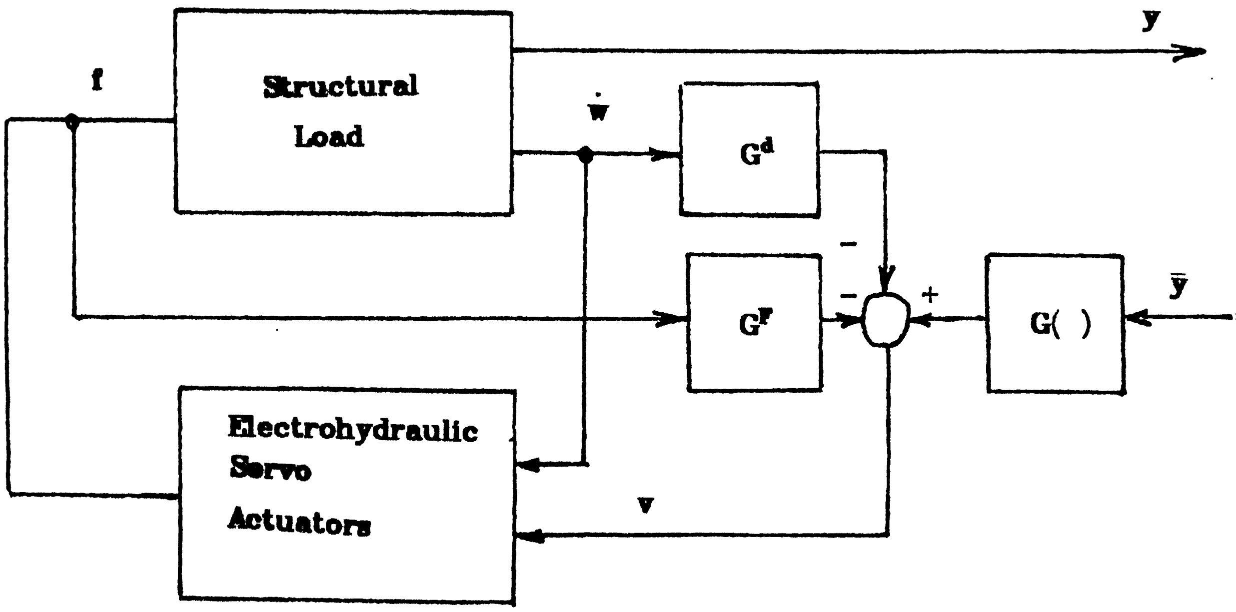

Figure 2

As mentioned previously, satisfaction of the control objectives depends primarily on selection of the feedback gains so that the closed loop is globally asymptotically stable. The main theoretical result to follow gives systems theoretic conditions on the feedback gain matrices G^d and G^f which guarantee that the closed loop is stable. Specification of the feedforward function $G:R^m \to R^m$ is also given so that there is no steady state error.

As a preliminary, assume the nonlinear algebraic vector equations

$$F(x) - Bf = 0 \tag{12}$$

$$G^d B^T x + G^f f = q \tag{13}$$

has a unique vector solution for each $q \in R^m$ denoted by $x = \mathfrak{X}(q)$, $f = \mathfrak{F}(q)$ where $\mathfrak{X}:R^m \to R^n$ and $\mathfrak{F}:R^m \to R^m$. Define the vector function $S:R^m \to R^m$ by

$$S(q) = C\mathfrak{X}(q) \ . \tag{14}$$

Theorem. Suppose that

(a) M is symmetric and positive definite, and $F:R^n \to R^n$ has incremental positive stiffness (as described previously);

(b) the transfer function matrix defined by

$$H_a(s) = \left[K_h^{-1}s + G_f\right]^{-1} \left[I + \frac{1}{s}G^d\right]$$

is strictly positive real (as defined in Appendix A).

Then the closed loop, defined by equations (10) and (11), is globally asymptotically stable, and for any constant displacement command vector $\bar{y}$

$$y(t) \to S(G(\bar{y})) \quad \text{as} \quad t \to \infty \ ,$$

for each initial state.

If in addition,

(c) $S(G(y)) = y$ for all $y \in R^m$

then for any constant displacement command vector $\bar{y}$

$$y(t) \to \bar{y} \ \ \text{as} \ \ t \to \infty \ ,$$

for each initial state.

The constant vector $\mathcal{X}(G(\bar{y}))$ and $\mathcal{F}(G(\bar{y}))$ are the equilibrium displacement vector and force vector corresponding to the displacement command vector $\bar{y}$. Although the closed loop, as shown in Figure 2, exhibits inherent coupling between the structural load dynamics and the actuator dynamics it is possible to define the "effective" actuator dynamics $H_a(s)$, as above, so that the closed loop in Figure 2 is equivalent to the closed loop as shown in Figure 3.

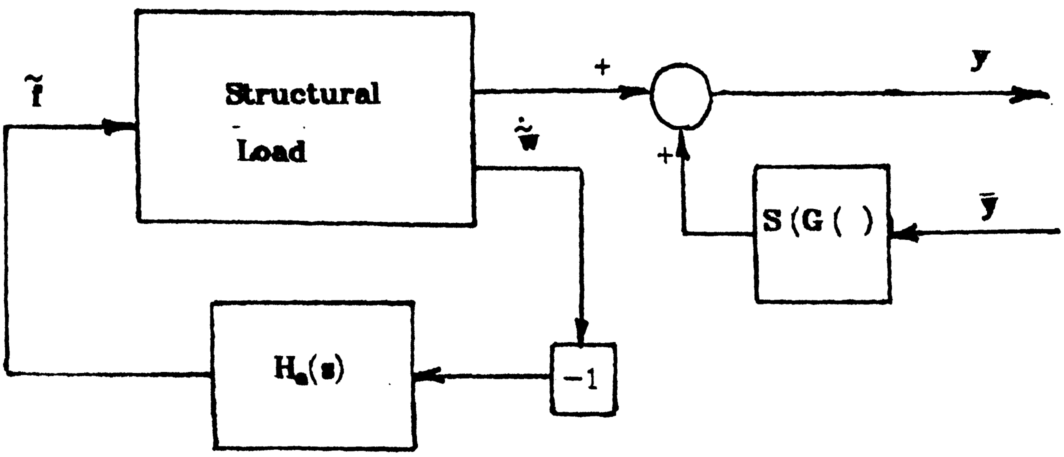

Figure 3

Based on condition (a) it can be shown that the structural load dynamics indicated in Figure 3 are positive. By condition (b), the effective actuator dynamics are strictly positive. On the basis of a standard argument in systems theory it follows that the closed loop, as a feedback connection of a positive system and a strictly positive system, is necessarily globally asymptotically stable. The details of the proof are given in Appendix B.

Conditions (a) and (b) in the Theorem constitute a set of sufficient conditions for stability of the closed loop. Note that condition (a) is a weak assumption about the qualitative properties of the structural load. Condition (b) depends only on the effective actuator compliance matrix K_h^{-1} and the feedback gain matrices of G^d and G^f. Thus the diagonal compliance matrix K_h^{-1} summarizes the actuator dynamic characteristics in a simple way. Note also that condition (b) is independent of the specific characteristics of the structural load. Thus, if the actuators and feedback gains are selected so that condition (b) is satisfied, it follows that the closed loop is globally asymptotically stable for **any** incremental positive stiff structure. This robustness property is particularly desirable since, as mentioned, detailed characteristics of the structural load are not often known. Conceptually, acceleration feedback could be used in place of force feedback, and stabilization conditions could be developed using positivity arguments. However, it turns out that the stabilization conditions for such feedback gains do depend on the properties of the structural load. Thus displacement and acceleration feedback does not have the same robustness property. Consequently, in applications where the structural load is uncertain a strong case is made that force (or pressure) feedback is preferred to acceleration feedback.

Condition (c) in the Theorem indicates that the feedforward function $G:R^m \to R^m$, should be selected as the inverse of the defined function $S:R^m \to R^m$, in order that the steady state error be zero for any constant displacement command. Such a specification of the feedforward control depends on the feedback gain matrices G^d and G^f and on the flexible structural restoring force function $F:R^n \to R^n$. Although closed loop stabilization can be achieved independently of the structural load characteristics, accurate displacement control of the structural load does require detailed knowledge of the structural restoring forces.

An especially important practical case is where the control relation (9) is based on decentralized feedback, that is the i^{th} electrohydraulic servo valve input v_i depends only on feedback of w_i and f_i for each $i = 1,\ldots,m$. Thus G^d and G^f are diagonal matrices

$$G^d = \text{diag}(g_1^d,\ldots,g_m^d) \tag{15}$$

$$G^f = \text{diag}(g_1^f,\ldots,g_m^f) \ . \tag{16}$$

Such decentralized feedback control is especially easy to implement since control of the i^{th} electrohydraulic servo valve depends only on local feedback of displacement and force. For such decentralized feedback condition (b) of the Theorem can be written in terms of simple inequalities. This result is expressed as follows.

Corollary. Suppose that

(a) M is symmetric and positive definite, and $F:R^n \to R^n$ has incremental positive stiffness:

(b) the inequalities

$$k_h^i g_i^f > g_i^d > 0, \quad i = 1,\ldots,m$$

are satisfied where

$$\frac{1}{k_h^i} = \frac{V_i}{B_i A_i^2} + \frac{1}{k_f^i}, \quad i = 1,\ldots,m \quad .$$

Then the closed loop, defined by equations (10) and (11), with decentralized feedback gain matrices as in (15) and (16), is globally asymptotically stable, and for any constant displacement command vector $\bar{y}$

$$y(t) \rightarrow S(G(\bar{y})) \quad \text{as} \quad t \rightarrow \infty$$

for each initial state.

If, in addition,

(c) $S(G(y)) = y$ for all $y \in R^m$

then for any constant displacement command vector $\bar{y}$

$$y(t) \rightarrow \bar{y} \quad \text{as} \quad t \rightarrow \infty \ ,$$

for each initial state.

The Corollary follows since the inequalities in condition (b) can be shown in guarantee satisfaction of condition (b) of the Theorem. (see Appendix C for proof).

Selection of particular values of the gain matrices G^d and G^f and the feedforward function $G:R^m \rightarrow R^m$ can be made using, in part, the guidance suggested by the above theoretical results.

If the feedforward controller suggested above cannot be used due to ignorance of the structural restoring force function then the controller may be based on explicit feedback of the error $\bar{y} - y$, where $C = B^T$ is assumed. In such a case the feedforward function can be chose as the linear function $G(y) = G^d y$ so that the controller is given by

$$u = G^d(\bar{y} - y) - G^f f \quad .$$

Selection of the control gain matrices G^d and G^f, as before, does not depend on explicit knowledge of the structural load to satisfy the stabilization condition. Note that if G^d and G^f are chose to be diagonal then the controller is completely decentralized. However, for this controller based on error feedback there is a steady state control error given by $\bar{y} - S(G^d \bar{y})$. Reduction of this steady state error can be achieved by use of high gain displacement feedback, i.e. "large G^d". But there is clearly a trade-off in selecting G^d to achieve some accuracy specification while also satisfying the stabilization condition.

5. Control of Multiple Structural Modes Using a Single Electrohydraulic Servo Actuator

In this section a particular case is considered where it is desired to control the displacement of a single point on the structural load using a single actuator. A schematic of such a configuration is shown in Figure 4. As a further simplification the flexible structural load is assumed to be elastic so that $F(x) = Kx$, where K is symmetric and nonnegative definite. Since the structural load is assumed to be elastic the feedforward function $G: R^1 \to R^1$ can be taken as a linear function $G(y) = g^s y$, where g^s is a feedforward gain parameter.

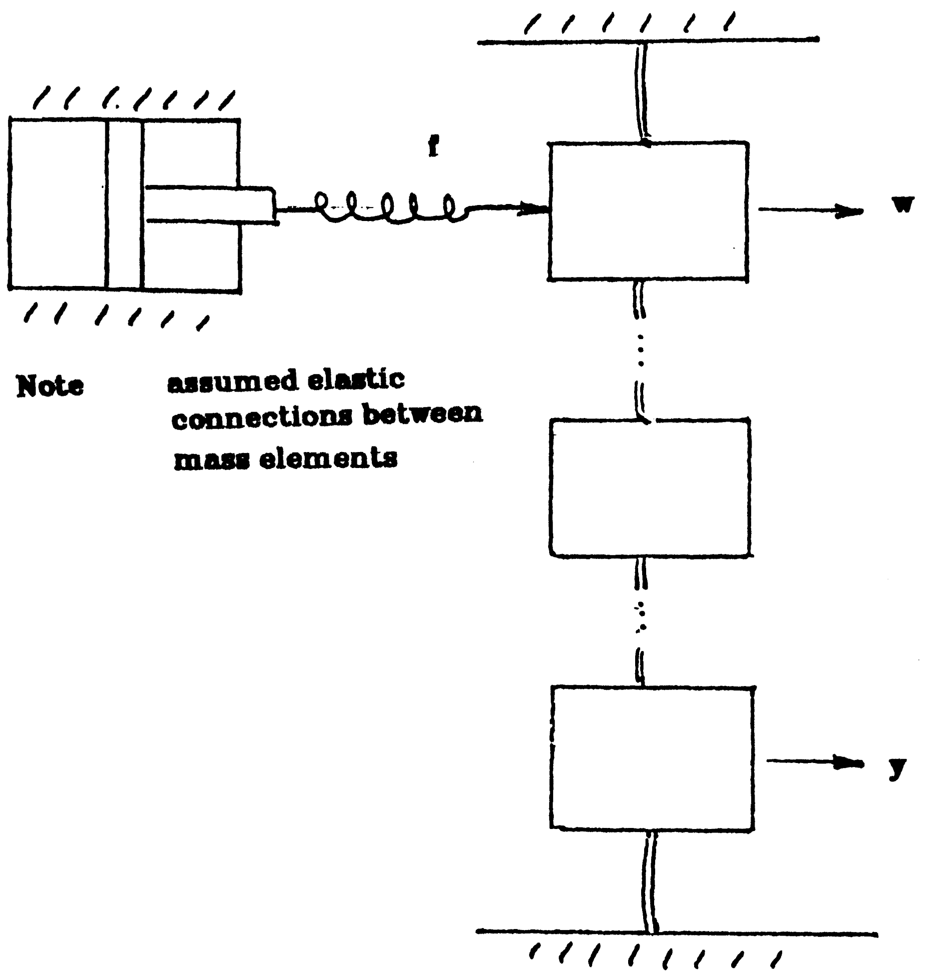

Figure 4

Thus the closed loop is described by the equations

$$M\ddot{x} + Kx = Bf$$

$$\frac{1}{k_h}\dot{f} = -B^T\dot{x} + g^s\bar{y} - g^d B^T x - g^f f$$

where f, $\bar{y}$, $B^T x$ are scalar valued and g^s, g^d, g^f are scalar gain parameters.

In this case, the closed loop is globally asymptotically stable if

$$k_h g^f > g^d > 0 \ .$$

If, in addition, the feedforward gain parameter is

$$g^* = \frac{g^f}{C\left[K + B\left(\frac{g^d}{g^f}\right)B^T\right]^{-1} B}$$

then for any command displacement vector $\bar{y}$

$$y(t) \to \bar{y} \text{ as } t \to \infty \ ,$$

for each initial state.

It is clear that selection of the feedback gain parameters to satisfy the stabilization condition does not depend on knowledge of the structural load, while the indicated choice of the feedforward gain parameter to achieve zero steady state error does require explicit knowledge of the stiffness of the structural load.

To illustrate the above special case consider an example.

Example 1. A simple example is shown in Figure 5, where it is desired to control the displacement of the second mas m_2 using a single electrohydraulic actuator, based on feedback of the displacement x_1 of the first mass and of the force f between the actuator and the first mass. The equations describing the closed loop are

$$m_1\ddot{x}_1 + k_1(x_1 - x_2) = f$$

$$m_2\ddot{x}_2 + k_1(x_2 - x_1) + k_2x_2 = 0$$

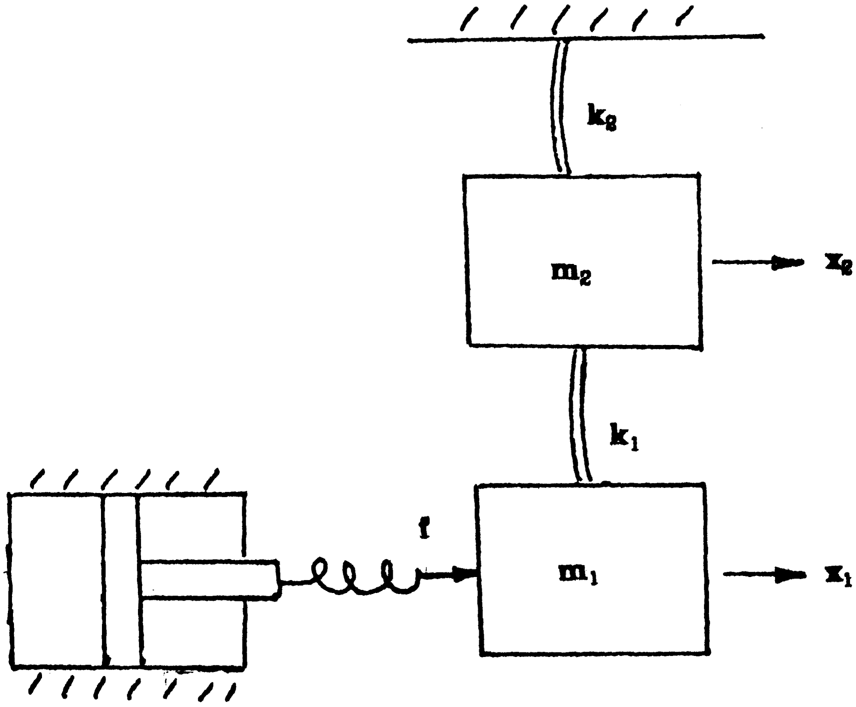

Figure 5

$$\frac{1}{k_h}\dot{f} = -\dot{x}_1 + g^s\bar{y} - g^d x_1 - g^f f \quad .$$

In this particular case the closed loop is stable if the feedback gain parameters satisfy

$$k_h g^f > g_d > 0 \quad .$$

If, in addition, the feedforward gain parameter is

$$g^s = g^d\left(1 + \frac{k_2}{k_1}\right) + g^f k_2$$

then for any command displacement $\bar{y}_2$

$$x_2(t) \to \bar{y}_2 \text{ as } t \to \infty \quad ,$$

for each initial state.

6. Control of Multiple Structural Modes

Using Equal Number of Electrohydraulic Servo Actuators

In this section it is desired to control the n-vector x of structural load displacements using n electrohydraulic servo actuators. A schematic of such a configuration is shown in Figure 6. As a further simplification the flexible structural load is assumed to be elastic so that $F(x) = Kx$, where K is

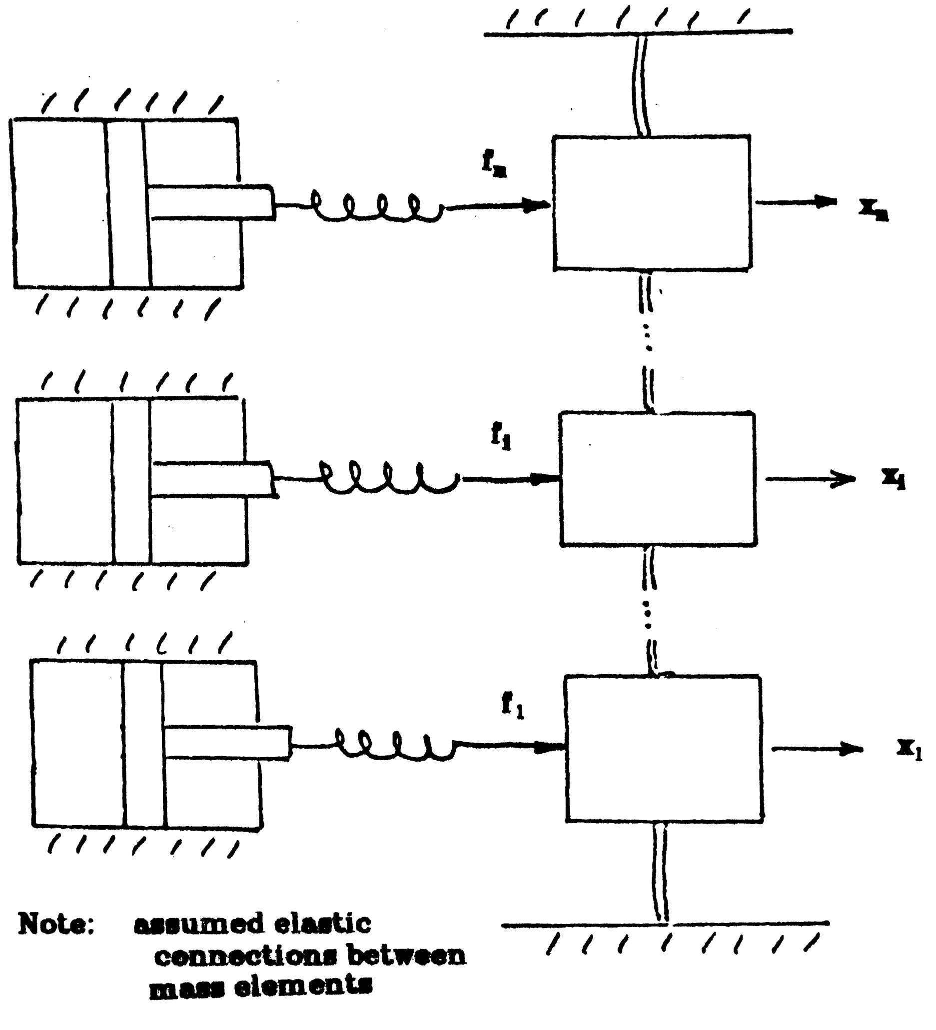

Figure 6

symmetric and nonnegative definite matrix. Since the structural load is assumed to be elastic the feedforward function $G: R^n \to R^n$ can be taken as a linear function $G(y) = G^s y$, where G^s is an $n \times n$ feedforward gain matrix.

Thus the closed loop is described by the equations

$$M\ddot{x} + Kx = f$$

$$K_h^{-1}\dot{f} = \dot{x} + G^s\bar{y} - G^d x - G^f f$$

where f, $\bar{y}$, are n-vectors, and decentralized feedback is used so that

$$G^d = \text{diag}\,(g_1^d, \ldots, g_n^d)$$

$$G^f = \text{diag}\,(g_1^f, \ldots, g_n^f)$$

In this case the closed loop is stabilized if the feedback gain parameters satisfy

$$k_h^i\, g_i^f > g_i^d > 0\ , \quad i = 1, \ldots n\ .$$

If, in addition, the feedforward gain matrix

$$G^s = G^d + G^f K$$

then for any command displacement vector $\bar{y}$

$$x(t) \to \bar{y} \quad \text{as} \quad t \to \infty ,$$

for each initial state.

It is clear that selection of the feedback gain parameters to satisfy the stabilization condition does not depend on knowledge of the structural load, while the indicated choice of the feedforward gain matrix to achieve zero steady state error does require explicit knowledge of the stiffness of the structural load. Note that, in general, the feedforward gain matrix is not diagonal, so that the controller is decentralized only in terms of the feedback. If the feedforward gain matrix were constrained to be diagonal then, in general, there would be a nonzero steady state error.

Example 2. Another example is shown in Figure 7, where it is desired to control, independently, the displacements of each of the two masses using two electrohydraulic actuators, based on decentralized feedback of the two displacements x_1 and x_2 of the two masses and feedback of the two forces f_1 and f_2 between the actuators and the masses. The equations describing the closed loop are

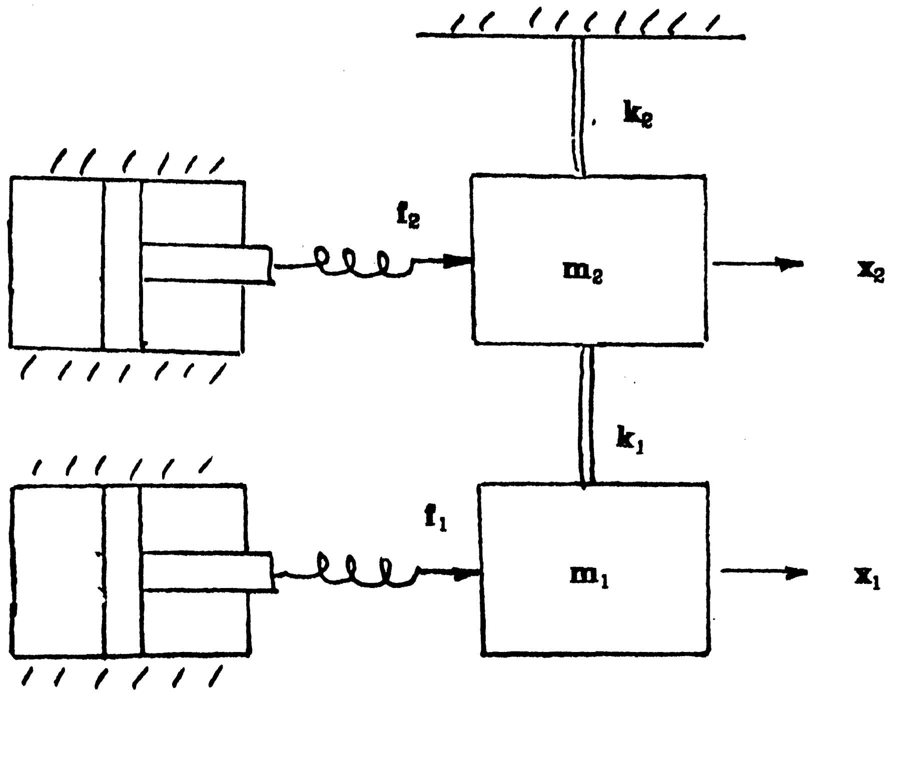

Figure 7

$$m_1\ddot{x}_1 + k_1(x_1 - x_2) = f_1$$

$$m_2\ddot{x}_2 + k_1(x_2 - x_1) + k_2x_2 = f_2$$

$$\frac{1}{k_h^1}\dot{f}_1 = -\dot{x}_1 + g_{11}^e\bar{y}_1 + g_{12}^e\bar{y}_2 - g_1^d x_1 - g_1^f f_1$$

$$\frac{1}{k_h^2}\dot{f}_2 = -\dot{x}_2 + g_{21}^e\bar{y}_1 + g_{22}^e\bar{y}_2 - g_2^d x_2 - g_2^f f_2 \ .$$

In this particular case the closed loop is stable if the feedback gain parameters satisfy

$$k_h^1 g_1^f > g_1^d > 0$$
$$k_h^2 g_2^f > g_2^d > 0 \ .$$

If, in addition, the feedforward gain parameters

$$g_{11}^e = g_1^d + g_1^f k_1$$
$$g_{12}^e = -g_1^f k_1$$

$$g_{21}^e = -g_2^f k_1$$
$$g_{22}^e = g_2^d + g_2^f (k_1 + k_2)$$

then for any command displacements $\bar{y}_1$ and $\bar{y}_2$

$$x_1(t) \to \bar{y}_1$$
$$x_2(t) \to \bar{y}_2 \quad \text{as} \quad t \to \infty \ ,$$

for each initial state.

7. Conclusions

A general approach to the displacement control of flexible structures using electrohydraulic servo actuators has been suggested. Under standard assumptions, mathematical models have been developed to describe the closed loop.

It is demonstrated, in terms of the developed theory, that use of decentralized force (or pressure) feedback and displacement feedback can stabilize any structure with incremental positive stiffness, if the feedback gains are chosen to satisfy certain simple inequalities. This result is significant, since a similar result for robust stabilization is not true if control is based on acceleration feedback instead of the force feedback. Careful attention has also been given to a proper selection of the feedforward function to achieve zero steady state error; but the suggested feedforward function does explicitly depend on the nonlinear structural load characteristics.

Specific details for choosing the values of the feedback gain parameters have not been presented here, but it is felt that the general framework developed here is an essential step before addressing specific parametric design issues.

8. Acknowledgements

Suggestions and comments by Lael Taplin and Yehia El-Ibiary of Sperry-Vickers, Troy, Michigan, are gratefully acknowledged.

9. Appendix A

Consider the abstract feedback relations

$$e = r - Gu$$
$$u = He \quad .$$

The following system theoretic results are will known [5,6,7].

Theorem A1. Let $\mathfrak{H} = L_2\,[0,\infty)$ be the Hilbert space of vector valued, square integrable, functions defined on $[0,\infty)$ and let $\mathfrak{H}_e$ be its extension. Let G and H be abstract causal operators which map $\mathfrak{H}_e$ into $\mathfrak{H}_e$ and $\mathfrak{H}$ into $\mathfrak{H}$. Let $r \in \mathfrak{H}$. If there exists $\delta > 0$ such that

$$\int_0^T f^T(t)H[f](t)dt \;\geq\; \delta \int_0^T f^T(t)f(t)dt$$

$$\int_0^T f^T(t)G[f](t)dt \;\geq\; 0$$

for all $f \in \mathfrak{H}_e$ and for all $T \geq 0$, then $e \in \mathfrak{H}$.

Theorem A2. Let $\mathfrak{H}$ and $\mathfrak{H}_e$ be as above and suppose

$$H[f](t) = \int_0^\infty h(t-\tau)f(\tau)d\tau \quad \text{for all } f \in \mathfrak{H}_e$$

where h is a square impulse response matrix, zero for negative arguments, with elements that are Fourier transformable. Then H satisfies the above strict positivity conditions if its Fourier transform, denoted by $H(j\omega)$, is strictly positive real, i.e. if

$$\tfrac{1}{2}\Big[H(j\omega) + H^*(j\omega)\Big] \geq \delta > 0 \text{ for all } \omega \;.$$

10. Appendix B

The proof of the main Theorem is given:

Define $\bar{x} = \mathfrak{H}(G(\bar{y}))$ and $\bar{f} = \mathfrak{F}(G(\bar{y}))$, and $\tilde{x} = x - \bar{x}$, $\tilde{f} = f - \bar{f}$.

Then equations (10) and (11) can be written as

$$M\ddot{\tilde{x}} + \tilde{F}(\tilde{x}) = B\tilde{f}$$

$$K_h^{-1}\dot{\tilde{f}} = -B^T\dot{\tilde{x}} - G^d B^T \tilde{x} - G^f \tilde{f} \ .$$

and

$$y = C\tilde{x} + S(G(\bar{y})) \ ,$$

where $\tilde{F}(\tilde{x}) = F(\bar{x} + \tilde{x}) - F(\bar{x})$. Also define $\tilde{w} = B^T\tilde{x}$. Then the equivalent black diagram of Figure 3 is obtained where the structural load dynamics are defined by the nonlinear equation

$$M\ddot{\tilde{x}} + \tilde{F}(\tilde{x}) = B\tilde{f}$$

$$\dot{\tilde{w}} = B^T\dot{\tilde{x}} \ .$$

The effective actuator dynamics are defined by the linear equation

$$K_h^{-1}\dot{\tilde{f}} = -\dot{\tilde{w}} - G^d\tilde{w} - G^f\tilde{f}$$

which has the transfer function matrix, from $\dot{\tilde{w}}$ to $-\tilde{f}$, given by $H_a(s)$ as defined in the Theorem. In the notation of Theorem A1 $\dot{\tilde{w}} = G[\tilde{f}](t)$ defines the nonlinear structural dynamics and $-\tilde{f}(t) = H[\dot{\tilde{w}}](t)$ defines the linear actuator dynamics.

Now, by condition (b) of the Theorem and Theorem A2 it follows that the operator H is strictly positive.

Now considering the operator G define the function

$$V(\tilde{x}) = \tfrac{1}{2}(\dot{\tilde{x}}^T M\dot{\tilde{x}} + Q(\tilde{x}))$$

so that

$$\frac{dV}{dt}(\tilde{x}) = \dot{\tilde{w}}^T\tilde{f} \ .$$

Thus

$$V(\tilde{x}(T)) = \int_0^T \tilde{f}^T(t) G[\tilde{f}](t)dt \quad .$$

Since M is symmetric and positive definite and $F:R^n \to R^n$ has incrementally positive stiffness it follows that $V(\tilde{x}(T)) \geq 0$; thus the operator G is positive.

Hence, from Theorem A1 it follows that if $r \in \mathfrak{H}$ then $e \in \mathfrak{H}$. As described in [7] this guarantees that $C\tilde{x}(t) \to 0$ as $t \to \infty$ for each initial state. Hence, $y(t) \to S(G(\bar{y}))$ as $t \to \infty$ for each initial state.

11. Appendix C

The proof of the Corollary is given:

Assume that G^d and G^f are diagonal as in (15), (16).

Since K_h^{-1} is diagonal then

$$H_a(s) = \text{diag}\left[\frac{k_h^1(s + g_1^d)}{s(s + k_h^1 g_1^f)}, \; \cdots, \; \frac{k_h^m(s + g_m^d)}{s(s + k_h^m g_m^f)}\right]$$

and

$$\tfrac{1}{2}\left[H_a(j\omega) + H_a^*(j\omega)\right] = \text{diag}\left[\frac{k_h^1(k_h^1 g_1^f - g_1^d)}{\omega^2 + (k_h^1 g_1^f)^2}, \; \cdots, \; \frac{k_h^m(k_h^m g_m^f - g_m^d)}{\omega^2 + (k_h^m g_m^f)^2}\right]$$

is thus strictly positive real from condition (b) of the Corollary; thus the Corollary follows from the Theorem.

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