RSD-TR-8-83

VIBRATION CONTROL OF FLEXIBLE STRUCTURES USING ELECTRO-MECHANICAL TRANSMISSION TYPE ACTUATORS[1]

N. Harris McClamroch
Department of Aerospace Engineering
The University of Michigan
Ann Arbor, Michigan 48109

May 1983

Center for Robotics and Integrated Manufacturing

Robot Systems Division
COLLEGE OF ENGINEERING
THE UNIVERSITY OF MICHIGAN
ANN ARBOR, MICHIGAN 48109

[1]This work was supported by the Charles Stark Draper Laboratory, Inc., Cambridge, Mass. under DARPA Contract No. F30602-81-C-0180.

Abstract

For certain classes of flexible structures, e.g. large flexible space structures, active methods for elimination or reduction of structural vibrations are required. Although much theoretical research on vibration control of flexible structures has been done, the important effects of physical transducers, especially actuators, in achieving the vibration control objectives have generally been ignored.

We consider a certain class of electromechanical actuators, where control forces are generated by servo motors and transmitted to the flexible structure; such electromechanical actuators are termed transmission type actuators. They serve as adequate models for many actuation devices, including so-called member dampers.

Our mathematical development focuses on lumped flexible structures controlled by multiple actuators. The key feature in the development is the modification of the dynamics of the flexible structure (alone) by the presence of the actuators.

The feasibility of such actuators used as vibration control devices is considered. A simple form of decentralized control of the actuators is presented, where the objective is to add effective damping to the structural modes. The decentralized control consists of internal compensation of each actuator plus feedback of structural velocities. Conditions for stabilization of the closed loop are presented. This feedback control scheme is illustrated by examining a generic example of a single vibration mode controlled by a single actuator; computations for this specific example demonstrate the limitation in damping that can be achieved.

1. INTRODUCTION

Many future spacecraft are expected to include large flexible structures; in order to meet stringent performance requirements in spite of environmental and other disturbances methods for elimination or reduction of structural vibration have been extensively studied. Although passive vibration control methods are effective in some cases, active vibration control technology may be necessary where high performance (e.g. pointing accuracy) of a spacecraft is required. Our emphasis here is on use of active feedback control for suppression of structural vibrations. There is a substantial literature on methods for achieving vibration control for large space structures; summaries of much of this work are available in the C. S. Draper Laboratory reports [1-3] and the survey articles [4,5].

The published literature on vibration control of large space structures is concerned, in the main, with the theoretical issues associated with the design of control laws or control strategies. Numerous suggestions have been made regarding the form of suitable control laws and the selection of control law parameters. Such investigations have generally based the design and analysis of control laws solely on the properties of the flexible structure to be controlled and on the vibration control objectives. The important effects of physical transducers, namely actuators and sensors, in achieving the vibration control objectives have been ignored. Of course, research has continued on the development of transducer hardware suitable for use in large space structure applications [6,7]. The objective of this paper is to reconcile the work on vibration control of large space structures (where transducer effects have generally been ignored) with the work on transducers (where application to the particular vibration control objectives have generally been ignored). Although our development will focus solely on the role of actuator transducers, a similar development likely holds with regard to the role of sensors. Thus, our main emphasis in this work is consideration of the role of actuators as an important and necessary part of the problem of vibration control of flexible structures.

Our work also has direct relevance to other technical areas where control of flexible structures is an important issue. In particular, we refer to several applications concerned with control of flexible structures through tensioned cables; such applications have appeared in control of civil engineering structures [8-10] and in control of manipulators [11-12].

In order for our conclusions to be as general as possible the detailed nature of actuator hardware is not considered. Rather our interest is in the dynamic effect that the actuators have on the flexible structure, with respect to the vibration control objectives. Thus the content of the following sections is concerned with the mathematical characterization of the actuators, the flexible structure and the feedback controller. This generality allows us to develop qualitative insight into the role of actuators in vibration control of flexible structures, for a wide variety of physical actuator implementations.

2. MODELLING

2.1. Actuators

Actuators are considered to represent the physical devices whereby forces and moments are actually applied to a flexible structure. An actuator can be characterized in terms of the physical nature of its input and output signals. Throughout, an actuator input will be considered to be a voltage signal; the actuator output will be considered to be a suitably defined generalized force or moment. If the structure is a truss connection of structural members, an actuator may be configured to provide a force at a joint connection of several members, an axial force along a member, a moment at a joint, or a bending moment on a member. If the structure is defined in terms of individual mass elements, an actuator may be configured to provide forces or moments on individual elements or between elements. Since the mathematical characterizations of all of these different physical actuator configurations are similar it suffices to consider the output of the actuator to be a suitably defined generalized force applied to the structure.

All electromechanical actuators consist of an electrical subsystem and a mechanical subsystem with some electromechanical interaction. The electrical subsystem is assumed to be of a simple resistive form so that its associated dynamics can be ignored. The mechanical subsystem is assumed to consist of an actuator inertia and stiffness that are significant in defining the actuator dynamics. The electromechanical interaction provides a force on the actuator inertia of electrical origin, with a resultant force being transmitted to the structure.

2.2. Modelling Assumptions

In this section the basic assumptions which hold throughout the paper are stated.

Although a flexible structure can often be described by a distributed mass model using partial differential equations, our development will make use of a lumped model for the structure. Thus the second order vector differential equation

$$M\ddot{x} + D\dot{x} + Kx = Bf \tag{2-1}$$

is chosen to represent the model for structural dynamics. Here x denotes the n-vector of generalized structural displacements, so the $\dot{x}$ and $\ddot{x}$ are structural velocity and acceleration vectors. The $n \times n$ mass matrix M is assumed to be symmetric and positive definite. The $n \times n$ structural stiffness matrix K is assumed to be symmetric; since our interest is in vibration motion rather than rigid body motion K is assumed to be positive definite. The $n \times n$ damping matrix D is often rather arbitrarily chosen since there is sparse theory for structural damping available for guidance. In this work, the damping matrix D is assumed to be symmetric and non-negative definite; the common choice $D=0$ is allowed. The right hand side of equation (2-1) characterizes the influence of the actuator forces on the structure; the m-vector f denotes the generalized actuator force vector while B is an $n \times m$ dimensionless influence matrix. The specific form of the actuator force vector is examined in considerable detail.

2.3. Model for Actuators and Structure

The basis for the development of the mathematical model of the actuators and structure is the schematic diagram in Figure 1. The flexible structure is assumed to be represented by a lumped mass model of which a single mass element is shown in Figure 1. Each electromechanical actuator is characterized by an actuator inertia m_a^i, and corresponding generalized displacement z_i of the actuator inertia element; the actuator force f^i is transmitted to the structure via an elastic transmission medium with stiffness k_a^i; internal damping in each actuator, with damping parameter c_a^i, is also assumed. The displacement of the structure at the location of application of the force f^i is given by the i^{th} component of $B^T x$.

In order to obtain an explicit characterization of the actuator force vector f in equation (2-1), the Lagrangian function for the actuators and structure is

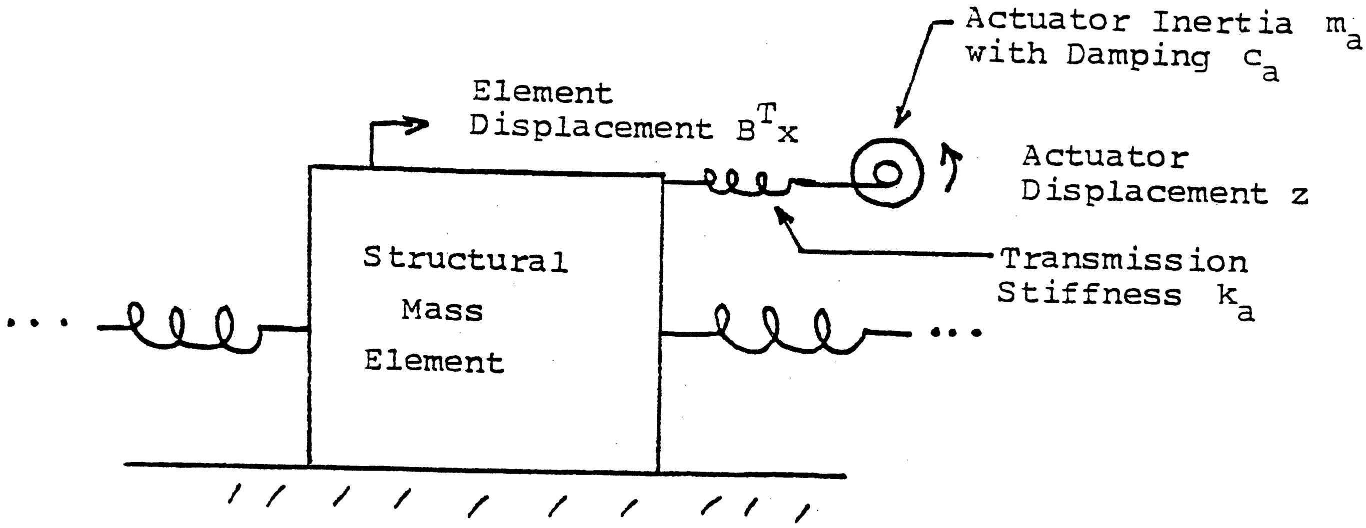

Figure 1. Actuator and Mass Element Interaction

$$L = \frac{1}{2}\dot{x}^T M \dot{x} + \frac{1}{2}\dot{z}^T M_a \dot{z} - \frac{1}{2}x^T K x - \frac{1}{2}(z - B^T x)^T K_a (z - B^T x) \qquad \textbf{(2-2)}$$

where the generalized actuator displacement vector $z = (z_1, \ldots, z_m)$ and the actuator inertia and stiffness matrices are

$$M_a = \ diag\ (m_a^1, \ldots, m_a^m)$$

$$K_a = \ diag\ (k_a^1, \ldots, k_a^m).$$

From Lagrange's equations obtain

$$M\ddot{x} + Kx - BK_a(z - B^T x) = Q_1 \qquad (2\text{-}3)$$

$$M_a \ddot{z} + K_a(z - B^T x) = Q_2 \qquad (2\text{-}4)$$

The generalized forces Q_1 and Q_2 are

$$Q_1 = -D\dot{x}$$

$$Q_2 = -C_a \dot{z} + B_a u$$

where $u = (u_1, \ldots, u_m)$ denotes the vector of actuator input voltages,

$$C_a = \text{diag} \; (c_a^1, \ldots, c_a^m)$$

and

$$B_a = \text{diag} \; (b_a^1, \ldots, b_a^m).$$

In summary, the mathematical model for the actuators and structure is given by

$$M\ddot{x} + D\dot{x} + Kx = BK_a(z - B^T x) \tag{2-5}$$

$$M_a \ddot{z} + C_a \dot{z} + K_a(z - B^T x) = B_a u. \tag{2-6}$$

Thus in comparison with equation (2-1) the actuator force vector is given by

$$f = K_a(z - B^T x). \tag{2-7}$$

An important feature of this mathematical model is that the actuator force vector f depends not only on the generalized actuator displacement z but also on the generalized structural displacement $B^T x$ at the locations of the actuators. Moreover, since equations (2-5) and (2-6) are inherently coupled neither the actuators nor the structure can be completely considered alone.

The above mathematical model should adequately characterize a large class of flexible structures controlled by multiple electromechanical actuators; the relevant assumptions on which equations (2-5), (2-6), (2-7) were derived are now explicitly stated. The structural dynamics are characterized by a lumped mass and stiffness model with proportional velocity damping; the dynamics of the structure are assumed to be linear and only flexible modes are included in the model, i.e. the rigid body motion of the structure is not included. The actuators are of electromechanical type and are connected to a single mass element of the structure through an elastic transmission medium with finite stiffness. The actuator dynamics are assumed linear, and the current voltage dynamic effects are ignored in comparison with the actuator inertia effects.

3. DECENTRALIZED CONTROL DESIGN VIEWPOINT

3.1. Approach

A natural approach to the design of a controller is to develop a controller for the flexible structure alone, ignoring all actuator dynamics. It is then necessary to provide local compensation for each actuator as a means of justifying the original assumption. The controller is thus separated into two parts: a force controller, typically based on the structural dynamics only, and an actuator servo controller for each actuator. This a priori assumed controller configuration is referred to here as a decentralized controller. A schematic of a closed loop, incorporating a decentralized controller, is shown in Figure 2.

Our objective is to consider a general class of procedures for designing such decentralized control schemes. Since much attention has been focused on design of force controllers this issue is generally ignored here; for purpose of illustration a force controller which depends on structural velocity feedback only is subsequently employed. Our main attention is focused on design of the actuator controllers.

This decentralized control design approach is conceptually appealing. However, the equations which describe the flexible structure and actuators, (2-5), (2-6), (2-7), are inherently coupled. This inability to completely separate structural dynamics from actuator dynamics makes design of a decentralized

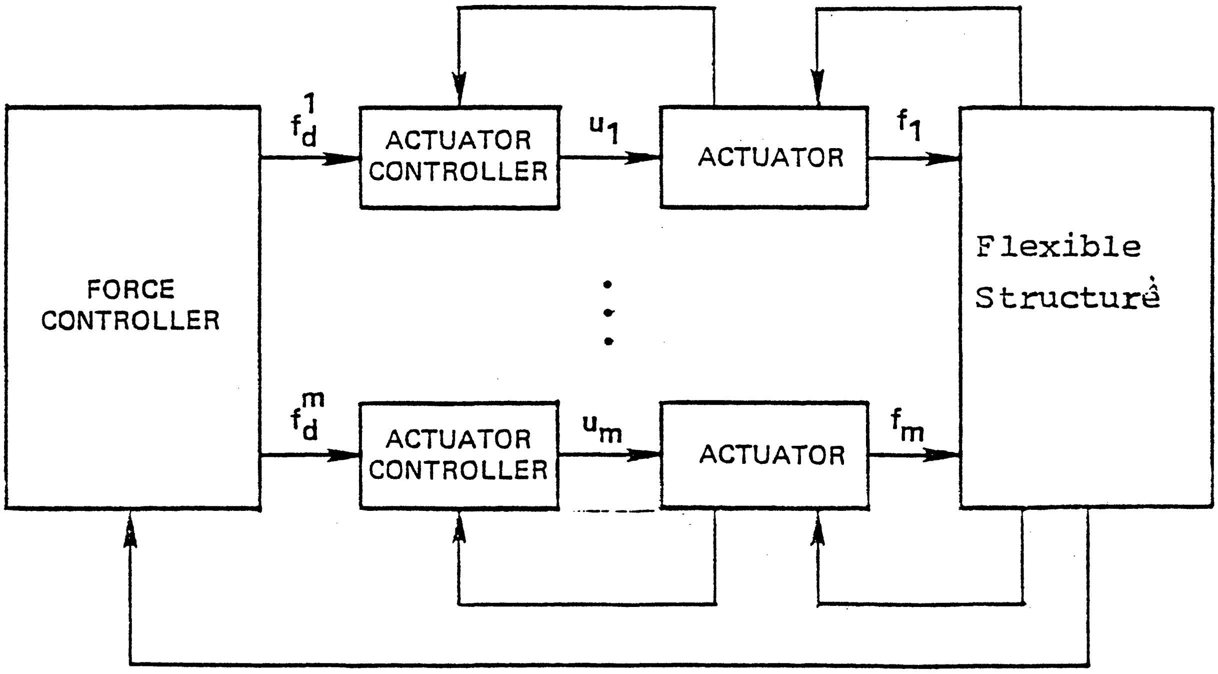

Figure 2. Decentralized Control Configuration

controller with guaranteed closed loop properties difficult. Hence, some attention is given to evaluation of closed loop stability properties.

3.2. The Actuator Controllers

For simplicity, a force controller which depends on co-located structural velocity feedback is assumed; such controllers are commonly suggested as a means of augmenting the structural damping using active feedback control [4,5]. Hence the command or desired actuator force vector f_d is given by

$$f_d = -C_d B^T \dot{x} \tag{3-1}$$

where $f_d = (f_d^1, \ldots, f_d^m)$ and C_d is the $m \times m$ feedback gain matrix, typically assumed to be symmetric and positive definite.

Our objective is to choose a servo controller for each actuator, the actuator controller, to cause the actual actuator force f to track the command actuator force f_d as closely as is possible. The decentralization constraint requires that control of the i^{th} actuator may depend only on f_d^i and local feedback of the z_i. Hence the actuator controllers may be viewed as integral to the actuators themselves; the claim is that such actuator controllers play a key role in achieving the closed loop structural vibration control objectives.

A basis for choosing an actuator controller is now developed. The structural transfer function from the actuator force vector f to the structural displacement vector $B^T x$ is

$$G_s(s) = B^T[Ms^2 + Ds + K]^{-1}B \tag{3-2}$$

and the transfer function from the actuator input voltage vector u to the actuator force vector f is

$$G_a(s) = [I + (M_a s^2 + C_a s)(K_a^{-1} + G_s(s))]^{-1}B_a. \tag{3-3}$$

Thus $G_a(s)$ represents the effective actuator transfer function, where the structural loading effects on the actuator are taken into account.

The actuator controller should be chosen to represent a decentralized and realizable approximation to the actuator inverse $G_a^{-1}(s)$. Although there may be many suitable forms for such an approximate system inverse, our subsequent development makes use of the actuator controller given by the transform relation

$$U(s) = B_a^{-1}[K_a + Cs]K_a^{-1}F_d(s) - B_a^{-1}C_f sZ(s) \tag{3-4}$$

where

$$C_f = diag\ (c_f^1, \ldots, c_f^m)$$

and

$$C = \; diag\; (c_a^1 + c_f^1, \ldots, c_a^m + c_f^m).$$

The above form for the actuator controller does satisfy the decentralization constraint since u_i depends only on f_d^i and $\dot{z}_i$. The actuator controller is realizable for a force controller given by (3-1) as

$$u = -B_a^{-1}C_dB^T\dot{x} - B_a^{-1}CK_a^{-1}C_dB^T\ddot{x} - B_a^{-1}C_f\dot{z} \qquad (3\text{-}5)$$

Thus the actuator controller can be realized using structural velocity feedback of $B^T\dot{x}$, structural acceleration feedback of $B^T\ddot{x}$, and decentralized actuator velocity feedback of $\dot{z}$.

Consequently, the effective transfer function from the command actuator force vector f_d to the actual actuator force vector f is given by

$$G_f(s) = [I + M_as^2(K_a + Cs)^{-1} + (M_as^2 + Cs)(K_a + Cs)^{-1}K_aG_s(s)]^{-1} \qquad (3\text{-}6)$$

This transfer function for the actuator, and actuator controller, is not in general diagonal due to the presence of the general nondiagonal structural transfer function $G_s(s)$ in the expression (3-6). Thus the effective actuator dynamics, including the actuator controller, involve crossfeed between the actuation channels, where the crossfeed effects are explicitly due to structural loading of the actuators.

The effective actuator transfer function $G_f(s)$ can be represented as a feedback connection of

$$K_a[M_as^2 + Cs][M_as^2 + Cs + K_a]^{-1}s^{-1}$$

and

$$sG_s(s).$$

Each of these transfer function matrices is positive real, with the former strictly positive real if $c_a^i + c_f^i > 0, i = 1, \ldots, m$. From results in [13,14], the actuator and actuator controller system, characterized by the transfer function matrix $G_f(s)$, is stable if $c_a^i + c_f^i > 0, i=1, \ldots, m$. Consequently, multiple actuators, of the electromechanical type considered, cannot be destabilized when connected to any flexible structure, in an open loop configuration.

3.3. The Closed Loop

The closed loop is now examined, where the force controller is given by (3-1), the actuator controller is given by (3-5) and the actuator and structure are described by (2-5), (2-6), (2-7). The closed loop system can be described by the second order vector equations

$$\begin{bmatrix} M & O \\ CK_a^{-1}C_dB^T & M_a \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{z} \end{bmatrix} + \begin{bmatrix} D & O \\ C_dB^T & C_a \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix}$$

$$+ \begin{bmatrix} K + BK_aB^T & -BK_a \\ -K_aB^T & K_a \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{3-7}$$

This set of closed loop equations does not have symmetric matrix coefficients so explicit conditions for closed loop stability cannot be stated. Nevertheless, equations (3-7) could form the basis for stability analysis of the closed loop.

An alternative description of the closed loop can be given in terms of transfer function matrices. The return difference function for the closed loop is given by

$$\det[I + G_f(s)G_p(s)] \tag{3-8}$$

where

$$G_p(s) = C_d s G_s(s).$$

Thus the closed loop is stable if and only if each zero of the return difference function has negative real part. Such necessary and sufficient conditions for closed loop stability are so complicated that little insight into the role of the actuator dynamics is obtained.

Simpler conditions, which do give explicit insight into the role of actuator dynamics on closed loop stability, can be developed using the robust stability results of [15]. Suppose that each zero of

$$det \;\; [I + G_p(s)] \tag{3-9}$$

has negative real part. If

$$\bar{\sigma}[G_f(j\omega) - I] \;<\; \underline{\sigma}[I + G_p^{-1}(j\omega)] \tag{3-10}$$

holds for all $\omega > 0$, the closed loop is guaranteed to be stable. Here $\bar{\sigma}[G]$ and $\underline{\sigma}[G]$ denote the maximum and minimum singular values of a matrix G, respectively.

The general assumption (3-9) is that the closed loop, ignoring all actuator dynamics, is stable; that is certainly the case if C_d is symmetric and positive definite [3]. The frequency condition inequality (3-10) has a simple interpretation and can be checked graphically [5]. The right hand side of (3-10) depends only on the structure dynamics and the force controller but not on the actuator dynamics; while the left hand side of (3-10) represents a measure of the variation of the compensated actuator dynamics from the ideal. Thus if the compensated actuator dynamics are sufficiently close to the ideal, as measured by (3-10), it is guaranteed that the actuator dynamics do not destabilize the closed loop. This sufficient condition for closed loop stability would, in general, be checked numerically; but it does expose in an explicit way the importance of proper compensation of the actuators.

In the case where multiple electromechanical actuators are used to control a flexible structure it can be shown that it is always possible to determine an actuator controller of the form (3-4) so that the closed loop is stable. Explicit guidelines for selection of the actuator parameters are difficult to give. It appears desirable that the actuator inertias be as small as possible while the stiffness of the elastic transmission medium be as large as possible. As is usual large actuator feedback gain parameters C_j^i may be required. Substantial trial and error may be required to select the actuator parameters effectively in a particular case.

3.4. Comments

The decentralized control design approach is to impose the a priori constraint that the controller consist of a force controller and a set of actuator controllers, one for each actuator, as shown in Figure 2. This approach is natural when the structure is viewed as the "plant" and the actuator dynamics are ignored; a controller obtained on the basis of this assumption is essentially a force controller. The actuator controllers can subsequently be developed.

The main advantage of this control design viewpoint is that, since all actuator dynamics are ignored in developing a force controller, there is a reduction in dimensionality and hence complexity of the control design problem. Such an order reduction is often desirable as a means of obtaining a computationally tractable control design problem. Specifically, this decentralized approach depends on obtaining a force controller based on n structural modes, plus m single loop actuator controllers, as opposed to the use of $n + m$ modes, i.e. $2(n + m)$ state variables, if a centralized control design approach is taken.

The major disadvantage with this approach is that there can be no guarantee of specific closed loop properties, e.g. stability. There is certainly the possibility that a closed loop system, with controller chosen to stabilize the closed loop ignoring actuator dynamics, may in fact be destabilized by actuator dynamics. Thus closed loop characteristics, where the controller is obtained using the decentralized design approach, should always be carefully analyzed. As indicated, a complete eigenvalue analysis of the closed loop can be performed using e.g. (3-7). An alternative, which often gives insight into the specific importance of the actuator parameters, is to make use of the robustness test as discussed.

4. VIBRATION CONTROL OF A TWO DEGREE OF FREEDOM SYSTEM USING AN ELECTROMECHANICAL ACTUATOR

4.1. The Example

The previously developed theory is now illustrated by examining in some detail a simple example of a two mass system, connected by a spring and damper, controlled by a single electromechanical actuator. A schematic of the system is given in Figure 3; the variable and parameter notation is also given in figure 3.

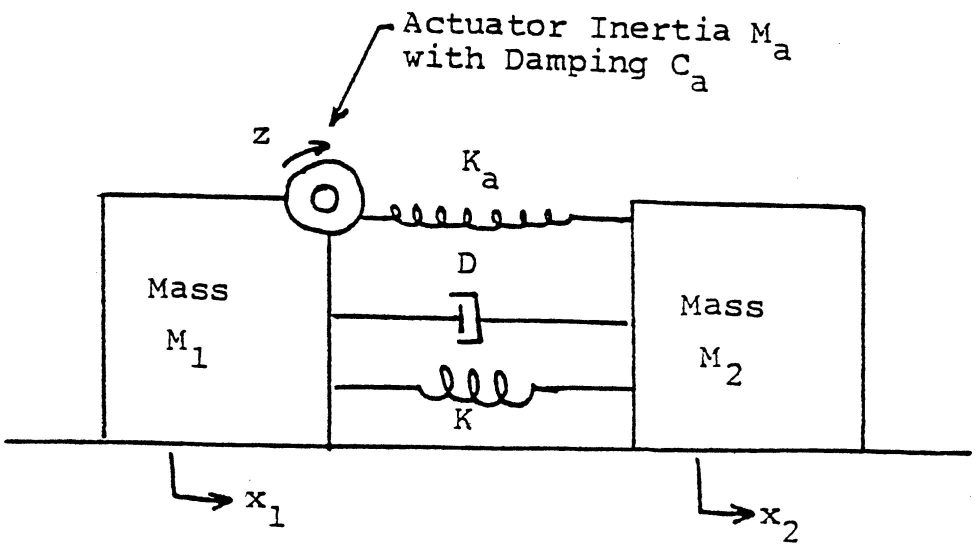

Figure 3. Two Mass Elements and One Actuator

Based on the assumptions made previously the spring mass-actuator system can be described by the equations

$$M_1\ddot{x}_1 + D(\dot{x}_1-\dot{x}_2) + K(x_1-x_2) = K_a(z-x_1+x_2)$$

$$M_2\ddot{x}_2 + D(\dot{x}_2-\dot{x}_1) + K(x_2-x_2) = -K_a(z-x_1+x_2)$$

$$M_a\ddot{z} + C_a\dot{z} + K_a(z-x_1+x_2) = B_a u$$

It is desirable to define the displacement $\bar{x}$ of the center of mass of the system and the relative displacement x between the two masses through

$$\bar{x} = \frac{M_1x_1 + M_2x_2}{M_1 + M_2}$$

$$x = x_1 - x_2.$$

Thus the equivalent equations

$$\ddot{\bar{x}} = O \qquad (4\text{-}1)$$

$$\ddot{x} + 2\zeta_s\omega_s\dot{x} + \omega_s^2 x = \rho\omega_a^2(z - x) \qquad (4\text{-}2)$$

$$\ddot{z} + 2\zeta_a\omega_a\dot{z} + \omega_a^2(z - x) = \beta_a u \qquad (4\text{-}3)$$

are obtained where the parameters are defined by

$$\omega_s^2 = \frac{K(M_1+M_2)}{M_1M_2}$$

$$2\zeta_s\omega_s = \frac{D(M_1+M_2)}{M_1M_2}$$

$$\omega_a^2 = \frac{K_a}{M_a}$$

$$\rho = \frac{M_a(M_1+M_2)}{M_1M_2}$$

$$2\zeta_a\omega_a = \frac{C_a}{M_a}$$

$$\beta_a = \frac{B_a}{M_a}$$

As seen from equation (4-1), the actuator does not influence the motion of the center of mass of the system. Thus our attention is focused on control of the relative vibration motion about the center of mass as described by equations (4-2) and (4-3). These equations are special cases of the previous equations (2-5) and (2-6).

Two different control laws are now examined and the corresponding closed loop characteristics are evaluated.

4.2. Vibration Control Without Relative Motion Feedback

First consider feedback control using only actuator velocity feedback, as given by

$$B_a u = -C_f \dot{z},$$

that is, the control law does not make explicit use of the relative velocity between the two masses. Such use of an actuator can be viewed as a passive damper [16]; the control law is especially simple to implement.

Define the damping parameter ζ_f from

$$2\zeta_f\omega_a = \frac{C_a+C_f}{M_a}.$$

The closed loop characteristic equation can be written as

$$1 + \frac{2\zeta_f\omega_a s\,[s^2+2\zeta_s\omega_s s+(1+\rho)\omega_s^2]}{(s^2+2\zeta_s\omega_s s+\omega_s^2)(s^2+\omega_a^2) + \rho s^2\omega_s^2} = 0. \qquad (4\text{-}4)$$

Using this characteristic equation, it is easily shown that the closed loop system is stable for any $\zeta_f > 0$. The structure can be stabilized by using only actuator velocity feedback as a passive damper. A root locus plot for (4-4), for a particular case, is shown in Figure 4; dependence of the closed loop poles on the actuator damping parameter ζ_f is indicated. The structure can be stabilized in this manner, but only modest damping of the closed loop can be achieved using feedback of the actuator velocity only. The maximum damping ratio for the dominant pole pair is 0.12, corresponding to selection of ζ_f = 0.78.

4.3. Vibration Control With Relative Motion Feedback

In order to achieve additional closed loop damping, feedback of the actuator velocity, plus feedback of the relative motion between the two masses, can be used. As developed previously in equation (3-5), such a controller is given by

$$B_a u = -C_f \dot{z} - C_d \dot{x} - \frac{(C_a + C_f)}{K_a} C_d \ddot{x}.$$

Define the damping parameter ζ_d from

$$2\zeta_d \omega_s = \frac{C_d(M_1 + M_2)}{M_1 M_2}$$

so that the effective actuator transfer function, based on (3-6), is

$$G_f(s) = \frac{(2\zeta_f \omega_a s + \omega_a^2)(s^2 + 2\zeta_s \omega_s s + \omega_s^2)}{(s^2 + 2\zeta_s \omega_s s + \omega_s^2)(s^2 + 2\zeta_f \omega_a s + \omega_a^2) + \rho\omega_s^2(s^2 + 2\zeta_f \omega_a s)}$$

and the transfer function $G_s(s)$, from (3-2), is given by

$$G_s(s) = \frac{1}{s^2 + 2\zeta_s \omega_s s + \omega_s^2} \quad .$$

Thus the closed loop characteristic equation is

$$1 + 2\zeta_d \omega_s s G_f(s) G_s(s) = 0 \quad . \tag{4-5}$$

The closed loop system can be shown to be stable for any $\zeta_f > 0$ and any $\zeta_d \geq 0$. A

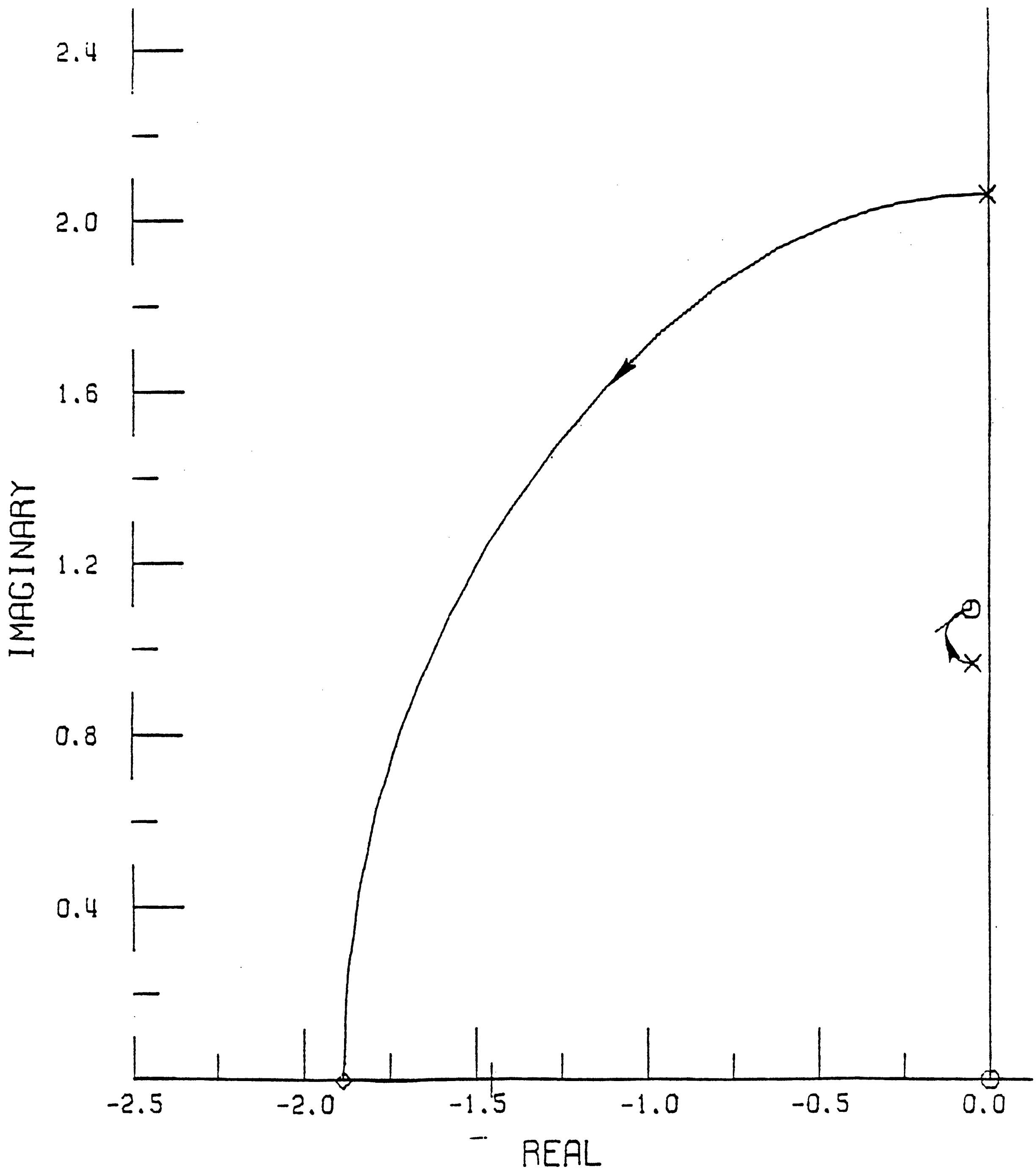

Figure 4. Closed Loop Root Locus: Control Without Relative Motion Feedback
Root Locus Parameter: ζ_f
Data: $\rho = 0.2, \omega_a^2 = 4.0,$
$\omega_s^2 = 1.0, \zeta_s = 0.05$,

root locus plot of (4-5), for the particular value $\zeta_f = 0.78$, is shown if Figure 5; dependence of the closed loop poles on the damping parameter ζ_d is indicated. It is clear that significant damping can be obtained using this actuator controller. As seen from Figure 5, there is a limit to the closed loop damping that can be obtained. If $\zeta_d = 0.32$ the damping ratios for the two pole pairs are 0.50 and 0.58.

Instead of a direct analysis of the closed loop poles an analysis of the closed loop using the robustness test (3-10) can be used. In this particular example the frequency response for the scalar actuator error function $|G_f(j\omega) - 1|$ is indicated in Figure 6 for the chosen values of the parameters. As an illustration the frequency response for the inverse return difference function $|1 - G_p^{-1}(j\omega)|$ is also indicated in Figure for the value $\zeta_d = 0.32$. Certain conclusions can be drawn from Figure 6. First, for the specific case where $\zeta_d = 0.32$, and in fact for any value $\zeta_d > 0$, the actuator dynamics do not destabilize the closed loop. Also, the closed loop is most sensitive to disturbances with frequency near 1.0 rad/sec, where the "loading peak" for the actuator error occurs. The low frequency stability margins are large since the actuator error is small. At high frequencies the actuator errors are relatively large, but good stability margin is maintained due to the force controller characteristics. These qualitative conclusions should be typical of a flexible structure controlled by a single transmission type actuator, using a properly chosen force controller. Experience has indicated that the magnitude of the load peak for the actuator error is strongly affected by the open loop structural damping; the robustness condition may not be satisfied if the open loop structural damping is sufficiently small.

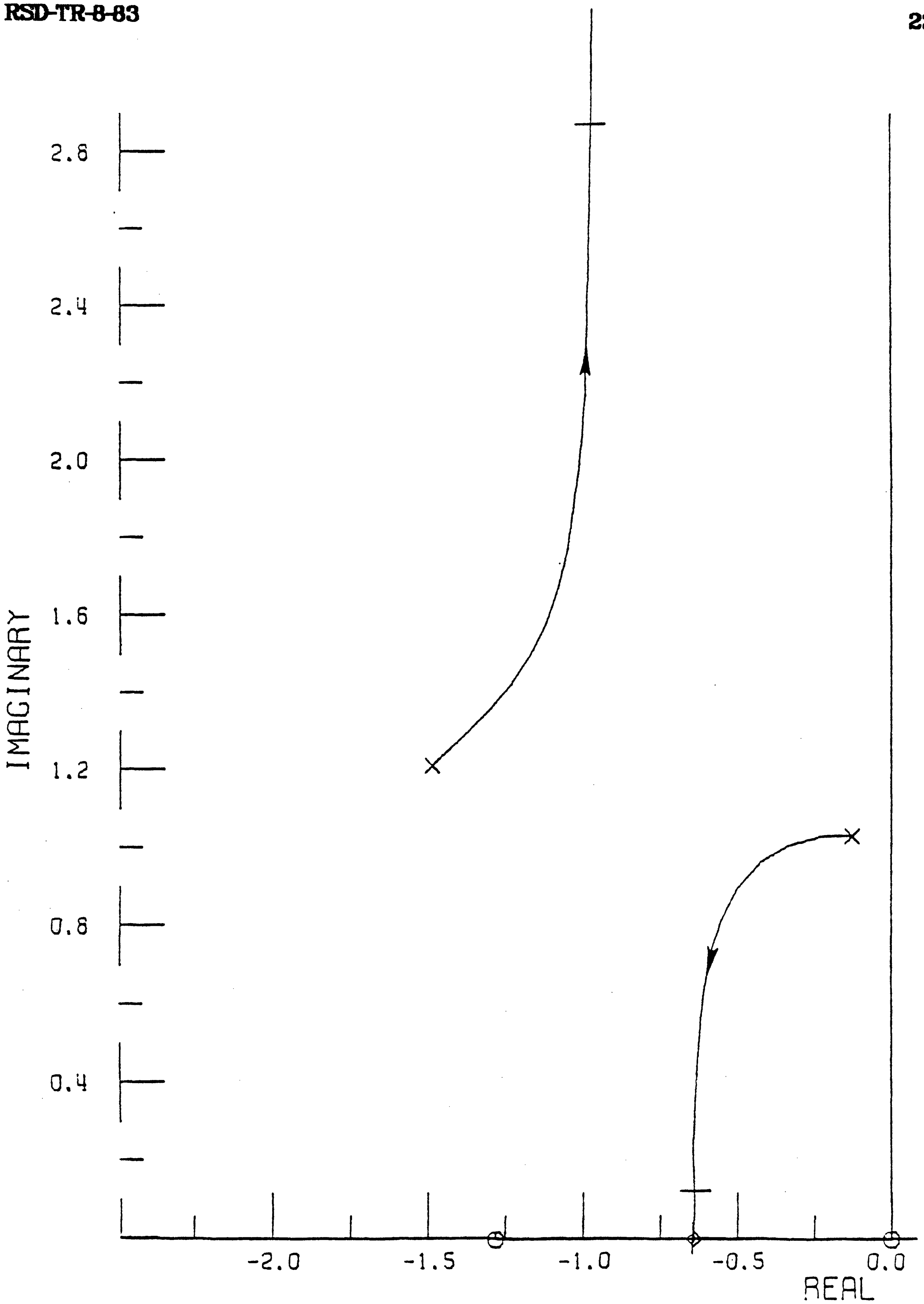

Figure 5. Closed Loop Root Locus: Control with Relative Motion Feedback
Root Locus Parameter: ζ_d
Data: $\rho = 0.2, \omega_a^2 = 4.0,$
$\omega_s^2 = 1.0, \zeta_s = 0.05,$
$\zeta_a = 0.78$

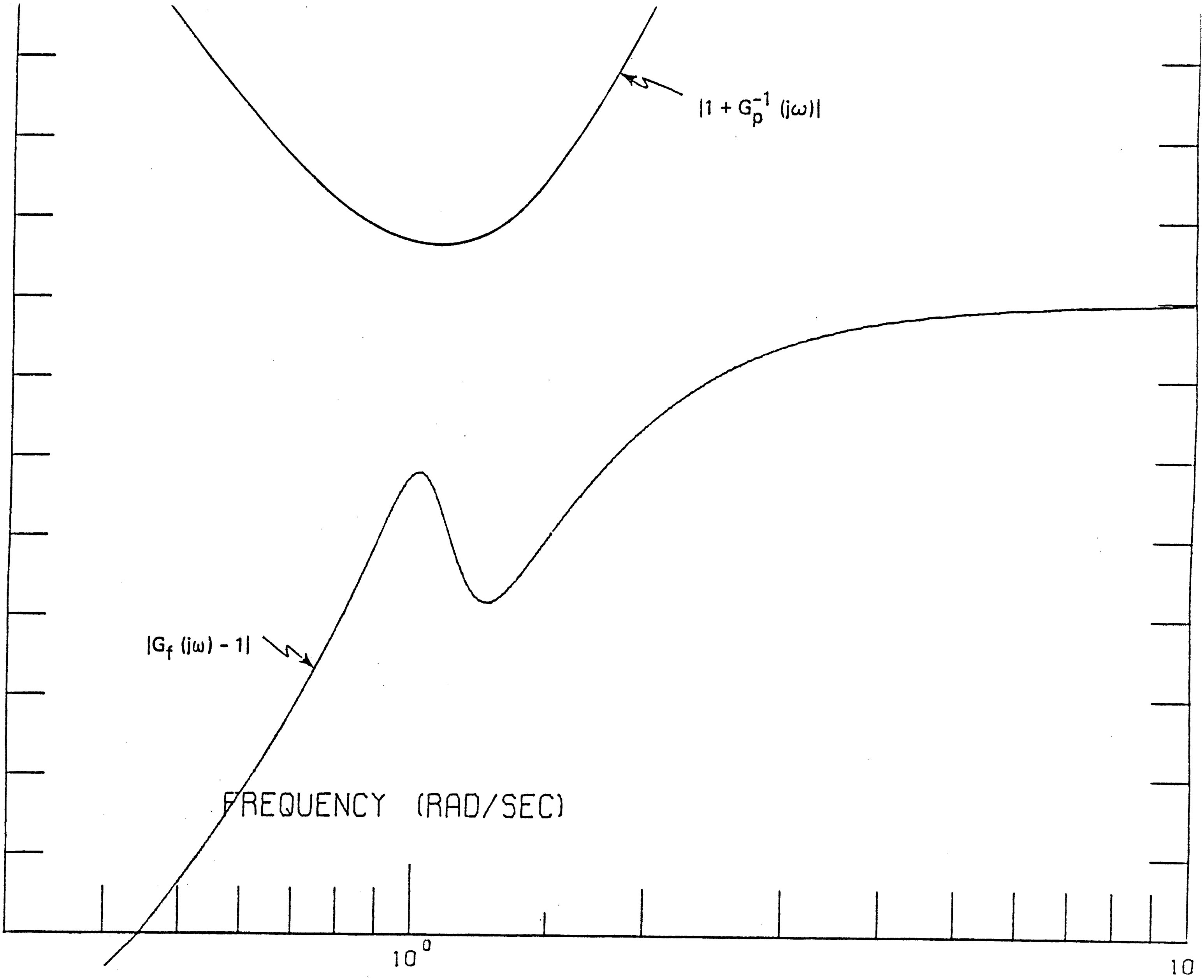

Figure 6. Example of Robustness Condition (3-10)
Data: $\rho = 0.2, \omega_a^2 = 4.0$,
$\omega_s^2 = 1.0, \zeta_s = 0.05$,
$\zeta_a = 0.78, \zeta_d = 0.32$

4.4. Comments

A simple example with one vibration mode, controlled by a single actuator, has been examined. Caution must be exercised in making general conclusions on the basis of such a simple example, but the following comments are suggested by the example.

It is possible to add damping to a structure by operating an electromechanical actuator as a passive damper using feedback of the actuator velocity only. It appears that this control approach can add only a modest amount of damping, however. In order to increase the closed loop damping, feedback of the vibrational motion to be controlled is required. By careful selection of the feedback gain parameters a substantial increase in the closed loop damping can be achieved.

5. CONCLUSIONS

5.1. Extensions

A theory has been developed for flexible structures controlled by a certain class of electromechanical actuators. This theory has been based on rather specific assumptions about the actuator characteristics and the assumed controller forms. The specifics of these assumptions not critical and extensions in several directions can be indicated. Our development has been based on specific assumptions about the transmission actuator characteristics. A similar development can be made if slightly different assumptions are made about the actuators.

Extension of the material on decentralized control is possible, in the sense that modifications to the developed forms for both force controllers and actuator controllers can be made. The development was carried out by considering a force controller defined in terms of constant gain structural velocity feedback; the form of actuator controllers suggested is consistent with that class of force controllers. However, other classes of force controllers could be used, including other output feedback forms or control based on optimal linear quadratic-gaussian theory. In principle the actuator controllers specified in Section 3 could be used in conjunction with any force controller; the only limitation arises from the fact that the force controller and actuator controllers must be realizable. Thus there are many extensions to the developments in Section 3 that could be made. The key idea is that the force controller be chosen to suitably control the structure, ignoring actuator dynamics; and the actuator controllers be chosen to suitably compensate for the actuator dynamics.

5.2. Summary

There has been substantial research into problems of active control of flexible structures. With few exceptions, the effects of actuator dynamics as part of the closed loop control scheme have been ignored. It is the premise of this work that actuator dynamics may play an important role in feedback control of flexible structures. Specifically, if actuator dynamics are ignored in the control design process their presence in an actual closed loop system may tend to be destabilizing. Such an undesirable possibility is thought to be more likely precisely in the class of problems considered, namely where a lightly damped elastic structure is coupled to actuators with damped oscillatory dynamics.

Our objective has been to develop a framework for the control design process where effects of actuator dynamics are not completely ignored. It is hoped that this work will serve to focus additional attention on the role of actuators, and other instrumentation, as a critical part of closed loop control of flexible structures.

ACKNOWLEDGEMENTS

Particular thanks are due to Dr. D.R. Hegg of The Charles Stark Draper Laboratory for his assistance.

6. REFERENCES

(1) "Actively Controlled Structures Theory-Final Report," *Charles Stark Draper Laboratory, Inc.* Report R-1338, Vol. 1, December, 1978.

(2) "Active Control of Space Structures-Final Report," *Charles Stark Draper Laboratory, Inc.* Report R-1454, February, 1981.

(3) "Across Eleven Semiannual Technical Report," *Charles Stark Draper Laboratory, Inc.* Report R-1536, Vol. 2, February, 1982.

(4) Balas, M.J., "Trends in Large Space Structure Control Theory: Fondest Hopes, Wildest Dreams," *IEEE Trans. Automatic Control*, Vol. AC-27, No. 3, June, 1982, pp. 522-535.

(5) Iwens, R.P., "Challenges in Stable and Robust Control System Design for Large Space Structures," *Proceedings of 19th IEEE Conference on Decision and Control*, Albuquerque, N.M., 1980, pp. 999-1002.

(6) Hardy, J.W., "Active Optics: A New Technology for the Control of Light," *Proceedings IEEE.* Vol. 66, No. 6, June, 1978, pp. 651-697.

(7) Murugeson, "An Overview of Electric Motors for Space Applications," *IEEE Trans. Indus. Electronics and Control Instrumentation*, Nov., 1981, pp. 260-265.

(8) Lund, R.A., "Active Damping of Large Structures in Wind," *Structural Control*, ed. H.H.E. Leipholz, North-Holland, 1980, pp. 459-470.

(9) Roberts, J.W. and M.C. Borgohain, "An Active System for Neutralization of Vibratory Force Interactions in Structures," *Structural Control*, ed. H.H.E. Leipholz, North-Holland, 1980, pp. 613-627.

(10) Roorda, J., "Trendon Control in Tall Structure," *ASCE J. of the Structural Division*, Vol. 101, No. ST3, 1975, pp. 505-521.

(11) Khalil, W. and A. Liegeois, "The Dynamics of Electrically-Actuated and Cable Driven Manipulators," *Dynamics of Multibody Systems*, ed. K. Magnus, Springer-Verlag, 1978, pp. 120-132.

(12) Riemenschneider, R., T.L. Johnson and K.H. Chen, "Final Position of a Single Degree of Freedom Tendon Arm," *Proceedings of 19th IEEE Conference on Decision and Control*, Albuquerque, N.M., 1980, pp. 1070-1075.

(13) Hughes, P.C. and R.E. Skelton, "Stability, Controllability and Observability of Matrix Second Order Systems," *Proceedings of Joint Automatic control Conference*, June, 1979, pp. 56-62.

(14) Benhabib, R.J., R.P. Iwens and R.L. Jackson, "Stability of Distributed Control for Large Flexible Structures Using Positivity Concepts," *AIAA Guidance and Control Conference*, Paper No. 79-1780, August, 1979.

(15) Kosut, R.L., H. Salzwedel, A. Emami-Naeini, "Robust Control of Flexible Spacecraft," *J. of Guidance, Control and Dynamics*, Vol. 6, No. 2, March/April, 1983, pp. 104-111.

(16) Hughes, P.C., "Passive Damper Analysis for Reducing Attitude Controller/Flexibility Interaction," *J. Spacecraft and Rockets*, Vol. 13, No. 5, May 1976, pp. 271-274.