T H E U N I V E R S I T Y O F M I C H I G A N

COLLEGE OF ENGINEERING
Department of Mechanical Engineering
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Technical Report No. 4

FINITE DIFFERENCE SOLUTION OF STRATIFICATION AND PRESSURE RISE IN CONTAINERS

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ORA Project 07461

under contract with:

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
GEORGE C. MARSHALL SPACE FLIGHT CENTER
CONTRACT NO. Nas-8-20228
HUNTSVILLE, ALABAMA

administered through:

OFFICE OF RESEARCH ADMINISTRATION ANN ARBOR

January 1968

TABLE OF CONTENTS

TABLE OF CONTENTS (Concluded)

LIST OF FIGURES

NOMENCLATURE

A - Area, Ft^2

a - Cylindrical tank radius, Ft, Acceleration, Ft/Sec^2

b - Total tank height, Ft

C_p, C_v - Specific heats $BTU/LB_m-°F$

$\frac{Df}{Dt}$ - Substantial derivative of function f

e - Specific internal energy, BTU/Lb_m

E - Total internal energy, BTU

g - Acceleration due to local gravity, Ft/Sec^2

h - Heat transfer coefficient, $BTU/HR-Ft^2-°F$, Enthalpy, BTU/Lb_m

h_{fg} - Latent heat, BTU/Lb_m

k - Thermal conductivity, BTU/HR-Ft-°F

P - Pressure, psia

q - Heat flux, $BTU/HR-Ft^2$

r - Radial coordinate direction, Ft

R - Gas constant, $\frac{Lb_f}{Lb_m} \cdot \frac{Ft}{°R}$

t - Time, Hour

T - Temperature, °F, °R

ΔT_w - $T_w - T_{sat}$, °F

ΔT_{wmax} - Specified maximum value of ΔT_w, °F

u - Axial component of velocity, Ft/Sec

U - Dimensionless Axial velocity - Eq. (85)

v - Raidal component of velocity, Ft/Sec

V - Volume, Ft^3; Dimensionless radial velocity - Eq. (85)

w - Mass flow rate, Lb_m/Sec

x - Axial coordinate direction, Ft.

X - Liquid height in container, Ft

Δx - Finite-difference grid spacing, Ft

Z - Compressibility factor, [1]

Greek Letters

α - Thermal diffusivity, $Ft^2/Hour$

β - Coefficient of thermal expansion, $°F^{-1}$

γ - Ratio of specific heats C_p/C_v, [1]

δ - Tank wall thickness, Ft

η - Dimensionless radius, [1] - Eq. (85)

Θ - Dimensionless temperature - Eq. (85)

ζ - Dimensionless axial distance, [1] - Eq. (85)

ψ - Dimensionless stream function - Eq. (85)

ψ',ψ'' - Liquid and vapor stream functions, Equations (50) and (56) [1]

τ - Dimensionless time - Eq. (85)

μ - Kinematic viscosity, $Ft^2/Hour$

ν - Dynamic viscosity, $Lb_m/Ft\text{-}Hour$

ρ - Density, Lb_m/Ft^3

ω' - Vorticity, Equation (49) $[Ft\text{-}Sec]^{-1}$

ω - Dimensionless vorticity - Eq. (85)

Subscripts

d - discharge

f - liquid or fluid

g - gas or vapor

i - liquid-vapor interface

ℓ - liquid

o - initial state

p - pressurization

r - reduced

s - saturation

w - wall

ABSTRACT

The processes of heat and mass transfer interactions between the gas and liquid phases of a single component in cylindrical containers with axial symmetry are considered. In the general formulation attention is given to the cases of external pressurization with and without liquid discharge as well as to the nonvented condition. The governing equations are cast into finite-difference form and numerical computations are carried out for the case of a non-vented container having an imposed head flux, using ideal gas relations for the vapor properties. Of specific interest is the calculation of the pressure-time history of the container under these conditions.

I. INTRODUCTION

A number of space missions of current interest will require the storage of liquid propellants for long periods of time, varying from hours to months. The ultimate goal is to maximize the quantity of useful propellant remaining at the end of this period of time. In the limit this maximum corresponds to the non-vented condition in which the original mass of propellant is retained. Whether or not non-venting is practical depends on the maximum internal pressure which results as a consequence of the thermal interaction between the storage container and its ambient.

Higher pressures require increased structural weight of the container, and a trade-off may become necessary between tank weight and propellant loss due to venting in order to maximize the mass of propellant remaining. Each case must be evaluated for the specific mission, storage time, environment and propellant. An additional factor is the method of discharge from the tank. Where the propellant discharges to the suction of a pump, some degree of subcooling is necessary to avoid cavitation and, depending on the fluid, may require additional external pressurization just prior to discharge. The pressure at which venting might take place would thus be less than the tank design pressure.

The pressure in a tank containing two phase, liquid and vapor, is related directly to the temperature at the liquid-vapor interface for a single component system. For a binary system, the pressure depends not only on the temperature at the interface but the relative liquid-vapor concentration of the two components as well. Pressure changes take place owing to heat and mass transfer interactions between the vapor, liquid and container walls.

The processes of heat and mass transfer interactions between the gas and liquid phases of a single component in cylinderical containers with axial symmetry are considered in this study. In the general formulation presented first attention is given to the cases of external pressurization with and without liquid discharge as well as to the non-vented case. Solutions are then presented, utilizing numerical finite-difference procedures for the non-vented case.

The initial conditions of the liquid and vapor are considered to be those at equilibrium, with uniform pressure P_o and saturation temperature T_o From these initial conditions, the walls of the container undergo a thermal perturbation, such as a change in temperature or an exposure to an external heat flux, either of which may be an arbitrary function of time and axial location. The perturbations in the boundary conditions lead to a series of non-equilibrium phenomena within the container. Natural convection currents are set up in the liquid and in the vapor spaces. The liquid-vapor system

tends to adjust to the new non-equilibrium conditions within the container by transferring mass and energy across the interface by either evaporation or condensation. The conditions at the liquid-vapor interface couple simultaneous transport processes in the liquid and gas phases. In the case of self-pressurization of non-vented tanks the rate of pressure rise within the tank is governed by the rate of heat and mass transfer from the walls and the liquid phase to the gas phase. The interfacial temperature is essentially that of equilibrium (saturation) conditions corresponding to the system pressure. At the same time, the temperature of the liquid-vapor interface affects both the interfacial mass and heat transfer, as well as the convective processes in both phases. These latter processes influence the temperature gradients within both phases and in turn will have an effect on the rate of pressure rise in the ullage space. Indeed, all the processes of heat transfer from the ambient to both phases, the natural convection within the container, the interfacial phenomena and the rate of ullage pressure rise are all mutually coupled. Such interactions have been the subject of many experimental investigations.

Any analytical approach that would adequately describe the phenomena taking place in propellant tanks must take into account these interactions. This requires the calculation of the transient velocity and temperature profiles in both the gas and liquid phases, as well as the corresponding concentration distributions in the case of multicomponent systems. Owing to the complexity of these phenomena, no analytical studies have been presented which consider their simultaneous interaction. The problem may be complicated further with the presence of turbulence or boiling of the liquid near the tank walls, as their nature is not yet fully understood nor adequately described. It is evident that some assumptions are necessary for the construction of models which reasonably represent the practical situations.

A review which covered much of the available literature on pressurization, stratification and interfacial phenomena in propellant tanks is given in Ref. (1). Therefore no further survey of the literature will be presented here. However, several studies which are pertinent to the present one will be mentioned. Thomas and Morse (2) and Knuth (3) have considered the phase change of suddenly pressurized single-component liquid-vapor system. Yang, et al., (4,5,6), have solved the phase change in a suddenly pressurized single-component and binary liquid-vapor system. The model adopted in these cases was one-dimensional, with the origin of the x-axis at the initial position of the liquid vapor interface. Initially, the gas and the liquid regions are at the same temperature T_o. The conditions in the gas space are suddenly changed to a pressure P_∞ and temperature T_∞. The differential equations describing the temperature and velocity transients in both phases are:

$$\frac{\partial T_g}{\partial t} + u \frac{\partial T_g}{\partial x} = \alpha_g \frac{\partial^2 T_g}{\partial x^2} \qquad (1)$$

$$\frac{\partial T_\ell}{\partial t} = \alpha_\ell \frac{\partial^2 T_\ell}{\partial x^2} \qquad (2)$$

These are coupled at the interface by the condition

$$\rho_\ell \, h_{fg} \, u_i = k_\ell \frac{\partial T_\ell}{\partial x} - k_g \frac{\partial T_g}{\partial x} \qquad (3)$$

Thermodynamic equilibrium is assumed at the interface, i.e., the interfacial temperature is always at the saturation temperature corresponding to the ullage pressure. Experience shows that this is a reasonable assumption (1). The details of the solution as well as the differential equations describing the concentration distribution for the binary case are given in Ref. (1).

The effect of the changes in the properties of the fluids with temperature on the accuracy of the results was investigated by O'Loughlin and Glenn (7). The same one-dimensional single component model was employed to study the case of interfacial phase change with variable ullage pressure. Finite-differences were used to solve Eqs. (1) through (3). The pressure in the ullage volume was either constant or a specified function of time. For the case in which the pressure decreases with time, the interfacial temperature also is reduced with time. In this case the liquid layers which may exist at a temperature higher than the interfacial temperature are made equal to the saturation temperature. The excess energy of this liquid over that of the saturated liquid is used to evaporate some liquid into the vapor region. Likewise any supersaturated vapor that may be present is allowed to condense a fraction of the vapor, thus providing the energy required to heat the supersaturated vapor to the saturation temperature. This supersaturation is a result of the expansion of the gas during the pressure decrease process, which is assumed to be isentropic. The results show that the variation in the thermal conductivity with temperature has very little effect. This one-dimensional model does not account for the effect of wall heat transfer.

Epstein, et al., (8), used finite-differences for the calculation of the pressurization process. In this model, the axial variation of temperature is considered while the radial variation is neglected. The wall to fluid heat transfer was accounted for by introducing an effective heat transfer coefficient between the fluid and the wall. Also, effective thermal conductivities and mass diffusivities between the adjacent fluid layers were used to simulate the effects of random fluid motion. Provisions for variable tank cross-section as well as variable heat transfer coefficients were made in the program. The effect of boiling near the walls was not considered, and momentum effects were neglected. The principal advantage of this program is its simplicity and the relatively small amount of machine time required to advance the solution to a particular time level. However, the determination of the effective thermal conductivities and heat transfer coefficients and their variation with location require the determination of a large number of emperical constants, which require considerable experimental experience with a particular system.

Although some analytical models have been used to study liquid stratification in self-pressurized propellant containers, none has considered the simultaneous interactions between the liquid and the vapor phases. Most of the available studies of the pressure and temperature transients in the vapor space are experimental in nature and have served to identify some factors influencing the rate of pressure rise within the containers. The studies of Huntley (9) show that liquid stratification causes the pressure to rise at a higher rate than that calculated using the average or mixed mean liquid temperature. These also showed that stirring the liquid causes a smaller rate of pressure rise, while stirring the vapor resulted in a substantial increase in pressure. Higher rates of pressure rise were obtained with smaller ullage volumes. Liebenberg and Edescuty have shown similar results (10).

II. FORMULATION

As mentioned earlier, the formulation of the most general case will be given first. This involves external pressurization, liquid discharge and the consideration of the vented and nonvented container. The heat capacity of the container side wall is also included in the formulation. The heat capacity of the end walls of the cylindrical container is neglected. The assumptions and limitations will be mentioned as appropriate and the results and problems encountered in the solution of the unvented, non-discharge case will be given.

A cylinderical container of diameter 2a, height b and with wall thickness δ, is partially filled with a liquid as shown in Figure 1. The initial height of the liquid is X_o, and that of the vappor is $b-X_o$. The origin of the coordinate system is taken at the center of the container base with x-positive in the direction of the liquid. At any time t, the location of the liquid-vapor interface is given by $X = X(t)$. The fluid in both phases is initially at rest in an equilibrium state at a uniform temperature T_o and pressure P_o. From these initial conditions the tank side walls are subjected to an arbitrary heat flux, $q_w(x,t)$. The tank ends are assumed adiabatic but this restriction can be removed if desired. The differential equation describing the temperature-time history of the tank wall obtained by lumping the wall radially, but not axially, is, Figure (2),

$$\left(\rho c_p \delta \frac{\partial T}{\partial t}\right)_w = -\left(k \frac{\partial T}{\partial r}\right)_f + \left(\delta k \frac{\partial^2 T}{\partial x^2}\right)_w + q_w(x,t), \qquad (4)$$

where the subscripts w and f refer to the wall and the fluid, respectively. The first term on the right-hand side of Eq. (4) is the rate of heat flow from the wall to the fluid, while the second term accounts for the net axial conduction. It is usual to calculate the heat flow from the wall to the liquid by introducing the heat transfer coefficient h according to;

$$\left(k \frac{\partial T}{\partial r_f}\right) = h(T_w - T_f) \quad . \qquad (5)$$

However, the determination of the heat transfer coefficient for these cases is difficult and subject to many uncertainties. Furthermore, its a priori use in studies of this kind is not relevant. The numerical procedure used in this study allows the direct evaluation of the rate of heat flow from the wall to the liquid, thus making the use of a heat transfer coefficient unnecessary.

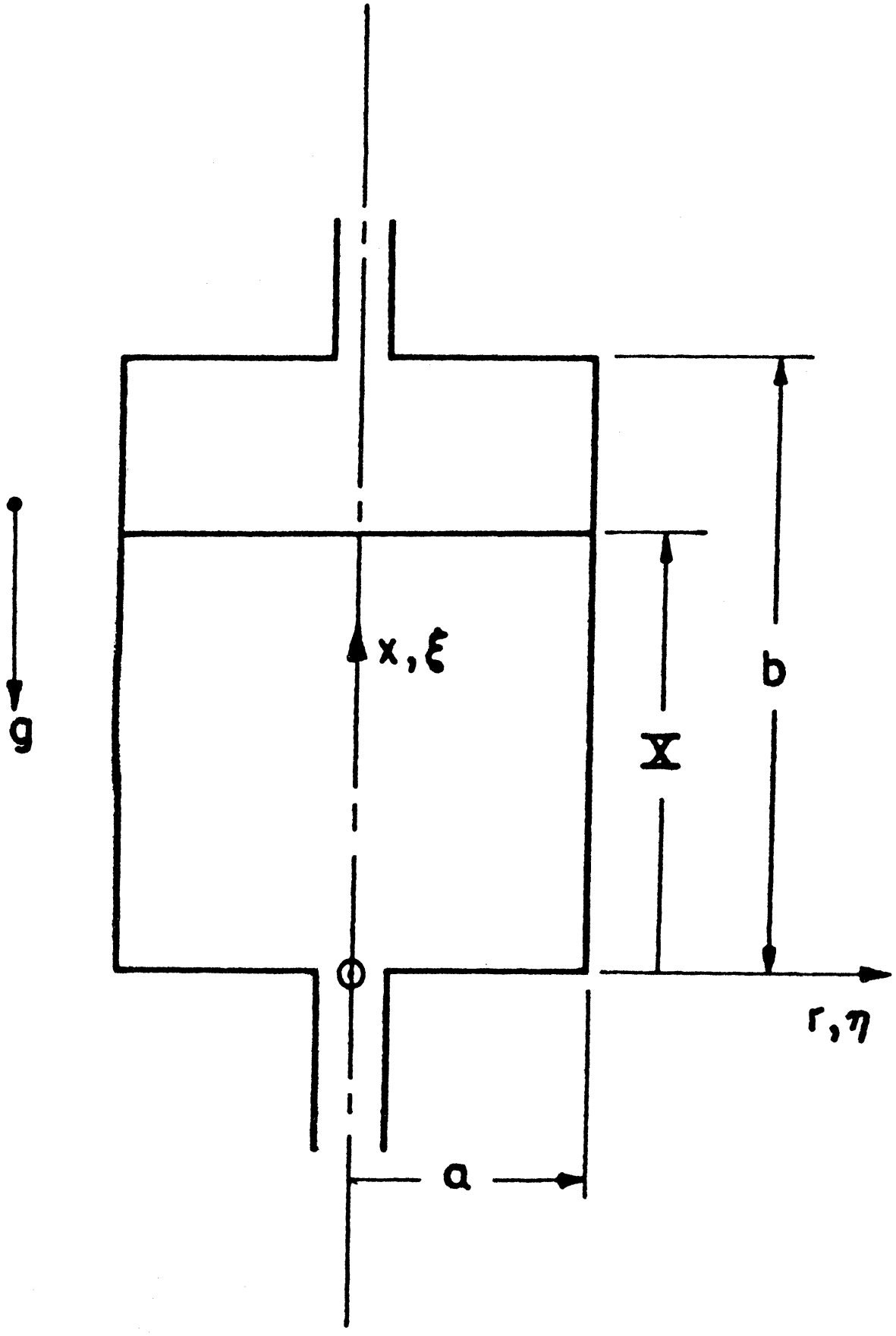


Figure 1. Container configuration and coordinate system.

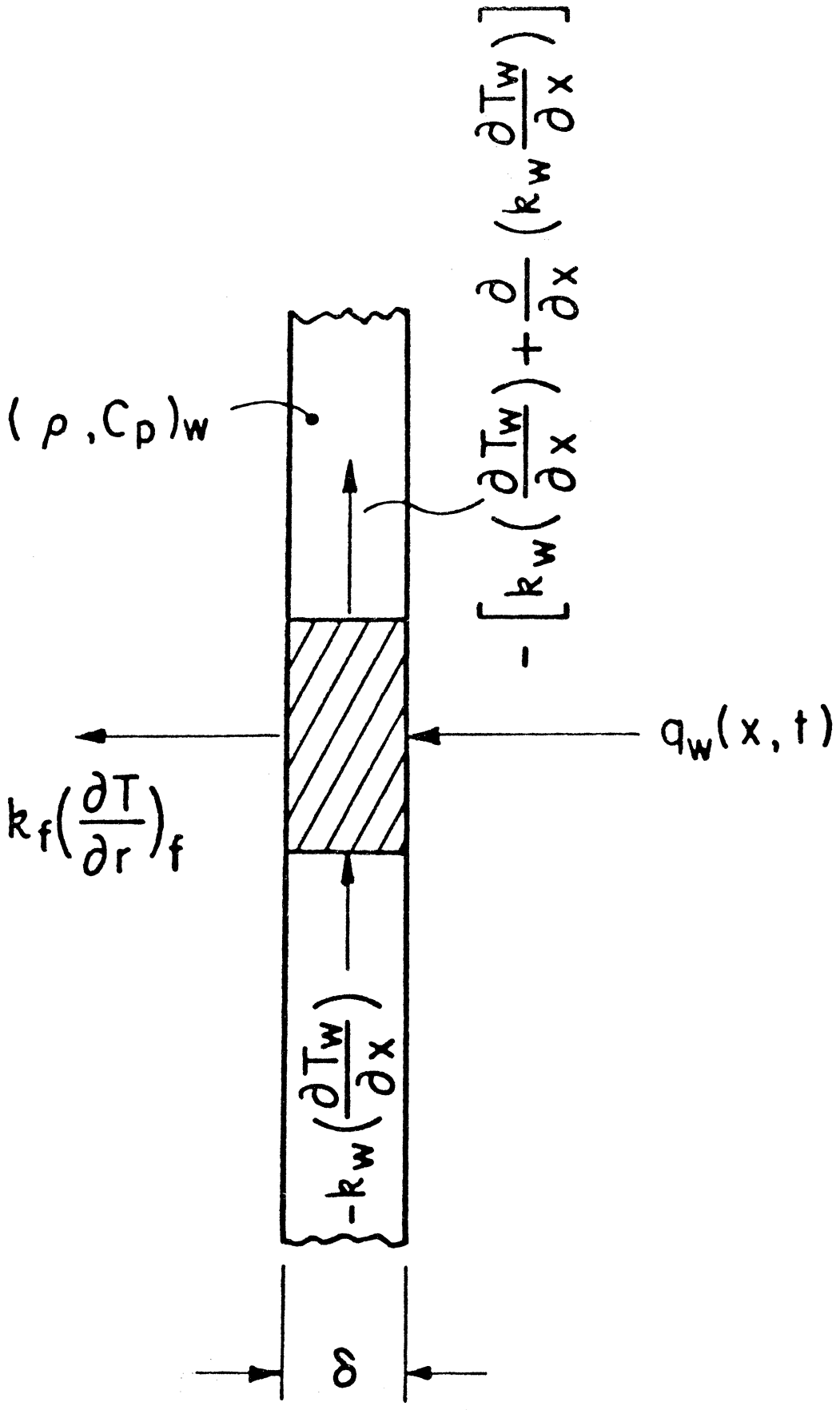


Figure 2. Container wall section.

A reversed process may be used, however, to determine a value for the heat transfer coefficient under these transient conditions. From the rate of heat flow from the wall to the fluid in the container as determined by Eq. (5), the transient heat transfer coefficient h may be computed. This procedure permits the determination of the variation of the heat transfer coefficient with time and space during this transient process.

The differential equations governing the velocity and temperature distribution in both phases are developed as follows.

A. LIQUID REGION

The following assumptions are made:

1. Constant thermal conductivity and viscosity.

2. Incompressible fluid. Density variations are introduced only in the body force term of the momentum equation. These variations are described by

$$\rho = \rho_o[1 + \beta(T_o - T)] \tag{6}$$

3. The influence of viscous dissipation is neglected.

Governing Equations

(i) x-momentum

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r}\right) = -\rho g - \frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2}\right) \tag{7}$$

(ii) radial momentum

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial r}\right) = -\frac{\partial P}{\partial r} + \mu\left(\frac{\partial^2 v}{\partial x^2} - \frac{v}{r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial r^2}\right) \tag{8}$$

(iii) continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} + \frac{v}{r} = 0 \tag{9}$$

(iv) energy

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial r} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2}\right) \tag{10}$$

Initial Conditions

$$T(x,r,o) = T_o \, , \; u(x,r,o) = 0, \; v(x,r,o) = 0 \tag{11}$$

Boundary Conditions

Velocity

$$u(o,r,t) = 0 \tag{12}$$

$$u(X_{-o},r,t) = - \frac{w_{\ell d}}{\rho_\ell \pi a^2} \tag{13}$$

where $w_{\ell d}$ is the rate of liquid discharge.

$$u(x,a,t) = 0 \tag{14}$$

$$\frac{\partial u(x,o,t)}{\partial r} = 0 \tag{15}$$

$$v(x,a,t) = 0 \tag{16}$$

$$v(x,o,t) = 0 \tag{17}$$

$$v(o,r,t) = 0 \tag{18}$$

$$\frac{\partial v(X,r,t)}{\partial x} = 0 \tag{19}$$

Temperature

$$T(X,r,t) = T_s(t) \tag{20}$$

$$T(x,a,t) = T_w(x,t) \tag{21}$$

$$\frac{\partial T(o,r,t)}{\partial x} = 0 \tag{22}$$

$$\frac{\partial T(x,o,t)}{\partial r} = 0 \tag{23}$$

where $T_s(t)$ is the saturation temperature corresponding to the system pressure and T_w is the wall temperature, Eq. (4).

B. VAPOR REGION

The differential equations for the vapor region are obtained assuming negligible changes in the viscosity and thermal conductivity within the vapor phase. The compressibility of the vapor is considered.

Governing Equations

(i) x-momentum

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r}\right) = -\rho g - \frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2}\right) + \frac{1}{3}\mu\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} + \frac{v}{r}\right) \tag{24}$$

(ii) radial momentum

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial r}\right) = -\frac{\partial P}{\partial r} + \mu\left(\frac{\partial^2 v}{\partial x^2} - \frac{v}{r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial r^2}\right) + \frac{1}{3}\mu\frac{\partial}{\partial r}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} + \frac{v}{r}\right) \tag{25}$$

(iii) continuity

$$\frac{D\rho}{Dt} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} + \frac{v}{r}\right) = 0 \quad , \tag{26}$$

(iv) energy

$$\rho c_v \left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r}\right) + \rho\frac{D(P/\rho)}{Dt}$$

$$= \frac{DP}{Dt} + k\left(\frac{\partial^2 T}{\partial x^2} + \frac{1}{r}\frac{\partial T}{\partial r} \quad \frac{\partial^2 T}{\partial r^2}\right) \quad . \tag{27}$$

(v) equation of state

$$P = z\rho RT \tag{28}$$

where z is the compressibility factor which can be evaluated from P, V, T, data or by using an equation of state which accounts for real gas effects. Using the Van der Waal equation of state the following equation relating z to the reduced pressure and temperature is obtained.

$$z^3 - \left(\frac{Pr}{8Tr} + 1\right) z^2 + \frac{27}{64}\frac{Pr}{Tr^2} z - \frac{27}{512}\frac{Pr^2}{Tr^3} = 0, \tag{29}$$

where P_r and T_r are the reduced pressure and temperature, respectively.

The value of z obtained from Eq. (29) is quite accurate for engineering applications (11). Other equations describing z or tabulated values of z can also be employed. However, calculation of z using an equation of the form of (29) offers less difficulties from the programming standpoint.

Initial Conditions

$$T(x,r,o) = T_o$$

$$\rho(x,r,o) = \rho_o$$

$$u(x,r,o) = 0$$

$$v(x,r,o) = 0 \tag{30}$$

Boundary Conditions

Velocity

$$u(b,r,t) = 0 \qquad (31\text{-a})$$

or

$$u(b,r,t) = u_{gp} = -\frac{w_{g\bar{p}}}{\rho_{gp}\pi a^2} \quad , \quad \chi \qquad (31\text{-b})$$

where w_{gp} is the rate of mass flow of the pressurant and ρ_{gp} is its density. Equation (31-b) is written for the case of uniform velocity at the inlet diffuser of the tank as in Fig. 3.

$$u(X_{+o},r,t) = u_{gi} \qquad (32)$$

$$\frac{\partial u(x,o,t)}{\partial r} = 0 \qquad (33)$$

$$u(x,a,t) = 0 \qquad (34)$$

$$v(b,r,t) = 0 \qquad (35)$$

$$v(x,a,t) = 0 \qquad (36)$$

$$u(x,o,t) = 0 \qquad (37)$$

$$\frac{\partial v(X,r,t)}{\partial x} = 0 \qquad (38)$$

where u_{gi} is the velocity of the vapor at the interface caused by simultaneous phase change and liquid discharge, Eq. (47). The boundary conditions given by Eqs. (19) and (38) assume a zero shear stress in both the liquid and vapor at the interface. These are reasonable and are made for the sake of simplification. Another approximation would be to take the vapor velocity at the interface equal to that of the liquid at that point. The interfacial shear stress would then be determined by the analysis. For flows with large radial gas velocities at the interface, this latter boundary condition becomes more realistic.

Temperature

$$T(X,r,t) = T_s(t) \tag{39}$$

$$T(x,a,t) = T_w(x,t) \tag{40}$$

$$\frac{\partial T(x,o,t)}{\partial r} = 0 \tag{41}$$

$$\frac{\partial T(b,r,t)}{\partial x} = 0 \tag{42-a}$$

or

$$T(b,r,t) = T_{gp}(r,t) \tag{42-b}$$

Equation (42-a) applies for the case with no external pressurization, and Eq. (42-b) for arbitrary external pressurization.

C. LIQUID-VAPOR COUPLING AT INTERFACE

The processes in the liquid and vapor regions are coupled by T_s in Eqs. (20) and (39) and by u_{gi} in Eq. (32), which is related to the rate of mass transfer at the interface.

The rate of mass transfer by evaporation or condensation across the liquid-vapor interface depends on the relative rates of heat transfer by diffusion from each phase at the interphase. Conservation of energy at the interface determines the rate of phase change as well as the interfacial displacement, according to;

$$h_{fg} w_i = \int_o^a \{[k \frac{\partial T(X,r,t)}{\partial x}]_\ell - [k \frac{\partial T(X,r,t)}{\partial x}]_v\} 2\pi r dr \quad , \tag{43}$$

where w_i is the rate of interfacial phase change. According to Eq. (43), w_i will be positive if condensation takes place. The interfacial displacement with no liquid discharge is then given by:

$$\rho_\ell \pi a^2 \frac{dX_i}{dt} = w_i \tag{44}$$

Should it be desirable to include the process of liquid discharge, Eq. (44) would be written

$$\rho_\ell \, \pi a^2 \frac{dX_d}{dt} = - w_d \tag{45}$$

and

$$u_i = \frac{dX}{dt} = \frac{dX_i}{dt} + \frac{dX_d}{dt} \tag{46}$$

where

$\frac{dX_i}{dt}$ = rate of interfacial displacement due to phase change,

$\frac{dX_d}{dt}$ = rate of interfacial displacement due to liquid discharge,

$\frac{dX}{dt}$ = combined rate of interfacial displacement due to phase change and liquid discharge.

Equation (46) is obtained assuming that the liquid surface remains flat during the discharge process. This is an approximation which neglects the influence of viscosity near the walls and surface tension effects. Such effects are negligible except for very low gravity levels and small Bond numbers.

The vapor velocity u_{gi} at the interface is related to the rate of interfacial displacement due to phase change and discharge by,

$$u_{gi} = -\left(\frac{\rho_\ell - \rho_{gs}}{\rho_{gs}}\right) \frac{dX_i}{dt} + \frac{dX_d}{dt} \tag{47}$$

D. SUMMARY OF FORMULATION

Taking a broad perspective of the foregoing formulation the following observations might be made. For convenience the liquid and vapor regions are considered separately, recognizing that they are coupled at the liquid-vapor interface.

1. Liquid Region

Four unknowns exist: u,v, T, P, each of which are functions of x, r, t. The four Eqs. (7)-(10), together with the boundary conditions, are available for the solution of these unknowns. The solution of this system of equations

by finite-difference procedures has been attempted with no success. The reason for this is believed due to the extremely small variations in pressure within the domain, $P(x,r,t)$, compared to the level of pressure. With the assumption of an incompressible fluid (except for changes in density due to temperatures in the body force term of the momentum equation) the pressure terms occur only in the x-and r-momentum equations. The pressure distribution is not of particular interest except that it is a factor in the fluid motion. This motion is influenced by the system pressure level only indirectly in that it establishes the temperature boundary condition at the liquid-vapor interface.

It is, however, possible to eliminate the pressure term by cross-differentiating and combining the two momentum equations. Introducing the definitions of vorticity and stream function, the system of 4 equations with 4 unknowns is reduced to a system of 3 equations in 3 unknowns, temperature $T(x,r,t)$, vorticity $\omega'(x,r,t)$, and stream function $\psi'(x,r,t)$. This procedure will be outlined below, and has been successfully applied to a single uncoupled domain (12,16).

2. Vapor Region

Since the assumption of an incompressible fluid is not a valid approximation in the vapor region, five unknowns exist: u, v, T, P, ρ, each of which are functions of x,r,t. The five Eqs. (24-28), together with the boundary conditions, are available for the solution of these unknowns. As in the case with the liquid region, the pressure distribution is of interest only insofar as it gives rise to the fluid motion. The influence of the spatial pressure variation within the vapor space on other properties can reasonably be neglected. Thus, the pressure term can be eliminated from the two momentum equations by combining them, and introducing the vorticity and stream function. The temporal variation of pressure, however, still remains in the energy equation. As will be demonstrated, the system is now described by 4 equations with 5 unknowns, i.e., $T(x,r,t)$, $\omega'(x,r,t)$, $\psi''(x,r,t)$, $\rho(x,r,t)$, $P(t)$. The additional needed relation is furnished by the energy equation written for the entire vapor space as a control volume. These procedures will now be described.

III. TRANSFORMATION OF THE PARTIAL DIFFERENTIAL EQUATIONS

The momentum and the continuity equations are combined to obtain the vorticity equation and an elliptic equation relating the vorticity and the stream function. This is accomplished in the same manner as described in Ref. (12). The x-momentum is differentiated with respect to r, the r-momentum equation is differentiated with respect to x and the two combined to eliminate the pressure terms.

A. LIQUID REGION

Equations (7) and (8) reduce to:

$$\frac{\partial \omega'}{\partial t} + u \frac{\partial \omega'}{\partial x} + v \frac{\partial \omega'}{\partial r} = - \frac{1}{\rho r} g \frac{\partial \rho}{\partial r} + \nu \left[\frac{\partial^2 \omega'}{\partial x^2} + \frac{3}{r} \frac{\partial \omega'}{\partial r} + \frac{\partial^2 \omega'}{\partial r^2} \right] \qquad (48)$$

where ω' is given by:

$$\omega' = \frac{1}{r} \left(\frac{\partial u}{\partial r} - \frac{\partial v}{\partial x} \right) \qquad (49)$$

Introducing the stream function ψ', defined by

$$u = \frac{1}{r} \frac{\partial \psi'}{\partial r} ; \quad v = - \frac{1}{r} \frac{\partial \psi'}{\partial x} \qquad (50)$$

Equations (10), (48), and (9) are transformed respectively to:

$$\frac{\partial T}{\partial t} + \frac{1}{r} \frac{\partial \psi'}{\partial r} \frac{\partial T}{\partial x} - \frac{1}{r} \frac{\partial \psi'}{\partial r} \frac{\partial T}{\partial r} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} \right) , \qquad (51)$$

$$\frac{\partial \omega'}{\partial t} + \frac{1}{r} \frac{\partial \psi'}{\partial r} \frac{\partial \omega'}{\partial x} - \frac{1}{r} \frac{\partial \psi'}{\partial r} \frac{\partial \omega'}{\partial r} = \frac{g\beta}{r} \frac{\partial T}{\partial r} + \nu \left(\frac{\partial^2 \omega'}{\partial x^2} + \frac{3}{r} \frac{\partial \omega'}{\partial r} + \frac{\partial^2 \omega'}{\partial r^2} \right), \qquad (52)$$

$$\frac{\partial^2 \psi'}{\partial x^2} - \frac{1}{r} \frac{\partial \psi'}{\partial r} + \frac{\partial^2 \psi'}{\partial r^2} = \omega' r^2 \qquad (53)$$

The initial and boundary conditions must also be transformed appropriately. Equations (51), (52), and (53) are solved numerically by finite differences to obtain the temperature, vorticity and stream function distributions. From

Eq. (50) the velocity distributions can then be computed.

B. VAPOR REGION

Combining the momentum Eqs. (24) and (25) in the manner described above, the following is obtained:

$$\frac{\partial \omega'}{\partial t} + u\frac{\partial \omega'}{dx} + v\frac{\partial \omega'}{\partial r} = \left[\frac{\omega'}{\rho}\frac{D\rho}{Dt} - \frac{1}{\rho r}\frac{Du}{Dt}\frac{\partial \rho}{\partial r} + \frac{1}{\rho r}\frac{Dv}{Dt}\frac{\partial \rho}{\partial x}\right] - \frac{g}{\rho r}\frac{\partial \rho}{\partial r} + \nu\left[\frac{\partial^2 \omega'}{\partial x^2} + \frac{3}{r}\frac{\partial \omega'}{\partial r}\frac{\partial^2 \omega'}{\partial r^2}\right] \tag{54}$$

where ω' is given by Eq. (49).

Equation (54) is the vorticity equation for this case. The first terms within the brackets on the right-hand side of this equation account for the compressibility effects due to density variation. Should the compressibility effects be neglected, Eq. (54) will reduce to Eq. (52). It should be noted that the presence of these terms does not introduce any additional difficulties as far as the solution of the vorticity equation is concerned. The difficulty will be in combining the continuity Eq. (26) and Eq. (49), in order to obtain an equation similar to Eq. (53). However, this difficulty can be overcome if the term $\partial\rho/\partial t$ is neglected in combining the continuity Eq. (26) with the definition of vorticity. The rigorous justification for this approximation has not yet been established. In this case Eq. (26) will be rewritten as:

$$\frac{\partial(\rho u)}{\partial x} + \frac{1}{r}\frac{\partial(\rho r v)}{\partial r} = 0. \tag{55}$$

The continuity Eq. (55) is combined with Eq. (49) by introducing the stream function ψ'', which satisfies (55). This stream function ψ'' is defined by:

$$u = \frac{1}{\rho r}\frac{\partial \psi''}{\partial r}\ ;\quad v = -\frac{1}{\rho r}\frac{\partial \psi''}{\partial x} \tag{56}$$

Substituting (56) and (49) into (55), the following equation which relates ψ'' to ω', is obtained,

$$\frac{\partial^2 \psi''}{\partial r^2} - \frac{1}{r}\frac{\partial \psi''}{\partial x} + \frac{\partial^2 \psi''}{\partial x^2} = \rho\omega' r^2 + r\left[u\frac{\partial \rho}{\partial r} - v\frac{\partial \rho}{\partial x}\right] \tag{57}$$

The last terms within the brackets in Eq. (57) account for the density changes in the vapor region. The terms $\partial\rho/\partial r$ and $\partial\rho/\partial x$ are calculated in the finite-difference procedure using the equation of state and the temperature distribution from the previous time step. The coefficients u and v in u $\partial\rho/\partial r$ and v $\partial\rho/\partial x$ will be taken equal to their values at the previous time step, which is the same procedure used in calculating the non-linear terms u $\partial T/\partial x$, v $\partial T/\partial r$, u $\partial\omega'/\partial x$, and v $\partial\omega'/\partial r$, thus in effect linearizing these terms.

The spatial variation of pressure in the vapor region can be neglected. Hence,

$$\frac{DP}{Dt} = \frac{dP}{dt} \tag{58}$$

The density terms in Eqs. (54) and (57) are evaluated using the equation of state (28), and the derivatives given by:

$$\frac{\partial\rho}{\partial r} = -\frac{P}{RZ^2T^2}\left(\frac{\partial(ZT)}{\partial T}\right)_p \frac{\partial T}{\partial r} \tag{59}$$

$$\frac{\partial\rho}{\partial x} = -\frac{P}{RZ^2T^2}\left(\frac{\partial(ZT)}{\partial T}\right)_p \frac{\partial T}{\partial x} \tag{60}$$

Substitution in Eq. (54) gives:

$$\begin{aligned}\frac{D\omega'}{Dt} - \frac{\omega'}{P}\frac{dP}{dt} + \frac{\omega'}{ZT}\left(\frac{\partial(ZT)}{\partial T}\right)_p \frac{DT}{Dt} &= -\frac{g}{rZT}\left(\frac{\partial(ZT)}{\partial T}\right)_p \frac{\partial T}{\partial r} + \frac{1}{rZT}\frac{\partial T}{\partial r}\left(\frac{\partial(ZT)}{\partial T}\right)_p \\ &\cdot \frac{Du}{Dt} - \frac{1}{rZT}\left(\frac{\partial(ZT)}{\partial T}\right)_p \frac{\partial T}{\partial x}\frac{Dv}{Dt} \\ &+ \nu\left[\frac{\partial^2\omega'}{\partial x^2} + \frac{3}{r}\frac{\partial\omega'}{\partial r}\frac{\partial^2\omega'}{\partial r^2}\right]\end{aligned} \tag{61}$$

Substitution of Eqs. (28), (59), and (60) into Eq. (57) gives:

$$\begin{aligned}\frac{\partial^2\psi''}{\partial r^2} - \frac{1}{r}\frac{\partial\psi''}{\partial r} + \frac{\partial^2\psi''}{\partial x^2} &= \frac{P}{RZT}\omega' r^2 \\ &- \frac{P\,r}{RZ^2T^2}\left(\frac{\partial(ZT)}{\partial T}\right)_p \left[u\frac{\partial T}{\partial r} - v\frac{\partial T}{\partial x}\right]\end{aligned} \tag{62}$$

The terms u and v in Eqs. (61) and (62) could also be expressed in terms of ψ'' by Eq. (56). Substitution of the equation of state (28) into the energy Eq. (27) gives:

$$\rho \left[C_v + R \left(\frac{\partial (ZT)}{\partial T} \right)_p \right] \frac{DT}{dt} = \frac{dP}{dt} + k \nabla^2 T \tag{63}$$

Taking the thermodynamic relation

$$C_p - C_v = T \left(\frac{\partial v}{\partial T} \right)_p \left(\frac{\partial p}{\partial T} \right)_v \tag{64}$$

and the ratio of specific heats

$$\frac{C_p}{C_v} = \gamma \tag{65}$$

and substituting into Eq. (63) and rearranging, another form of the energy equation is:

$$\frac{DT}{Dt} = \frac{1}{1 + \dfrac{(\gamma - 1)Z}{\left(\dfrac{\partial (ZT)}{\partial T} \right)_v}} \cdot \frac{RZT}{C_v P} \frac{dP}{dt} + \frac{\gamma}{1 + \dfrac{(\gamma - 1)Z}{\left(\dfrac{\partial (ZT)}{\partial T} \right)_v}} \alpha \nabla^2 T \tag{66}$$

It might be noted that the body force term in Eq. (61) could also be expressed as:

$$\frac{g}{rZT} \left(\frac{\partial (ZT)}{\partial T} \right)_p \frac{\partial T}{\partial r} = \frac{g}{r} \beta \frac{\partial T}{\partial r} \tag{67}$$

Since the assumption of ideal gas behavior is adequate for the vapor,* Eqs. (61), (62), and (66) reduce, respectively, to:

$$\frac{D\omega'}{Dt} - \frac{\omega'}{P} \frac{dP}{dt} + \frac{\omega'}{T} \frac{DT}{Dt} = -\frac{g}{rT} \frac{\partial T}{\partial r} + \frac{1}{rT} \frac{\partial T}{\partial r} \frac{Du}{Dt} - \frac{1}{rT} \frac{\partial T}{\partial x} \frac{Dv}{Dt} + \nu \left[\frac{\partial^2 \omega'}{\partial x^2} + \frac{3}{r} \frac{\partial \omega'}{\partial r} + \frac{\partial^2 \omega'}{\partial r^2} \right] \tag{68}$$

*For saturated O_2 vapor at atmospheric pressure, Z = 0.97, and $[\partial(ZT)/\partial T]_p$ = 1.015. For an ideal gas, Z = 1.00, and $\partial(ZT)/\partial T$ = 1.00.

$$\frac{\partial^2\psi''}{\partial r^2} - \frac{1}{r}\frac{\partial\psi''}{\partial r} + \frac{\partial^2\psi''}{\partial x^2} = \frac{P}{RT}\omega' r^2$$

$$- \frac{Pr}{RT^2}\left[u\frac{\partial T}{\partial r} - v\frac{\partial T}{\partial x}\right] \qquad (69)$$

$$\frac{DT}{Dt} = \frac{\gamma - 1}{\gamma}\frac{T}{P}\frac{dP}{dt} + \alpha\nabla^2 T \qquad (70)$$

Equations (61), (62) and (66) (or (68)-(70) for ideal gases) together with the transformed boundary conditions constitute the system of 3 equations with 4 unknowns for the vapor space, $\omega'(x,r,t)$, $\psi''(x,r,t)$, $T(x,r,t)$, and $P(t)$ referred to earlier. The finite-difference solution of Eqs. (61), (62), and (66) for ω', ψ'' and T requires that a function for dP/dt be available. This is obtained from the First Law of Thermodynamics written as an instantaneous rate equation, taking the vapor space as the control volume. Referring to Figure 3 this formulation is written:

$$\frac{d}{dt}\int_{V_g(t)}(\rho_g e_g)dV - \int_{A_i} h_{gs}\rho_{gs}(u_{gi} - u_i)dA$$

$$+ \int_{A_p} h_{gp}\rho_{gp}u_{gp}dA = \int_{A_{c.s.}} qdA - P\frac{dV_g}{dt} \qquad (71)$$

This formulation includes the following generalized variation:

$$e_g = e(x,r,t)$$

$$\rho_g = \rho(x,r,t)$$

$$u_{gi} = u(r,t)$$

h_{gs}, ρ_{gs} are uniform over the liquid-vapor interface

u_i is uniform for a flat interface

$h_{gp}, \rho_{gp}, u_{gp}$ are specified by the particular pressurization process.

q is the local heat flux on the control surface, and its integral includes that from the tank wall and through the liquid-vapor interface.

Given property information on $e_g = f_1(P,T)$, $\rho_g = f_2(P,T)$, $h_{gs} = f_3(P)$, $\rho_{gs} = f_4(P)$, the desired term dP/dt can be extracted from the first term of Eq. (71). The necessary properties can come either from specific heat data plus an equation of state, or from tabulated values. It may be anticipated that different computational procedures would be required for each of these.

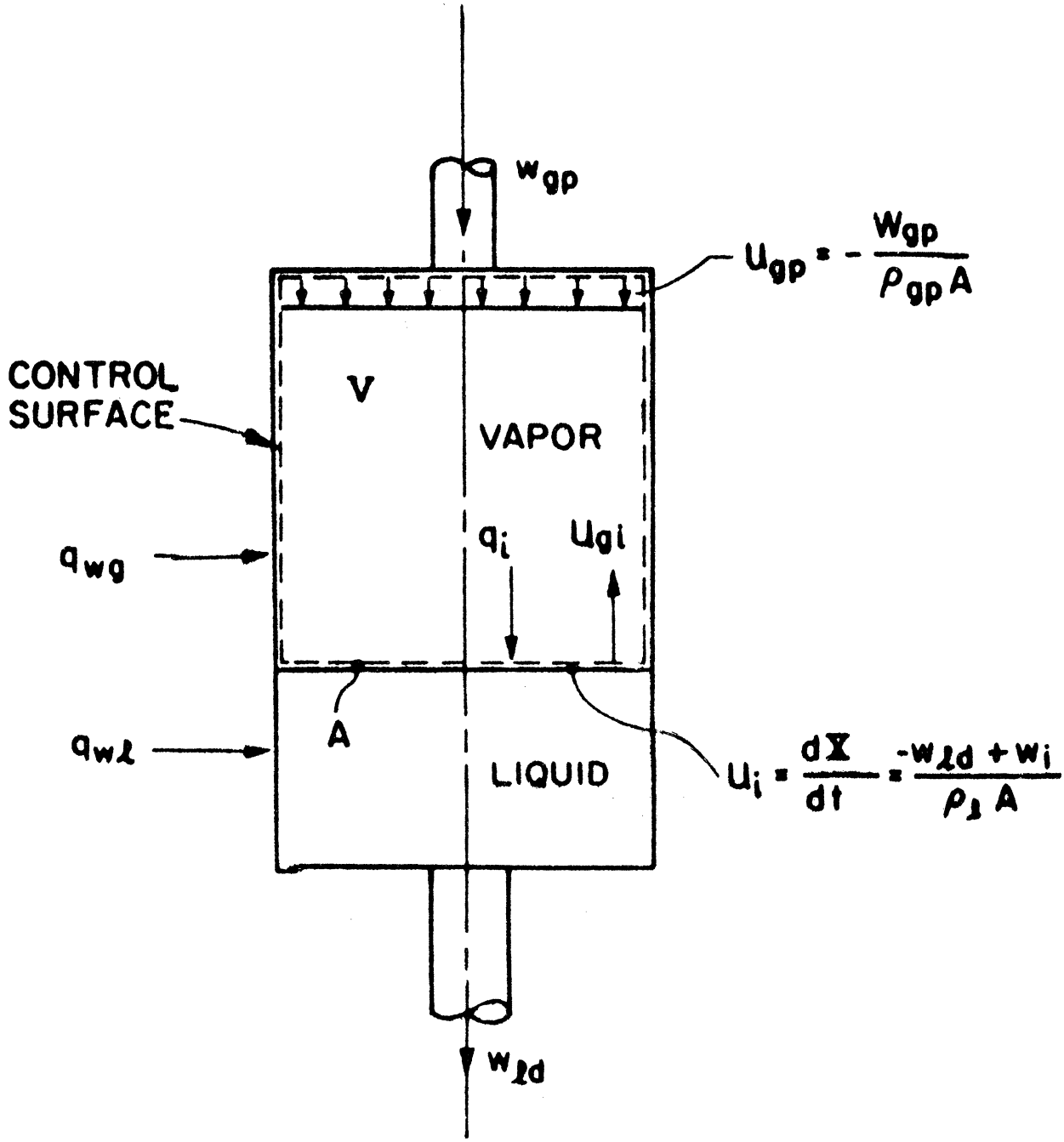


Figure 3. Control volume of vapor region.

The procedure will be demonstrated for the case of self-pressurization, that is with no liquid discharge, no external pressurization, and a flat interface. Thus, $w_d = 0$, $w_p = 0$, and $u_{gp} = 0$. Then,

$$\frac{dX}{dt} = u_i = \frac{dX_i}{dt} \tag{72}$$

and from Eq. (47), assuming a uniform vapor velocity across the interface

$$u_{gi} - u_i = \frac{\rho_\ell}{\rho_\ell - \rho_{gs}} u_{gi} \tag{73}$$

For the coordinate frame selected

$$\frac{dV_g}{dt} = - A_i \frac{dX}{dt} = - A_i u_i \tag{74}$$

For brevity

$$q_{tot} = \int_{A_{c.s.}} q \, d \, A \tag{75}$$

Substituting Eqs. (73)-(75) into Eq. (71)

$$\frac{d}{dt} \int_{V_g(t)} (\rho_g e_g) dV = q_{tot}$$

$$+ h_{gs} \rho_{gs} \frac{\rho_\ell}{\rho_\ell - \rho_{gs}} u_{gi} A_i + P A_i u_i \tag{76}$$

Using Leibnitz's rule for differentiation of integrals having variable limits the first term of Eq. (76) can be written as

$$\frac{d}{dt} \int_{V_g(t)} (\rho_g e_g) dV = \int_{V_g} \rho_g \frac{\partial e_g}{\partial t} dV$$

$$+ \int_{V_g} e_g \frac{\partial \rho_g}{\partial t} dV - \rho_{gs} e_{gs} u_i A_i \tag{77}$$

Substituting Eq. (77) in Eq. (76) and rearranging with the use of Eq. (47) gives,

$$\int_{V_g} \rho_g \frac{\partial e_g}{\partial t}\, dV + \int_{V_g} e_g \frac{\partial \rho_g}{\partial t}\, dV = q_{tot}$$

$$+ h_{gs} A_i u_i (\rho_{gs} - \rho_\ell) \qquad (78)$$

For the case in which the use of an equation of state (28) is desired, the entire left-hand side of Eq. (78) can be expressed as the following by means of general thermodynamic property relations:

$$\frac{C_v}{R} \frac{dP}{dt} \int_{V_g} \frac{dV}{\left[\frac{\partial(ZT)}{\partial T}\right]_v} + \int_{V_g} \left\{ \frac{PC_v}{R} \frac{1}{\left[\frac{\partial(ZT)}{\partial T}\right]_p} + \frac{T}{Z^2}\left(\frac{\partial P}{\partial T}\right)_v \right\} \frac{1}{ZT} \left[\frac{\partial(ZT)}{\partial T}\right] dV$$

$$- \frac{P}{R} \int_{V_g} \frac{h_g}{Z^2 T^2} \left[\frac{\partial(ZT)}{\partial t}\right] dV \qquad (79)$$

The enthalpy terms in Eqs. (78) and (79) can be expressed by:

$$h - h_R = \int_{T_R}^{T} C_{pR}\, dT + \int_{P_R}^{P} \frac{RT}{P} \left\{ Z - \left[\frac{\partial(ZT)}{\partial T}\right]_p \right\}_T dp \qquad (80)$$

where the subscript R refers to a low pressure reference condition.

For an ideal gas with constant specific heat expression (79) reduces to:

$$\frac{C_v}{R} \left(\frac{dP}{dt}\right) V_g \qquad (81)$$

Substituting expression (81) for the left-hand side of Eq. (78), writing the enthalpy by a specific heat term, and rearranging, Eq. (78) reduces to:

$$\frac{dP}{dt} = \frac{R}{C_v V_g} q_{tot} + \frac{RTs\gamma}{V_g} A_i u_i (\rho_{gs} - \rho_\ell) \qquad (82)$$

For the case far from the region of the critical state, such that $\rho_\ell \gg \rho_{gs}$, Eq. (82) can be written as

$$\frac{dP}{dt} = \frac{R}{C_v V_g} q_{tot} - \frac{w_i R \gamma T_s}{V_g} \qquad (83)$$

where w_i is the rate of interfacial mass transfer (having a negative value for evaporation) and is given by

$$w_i = - \frac{\rho_\ell \rho_{gs}}{(\rho_\ell - \rho_{gs})} A_i u_{gi} \tag{84}$$

IV. DIMENSIONLESS FORM OF THE EQUATIONS

The governing equations for the wall, liquid and vapor regions (4), (51), (52), (53), (61), (62), and (66) are next made nondimensional, along with the boundary conditions.

The substitutions necessary to nondimensionalize the differential equations are:

$$\left.\begin{aligned} u &= \frac{\alpha_\ell b}{a^2} U; \quad v = \frac{\alpha_\ell}{a} V; \quad T-T_o = \frac{\nu_\ell \alpha_\ell b}{\beta_\ell q a^4} \Theta \\ t &= \frac{a^2}{\alpha_\ell} \tau; \quad x = b\zeta \ ; \quad r = a\eta \\ \omega'' &= \frac{\alpha_\ell b}{a^4} \omega; \quad \psi' = \alpha b \psi \end{aligned}\right\} \tag{85}$$

The resulting dimensionless equations are given below.

A. WALL TEMPERATURE

The dimensionless equations describing the wall temperature, from Eq. (4), are given by:

$$\text{(i)} \quad 0 \le \zeta \le \zeta_i = \frac{X(t)}{b}$$

$$\frac{\partial \Theta_w}{\partial \tau} = -\frac{a}{\delta_w} \cdot \frac{(\rho C_p)_\ell}{(\rho C_p)_w} \left(\frac{\partial \Theta_\ell}{\partial \eta}\right)_{\eta=1} + \frac{a^2}{b^2} \cdot \frac{(\rho C_p)_\ell}{(\rho C_p)_w} \cdot \frac{k_w}{k_\ell} \frac{\partial^2 \Theta_w}{\partial \zeta^2}$$

$$+ \frac{a}{b} \cdot \frac{a}{\delta_w} \cdot \frac{(\rho C_p)_\ell}{(\rho C_p)_w} \cdot Pr_\ell \cdot Gr_w^*(\zeta,t) \tag{86}$$

$$\text{(ii)} \quad \frac{X(t)}{b} \le \zeta \le 1$$

$$\frac{\partial \Theta_w}{\partial \tau} = -\frac{a}{\delta_w} \cdot \frac{(\rho C_p)_\ell}{(\rho C_p)_w} \cdot \frac{k_g}{k_\ell} \cdot \left(\frac{\partial \Theta_g}{\partial \eta}\right)_{\eta=1} + \frac{a^2}{b^2} \cdot \frac{(\rho C_p)_\ell}{(\rho C_p)_w} \cdot \frac{k_w}{k_\ell} \cdot \frac{\partial^2 \Theta_w}{\partial \zeta^2}$$

$$+ \frac{a}{b} \cdot \frac{a}{\delta_w} \cdot \frac{(\rho C_p)_\ell}{(\rho C_p)_w} \cdot Pr_\ell \cdot Gr_w^*(\zeta,t) \tag{87}$$

where $Gr^*_w(\zeta,t)$ is the modified Grashof number and is given by:

$$Gr^*_w(\zeta,t) = \frac{g\beta a^4}{k_\ell \nu_\ell^2} q_w(\zeta,t) \tag{88}$$

(iii) Boundary and Initial Conditions

$$\frac{\partial \theta_w(0,\tau)}{\partial \zeta} = \frac{\partial \theta_w}{\partial \zeta}(1,\tau) = 0 \tag{89}$$

$$\theta(\zeta,0) = 0 \tag{90}$$

B. LIQUID REGION

(i) The energy equation, Eq. (51)

$$\frac{\partial \theta}{\partial \tau} + \frac{1}{\eta} \cdot \frac{\partial \psi}{\partial \eta} \cdot \frac{\partial \theta}{\partial \xi} - \frac{1}{\eta} \frac{\partial \psi}{\partial \xi} \cdot \frac{\partial \theta}{\partial \eta} = \frac{a^2}{b^2} \frac{\partial^2 \theta}{\partial \zeta^2} + \frac{1}{\eta} \frac{\partial \theta}{\partial \eta} + \frac{\partial^2 \theta}{\partial \eta^2} \tag{91}$$

(ii) The vorticity equation, Eq. (52)

$$\frac{\partial \omega}{\partial t} + \frac{1}{\eta} \frac{\partial \psi}{\partial \eta} \frac{\partial \omega}{\partial \xi} - \frac{1}{\eta} \frac{\partial \psi}{\partial \xi} \frac{\partial \omega}{\partial \eta} = Pr_\ell \left[\frac{1}{\eta} \frac{\partial \theta}{\partial \eta} + \frac{a^2}{b^2} \frac{\partial^2 \omega}{\partial \xi^2} + \frac{3}{\eta} \frac{\partial \omega}{\partial \eta} + \frac{\partial^2 \omega}{\partial \eta^2} \right] \tag{92}$$

(iii) The vorticity-stream function equation, Eq. (53)

$$\frac{a^2}{b^2} \frac{\partial^2 \omega}{\partial \xi^2} - \frac{1}{\eta} \frac{\partial \psi}{\partial \eta} + \frac{\partial^2 \psi}{\partial \eta^2} = \omega \tag{93}$$

$$U = \frac{1}{\eta} \frac{\partial \psi}{\partial \eta} \; ; \quad V = -\frac{1}{\eta} \frac{\partial \psi}{\partial \xi} \tag{94}$$

(iv) The Boundary Conditions

(1) Stream function boundary conditions

$$\psi(0,\eta,\tau) = \frac{\partial \psi(0,\eta,\tau)}{\partial \xi} = 0 \tag{95}$$

$$\psi(\xi_i,\eta,\tau) = \frac{\partial^2 \psi(\xi_i,\eta,\tau)}{\partial \xi^2} = 0 \tag{96}$$

$$\psi(\xi,0,\tau) = \frac{\partial}{\partial\eta}\left[\frac{1}{\eta}\frac{\partial\psi(\xi,0,\tau)}{\partial\eta}\right] = 0 \tag{97}$$

$$\psi(\xi,1,\tau) = \frac{\partial\psi(\xi,1,\tau)}{\partial\eta} = 0 \tag{98}$$

(2) Vorticity boundary conditions

$$\omega(1,\eta,\tau) = 0 \tag{99}$$

$$\omega(\xi,0,\tau) = 0 \tag{100}$$

Two additional vorticity boundary conditions are required at the tank wall and bottom. An explicit expression for the vorticity at any of these locations is difficult to obtain. The method of solution used in this work, which follows that of Ref. (16), overcomes this difficulty.

(3) Thermal boundary conditions

$$\Theta(\xi,1,\tau) = \Theta_w(\xi,\tau) \tag{101}$$

$$\Theta(\xi_i,\eta,\tau) = \Theta_s(\tau) \tag{102}$$

$$\frac{\partial\Theta(0,\eta,\tau)}{\partial\xi} = 0 \tag{103}$$

$$\frac{\partial\Theta(\xi,0,\tau)}{\partial\eta} = 0 \tag{104}$$

where Θ_s is given by:

$$\Theta_s(\tau) = \frac{a}{b}\,Pr_\ell\left[T_s(t) - T_o\right]\frac{g\beta_\ell a^3}{\nu_\ell^2} \tag{105}$$

(4) Initial Conditions

$$\Theta(\xi,\eta,0) = U(\xi,\eta,0) = V(\xi,\eta,0)$$

$$= \omega(\xi,\eta,0) = \psi(\xi,\eta,0) = 0 \tag{106}$$

C. VAPOR REGION

(i) The energy equation, Eq. (66)

$$\frac{\partial\theta}{\partial\tau} + \frac{1}{\rho\eta}\frac{\partial\psi}{\partial\eta}\frac{\partial\theta}{\partial\xi} - \frac{1}{\rho\eta}\frac{\partial\psi}{\partial\xi}\frac{\partial\theta}{\partial\eta} = \frac{1}{1+(\gamma-1)Z/\left(\frac{\partial(ZT)}{\partial T}\right)_v} \cdot \frac{RZ}{C_v} \cdot \frac{(\theta+\theta_o)}{P} \cdot \frac{\partial P}{\partial\tau}$$

$$+ \frac{\gamma}{1+(\gamma-1)Z/\left(\frac{\partial(ZT)}{\partial T}\right)_v} \frac{\alpha_g}{\alpha_\ell}\left[\frac{a^2}{b^2}\frac{\partial^2\theta}{\partial\xi^2} + \frac{1}{\eta}\frac{\partial\theta}{\partial\eta} + \frac{\partial^2\theta}{\partial\eta^2}\right] \tag{107}$$

where $\theta = \frac{g\,\beta_\ell a^4}{\nu_\ell \alpha_\ell b}\,T_o$ (108)

(ii) The vorticity equation, Eq. (61)

$$\frac{\partial\omega}{\partial\tau} + \frac{1}{\rho\eta}\cdot\frac{\partial\psi}{\partial\eta}\cdot\frac{\partial\omega}{\partial\xi} - \frac{1}{\rho\eta}\cdot\frac{\partial\psi}{\partial\xi}\cdot\frac{\partial\omega}{\partial\eta} = Pr_\ell\frac{\nu_g}{\nu_\ell}\left[\frac{a^2}{b^2}\frac{\partial^2\omega}{\partial\xi^2} + \frac{3}{\eta}\frac{\partial\omega}{\partial\eta} + \frac{\partial^2\omega}{\partial\eta^2}\right] + \frac{ga^4}{\alpha_\ell^2 bZ\eta(\theta+\theta_o)}\left(\frac{\partial(ZT)}{\partial T}\right)_P \frac{\partial\theta}{\partial\eta}$$

$$+ \frac{1}{\eta Z(\theta+\theta_o)}\left(\frac{\partial(ZT)}{\partial T}\right)_P \frac{\partial\theta}{\partial\eta}\frac{DU}{D\tau} - \frac{1}{\eta Z(\theta+\theta_o)}\left(\frac{\partial(ZT)}{\partial T}\right)_P \frac{\partial\theta}{\partial\xi}\frac{DV}{D\tau} + \frac{\omega}{P}\left(\frac{dP}{d\tau}\right) \tag{109}$$

where β is given by:

$$\beta_g = \frac{1}{ZT}\left(\frac{\partial(ZT)}{\partial T}\right)_P \tag{110}$$

(iii) The vorticity-stream function equation, Eq. (62)

$$\frac{a^2}{b^2}\frac{\partial^2\psi}{\partial\xi^2} - \frac{1}{\eta}\frac{\partial\psi}{\partial\eta} + \frac{\partial^2\psi}{\partial\eta^2} = \frac{P}{ZR(\theta+\theta_o)}\left(\frac{g\beta_\ell a^4}{\nu_\ell\alpha_\ell b}\right)\omega\eta^2$$

$$- \left(\frac{g\beta_\ell a^4}{\nu_\ell\alpha_\ell b}\right)\frac{P}{Z^2R(\theta+\theta_o)}\left(\frac{\partial(ZT)}{\partial T}\right)_p \eta\left[U\frac{\partial\theta}{\partial\eta} - \frac{a^2}{b^2}V\frac{\partial\theta}{\partial\xi}\right] \tag{111}$$

(iv) Boundary Conditions

(1) Stream function boundary conditions

$$\psi(1,\eta,\tau) = \frac{2\psi(1,\eta,\tau)}{2\xi} = 0 \tag{112}$$

$$\psi(\xi_i,\eta,\tau) = (\rho_\ell - \rho_{g,s})\frac{w_i}{\rho_\ell\alpha_\ell}\frac{\eta^2}{2\pi b} = \frac{\rho_{gs}U_{gi}\eta^2}{2} \tag{113}$$

$$\psi(\xi,0,\tau) = \frac{\partial}{\partial\eta}\left(\frac{1}{\rho_{gi}\eta}\frac{2\psi(\xi,0,\tau)}{2\eta}\right) = 0 \tag{114}$$

$$\psi(\xi,1,\tau) = \frac{2\psi(\xi,1,\tau)}{2\eta} = 0 \tag{115}$$

(2) Vorticity boundary conditions

$$\omega(\xi,0,\tau) = 0 \tag{116}$$

$$\omega(\xi_i,\eta,\tau) = 0 \tag{117}$$

(3) Temperature boundary conditions

$$\theta(\xi_i,\eta,\tau) = \theta_s(\tau) \tag{118}$$

$$\frac{\partial\theta(1,\eta,\tau)}{\partial\xi} = 0 \tag{119}$$

$$\theta(\xi,1,\tau) = \theta_w(\xi,\tau) \tag{120}$$

$$\frac{\partial\theta(\xi,0,\tau)}{\partial\eta} = 0 \tag{121}$$

(4) Initial Conditions

$$\theta(\xi,\eta,0) = U(\xi,\eta,0) = V(\xi,\eta,0) = \psi(\xi,\eta,0) = \omega(\xi,\eta,0) = 0 \tag{122}$$

From the above results, it is clear that the temperature, pressure and velocity within the container are functions of the parameters Gr_w^*, Pr, $(\rho C_p)_\ell/(\rho C_p)_w$, (k_w/k_ℓ), (α_g/α_ℓ), (ν_g/ν_ℓ) (β_g/β_ℓ), (a/b) and (a/δ). Gr_w^* is defined in Eq. (88).

V. METHOD OF SOLUTION

A. FINITE-DIFFERENCE FORMS

The finite-difference method of solution used in Ref. (16) is adopted here. A complete discussion of the application of difference methods for the solution of the energy and vorticity equations is given in this reference, along with the problem of stability of the difference equations. A brief discussion of the method of solution will be made here.

The basic concept in the application of finite-difference methods for the solution of partial differential equations is the use of Taylor Series Expansion to approximate the derivatives at a point in terms of the value of the function at that point and/or at its neighboring points. This may be demonstrated as follows:

1. The time derivative is represented by

$$\frac{\partial f}{d\tau} = \frac{f(\tau+\Delta\tau) - f(\tau)}{\Delta\tau} + O(\Delta\tau) \tag{123}$$

2. The first order derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial R}$ can be approximated by

$$\text{(i)}\ \frac{\partial f}{\partial x} = \frac{f(x+\Delta x,R,\tau) - f(x,R,\tau)}{\Delta x} + O(\Delta x)\ \text{(forward differences)} \tag{124}$$

or

$$\text{(ii)}\ \frac{\partial f}{\partial x} = \frac{f(x,R,\tau) - f(x-\Delta x,R,\tau)}{\Delta x} + O(\Delta x)\ \text{(backward differences)} \tag{125}$$

or

$$\text{(iii)}\ \frac{\partial f}{\partial x} = \frac{f(x+\Delta x,R,\tau) - f(x-\Delta x,R,\tau)}{2\Delta x} + O(\Delta x)^2\ \text{(central differences)} \tag{126}$$

3. The second order derivatives are replaced by finite-differences according to the formula.

$$\frac{\partial^2 f}{\partial x^2} = \frac{f(x+\Delta x,R,\tau) - 2f(x,R,\tau) + f(x-\Delta x,R,\tau)}{(\Delta x)^2} + O(\Delta x)^2 \qquad (127)$$

The function f represents either Θ or ω, and Δx is the spatial increment in the x-direction. The last term on the righthand side of Eqs. (123) through (127) indicates the order of the truncation error involved in replacing the derivatives by finite-differences. It is clear that the central differences offer a better representation of the first order derivative $\frac{\partial f}{\partial x}$ than the forward or backward differences. However, the form used to approximate the first order derivatives is usually determined by stability considerations. Similar formulations can be written for the derivatives $\frac{\partial f}{\partial R}$ and $\frac{\partial^2 f}{\partial R^2}$.

The substitution of the above formulae in the energy and vorticity equations produces a set of explicit difference equations. However, if the values of the function f in Eqs. (124) through (127) are taken at the time level $\tau+\Delta\tau$ instead of being taken at time level τ, the resulting finite-difference methods may require the use of small time increments and consequently large machine time. Certain implicit formulations may permit the use of large increments. The application of both explicit and implicit methods to the present problem has been extensively investigated in Ref. (16). It was concluded that the lack of explicit boundary conditions for the vorticity at the solid boundaries prevent the use of large time increments, i.e., implicit methods. Therefore, it was decided to employ explicit methods.

It is clear from Eqs. (124) through (126) that more than one explicit finite-difference formulation can be constructed for each of the vorticity and energy equations. The finite-difference formulation chosen for the solution of the present problem is dictated by stability, as well as practical considerations, which will be shown below in studying the stability of the finite-difference equations.

The method of solution used in the present problem can be summarized as follows:

1. The time derivatives $\frac{\partial \Theta}{\partial \tau}$ and $\frac{\partial \omega}{\partial \tau}$ are approximated by Eq. (123).

2. The nonlinear terms $U\frac{\partial \Theta}{\partial x}$, $V\frac{\partial \Theta}{\partial R}$, $U\frac{\partial \omega}{\partial x}$ and $V\frac{\partial \omega}{\partial R}$ are linearized by considering the velocity components U and V to be known and are taken equal to their values at time level τ. The order of the error introduced by this linearization can be obtained from Taylor Series Expansion. If U_o and U are the values of the axial velocity component at time levels τ_o and $\tau_o+\Delta\tau_o$, respectively, then

$$U\frac{\partial \Theta}{\partial x} = U_o\frac{\partial \Theta}{\partial x} + \Delta\tau\left(\frac{\partial U}{\partial \tau}\right)_{\tau_o} + \beta\frac{\partial \Theta}{\partial x}, \qquad (128)$$

where

$$0 \leq \beta \leq \Delta\tau \ .$$

The last term on the right side of Eq. (128) represents the linearization error and is of order $O(\Delta\tau)$.

3. The nonlinear terms $U \frac{\partial\theta}{\partial x}$ and $U \frac{\partial\omega}{\partial x}$ are approximated by backward differences, Eq. (125), if the coefficient velocity U is positive and by forward differences, Eq. (124), if U is negative. The same procedure is followed for approximating the terms $V \frac{\partial\theta}{\partial R}$ and $V \frac{\partial\omega}{\partial R}$ according to the sign of the velocity component V.

4. Central differences are used to approximate the first order terms $\frac{1}{R}\frac{\partial\theta}{\partial R}$, $\frac{1}{R}\frac{\partial\omega}{\partial R}$ and $\frac{1}{R}\frac{\partial\psi}{\partial R}$. No stability problems will be encountered in this case. At the centerline where both R and $\frac{\partial\theta}{\partial R}$ are zero, $\frac{1}{R}\frac{\partial\theta}{\partial R}$ is replaced by its limit according to

$$\underset{R\to 0}{\text{Limit}} \frac{1}{R}\frac{\partial\theta}{\partial R} = \left.\frac{\partial^2\theta}{\partial R^2}\right|_{R=0} \tag{129}$$

5. The second order terms $\frac{\partial^2\theta}{\partial x^2}$. $\frac{\partial^2\theta}{\partial R^2}$. $\frac{\partial^2\omega}{\partial x^2}$. $\frac{\partial^2\omega}{\partial R^2}$. $\frac{\partial^2\psi}{\partial x^2}$ and $\frac{\partial^2\psi}{\partial R^2}$ are represented by Eq. (127).

6. Although the first order derivative $\left.\frac{\partial\theta}{\partial\eta}\right|_{\eta=1}$ in Eqs. (86) and (87) can be represented by any of Eqs. (124), (125) or (126), the following formula, which has a higher truncation error is used,

$$\left.\frac{\partial\theta}{\partial\eta}\right|_{\eta=1} = \frac{11\theta(\xi,1)-18\theta(\xi,\eta-\Delta\eta)+9\theta(\xi,\eta-2\Delta\eta)-2\theta(\xi,\eta-3\Delta\eta)}{6\Delta\eta} \tag{130}$$

Similarly

$$\left.\frac{\partial\theta_\ell}{\partial\xi}\right|_{\xi=\xi_i} = \frac{11\theta(\xi_i,\eta)-18\theta(\xi_i-\Delta\xi,\eta)+9\theta(\xi_i-2\Delta\xi,\eta)-2\theta(\xi_i-3\Delta\xi,\eta)}{6\Delta\eta} \tag{131}$$

B. STABILITY OF FINITE-DIFFERENCE EQUATIONS

The stability of the finite-difference equations is an important consideration in establishing the size of the grid and the time steps, and the type of differencing used. The details are given in Ref. (16). As might be anticipated, the size of the grid and time steps will also be governed by the storage capacity of the machine and limitations of time and cost. The necessary requirements for stability are given by the following:

$$\Delta\tau\left(\frac{2a^2}{b^2(\Delta\xi)^2}+\frac{2}{(\Delta\eta)^2}+\frac{|U_{i,j}|}{\Delta\xi}+\frac{|V_{i,j}|}{\Delta\eta}\right)\leq 1 \qquad (132)$$

$$\Delta\tau\left(\frac{2a^2Pr}{b^2(\Delta\xi)^2}+\frac{2\ Pr}{(\Delta\eta)^2}+\frac{|U_{i,j}|}{\Delta\xi}+\frac{|V_{i,j}|}{\Delta\eta}\right)\leq 1 \qquad (133)$$

For Prandtl number less than unity, inequality (132) is more restrictive and therefore should be used. For Prandtl numbers greater than unity inequality (133) must be used.

C. COMPUTATIONAL PROCEDURES

The sequence of steps in establishing the numerical calculation is as follows:

(1) A convenient grid size is selected, dictated by the machine storage capacity.

(2) A suitable time increment is chosen. This may be altered during the course of computation as necessary to maintain numerical stability.

(3) The temperature distribution is computed using velocities, temperatures and pressure from the previous time step.

(4) These temperatures are used to compute the vorticity at the interior nodal points for the current time step.

(5) The stream function is computed at the interior nodal points.

(6) The vorticities at the solid boundaries are calculated.

(7) The velocity components are calculated.

(8) The rate of phase change at the interface is determined using the computed temperature distribution, with Eq. (43).

(9) The pressure rise is computed with an equation such as (83) using the parameters from the previous time step.

(10) The above procedures are repeated successively.

A problem inherent in the use of numerical methods is the accurate determination of temperature gradients. In the present application temperature gradients are computed in the liquid and vapor at the liquid-vapor interface to determine the rate of phase change, from Eq. (43), and in the liquid and vapor at the wall to determine the heat transfer to the bulk liquid and vapor, respectively.

The general problem of the effect of grid size on the interfacial heat and mass transfer is presented in some detail here.

The formulation with Eq. (43) was used by a number of investigators (2 through 8) to determine the rate of interfacial phase change for the case of a suddenly pressurized one-dimensional model. However, the determination of the temperature gradients by numerical differentiation of the calculated temperature distribution in the case of self-pressurized containers may be

difficult for two-dimensional problems. This would occur in cases in which the temperature gradients near the interface, that cause the phase change, are large. These large temperature gradients exist in a very thin layer near the interface, as has been shown by experimental measurements (13,14). This requires the use of a very small grid size in order to obtain an acceptable approximation for the temperature gradients near the interface.

Computations have shown that the spatial grid size has a considerable effect on the calculated rates of heat transfer at the interface. These calculation have been performed for the one-dimensional problem described by Eqs.(1), (2), (3), and the results are shown in Figures 4, 5, 6. Figure 4 gives the interfacial velocity and indicates that the effect of grid size is large at early times. As time progresses the interface velocity becomes less influenced by choice of grid size. This is also seen in Figure 5, which shows that for suitably small grid size the rate of condensation approaches a value independent of grid size. However, the influence on total mass condensed (or evaporated) is a cumulative one, as also noted in Figure 5, and can give rise to serious error in cases where the ratio of interfacial area to vapor volume is large or for small times. Figure 6 shows the influence of grid size on the computed temperatures for two different time periods of 10 and 100 seconds. As the grid size decreases the finite-difference calculations of the rate of interfacial phase change may be expected to approach the exact solution.

The example presented above dealt with an externally pressurized system, and the severity of the temperatures gradients at the interface will depend on the magnitudes of the pressure change and the superheat of the pressurant. In the case of self-pressurized containers the liquid is considered initially saturated. Any disturbances from these conditions such as imposed heat transfer from the ambient will cause the liquid to become superheated. This is an unstable nonequilbrium state, and the liquid will attempt to adjust to a new equilibrium condition by convection and evaporation. The evaporation of the liquid will cause the pressure in the ullage space to rise. The net effect will be to decrease the degree of superheat of the liquid, and increase the mass of the vapor in the ullage volume. If the rate of heating is low, the conditions in the liquid will be very close to equilibrium conditions. In this case the degree of superheat will be small. This condition may be expected to exist in high altitude flights with a well-insulated container. Under these circumstances evaporation takes place such that the pressure in the ullage volume rises at a rate which keeps the liquid near an equilibrium state. It thus might be anticipated that the temperature gradients in the liquid and vapor at the liquid-vapor interface will be relatively small, and the errors associated with reasonably sized grid spacing will be small. The only true test available is to vary the grid size for given conditions and compare the computed results.

In the case of the temperature gradients in the fluid adjacent to the wall two aspects must be considered when either the heat flux or the wall temperatures are imposed. One is similar to the liquid-vapor interface problem in that the grid spacing should again be small enough to permit sufficiently accurate representation of the temperature distribution in the

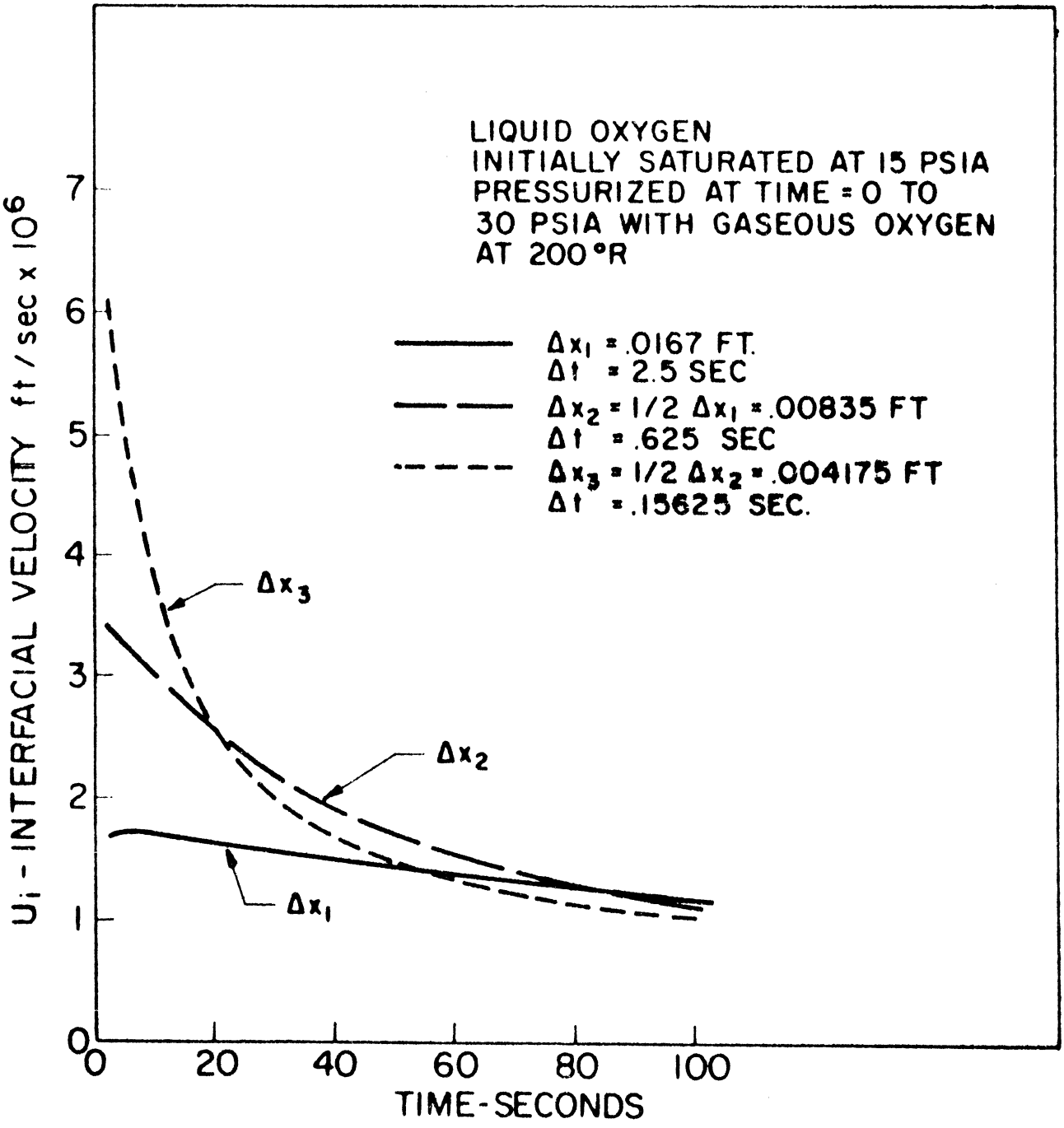


Figure 4. Effect of grid size on computed interfacial velocity.

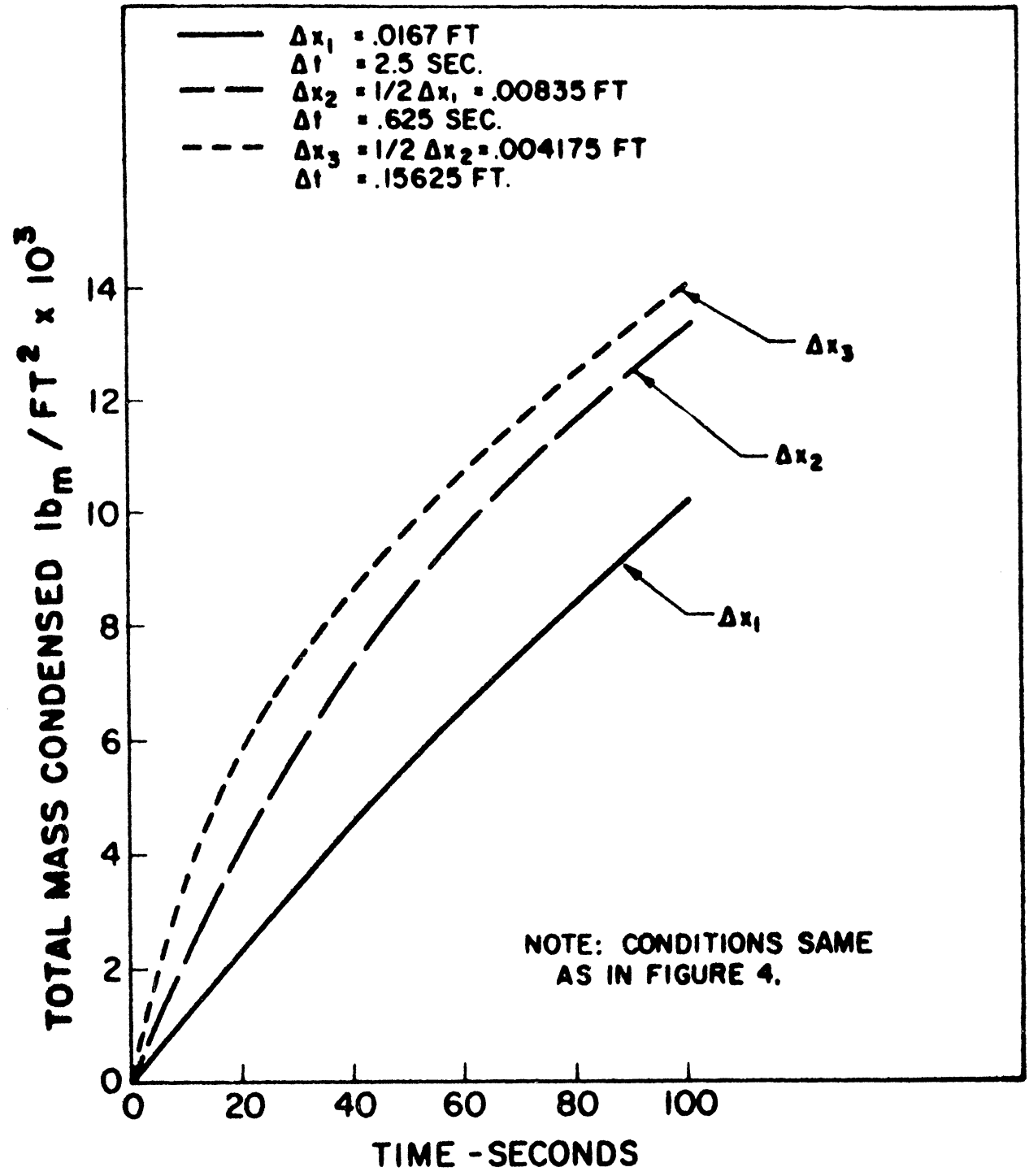


Figure 5. Effect of grid size on computed condensed mass.

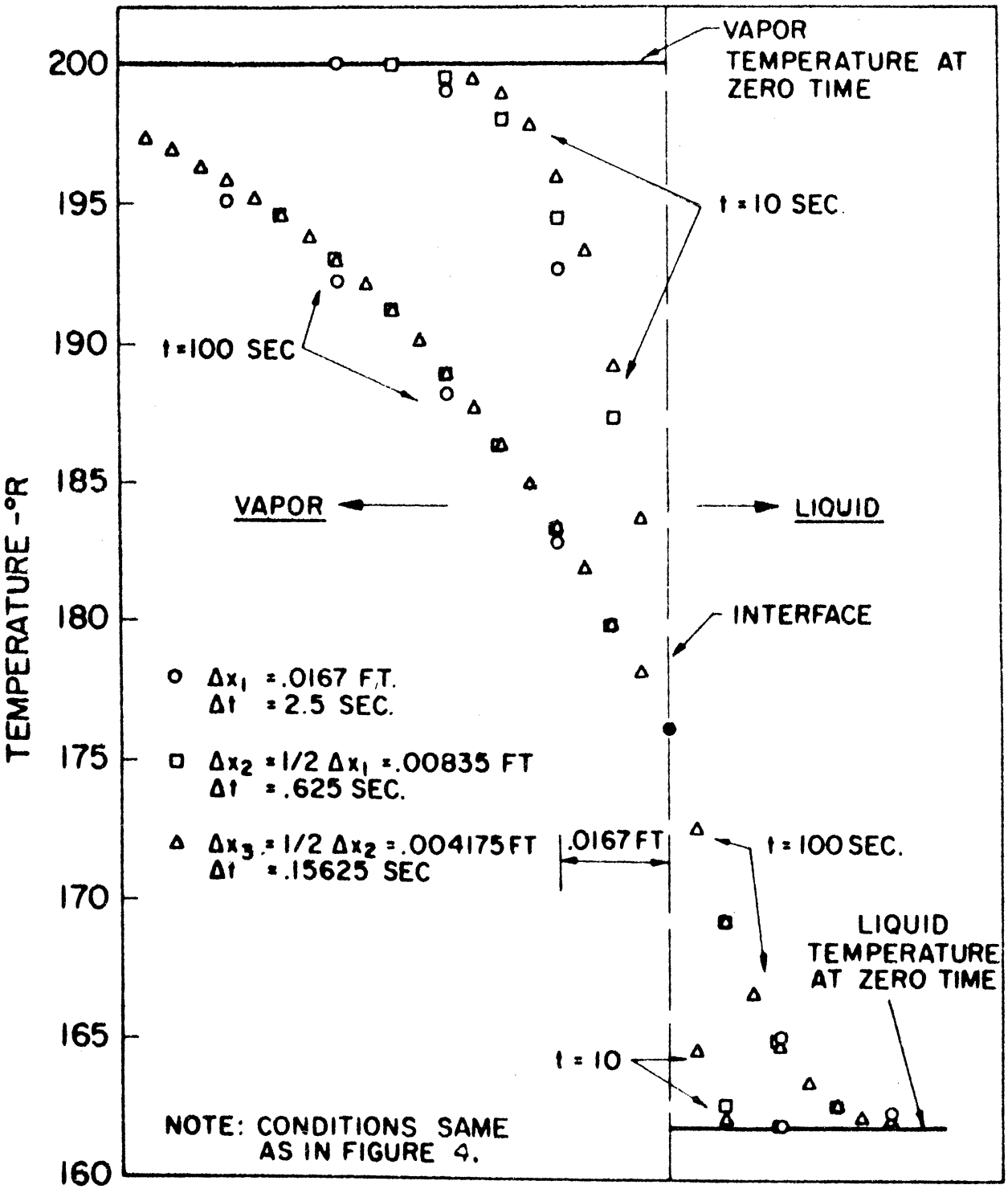


Figure 6. Effect of grid on computed temperature distribution.

fluid, either liquid or vapor, and in general dictates that the grid spacing be as small as possible. The degree of being sufficiently small will be dictated in part by the magnitude of the imposed heat flux or the imposed temperature, and the response of the transient convective process to these disturbances under the prevailing effective gravity level. Again, the only true test available is to vary the grid size for given conditions and compare the coupled results.

The other aspect to be considered is a physical one involving only the liquid, but is also related to the problem of grid spacing. If the temperature of the solid wall in contact with the liquid exceeds the saturation temperature by some amount, dependent upon various parameters including liquid and solid properties and the configuration, nucleate boiling will be initiated. This particular heating surface superheat might be called the incipient boiling point. If information on the incipient boiling point is available for the prevailing conditions, an imposed wall temperature below this point then represents no additional problem beyond that of having sufficiently small grid sizes, as discussed above. For the case of a heat flux imposed on the outer surface of the container, however, the resulting wall temperature is a variable dependent upon a number of parameters such as wall thickness and heat capacity, fluid properties, acceleration level and container geometry. Whether the wall temperature will exceed the incipient boiling point will not be known a priori. since the wall temperature is computed during the course of the computations.

The procedure by which the possibility of nucleate boiling is taken into account is based on the following physical assumptions:

(1) should nucleate boiling begin, further increases in heat flux generally result in relatively small increases in surface temperature as compared to nonboiling convection. This has been observed widely (e.g., Ref. (15)).

(2) The vapor bubbles formed are transported by bouyant forces to the ullage volume quite rapidly. The extent to which this may occur is as yet uncertain, but may be anticipated to depend upon the degree of subcooling present, the pressure and the effective gravity level. These are implemented in the computational procedure by considering that should the tank wall temperature, and hence the liquid adjacent to the wall, exceed the existing saturation temperature by some arbitrary amount, this excess is eliminated by the evaporation of the appropriate amount of liquid directly into the ullage space. In effect, then, a portion of the vapor by passes the liquid-vapor interface. The arbitrary amount referred to above, the symbol for which is given as ΔT_{wmax} might be considered as the incipient boiling point, and no longer is arbitrary when sufficient information on its behavior is available.

They physical phenonmena described above is simulated in the computer program as follows:

(1) The container wall temperature is calculated using Eq. (4).

(2) The liquid temperature is obtained using Eq. (51).

(3) The calculated wall temperature in the liquid region is examined. If it exceeds the saturation temperature by more than the prescribed temperature difference ΔT_{wmax}, it then is reduced such that it equals the saturation temperature plus the prescribed temperature difference.

(4) Part of the heat added to the liquid region appears as enthalpy in the wall and liquid and the rest is used for evaporating some of the liquid. The portion of the heat transferred to the liquid and resulting in evaporation is determined by setting an energy balance according to

$$\int_0^X 2\pi a\, q_w(x,t)dxdt + \int_0^a 2\pi r q_{i\ell} dr$$

$$= \int_0^a \int_0^X 2\rho_\ell c_{p\ell} \pi r \frac{\partial T_\ell}{\partial t} dxdr + \int_0^X 2\pi a \delta \rho_w c_{pw} \frac{\partial Tw}{\partial t} dx + w_i h_{fg} \tag{134}$$

where $q_{i\ell}$ is the rate of heat flow from the interface to the liquid and is given by,

$$q_{i\ell} = k_\ell \left(\frac{\partial T_\ell}{\partial x}\right)_{x=X} \tag{135}$$

Equations (134) and (135) are used to determine the rate of evaporation from the interface, w_i.

If the difference between the wall temperature and the saturation temperature is less than the specified maximum, ΔT_{wmax}, then the procedure above is bypassed and computations proceed as described earlier. An implicit assumption in the use of this procedure is that the laminar flow conditions described by the momentum and energy equations are not affected. This may be reasonable if the container is relatively large compared to the "bubble boundary layer" region next to the wall. In other words, if the vapor bubbles remain in the vicinity of the wall, the major bulk laminar motion of the liquid will not be influenced by their movement to the ullage space. An accurate physical description of this behavior requires additional analytical and experimental investigation in incipient boiling and the departure and motion of vapor bubbles under low gravity fields with various patterns of subcooling.

D. COMPUTER PROGRAM

The computer program listing with the assumption of ideal gas behavior is given in Appendix A and a simplified flow sheet is shown in Appendix B. The MAD (Michigan Algorithmic Decoder) language is used here. In Appendix C are listed the meanings of the symbols employed, and Appendix D lists the

required program inputs. Appendix E presents the results for a single time step during the course of computations of a typical run, and includes the input conditions used.

VI. COMPUTATIONS

Computations were carried out using the program listing of Appendix A, varying several different parameters so as to demonstrate general trends. As a review, the general assumptions incorporated into this particular program are listed below.

A. GENERAL ASSUMPTIONS

1. Cylindrical tank having flat ends.
2. Acceleration or body forces act along the axis.
3. Two dimensional conditions prevail, with variations only along the axis and radially.
4. Laminar flow conditions within the entire container.
5. The vapor behaves as an ideal gas.
6. The liquid has constant properties except in the body force terms.
7. The tank side-wall is uniform in thickness and has constant properties. The wall is lumped in the radial direction but axial conduction is taken into consideration.
8. The imposed heat flux on the outside of the tank wall is uniform, but may differ in those portions in contact with the liquid and vapor.
9. The ends of the tank are adiabatic, and the heat capacity of the ends is neglected.
10. The grid size varies in the liquid and vapor region as liquid fraction changes, in order that a nodal point always exist at the liquid-vapor interface.
11. Initial conditions of uniform temperature and zero velocity exists within the container.
12. The Bond number is sufficiently large that a reasonable approximation to a flat liquid-vapor interface exists.

B. VARIATIONS POSSIBLE IN PROGRAM

By relatively minor modifications to the program, some of the above listed general assumptions can be relaxed, providing additional flexibility. In any case, however, axial symmetry must be maintained.

1. The imposed heat flux can be varied axially and with time.
2. Specified initial conditions of temperature and velocity can be utilized.
3. Specified heat flux to the tank ends, including variations with radius and time, and including heat capacity can be incorporated.
4. If the description of the physical process warrants, imposed temperatures of the container walls can be utilized, with variation axially and with time.

5. Axial variation of tank side wall thickness and variation of specific heat with temperature can be accounted for.

6. Radial variations in the tank wall temperature can be taken into consideration. This may be particularly desirable if the wall is of composite construction.

Although the influence of variations in liquid properties with temperature and pressure can be incorporated with minor changes, the use of real gas properties in the vapor space will require major modifications to the program.

Major modifications also are necessary to handle the spacewise variation of grid size in either the liquid or vapor domains.

C. VARIABLES

The variables listed below were maintained constant for the computations presented here:

1. Grid size: 21 Radial x 31 Axial Nodel Points
2. Tank Diameter: 5 feet
3. Tank Height: 10 feet
4. Tank Wall: Aluminum—0.01 feet thickness
5. Fluid: Liquid and Gaseous Oxygen, initially saturated at 15 psia, with zero velocity.

The grid size was made as small as possible for reasonable computational times on the IBM 7090 computer. It might be noted that decreasing the grid size by one-half would result in an increase in computational time by a factor of approximately 16.

The relation between real physical system time and computational time depends on a number of factors, but primarily on a/g level and heat flux. This arises from the stability requirement on the computational time interval—the higher the fluid velocity the smaller the time steps, hence the greater computer time required to cover a given amount of real time. For example, at $a/g = 10^{-5}$ and $q/A = 1$ BTU/hr-ft^2, 30 minutes of computational time results in 200 minutes of real time, for a ratio of approximately 7 to 1. On the other hand, at $a/g = 10^{-1}$ and $q/A = 1$ BTU/hr-ft^2, this ratio was 1 to 3.

The parameters listed below were varied in order to demonstrate their influence on the pressure rise and the mass of liquid evaporated:

1. ΔT_{wmax}: 0 to 4°F. This is the maximum permissible wall superheat. The value of zero corresponds to neglecting the the heat capacity of the container walls.
2. q/A: 1 to 72 BTU/hr-ft^2 imposed on wall. The heat flux to the liquid and vapor portions of the tank was made the same.
3. Ullage Fraction: 33% to 67%.
4. a/g: 10^{-1} to 10^{-5}.

D. RESULTS

1. Influence of ΔT_{wmax}

To demonstrate the influence of the specified maximum wall superheat, ΔT_{wmax}, on the computed pressure, computations were carried on for several values of this parameter at a relatively high and low heat flux.

Figure 7 shows the pressure rise and total mass evaporated for the heat flux of $q/A = 72$ BTU/hr-ft^2. A continuing effect of ΔT_{wmax} up to 4°F is noted. The computer output listed finite nonzero values of the quantity designated DMB, indicating that evaporation was by-passing the liquid-vapor interface. It is anticipated that a sufficiently large value of ΔT_{wmax} exists such that a further change in it would produce no change in the pressure-time behavior. This resulting wall temperature would probably be considerably above the incipient boiling point, so that the results would have no physical significance.

Figure 8 shows the corresponding results with a relatively low heat flux $q/A = 1$ BTU/hr-ft^2. No differences accrue when ΔT_{wmax} is changed from 1°F to 2°F, indicating that the "natural" wall superheat is less than 1°F for the given conditions. This is also demonstrated by the fact that the quantity DMB is zero for ΔT_{wmax} of 1°F and 2°F. The results for $\Delta T_{wmax} = 0$ correspond to the case where the heat capaicty of the wall is neglected. After 120 minutes, it is noted that the rate of pressure rise is approximately the same for both cases where the wall heat capacity is considered and neglected, although the pressure level is somewhat different. It is also noted in the lower part of Figure 8 that the total mass evaporated levels off for $\Delta T_{wmax} = 0$. This also occurs with the high heat flux shown in Figure 7, and it is believed will occur with the low heat flux case for $\Delta T_{wmax} = 1$°F, in Figure 8, at longer periods of time. Even though the rate of evaporation levels off, the rate of pressure rise continues. This is attributed to the increasing effect on pressure rise of the heat transfer to the ullage space as compared to the heat transfer to the liquid.

2. INFLUENCE OF HEAT FLUX

Figure 9 shows the influence of heat flux on pressure and mass evaporated for the case with $\Delta T_{wmax} = 0$°F, which corresponds to neglecting the wall heat capacity. An additional heat flux of $q/A = 1$ BTU/hr-ft^2 was used, but the effects were too small to be shown on Figure 9 because of the scale. A cross-plot of Figure 9 is shown on a log-log scale in Figure 10 for various time periods, with this additional data showing the effect of heat flux. It is noted that the pressure rise is approximately a linear function of heat flux. Whether this would hold for other values of initial ullage cannot be stated at this time. It might be anticipated that these results would be modified at the early periods of time with values of ΔT_{wmax} other than zero.

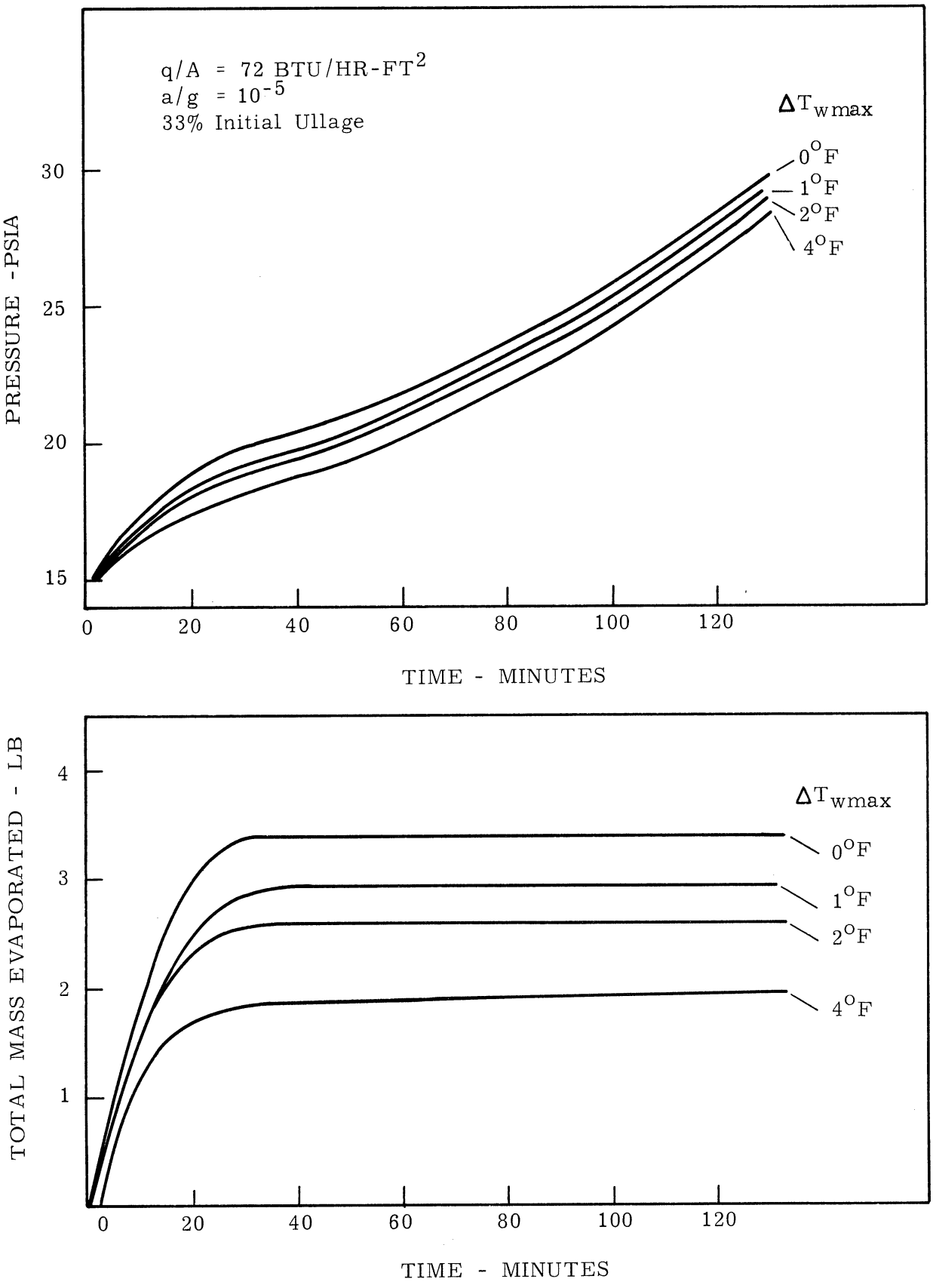
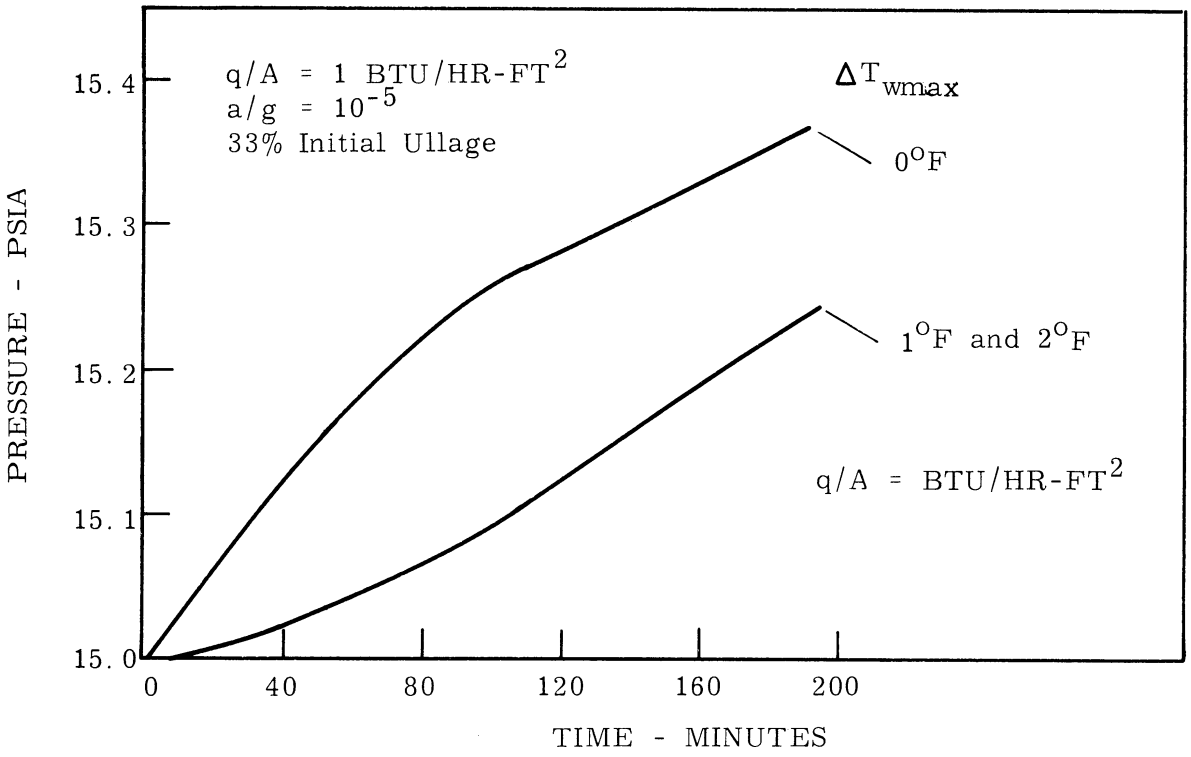


Figure 7. Total mass evaporated and pressure rise under high heat flux.



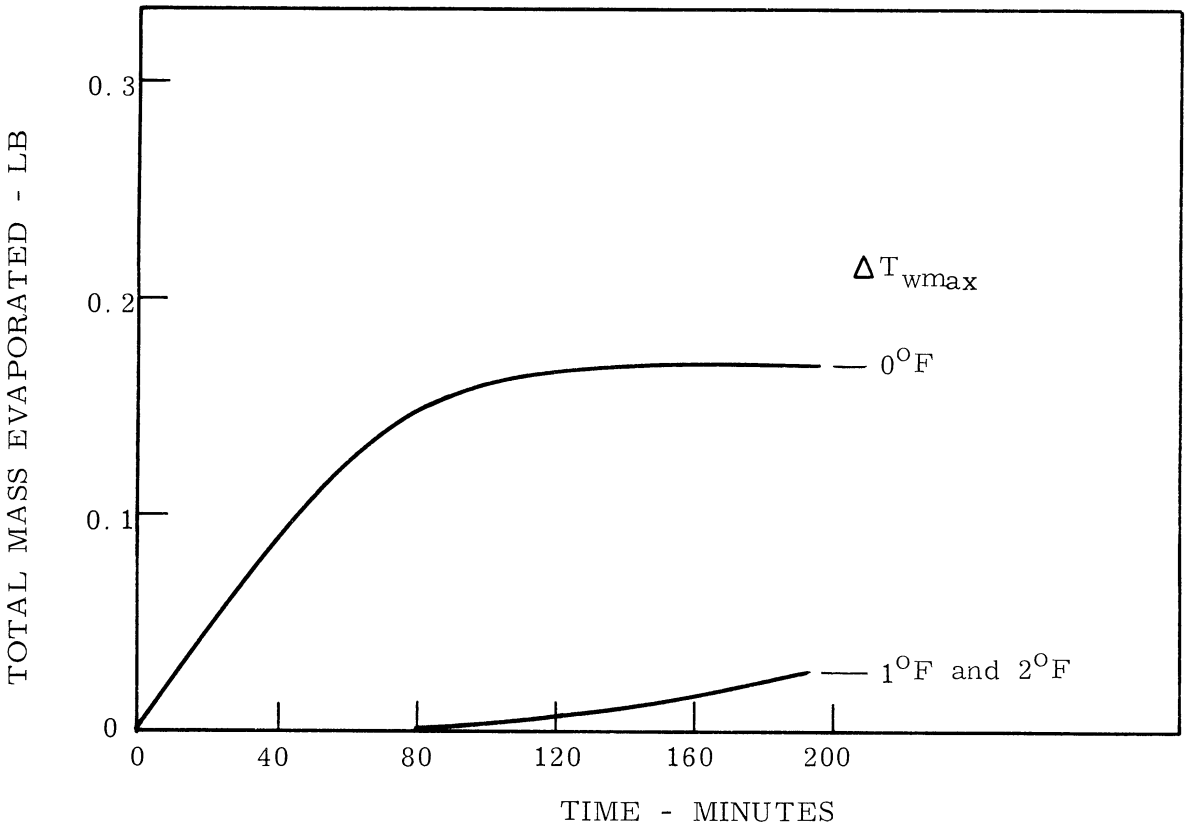


Figure 8. Total mass evaporated and pressure rise under low heat flux.

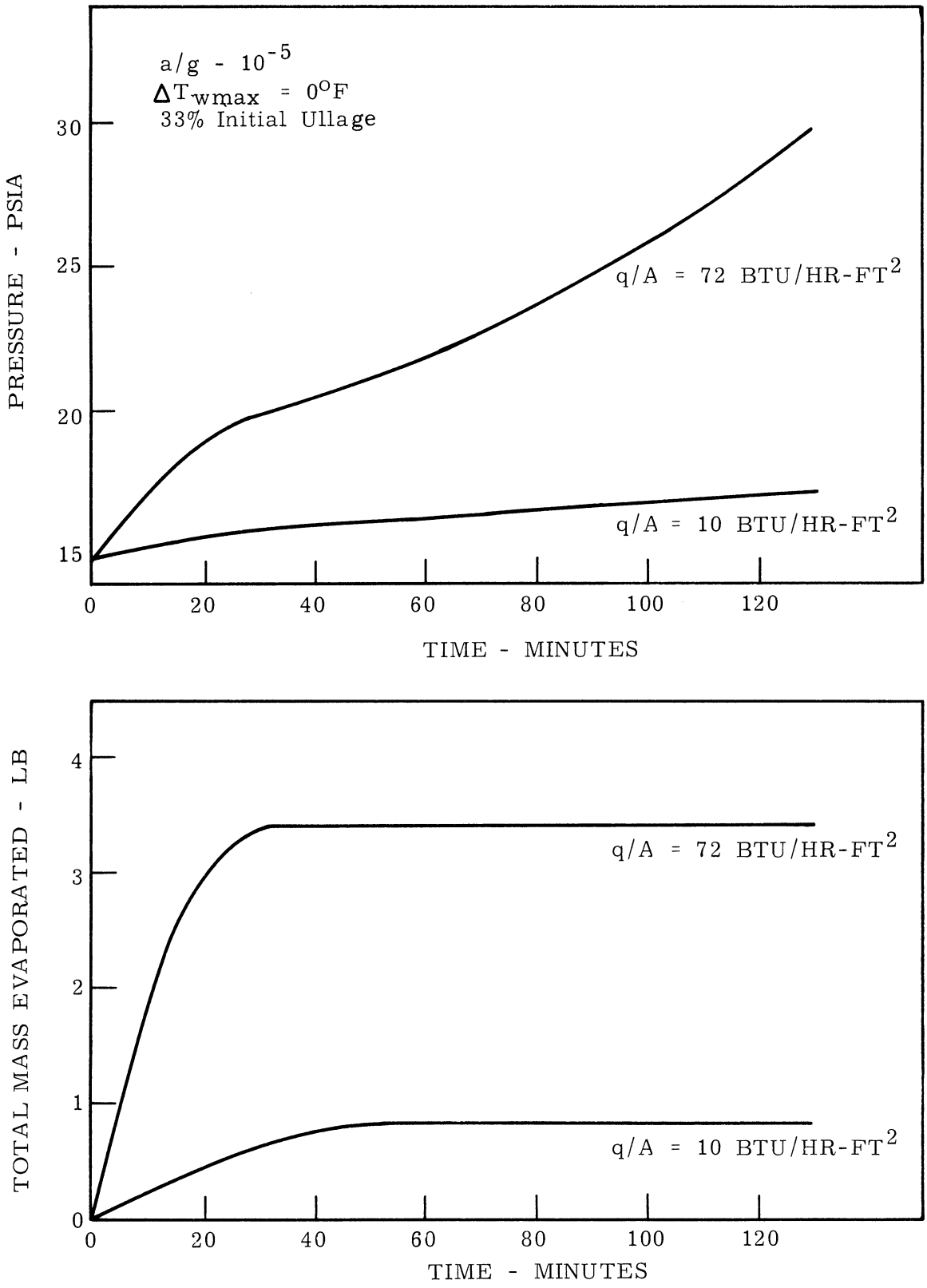


Figure 9. Effect of heat flux on pressure rise and mass evaporated.

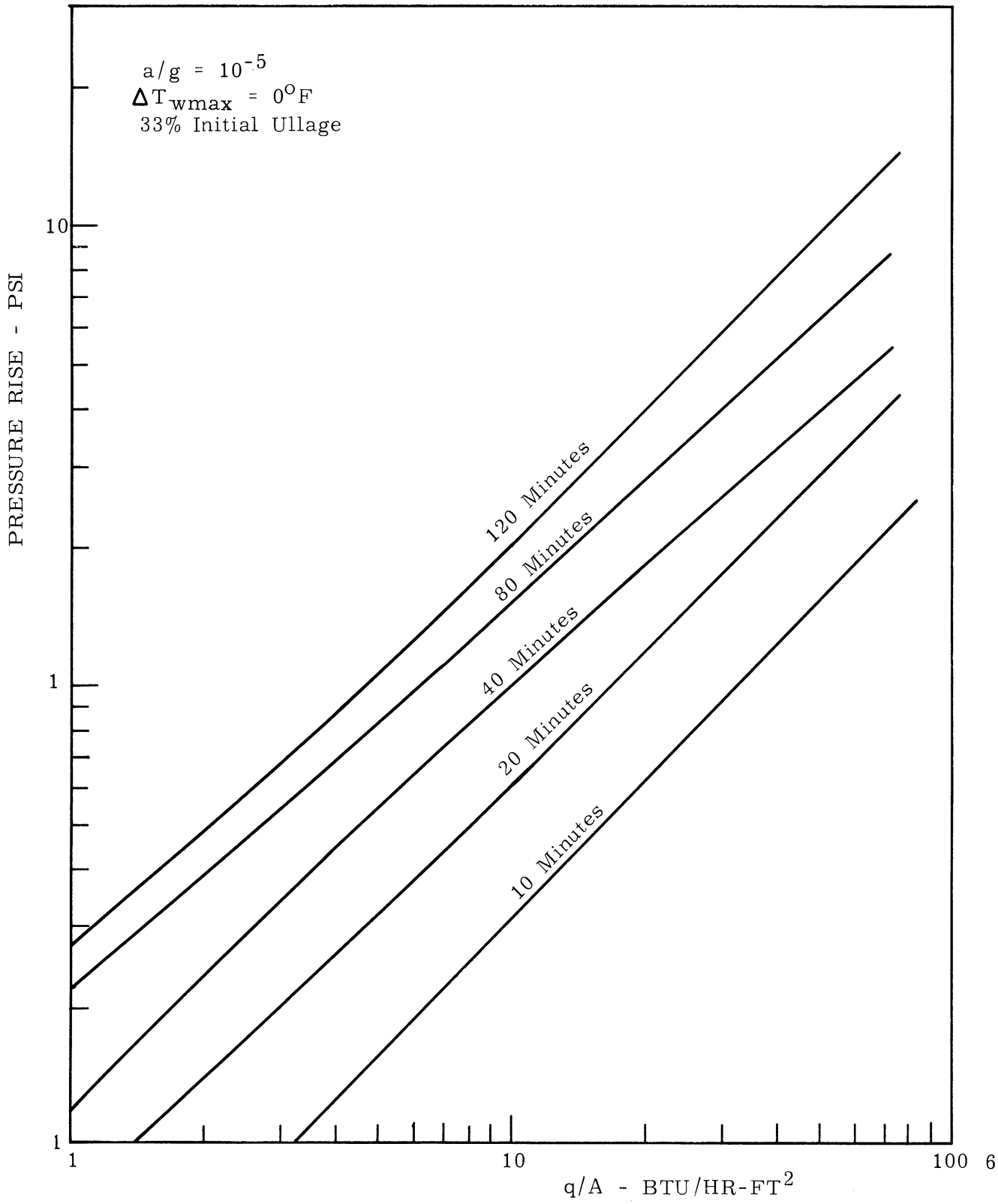


Figure 10. Effect of heat flux on pressure rise.

3. INFLUENCE OF ULLAGE FRACTION

Figure 11 shows that the influence of initial ullage volume on pressure rise and mass evaporated for the case where $q/A = 1$ BTU/hr-ft^2, $a/g = 10^{-5}$, and $\Delta T_{wmax} = 1°F$. In this case the wall superheat never reaches the specified maximum value of $\Delta T_{wmax} = 1°F$, and hence all of the evaporation takes place at the liquid-vapor interface. In terms of the computer program the quantity DMP is always zero.

Figure 12 is a crossplot of Figure 11 for three levels of time. It is noted that the rate of pressure rise increases as the initial ullage fraction increases. This is contrary to the trend observed in an experimental study (9), and may indicate that the level of heat flux is an additional factor governing the effect of ullage volume. Added to the upper part of Figure 11 are the cases for 100% ullage, as computed from Eq. (83), and 0% ullage consistent with the assumption of an incompressible liquid. The behavior of the intermediate values of ullage is consistent with these limits.

It is further noted from Figure 11 that after an initial starting transient of approximately 100 minutes that both the pressure and mass evaporated become essentially linear functions of time.

4. INFLUENCE OF ACCELERATION LEVEL

The effect of acceleration or body force level on the pressure rise and mass evaporated is shown in Figure 13. Again, the wall superheat never reaches the specified maximum value of $\Delta T_{wmax} = 1°F$. As might be anticipated, increasing the acceleration level increases both the pressure rise rate and the interface evaporation rate, owing to the increasing convective velocities induced in both the liquid and vapor.

Figure 13 also demonstrates, as discussed earlier, the relation between computational time and real time. In each of the three cases shown the computational time was 30 minutes. The increased fluid velocities associated with the larger body forces result in smaller incremental time steps from the stability requirements. If the body forces present in a particular physical application are considerably smaller than $a/g = 10^{-5}$, as in deep space probes, it can be anticipated that the behavior for long periods of real time can be described with reasonably small amounts of total computer time.

5. REPRESENTATIVE TEMPERATURE PROFILES

In Figure 14 a-e are plotted the axial temperature distribution for several levels of time, for five different radial locations from the centerline to the wall. All temperatures are expressed as the temperature increase above the initial temperature.

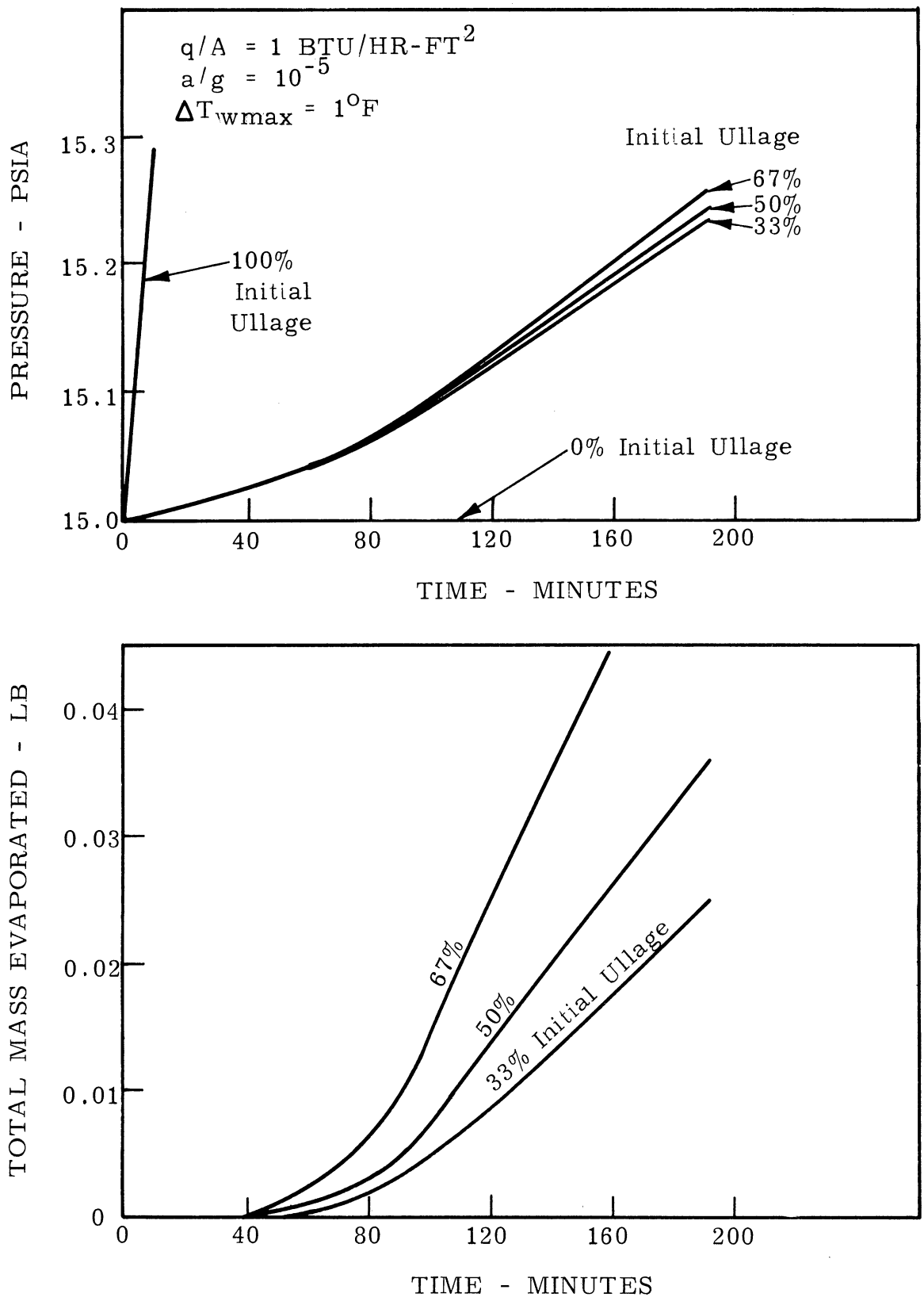


Figure 11. Effect of initial ullage on pressure rise and mass evaporated.

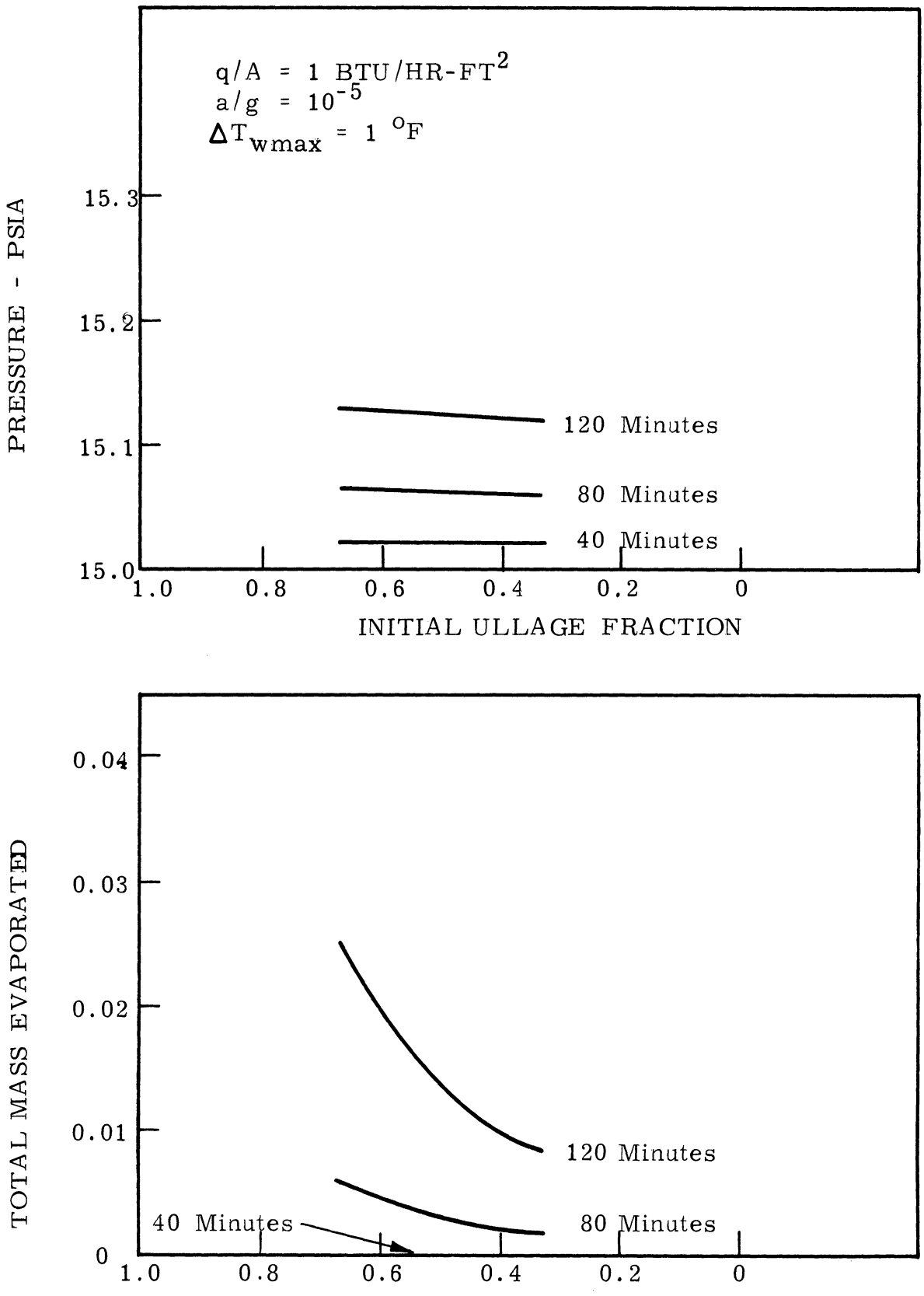


Figure 12. Effect of initial ullage on pressure rise and mass evaporated.

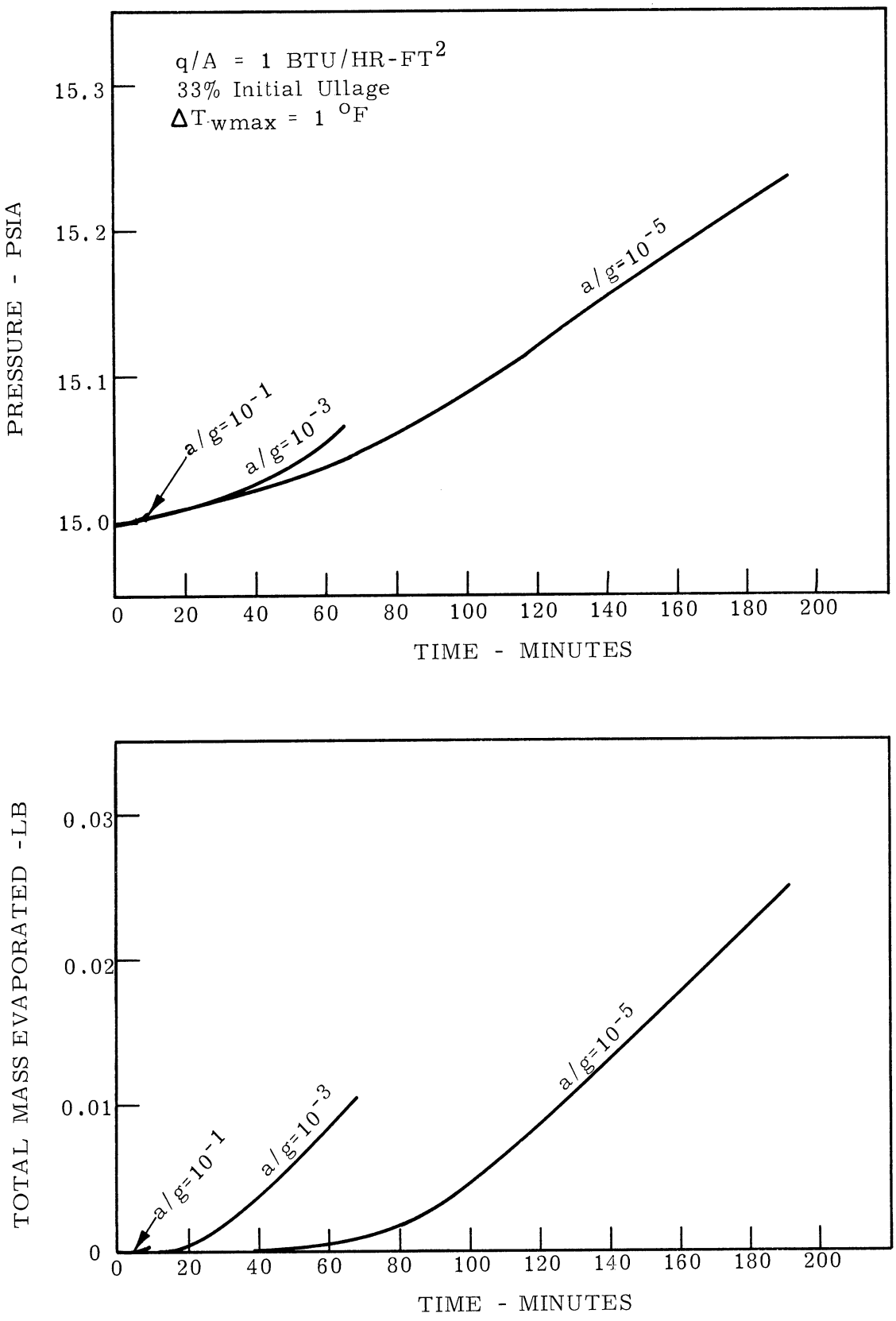


Figure 13. Effect of acceleration on pressure rise and mass evaporated.

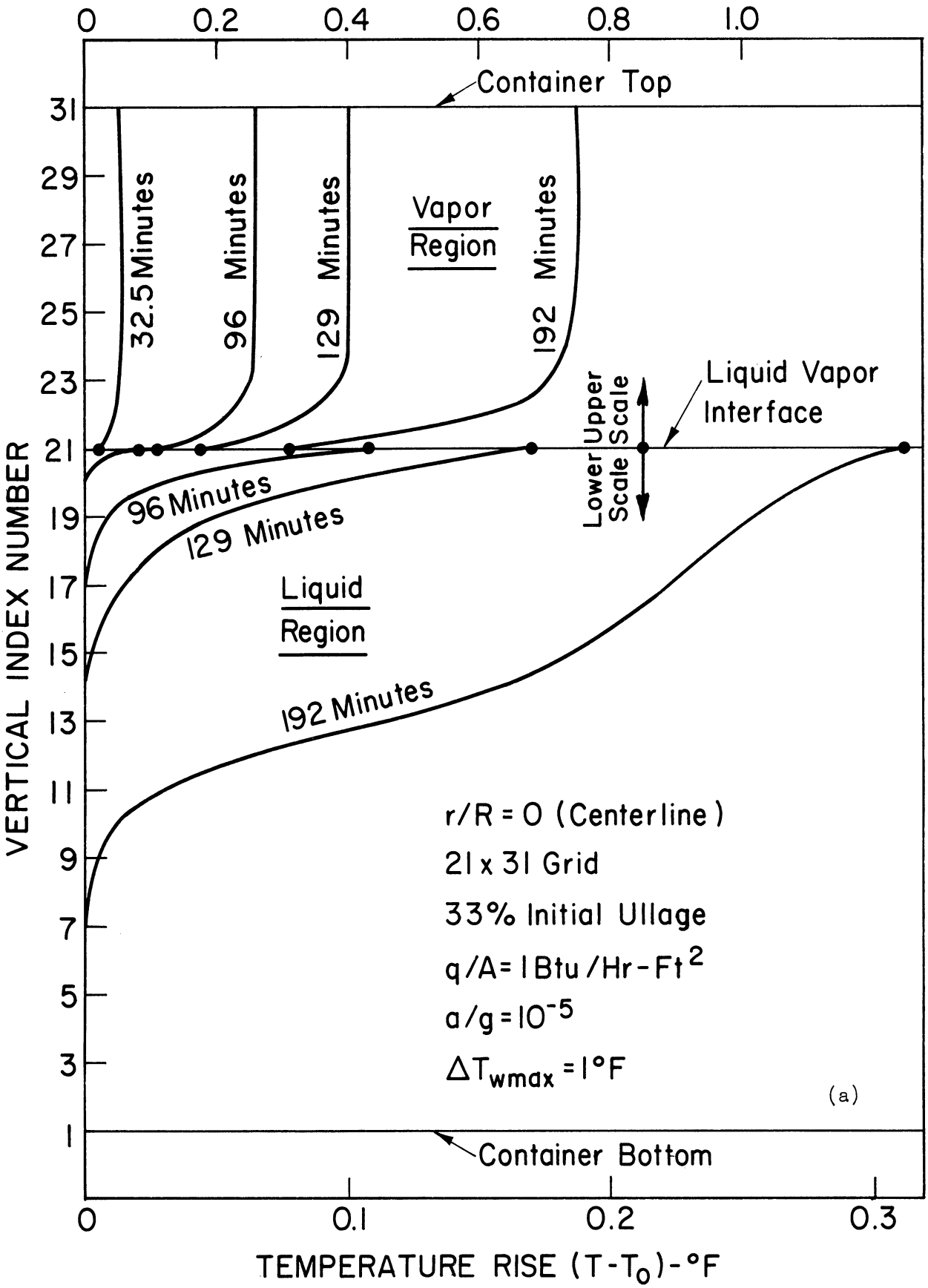


Figure 14. Axial temperature distribution.

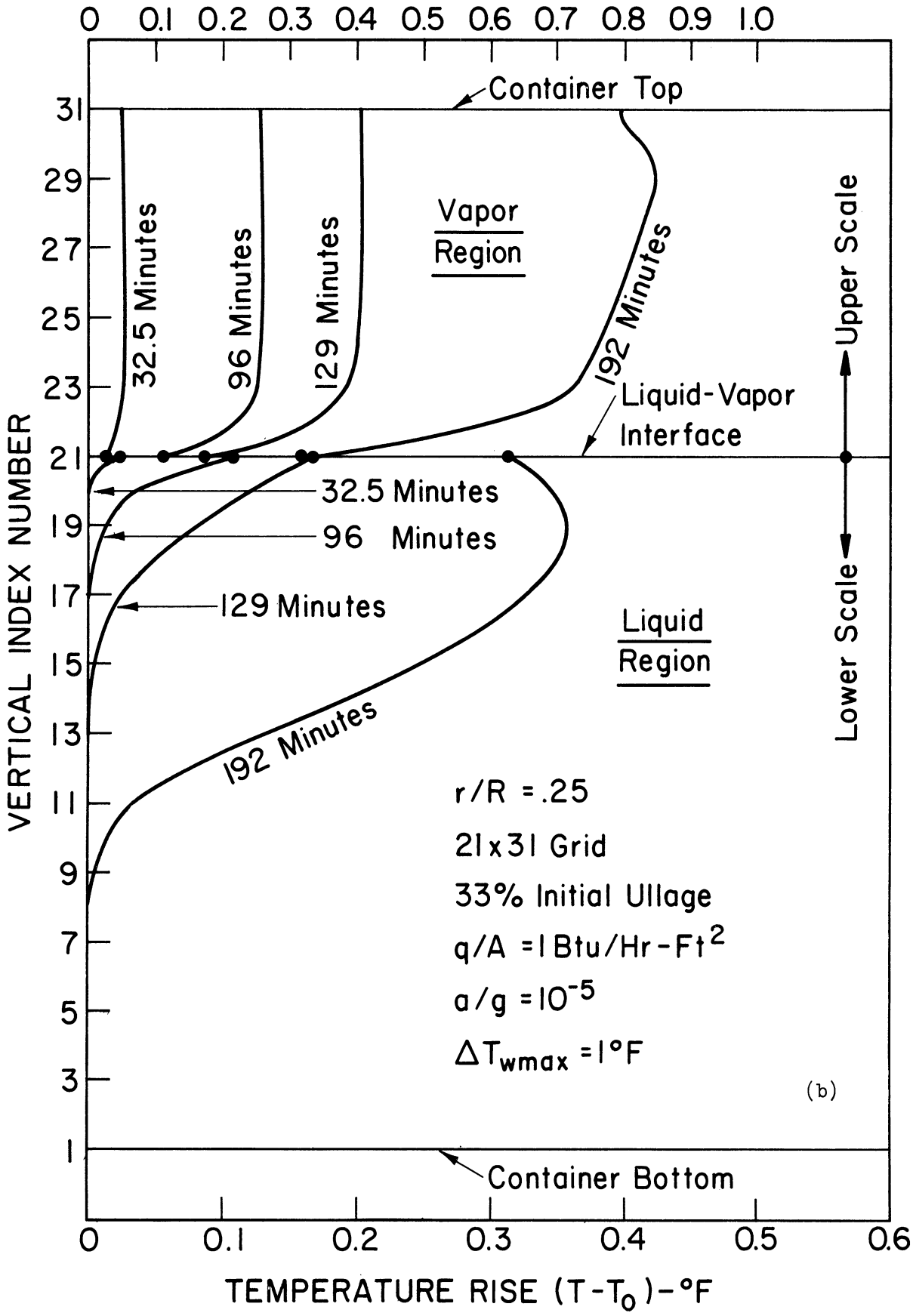


Figure 14. (Continued).

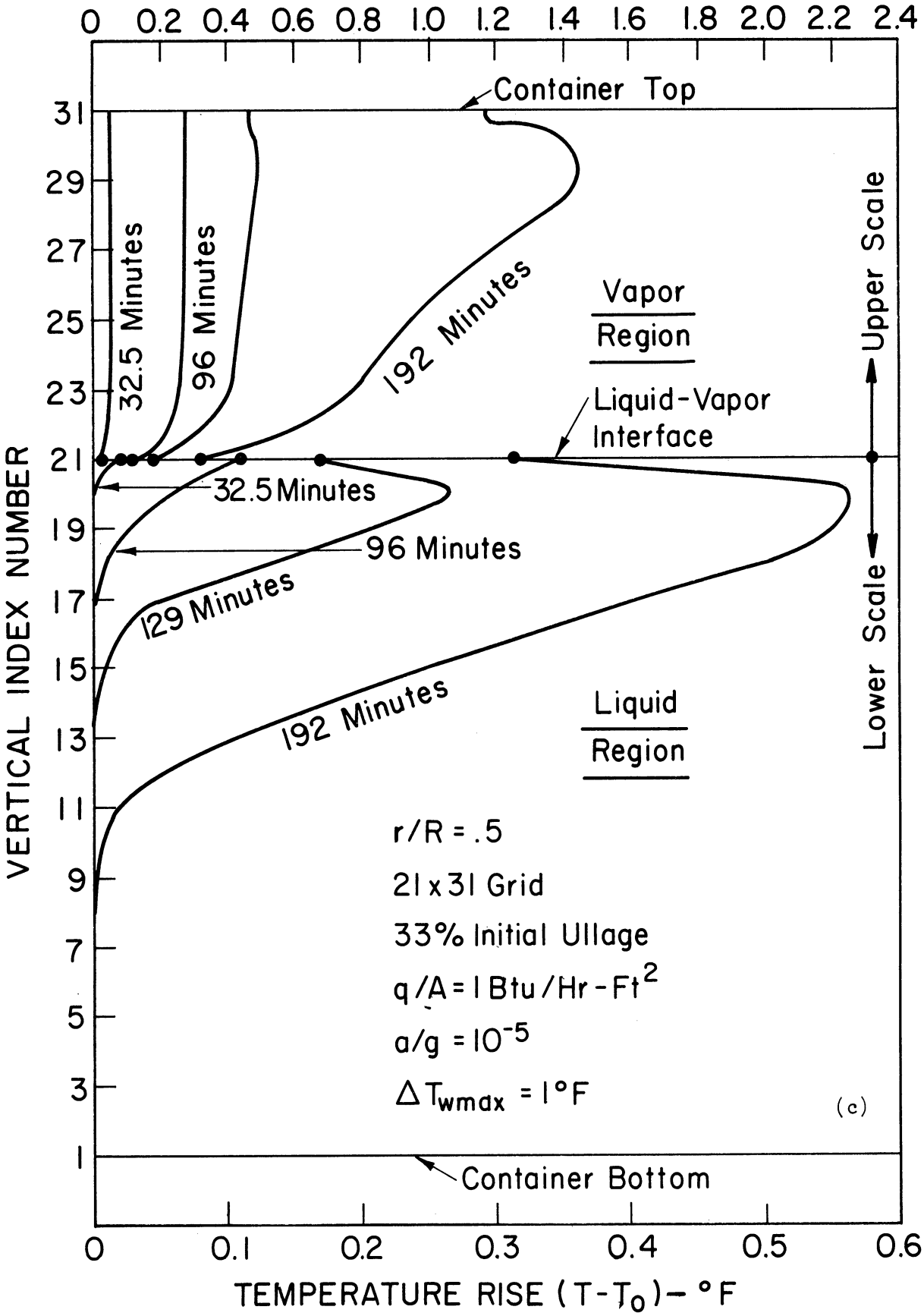


Figure 14. (Continued).

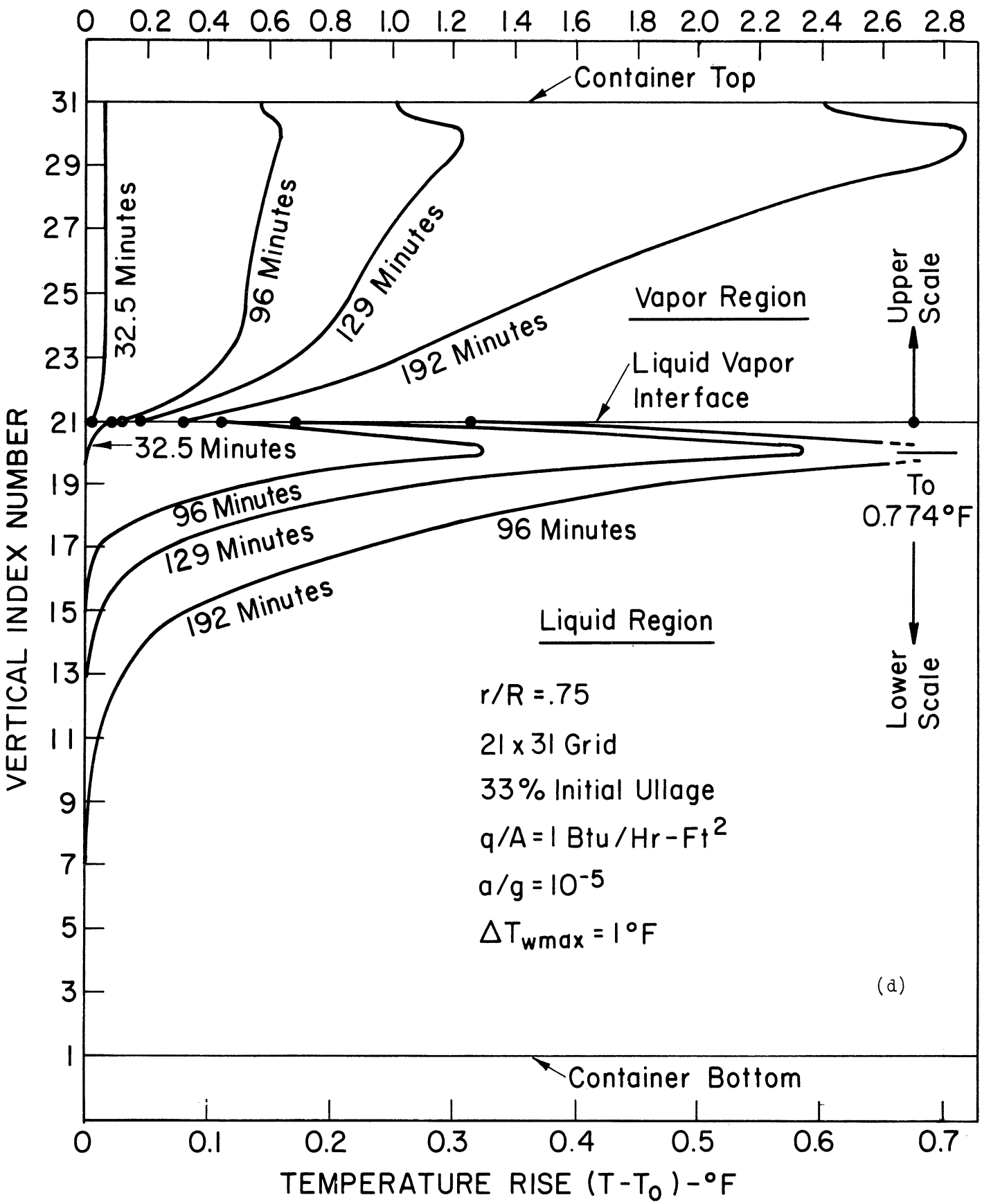


Figure 14. (Continued).

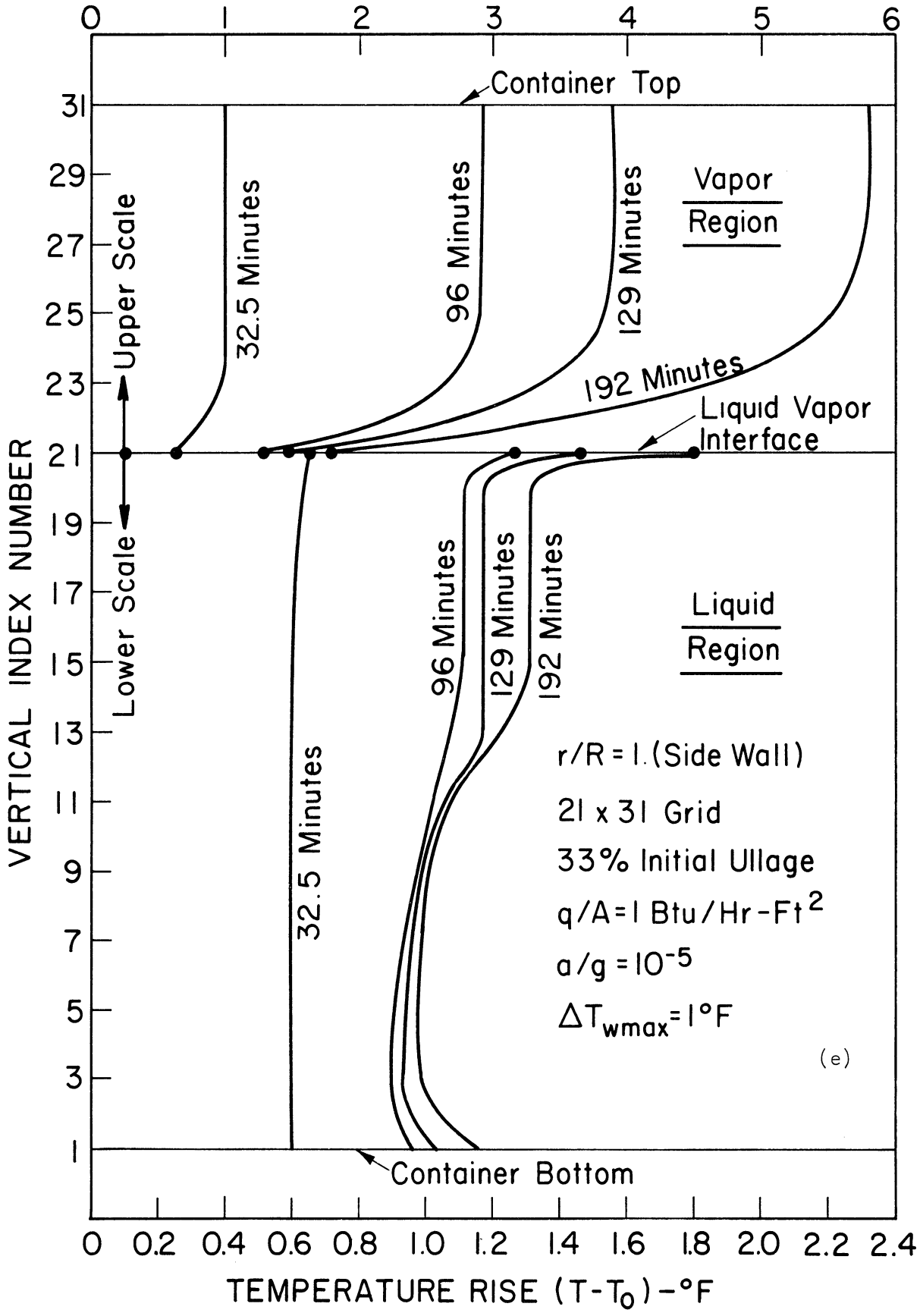


Figure 14. (Concluded).

The relatively rapid increase in temperature of the ullage space as compared to the liquid region is noted. In fact, about one half of the liquid undergoes no change in temperature at all during this period of time. The temperature gradients near the centerline are such that condensation may be taking place there simultaneously with evaporation toward the walls. Of particular interest is the superheated layer of liquid existing just below the liquid-vapor interface at the later time periods in Figures 14 b-d.

The influence of the stagnation region in the tank is observed in the lower part of Figure 14e.

In Appendix E the computer input and output are reproduced for the case represented in Figures 14 a-e, for computational step number 154, which corresponds to a total elapsed real time of 192 minutes.

APPENDIX A

COMPUTER PROGRAM LISTING

MAD (17 MAY 1967 VERSION) PROGRAM LISTING ... ...

```
                DIMENSION U( 980,DIM),V( 980,DIM),T( 980,DIM),SF( 980,DIM),                *C01
               1 W( 980,DIM),TO( 980,DIM),WC( 980,DIM),SFI(31),                            *C01
               2 EJ(31),F(31),DI(31),DR1(31),DR2(31),DR3(31),DR4(31),R1(31),R              *C01
               3 2(31),R3(31),R4(31),R5(31),A1(31),A2(31),A3(31),C2(31                     *C01
               4 ),C1( 980,DIM),C3( 980,DIM),EJ1(31),D1(31),D2(31),R6(31),DR6              *C01
               5 (31),UL(31),DR5(31),DR6(31),DR7(31),DR8(31),R6(31),A4(31)                 *001
               6 ,VG(31),RO( 980,DIM) , DTDRW(46),DTDXG(31),UO( 980,DIM),VC( 9             *C01
               7 80,DIM),DTDXL(31),HL(31),HG(31),C4( 980,DIM)                              *C01
                 VECTOR VALUES DIM=2,1,0                                                   *C02
                INTEGER I,J,M,N,NT,NE,NE1,NE2, L ,NR,NC,H,P,J1,J2,I1,I4,NP,               *C03
               1 N1,N2,N3,TAU1,NMAX                                                        *C03
                EXECUTE FTRAP.                                                           *C04
START           READ AND PRINT DATA                                                      *C05
                DIM(2)=N+1                                                               *C06
                EXECUTE ZERO.(U(1,1)...U(P+1,N+1),V(1,1)...V(P+1,N+1),T(1,1)             *C07
               1 ...T(P+1,N+1), SF(1,1)...SF(P+1,N+1), W(1,1)...W(P+1,N+1),              *C07
               2 TO(1,1)...TO(P+1,N+1),NT,TIME,NE1,NE2,N2,U,TAU,                         *C07
               3 UL(0)...UL(N+1),DTDXG(0)...DTDXG(N+1),DTDRW(0)...DTDRW(N+1),            *C07
               4 RO(1,1)...RO(P+1,N+1),P1,UO(1,1)...UO(P+1,N+1),VO(1,1)...VO             *C07
               5 (P+1,N+1),DMT,DMBT,DM,A6,M6,C4(1,1)...C4(M+1,N+1),NP1,DMB )             *C07
                I1=M/2+1                                                                 *C08
                Y=AOVERB                                                                 *C09
                PR=NEW/ALPHA                                                             *C10
                PRV=NEWV/ALPHAV                                                          *C11
                RNEW=NEWV/NEW                                                            *C12
                RALPHA=ALPHAV/ALPHA                                                      *C13
                DR=1.0/N                                                                 *C14
                DR2=DR*DR                                                                *C15
                GAM=CPV/CV                                                               *C16
                DYY=2.0*DR2                                                              *C17
                DR6=DR2/2.0                                                              *C18
                PRESS=PC                                                                 *C19
                TSAT=C5*(PRESS.P.C6)                                                     *C20
                TINIT=C5*(PO.P.C6)                                                       *C21
                CONST = A*A/ALPHA                                                        *C22
                CONST1=Y*PR*G*BEITA*(A.P.3)/NEW/NEW                                      *C23
                CONST2=1.0/CONST                                                         *C24
                CONST3=1.0/CONST1                                                        *C25
                PRINT RESULTS CONST , CONST2 , CONST1 , CONST3,PR,PRV                    *C26
                M1=AOVERB*QSURFL*(A.P.5)*BEITA*G/ALPHA/ALPHA/NEW/ROW/CPW                 *C27
               1 /DELTA                                                                  *027
                M2=KL*A/ALPHA/ROW/CPW/DELTA                                              *028
                M3=KG*A/ALPHA/ROW/CPW/DELTA                                              *C29
                M7=AOVERB*QSURFG*(A.P.5)*BEITA*G/ALPHA/ALPHA/NEW/ROW/CPW                 *C30
               1 /DELTA                                                                  *030
                M5=CONST1*TINIT                                                          *031
                D2R=DR/2.0                                                               *032
                AOVRB2=AOVERB*AOVERB                                                     *033
                THROUGH CC,FOR J=2,1,J.G.N+1                                             *034
                A1(J)=DR2*DR2*(J-1.)*(J-1.)                                              *C35    01
                C2(J)=1.+1./(2.*(J-1.))                                                  *C36    01
                A4(J)=1.0/(J-1.0)/DR/4.0                                                 *037    01
CC              R1(J)=12*(J-1.0)*DR2                                                     *038    01
```

```
      THROUGH GAST,FOR I=M+1,1,I.G.P+1                                  *C39
      THROUGH GAST,FOR J=1,1,J.G.N                                      *C40          01
GAST  T(I,J)=0                                                          *C41          02
      WHENEVER CODE .E. 1                                               *C42
      READ FORMAT INPUT,A,PRESS, PC, X, NT, TIME, U,                    *C43    01
     1 T(1,1)...T(P+1,N+1),SF(1,1)...SF(P+1,N+1)                        *C43
      THROUGH HUSS,FOR I=1,1,I.G.P+1                                    *C44    01
      THROUGH HUSS , FOR J=1,1,J.G. N+1                                 *C45    01    01
HUSS  TO(I,J)=T(I,J)                                                    *C46    01    02
      END OF CONDITIONAL                                                *C47    01
      N3=1                                                              *C48
      P1=PRESS                                                          *C49
BACK   NT=NT+1                                                          *C50
      HFG=.00745*(-1.1383*(TSAT-180)*(TSAT-198)*(TSAT-216)+3.238*       *C51
     1 (TSAT-162)*(TSAT-198)*(TSAT-216)-3.025*(TSAT-162)*(TSAT-180)     *C51
     2 *(TSAT-216)+.925*(TSAT-162)*(TSAT-180)*(TSAT-198))               *C52
      RP=PRESS/PO                                                       *C53
      I=(P+M)/2+1                                                       *C54
      J=N/2+1                                                           *C55
      BETTAG = 1./(TINIT+T(I,J)/CONST1)                                 *C56
      M4=(GAM-1)/GAM                                                    *C57
      CT=1.                                                             *C58
      DX=X/M                                                            *C59
      DX2=DX*DX                                                         *C60
      DZ=(1-X)/(P-M)                                                    *C61
      DZ2=DZ*DZ                                                         *C62
      DRZ=DR2/DZ2                                                       *C63
      A3=DRZ*Y*Y                                                        *C64
      BJ1=2.*(1+A3)                                                     *C65
      DXR=DX2/DR2                                                       *C66
      DRX=DR2/DX2                                                       *067
      A1=DRX*Y*Y                                                        *C68
      A2=DXR/Y/Y                                                        *C69
      D2Z=.5                                                            *C70
      ZOVR2=Z/2.0                                                       *C71
       BJ=2.*(1.+A1)                                                    *C72
      BO=2.0*PR*Y*Y/DR2+2.C*PR/DR2                                      *C73
      EJ(1)=0                                                           *C74
      EJ1(1)=0                                                          *C75
      THROUGH JE, FOR J = 2,1, J .G. N+1                                *C76          01
      A2(J)=DX2*DR2*(J-1.)*(J-1.)/Y/Y                                   *C77          01
       A3(J)=DX2/Y/Y/(J-1.)/DR2/2.                                      *C78          01
       D1(J)=BJ-C2(J)*EJ(J-1)                                           *C79          01
      R2(J)=12*(J-1.)*DR*DX                                             *C80          01
      D2(J)=BJ1-C2(J)*EJ1(J-1)                                          *C81          01
      EJ1(J)=(1-1/(2.C*(J-1.C)))/D2(J)                                  *C82          01
      R4(J)=Y*Y/(J-1.)/(J-1.)/DR2/DX2/2.                                *C83          01
      R5(J)=Y*Y/(J-1.)/(J-1.)/DR2/DZ2/2.                                *C84          01
      R6(J)=12*(J-1.)*DR*DZ                                             *C85          01
JE    EJ(J)=(1-1/(2.0*(J-1.0)))/D1(J)                                   *C86
      WHENEVER CODE .E. 1                                               *087    01
      THROUGH FOUR, FOR J=2,1,J.G.N                                     *C88    01    01
FOUR  SF(M+1,J)=0                                                       *C89    01
      THROUGH MMM, FOR I=2,1,I.G.M                                      *C90    01    01
      THROUGH MMM, FOR J=2,1,J.G.N                                      *C91    01    02
MMM   W(I,J)=((SF(I+1,J)-2 *SF(I,J)+SF(I-1,J))*Y*Y/DX2-(SF(I,J+1)-      *C91
     1 SF(I,J-1))/DR2/(J-1.0)/2+(SF(I,J+1)-2.C*SF(I,J)+SF(I,J-1))/      *C91
     2 DR2)/DR2/(J-1.0)/(J-1.C)                                         *C92    01
      THROUGH QQQ, FOR I=2,1,I.G.M                                      *C93    01    01
QQQ   W(I,N+1)= (8.*SF(I,N)-SF(I,N-1)-7.*SF(I,N+1))/DYY
```

```
      THROUGH QQQQQ, FOR I=M+1,1,I.G.P+1
      THROUGH QQQQQ, FOR J=1,1,J.G.N+1
QQQQQ RO(I,J)=CONST1*PRESS*144./RGAS/(T(I,J)+M5)
      THROUGH QQQQ, FOR I=M+2,1,I.G.P
QQQQ  W(I,N+1)=(8*SF(I,N)-SF(I,N-1)-7*SF(I,N+1))/DYY/RC(I,N+1)
      UG=-U*(ROL/RO(M+1,1)-1.0)
      THROUGH FIVE, FOR J=2,1,J.G.N+1
FIVE  SF(M+1,J)=RO(M+1,J)*UG*DR2*(J-1.0)*(J-1.0)/2.0
      THROUGH WWW, FOR I=M+1,1,I.G.P
      U(I,N)=2.*(3*SF(I,N)-6*SF(I,N-1)+SF(I,N-2))/R1(N)/RC(I,N)
      U(I,1)=SF(I,2)/DR6/RO(I,1)
WWW   U(I,2)=2.*(6.*SF(I,3)-SF(I,4)-3.*SF(I,2))/R1(2)/RO(I,2)
      THROUGH WWWW, FOR I=M+1,1,I.G.P
      THROUGH WWWW, FOR J=3,1,J.G.N-1
WWWW  U(I,J)=(SF(I,J-2)-8*SF(I,J-1)+8*SF(I,J+1)-SF(I,J-2))/R1(J)
     1 /RO(I,J)
      THROUGH EFF, FOR J=2,1,J.G.N
      V(M+2,J)=2.*(3.*SF(M+2,J)+SF(M+4,J)-6.*SF(M+3,J)+2.*SF(M+1,J)
     1 )/R6(J)/RO(M+2,J)
      VG(J)=2.*(SF(M+3,J)-8.*SF(M+2,J)+7.*SF(M+1,J))/R6(J)/RC(M+1,J
     1 )
EFF   V(P,J)=2.*(6.*SF(P-1,J)-3.*SF(P,J)-SF(P-2,J))/R6(J)/RC(P,J)
      THROUGH EFFF, FOR J=2,1,J.G.N
      THROUGH EFFF, FOR I=M+3,1,I.G.P
EFFF  V(I,J)=(SF(I+2,J)-8*SF(I+1,J)+8*SF(I-1,J)-SF(I-2,J))/R1(J)/RC
     1 (I,J)
      THROUGH SSS, FOR I=M+2,1,I.G.P
      THROUGH SSS, FOR J=2,1,J.G.N
SSS   W(I,J)=((SF(I+1,J)-2.*SF(I,J)+SF(I-1,J))*Y*Y/DZ2-(SF(I,J+1)-
     1 SF(I,J-1))/DR2/(J-1.)/2+(SF(I,J+1)-2*SF(I,J)+SF(I,J-1))/DR2)/
     2 DR2/(J-1.)/(J-1.)/RO(I,J)-(U(I,J)*(RC(I,J+1)-
     3 RO(I,J-1))/D2R-V(I,J)*(RC(I+1,J)-RC(I-1,J))/D2Z)*A4(J)/RC(I,J
     4 )
      THROUGH PPP, FOR J=2,1,J.G.N
      W(P+1,J)=(-SF(P-1,J)+8.*SF(P,J)-7.*SF(P+1,J))
     1 *R5(J)/RC(P+1,J)
PPP   W(1,J)=(-SF(3,J)+8.*SF(2,J))*R4(J)
      CODE=2
      TRANSFER TO ML
      END OF CONDITIONAL
      SMAX=0
      THROUGH CDD,FOR I=2,1,I.G.M+1
      THROUGH CDD,FOR J=1,1,J.G.N
      S=BO+.ABS.U(I,J)/DX+.ABS.V(I,J)/DR
CDD   WHENEVER SMAX .L.S, SMAX=S
      WHENEVER PRV.G.1.
      BOO=4.*PRV/DR2+2.*PRV*Y*Y/DZ2
      BO=2.*PRV*(Y*Y/DZ2+1./DR2)
      OTHERWISE
      BOO=4./DR2+2.*Y*Y/DZ2
      BO=2.*(Y*Y/DZ2+1./DR2)
      END OF CONDITIONAL
      THROUGH DCC, FOR I=M+2,1,I.G.P
      THROUGH DCC, FOR J=2,1,J.G.N
      S=BO+.ABS.U(I,J)/DZ+.ABS.V(I,J)/DR
DCC   WHENEVER SMAX .L. S, SMAX=S
      THROUGH MAXS,FOR I=M+2,1,I.G.P
      S=BOO+.ABS.U(I,1)/DZ
      WHENEVER (SMAX*DT) .G. 0.8,DT=0.8/SMAX
MAXS  WHENEVER SMAX .L. S,SMAX=S
```

```
*094    01
*095    01    01
*096    01    02
*097    01
*098    01    01
*099    01
*100    01
*101    01    01
*102    01
*103    01    01
*104    01    01
*105    01    01
*106    01
*107    01    01
*108    01    02
*108
*109    01
*110    01    01
*110
*111    01    01
*111
*112    01    01
*113    01
*114    01    01
*115    01    02
*115
*116    01
*117    01    01
*118    01    02
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*119    01
*120    01    01
*120
*121    01    01
*122    01
*123    01
*124    01
*125
*126
*127          01
*128          02
*129          02
*130
*131    01
*132    01
*133    01
*134    01
*135    01
*136    01
*137
*138          01
*139          02
*140          02
*141
*142          01
*143          01
*144          01
```

```
      TIME=TIME+DT                                                      *145
      TAU=TIME * CONST                                                  *146
      X=X+U*DT                                                          *147
      DX1=DT/DX                                                         *148
      DZ1=DT/DZ                                                         *149
      DZ3=CT*DT*Y*Y*RALPHA/DZ2                                          *150
      DZ5=2.*DZ3                                                        *151
      DZ7 = DT *Y * Y * PR * RNEW / DZ2                                 *152
      DZ8=DT*Y*Y*ALPHAW/ALPHA/DZ2                                       *153
      DY3=CT*DT*RALPHA/DR2                                              *154
      DY1=2.*DY3                                                        *155
      DY7 = DT * RNEW * PR / DR2                                        *156
      DY5=4.*DY3                                                        *157
      DX3=DT*Y*Y/DX2                                                    *158
      DR3=N*N*DT                                                        *159
      DR4=N*DT                                                          *160
      DR5=4.*DR3                                                        *161
      DX5=2.*DX3                                                        *162
      DR1=2*DR3                                                         *163
      DX7=DX3*PR                                                        *164
      DX8=DT*Y*Y*ALPHAW/ALPHA/DX2                                       *165
      DR7=DR3*PR                                                        *166
      C1=1.0-DX5-DR5                                                    *167
      C2=1.0-DX5-DR1                                                    *168
      C3=1.0-2.C*DX7-2.0*DR7                                            *169
      THROUGH CD,FOR J=2,1, J.G.N+1                                     *170
      FI=0.5/(J-1.0)                                                    *171  01
      DI=1.5/(J-1.0)                                                    *172  01
      DR1(J)=DR3*(1.0-FI)                                               *173  01
      DR2(J)=DR3*(1.0+FI)                                               *174  01
      DR3(J)=DR3*(1.0-DI)                                               *175  01
      DR4(J)=DR3*(1.0+DI)                                               *176  01
      R3(J)=PR*DR3/(J-1.C)/2.C                                          *177  01
      DR5(J)=CT*RALPHA*DR1(J)                                           *178  01
      DR6(J)=CT*RALPHA*DR2(J)                                           *179  01
      DR7(J) = DR3(J) * PR * RNEW                                       *180  01
CD    DR8(J) = DR4(J) * PR * RNEW                                       *181  01
      THROUGH BBB,FOR I=2,1,I.G.M                                       *182
      THROUGH BBB ,FOR J=1,1,J.G. N                                     *183  01
      C1(I,J)=U(I,J)*DX1                                                *184  02
BBB   C3(I,J)=V(I,J)*DR4                                                *185  02
      T(1,N+1)=TO(1,N+1)+DT*M1-DT*M2*(11*T(1,N+1)-18*T(1,N)+9*          *186
     1 T(1,N-1)-2*T(1,N-2))/6/DR+2*DX8*(TO(2,N+1)-TO(1,N+1))            *186
      THROUGH CBA,FOR I=2,1,I.G.M+1                                     *187
CEA   T(I,N+1)=T(I,N+1)+DT*M1-DT*M2*(11*T(I,N+1)-18*T(I,N)+9*T(I,N-     *188  01
     1 1)-2*T(I,N-2))/6/DR+DX8*(TO(I+1,N+1)-2*TO(I,N+1)+TO(I-1,N+1))    *188
      THROUGH BAAA,FOR I=M+2,1,I.G.P                                    *189
BAAA  T(I,N+1)=TO(I,N+1)+DT*M7-DT*M3*(11*T(I,N+1)-18*T(I,N)+9*T(I,      *190  01
     1 N-1)-2.*T(I,N-2))/6/DR+DZ8*(TO(I+1,N+1)-2.*TO(I,N+1)+TO(I-1,N    *190
     2 +1))                                                             *190
      T(P+1,N+1)=TO(P+1,N+1)+DT*M7-DT*M3*(11*T(P+1,N+1)-18*T(P+1,N)     *191
     1 +9*T(P+1,N-1)-2*T(P+1,N-2))/6/DR+2*DZ8*(TO(P,N+1)-TO(P+1,N+1)    *191
     2 )                                                                *191
      THROUGH BA,FOR I=1,1,I.G.P+1                                      *192
      THROUGH BA ,FOR J=1,1,J.G. N+1                                    *193  01
      RO(I,J)=CONST1*PRESS*144.0/RGAS  /(T(I,J)+M5)                     *194  02
      WO(I,J)=W(I,J)                                                    *195  02
EA    TO(I,J)=T(I,J)                                                    *196  02
      THROUGH GRAD, FOR J= 1,1, J .G. N                                 *197
      DTDXL(J)=(11*T(M+1,J)-18*T(M,J)+9*T(M-1,J)-2*T(M-2,J))/6/DX       *198  01
```

```
          DTDXG(J)=(-11*T(M+1,J)+18*T(M+2,J)-9*T(M+3,J)+2*T(M+4,J))/DZ/          *199                 01
         1 6                                                                     *199
CRAD      UL(J)=(DTDXL(J)-DTDXG(J)*KG/KL)*CP*NEW*NEW*AOVERB/PR/                  *200                 01
         1 BEITA/G/HFG/(A.P.3)                                                   *200
          U=0                                                                    *201
          THROUGH AVER,FOR J = 1,1, J .G. N                                      *202
AVER      U=U+(UL(J+1)+UL(J))*(2.C*J-1.C)*DR2/2.C                                *203                 01
          THROUGH ACAA,FOR    I=1,1,I.G.P+1                                      *204
ACAA      DTDRW(I)=(11*T(I,N+1)-18*  T(I,N)+9*T(I,N-1)-2*T(I,N-2))/6/DR          *205                 01
          QW=0                                                                   *206
          THROUGH CAAA, FOR I=M+1,1,I.G.P                                        *207
CAAA      QW=QW+(DTDRW(I)+DTDRW(I+1))*DZ/2.                                      *208                 01
          QI=0                                                                   *209
          DQIL = 0                                                               *210
          THROUGH DAAA, FOR J=1,1,J.G.N                                          *211
          DQIL = DQIL+(DTDXL(J)+DTDXL(J+1))*(2.0*J-1.0)*DR2/2.0                  *212                 01
DAAA      QI=QI+(DTDXG(J)+DTDXG(J+1))*(2.C*J-1.C)*DR2/2.C                        *213                 01
          DQWDT=QW*KG*6.28*NEW*ALPHA/BEITA/G/A/A/AOVERB/AOVERB                   *214
          RQIN=DQWDT*AOVERB/6.28/A/A/CSURFG/(1-X)                                *215
          DQILDT = 3.14*DQIL*KL*NEW*ALPHA/BEITA/G/A/A                            *216
          DQIDT=3.14*QI*KG*NEW*ALPHA/BEITA/G/A/A                                 *217
          DME = DT*U*ROL*(A.P.3) * 3.14/AOVERB                                   *218
          DMT=DMT+DME                                                            *219
          DM=-DMB+DME                                                            *220
          PRESS=PRESS+RGAS*AOVERB/(3.14*144.*A.P.3.*  (1.-X))*((DQWDT-           *221
         1 DQIDT)*DT*A.P.2./(CV*ALPHA)-DM*GAM*TSAT)                              *221
          A6=(PRESS-P1)/PRESS                                                    *222
          M6=(PRESS-P1)/P1                                                       *223
          P1=PRESS                                                               *224
          TSAT=C5*(PRESS.P.C6)                                                   *225
          A5=2.*DT*AOVERB*G*(A.P.3.)/ALPHA/ALPHA                                 *226
          THROUGH INTERF,FOR J=1,1,J.G.N                                         *227
INTERF    T(M+1,J)=(TSAT-TINIT)*CONST1                                           *228                 01
          T(1,1)=TO(1,1)*C1+TO(1,2)*DX5+TO(2,1)*DR5                              *229
          THROUGH DD,FOR I=2,1,I.G.M                                             *230
          WHENEVER U(I,1) .G. 0                                                  *231                 01
          C=(TO(I-1,1)-TO(I,1))*C1(I,1)                                          *232            C1   01
          OTHERWISE                                                              *233            01   01
          C=(TO(I,1)-TO(I+1,1))*C1(I,1)                                          *234            01   01
          END OF CONDITIONAL                                                     *235            01   01
DD        T(I,1)=TO(I,1)*C1+TO(I,2)*DR5+(TO(I+1,1)+TO(I-1,1))*DX3 +C             *236                 01
          THROUGH EE,FOR J=2,1,J.G.N                                             *237
EE        T(1,J)=TO(1,J)*C2+TO(2,J)*DX5+TO(1,J-1)*CR1(J)                         *238                 01
         1 +TO(1,J+1)*DR2(J)                                                     *238
          THROUGH FF,FOR J=2,1,J.G.N                                             *239
          THROUGH FF,FOR I=2,1,I.G.M                                             *240                 01
          WHENEVER U(I,J) .G. C                                                  *241                 02
          C=(TO(I-1,J)-TO(I,J))*C1(I,J)                                          *242            01   02
          OTHERWISE                                                              *243            01   02
          C=(TO(I,J)-TO(I+1,J))*C1(I,J)                                          *244            01   02
          END OF CONDITIONAL                                                     *245            01   02
          WHENEVER V(I,J) .G. 0                                                  *246                 02
          D=(TO(I,J-1)-TO(I,J))*C3(I,J)                                          *247            01   02
          OTHERWISE                                                              *248            01   02
          D=(TO(I,J)-TO(I,J+1))*C3(I,J)                                          *249            01   02
          END OF CONDITIONAL                                                     *250            01   02
FF        T(I,J)= TO(I,J)*C2+(TO(I+1,J)+TO(I-1,J))*DX3+                          *251                 02
         1 TO(I,J-1)*DR1(J)+TO(I,J+1)*DR2(J)+C+D                                 *251
          THROUGH GGG,FOR I=2,1,I.G.M                                            *252
          THROUGH GGG,FOR J=2,1,J.G.N                                            *253                 01
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      FI=R3(J)*(T(I,J+1)-T(I,J-1))                                          *254          02
      WHENEVER U(I,J) .G. 0                                                 *255          02
      C=(WO(I-1,J)-WO(I,J))*C1(I,J)                                         *256    01    02
      OTHERWISE                                                             *257    01    02
      C=(WO(I,J)-WO(I+1,J))*C1(I,J)                                         *258    01    02
      END OF CONDITIONAL                                                    *259    01    02
      WHENEVER V(I,J) .G. 0                                                 *260          02
      D=(WO(I,J-1)-WO(I,J))*C3(I,J)                                         *261    01    02
      OTHERWISE                                                             *262    01    02
      D=(WO(I,J)-WO(I,J+1))*C3(I,J)                                         *263    01    02
      END OF CONDITIONAL                                                    *264    01    02
GGG   W(I,J)=WO(I,J)*C3+(WO(I+1,J)+WO(I-1,J))*DX7+C+D+                      *265          02
     1 WO(I,J-1)*DR3(J)+WO(I,J+1)*DR4(J)+FI                                 *265
      THROUGH MMMM, FOR J=2,1,J.G.N+1                                       *266
MMMM  SF(M+1,J)=0                                                           *267          01
      NE1=0                                                                 *268
JJ    NE1=NE1+1                                                             *269
      WHENEVER NE1 .G. NE , TRANSFER TO PRINT                               *270
      THROUGH LL,FOR I=1,1,I.G.M+1                                          *271
      THROUGH LL,FOR J=1,1,J.G.N+1                                          *272          01
LL.   C3 (I,J)=SF(I,J)                                                      *273          02
      I=1                                                                   *274
II     I=I+1                                                                *275
      F(1)=0                                                                *276
      THROUGH LM,FOR J=2,1,J.G.N                                            *277
      DI(J)=(C3(I+1,J)+C3(I-1,J))*A1-W(I,J)*A1(J)                           *278          01
LM     F(J)=(DI(J)+F(J-1)*C2(J))/D1(J)                                      *279          01
      THROUGH LN,FOR J=1,1,J.G.N-1                                          *280
      H=N+1-J                                                               *281          01
LN    SF(I,H)=EJ(H)*SF(I,H+1)+F(H)                                          *282          01
      WHENEVER I.L.M,TRANSFER TO II                                         *283
      RMAX=0                                                                *284
      THROUGH AAA,FOR I=2,NR,I.G.M                                          *285
      THROUGH AAA,FOR J=2,NR,J.G.N                                          *286          01
      R3=.ABS.((SF(I,J)-C3(I,J))/(C3(I,J)+1.0E-20))                         *287          02
AAA   WHENEVER RMAX .L. R3 ,RMAX=R3                                         *288          02
      WHENEVER RMAX .G. EPSLON , TRANSFER TO JJ                             *289
      THROUGH GFH , FOR I=1,1,I.G.M                                         *290
      WHENEVER  T(I,N+1) .G. (T(M+1,N)+DTW*CONST1),T(I,N+1)=T(M+1,N)+DTW    *291          01
     1 *CONST1                                                              *291
GFH   WHENEVER T(I,N).G.(T(M+1,N)+DTS*CONST1),T(I,N)=T(M+1,N)+DTS*CONST1    *292          01
      DQS = 0                                                               *293
      DQL = 0                                                               *294
      DMB=0                                                                 *295
      CT2=6.28*A*A*DX*ROW*CPW*DELTA/AOVERB/2.0/CONST1                       *296
      THROUGH SQD, FOR I=1,1,I.G.M                                          *297
      C=T(I,N+1)+T(I+1,N+1)                                                 *298          01
      DQS=DQS+(C-C4(I,N+1))*CT2                                             *299          01
SQD   C4(I,N+1)=C                                                           *300          01
      THROUGH GHF,FOR J=1,1,J.G.N+1                                         *301
GHF   DR1(J)=3.14*ROL*CP*(A.P.3)*DX*DR2*(2.0*J-1.0)/AOVERB                  *302          01
      THROUGH FHG, FOR I=1,1,I.G.M                                          *303
      THROUGH FHG, FOR J=1,1,J.G.N                                          *304          01
      C = (T(I,J)+T(I,J+1)+T(I+1,J)+T(I+1,J+1))/4.C/CONST1                  *305          02
      DQL = DQL +(C-C4(I,J))*DR1(J)                                         *306          02
FHG   C4(I,J) = C                                                           *307          02
      DQI=(6.28*A*A*X*QSURFL/AOVERB+CQILOT)*DT*CONST                        *308
      DMB = (DQI-DQL-DQS)/HFG                                               *309
      WHENEVER DMB .L. 0.0 , DMB=C.0                                        *310
      DMBT=DMBT+DMB                                                         *311
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                DMTT=DMT-DMBT                                                    *312
                U=U-AOVERB*DMB/3.14/(A.P.3)/ROL/DT                               *313
                DWDT=-.581*PRESS*ALPHA*A*U/ACVERB                                *314
                UG=-U*(ROL/RO(M+1,1)-1.0)                                        *315
                C1=1-DZ5-DY5                                                     *316
                C2=1-DZ5-DY1                                                     *317
                C3=1-2*DZ7-2*DY7                                                 *318
                THROUGH BBBB,FOR I=M+2,1,I.G.P                                   *319
                THROUGH BBBB,FOR J=1,1,J.G.N                                     *320      01
                C1(I,J)=DZ1*U(I,J)                                               *321      02
BBBB            C3(I,J)=V(I,J)*DR4                                               *322      02
                T(P+1,1)=TO(P+1,1)*C1+TO(P,1)*DZ5+TO(P+1,2)*DY5                  *323
               1 +M4*M6*(M5+TO(P+1,1))                                           *323
                THROUGH BIB, FOR J=2,1,J.G.N                                     *324
BIB             T(P+1,J)=TO(P+1,J)*C2+TO(P,J)*DZ5+TO(P+1,J-1)*DR5(J)+TO(P+1,J    *325      01
               1 +1)*DR6(J)+M4*M6*(M5+TO(P+1,J))                                 *325
                THROUGH TWO, FOR I = M+2,1,I.G.P                                 *326
                WHENEVER U(I,1) .G. 0                                            *327      01
                C=(TO(I-1,1)-TO(I,1))*C1(I,1)                                    *328      01 01
                OTHERWISE                                                        *329      01 01
                C=(TO(I,1)-TO(I+1,1))*C1(I,1)                                    *330      01 01
                END OF CONDITIONAL                                               *331      01 01
TWO             T(I,1)=TO(I,1)*C1+TO(I,2)*DY5+(TO(I+1,1)+TO(I-1,1))*DZ3 +C       *332      01
               1  +M4*M6*(M5+TO(I,1))                                            *332
                THROUGH ONE ,FOR I=M+2,1,I.G.P                                   *333
                THROUGH ONE,FOR J=2,1,J.G.N                                      *334      01
                WHENEVER U(I,J) .G. C                                            *335      02
                C=(TO(I-1,J)-TO(I,J))*C1(I,J)                                    *336      01 02
                OTHERWISE                                                        *337      01 02
                C=(TO(I,J)-TO(I+1,J))*C1(I,J)                                    *338      01 02
                END OF CONDITIONAL                                               *339      01 02
                WHENEVER V(I,J) .G. C                                            *340      02
                D=(TO(I,J-1)-TO(I,J))*C3(I,J)                                    *341      01 02
                OTHERWISE                                                        *342      01 02
                D=(TO(I,J)-TO(I,J+1))*C3(I,J)                                    *343      01 02
                END OF CONDITIONAL                                               *344      01 02
CNE             T(I,J)= TO(I,J)*C2+(TO(I+1,J)+TO(I-1,J))*DZ3+                    *345      02
               1 TO(I,J-1)*DR5(J)+TO(I,J+1)*DR6(J)+C+D +M4*M6*(M5+TO(I,J))       *345
                THROUGH THREE,FOR I=M+2,1,I.G.P                                  *346
                THROUGH THREE,FOR J=2,1,J.G.N                                    *347      01
                FI=A4(J)*(-(2*(U(I,J)-UO(I,J))+C1(I,J)*(U(I+1,J)-U(I-1,J))+      *348      02
               1 C3(I,J)*(U(I,J+1)-U(I,J-1)))*(RO(I,J+1)-RO(I,J-1))+ACVRB2*(2*   *348
               2 (V(I,J)-VO(I,J))+C1(I,J)*(V(I+1,J)-V(I-1,J))+C3(I,J)*(V(I,J+1   *348
               3 )-V(I,J-1)))*(RO(I+1,J)-RO(I-1,J)))/RO(I,J)-W(I,J)*(2*(T(I,J)   *348
               4 -TO(I,J))+C1(I,J)*(T(I+1,J)-T(I-1,J))+C3(I,J)*(T(I,J+1)-T(I,J   *348
               5 -1)))/ZOVR2/(T(I,J)+M5)+A6*W(I,J)                               *348
                WHENEVER U(I,J) .G. 0                                            *349      02
                C=(WO(I-1,J)-WO(I,J))*C1(I,J)                                    *350      01 02
                OTHERWISE                                                        *351      01 02
                C=(WO(I,J)-WO(I+1,J))*C1(I,J)                                    *352      01 02
                END OF CONDITIONAL                                               *353      01 02
                WHENEVER V(I,J) .G. C                                            *354      02
                D=(WO(I,J-1)-WO(I,J))*C3(I,J)                                    *355      01 02
                OTHERWISE                                                        *356      01 02
                D=(WO(I,J)-WO(I,J+1))*C3(I,J)                                    *357      01 02
                END OF CONDITIONAL                                               *358      01 02
THREE           W(I,J)=WO(I,J)*C3+(WO(I+1,J)+WO(I-1,J))*DZ7 +C+D +               *359      02
               1 WO(I,J-1)*DR7(J)+WO(I,J+1)*DR8(J)+FI                            *359
               2 +A5*A4(J)*(T(I,J+1)-T(I,J-1))/(T(I,J)+M5)                       *359
                THROUGH NNN, FOR J=2,1,J.G.N                                     *360
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                SF(M+1,J)=RO(M+1,J)*UG*DR2*(J-1.0)*(J-1.C)/2.C                    *361   01
NNN             SFI(J)=SF(M+1,J)                                                  *362   01
                THROUGH DDAA, FOR I=M+2,1,I.G.P                                   *363
DDAA            SF(I,N+1)=SF(M+1,N+1)                                             *364   01
                NE2=0                                                             *365
JJJ             NE2=NE2+1                                                         *366
                WHENEVER NE2 .G.NE,TRANSFER TO PRINT                              *367
                THROUGH LNL, FOR I = M+1,1, I .G. P +1                            *368
                THROUGH LNL, FOR J=1,1,J .G. N+1                                  *369   01
LNL             C3(I,J)=SF(I,J)                                                   *370   02
                I=M+1                                                             *371
III             I=I+1                                                             *372
                THROUGH LLM, FOR J = 2,1, J .G. N                                 *373
                DI(J)=(C3(I+1,J)+C3(I-1,J))*A3-(W(I,J)*RC(I,J)+A4(J)*(U(I,J)*     *374   01
              1 (RO(I,J+1)-RO(I,J-1))/D2R-V(I,J)*(RO(I+1,J)-RO(I-1,J))/D2Z))*     *374
              2 A1(J)                                                             *374
LLM             F(J)=(DI(J)+F(J-1)*C2(J))/D2(J)                                   *375   01
                THROUGH LLN, FOR J = N,-1, J .L. 2                                *376
LLN             SF(I,J)=EJ1(J)*SF(I,J+1)+F(J)                                     *377   01
                WHENEVER I .L. P, TRANSFER TO III                                 *378
                WHENEVER NT .E. 1 .AND. NE2 .E. 1, TRANSFER TO JJJ                *379
                RMAXX=0                                                           *380
                THROUGH AAAA, FOR I=M+2,NR,I .G. P                                *381
                THROUGH AAAA, FOR J = 2,NR, J .G. N                               *382   01
                R3=.ABS.((SF(I,J)-C3(I,J))/(C3(I,J)+1.CE-2C))                     *383   02
AAAA            WHENEVER RMAXX .L. R3, RMAXX= R3                                  *384   02
                WHENEVER RMAXX .G. EPSLON, TRANSFER TO JJJ                        *385
                THROUGH BEE,FOR I=2,1,I.G.M                                       *386
BEE             W(I,N+1)=(8.*SF(I,N)-SF(I,N-1)-7.*SF(I,N+1))/DYY                  *387   01
                THROUGH UUUU, FOR I= M+2,1,I.G.P                                  *388
UUUU            W(I,N+1)=(8*SF(I,N)-SF(I,N-1)-7*SF(I,N+1))/DYY/RC(I,N+1)          *389   01
                THROUGH CEE , FOR J=2,1,J .G. N                                   *390
                W(P+1,J)=(-SF(P-1,J)+8.*SF(P,J)-7.*SF(P+1,J))*R5(J)/RO(P+1,J)     *391   01
CEE             W(1,J)=(-SF(3,J)+8.0*SF(2,J))*R4(J)                               *392   01
ML              THROUGH UUU, FOR I=M+1, 1, I.G.P+1                                *393
                THROUGH UUU, FOR J=1,1,J.G.N+1                                    *394   01
                VO(I,J)=V(I,J)                                                    *395   02
UUU             UO(I,J)=U(I,J)                                                    *396   02
                THROUGH BE, FOR I=2,1,I.G.M                                       *397
                U(I,N)=2.0*(3.*SF(I,N)-6.0*SF(I,N-1)+SF(I,N-2))/R1(N)             *398   01
                U(I,1)=SF(I,2)/DR6                                                *399   01
BE              U(I,2)=2.0*(6.C*SF(I,3)-3.C*SF(I,2)-SF(I,4))/R1(2)                *400   01
                THROUGH VVV, FOR I=M+1,1,I.G.P                                    *401
                U(I,N)=2.*(3*SF(I,N)-6*SF(I,N-1)+SF(I,N-2))/R1(N)/RO(I,N)         *402   01
                U(I,1)=SF(I,2)/DR6/RO(I,1)                                        *403   01
VVV             U(I,2)=2.*(6.*SF(I,3)-SF(I,4)-3.*SF(I,2))/R1(2)/RO(I,2)           *404   01
                THROUGH BBETA, FOR I=2,1,I.G.M                                    *405
                THROUGH BBETA,FOR J=3,1,J.G.N-1                                   *406   01
BBETA           U(I,J)=(SF(I,J-2)-8*SF(I,J-1)+8*SF(I,J+1)-SF(I,J+2))/R1(J)        *407   02
                U(M+1,N)=U(M+1,N-1)                                               *408
                THROUGH VVVV, FOR I=M+1,1,I.G.P                                   *409
                THROUGH VVVV, FOR J=3,1,J.G.N-1                                   *410   01
VVVV            U(I,J)=(SF(I,J-2)-8*SF(I,J-1)+8*SF(I,J+1)-SF(I,J-2))/R1(J)        *411   02
              1 /RO(I,J)                                                          *411
                THROUGH BFF, FOR J=2,1,J.G.N                                      *412
                V(M+2,J)=2.*(3.*SF(M+2,J)+SF(M+4,J)-6.*SF(M+3,J)+2*SF(M+1,J))     *413   01
              1 /R6(J)/RO(M+2,J)                                                  *413
                V(P,J)=2.*(6.*SF(P-1,J)-3.*SF(P,J)-SF(P-2,J))/R6(J)/RO(P,J)       *414   01
BFF             VG(J)=2.*(SF(M+3,J)-8.*SF(M+2,J)+7.*SF(M+1,J))/R6(J)/RO(M+1,J     *415   01
              1 )                                                                 *415
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            THROUGH BFFF, FOR I=M+3,1,I.G.P-1                                    *416
            THROUGH BFFF, FOR J=2,1,J.G.N                                        *417                01
BFFF        V(I,J)=(SF(I+2,J)-8*SF(I+1,J)+8*SF(I-1,J)-SF(I-2,J))/R6(J)/RC        *418                02
          1 (I,J)                                                                *418
            THROUGH FBF, FOR J=2,1,J.G.N+1                                       *419
FBF         SF(M+1,J)=0                                                          *420                01
            THROUGH BF,FOR J=2,1,J.G.N                                           *421
            V(2,J)=2.0*(SF(4,J)-6.0*SF(3,J)+3.0*SF(2,J))/R2(J)                   *422                01
            V(M,J)=2.0*(6.0*SF(M-1,J)-3.0*SF(M,J)-SF(M-2,J))/R2(J)               *423                01
BF          V(M+1,J)=2.0*(8*SF(M,J)-SF(M-1,J))/R2(J)                             *424                01
            THROUGH BETA,FOR I=3,1,I.G.N-1                                       *425
            THROUGH BETA ,FOR J=2,1,J.G.N                                        *426                01
BETA        V(I,J)=(SF(I+2,J)-8*SF(I+1,J)+8*SF(I-1,J)-SF(I-2,J))/R2(J)           *427                02
            WHENEVER NT.L.N1,TRANSFER TO PRINT                                   *428
            WHENEVER NT .E.NMAX                                                  *429
            PUNCH FORMAT DDATA,A,PRESS, PO, X, NT, TIME, U,                      *430          01
          1 T(1,1)...T(P+1,N+1),SF(1,1)...SF(P+1,N+1)                            *430
            TRANSFER TO PRINT                                                    *431          01
            END OF CONDITIONAL                                                   *432          01
            NP=NP1                                                               *433
            NP1=TAU/(TAU1*N3)                                                    *434
            WHENEVER NP1-NP-1.G.0, TRANSFER TO PRINT                             *435
            TRANSFER TO BACK                                                     *436
PRINT       WHENEVER NT .L. NC,TRANSFER TO NEXT                                  *437
            R1=0                                                                 *438
            THROUGH YZ,FOR I=2,1,I.G.M                                           *439
            THROUGH YZ,FOR J=2,1,J.G.N+1                                         *440                01
            R2=.ABS.((T(I,J)-TO(I,J))/TO(I,J))                                   *441                02
YZ          WHENEVER R1 .L. R2 ,R1=R2                                            *442                02
NEXT        NUG=0                                                                *443
            NUL=0                                                                *444
            RAL = 0                                                              *445
            RAG = 0                                                              *446
            THROUGH CLH, FOR I=1,1,I.G.M+1                                       *447
            HL(I)=.ABS.(KL*DTDRW(I)/A/(T(I,N+1)-T(I,1)))                         *448                01
            RAL = RAL+DX*T(I,N+1)                                                *449                01
CLH         NUL = NUL+DX*HL(I)*A*X/AOVERB/KL                                     *450                01
            RAL = RAL*(X.P.3)/(AOVERB.P.4)                                       *451
            THROUGH GLH, FOR I=M+1,1,I.G.P+1                                     *452
            J=I-M                                                                *453                01
            HG(J)=.ABS.(KG*DTDRW(I)/A/(T(I,N+1)-T(I,1)))                         *454                01
            RAG = RAG+DZ*T(I,N+1)                                                *455                01
GLH         NUG = NUG+DZ*HG(J)*A*(1-X)/AOVERB/KG                                 *456                01
            RAG = RAG * ((1-X).P.3) * BEITAG/RALPHA/RNEW/BEITA/                  *457
          1 (AOVERB.P.4)                                                         *457
            CL = NUL/(RAL.P.0.25)                                                *458
            CG = NUG/(RAG.P.0.25)                                                *459
            UR=U(I1,1)*R1(1)/2.0                                                 *460
            THROUGH DAL , FOR J=2,1,J.G.N+1                                      *461
DAL         UR=(UR+U(I1,J)*R1(J))/12.0                                           *462                01
            PRINT RESULTS NT,TIME,TAU,DT,UR,U,PRESS,Z,X,DX,                      *463
          1 DME,DMT,DMB,DMBT,DM,DMTT,                                            *463
          2 RQIN,DQIDT,DWDT,DQWDT,HFG,   U(1,1)...U(P+1,N+1),V(1,1)...V(         *463
          3 P+1,N+1),T(1,1)...T(P+1,N+1),SF(1,1)...SF(P+1,N+1),VG(1)...          *463
          4 VG(N+1),UL(1)...UL(N+1),HL(1)...HL(M+1),HG(1)...HG(P+1-M),           *463
          5 NUL,NUG,RMAX,R1,NE1,NE2,RAL,RAG,CL,CG,RP,R1,                         *463
          6 SFI(1)...SFI(N+1),DQILDT                                             *463
            WHENEVER NE1 .G. NE , TRANSFER TO END                                *464
            WHENEVER NT .E. NMAX , TRANSFER TO END                               *465
            WHENEVER N2.E.1                                                      *466
```

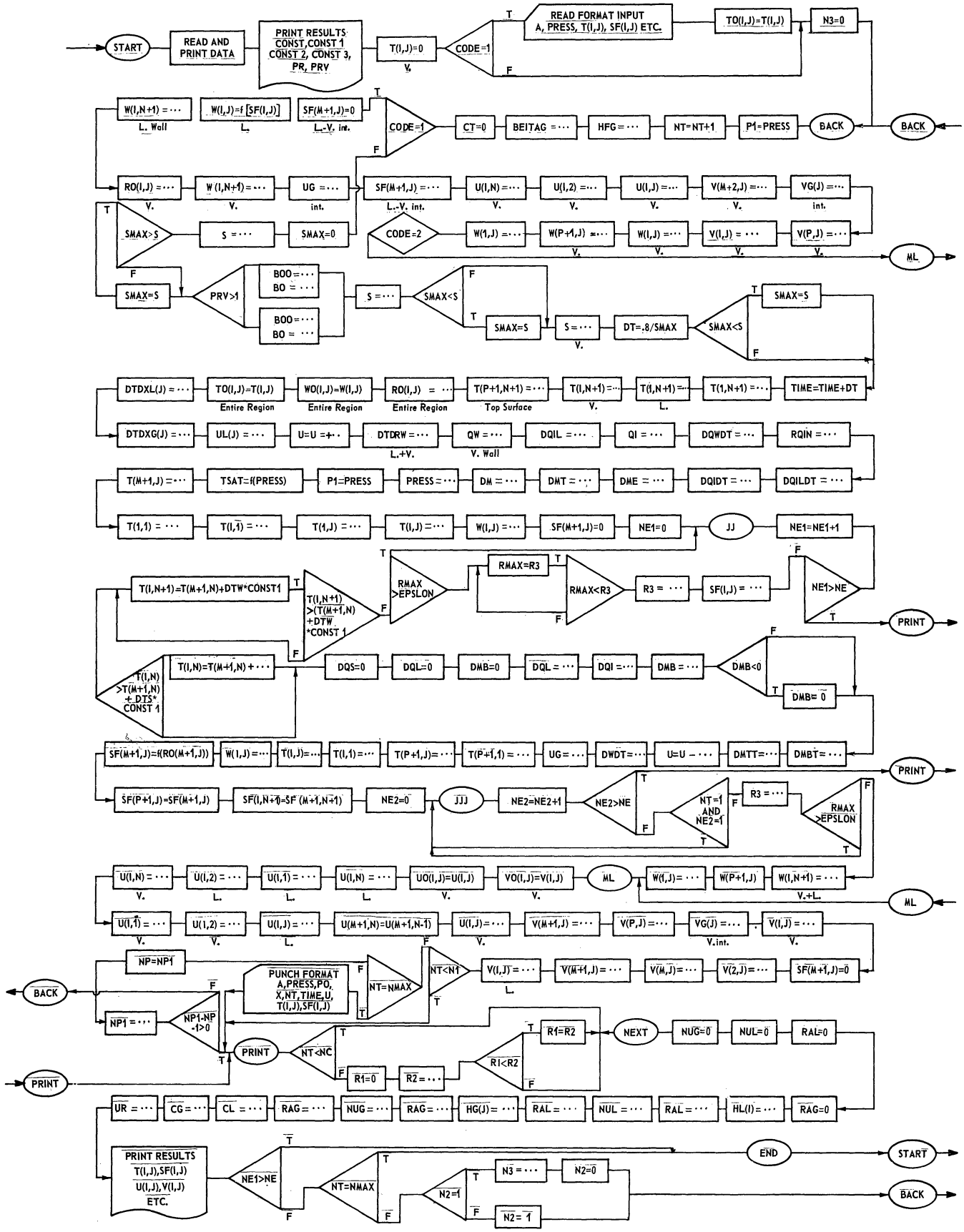
```
                N3=2.*N3                                                        *467        01
                N2=0                                                            *468        01
                OTHERWISE                                                       *469        01
                N2=1                                                            *470        01
                END OF CONDITIONAL                                              *471        01
                TRANSFER TO BACK                                                *472
      END       CONTINUE                                                        *473
                 TRANSFER TO START                                              *474
                VECTOR VALUES INPUT=$3F12.4,F12.8,I 4,F12.8,E16.8/(5E16.8)*$    *475
                VECTOR VALUES DDATA=$3F12.4,F12.8,I 4,F12.8,E16.8/(5E16.8)*$    *476
                END OF PROGRAM                                                  *477

THE FOLLOWING NAMES HAVE OCCURRED ONLY ONCE IN THIS PROGRAM.
COMPILATION WILL CONTINUE.

      CPV           *016
      N1            *428
      NC            *437
      TAU1          *434
```

APPENDIX B

FLOW CHART


START
READ AND PRINT DATA
PRINT RESULTS CONST,CONST 1 CONST 2, CONST 3, PR, PRV
T(I,J)=0
V.
CODE=1
READ FORMAT INPUT A, PRESS, T(I,J), SF(I,J) ETC.
TO(I,J)=T(I,J)
N3=0
W(I,N+1) = ···
L. Wall
W(I,J)=f [SF(I,J)]
L.
SF(M+1,J)=0
L.-V. int.
CODE=1
CT=0
BEITAG = ···
HFG = ···
NT=NT+1
P1=PRESS
BACK
BACK
RO(I,J) = ···
V.
W (I,N+1) = ···
V.
UG = ···
int.
SF(M+1,J) = ···
L.-V. int.
U(I,N) = ···
V.
U(I,2) = ···
V.
U(I,J) = ···
V.
V(M+2,J) = ···
V.
VG(J) = ···
int.
SMAX>S
S = ···
SMAX=0
CODE=2
W(1,J) = ···
W(P+1,J) = ···
V.
W(I,J) = ···
V.
V(I,J) = ···
V.
V(P,J) = ···
V.
ML
SMAX=S
PRV>1
BOO=···
BO = ···
BOO=···
BO = ···
S = ···
SMAX<S
SMAX=S
S = ···
V.
DT=.8/SMAX
SMAX<S
SMAX=S
DTDXL(J) = ···
TO(I,J)=T(I,J)
Entire Region
WO(I,J)=W(I,J)
Entire Region
RO(I,J) = ···
Entire Region
T(P+1,N+1) = ···
Top Surface
T(I,N+1) = ···
V.
T(I,N+1) = ···
L.
T(1,N+1) = ···
TIME=TIME+DT
DTDXG(J) = ···
UL(J) = ···
U=U +··
DTDRW = ···
L.+V.
QW = ···
V. Wall
DQIL = ···
QI = ···
DQWDT = ···
RQIN = ···
T(M+1,J) = ···
TSAT=f(PRESS)
P1=PRESS
PRESS = ···
DM = ···
DMT = ···
DME = ···
DQIDT = ···
DQILDT = ···
T(1,1) = ···
T(I,1) = ···
T(1,J) = ···
T(I,J) = ···
W(I,J) = ···
SF(M+1,J)=0
NE1=0
JJ
NE1=NE1+1
T(I,N+1)=T(M+1,N)+DTW*CONST1
T(I,N+1) >(T(M+1,N) +DTW *CONST 1
RMAX >EPSLON
RMAX=R3
RMAX<R3
R3 = ···
SF(I,J) = ···
NE1>NE
PRINT
T(I,N) >T(M+1,N) + DTS* CONST 1
T(I,N)=T(M+1,N) + ···
DQS=0
DQL=0
DMB=0
DQL = ···
DQI = ···
DMB = ···
DMB<0
DMB= 0
SF(M+1,J)=f(RO(M+1,J))
W(I,J) = ···
T(I,J)=···
T(1,1) = ···
T(P+1,J)= ···
T(P+1,1) = ···
UG = ···
DWDT = ···
U=U − ···
DMTT = ···
DMBT = ···
PRINT
SF(P+1,J)=SF(M+1,J)
SF(I,N+1)=SF (M+1,N+1)
NE2=0
JJJ
NE2=NE2+1
NE2>NE
NT=1 AND NE2=1
R3 = ···
RMAX >EPSLON
U(I,N) = ···
V.
U(I,2) = ···
L.
U(I,1) = ···
L.
U(I,N) = ···
L.
UO(I,J)=U(I,J)
V.
VO(I,J)=V(I,J)
V.
ML
W(I,J) = ···
W(P+1,J)
W(I,N+1) = ···
V.+L.
ML
U(I,1) = ···
V.
U(1,2) = ···
V.
U(I,J) = ···
L.
U(M+1,N)=U(M+1,N-1)
U(I,J) = ···
V.
V(M+1,J) = ···
V(P,J) = ···
VG(J) = ···
V.int.
V(I,J) = ···
V.
NP=NP1
BACK
PUNCH FORMAT A,PRESS,PO, X,NT,TIME,U, T(I,J),SF(I,J)
NT=NMAX
NT<N1
V(I,J) = ···
L.
V(M+1,J) = ···
V(M,J) = ···
V(2,J) = ···
SF(M+1,J)=0
NP1 = ···
NP1-NP -1>0
PRINT
NT<NC
R1=R2
R1=0
R2 = ···
R1<R2
NEXT
NUG=0
NUL=0
RAL=0
PRINT
UR = ···
CG = ···
CL = ···
RAG = ···
NUG = ···
RAG = ···
HG(J) = ···
RAL = ···
NUL = ···
RAL = ···
HL(I) = ···
RAG=0
PRINT RESULTS T(I,J),SF(I,J) U(I,J),V(I,J) ETC.
NE1>NE
NT=NMAX
N2=1
N3 = ···
N2=0
N2= 1
END
START
BACK

APPENDIX C

PROGRAM NOTATION AND NOMENCLATURE

Symbols preceeding the expression indicate the following:

* - Input data
** - Printout

Symbols following the expression indicate the following:

[0] - Integer
[1] - Dimensionless or units
(150) - e.g., - This expression appears in statement number 150 in the program of Appendix A.

A

*A - [ft] - Tank radius
*AOVERB - [1] - A/B
AOVERB2 - [1] - $(A/B)^2$
A1 - [1] - (67)
A2 - [1] - (68)
A3 - [1] - (63)
A5 - [1] - (226,359)
A6 - [1] - (222,348)
A1(J) - [1] - (35)
A2(J) - [1] - (76)
A3(J) - [1] - (77)
A4(J) - [1] - (37)
ALPHA - [ft^2/sec] - Thermal diffusivity of liquid
ALPJAW - [ft^2/sec] - Thermal diffusivity of wall
ALPHAV - [ft^2/sec] - Thermal diffusivity of vapor

B

B - [ft] - Total tank height
BJ - [1] - (71)
BJ1 - [1] - (64)
BO - [1] - (72,132,135)
BOO - [1] - (131,134)
BEITA - [$°F^{-1}$] - Volumetric coeff. of expansion-liquid
BEITAG - [$°F^{-1}$] - (55) - Volumetric coeff. of expansion-vapor

C

C - [1] - Temporary storage
C1 - [1] - (167,316)
C2 - [1] - (317)
C3 - [1] - (318)

C

*C5	- [°F]	- (20,225)	- Coefficient for computing TSAT as function of pressure
*C6	- [1]	- (20,225)	- Exponent for computing TSAT as function of pressure
C1(I,J)	- [1]	- (184,242,321)	- Temporary storage
C2(J)	- [1]	- (36)	
C3(I,J)	- [1]		- Temporary storage
C4(I,J)	- [1]		- Temporary storage
**CL	- [1]	- (458)	- Coefficient of Rayleigh number correlation-liquid
**CG	- [1]	- (459)	- Coefficient of Rayleigh number correlation-vapor
*CP	- [BTU/lbm-°F]		- specific heat-liquid
*CPV	- [BTU/lbm-°F]		- specific heat-vapor-constant pressure
*CV	- [BTU/lbm-°F]		- specific heat-vapor-constant volume
*CPW	- [BTU/lbm-°F]		- specific heat-wall
CT	- [1]	- (57)	- Property term for vapor (=1 for ideal gas)
CT2	- [BTU]	- (296)	- Property term for wall
*CODE	- [0]	- (86)	- = 2 for a new run = 1 for continuation of prior run vs READ FORMAT INPUT
**CONST	- [SECONDS]	- (22)	- conversion constant for dimension-less time
**CONST1	- [°F^{-1}]	- (23)	- conversion constant for dimension-less temperature
**CONST2	- [SECONDS^{-1}]		- = 1/CONST
**CONST3	- [°F]		- = 1/CONST1

D

D	- [1]		- Temporary storage
D1(J)	- [1]	- (78)	
D2(J)	- [1]	- (80)	
DI(J)	- [1]	- (278,374)	
DR1(J)	- [1]	- (173,302)	
DR2(J)	- [1]	- (174)	
DR3(J)	- [1]	- (175)	
DR4(J)	- [1]	- (176)	
DR5(J)	- [1]	- (178)	
DR6(J)	- [1]	- (179)	
DR7(J)	- [1]	- (180)	
DR8(J)	- [1]	- (181)	
DI	- [1]	- (172)	

D

DR - [1] - (14) - Radial grid space
D2R - [1] - (32) - DR/2
DR1 - [1] - (163) -
DR2 - [1] - (15) - DR^2
DR3 - [1] - (159)
DR4 - [1] - (160)
DR5 - [1] - (161)
DR6 - [1] - (18)
DR7 - [1] - (166)
DRZ - [1] - (62) - $(DR/DZ)^2$
**DX - [1] - (58) - Axial grid space-liquid
DX1 - [1] - (148) - DT/DX
DX2 - [1] - (59) - DX^2
DX3 - [1] - (158)
DX5 - [1] - (162)
DX7 - [1] - (164)
DX8 - [1] - (165)
DXR - [1] - (65) - $(DX/DR)^2$
DRX - [1] - (66) - 1/DXR
DYY - [1] - (17) - $2(DR)^2$
DY1 - [1] - (155)
DY3 - [1] - (154)
DY5 - [1] - (157)
DY7 - [1] - (156)
DZ - [1] - (60) - Axial grid space-vapor
DZ1 - [1] - (149) - DT/DZ
DZ2 - [1] - (61) - DZ^2
D2Z - [1] - (69) - DZ/2
DZ3 - [1] - (150)
DZ5 - [1] - (151)
DZ7 - [1] - (152)
DZ8 - [1] - (153)
**DM - [lbm] - (220) - Net phase change in time DT. (minus sign for evaporation)
**DMB - [lbm] - (309) - Phase change in time DT from energy balance on tank wall and liquid system (plus sign for evaporation)
**DME - [lbm] - (218) - Phase change at L-V interface in time DT (minus sign for evaporation)
**DMT - [lbm] - (219) - Cumulative total of DME (minus sign for evaporation)
**DMBT - [lbm] - (311) - Cumulative total of DMB
**DMTT - [lbm] - (312) - Cumulative total of DM (minus sign for evaporation)

D

***DT	- [1]		Time step interval
*DTS	- [°F]	- (292)	Max. permissible liquid superheat at node adjacent to wall
*DTW	- [°F]	- (291)	Max. permissible superheat of wall in contact with liquid
DTDRW(I)	- [1]	- (205,448,454)	Temperature gradient in fluid at wall
DTDXG(J)	- [1]	- (199)	Temperature gradient in vapor at Liquid-vapor interface
DTDXL(J)	- [1]	- (198)	Temperature gradient in liquid at liquid-vapor interface
DQIL	- [1]	- (212)	Mean temperature gradient in liquid at L-V interface
DQILDT	- [BTU/sec]	- (216)	Heat transfer rate in liquid at L-V interface (plus for heat transfer to liquid
**DQIDT	- [BTU/sec]	- (217)	Heat transfer rate in vapor at L-V interface (plus for heat transfer out of vapor)
**DQWDT	- [BTU/sec]	- (214)	Heat transfer rate to vapor from wall (plus for heat transfer to vapor)
**DWDT	- [BTU/sec]	- (314)	Work rate by vapor control volume
DQI	- [BTU]	- (308)	Sum of heat transfer to wall in contact with liquid and to liquid at L-V interface in DT (plus for heat transfer in)
DQL	- [BTU]	- (306)	Enthalpy rise of liquid in DT
DQS	- [BTU]	- (299)	Enthalpy rise of wall in contact with liquid in DT
*DELTA	- [feet]		Wall thickness

E

EJ(J)	- [1]	- (85,282)	
EJ1(J)	- [1]	- (81)	
*EPSLON	- [1]	- (289,385)	Maximum fractional change in stream function between iterations before iteration is terminated

F

F(J)	- [1]	- (279,375)	Temporary storage
FI	- [1]	- (171,254,348)	Temporary storage

G

*G	- [ft/sec^2]		- Acceleration corresponding to effective body force acting on container
GAM	- [1]		- Ratio of specific heats-vapor

H

H	- [0]	- (281)	- Index
**HFG	- [BTU/lbm]	- (51)	- Latent heat of vaporization corresponding to TSAT
**HG(I)	- [BTU/sec-ft^2-°F]	- (454)	- Local heat transfer coefficient of vapor based on ΔT between wall and centerline
**HL(I)	- [BTU/sec-ft^2-°F]	- (448)	- Local heat transfer coefficient of liquid based on ΔT between wall and centerline

I

I	- [0]	- Axial nodel index number
I1	- [0]	- Mid vertical height of liquid

J

J	- [0]	- Radial nodel index number

K

*KG	- [BTU/sec-ft-°F]	- Thermal conductivity-vapor
*KL	- [BTU/sec-ft-°F]	- Thermal conductivity-liquid

M

*M	- [0]		- Number of vertical divisions in liquid (M+1 = L-V interface)
M1	- [1]	- (27)	- Liquid-wall property
M2	- [1]	- (28)	- Liquid-wall property
M3	- [1]	- (29)	- Vapor-wall property
M4	- [1]	- (56)	- Vapor property
M5	- [1]	- (31)	- Initial temperature
M6	- [1]	- (223)	- Pressure rise ratio in DT
M7	- [1]	- (30)	- Vapor-wall property

N

*N	- [0]		- Number of radial divisions (N+1 = wall)
*N1	- [1]	- (428)	- Minimum number of time steps before punch is permitted

N

N2	- [0]	(466,468)	Control number for establishing time steps between printouts.
N3	- [0]	(48,467)	Control number for establishing time steps between printouts
*NC	- [0]	(437)	Minimum number of time steps NT before fractional temperature changes between time steps is computed
*NE	- [0]	(270,367)	Maximum number of iterations permitted in computations of stream function
**NE1	- [0]	(269,270)	counter on iterations on stream function-liquid
**NE2	- [0]	(366,367)	counter on iterations on stream function-vapor
NP	- [0]	(433,435)	control for printout instruction
NP1	- [1]	(433,434,435)	control for printout instruction
*NR	- [0]	(285,381)	Index steps of grid spaces for which fractional change in stream function with iterations is computed
**NT	- [0]	(50)	Time step number
*NMAX	-	(465)	Maximum number of time steps NT permitted before program is terminated
*NEW	- [ft^2/sec]		kinematic viscosity-liquid
*NEWV	- [ft^2/sec]		kinematic viscosity-vapor
**NUG	- [1]		Mean Nusselt number-vapor
**NUL	- [1]		Mean Nusselt number-liquid

P

*P	- [0]		Total number of vertical divisions
*P0	- [lb_f/in^2]		Initial system pressure
P1	- [lb_f/in^2]	(49,224)	Pressure previous time step
**PRESS	- [lb_f/in^2]	(221)	Current system pressure
**PR	- [1]	(10)	Prandtl number-liquid
**PRV	- [1]	(11)	Prandtl number-vapor

Q

QI	- [1]	(213)	Mean temperature gradient in vapor at L-V interface
QW	- [1]	(208)	Mean temperature gradient in vapor at wall
*QSURFG	- [BTU/sec-ft^2]		Imposed heat flux on container exterior on vapor portion
*QSURFL	- [BTU/sec-ft^2] -		Imposed heat flux on container exterior on liquid portion

R

**R1	- [1]	- (442)	- Maximum value of R2
R2	- [1]	- (441)	- Fractional change in temperature between time steps
R3	- [1]	- (287,383)	- Fractional change in stream function with iteration
R1(J)	- [1]	- (38)	
R2(J)	- [1]	- (79)	
R3(J)	- [1]	- (177)	
R4(J)	- [1]	- (82)	
R5(J)	- [1]	- (83)	
R6(J)	- [1]	- (84)	
RO(I,J)	- [lbm/ft^3]	- (96)	- Local vapor density
*ROL	- [lbm/ft^3]		- Liquid density
*ROW	- [lbm/ft^3]		- Wall density
**RP	- [1]	- (52)	- Ratio of current pressure to initial
**RMAX	- [1]	- (289)	- Maximum value of change in stream function with iteration-liquid
RMAXX	- [1]	- (385)	- Maximum value of change in stream function with iteration-vapor
**RAG	- [1]	- (455)	- Mean Rayleigh number-vapor
**RAL	- [1]	- (451)	- Mean Rayleigh number-liquid
*RGAS	- [ft-lb$_f$/lbm-°R]		- Gas constant-vapor
**RQIN	- [1]	- (215)	- Fraction of total heat transfer rate to vapor part of tank going into vapor
RALPHA	- [1]	- (13)	- Ratio-thermal diffusivity of vapor to liquid
RNEW	- [1]	- (12)	- Ratio-kinematic viscosity of vapor to liquid

S

S	- [1]	- (139,142)	- stability criteria
SMAX	- [1]	- (143,144)	- Maximum value of S
**SF(I,J)	- [1]		- stream function
**SFI(J)	- [1]		- vapor stream function at L-V interface

T

**T(I,J)	- [1]		- Current temperature
TO(I,J)	- [1]		- Temperature of prior time step
**TAU	- [seconds]	- (146)	- Current time
*TAU1	- [seconds]	- (434)	- Basic multiple of time for which printout occurs
**TIME	- [1]	- (145)	- Current time
TINIT	- [°F]	- (21)	- Initial saturation temperature corresponding to initial pressure

T

TSAT	- [°F]	-(20,225)	- Current saturation temperature corresponding to current pressure

U

**U(I,J)	- [1]		- Axial component of velocity
**U	- [1]	- (147,203,313)	- Mean velocity of liquid-vapor interface
UG	- [1]	- (99,315)	- Mean velocity of vapor at liquid-vapor interface
**UL(J)	- [1]	- (200)	- Local L-V interface velocity due to interface phase change
UO(I,J)	- [1]	- (396)	- Temporary storarge of U(I,J)
**UR	- [1]	- (460,462)	- Mean axial velocity at mid-point of liquid (serves as check on continuity)

V

**V(I,J)	- [1]		- Radial component of velocity
VO(I,J)	- [1]	- (395)	- Temporary storage of V(I,J)
**VG(J)	- [1]	- (111,415)	- Radial velocity of vapor at L-V at interface

W

W(I,J)	- [1]		- Vorticity
WO(I,J)	- [1]		- Vorticity of prior time step

X

***X	- [1]	- (147)	- Liquid fraction in container

Y

Y	- [1]	- (9)	- A/B

Z

Z	- [1]		- Compressibility factor
ZOVRZ	- [1]	- (70)	- = Z/2, = 0.5 for ideal gas

APPENDIX D

COMPUTER PROGRAM DATA INPUTS

P - Total number of vertical grid spaces
M - Number of vertical grid spaces in liquid
N - Number of radial grid spaces
CODE - = 2 for a new run
= 1 for continuation of prior run via READ FORMAT INPUT
N1 - Minimum number of time steps NT before PUNCH FORMAT DDATA is permitted
NC - Minimum number of time steps NT before fractional temperature changes between time steps is computed
NMAX - Maximum number of time steps NT permitted before program is terminated
NE - Maximum number of iterations permitted in computations of stream function
EPSLON - Maximum fractional change in stream function between iterations before iteration is terminated
NR - Index steps of grid spaces for which fractional change in stream function with iterations is computed
DT - Initial estimate of dimensionless time step
TAU1 - seconds - Basic multiple of real time for which printout occurs

A - feet - Tank radius
APVERB - Ratio of tank radius to height
X - Initial fraction of liquid in tank
QSURFG - BTU/sec-ft^2-Imposed heat flux on container exterior on vapor portion
QSURFL - BTU/sec-ft^2 - Imposed heat flux on container exterior on liquid portion
G - ft/sec^2 - Acceleration corresponding to effective body force acting on container
PO - lb_f/in^2 - Initial system pressure
DTW - °F - Imposed maximum permissible temperature difference between container wall and saturation
DTS - °F - Imposed maximum permissible temperature difference between liquid nodes adjacent to container wall and saturation
CPW - BTU/lbm-°F - specific heat of container wall
ROW - lbm/ft^3 - Density of container wall
ALPHAW - ft^2/sec - Thermal diffusivity of container wall
DELTA - ft - container wall thickness
ALPHA - ft^2/sec - Thermal diffusivity of liquid
ROL - lbm/ft^3 - Liquid density
CP - BTU/lbm - °F - Specific heat of liquid
KL - BTU/sec - ft - °F - Thermal conductivity of liquid
BEITA - $°F^{-1}$ - Volumetric coefficient of expansion of liquid
C5 - Coefficient for computing TSAT
C6 - Exponent for computing TSAT
NEW - ft^2/sec - kinematic viscosity of liquid
ALPHAV - ft^2/sec - Thermal diffusivity of vapor
CPV - BTU/lbm - °F - Specific heat of vapor, constant pressure
CV - BTU/lbm - °F - Specific heat of vapor, constant volume
KG - BTU/sec - ft - °F - Thermal conductivity of vapor
RGAS - $ft\text{-}lb_f/lbm$ - °R Gas constant of vapor
Z - Compressibility factor

APPENDIX E

TYPICAL OUTPUT

```
    $ DATA
MAP
     ACCEPT 00000*     DPUNCH 00000*     SPRINT 00000*     ERROR  00000*     SKIP6  00000*     SPEEK  00000*
     SCARDS 00000*     FTRAP  00000*     SYSTEM 00000*     (PGCM) 10000      (MAIN) 10000      .IR4   64412*
     .IOH   64412*     VER.87 64412*     .LINK  64412*     .IR1   64412*     .IR2   64412*     (SUBT) 67622
     .SCOM. 71135*     DFMIOH 71466*     DFDIOH 71466*     .ERR   71541*     .RTN   71541*     .ERPRT 71541*
     ATLOC  71666*     .03311 71714*     .ICSET 71733*     .IOFMT 71751*     .RST4  71751*     .PRINT 72014*
     .WR.   72025*     .READ  72041*     .RD.   72056*     .RDSW  72056*     GTCTLW 72140*     .GTCLW 72140*
     .EOF7  72153*     .PUNCH 72211*     .PC.   72222*     .LBUFF 72236*     .PRSLT 72367*     .RPDTA 73207*
     USQRT  74423*     SQRT   74473*     .EXIT  74515*     ELOG   74572*     .01300 74705*     .C1301 74773*
     EXP    75071*     ZERO   75204*     (PROG) 75240      (ERAS) 77741
EXECUTION BEGINS WITH ROUTINE      (MAIN)
02501 LOCS. CAN BE SAFELY USED IN EXPANDING PROG. (OCTAL)
FULL CORE LOADING PROCEDURE USED
LOADER OVERLAPPED BY    05416 LOCS. (OCTAL), FULL CORE LOADING PROCEDURE USED

          A=2.5,AOVERB=.25,P=30,M=20,N=20,X=.67,CODE=2,DT=10.E-6,

          NMAX=500,PO=15.,G=32.2E-5,QSURFG=2.78E-4,QSURFL=2.78E-4,

          DTW=1.,DTS=1.,C5=116.,C6=.123,ALPHA=8.35E-7,NEW=1.63E-6,

          KL=2.33E-5,BEITA=.49E-2,ROL=70.,CP=.4,ALPHAV=2.14E-5,

          NEWV=2.11E-5,KG=1.4E-6,CPV=.216,CV=.153,RGAS=48.3,

          ALPHAW=4.16E-5,CPW=.11,ROW=488.,DELTA=.C1,TAU1=30,N1=6,

          NE=50,EPSLON=.01,NC=5,NR=4,Z=1.0*
    CONST =  7.485030E+06,      CONST2 =  1.336000E-07,      CONST1 =  4.528328E+06,      CONST3 =  2.208320E-07
       PR =      1.952096,         PRV =       .985981
       NT =           154,        TIME =  1.539999E-03,         TAU =  1.152694E+04,          DT =  1.000000E-05
       UR =      2.329724,      U(1,0) = -2.198956E-03,       PRESS =     15.235698,           Z =      1.000000
        X =       .669998,          DX =       .033500,         DME = -3.020815E-04,         DMT =      -.024537
      DMB =       .000000,        DMBT =       .000000,          DM = -3.020815E-04,        DMTT =      -.024537
     RQIN =       .058085,        DQIDT =  6.577250E-05,        DWDT =  1.625330E-07,       DQWDT =  8.366078E-04
      HFG =    296.621334

          U(1,1)...U(31,21)

 .CCCCCCE+00    .CCCCCCE+00    .CCCCCCE+00    .CCCCCCE+00    .CCCCCCE+00    .CCCCCCE+00    .CCCCCCE+00    .CCCCCCE+00
 .CCCCCCE+00    .CCCCCCE+00    .CCCCCCE+00    .CCCCCCE+00    .CCCCCCE+00    .CCCCCCE+00    .CCCCCCE+00    .CCCCCCE+00
 .CCCCCCE+00    .CCCCCCE+00    .CCCCCCE+00    .CCCCCCE+00    .CCCCCCE+00
```

-2.211137E+01	-2.211628E+01	-2.225188E+01	-2.244571E+01	-2.272659E+01	-2.310834E+01	-2.361043E+01	-2.425956E+01
-2.509219E+01	-2.615838E+01	-2.752779E+01	-2.929880E+01	-3.161301E+01	-3.467842E+01	-3.880801E+01	-4.449614E+01
-5.269644E+01	-6.635945E+01	-5.981325E+01	1.179201E+02	.000000E+00			
-4.683610E+01	-4.685800E+01	-4.708505E+01	-4.740008E+01	-4.785116E+01	-4.845818E+01	-4.924635E+01	-5.024781E+01
-5.150318E+01	-5.306329E+01	-5.499125E+01	-5.736472E+01	-6.027796E+01	-6.384294E+01	-6.818893E+01	-7.346853E+01
-7.995209E+01	-8.827012E+01	-7.033465E+01	2.037808E+02	.000000E+00			
-7.013079E+01	-7.015323E+01	-7.037743E+01	-7.068790E+01	-7.113194E+01	-7.172829E+01	-7.249926E+01	-7.347136E+01
-7.467518E+01	-7.614510E+01	-7.791836E+01	-8.003371E+01	-8.252880E+01	-8.543651E+01	-8.878148E+01	-9.258206E+01
-9.682913E+01	-1.003642E+02	-6.765599E+01	2.721730E+02	.000000E+00			
-9.198101E+01	-9.199214E+01	-9.211884E+01	-9.229820E+01	-9.256296E+01	-9.293085E+01	-9.342255E+01	-9.406132E+01
-9.487201E+01	-9.587981E+01	-9.710866E+01	-9.857919E+01	-1.003064E+02	-1.022977E+02	-1.045527E+02	-1.070612E+02
-1.096186E+02	-1.092829E+02	-6.147666E+01	3.297831E+02	.000000E+00			
-1.132106E+02	-1.131973E+02	-1.131234E+02	-1.130357E+02	-1.129417E+02	-1.128634E+02	-1.128279E+02	-1.128651E+02
-1.130063E+02	-1.132824E+02	-1.137217E+02	-1.143478E+02	-1.151781E+02	-1.162229E+02	-1.174879E+02	-1.189573E+02
-1.202147E+02	-1.165049E+02	-5.393336E+01	3.804950E+02	.000000E+00			
-1.355505E+02	-1.354936E+02	-1.350659E+02	-1.345101E+02	-1.337946E+02	-1.329568E+02	-1.320439E+02	-1.311096E+02
-1.302104E+02	-1.294025E+02	-1.287380E+02	-1.282618E+02	-1.280097E+02	-1.280079E+02	-1.282771E+02	-1.287985E+02
-1.289337E+02	-1.222121E+02	-4.553632E+01	4.266123E+02	.000000E+00			
-1.619037E+02	-1.617711E+02	-1.607155E+02	-1.593285E+02	-1.575151E+02	-1.553476E+02	-1.529197E+02	-1.503402E+02
-1.477259E+02	-1.451929E+02	-1.428477E+02	-1.407794E+02	-1.390547E+02	-1.377175E+02	-1.367927E+02	-1.362332E+02
-1.351375E+02	-1.253352E+02	-3.555222E+01	4.690483E+02	.000000E+00			
-1.971351E+02	-1.968796E+02	-1.946884E+02	-1.917840E+02	-1.879560E+02	-1.833471E+02	-1.781493E+02	-1.725956E+02
-1.669441E+02	-1.614516E+02	-1.563457E+02	-1.518035E+02	-1.479428E+02	-1.448252E+02	-1.424738E+02	-1.408340E+02
-1.388662E+02	-1.271841E+02	-2.547719E+01	5.101930E+02	.000000E+00			
-2.483276E+02	-2.479085E+02	-2.437584E+02	-2.381751E+02	-2.307176E+02	-2.216422E+02	-2.113332E+02	-2.003016E+02
-1.891394E+02	-1.784355E+02	-1.686878E+02	-1.602406E+02	-1.532615E+02	-1.477534E+02	-1.435848E+02	-1.404458E+02
-1.367071E+02	-1.201905E+02	-8.311547E+00	5.455704E+02	.000000E+00			
-3.232453E+02	-3.226866E+02	-3.155704E+02	-3.057719E+02	-2.923645E+02	-2.757001E+02	-2.564844E+02	-2.358041E+02
-2.149971E+02	-1.954091E+02	-1.781428E+02	-1.638970E+02	-1.529398E+02	-1.451973E+02	-1.404146E+02	-1.382898E+02
-1.374097E+02	-1.230529E+02	-6.175006E-01	5.894478E+02	.000000E+00			
-4.241774E+02	-4.235865E+02	-4.128358E+02	-3.975921E+02	-3.759794E+02	-3.482250E+02	-3.154311E+02	-2.796627E+02
-2.436424E+02	-2.101698E+02	-1.815030E+02	-1.589480E+02	-1.427801E+02	-1.324584E+02	-1.269832E+02	-1.252167E+02
-1.248605E+02	-1.050002E+02	9.428528E+01	6.517009E+02	.000000E+00			
-5.405803E+02	-5.400068E+02	-5.257460E+02	-5.049056E+02	-4.741055E+02	-4.330652E+02	-3.832464E+02	-3.280022E+02
-2.720399E+02	-2.203770E+02	-1.771503E+02	-1.447378E+02	-1.235249E+02	-1.123189E+02	-1.091181E+02	-1.118555E+02
-1.175967E+02	-9.864233E+01	1.462095E+02	6.995412E+02	.000000E+00			
-6.497908E+02	-6.491102E+02	-6.320017E+02	-6.063197E+02	-5.668796E+02	-5.127497E+02	-4.457560E+02	-3.705795E+02
-2.940208E+02	-2.235700E+02	-1.656213E+02	-1.239646E+02	-9.921260E+01	-8.937681E+01	-9.114832E+01	-1.011170E+02
-1.149081E+02	-9.488580E+01	2.227802E+02	7.536559E+02	.000000E+00			
-7.280661E+02	-7.271375E+02	-7.079985E+02	-6.786406E+02	-6.323588E+02	-5.679241E+02	-4.876210E+02	-3.971589E+02
-3.048780E+02	-2.202081E+02	-1.515213E+02	-1.040198E+02	-7.861821E+01	-7.256481E+01	-8.132342E+01	-1.002901E+02
-1.232050E+02	-9.917574E+01	2.966782E+02	8.065073E+02	.000000E+00			
-7.595894E+02	-7.584945E+02	-7.385789E+02	-7.075731E+02	-6.682059E+02	-5.895087E+02	-5.041374E+02	-4.081042E+02
-3.100772E+02	-2.200262E+02	-1.471928E+02	-9.783018E+01	-7.337138E+01	-7.058148E+01	-8.382255E+01	-1.073184E+02
-1.316798E+02	-7.827946E+01	3.904505E+02	8.410642E+02	.000000E+00			

-7.365917E+02	-7.357071E+02	-7.171696E+02	-6.880736E+02	-6.418908E+02	-5.781897E+02	-4.996360E+02	-4.116660E+02
-3.218854E+02	-2.389192E+02	-1.708284E+02	-1.235208E+02	-9.871720E+01	-9.338325E+01	-1.014688E+02	-1.169637E+02
-1.201232E+02	1.846268E+01	5.125904E+02	8.235279E+02	.000000E+00			
-6.536520E+02	-6.531456E+02	-6.389111E+02	-6.163503E+02	-5.805661E+02	-5.314536E+02	-4.711419E+02	-4.037703E+02
-3.350413E+02	-2.712689E+02	-2.180214E+02	-1.787402E+02	-1.537808E+02	-1.401121E+02	-1.308327E+02	-1.261681E+02
-3.878747E+01	2.141488E+02	6.078245E+02	7.137864E+02	.000000E+00			
-5.025515E+02	-5.024320E+02	-4.944684E+02	-4.814649E+02	-4.604321E+02	-4.312205E+02	-3.949327E+02	-3.538452E+02
-3.111886E+02	-2.705911E+02	-2.351936E+02	-2.064881E+02	-1.827778E+02	-1.568788E+02	-1.131325E+02	-2.789796E+01
1.138102E+02	3.117109E+02	5.186524E+02	4.901771E+02	.000000E+00			
-2.788324E+02	-2.789231E+02	-2.764835E+02	-2.721145E+02	-2.645342E+02	-2.534676E+02	-2.390216E+02	-2.216443E+02
-2.020164E+02	-1.808089E+02	-1.583320E+02	-1.341289E+02	-1.065836E+02	-7.265148E+01	-2.794357E+01	3.272106E+01
1.126064E+02	2.041811E+02	2.735197E+02	2.315450E+02	.000000E+00			
5.473984E-01	5.473984E-01	7.298645E-01	7.298645E-01	7.298645E-01	7.298645E-01	7.298645E-01	7.298645E-01
7.298645E-01	7.298645E-01	7.298645E-01	7.298645E-01	7.298645E-01	7.298645E-01	7.298645E-01	7.298645E-01
7.298645E-01	7.298645E-01	7.298645E-01	7.305762E-01	.000000E+00			
-5.388525E+01	-5.387820E+01	-7.230521E+01	-7.278958E+01	-7.333054E+01	-7.373435E+01	-7.373837E+01	-7.300918E+01
-7.114455E+01	-6.768447E+01	-6.213350E+01	-5.399423E+01	-4.280847E+01	-2.819807E+01	-9.858295E+00	1.219124E+01
3.776487E+01	6.496241E+01	8.625951E+01	6.400463E+01	.000000E+00			
-1.032714E+02	-1.033163E+02	-1.388776E+02	-1.399729E+02	-1.411530E+02	-1.420267E+02	-1.420608E+02	-1.405781E+02
-1.367669E+02	-1.297086E+02	-1.184231E+02	-1.019304E+02	-7.932722E+01	-4.988296E+01	-1.318201E+01	3.056650E+01
7.976737E+01	1.289958E+02	1.619013E+02	1.132115E+02	.000000E+00			
-1.435387E+02	-1.436568E+02	-1.934623E+02	-1.953036E+02	-1.972830E+02	-1.987956E+02	-1.990207E+02	-1.969252E+02
-1.912928E+02	-1.807804E+02	-1.639973E+02	-1.395962E+02	-1.063793E+02	-6.345126E+01	-1.051333E+01	5.146846E+01
1.189829E+02	1.826529E+02	2.189738E+02	1.464104E+02	.000000E+00			
-1.706188E+02	-1.708263E+02	-2.306679E+02	-2.334531E+02	-2.365116E+02	-2.390278E+02	-2.398944E+02	-2.377286E+02
-2.309281E+02	-2.177695E+02	-1.965354E+02	-1.656456E+02	-1.237893E+02	-7.011673E+01	-4.667783E+00	7.060270E+01
1.501719E+02	2.212577E+02	2.557183E+02	1.650289E+02	.000000E+00			
-1.803302E+02	-1.806508E+02	-2.449155E+02	-2.488407E+02	-2.532972E+02	-2.573056E+02	-2.595247E+02	-2.582754E+02
-2.516187E+02	-2.374921E+02	-2.138921E+02	-1.790706E+02	-1.317248E+02	-7.122623E+01	1.876205E+00	8.452577E+01
1.693958E+02	2.413482E+02	2.703626E+02	1.692162E+02	.000000E+00			
-1.690696E+02	-1.695218E+02	-2.311775E+02	-2.362296E+02	-2.421800E+02	-2.480008E+02	-2.522555E+02	-2.531150E+02
-2.484290E+02	-2.358650E+02	-2.131226E+02	-1.782149E+02	-1.297996E+02	-6.757025E+01	7.124181E+00	9.002655E+01
1.725239E+02	2.386885E+02	2.600340E+02	1.584337E+02	.000000E+00			
-1.355081E+02	-1.360790E+02	-1.871253E+02	-1.927695E+02	-1.996694E+02	-2.069481E+02	-2.133281E+02	-2.171215E+02
-2.162533E+02	-2.083408E+02	-1.908680E+02	-1.615086E+02	-1.186343E+02	-6.200152E+01	6.426422E+00	8.152475E+01
1.543793E+02	2.102805E+02	2.250899E+02	1.348011E+02	.000000E+00			
-8.387775E+01	-8.452203E+01	-1.176191E+02	-1.225290E+02	-1.287539E+02	-1.357908E+02	-1.428391E+02	-1.487704E+02
-1.520871E+02	-1.508894E+02	-1.428992E+02	-1.256437E+02	-9.693591E+01	-5.575205E+01	-3.428022E+00	5.509378E+01
1.117539E+02	1.548514E+02	1.665842E+02	1.000982E+02	.000000E+00			
-2.873880E+01	-2.936403E+01	-4.163849E+01	-4.404742E+01	-4.720672E+01	-5.102794E+01	-5.530838E+01	-5.972610E+01
-6.379774E+01	-6.679627E+01	-6.763685E+01	-6.478414E+01	-5.630510E+01	-4.027421E+01	-1.578596E+01	1.528582E+01
4.768112E+01	7.417348E+01	8.506414E+01	5.398312E+01	.000000E+00			
.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00
.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00
.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00			

V(1,1)...V(31,21)

.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00
.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00
.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00			
.000000E+00	1.797913E+01	3.605490E+01	5.427706E+01	7.272249E+01	9.147821E+01	1.106439E+02	1.303354E+02
1.506878E+02	1.718600E+02	1.940366E+02	2.174298E+02	2.422754E+02	2.688196E+02	2.972836E+02	3.277821E+02
3.600955E+02	3.926962E+02	4.165739E+02	3.752857E+02	.000000E+00			
.000000E+00	1.817011E+01	3.638380E+01	5.465256E+01	7.300314E+01	9.146446E+01	1.100661E+02	1.288359E+02
1.477965E+02	1.669581E+02	1.863070E+02	2.057866E+02	2.252661E+02	2.444898E+02	2.629933E+02	2.799569E+02
2.938736E+02	3.013769E+02	2.922639E+02	2.389425E+02	.000000E+00			
.000000E+00	1.679483E+01	3.355145E+01	5.024855E+01	6.685710E+01	8.334622E+01	9.968163E+01	1.158229E+02
1.317190E+02	1.473027E+02	1.624817E+02	1.771288E+02	1.910681E+02	2.040611E+02	2.157917E+02	2.258431E+02
2.335592E+02	2.370666E+02	2.280949E+02	1.760413E+02	.000000E+00			
.000000E+00	1.591571E+01	3.171957E+01	4.735108E+01	6.273177E+01	7.778468E+01	9.243477E+01	1.066082E+02
1.202308E+02	1.332259E+02	1.455114E+02	1.569969E+02	1.675813E+02	1.771523E+02	1.855864E+02	1.927354E+02
1.982667E+02	2.006018E+02	1.918786E+02	1.435951E+02	.000000E+00			
.000000E+00	1.596915E+01	3.173353E+01	4.718525E+01	6.218840E+01	7.661750E+01	9.036083E+01	1.033226E+02
1.154239E+02	1.266028E+02	1.368135E+02	1.460248E+02	1.542193E+02	1.613920E+02	1.675497E+02	1.726900E+02
1.766260E+02	1.779213E+02	1.691638E+02	1.237354E+02	.000000E+00			
.000000E+00	1.768606E+01	3.501280E+01	5.179208E+01	6.779265E+01	8.281035E+01	9.667588E+01	1.092609E+02
1.204823E+02	1.303025E+02	1.387284E+02	1.458051E+02	1.516087E+02	1.562375E+02	1.598000E+02	1.623805E+02
1.638483E+02	1.628217E+02	1.523781E+02	1.088075E+02	.000000E+00			
.000000E+00	2.223636E+01	4.382790E+01	6.442394E+01	8.360399E+01	1.010071E+02	1.163475E+02	1.294315E+02
1.401694E+02	1.485796E+02	1.547798E+02	1.589686E+02	1.614004E+02	1.623598E+02	1.621408E+02	1.610212E+02
1.591273E+02	1.555695E+02	1.445100E+02	1.021438E+02	.000000E+00			
.000000E+00	3.132454E+01	6.151852E+01	8.989685E+01	1.156570E+02	1.381066E+02	1.567024E+02	1.711050E+02
1.812234E+02	1.872278E+02	1.895190E+02	1.886605E+02	1.852918E+02	1.800387E+02	1.734282E+02	1.657946E+02
1.570658E+02	1.458218E+02	1.259425E+02	8.336030E+01	.000000E+00			
.000000E+00	4.643099E+01	9.108867E+01	1.326891E+02	1.697371E+02	2.008586E+02	2.249270E+02	2.412485E+02
2.497205E+02	2.508809E+02	2.458202E+02	2.359900E+02	2.229793E+02	2.083248E+02	1.934004E+02	1.793943E+02
1.672834E+02	1.569816E+02	1.423681E+02	1.106457E+02	.000000E+00			
.000000E+00	6.612913E+01	1.299616E+02	1.894636E+02	2.421613E+02	2.854941E+02	3.171701E+02	3.356802E+02
3.407357E+02	3.334292E+02	3.160430E+02	2.915811E+02	2.632056E+02	2.337640E+02	2.055158E+02	1.800576E+02
1.582585E+02	1.386515E+02	1.045679E+02	-7.327448E+00	.000000E+00			
.000000E+00	8.316054E+01	1.639102E+02	2.398027E+02	3.074459E+02	3.628215E+02	4.020263E+02	4.222702E+02
4.227358E+02	4.049808E+02	3.727080E+02	3.309638E+02	2.850687E+02	2.396650E+02	1.981469E+02	1.624870E+02
1.331311E+02	1.069411E+02	6.023417E+01	-8.509604E+01	.000000E+00			
.000000E+00	8.707033E+01	1.719168E+02	2.524018E+02	3.246708E+02	3.837201E+02	4.245281E+02	4.433533E+02
4.388508E+02	4.127171E+02	3.695947E+02	3.161538E+02	2.596420E+02	2.064842E+02	1.614406E+02	1.274284E+02
1.055203E+02	9.265724E+01	6.290506E+01	-5.734726E+01	.000000E+00			
.000000E+00	7.241778E+01	1.428690E+02	2.099681E+02	2.699878E+02	3.181486E+02	3.498324E+02	3.616535E+02
3.523636E+02	3.235230E+02	2.796525E+02	2.275508E+02	1.748180E+02	1.282046E+02	9.261472E+01	7.090565E+01
6.360238E+01	6.503634E+01	4.023841E+01	-1.089186E+02	.000000E+00			
.000000E+00	4.243656E+01	8.348672E+01	1.225113E+02	1.567647E+02	1.832560E+02	1.994633E+02	2.036689E+02
1.952523E+02	1.750988E+02	1.459855E+02	1.125707E+02	8.059971E+01	5.529744E+01	4.020561E+01	3.692125E+01

4.447415E+01	5.365844E+01	1.589166E+01	-1.289977E+02	.000000E+00			
.000000E+00	4.001483E+00	8.087498E+00	1.229838E+01	1.619074E+01	1.994337E+01	2.419991E+01	2.958566E+01
3.629777E+01	4.393424E+01	5.168775E+01	5.900409E+01	6.635274E+01	7.447852E+01	8.341156E+01	9.161104E+01
9.408867E+01	7.317708E+01	-6.125195E+01	-1.816710E+02	.000000E+00			
.000000E+00	-3.868078E+01	-7.508905E+01	-1.079223E+02	-1.344901E+02	-1.512252E+02	-1.545498E+02	-1.418101E+02
-1.120631E+02	-6.667315E+01	-9.568711E+00	5.308745E+01	1.139443E+02	1.661068E+02	2.037071E+02	2.202396E+02
2.079497E+02	5.676240E+01	-2.094771E+02	-2.392325E+02	.000000E+00			
.000000E+00	-8.654035E+01	-1.689059E+02	-2.441496E+02	-3.071017E+02	-3.520374E+02	-3.738814E+02	-3.692818E+02
-3.374649E+02	-2.807693E+02	-2.047398E+02	-1.178885E+02	-3.147351E+01	4.073254E+01	7.951935E+01	5.433086E+01
-7.572167E+01	-3.089484E+02	-4.758461E+02	-2.976958E+02	.000000E+00			
.000000E+00	-1.406700E+02	-2.767917E+02	-4.047851E+02	-5.188198E+02	-6.128597E+02	-6.818697E+02	-7.229282E+02
-7.362085E+02	-7.255982E+02	-6.986833E+02	-6.660067E+02	-6.398136E+02	-6.327979E+02	-6.567641E+02	-7.201281E+02
-8.251856E+02	-8.501942E+02	-6.861803E+02	-2.917352E+02	.000000E+00			
.000000E+00	-1.896971E+02	-3.763765E+02	-5.572607E+02	-7.276717E+02	-8.826922E+02	-1.017989E+03	-1.130626E+03
-1.219845E+03	-1.287602E+03	-1.338571E+03	-1.379340E+03	-1.416343E+03	-1.451701E+03	-1.475672E+03	-1.456473E+03
-1.337459E+03	-1.091213E+03	-7.011001E+02	-2.404403E+02	.000000E+00			
.000000E+00	-2.149394E+02	-4.286857E+02	-6.396252E+02	-8.445816E+02	-1.039877E+03	-1.221690E+03	-1.386379E+03
-1.530739E+03	-1.652030E+03	-1.747620E+03	-1.814185E+03	-1.846498E+03	-1.836060E+03	-1.770140E+03	-1.632076E+03
-1.402658E+03	-1.066091E+03	-6.395190E+02	-2.160580E+02	.000000E+00			
.000000E+00	3.980614E+01	7.986476E+01	1.203395E+02	1.613177E+02	2.027352E+02	2.443104E+02	2.854742E+02
3.253002E+02	3.624434E+02	3.950967E+02	4.209737E+02	4.373268E+02	4.410090E+02	4.285964E+02	3.966111E+02
3.420188E+02	2.633540E+02	1.639575E+02	6.080399E+01	.000000E+00			
.000000E+00	3.444746E+01	6.925409E+01	1.045782E+02	1.405059E+02	1.769641E+02	2.136495E+02	2.499566E+02
2.849144E+02	3.171420E+02	3.448324E+02	3.657638E+02	3.773385E+02	3.766593E+02	3.606926E+02	3.266378E+02
2.727886E+02	2.000551E+02	1.151421E+02	3.637832E+01	.000000E+00			
.000000E+00	2.602231E+01	5.248262E+01	7.957383E+01	1.074341E+02	1.360316E+02	1.650905E+02	1.940211E+02
2.218672E+02	2.472875E+02	2.685802E+02	2.837467E+02	2.905806E+02	2.867782E+02	2.701239E+02	2.389229E+02
1.929744E+02	1.349641E+02	7.265779E+01	2.102769E+01	.000000E+00			
.000000E+00	1.442435E+01	2.938584E+01	4.511662E+01	6.184262E+01	7.963665E+01	9.834560E+01	1.175219E+02
1.363748E+02	1.537600E+02	1.682225E+02	1.780979E+02	1.816580E+02	1.772745E+02	1.636082E+02	1.399774E+02
1.073195E+02	6.909727E+01	3.235465E+01	7.129430E+00	.000000E+00			
.000000E+00	-2.775363E-01	5.090634E-02	1.212859E+00	3.522747E+00	7.204759E+00	1.233589E+01	1.878632E+01
2.616520E+01	3.378378E+01	4.065572E+01	4.556467E+01	4.722914E+01	4.456714E+01	3.702926E+01	2.502154E+01
1.049882E+01	-2.815433E+00	-9.726333E+00	-6.659998E+00	.000000E+00			
.000000E+00	-1.716295E+01	-3.381292E+01	-4.983201E+01	-6.489109E+01	-7.862297E+01	-9.063675E+01	-1.005472E+02
-1.080282E+02	-1.128973E+02	-1.152216E+02	-1.153902E+02	-1.140525E+02	-1.118095E+02	-1.086485E+02	-1.033523E+02
-9.353255E+01	-7.633154E+01	-5.007075E+01	-1.883813E+01	.000000E+00			
.000000E+00	-3.345687E+01	-6.679347E+01	-1.002028E+02	-1.335400E+02	-1.664423E+02	-1.982659E+02	-2.280391E+02
-2.544612E+02	-2.759855E+02	-2.910132E+02	-2.981810E+02	-2.966315E+02	-2.860587E+02	-2.663245E+02	-2.367891E+02
-1.961341E+02	-1.441409E+02	-8.426626E+01	-2.803569E+01	.000000E+00			
.000000E+00	-4.325045E+01	-8.705460E+01	-1.321944E+02	-1.790024E+02	-2.274900E+02	-2.771912E+02	-3.269973E+02
-3.750058E+02	-4.184328E+02	-4.536587E+02	-4.764983E+02	-4.827701E+02	-4.691332E+02	-4.339582E+02	-3.777586E+02
-3.027076E+02	-2.127670E+02	-1.176122E+02	-3.644418E+01	.000000E+00			
.000000E+00	-3.554796E+01	-7.242999E+01	-1.112670E+02	-1.527822E+02	-1.974746E+02	-2.454674E+02	-2.963295E+02
-3.488698E+02	-4.009089E+02	-4.490613E+02	-4.886143E+02	-5.136775E+02	-5.178313E+02	-4.954398E+02	-4.434598E+02

-3.630791E+02	-2.597532E+02	-1.463572E+02	-4.689714E+01	.000000E+00			
.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00
.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00
.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00			

T(1,1)...T(31,21)

4.300608E-05	3.495397E-06	3.406996E-06	3.707045E-06	4.161366E-06	4.822198E-06	5.768783E-06	7.252350E-06
1.217409E-05	7.695154E-05	1.271062E-03	2.216198E-02	3.537634E-01	5.063138E+00	6.408640E+01	7.059813E+02
6.633168E+03	5.177224E+04	3.239763E+05	1.546724E+06	5.185355E+06			
7.674411E-04	7.783252E-04	8.193104E-04	8.853089E-04	9.833987E-04	1.123774E-03	1.321699E-03	1.600363E-03
1.995269E-03	2.564005E-03	3.420809E-03	5.094375E-03	1.329515E-02	1.003278E-01	1.138668E+00	1.309884E+01
1.506630E+02	1.912554E+03	3.025189E+04	6.188326E+05	4.690227E+06			
2.871840E-02	2.908460E-02	3.030168E-02	3.225529E-02	3.509364E-02	3.903409E-02	4.438396E-02	5.157609E-02
6.122528E-02	7.421677E-02	9.186434E-02	1.166033E-01	1.592966E-01	3.306734E-01	1.936052E+00	1.985418E+01
2.202270E+02	2.545403E+03	3.248546E+04	4.775746E+05	4.453294E+06			
5.248538E-01	5.306073E-01	5.471323E-01	5.738845E-01	6.119075E-01	6.631308E-01	7.301706E-01	8.164493E-01
9.264603E-01	1.066175E+00	1.243558E+00	1.470293E+00	1.777847E+00	2.411907E+00	6.075274E+00	4.155294E+01
3.955206E+02	3.869630E+03	3.966219E+04	4.576776E+05	4.385143E+06			
6.422473E+00	6.481852E+00	6.617509E+00	6.843161E+00	7.156796E+00	7.566233E+00	8.062343E+00	8.718040E+00
9.488847E+00	1.041438E+01	1.151933E+01	1.283687E+01	1.443739E+01	1.676140E+01	2.434161E+01	8.310969E+01
6.177296E+02	5.335430E+03	4.759061E+04	4.700313E+05	4.388604E+06			
5.947374E+01	5.993047E+01	6.061629E+01	6.184987E+01	6.352008E+01	6.561487E+01	6.813218E+01	7.106361E+01
7.439723E+01	7.813045E+01	8.227758E+01	8.688036E+01	9.205595E+01	9.849964E+01	1.129650E+02	2.010087E+02
9.353266E+02	6.930285E+03	5.570224E+04	4.936915E+05	4.420132E+06			
4.406950E+02	4.434953E+02	4.448417E+02	4.484357E+02	4.530376E+02	4.582313E+02	4.636433E+02	4.688275E+02
4.733346E+02	4.768606E+02	4.793369E+02	4.809604E+02	4.822134E+02	4.844659E+02	4.973095E+02	6.075071E+02
1.536939E+03	8.768130E+03	6.392758E+04	5.223042E+05	4.462839E+06			
2.664961E+03	2.679386E+03	2.669586E+03	2.664593E+03	2.655595E+03	2.638986E+03	2.611385E+03	2.569306E+03
2.510136E+03	2.433354E+03	2.341024E+03	2.237381E+03	2.127870E+03	2.018793E+03	1.925680E+03	1.956959E+03
2.967320E+03	1.117732E+04	7.170480E+04	5.509611E+05	4.510352E+06			
1.306714E+04	1.313308E+04	1.303303E+04	1.292740E+04	1.276979E+04	1.253650E+04	1.220414E+04	1.175220E+04
1.117195E+04	1.047359E+04	9.686345E+03	8.851478E+03	8.012171E+03	7.205527E+03	6.468265E+03	5.931499E+03
6.544982E+03	1.511582E+04	8.101297E+04	5.852914E+05	4.564790E+06			
5.054096E+04	5.081729E+04	5.043146E+04	4.998930E+04	4.929865E+04	4.822071E+04	4.660716E+04	4.433867E+04
4.138460E+04	3.783737E+04	3.389757E+04	2.981866E+04	2.584144E+04	2.214681E+04	1.884830E+04	1.609987E+04
1.485732E+04	2.199694E+04	8.639729E+04	6.006047E+05	4.631771E+06			
1.484046E+05	1.494066E+05	1.490015E+05	1.486647E+05	1.477733E+05	1.456681E+05	1.415709E+05	1.348253E+05
1.252014E+05	1.130617E+05	9.928182E+04	8.497587E+04	7.117965E+04	5.863324E+04	4.772458E+04	3.864911E+04
3.228273E+04	3.576850E+04	1.032260E+05	6.882082E+05	4.786970E+06			
3.227506E+05	3.255672E+05	3.276763E+05	3.313884E+05	3.352458E+05	3.370738E+05	3.339841E+05	3.232280E+05
3.033167E+05	2.747695E+05	2.400400E+05	2.026592E+05	1.660939E+05	1.328873E+05	1.043622E+05	8.090060E+04
6.311396E+04	5.832360E+04	1.433713E+05	1.082424E+06	5.167368E+06			
5.307542E+05	5.366651E+05	5.462996E+05	5.624808E+05	5.829203E+05	6.028320E+05	6.150108E+05	6.117041E+05
5.873684E+05	5.410229E+05	4.768951E+05	4.028932E+05	3.277559E+05	2.584516E+05	1.989384E+05	1.504593E+05
1.131305E+05	9.221266E+04	1.844909E+05	1.345909E+06	5.558392E+06			

7.090756E+05	7.188928E+05	7.402508E+05	7.769053E+05	8.263311E+05	8.807699E+05	9.272671E+05	9.505715E+05
9.378320E+05	8.834446E+05	7.917186E+05	6.755515E+05	5.515182E+05	4.341975E+05	3.326561E+05	2.502038E+05
1.858518E+05	1.385979E+05	2.354474E+05	1.766377E+06	5.934748E+06			
8.384305E+05	8.524022E+05	8.869792E+05	9.476352E+05	1.032024E+06	1.129638E+06	1.221966E+06	1.286098E+06
1.300738E+06	1.253270E+06	1.145221E+06	9.927960E+05	8.210281E+05	6.545376E+05	5.096433E+05	3.922390E+05
2.959592E+05	2.035455E+05	2.935527E+05	2.178164E+06	5.935200E+06			
9.331344E+05	9.511018E+05	9.981679E+05	1.081479E+06	1.198567E+06	1.336762E+06	1.473957E+06	1.582509E+06
1.635404E+06	1.613903E+06	1.514497E+06	1.351832E+06	1.154685E+06	9.567557E+05	7.809468E+05	6.322913E+05
4.852531E+05	2.985756E+05	5.400638E+05	2.655721E+06	5.935200E+06			
1.010045E+06	1.031529E+06	1.088544E+06	1.188565E+06	1.328293E+06	1.494079E+06	1.663620E+06	1.809749E+06
1.905952E+06	1.932941E+06	1.884170E+06	1.766743E+06	1.601143E+06	1.417860E+06	1.233248E+06	1.043744E+06
7.851053E+05	3.939107E+05	1.116459E+06	3.193703E+06	5.935200E+06			
1.080623E+06	1.103591E+06	1.164412E+06	1.269827E+06	1.416635E+06	1.593233E+06	1.781239E+06	1.958066E+06
2.100835E+06	2.191275E+06	2.219856E+06	2.186750E+06	2.098170E+06	1.958723E+06	1.766119E+06	1.520481E+06
1.055600E+06	1.179014E+06	2.036500E+06	3.798874E+06	5.935200E+06			
1.154258E+06	1.175066E+06	1.229921E+06	1.324585E+06	1.458060E+06	1.624365E+06	1.813165E+06	2.010390E+06
2.199476E+06	2.363194E+06	2.485310E+06	2.551067E+06	2.546392E+06	2.459478E+06	2.296918E+06	2.131096E+06
2.142720E+06	2.422508E+06	3.136242E+06	4.410170E+06	5.935200E+06			
1.245637E+06	1.259120E+06	1.294602E+06	1.356413E+06	1.446475E+06	1.565582E+06	1.713427E+06	1.888371E+06
2.087346E+06	2.305938E+06	2.538417E+06	2.777359E+06	3.012465E+06	3.228357E+06	3.401934E+06	3.500320E+06
3.594831E+06	3.825617E+06	4.271677E+06	5.014033E+06	5.935200E+06			
1.406871E+06	1.406871E+06	1.406871E+06	1.406871E+06	1.406871E+06	1.406871E+06	1.406871E+06	1.406871E+06
1.406871E+06	1.406871E+06	1.406871E+06	1.406871E+06	1.406871E+06	1.406871E+06	1.406871E+06	1.406871E+06
1.406871E+06	1.406871E+06	1.406871E+06	1.406871E+06	8.098308E+06			
2.757322E+06	2.761148E+06	2.773917E+06	2.782206E+06	2.790452E+06	2.800415E+06	2.813239E+06	2.829546E+06
2.851579E+06	2.879268E+06	2.914367E+06	2.958854E+06	3.016393E+06	3.094863E+06	3.212158E+06	3.408915E+06
3.798092E+06	4.640331E+06	6.451624E+06	1.013947E+07	1.626798E+07			
3.207775E+06	3.210762E+06	3.220469E+06	3.230277E+06	3.243468E+06	3.262375E+06	3.289392E+06	3.327132E+06
3.378359E+06	3.445959E+06	3.533225E+06	3.644864E+06	3.789244E+06	3.982990E+06	4.260600E+06	4.694864E+06
5.458455E+06	6.902468E+06	9.631520E+06	1.442168E+07	2.107286E+07			
3.327484E+06	3.330259E+06	3.339277E+06	3.353029E+06	3.374766E+06	3.408081E+06	3.457124E+06	3.526258E+06
3.619502E+06	3.740197E+06	3.891572E+06	4.078997E+06	4.314502E+06	4.624101E+06	5.059841E+06	5.722315E+06
6.808566E+06	8.683138E+06	1.196651E+07	1.704966E+07	2.369652E+07			
3.360504E+06	3.364249E+06	3.376527E+06	3.398260E+06	3.434218E+06	3.490326E+06	3.573340E+06	3.689745E+06
3.844240E+06	4.038626E+06	4.272488E+06	4.547213E+06	4.874233E+06	5.286979E+06	5.855696E+06	6.707394E+06
8.042191E+06	1.019902E+07	1.364171E+07	1.874366E+07	2.503187E+07			
3.375419E+06	3.380844E+06	3.398690E+06	3.431861E+06	3.487733E+06	3.575731E+06	3.706410E+06	3.889290E+06
4.129902E+06	4.427300E+06	4.774237E+06	5.162592E+06	5.595781E+06	6.107355E+06	6.782237E+06	7.777661E+06
9.282379E+06	1.159067E+07	1.505635E+07	1.989007E+07	2.566946E+07			
3.386461E+06	3.393946E+06	3.418422E+06	3.465375E+06	3.545480E+06	3.672663E+06	3.862455E+06	4.128682E+06
4.479012E+06	4.910935E+06	5.410870E+06	5.959756E+06	6.547943E+06	7.199447E+06	7.999509E+06	9.107521E+06
1.070154E+07	1.302065E+07	1.631730E+07	2.073170E+07	2.595989E+07			
3.394013E+06	3.403259E+06	3.433345E+06	3.492558E+06	3.594916E+06	3.759207E+06	4.006804E+06	4.357545E+06
4.824174E+06	5.406989E+06	6.091371E+06	6.851277E+06	7.661646E+06	8.521216E+06	9.482237E+06	1.067147E+07
1.226436E+07	1.446209E+07	1.746120E+07	2.139431E+07	2.609094E+07			

3.395665E+06	3.405487E+06	3.437397E+06	3.501316E+06	3.613208E+06	3.795233E+06	4.073756E+06	4.475427E+06
5.021635E+06	5.722715E+06	6.573977E+06	7.555459E+06	8.636866E+06	9.788871E+06	1.100162E+07	1.230784E+07
1.387508E+07	1.588638E+07	1.852366E+07	2.195796E+07	2.615204E+07			
3.389088E+06	3.397350E+06	3.424282E+06	3.478352E+06	3.573698E+06	3.730777E+06	3.975563E+06	4.337209E+06
4.844132E+06	5.519400E+06	6.376798E+06	7.418684E+06	8.635669E+06	1.000723E+07	1.150276E+07	1.308518E+07
1.479170E+07	1.677615E+07	1.921145E+07	2.231454E+07	2.617683E+07			
3.378047E+06	3.383307E+06	3.400382E+06	3.433839E+06	3.491631E+06	3.585548E+06	3.731401E+06	3.948686E+06
4.259613E+06	4.687600E+06	5.255606E+06	5.984868E+06	6.894636E+06	8.003653E+06	9.334601E+06	1.092369E+07
1.283851E+07	1.518814E+07	1.811242E+07	2.177403E+07	2.617446E+07			

SF(1,1)...SF(31,21)

.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00
.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00
.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00			
.000000E+00	-2.763921E-02	-1.108687E-01	-2.505503E-01	-4.481759E-01	-7.059832E-01	-1.027141E+00	-1.416017E+00
-1.878569E+00	-2.422901E+00	-3.060085E+00	-3.805385E+00	-4.680109E+00	-5.714486E+00	-6.952208E+00	-8.457599E+00
-1.032565E+01	-1.267874E+01	-1.542704E+01	-1.542543E+01	.000000E+00			
.000000E+00	-5.854513E-02	-2.347553E-01	-5.300263E-01	-9.467464E-01	-1.488467E+00	-2.160139E+00	-2.968442E+00
-3.922210E+00	-5.032993E+00	-6.315781E+00	-7.789923E+00	-9.480276E+00	-1.141859E+01	-1.364505E+01	-1.620939E+01
-1.916731E+01	-2.253579E+01	-2.587929E+01	-2.488746E+01	.000000E+00			
.000000E+00	-8.766349E-02	-3.512246E-01	-7.920587E-01	-1.412516E+00	-2.216086E+00	-3.207607E+00	-4.393541E+00
-5.782291E+00	-7.384573E+00	-9.213820E+00	-1.128660E+01	-1.362300E+01	-1.624692E+01	-1.918603E+01	-2.247054E+01
-2.612373E+01	-3.006661E+01	-3.363662E+01	-3.138792E+01	.000000E+00			
.000000E+00	-1.149763E-01	-4.602233E-01	-1.036529E+00	-1.845290E+00	-2.888671E+00	-4.169807E+00	-5.693040E+00
-7.464178E+00	-9.490761E+00	-1.178230E+01	-1.435046E+01	-1.720916E+01	-2.037452E+01	-2.386446E+01	-2.769679E+01
-3.187520E+01	-3.628284E+01	-3.993589E+01	-3.644811E+01	.000000E+00			
.000000E+00	-1.415132E-01	-5.658488E-01	-1.272593E+00	-2.261230E+00	-3.531351E+00	-5.082917E+00	-6.916571E+00
-9.033966E+00	-1.143808E+01	-1.413350E+01	-1.712661E+01	-2.042570E+01	-2.404093E+01	-2.798392E+01	-3.226537E+01
-3.687636E+01	-4.164970E+01	-4.535546E+01	-4.068066E+01	.000000E+00			
.000000E+00	-1.694381E-01	-6.766260E-01	-1.519038E+00	-2.692842E+00	-4.193256E+00	-6.015022E+00	-8.152959E+00
-1.060256E+01	-1.336059E+01	-1.642554E+01	-1.979811E+01	-2.348138E+01	-2.748089E+01	-3.180431E+01	-3.645852E+01
-4.142600E+01	-4.649507E+01	-5.020852E+01	-4.439052E+01	.000000E+00			
.000000E+00	-2.023796E-01	-8.067628E-01	-1.806871E+00	-3.193003E+00	-4.952732E+00	-7.071844E+00	-9.535461E+00
-1.232928E+01	-1.544080E+01	-1.886034E+01	-2.258183E+01	-2.660318E+01	-3.092630E+01	-3.555648E+01	-4.049845E+01
-4.572743E+01	-5.098893E+01	-5.461434E+01	-4.767278E+01	.000000E+00			
.000000E+00	-2.464188E-01	-9.800253E-01	-2.187701E+00	-3.848957E+00	-5.937307E+00	-8.422153E+00	-1.127120E+01
-1.445323E+01	-1.794084E+01	-2.171267E+01	-2.575478E+01	-3.006106E+01	-3.463280E+01	-3.947742E+01	-4.460383E+01
-4.999032E+01	-5.536073E+01	-5.887678E+01	-5.083165E+01	.000000E+00			
.000000E+00	-3.104096E-01	-1.231168E+00	-2.737149E+00	-4.788409E+00	-7.332578E+00	-1.030850E+01	-1.365154E+01
-1.730011E+01	-2.120214E+01	-2.531988E+01	-2.963211E+01	-3.413350E+01	-3.883170E+01	-4.374248E+01	-4.888018E+01
-5.421573E+01	-5.941704E+01	-6.241799E+01	-5.320110E+01	.000000E+00			
.000000E+00	-4.040566E-01	-1.598828E+00	-3.540497E+00	-6.157306E+00	-9.352361E+00	-1.301001E+01	-1.700717E+01
-2.122856E+01	-2.558191E+01	-3.000868E+01	-3.448766E+01	-3.903170E+01	-4.368009E+01	-4.848979E+01	-5.352541E+01
-5.882017E+01	-6.409276E+01	-6.707186E+01	-5.678192E+01	.000000E+00			

.000000E+00	-5.302218E-01	-2.095428E+00	-4.628107E+00	-8.012929E+00	-1.208694E+01	-1.664859E+01	-2.147965E+01
-2.637702E+01	-3.118519E+01	-3.581875E+01	-4.026767E+01	-4.458589E+01	-4.886952E+01	-5.323277E+01	-5.778392E+01
-6.256811E+01	-6.720474E+01	-6.866632E+01	-5.301947E+01	.000000E+00			
.000000E+00	-6.757254E-01	-2.669671E+00	-5.890747E+00	-1.017541E+01	-1.528016E+01	-2.089230E+01	-2.666607E+01
-3.227836E+01	-3.748813E+01	-4.217918E+01	-4.637087E+01	-5.019523E+01	-5.385306E+01	-5.756774E+01	-6.154656E+01
-6.591363E+01	-7.028526E+01	-7.103087E+01	-5.217691E+01	.000000E+00			
.000000E+00	-8.122385E-01	-3.208744E+00	-7.079095E+00	-1.221475E+01	-1.829152E+01	-2.488261E+01	-3.150953E+01
-3.772277E+01	-4.319106E+01	-4.776980E+01	-5.152193E+01	-5.468336E+01	-5.759123E+01	-6.060770E+01	-6.405967E+01
-6.814855E+01	-7.241537E+01	-7.229745E+01	-4.885914E+01	.000000E+00			
.000000E+00	-9.100826E-01	-3.594322E+00	-7.928624E+00	-1.366842E+01	-2.042525E+01	-2.768425E+01	-3.486661E+01
-4.142758E+01	-4.696853E+01	-5.132882E+01	-5.462038E+01	-5.718602E+01	-5.949959E+01	-6.205999E+01	-6.531188E+01
-6.952848E+01	-7.411692E+01	-7.336196E+01	-4.540009E+01	.000000E+00			
.000000E+00	-9.494868E-01	-3.749327E+00	-8.270087E+00	-1.425007E+01	-2.127383E+01	-2.879453E+01	-3.619953E+01
-4.291095E+01	-4.850327E+01	-5.280595E+01	-5.595241E+01	-5.834629E+01	-6.054203E+01	-6.310914E+01	-6.652848E+01
-7.103245E+01	-7.563523E+01	-7.275680E+01	-4.040076E+01	.000000E+00			
.000000E+00	-9.207396E-01	-3.638183E+00	-8.031273E+00	-1.385410E+01	-2.072142E+01	-2.813248E+01	-3.552989E+01
-4.238735E+01	-4.831056E+01	-5.312532E+01	-5.692545E+01	-6.005801E+01	-6.301024E+01	-6.626544E+01	-7.016535E+01
-7.468199E+01	-7.770907E+01	-6.895608E+01	-3.377397E+01	.000000E+00			
.000000E+00	-8.170650E-01	-3.234513E+00	-7.158698E+00	-1.239840E+01	-1.865662E+01	-2.554995E+01	-3.265305E+01
-3.956429E+01	-4.597961E+01	-5.175258E+01	-5.691676E+01	-6.165507E+01	-6.621523E+01	-7.077929E+01	-7.526680E+01
-7.911785E+01	-7.614805E+01	-5.921128E+01	-2.523016E+01	.000000E+00			
.000000E+00	-6.281894E-01	-2.494376E+00	-5.545207E+00	-9.668041E+00	-1.468567E+01	-2.036620E+01	-2.644740E+01
-3.267398E+01	-3.884043E+01	-4.482520E+01	-5.059757E+01	-5.617299E+01	-6.148169E+01	-6.609819E+01	-6.881643E+01
-6.733522E+01	-5.882383E+01	-4.078633E+01	-1.539789E+01	.000000E+00			
.000000E+00	-3.485405E-01	-1.388866E+00	-3.103731E+00	-5.452514E+00	-8.367415E+00	-1.175423E+01	-1.549735E+01
-1.946813E+01	-2.353326E+01	-2.755756E+01	-3.139438E+01	-3.485750E+01	-3.767026E+01	-3.939451E+01	-3.935639E+01
-3.661024E+01	-3.012061E+01	-1.955937E+01	-7.081754E+00	.000000E+00			
.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00
.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00
.000000E+00	.000000E+00	.000000E+00	.000000E+00	.000000E+00			
.000000E+00	-1.883043E-02	-7.547617E-02	-1.704313E-01	-3.043983E-01	-4.779258E-01	-6.908779E-01	-9.417187E-01
-1.226607E+00	-1.538326E+00	-1.865124E+00	-2.189573E+00	-2.487615E+00	-2.727929E+00	-2.871741E+00	-2.873117E+00
-2.680200E+00	-2.240558E+00	-1.525399E+00	-6.209369E-01	.000000E+00			
.000000E+00	-3.606675E-02	-1.447103E-01	-3.270738E-01	-5.846578E-01	-9.186059E-01	-1.328631E+00	-1.811557E+00
-2.359483E+00	-2.957655E+00	-3.582190E+00	-4.197869E+00	-4.756251E+00	-5.194448E+00	-5.434967E+00	-5.387527E+00
-4.955558E+00	-4.054426E+00	-2.669462E+00	-1.027512E+00	.000000E+00			
.000000E+00	-5.012177E-02	-2.012935E-01	-4.554421E-01	-8.150496E-01	-1.282054E+00	-1.856192E+00	-2.532760E+00
-3.299854E+00	-4.135272E+00	-5.003333E+00	-5.851959E+00	-6.610314E+00	-7.187457E+00	-7.472903E+00	-7.341283E+00
-6.666780E+00	-5.356405E+00	-3.437488E+00	-1.274887E+00	.000000E+00			
.000000E+00	-5.957515E-02	-2.395523E-01	-5.428205E-01	-9.731524E-01	-1.533723E+00	-2.224867E+00	-3.041065E+00
-3.967291E+00	-4.975048E+00	-6.018517E+00	-7.031312E+00	-7.924196E+00	-8.584228E+00	-8.876560E+00	-8.652452E+00
-7.772203E+00	-6.151407E+00	-3.868283E+00	-1.395964E+00	.000000E+00			
.000000E+00	-6.296487E-02	-2.536425E-01	-5.760442E-01	-1.035563E+00	-1.637200E+00	-2.382845E+00	-3.267605E+00
-4.275376E+00	-5.374086E+00	-6.511241E+00	-7.610477E+00	-8.569703E+00	-9.261371E+00	-9.536089E+00	-9.233982E+00
-8.215826E+00	-6.417412E+00	-3.965267E+00	-1.399005E+00	.000000E+00			

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  .000000E+00   -5.903224E-02   -2.384484E-01   -5.433192E-01   -9.807094E-01   -1.557825E+00   -2.279108E+00   -3.142242E+00
-4.133248E+00   -5.221071E+00   -6.352386E+00   -7.447550E+00   -8.398859E+00   -9.072272E+00   -9.314318E+00   -8.968440E+00
-7.910605E+00   -6.105229E+00   -3.714797E+00   -1.286512E+00    .000000E+00

  .000000E+00   -4.731345E-02   -1.918993E-01   -4.393091E-01   -7.975822E-01   -1.275682E+00   -1.880929E+00   -2.615343E+00
-3.470928E+00   -4.424108E+00   -5.429822E+00   -6.416303E+00   -7.282268E+00   -7.899079E+00   -8.120812E+00   -7.805601E+00
-6.852285E+00   -5.248792E+00   -3.162845E+00   -1.083864E+00    .000000E+00

  .000000E+00   -2.928635E-02   -1.196112E-01   -2.756771E-01   -5.045935E-01   -8.149866E-01   -1.215407E+00   -1.711910E+00
-2.304694E+00   -2.983648E+00   -3.722815E+00   -4.474210E+00   -5.162557E+00   -5.684219E+00   -5.915314E+00   -5.734128E+00
-5.059907E+00   -3.887204E+00   -2.348073E+00   -8.091169E-01    .000000E+00

  .000000E+00   -1.003438E-02   -4.173250E-02   -9.726093E-02   -1.801322E-01   -2.950050E-01   -4.472661E-01   -6.423445E-01
-8.846550E-01   -1.175938E+00   -1.512600E+00   -1.881630E+00   -2.255088E+00   -2.584461E+00   -2.798708E+00   -2.813471E+00
-2.561618E+00   -2.026410E+00   -1.264174E+00   -4.560396E-01    .000000E+00

  .000000E+00    .000000E+00    .000000E+00    .000000E+00    .000000E+00    .000000E+00    .000000E+00    .000000E+00
  .000000E+00    .000000E+00    .000000E+00    .000000E+00    .000000E+00    .000000E+00    .000000E+00    .000000E+00
  .000000E+00    .000000E+00    .000000E+00    .000000E+00    .000000E+00

                VG(1)...VG(21)

 1.884517E-37   4.180647E+01   8.375581E+01   1.260429E+02   1.687858E+02   2.119452E+02   2.552647E+02   2.982064E+02
 3.398825E+02   3.789929E+02   4.137776E+02   4.420003E+02   4.609725E+02   4.676224E+02   4.585959E+02   4.303634E+02
 3.793518E+02   3.024777E+02   1.997301E+02   8.398052E+01   1.884517E-37

                UL(1)...UL(21)

 5.176800E-05   4.222789E-05   1.678685E-05  -2.725246E-05  -9.368162E-05  -1.868194E-04  -3.115143E-04  -4.727127E-04
-6.749414E-04  -9.217645E-04  -1.215196E-03  -1.554897E-03  -1.936269E-03  -2.344234E-03  -2.734328E-03  -2.992454E-03
-3.067164E-03  -3.275801E-03  -3.664973E-03  -4.355241E-03    .000000E+00

                HL(1)...HL(21)

 1.925877E-04   2.701233E-04   2.840120E-04   2.860186E-04   2.849335E-04   2.828131E-04   2.802910E-04   2.779480E-04
 2.756607E-04   2.772220E-04   2.760033E-04   2.478657E-04   2.378216E-04   2.107919E-04   1.759380E-04   1.403507E-04
 1.144092E-04   9.059059E-05   6.992228E-05   5.330813E-05   3.424333E-04

                HG(1)...HG(11)

 2.057539E-05   6.322576E-06   4.800074E-06   4.094149E-06   3.589955E-06   3.196971E-06   2.881997E-06   2.621537E-06
 2.393444E-06   2.250477E-06   2.493679E-06

        NUL =     44.996140,          NUG =     4.295404,          RMAX =  9.006338E-03,          R1(0) =      .261686

        NE1 =           2,            NE2 =           6,           RAL =  2.859151E+08,           RAG =  2.870898E+05

         CL =      .346032,             CG =      .185567,           RP =      1.015559,          R1(0) =      .261686

                SFI(1)...SFI(21)

 1.884517E-37   1.916388E-04   7.665551E-04   1.724749E-03   3.066220E-03   4.790969E-03   6.898996E-03   9.390300E-03
 1.226488E-02   1.552274E-02   1.916388E-02   2.318829E-02   2.759598E-02   3.238695E-02   3.756120E-02   4.311872E-02
 4.905953E-02   5.538361E-02   6.209096E-02   6.918160E-02   1.884517E-37

 DQILDT = -1.127238E-03
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