Report U M M - 24

A New Servomechanisms Technique

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A Simple Experimental Method
for Improving the Response
of Servomechanisms

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The development of new techniques for the improvement of the response of servomechanisms has made it possible to apply servomechanisms to an ever increasing number of military and industrial uses. These techniques are based upon: (1) improved methods of analysis which show the factors that limit servomechanism response, and (2) the design, or redesign, of components which improve the response.

This article describes a new and simple method for designing stabilization circuits which may be used to improve servomechanism response, and offers a new approach to the design of stabilization circuits in general. The design of these stabilization circuits is based upon the following concepts: a circuit, having a transfer function which is the inverse of the transfer function of the servomechanism, can be used as a stabilization network; and this circuit can be approximated by a feedback amplifier having in its feedback path a network with a transfer function proportional to that of the servomechanism.

Consequently the first step in obtaining such a stabilization circuit is to design a circuit having a transfer function proportional to the transfer function of the unimproved servomechanism. This latter circuit will be referred to as the Equivalent Servomechanism Circuit or the Equivalent Circuit.

Equivalent Circuit

The first step in designing the Equivalent Circuit is to set up the general form of the equivalent network. The second step is then to determine the parameters of this network. These two steps are illustrated by the examples of Figures 1, 2, and 3.

In figure 1-A is shown the schematic of a simple servomechanism, using a motor generator set as the power control component of the servomechanism. Figure 1-B is the electro-mechanical equivalent of 1-A. The transfer functions of the system are expressed by the equations of Figure 1.

Figure 2 shows the general form of an electrical network having a transfer function of the same type as the velocity transfer function of the physical servomechanism. Comparison of the analytical expressions for the transfer function of the Equivalent Circuit of Figure 2, and for the physical servomechanism, demonstrates the equivalence of the network.

Figure 3 is a schematic of the set-up used in determining the values of the parameters of the Equivalent Circuit. This set-up consists of the unimproved open-cycle servomechanism, the Equivalent Circuit, a variable-frequency-sinusoidal-voltage source, a d.c. tachometer, and an oscilloscope having d.c. amplifiers to drive its deflection plates.[1] The following paragraphs describe the operation and theory of the setup of Figure 3:

With Switch S-2 open, and selector Switches S-3 and S-4 in Position 1, the voltage E_1 of the Equivalent Circuit and the open-circuit output-voltage E_g of the generator are placed across the deflection plates of the oscilloscope. By adjusting Resistor R_1, it is possible to reduce the elliptical figure on the oscilloscope screen to a straight line. Then the first factors of the velocity transfer functions of the actual servomechanism and of the Equivalent Circuit are the same.

In essence, when the ellipse becomes a straight line, the phase shift of the first stage of the Equivalent Circuit has been made equal to the phase shift of the first stage of the actual servomechanism. Therefore, the first factors of the velocity transfer functions of the actual servomechanism and of the Equivalent Circuit are the same because the servomechanism contains only minimum phase networks.[2]

The frequency of the variable-frequency-sinusoidal-voltage source should be varied over a large frequency range to insure that the first factors of the velocity transfer functions of the actual servomechanism and of the Equivalent Circuit are the same for all frequencies. Non-linearities in the servomechanism may make it impossible to obtain one value of R_1, which reduces the elliptical figure to a straight line for all frequencies. If this occurs, then that value of R_1 which minimizes the width of the ellipse over the frequency range of interest should be chosen.

With Switch S-2 closed, selector Switches S-3 and S-4 in Position 2, and with the rotor of the servo motor locked, the voltage E_2 of the Equivalent Circuit and a voltage proportional to the current i_a of Figure 1 are placed across the deflection plates of the oscilloscope. By adjusting resister R_2 it is possible to reduce the elliptical figure to a straight line,

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[1] This set-up, in a simplified form, was first developed by the author in 1945 while a member of the M.I.T. Servomechanisms Laboratory.

[2] Feedback Amplifier Design, by H.W. Bode; Bell Telephone System Technical Publications, Monograph B-1239

whereupon the second factors of the velocity transfer functions of the actual servomechanism and of the Equivalent Circuit are the same.

In a similar fashion, with Switch S-2 closed and selector Switches S-3 and S-4 in Position 3, the voltage E_3 of the Equivalent Circuit and a voltage proportional to pA_o of the servo motor are placed across the deflection plates of the oscilloscope. By proper adjustment of the resistor R_3 it is again possible to minimize the width of the ellipse and make the third factors of the velocity transfer functions of the actual servomechanism and of the Equivalent Circuit the same.

Upon conclusion of these three steps, the transfer function of the Equivalent Circuit is proportional to the velocity transfer function of the unimproved servomechanism. As mentioned before, non-linearities in the servomechanism may make it necessary to choose values of R_1, R_2, and R_3 which tend to minimize the width of the ellipse over the frequency range of interest.

From simple network considerations of the Equivalent Circuit, it is possible to determine what factors in the unimproved servomechanism limit its response. (Reference 1 and 2) If these factors cannot be removed by redesign, then a stabilization network must be used.

Application of the Equivalent Circuit as a Stabilization Network

The schematic diagram of Figure 4 shows how the Equivalent Circuit can be incorporated into a feedback amplifier and thus create a stabilization network. The resulting feedback amplifier will be referred to as the Stabilization Feedback Amplifier.

In Figure 4 the unimproved open-cycle servomechanism, represented by the transfer function K_1G_1 of Figure 1, is cascaded with the stabilization Feedback Amplifier having a transfer function K_4G_4. The Stabilization Feedback Amplifier has a constant gain amplifier K_3 in its forward branch, and the Equivalent Circuit of Figure 2 in its feedback branch.

The expressions of Figure 4 demonstrate that if K_3 is large enough, K_4G_4 approaches $1/K_2G_2$. It is further shown that the overall transfer function K_5G_5 reduces to $1/bp$ and A_o/A_i to $1/(1+bp)$.

Thus by combining the Equivalent Circuit of the velocity transfer function of a servomechanism with an amplifier, it is possible to obtain a stabilization network which tends to reduce both the transfer function of the compensated servomechanism to 1/bp, and the output-to-input ratio to 1/(1 + bp).

If the output-to-input ratio A_o/A_i of the compensated servomechanism of Figure 4 is considered to be a transfer function K_6G_6, it is possible to design an equivalent network for this transfer function. This equivalent network may be obtained by comparing K_6G_6 to a single RC stage in a manner similar to that demonstrated in Figure 3.

This single stage equivalent RC network can be used as the feedback branch of a Stabilization Feedback Amplifier and the resulting feedback amplifier cascaded with K_6G_6, as shown in Figure 5. If this is done, the expressions of Figure 5 show that K_7G_7, the transfer function of this Stabilization Feedback Amplifier, approaches 1 + bp, and the overall transfer function K_0G_0 of the twice-compensated servomechanism approaches 1. Therefore, in the limit, A_o becomes directly proportional to A_i

In the examples of Figures 4 and 5, it was assumed that the transfer function K_6G_6 approached 1/(1 + bp) and that, therefore, the RC product of the equivalent circuit of K_6G_6 should equal b. In general, it may not be possible to reduce K_5G_5 to exactly 1/bp, and therefore K_6G_6 will be slightly more complex than 1/(1 + bp). Consequently, when comparing the equivalent RC network to K_6G_6, a value of R should be chosen which tends to minimize the width of the ellipse over the frequency range of interest.

Figures 4 and 5 show that it is possible to enhance the stability of a servomechanism by first cascading the open-cycle servomechanism with a Stabilization Feedback Amplifier containing the equivalent circuit of its transfer function, and then by closing the servo feedback loop around these two cascaded components. This procedure may be repeated successively until the desired improvement in the response of the servomechanism has been achieved.

Some General Considerations

In any servomechanism it may not be necessary to compensate for every time delay or its equivalent. Consequently, the Equivalent Circuit used in the Stabilization Feedback Amplifier need not always contain all of the terms of the complete velocity transfer function. In fact, it may be more practical not only to reduce the number of terms in the equivalent transfer function, but also to approximate these terms.

Following this philosophy, an Equivalent Circuit can be designed which is composed of only one or two RC networks or an L C R network. With the setup of Figure 3 these RC networks can be made equivalent for only those frequencies where the maximum phase shift and resonance in the servomechanism response has been observed. Then an approximate Stabilization Feedback Amplifier can be constructed, and the procedure of Figure 4 followed. The remaining uncompensated factors which limit the response of the Servomechanism can either be neglected or removed, by another approximate Equivalent Circuit and Stabilization Feedback Amplifier, as was done in Figure 5.

The number of cathode followers used in the Equivalent Circuit may be reduced if the impedance of one RC network is several times greater than the impedance of the preceding network, thus making it unnecessary to isolate these two networks.

In some cases the inertia of the d.c. tachometer of Figure 3, might be comparable to the inertia of the servo motor which would change the effective characteristics of the servo motor. If this condition arises, the d.c. tachometer should be coupled to the motor through a gear reduction. This gearing down usually can be accomplished by coupling the d.c. tachometer into the output gear train at some convenient gear mesh.

It is a worthwhile precaution, when combining the Equivalent Circuit with an amplifier, to form a Stabilization Feedback Amplifier to use no more than two time constants, or their equivalent, in one feedback branch. Otherwise the possibility exists that the Stabilization Feedback Amplifier may oscillate. If there are more than two time constants, or their equivalent, in an Equivalent Circuit, then several Stabilization Feedback Amplifiers should be used.

Good results can be obtained from a Stabilization Feedback Amplifier with a value of K_3, of Figure 4, of only 7 to 10 for each time constant. The overall gain K_4 of the Stabilization Feedback Amplifier can be made any value desired by proper choice of the value of K_2 of the Equivalent Circuit. It is worth noting that the amplifier K_3 does not have to be an additional amplifier, but may be a stage of K_1.

Further Equivalent Circuits and Stabilization Feedback Amplifiers

In many applications the output motion of the servo motor is restrained by a spring, as in the case of a servo torque motor actuating the pilot valve of a hydraulic system. Figure 6-A shows the electro-mechanical circuit of a servo motor restrained by a spring of stiffness K, and Figure 6-B shows the Equivalent Circuit of Figure 6-A. In this example the servomechanism amplifier stages are not shown, and the servo motor is assumed to operate on d.c.

The output voltage E_o of the Equivalent Circuit may be made proportional to the output voltage E_o of the physical system in a manner similar to that demonstrated in Figure 3. In this case it is not possible to subdivide the Equivalent Circuit or the electro-mechanical circuit so that only one variable may be adjusted, or compared, at a time. This difficulty may be overcome by noting that at high frequencies the voltage E_o is primarily a function of the LC product, and that at low frequencies it is primarily a function of the RC product. Therefore, with the frequency of the variable frequency sinusoidal voltage source high, C should be adjusted until the width of the ellipse has been minimized. Then the frequency should be reduced until the width of the ellipse is quite large. At this frequency the value of R should be adjusted until the width of the ellipse has again been minimized. Some estimate of the approximate value of L and C may be obtained by visual observation of the natural frequency of the unexcited servo motor and spring.

So far, the servomechanisms described have been d.c. servomechanisms. Figure 7 shows part of a 2-phase induction motor servemechanism and its electro-mechanical equivalent. The amplifier components have not been considered because, in general, straightforward amplification stages act upon the side bands of the modulated suppressed carrier signals and therefore produce only very small time delays. On the other hand, the servo motor essentially demodulates the modulated suppressed carrier signal and acts only upon the envelope or modulating signal, and thus may produce large time delays.

Figure 8 is a schematic of the Equivalent Circuit of the 2-phase induction servo motor of Figure 7. As will be noted from the analytical expressions of Figure 8, the equivalence of this Circuit holds only over a limited region of the modulating signal frequencies. This situation is generally true of any a.c. network containing only linear elements.

The parameters of this Equivalent Circuit may be determined by the procedure demonstrated in Figure 3. As there are two variables, L and R_s, to be adjusted together, tuning the LC components so that they resonate at the carrier frequency leaves only R to be adjusted.

As mentioned before, the a.c. servo motor demodulates the modulated suppressed carrier signal, and, therefore, it is difficult with linear elements to synthesize a network which is the equivalent of the a.c. servo motor. This factor makes it quite difficult to obtain as much stabilization from linear a.c. networks as from linear d.c. networks. It is, therefore, suggested that a small 2-phase motor, directly coupled to a small a.c. rate generator, may be used as the Equivalent Circuit for any a.c. servomechanism. This small 2-phase motor and generator combination can then be used as the feedback branch of a Stabilization Feedback Amplifier, and the resulting amplifier will serve as a stabilization network for 2-phase servomechanisms. Figure 9 demonstrates the method by which this may be accomplished.

It may be noted that if an additional inertia is coupled to the combination shaft of the small servo motor and rate generator, the time constant of the feedback path of the Stabilization Feedback Amplifier may be made large enough to compensate for more than one a.c. time delay or servo motor. The A.C. Amplifier No. 2 of Figure 9 does not need to be a separate amplifier, but may be a stage from either K_3 or A.C. Amplifier No. 3.

In review, the response of a servomechanism can be improved with the aid of an approximate Stabilization Feedback Amplifier as shown in Figure 4. Furthermore, if the response is not sufficiently improved after the use of a single stabilization network, additional networks may be added. The design of these additional networks usually is simplified by the use of a minor feedback loop around the first stabilization network and unimproved servomechanism in cascade. This is demonstrated in Figure 5.

REFERENCES

(1) "Application of Circuit Theory to Design of Servomechanisms"
By A.C. Hall, Journal of Franklin Institute. V-242, P-279.

(2) "Electrical Analogy Method Applied to Servomechanism Problems"
By G.D. McConn, S.W. Herwald, H.S. Kirschbaum
(Translation AIEE V-65, P-91, 1946)

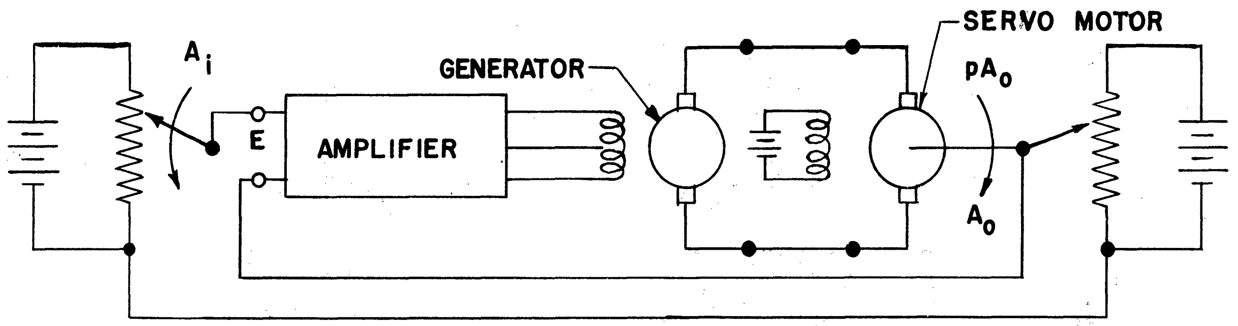


FIG. 1a

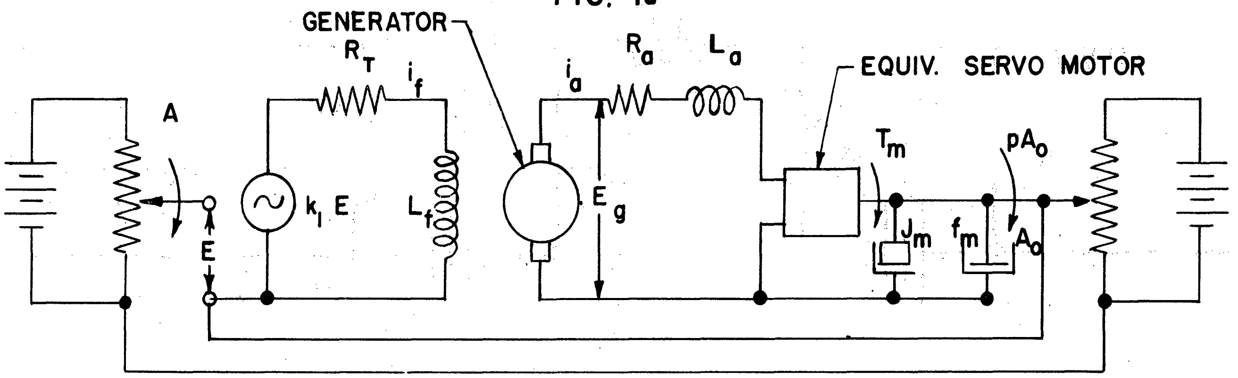


FIG. 1b

$$i_f = \frac{k_1 \frac{E}{R_T}}{1 + \frac{L_f}{R_T} p}$$

$$E_g = k_2 i_f$$

$$i_a = \frac{\frac{E_g}{R_a}}{1 + \frac{L_a}{R_a} p}$$

$$T_m = k_3 i_a$$

$$T_m = \left(1 + \frac{J_m}{f_m} p\right) f_m p A_o$$

$$\frac{pA_o}{E} = \frac{\frac{k_1}{R_T}}{1 + \frac{L_f}{R_T} p} \cdot \frac{\frac{k_2}{R_a}}{1 + \frac{L_a}{R_a} p} \cdot \frac{\frac{k_3}{f_m}}{1 + \frac{J_m}{f_m} p}$$

$K_1' G_1' = \frac{pA_0}{E}$ = VELOCITY TRANSFER FUNCTION

$K_1 G_1 = \frac{A_0}{E}$ = TRANSFER FUNCTION

$K_1 G_1 = \frac{1}{p} K_1' G_1'$

WHERE : $p = \frac{d}{dt}$

J_m = MOMENT OF INERTIA OF MOTOR

f_m = EQUIV. DAMPING COEFF. OF MOTOR.

R_a = RESISTANCE OF GENERATOR AND MOTOR ARMATURES.

L_a = INDUCTANCE OF GENERATOR AND MOTOR ARMATURES.

L_f = INDUCTANCE OF GENERATOR FIELD.

R_T = EQUIV. SOURCE RESISTANCE OF AMPLIFIER AND FIELD.

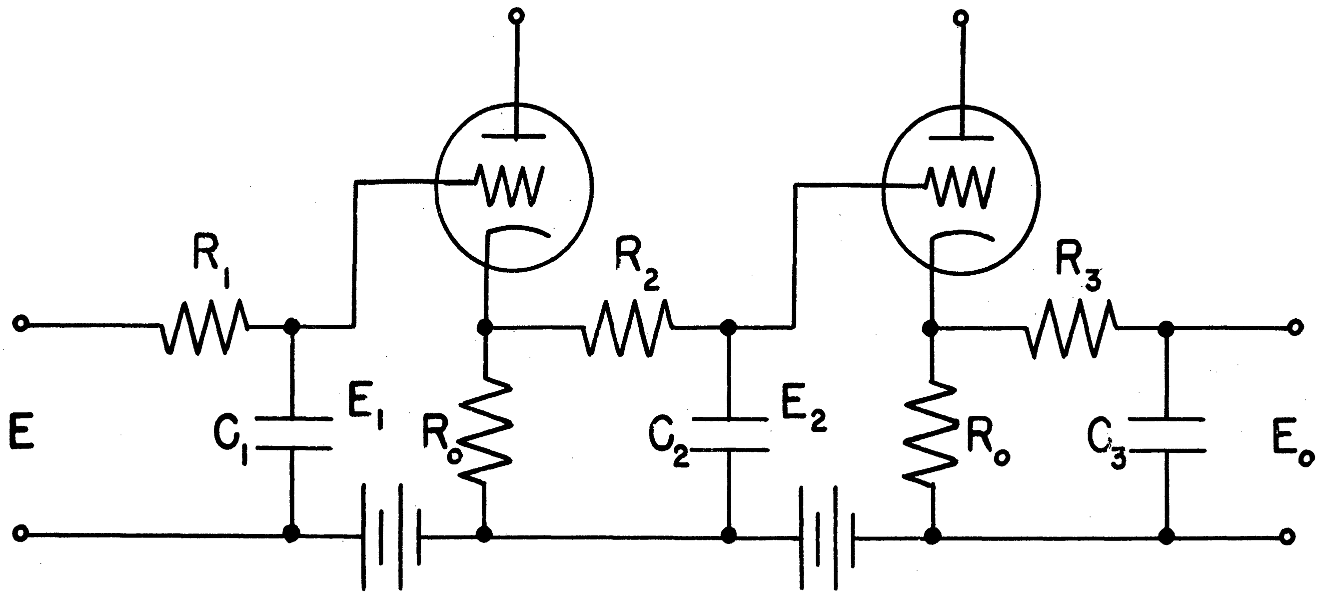
$k_1, k_2, \& k_3$ = CONSTANTS OF PROPORTIONALITY.

$K_1' \& K_1$ = REAL GAIN FACTORS OF TRANSFER FUNCTIONS AND ARE NOT A FUNCTION OF FREQUENCY

$G_1' \& G_1$ = COMPLEX FACTORS OF TRANSFER FUNCTION AND ARE FUNCTIONS OF FREQUENCY

FIG. 1

pA_0/E is approximate. This approximation is justifiable for the operating range of interest, because, in general, the armature circuit time constant is small compared to the motor time constant.



$$E_1 = \frac{E}{1+R_1C_1p} \qquad E_2 = \frac{aE_1}{1+R_2C_2p} \qquad E_0 = \frac{aE_2}{1+R_3C_3p}$$

$$\frac{E_0}{E} = \frac{1}{1+R_1C_1p} \cdot \frac{a}{1+R_2C_2p} \cdot \frac{a}{1+R_3C_3p} = K_2\,G_2$$

HERE THE SOURCE IMPEDANCE OF THE CATHODE FOLLOWERS HAS BEEN ASSUMED TO BE ZERO.

FIG. 2

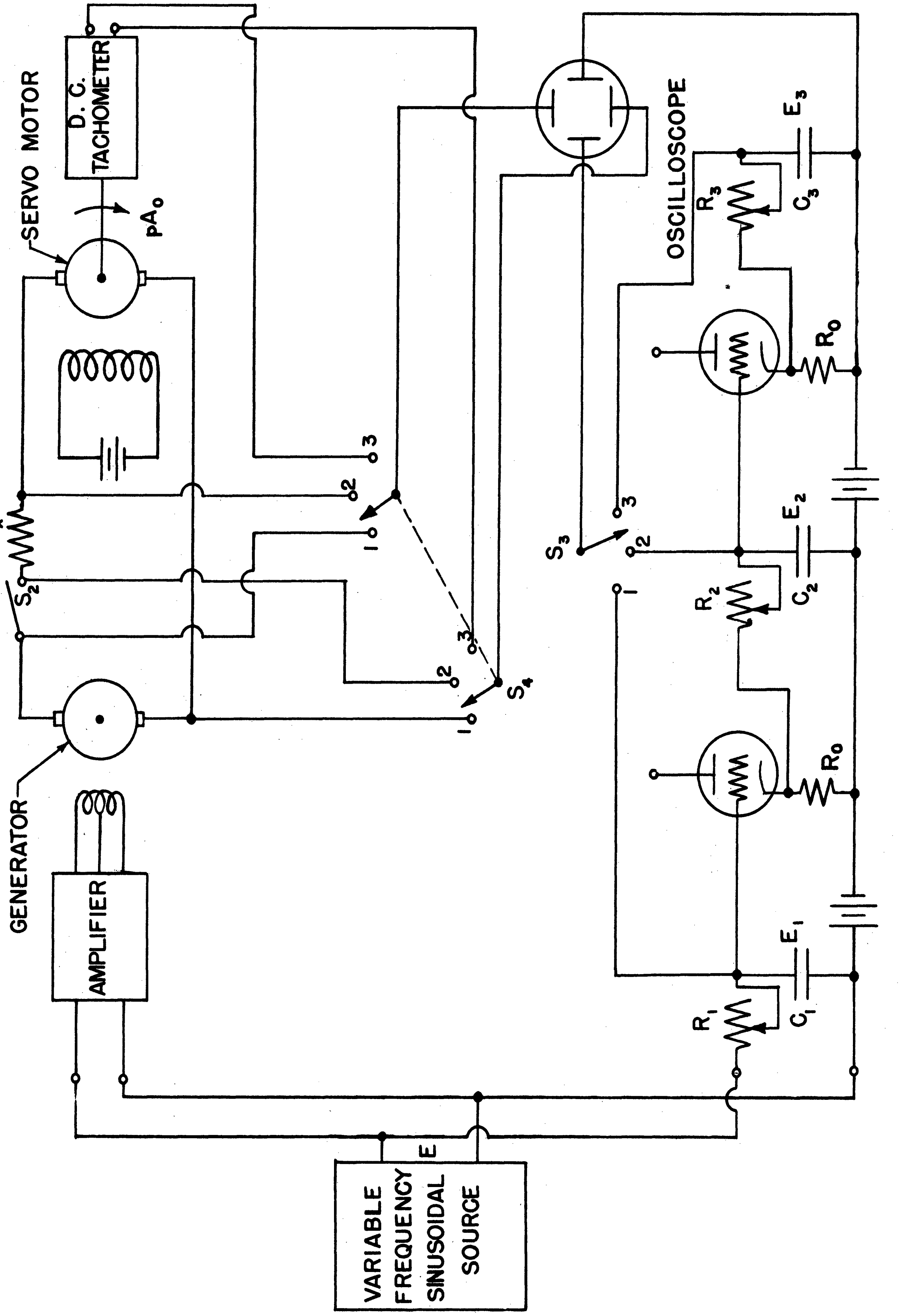
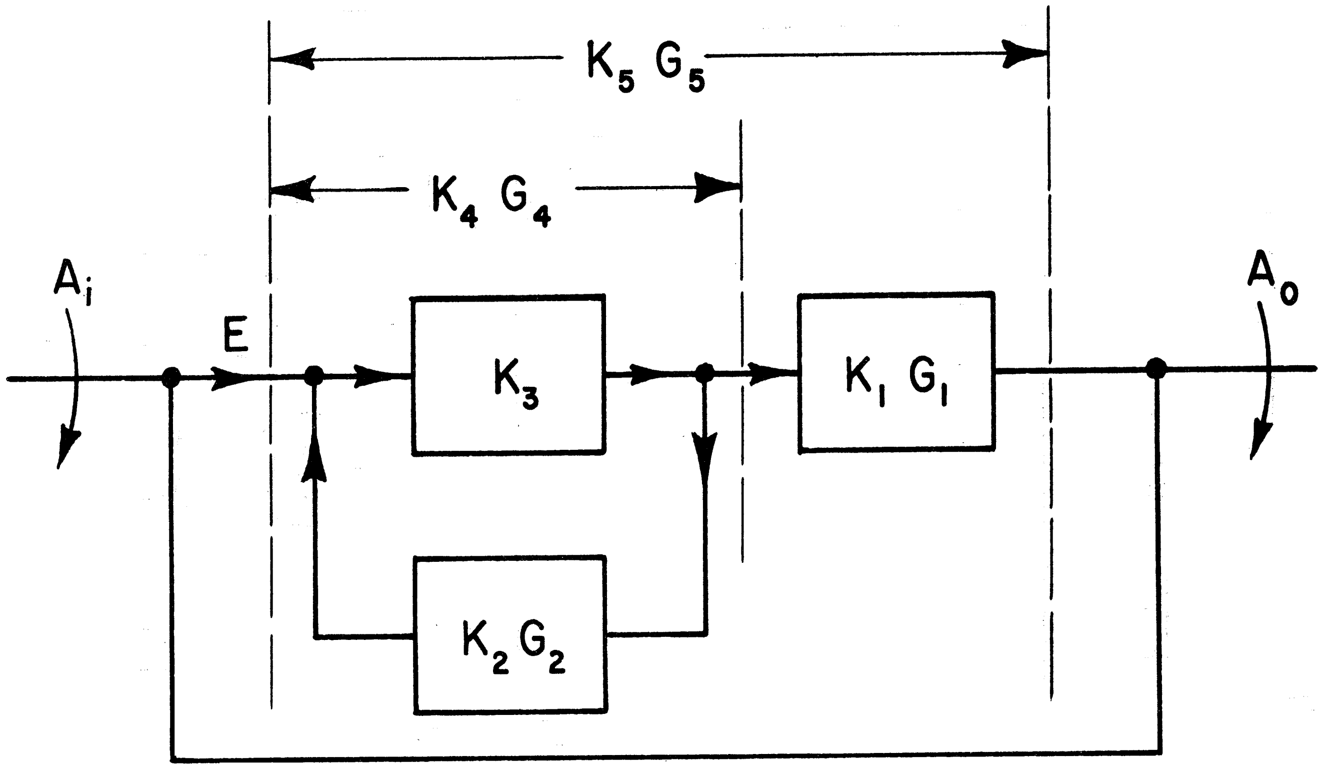

D. C. TACHOMETER
SERVO MOTOR
pA$_o$
GENERATOR
AMPLIFIER
R$_x$
S$_2$
S$_3$
S$_4$
VARIABLE FREQUENCY SINUSOIDAL SOURCE
E
OSCILLOSCOPE
R$_1$
R$_2$
R$_3$
C$_1$
C$_2$
C$_3$
E$_1$
E$_2$
E$_3$
R$_o$
R$_o$

FIG. 3



$$K_4 G_4 = \frac{K_3}{1 + K_3 K_2 G_2}$$

IF: $K_3 K_2 G_2 >> 1$

$$K_4 G_4 \equiv \frac{1}{K_2 G_2}$$

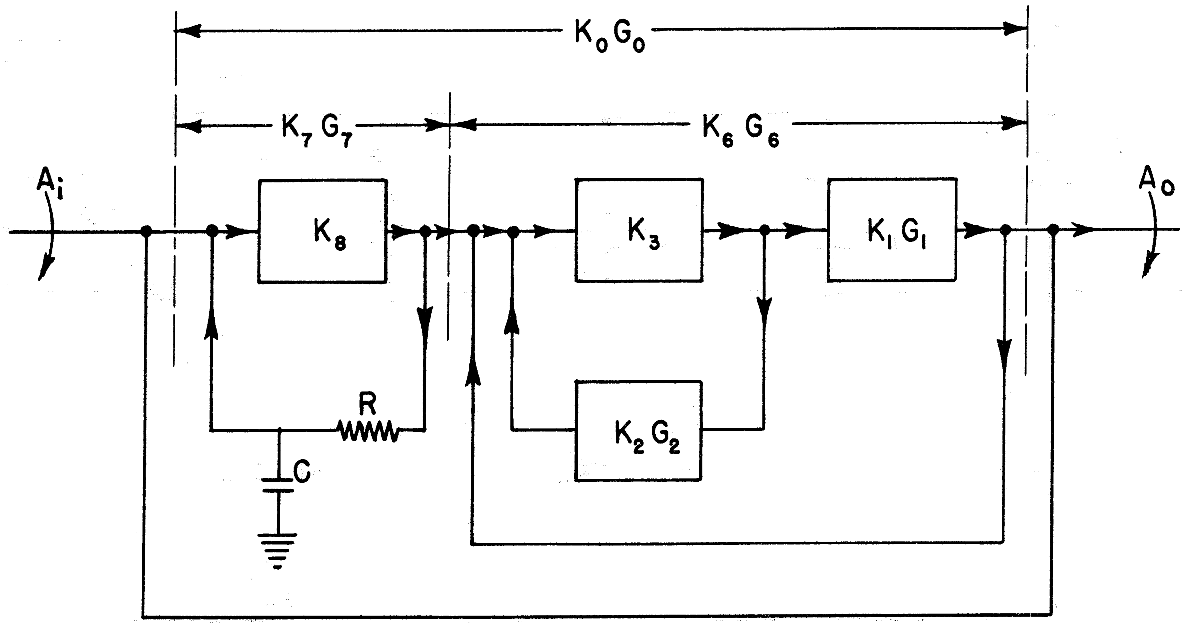
THEN: $K_5 G_5 = K_1 G_1 K_4 G_4 \equiv \frac{K_1 G_1}{K_2 G_2} = \frac{1}{p} \cdot \frac{K_1' G_1'}{K_2 G_2}$

FINALLY IF: $K_2 G_2 \equiv b K_1' G_1'$

$$K_5 G_5 = \frac{1}{b p}$$

$$\frac{A_0}{A_i} \equiv \frac{K_5 G_5}{1 + K_5 G_5} = \frac{1}{1 + b p} \equiv K_6 G_6$$

FIG. 4



$$K_7 G_7 = \frac{K_8}{1 + \frac{K_8}{1 + pRC}}$$

IF $\frac{K_8}{1 + pRC} >> 1$

$K_7 G_7 \equiv 1 + pRC$

AND IF $RC = b$

$K_7 G_7 \equiv 1 + bp$

FROM FIG. 4

$$K_6 G_6 \equiv \frac{1}{1 + bp}$$

THEN, $K_0 G_0 = K_7 G_7 K_6 G_6 \equiv$

$$\frac{A_0}{A_i} = \frac{K_0 G_0}{1 + K_0 G_0} = \frac{1}{2}$$

FIG. 5

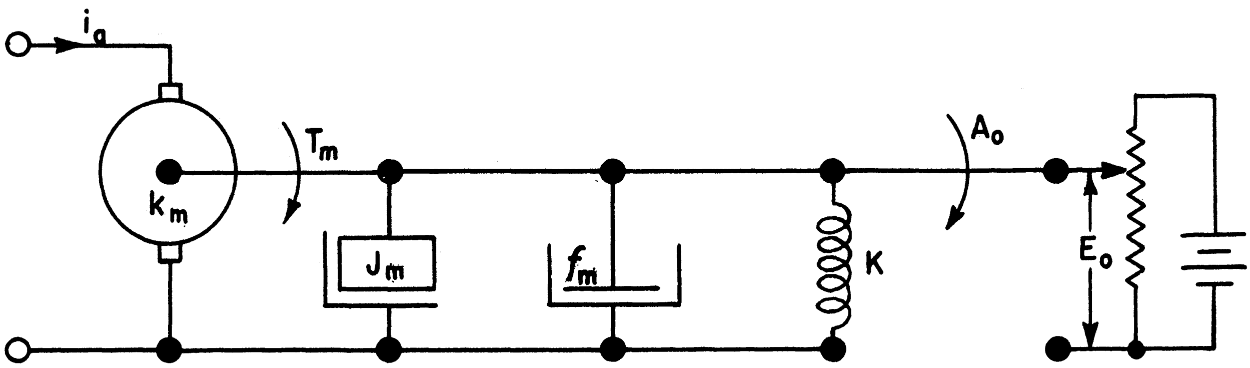


FIG 6a

$$T_m = k_m i_a = (J_m p^2 + f_m p + K)\ A_o \qquad \text{WHERE } \omega_n^2 = \frac{K}{J_m}$$

$$\frac{A_o}{i_a} = \frac{k_m}{J_m p^2 + f_m p + K}, \qquad \frac{A_o}{i_a} = \frac{\frac{k_m}{J_m}}{[\omega_n^2 - \omega^2] + j \frac{f_m}{J_m} \omega}$$

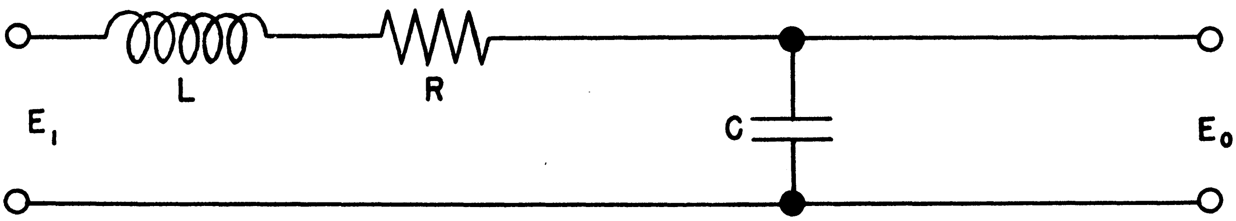


FIG. 6b

$$\frac{E_o}{E_1} = \frac{1}{LCp^2 + RCp + 1}, \qquad \frac{E_o}{E_1} = \frac{\omega_n^2}{(\omega_n^2 - \omega^2) + j \frac{R}{L} \omega}$$

$$\text{WHERE } \omega_n^2 = \frac{1}{LC}$$

FIG. 6

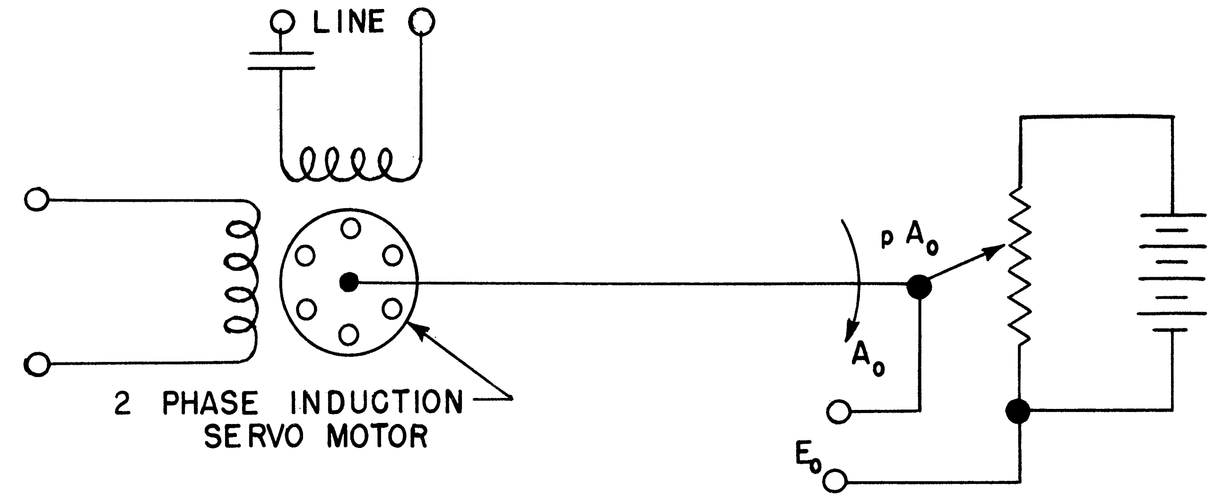


FIG 7a

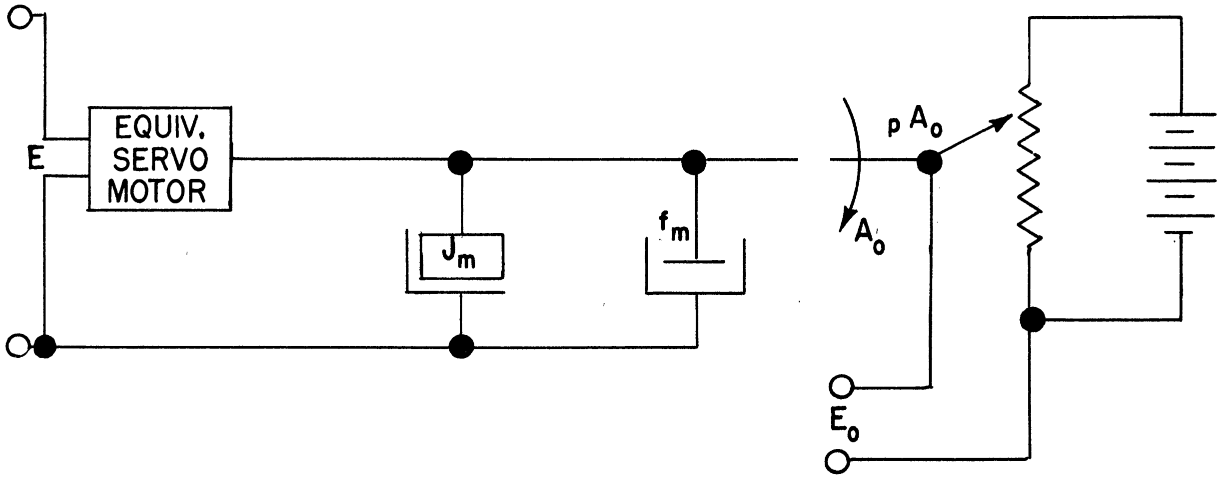


FIG. 7b

$$T_m = K_3 E = \left(1 + \frac{J_m}{f_m} p\right) f_m p A_0$$

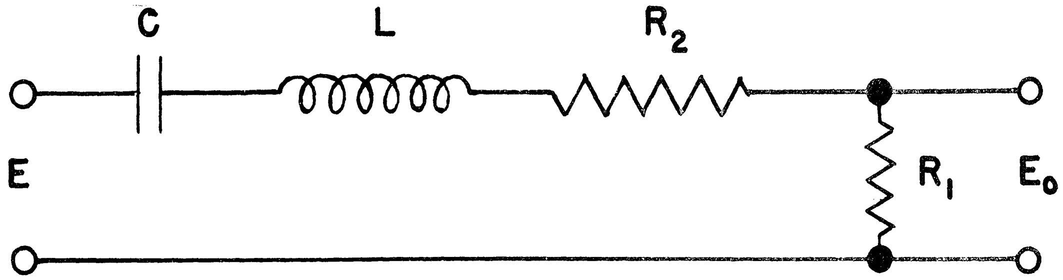
$$\frac{pA_0}{E} = \frac{\frac{K_3}{f_m}}{1 + \frac{J_m}{f_m} P} = K_1' G_1'$$

WHERE : $p = \frac{d}{dt}$

J_m = MOMENT OF INERTIA OF MOTOR.

f_m = EQUIV. DAMPING COEFF. OF MOTOR.

FIG. 7



R_2 = SELF RESISTANCE OF INDUCTANCE L

$$\frac{E_0}{E} = \frac{aRCp}{LCp^2 + RCp + 1}$$

WHERE $R = R_1 + R_2 = \frac{1}{a} R_2$

ON A FREQUENCY BASIS:

$$\frac{E_0}{E} = \frac{jaRC\omega}{(1-\omega^2 LC) + jRC\omega}$$

IF $LC = \frac{1}{\omega_c^2}$

ω_c = CARRIER FREQUENCY

$$E = e \sin\omega_s t \sin\omega_c t = e'\left[\cos(\omega_c+\omega_s)\,t + \cos(\omega_c-\omega_s)\,t\right]$$

THEN $\omega = \omega_c \pm \omega_s$

$$\frac{E_0}{E} = \frac{jaRC\omega}{1-\frac{\omega^2}{\omega_c^2} + jRC\omega_c\frac{\omega}{\omega_c}} = \frac{jaRC\omega_c\left(\frac{\omega_c \pm \omega_s}{\omega_c}\right)}{1-\frac{(\omega_c\pm\omega_s)^2}{\omega_c^2} + jRC\omega_c\left(\frac{\omega_c\pm\omega_s}{\omega_c}\right)}$$

IF $\omega_s << \omega_c$

$$\frac{E_0}{E} = \frac{a}{1 \pm j\left(\frac{2}{RC\omega_c^2}\right)\omega_s}$$

THIS APPROXIMATE RELATION HOLDS ONLY AS LONG AS

$\omega_s << \omega_c$

FIG 8

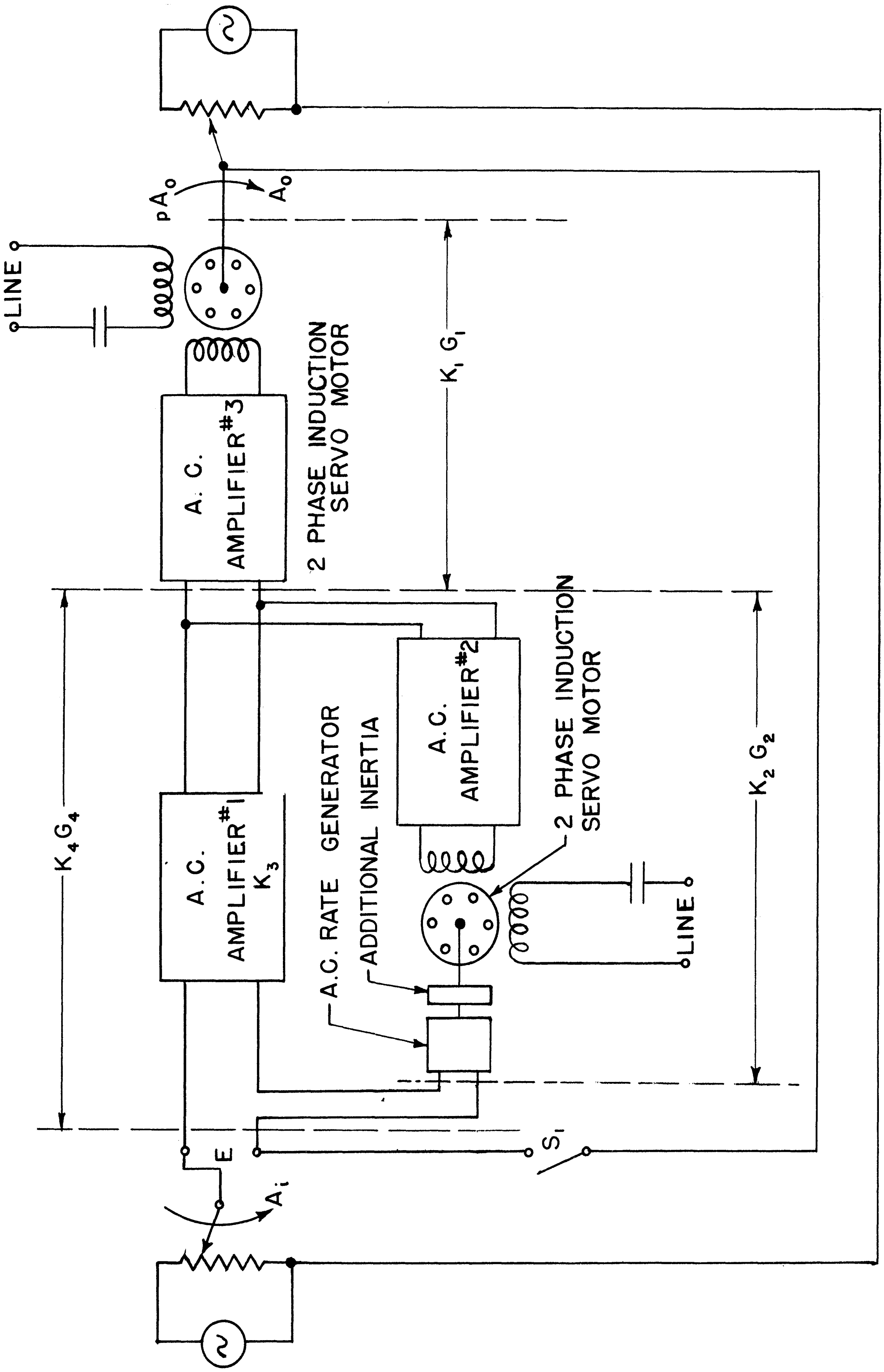

LINE
pA_o
A_o
A. C. AMPLIFIER #3
2 PHASE INDUCTION SERVO MOTOR
$K_1 G_1$
A. C. AMPLIFIER #2
2 PHASE INDUCTION SERVO MOTOR
LINE
$K_2 G_2$
A. C. RATE GENERATOR
ADDITIONAL INERTIA
A. C. AMPLIFIER #1 K_3
$K_4 G_4$
E
A_i
S_1

FIG. 9