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TESTS ON MODELS OF NUCLEAR REACTOR ELEMENTS

II. Studies of Diffusion

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A. SUMMARY

To estimate the distribution of temperature in the proposed nuclear reactor, one must determine a coefficient of eddy diffusivity and devise a suitable method of computing the heat transfer. Measurements of diffusion in a model of a blanket element for the proposed reactor indicated a gross eddy diffusion coefficient of about $0.002 (\pm .0005)$ ft²/sec. Thus, the apparent eddy diffusion for the test conditions is about 200 times the molecular diffusivity of water and about twice that of the liquid sodium. Even approximate methods of applying this result require an elaborate calculation if the primary characteristics of the flow system are to be taken into account.

The dispersion of dye in flowing water provided an indication of the diffusion in the model tests. The presence and arrangement of the rods, the effect on the flow of the spiral wire spacers, and the existence of a comparatively large area on which a laminar sub-layer develops made it impossible to get simple turbulence criteria like those obtained downstream from a screen. Although the results are consequently somewhat unsystematic, they do establish reliably the approximate magnitude of the coefficient of eddy diffusivity. The data were obtained from both line and sectional traverses, the two results being approximately equal. Preliminary data were also obtained for a core element for which $\epsilon \approx 0.0013$, only slightly less than for the blanket element.

Determination of the diffusion coefficient makes it possible to compute the temperature for an array of spatially variable heat sources, as occur in any element. Because of the extreme complexity of the problem, two alternative simplifying assumptions are proposed. In one, the heat sources are assumed to be concentrated along their axes. In the other, the heat is assumed to pass to the fluid immediately at the surface of each circular rod and then to diffuse as though no other rods were present. In each case the effect of the rods on the pattern of diffusion is taken into account only by the aforementioned ratio between the local and the apparent diffusivities. The calculations involve a doubly infinite summation to account for the rods and for the solid walls of the container which are assumed to be insulated. An effect of the rods is to make the local diffusivity much more than the apparent diffusivity, which was observed. In a calculation based on an analogy with heat transfer, the former was found to be 5.3 times the latter.

B. EXPERIMENTATION

Experiments were required to determine the coefficient of eddy diffusivity for the complex geometry and system of flow in the elements. In these experiments the full-scale models of elements and the system for circulating water were employed much as in the study of losses.¹ The flow system and the model are shown in Fig. 1. For convenience, the diffusion of dye rather than of heat was observed. A continuous supply of dye in solution was injected, samples were taken at various points downstream, and the concentrations of dye in these samples were determined by means of a colorimeter.

EQUIPMENT

Various additions to the equipment employed previously were required in the study of diffusion. A dye solution was injected by means of a hypodermic needle inserted through the side wall of the sub-assembly. The No. 20 needle used was bent at right angles so that 2-1/2 in. of it projected downstream, and was supported so that it could be moved laterally within a channel-type guide fabricated from a No. 17 needle. The guide extended entirely across the sub-assembly so that disturbances to the flow caused by the presence of the injection needle were symmetrical. A pot of dye mounted as shown in Fig. 1 was connected to the injection needle by means of flexible tubing. The pot was placed about one foot above the average piezometric head line at the point of injection to insure a continuous flow. After some initial trials with other substances, a solution of five grams of Nigrosin dye per liter of water was used. (Nigrosin is manufactured by the National Analine Division of the Allied Chemical and Dye Corporation.)

In the initial tests, samples of water-dye mixture were collected at various positions downstream from the point of supply through another No. 20 hypodermic needle inserted through the wall of the sub-assembly. The tip of this needle was bent at right angles so as to face upstream. The needle fitted through a guide designed to keep it correctly aligned and could be moved completely across the model so as to obtain a line traverse of the distribution of dye. Later tests were made using an array of No. 20 needles mounted in a bracket attached to the downstream end of the sub-assembly. The ends of these needles faced upstream and were 1-1/2 in. from the end of the sub-assembly.

The concentrations of dye in the various samples were determined by several methods. Initially a Bausch and Lomb Duboscq Colorimeter No. 5 was used to compare the sample with clear water. The readings were calibrated by means of a series of standard samples of known concentrations. Later a special photocell system was built to compare any two samples. This was done partly to eliminate eye strain and human error involved in using the colorimeter. Also, the sump of the circulating system gradually accumulated enough dye during a test that a background reading was necessary to determine the net concentration of dye in any sample. Hence reference samples were taken periodically as bases

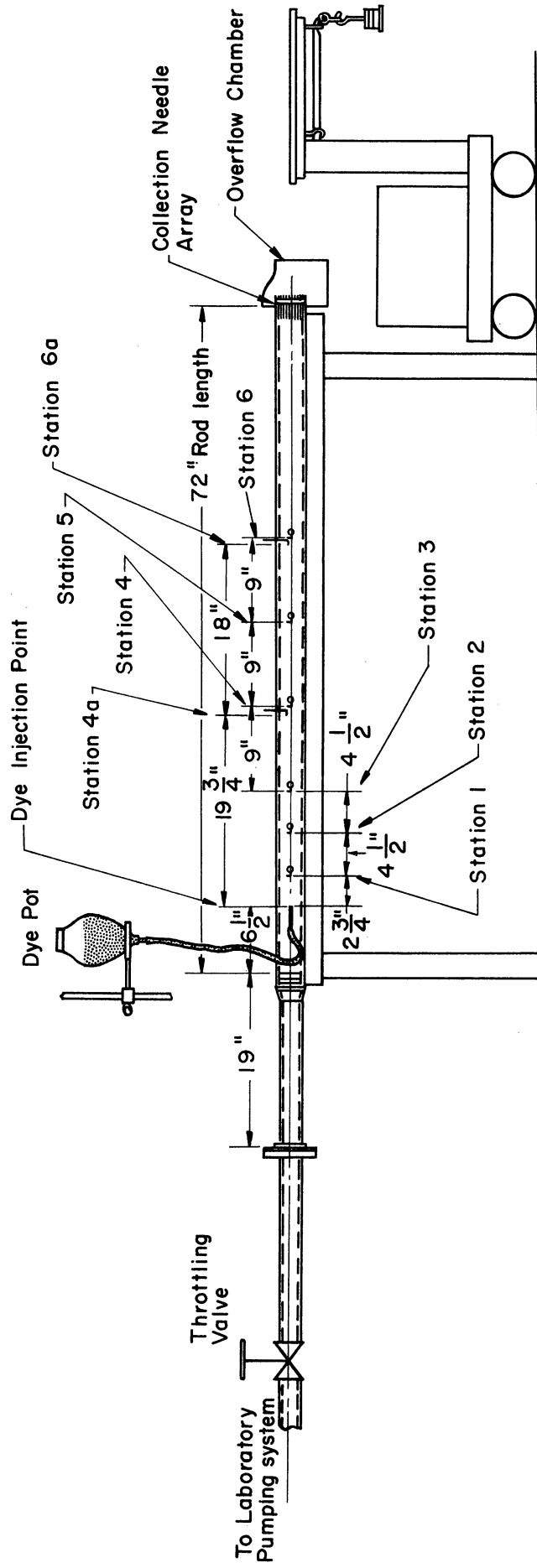


Fig. 1. Blanket sub-assembly model showing locations of dye injection and sampling points.

for comparison. Minor difficulties with the original photocell system were eliminated by changing from the original unit to a commercial instrument, a model 520M photocell circuit purchased subsequently by the department from the Photovolt Corporation of New York, N. Y.

LINE TRAVERSES

The first comprehensive tests consisted of line traverses of dye concentration at various distances downstream from the point of injection. The injection needle was placed 6-1/2 in. from the upstream end of the rods. The traverses were made with a single collection needle at the first four stations shown in Fig. 1. Because of the rods and the spiral spacers, traverses with the collection needle could be made only at specific points. The 4-1/2-in. modulus was a consequence of the 9-in. pitch for the spiral winding of the spacer wires. Stations 5 and 6 farther downstream, and stations 4a and 6a for vertical traverses were added subsequently. Stations 4a and 6a were one inch upstream from 4 and 6, respectively, again because of the spiral windings. The two positions A and B selected for the injection needle are shown in Fig. 2.

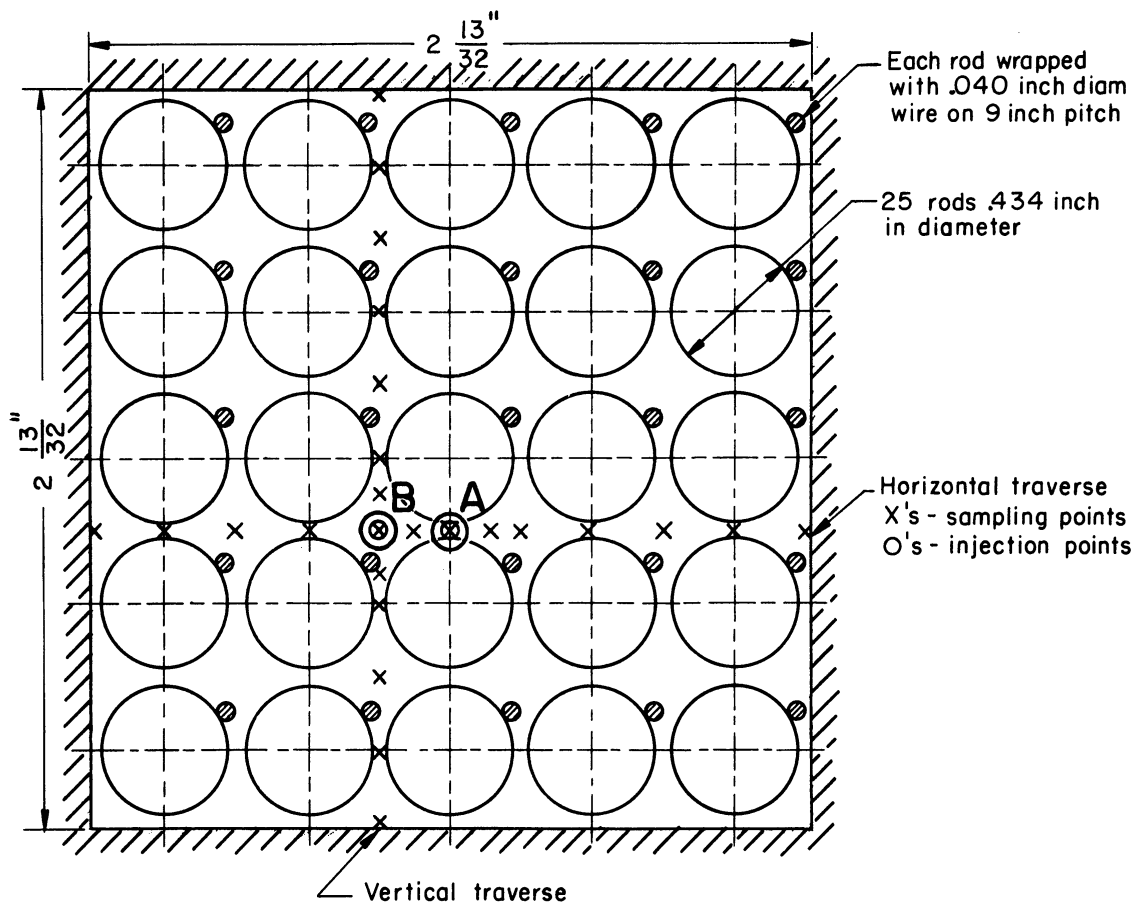


Fig. 2. Section of blanket sub-assembly showing locations of dye injection and sampling points for line traverses.

The initial experiments were conducted with the injection needle at position A and with a mean velocity of 4 ft/sec. The collection needle was

used to obtain samples along line traverses between the same two tiers of rods at the several stations. Figure 3a shows the net concentration in grams per liter at each of the first four stations with the injection needle on the vertical centerline (A). Figure 3b shows the net concentration at all six stations with the injection needle at position B (Fig. 2). The change in position was made to determine whether the proximity of the rods for position A altered significantly the pattern of diffusion. Although some effect was noted in the distribution at the first station, other discrepancies in the data were much more important.

The area under the concentration curves is a measure of the total amount of dye at any section, and the shape of the curves from station to station indicates the amount of diffusion which has taken place. If the diffusion had been symmetrical with respect to horizontal and vertical (longitudinal) planes through the point of injection, the line traverses would have been sufficient to show the amount of spread of the dye. On the contrary, the curves for the single horizontal line traverses showed a lack of symmetry in that the samples did not account for all the dye injected and varied by as much as 30%.

To obtain more information, vertical traverses were made using stations 4a and 6a and positions A and B for the injection needle. The horizontal traverses at stations 4 and 6 are compared with the vertical traverses at stations 4a and 6a in Fig. 3c. For symmetrical diffusion, each pair of comparable vertical and horizontal traverses should have been much more nearly the same than they were. Even the horizontal and vertical line traverses together did not indicate completely how the dye had diffused. It was therefore decided to obtain complete sectional traverses to obtain more dependable information on the diffusion.

SECTIONAL TRAVERSES

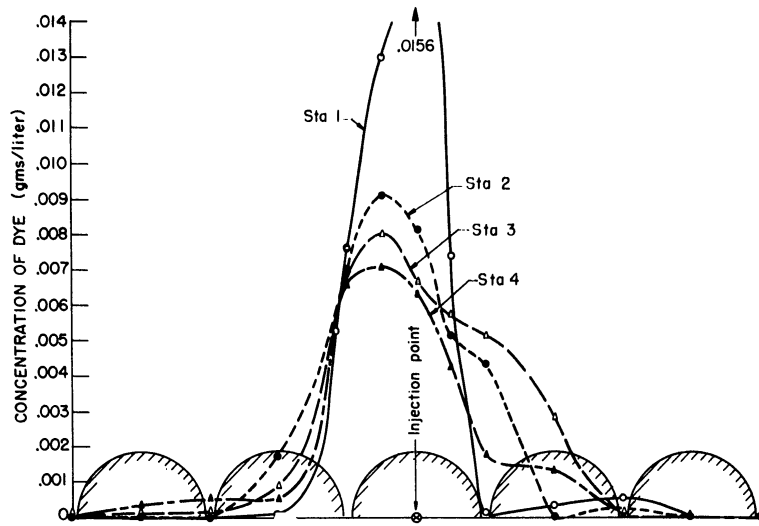
The sampling of sufficient points at any one section to obtain a complete picture of how the dye was distributed was accomplished by means of the array of needles at the downstream end of the model. For these tests the dye was again injected at A and B, but at stations 4, 5, and 6. Only 36 needles were used, but by the shifting of the end bracket horizontally or vertically, samples were obtained at 156 points in all. Figure 4a shows the data with injection at station 5 and Fig. 4b that obtained with injection at station 6. In both cases the average velocity was 4.65 ft/sec. The data in Fig. 4a are shown for 156 points, and those in Fig. 4b were averaged for 100 repeating areas.

Two preliminary tests were made for a core element (Hydraulic Model No. 4). The dye was fed at the center and near one corner of the upstream end and 16 samples were taken at the downstream end.

DATA REDUCTION

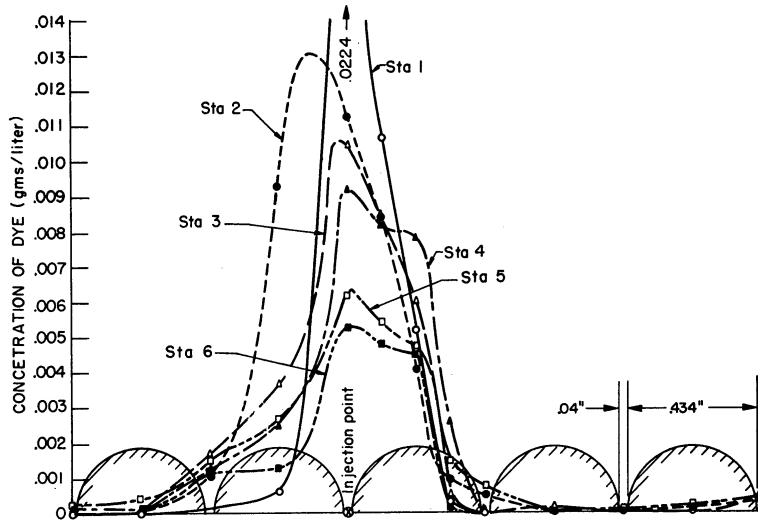
For both line and sectional traverses, the measured values of concentration of the various positions were used to obtain the value of the standard

(a)



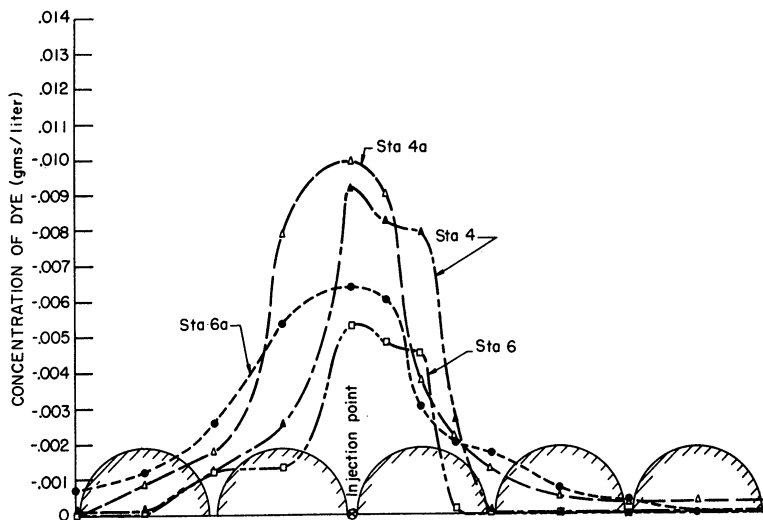
(a) Horizontal traverses, data at 4 stations with injection on vertical centerline

(b)



(b) Horizontal traverse data at 6 stations with injection needle displaced

(c)



(c) Comparison of horizontal and vertical traverses at two stations

Fig. 3. Line traverses of dye concentration ($V = 4$ ft/sec).

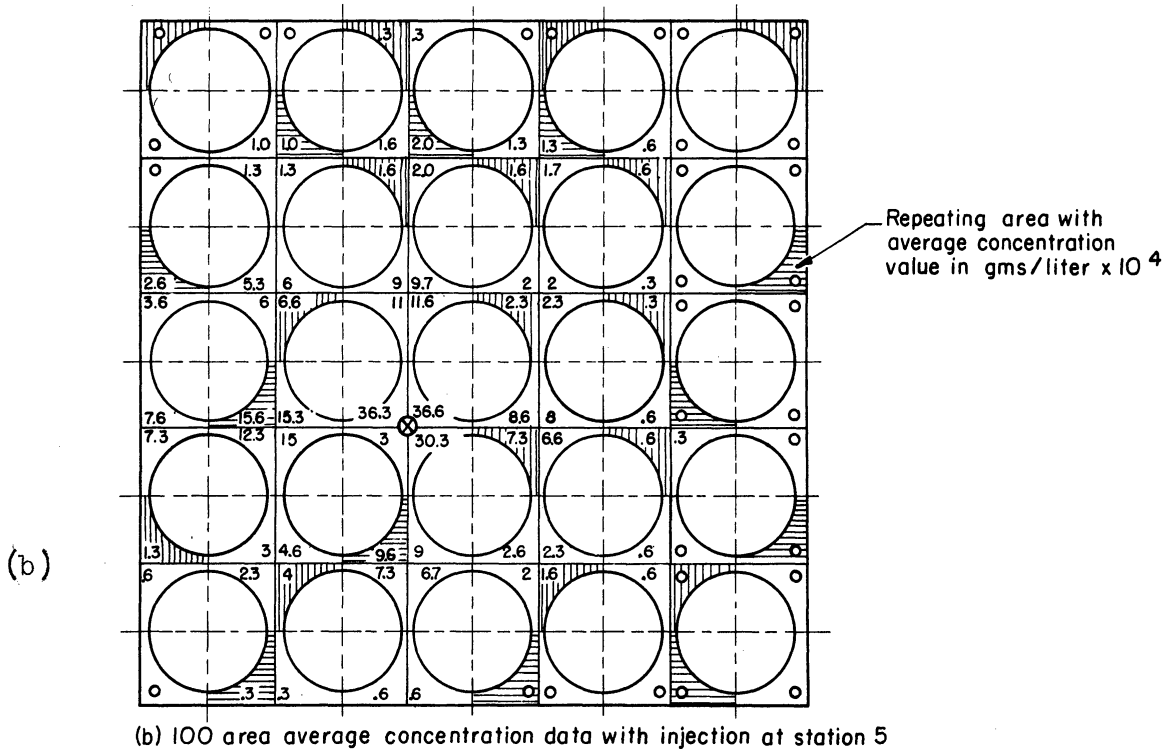
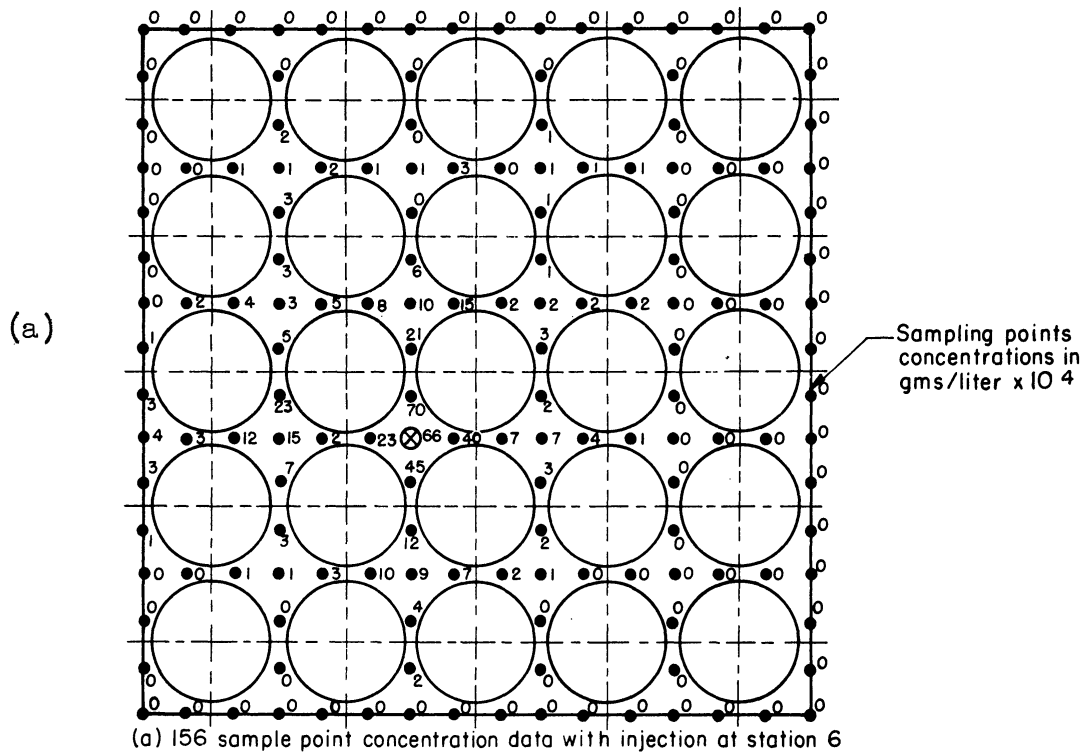


Fig. 4. Variations of dye concentration over a section of the blanket sub-assembly ($V = 4.65$ ft/sec).

deviation. The rate of change of the square of this value with position along the sub-assembly, $d\sigma^2/dx$ (ft), together with the mean velocity V (ft/sec) was then sufficient to estimate the diffusivity ϵ (ft²/sec):

$$\epsilon = \frac{V}{2} \frac{d\sigma^2}{dx} . \quad (1)$$

The relationship is based on G. I. Taylor's statistical theory of turbulence² and is presented in useful form by Baines and Peterson.³

Three different approaches were used for the reduction of the data from the sectional traverses. The value of σ^2 was determined in three ways: (a) using all 156 points such as are shown in Fig. 4a; (b) using 100 values based on averages to represent each of the 100 repeating areas shown in Fig. 4b; and (c) using only the 36 values read at the centers of the spaces. The results showed surprisingly little variation. For the data shown in Fig. 4b, for example, the values of σ^2 obtained in the three ways were 0.0019, 0.0018, and 0.0019 ft², respectively. The values of σ^2 for the sectional traverses and typical values for both horizontal and vertical line traverses are shown in Fig. 5. For the line traverses the values plotted are those for an axially symmetric distribution which are twice the values for the (plane) areas shown in Fig. 3. The dye concentration values obtained for any one line traverse were considered the same as those which would have been read along any other central line, and the volume of revolution thus included equivalent to a sectional traverse.

Various values of $d\sigma^2/dx$ were obtained for the several methods of obtaining σ^2 , and corresponding values of ϵ were evaluated from Eq. (1). Thus each set of data of the type shown in Fig. 5 gives a different slope, and the curve actually shown in Fig. 5 represents only generally the slope expected as four different velocities are represented. The curve differs in shape from standard diffusion results. Obviously the initial stages of diffusion differ markedly from the average. Hence, the final points are given more weight than are the initial ones. The longitudinal distances may be too short to establish a truly representative value of ϵ .

The values of ϵ for the blanket element were between .0017 and .0025 ft²/sec, and those for the core element were 0.0012 and 0.0013 ft²/sec. Although the range is considerable, it was present in spite of extreme care in collecting data and of lengthy data reduction methods as well as in the relatively simple line traverse data. A certain amount of variation in the system is to be expected for this kind of data and particularly for such a complex flow system.

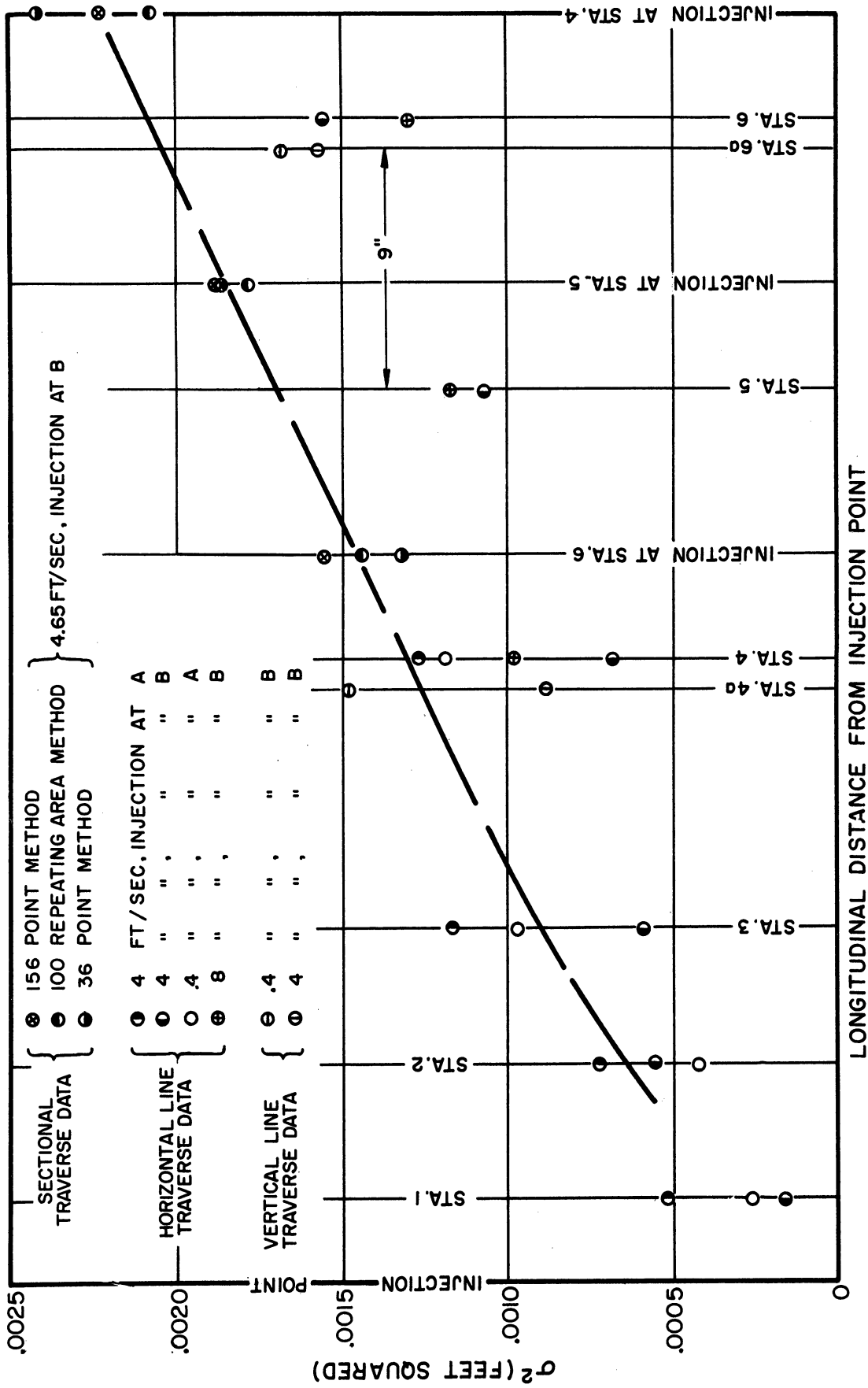


Fig. 5. Composite plot of standard deviations obtained by the several methods.

C. ANALYSIS

MATHEMATICAL FORMULATION OF THE PROBLEM

Prediction of the temperature distribution in an element of the reactor is complicated by the distribution of the heat sources, by the difference in diffusivity of the solid and of the liquid, and by the variations of fluid velocity and of eddy diffusivity across a section. To obtain a solution for the temperature distribution without excessive labor, a few simplifying assumptions must be made. If the velocity V and the eddy diffusivity ϵ are assumed to be constants, and if the longitudinal diffusion is neglected, the differential equation for the temperature T is

$$V \frac{\partial T}{\partial x} = \alpha \left(\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right), \quad (2)$$

in which α is the sum of the molecular diffusivity α_m and ϵ ; Cartesian coordinates are used, with x measured longitudinally from the entrance section. The boundary conditions at the walls are determined from the assumption that they are insulated.

FIRST SOLUTION FOR THE TEMPERATURE DISTRIBUTION

The first solution for the temperature distribution is based on the assumption that the heat sources are concentrated at the center lines of the 25 rods in each blanket element of the reactor arranged as in Fig. 6. The solution of Eq. (2) for the steady source of Q units of heat per unit of time located at (x_0, y_0, z_0) is

$$T - T_0 = \frac{Q(x_0, y_0, z_0)}{4\pi k(x-x_0)} \exp \left\{ \frac{-V[(y-y_0)^2 + (z-z_0)^2]}{4\alpha(x-x_0)} \right\}, \quad (3)$$

in which T_0 is the temperature of the fluid at the entrance section and k is the thermal diffusivity. The boundary conditions arising from the insulation of the walls can be satisfied by using the principle of reflections, i.e., by including the effects of successive images of the sources across the walls and their extensions.

If the strength of the source at $x = x_0$ for a rod in the i -th column (Fig. 6) is $Q_i(x_0)$ in heat per unit length per unit time, the temperature distribution can be calculated from the formula

$$T - T_0 = 4 \sum_{i=0}^4 \sum_{m_i} \sum_{n_i} \int_0^x \frac{Q_i(x_0)}{4\pi k(x-x_0)} \exp \left\{ \frac{-V[(y-m_i s)^2 + (z-n_i s)^2]}{4\alpha(x-x_0)} \right\} dx_0, \quad (4)$$

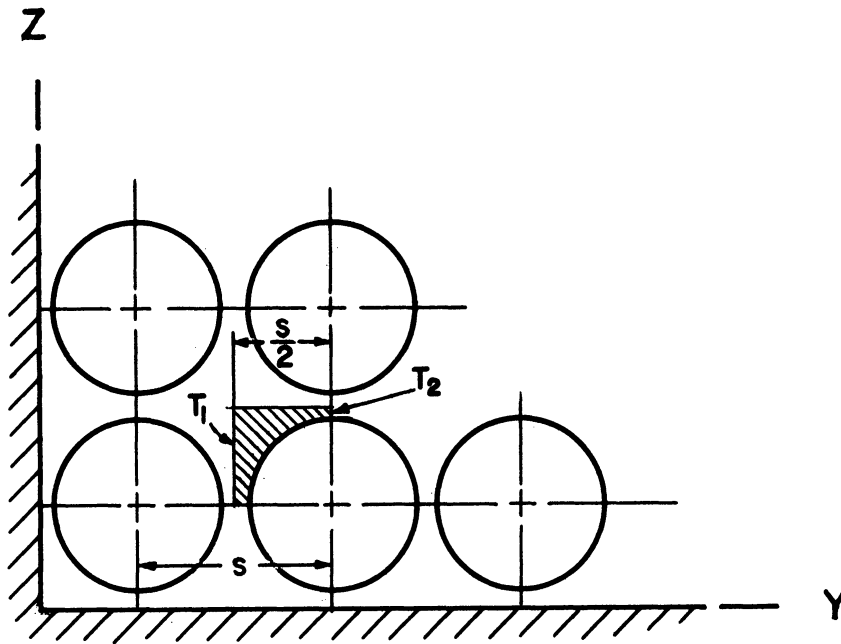


Fig. 6. Geometry for heat conduction problem.

in which

$$m_i = 5 + 10p \pm \left(i + \frac{1}{2}\right) \quad (p = 0, 1, 2, \dots) ,$$

and

$$n_i = p + \frac{1}{2} .$$

The factor 4 enters because symmetry with respect to the two coordinate planes has been utilized. The actual evaluation of the temperature from Eq. (4) for a given function $Q_i(x_0)$ can be facilitated by using either the tabulation of the heat-source function⁴ or, if greater accuracy is necessary, the tabulation of the error function in the Standard Mathematical Tables.⁵

SECOND SOLUTION FOR THE TEMPERATURE DISTRIBUTION

The second solution for the temperature distribution is based on the assumption that an outward heat flux equal to the total strength of the local sources is maintained on the surface of each rod. The temperature distribution due to one rod with an outward heat flux $Q(x_0)$ per unit time per unit area at $x = x_0$ is, in the absence of the other rods, given by Carslaw and Jaeger:⁶

$$T - T_0 = \int_0^x \phi(r, x-x_0) \frac{dQ(x_0)}{dx_0} dx_0 + Q(0) \phi(r, x) \quad (5)$$

in which

$$\phi(r,x) = -\frac{2}{\pi k} \int_0^{\infty} \left[1 - e^{-\frac{\alpha u^2 x}{V}} \right] \frac{J_0(ur)Y_1(ua) - Y_0(ur)J_1(ua)}{u^2[J_1^2(ua) + Y_1^2(ua)]} du .$$

If the diffusivity of the fluid is assumed to be the same as that of the solid, heat originating from one bar will go unobstructed through the others, and the principle of superposition can be used. Thus, the second solution for the temperature distribution is obtained by the appropriate summation of expressions obtained from (5) for each rod.

DISCUSSION OF THE TWO SOLUTIONS

The two solutions are based on two extreme conditions as far as the locations of the heat sources are concerned. The true distribution of the heat sources is neither along the centerlines of the rods nor such that the total heat efflux from a rod is equal to the total strength of the heat sources at the same section. Which solution is more nearly correct? This question can be partially answered by considering the effects on temperature prediction of the assumption of equal diffusivities taken together with each particular assumption as to the location of the heat sources.

If the calculations were based on the exact diffusivities of the solid and the liquid, the assumption that the heat sources are concentrated at the centerlines would have the effect of making the calculated heat efflux from each rod smaller than it should be, whereas that of immediate transfer of heat to the fluid (on which the second solution is based) would have an opposite effect. On the other hand, since the diffusivity of the solid is less than that of the liquid, the assumption of equal diffusivities (liquid diffusivity being used) has the effect of making the calculated heat flux larger than it should be. Thus, the errors arising from the assumption of centerline concentration of heat sources and that of equal diffusivities, on which the first solution is based, are compensatory, whereas the errors known to be introduced into the second solution are additive. For this reason, and because Eq. (4) is mathematically simpler than Eq. (5), the first solution is recommended for the calculation of the temperature distribution. The second solution could be used to give an upper bound to the result if needed.

CORRELATION OF THE APPARENT WITH THE LOCAL EDDY DIFFUSIVITY

To calculate the temperature distribution in the liquid, one must know the value of the total diffusivity α . Since the molecular diffusivity is known, the value of α is known if the eddy diffusivity is known. This latter can only be obtained by experiment. As described in the preceding section, the apparent eddy diffusivity was obtained by measuring the diffusion of dye injected into the fluid. The local eddy diffusivity remains to be evaluated from the apparent value obtained experimentally.

Because dye was used in the experiments and because the boundaries are impervious to dye, to correlate the apparent with the local eddy diffusivity one can consider a hypothetical problem of heat conduction with insulated boundaries. Furthermore, since the object is only to correlate the two diffusivities, one may, for simplicity, consider steady heat conduction only. If the change of apparent diffusivity with direction is assumed to be small, the problem can be idealized to one of steady heat conduction, principally in the y-direction (Fig. 6), through the fluid between two insulated (because of symmetry) boundaries and surrounding insulated cylinders. The symmetry in geometry allows one to concentrate on heat conduction in the shaded area (Fig. 6) only.

For a given fluid, the ratio of the local to the apparent diffusivity is the same as that of the local to the apparent conductivity. The latter, in turn, is the same as the ratio of the heat flux in the y-direction in the absence of the rods to that in their presence for the same temperature difference ($T_1 - T_2$) (Fig. 6). In the numerical calculation (relaxation) ECD and AB (Fig. 7) are taken as the streamlines of the heat flow from the isotherm AE to the isotherm BC. Under the assumption that the heat flux is 100 units and that the conductivity is unity, the difference in temperature between AE and BC is found to be approximately 540 units. The same difference would give rise to a heat flux of 540 units in the absence of the cylinder. As a consequence of the smallness of the molecular diffusivity in water (which was used in the experiments), the apparent total diffusivity α_a determined experimentally is essentially the apparent eddy diffusivity ϵ_a . Thus, the ratio of the local eddy diffusivity to the apparent eddy diffusivity is 5.4. This is assumed to be the same as for the prototype in which liquid sodium is used.

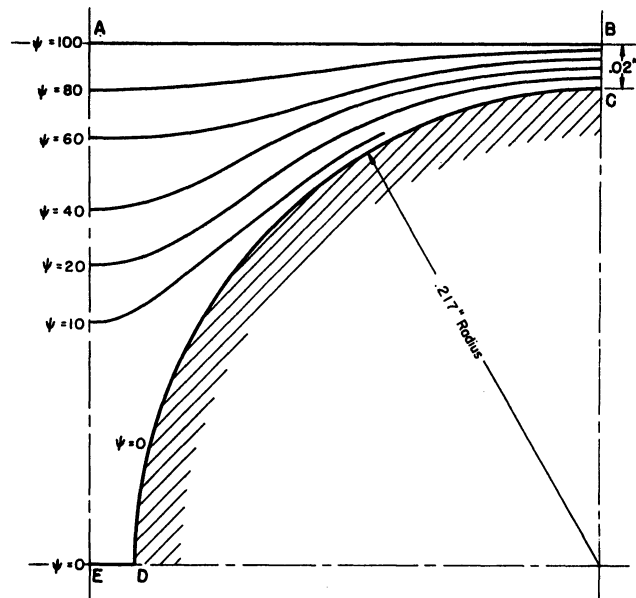


Fig. 7. Pattern of heat flow.

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