

Phase Stability and Physical Properties of Cubic and Tetragonal ZrO_2 in the System ZrO_2 - Y_2O_3 - Ta_2O_5

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The cubic (c - ZrO_2) and tetragonal zirconia (t - ZrO_2) phase stability regions in the system ZrO_2 - Y_2O_3 - Ta_2O_5 were delineated. The c - ZrO_2 solid solutions are formed with the fluorite structure. The t - ZrO_2 solid solutions having a c/a axial ratio (tetragonality) smaller than 1.0203 display high fracture toughness (5 to 14 $MPa \cdot m^{1/2}$), and their instability/transformability to monoclinic zirconia (m - ZrO_2) increases with increasing tetragonality. On the other hand, the t - ZrO_2 solid solutions stabilized at room temperature with tetragonality greater than 1.0203 have low toughness values (2 to 5 $MPa \cdot m^{1/2}$), and their transformability is not related to the tetragonality. [Key words: zirconia, solid solution, lattice, phases, physical properties.]

I. Introduction

THE alloying of Y_2O_3 into ZrO_2 stabilizes the high-temperature phases, cubic ZrO_2 (c - ZrO_2) and tetragonal ZrO_2 (t - ZrO_2), by suppressing the deleterious phase transformation to monoclinic ZrO_2 (m - ZrO_2) which is the room-temperature-stable form of the pure ZrO_2 . The substitution of Y^{3+} for Zr^{4+} in the formation of the solid solutions results in a creation of oxygen-ion vacancies to balance the change in valence. The fluorite structure c - ZrO_2 , alloyed with more than 8 mol% Y_2O_3 , is a stable phase at ambient temperature and has been used as ionic conductors. The distorted fluorite structure t - ZrO_2 , alloyed with about 2 to 3 mol% Y_2O_3 , is a metastable phase at room temperature and has been considered as a structural ceramic because of its excellent mechanical properties, such as fracture toughness, strength, and hardness.

The stability regions and the ionic conductivities of c - ZrO_2 in the system ZrO_2 - Y_2O_3 - Ta_2O_5 ¹ and the related ternary system ZrO_2 - MgO - Ta_2O_5 ² are of interest mainly because the alloying of Ta_2O_5 into c - ZrO_2 in the binary systems is expected to suppress the oxygen-ion vacancy formation as a result of the substitution of Ta^{5+} for Zr^{4+} . On the other hand, recently a small amount of Ta_2O_5 or Nb_2O_5 alloying into t - ZrO_2 in the system ZrO_2 - Y_2O_3 was found to remarkably increase its fracture toughness, which is related to the increase in the tetragonality of t - ZrO_2 symmetry.³ While technologically important, the studies of phase relationships in ternary systems with pentavalent oxides are either incomplete or lack details about the stability of t - ZrO_2 solid solutions.^{1,2,4}

The purpose of this study is to investigate the phase stability regions of c - ZrO_2 and t - ZrO_2 and the mechanical and

crystallochemical properties of t - ZrO_2 alloys in the system ZrO_2 - Y_2O_3 - Ta_2O_5 .

II. Experimental Procedure

The starting powders were prepared by the coprecipitation from aqueous solutions of $ZrOCl_2 \cdot 8H_2O$, $Y(NO_3)_3 \cdot 6H_2O$, and $TaCl_5$ using ammonia. The coprecipitation was accomplished at pH 7.6 by pouring the aqueous mixture into vigorously stirred ammonia. The precipitates were dried for 20 h at 120°C after washing with distilled water and decomposed to the oxides by calcination for 3 h at 920°C. The oxide agglomerates were attrition-milled using ZrO_2 balls. Pellets of each composition were fired on platinum foil at 1500°C for up to 165 h. The fired pellets were examined by X-ray diffraction (XRD) for phase equilibria. The fracture toughness was determined by the indentation technique using the equation proposed by Anstis *et al.*⁵ The Young's modulus was calculated from the bulk density, and the longitudinal and transverse wave velocity measured with an ultrasonic transducer.⁶ The bulk density of specimens was determined by the Archimedes method. A dilatometer was used to measure the phase transformation temperatures of m - $ZrO_2 \rightarrow t$ - ZrO_2 (A_s) and t - $ZrO_2 \rightarrow m$ - ZrO_2 (M_s) which correspond to the starting points of the discontinuous volume shrinkage and expansion of the samples during heating and cooling, respectively. The dilatometry samples, which were prepared by pressing the powders isostatically at 170 MPa, were heated from room temperature to 1200°C and cooled at the same rate of 7.5°C/min. For lattice parameter determination, X-ray powder diffraction data from the pressed and sintered samples coated with a thin layer of Si powder (SRM640b) were obtained on a goniometer (2 θ / θ , Scintag, Santa Clara, CA) equipped with a solid-state detector and a Cu X-ray tube, $\lambda(CuK\alpha_1) = 1.5405981 \text{ \AA}$. A detail of the procedure and the X-ray powder diffraction data for t - ZrO_2 solid solutions in the system ZrO_2 - $YTaO_4$ are described elsewhere.⁷ The Raman spectra were obtained with the 514.5-nm line of a 0.35-W argon laser.

III. Results and Discussion

An isothermal section of the phase diagram for the system ZrO_2 - Y_2O_3 - Ta_2O_5 at 1500°C is determined and illustrated in Fig. 1. The dome-shape c - ZrO_2 phase region is consistent with the stability boundary reported by Choudhary and Subbarao,¹ and a similar ternary t - ZrO_2 phase field is also found in the system ZrO_2 - MgO - Ta_2O_5 .² In Fig. 2 the lattice parameters of c - ZrO_2 solid solutions in the system ZrO_2 - Y_2O_3 - Ta_2O_5 , measured by Choudhary and Subbarao,¹ are compared with the calculated values from the empirical equation for the fluorite structure c - ZrO_2 solid solutions.⁸ With the ionic radii in eightfold coordination, that is, $r_{Zr^{4+}} =$

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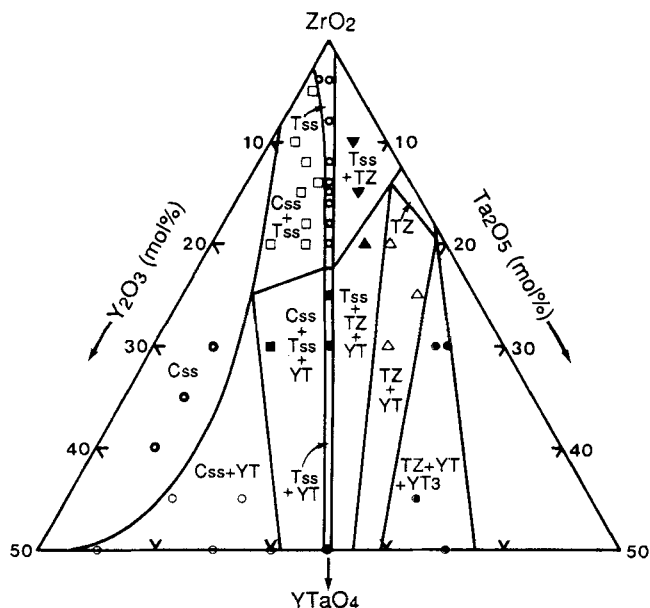


Fig. 1. Part of the ternary phase diagram for the system ZrO_2 - Y_2O_3 - Ta_2O_5 at $1500^\circ C$. T_{ss} is t - ZrO_2 solid solution, C_{ss} is c - ZrO_2 solid solution, TZ is $Ta_2Zr_6O_{17}$, YT is $YTao_4$, YT_3 is $YTao_3O_9$.

0.84 \AA , $r_{Y^{3+}} = 1.019 \text{ \AA}$, and $r_{Ta^{5+}} = 0.74 \text{ \AA}$,⁹ the equation for the lattice constants (in \AA) can be written as

$$d = 5.120 + 0.00141m_Y + 0.00027m_{Ta} \quad (1)$$

where m_Y and m_{Ta} are mol% of $YO_{1.5}$ and $TaO_{2.5}$, respectively. The remarkable agreement between the measured and the calculated lattice constants in Fig. 2 suggests that Y^{3+} and Ta^{5+} are randomly substituted for Zr^{4+} in 8-fold lattice sites of the fluorite structure.

The effect of Ta_2O_5 alloying on the phase stability of c - ZrO_2 is shown in Fig. 3(A), where Ta_2O_5 was increasingly added to 8 mol% Y_2O_3 -stabilized c - ZrO_2 . The Ta_2O_5 alloying in the c - ZrO_2 results in the decrease of the phase stability of c - ZrO_2 , which is indicated by the diminishing XRD peak corresponding to c - ZrO_2 with increasing amount of Ta_2O_5 . On the other hand, Fig. 3(B) shows the decrease in t - ZrO_2 stability by Ta_2O_5 alloying. The intensity of the XRD peak assigned to t - ZrO_2 becomes weak and those related to m - ZrO_2

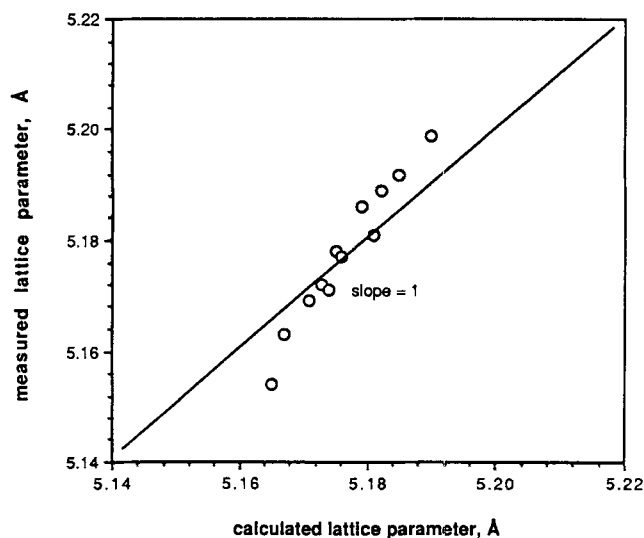


Fig. 2. Comparison of measured (Ref. 1) and calculated lattice parameters of cubic ZrO_2 solid solutions in the system ZrO_2 - Y_2O_3 - Ta_2O_5 .

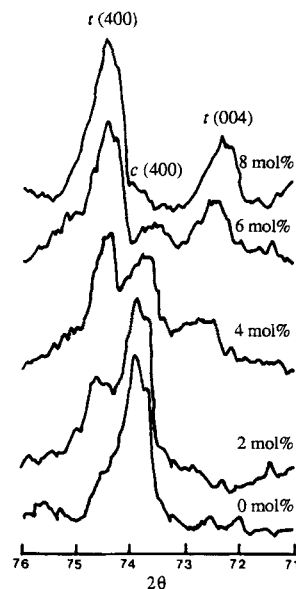


Fig. 3(a). Ta_2O_5 alloying effect on phase stability of 8 mol% Y_2O_3 -stabilized cubic ZrO_2 after sintering for 5 h in $1500^\circ C$.

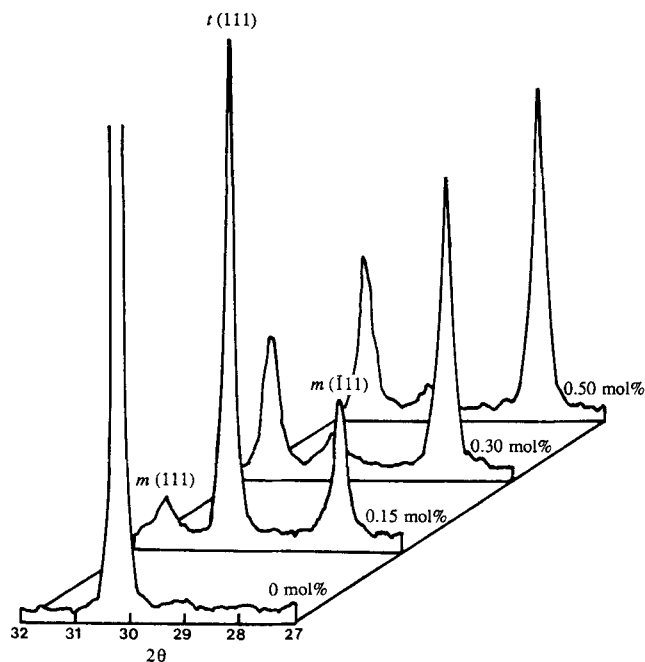


Fig. 3(b). Ta_2O_5 alloying effect on phase stability of 2 mol% Y_2O_3 -stabilized tetragonal ZrO_2 after sintering for 5 h at $1500^\circ C$.

become strong as the Ta_2O_5 alloying into 2 mol% Y_2O_3 -stabilized t - ZrO_2 increases. The Ta_2O_5 alloying is reported to have little effect on the grain size of Y_2O_3 -stabilized t - ZrO_2 .³ Thus, the Ta_2O_5 behaves as a destabilizer for the high-temperature phases of ZrO_2 .

The quasi-binary system ZrO_2 - $YTao_4$ is unique because the crystallochemical characteristics of ZrO_2 are similar to those of $YTao_4$, which has a monoclinic distorted scheelite structure at ambient temperature and transforms to a tetragonal scheelite form above $1400^\circ C$.¹⁰ The t - ZrO_2 single-phase region in Fig. 1 extends to about 22 mol% $YTao_4$, which corresponds to $0.78ZrO_2 \cdot 0.11Y_2O_3 \cdot 0.11Ta_2O_5$. This t - ZrO_2 stability boundary is more extensive than that in the analogous system ZrO_2 - $MgTa_2O_6$, where the t - ZrO_2 single phase exists up to only 5 mol% $MgTa_2O_6$ at $1500^\circ C$.² The t - ZrO_2 solid solutions containing up to 11 mol% $YTao_4$ transform to

m-ZrO₂ with large volume expansion during cooling from 1500°C. As shown in Fig. 4, the increase in the YTaO₄ content decreases the martensitic (*t* → *m*) and the reverse (*m* → *t*) transformation temperatures, which are designated by *M*_s and *A*_s, respectively. The lowered transformation temperatures result in a decrease of the chemical driving force for the *t* → *m* transformation so that *t*-ZrO₂ phase becomes stable. Accordingly, YTaO₄ behaves as a stabilizer for *t*-ZrO₂.

Two phases of *t*-ZrO₂ and *m*-ZrO₂ are observed at concentrations between 12 and 14 mol% YTaO₄ after sintering for 16 h at 1500°C. A single phase of *t*-ZrO₂ is stabilized from 15 to 22 mol% YTaO₄ after the same sintering schedule. The *t*-ZrO₂ with 15 mol% YTaO₄ transforms to *m*-ZrO₂ during grinding. However, the *t*-ZrO₂ solid solutions in the composition range of 16 to 22 mol% YTaO₄ do not transform to *m*-ZrO₂ even for a wide range of grain sizes or aging temperatures. A typical grain size of *t*-ZrO₂ with 16 mol% YTaO₄ is determined to be 5 μm after sintering at 1500°C for 120 h.¹¹ The *t* → *m* transformation during aging at low temperatures^{12,13} and the grain size dependence of the transformations¹⁴ are characteristic of transformable *t*-ZrO₂. Thus, the *t*-ZrO₂ solid solutions stabilized in the system ZrO₂-YTaO₄ are different from the *t*-ZrO₂, which possesses excellent mechanical properties. From 22 mol% YTaO₄, tetragonal scheelite YTaO₄ starts to form. The room-temperature lattice parameters of the fergusonite structure YTaO₄ are determined to be *a* = 5.144 ± 0.006 Å and *c* = 10.774 ± 0.042 Å in this study. The two-phase field of *t*-ZrO₂ and tetragonal YTaO₄ extends up to about 80 mol% YTaO₄, which is not shown in Fig. 1. From this composition, a single phase of monoclinic YTaO₄ exists.

The tetragonality of *t*-ZrO₂ at concentrations between 14 and 22 mol% YTaO₄ increases with the YTaO₄ content, as shown in Fig. 5 and Table I. Since the fracture toughness of *t*-ZrO₂ alloys, stabilized at room temperature in the ZrO₂-rich region in Fig. 1, proportionally increases with their tetragonality,³ it is expected that the toughness of *t*-ZrO₂ in the system ZrO₂-YTaO₄ should increase with increasing the YTaO₄ concentration in ZrO₂. As illustrated in Table I, however, the toughness decreases with increasing the tetragonality. This is because the tetragonal phase in this composition range is so stable that there is no stress-induced martensitic phase transformation, regardless of the tetragonality. Additionally, the decrease in the Young's modulus with YTaO₄ alloying causes the low calculated fracture toughness. For comparison, the fracture toughness of the transformable *t*-ZrO₂ alloys in the

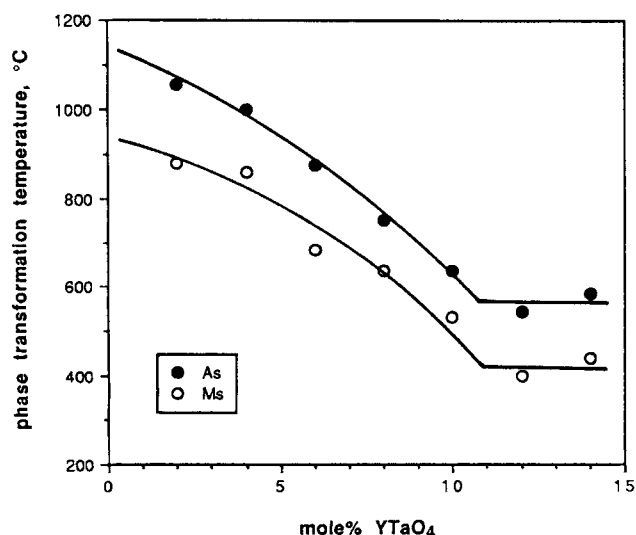


Fig. 4. *m*-ZrO₂ → *t*-ZrO₂ (*A*_s) and *t*-ZrO₂ → *m*-ZrO₂ (*M*_s) phase transformation temperatures in the quasi-binary system ZrO₂-YTaO₄ as a function of mol% YTaO₄.

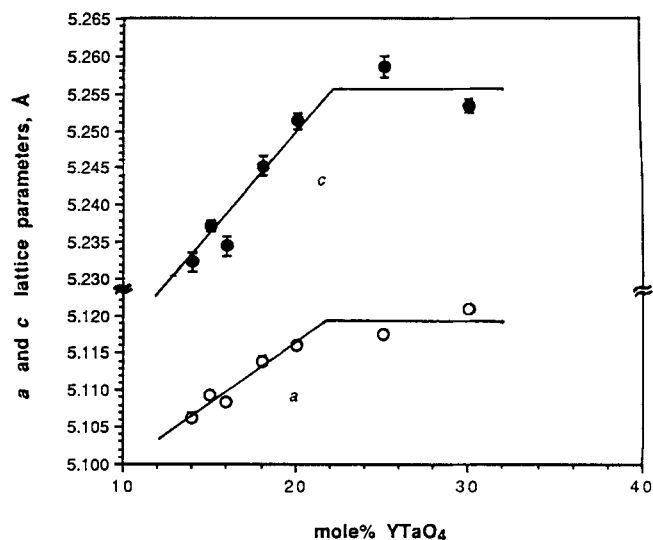


Fig. 5. *a* and *c* lattice parameters of *t*-ZrO₂ solid solutions in the system ZrO₂-YTaO₄.

ZrO₂-rich field of the system ZrO₂-Y₂O₃-Ta₂O₅ is determined to be 5 to 14 MPa·m^{1/2} for tetragonality of 1.0141 to 1.0188, respectively, with the average Young's modulus and hardness of 210 GPa and 12 GPa.³

The value of 1.0203 is the largest possible room-temperature tetragonality of *t*-ZrO₂, which belongs to the space group *P*₄/nmc.³ The tetragonality of *t*-ZrO₂ solid solutions in the binary systems of ZrO₂ rare-earth and alkaline-earth element oxides (stabilizers) decreases toward unity, which corresponds to *c*-ZrO₂, as the stabilizer content increases. The lowered tetragonality by the alloying of stabilizers is consistent with an enhanced phase stability of *t*-ZrO₂, which is metastable at room temperature. The phase stability of *t*-ZrO₂ with the extraordinarily high tetragonality in the system ZrO₂-YTaO₄, which ranges from 1.0246 to 1.0264, implies that there is a difference in details of the crystal structure between two *t*-ZrO₂ solid solutions. Besides their lattice parameters, XRD patterns exhibit no difference in crystal structure between the transformable *t*-ZrO₂, alloyed with 2 mol% Y₂O₃, and the nontransformable *t*-ZrO₂, alloyed with 16 mol% YTaO₄, as shown in Fig. 6. The nontransformability of a similar *t*-ZrO₂ with an exceptionally high tetragonality of 1.0327 in the system ZrO₂-TiO₂-CeO₂ has been reported.¹⁵ However, the nontransformability of the *t*-ZrO₂ in this system was related to the presence of short-range ordering of the anion lattice based on the observation of extra peaks on the XRD pattern. No such extra peaks are found from the XRD patterns of *t*-ZrO₂ solid solutions in the system ZrO₂-YTaO₄ in Fig. 6.

Assuming the *t*-ZrO₂ in the system ZrO₂-YTaO₄ was formed by a random substitution of Y³⁺ and Ta⁵⁺ for Zr⁴⁺ in eightfold coordination lattice sites, the changes in the lattice parameters (Å) as a function of mol% YTaO₄ are expected to follow the equations³

$$a = 5.086 + 0.00201m \quad (2)$$

$$c = 5.189 + 0.00168m \quad (3)$$

where *m* is calculated by $m = 100(\text{mol\% YTaO}_4) / (100 + \text{mol\% YTaO}_4/2)$. The slope for *a* axis in Eq. (2) is predicted to be larger than that for *c* axis in Eq. (3) but the experimental measurements in Fig. 5 exhibit a reverse trend. The poor prediction of the lattice parameters by using Eqs. (2) and (3) may be derived from a difference in either space group or mechanism for solid-solution formation between the *t*-ZrO₂ in the system ZrO₂-YTaO₄ and the *t*-ZrO₂ stabilized by rare-earth element oxides. Nevertheless, the least-squares plots of the *a* and *c* lattice constants in Fig. 5

Table I. Selected Mechanical Properties and c/a Axial Ratio of t -ZrO₂ in the System ZrO₂-YTaO₄ Prepared by Sintering for 16 h at 1500°C

YTaO ₄ content (mol%)	Modulus (GPa)	Hardness (GPa)	K_{IC} (MPa·m ^{1/2})	c/a
14				1.0246
15	140			1.0250
16	157	11.3	5.4	1.0247
18	152	9.4	2.9	1.0257
20	144	8.8	2.5	1.0264

extrapolate to 5.083 and 5.188 Å for pure t -ZrO₂, respectively, which are in good agreement with the values in Eqs. (2) and (3). This indicates that the stable t -ZrO₂ solid solutions in the system ZrO₂-YTaO₄ are related to the crystal structure of the space group $P4_2/nmc$ so that the XRD patterns in Fig. 6 coincide. Despite the XRD patterns, the Raman spectroscopy

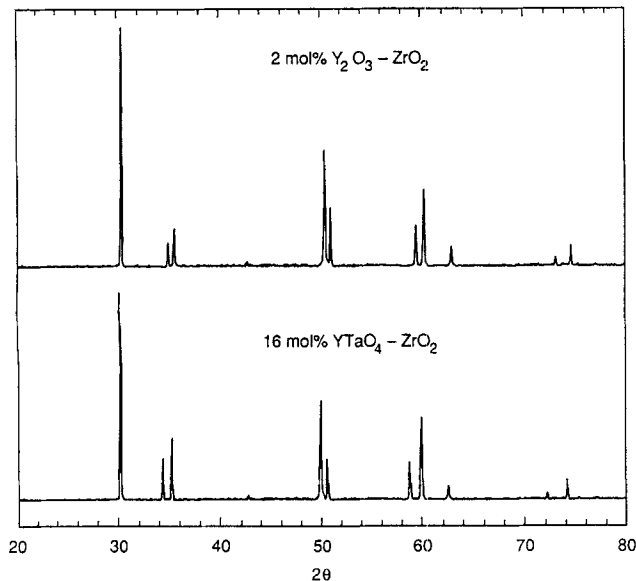


Fig. 6. Comparison of XRD patterns of 2 mol% Y₂O₃-stabilized t -ZrO₂ and 16 mol% YTaO₄-stabilized t -ZrO₂.

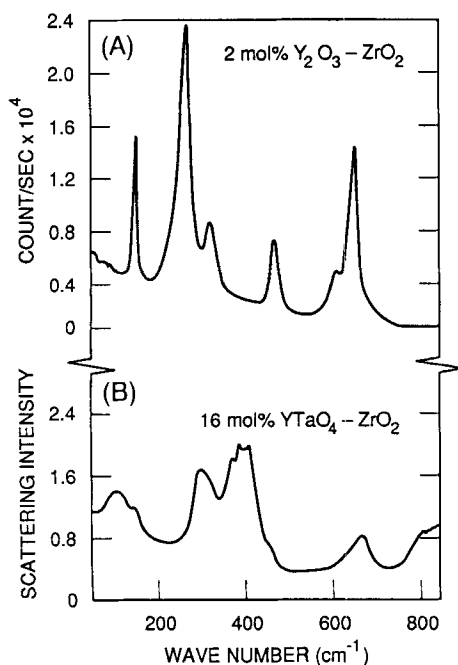


Fig. 7. Comparison of Raman spectra of (A) 2 mol% Y₂O₃-stabilized t -ZrO₂ and (B) 16 mol% YTaO₄-stabilized t -ZrO₂.

reveals a dissimilarity in the spectra between the transformable (with 2 mol% Y₂O₃) and the nontransformable (with 16 mol% YTaO₄) t -ZrO₂ solid solutions as shown in Fig. 7. The spectra, obtained from the 2 mol% Y₂O₃-stabilized t -ZrO₂, are typical of the distorted fluorite structure t -ZrO₂.¹⁶ The Raman line of 466 cm⁻¹ in the 2 mol% Y₂O₃-stabilized t -ZrO₂, which is unique for MO₂ oxides (M = Zr⁴⁺, Ce⁴⁺, Th⁴⁺, U⁴⁺) with the fluorite structure,^{16,17} is absent in the t -ZrO₂ stabilized with 16 mol% YTaO₄. This implies that there are differences in the strength and the length of cation-oxygen ion bonds and the cation coordination between the two types of t -ZrO₂ solid solutions, although a close investigation should be pursued to understand the crystal structure of the t -ZrO₂ phase in the system ZrO₂-YTaO₄. Indeed, a small Nb⁵⁺ ($r_{Nb^{5+}} = 0.74$ Å), which is similar to Ta⁵⁺, most likely takes an interstitial position in the lattice of the t -ZrO₂, stabilized by 2 mol% Y₂O₃.¹⁸

In each binary system of ZrO₂ rare earth or alkaline earth, c -ZrO₂ starts to form as stabilizer content exceeds an amount for t -ZrO₂ single phase. In these systems, t -ZrO₂ is metastable at room temperature, and the tetragonality represents a degree of cubic fluorite structure distortion or microstrain due to the lattice anisotropy. Accordingly, a small tetragonality by an increased amount of stabilizer in t -ZrO₂ results in low phase transformability. The decreased transformability causes a small volume fraction of t -ZrO₂ to transform to m -ZrO₂ during the stress-induced transformation process and contributes to a low fracture toughness. In the system ZrO₂-YTaO₄, however, the tetragonal fergusonite structure YTaO₄ starts to form as YTaO₄ content exceeds the amount for t -ZrO₂ single phase, which is about 22 mol% YTaO₄ in Fig. 1. Thus, the room-temperature stable phase of t -ZrO₂ with the tetragonality larger than 1.0203 in the system ZrO₂-YTaO₄ is not related to a distorted form of the fluorite structure, so that the tetragonality does not represent the degree of the cubic lattice distortion. As a result, the fracture toughness of the t -ZrO₂ in Table I is not governed by the tetragonality.

IV. Conclusions

The phase stability of both c -ZrO₂ and t -ZrO₂ in the binary system ZrO₂-Y₂O₃ decreases as a result of Ta₂O₅ alloying. The t -ZrO₂ solid solutions in the quasi-binary system ZrO₂-YTaO₄ are characterized by their high tetragonality (>1.0203) at room temperature and are not related to the cubic fluorite structure. In this case the fracture toughness is low (2 to 5 MPa·m^{1/2}) and does not depend on the tetragonality. On the other hand, the t -ZrO₂ solid solutions in the system ZrO₂-Y₂O₃-Ta₂O₅, where Ta₂O₅ is added as a minor phase, exhibit relatively low tetragonality (<1.0203) at room temperature and possess a distorted cubic fluorite structure. The fracture toughness of these t -ZrO₂ solid solutions having tetragonality smaller than 1.0203 is high (5 to 14 MPa·m^{1/2}) and increases with increasing tetragonality.

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